Complementarity-Guided Epsilon Lexicase Selection for Genetic Programming in Symbolic Regression

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Outline

Introduction

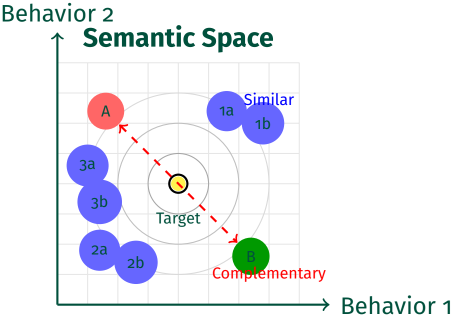
The Problem with Independent Selection

Current Approach:

- Selection operators (tournament, lexicase) select parents independently
- Crossover applied with high probability (80-90%)
- No consideration of parent interaction during crossover

Consequences:

- Parents may be semantically similar or identical
- Crossover fails to combine diverse strengths
- Computational resources wasted
- Evolution can stagnate



Complementarity is important!

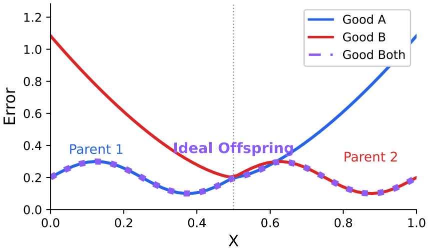
Key Insight: Selection should be aware of crossover behavior!

Ideal Scenario:

- Parent 1 excels on cases A, struggles on cases B
- Parent 2 excels on cases B, struggles on cases A
- Crossover combines strengths from both
- Offspring potentially excels on A **and** B

Challenge:

- Tournament selection favors generalists
- Hard to find truly complementary pairs
- Need specialists that excel in different areas



Background

Tournament Selection:

- Random subsets, select fittest
- Favors generalists

Lexicase Selection [1]:

- Random test case ordering
- Filter by case-specific thresholds
- Favors specialists

Complementary Phenotype Selection [2]:

- First parent: roulette wheel/tournament selection
- Second parent: maximize complementarity
- Limited by generalist first parent

Our Contribution:

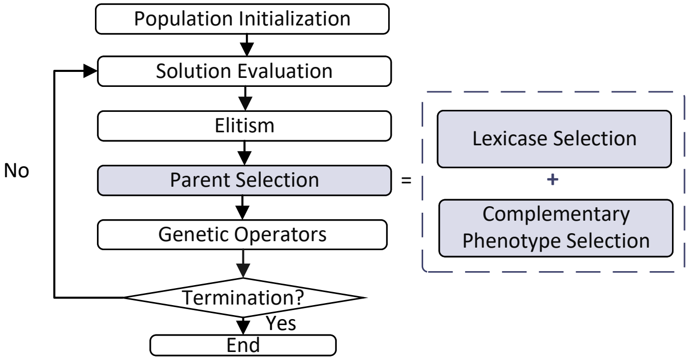
- First parent: lexicase selection
- Second parent: complementarity-guided
- Specialists paired with complementary specialists

[1] La Cava, William et al., 2019“, ‘A probabilistic and multi-objective analysis of lexicase selection and ε-lexicase selection’”, *Evolutionary Computation*

[2] Dolin, Brad, Maribel García Arenas, and Juan J Merelo, 2002“, ‘Opposites attract: Complementary phenotype selection for crossover in genetic programming’”, *Parallel Problem Solving from Nature—PPSN VII*

Proposed Method

Algorithm Framework



Key Features:

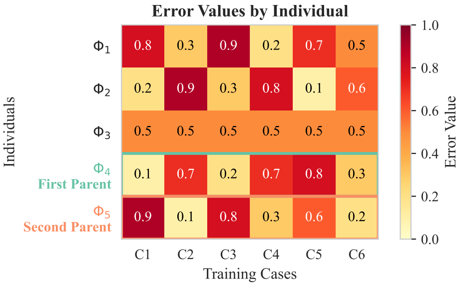
- Standard GP framework with novel selection strategy
- Linear scaling with regularization for fitness evaluation
- Focus on parent complementarity during selection

Step 1 - First Parent Selection:

- Use automatic epsilon-lexicase selection
- Adaptive threshold:
  $\tau_t = \min(e_t) + \text{median}(|e_t - \text{median}(e_t)|)$
- Selects specialists, not generalists

Step 2 - Second Parent Selection:

- Evaluate complementarity with first parent
- Minimize:
  $\mathcal{L}(\Phi_a, \Phi_b) = \frac{1}{m}\sum_{j=1}^{m} \min(E_a[j], E_b[j])$
- Choose parent that best complements Φ_a



Parent selection based on case-wise error patterns

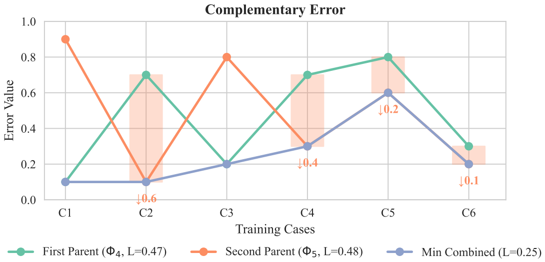
Complementarity Metric:

$$\mathcal{L}(\Phi_a, \Phi_b) = \frac{1}{m} \sum_{j=1}^{m} \min(E_a[j], E_b[j])$$

Interpretation:

- $E_a[j]$, $E_b[j]$: errors on case j
- $\min(E_a[j], E_b[j])$: best performance on case j
- $\mathcal{L}$: average of best case-wise performances
- Lower $\mathcal{L}$ = better complementarity

Tie-breaking: Prefer lower total error



Ideal combination leverages strengths of both parents

Experimental Setup

Datasets:

- 98 regression datasets from PMLB (Fewer than 2000 instances)
- 80:20 train/test split
- Features standardized

GP Parameters:

- Population: 200
- Generations: 100
- Max depth: 10
- Crossover/Mutation: 0.9/0.1
- Functions: +, -, ×, AQ, $\sqrt{\cdot}$, etc.

Baseline Methods:

- **Tournament-3/7**: Tournament selection
- **Lexicase**: Automatic epsilon-lexicase
- **CPS**: Complementary phenotype selection

Evaluation:

- R^2 coefficient of determination
- 30 independent runs per dataset
- Wilcoxon signed-rank test
- Significance level: 0.05

Experimental Results

Table: Statistical comparison of **training** R^2 **scores**

	Lexicase	**Tournament-3**	**Tournament-7**	**CPS**
CLS	64(+)/34(∼)/0(-)	78(+)/18(∼)/2(-)	66(+)/25(∼)/7(-)	36(+)/61(∼)/1(-)
Lexicase	—	67(+)/18(∼)/13(-)	44(+)/36(∼)/18(-)	8(+)/69(∼)/21(-)
Tournament-3	—	—	0(+)/78(∼)/20(-)	1(+)/36(∼)/61(-)
Tournament-7	—	—	—	10(+)/63(∼)/25(-)

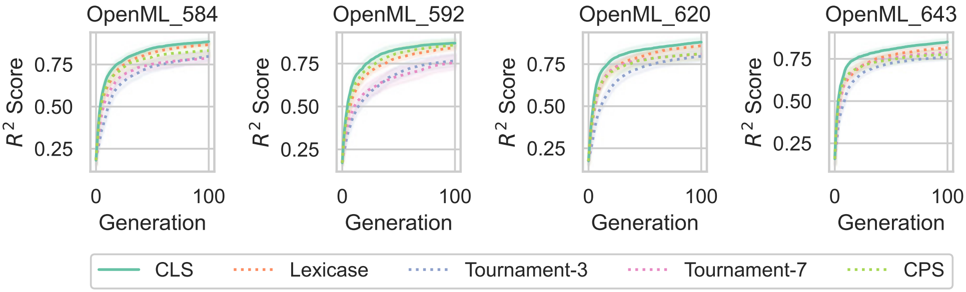
- **Significant improvements**: CLS outperforms baselines on majority of datasets
- **vs Tournament**: 78/98 and 66/98 datasets improved
- **vs Lexicase**: 64/98 datasets improved, 0 datasets worse
- **vs CPS**: 36/98 datasets improved, consistent performance

Table: Statistical comparison of **test** R^2 **scores**

	Lexicase	Tournament-3	Tournament-7	CPS
CLS	23(+)/74(∼)/1(-)	56(+)/39(∼)/3(-)	57(+)/40(∼)/1(-)	25(+)/72(∼)/1(-)
Lexicase	—	55(+)/41(∼)/2(-)	49(+)/48(∼)/1(-)	8(+)/89(∼)/1(-)
Tournament-3	—	—	0(+)/94(∼)/4(-)	0(+)/65(∼)/33(-)
Tournament-7	—	—	—	1(+)/68(∼)/29(-)

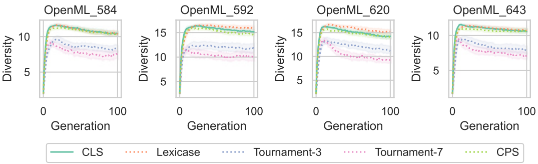
- **Generalization improvement**: Strong test performance across methods
- **Robust performance**: Very few cases where CLS performs worse
- **Consistent gains**: Improvements maintained on unseen data

Evolutionary Performance Trajectories

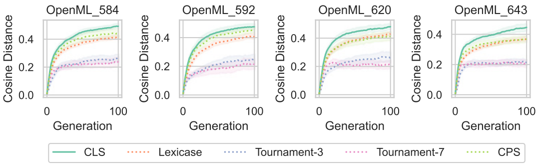


- **Consistent advantage:** CLS performs better from early generations
- **Sustained improvement:** Advantage maintained throughout evolution

Population Diversity Analysis

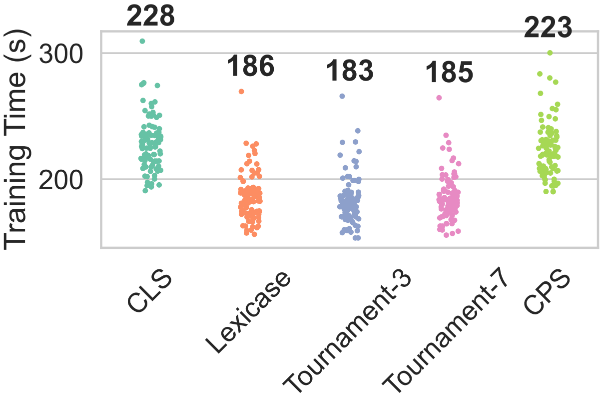


Euclidean Distance Diversity



Cosine Distance Diversity

- **High diversity maintained:** CLS preserves population diversity
- **Functional diversity:** Higher cosine diversity indicates distinct functional behaviors



Time Complexity:

- CLS: $\mathcal{O}(|P|^2 N)$
- Lexicase: $\mathcal{O}(|P|N)$
- Tournament: $\mathcal{O}(|P|)$

Practical Impact:

- Evaluation dominates runtime
- Selection overhead is relatively small

Key Finding: Despite higher theoretical complexity, the practical overhead is relatively small.

Conclusions

Key Contributions and Findings

- **Problem Identified**: Independent parent selection ignores crossover dynamics
- **Solution Proposed**: Complementarity-Guided Lexicase Selection (CLS)
  - First parent: lexicase selection (specialist)
  - Second parent: complementarity-guided selection
  - Leverages case-wise performance differences
- **Experimental Results**:
  - Significant improvements across 98 PMLB datasets
  - Superior training and generalization performance
  - Higher functional diversity in evolved populations
  - Relatively small computational overhead in practice

Future Research Directions

- **Lexicase Selection $\rightarrow$ Batch Lexicase Selection** [1]
- **Lexicase Selection $\rightarrow$ DALex** [2]
- **Lexicase Selection $\rightarrow$ PLex** [3]

[1] Aenugu, Sneha and Lee Spector, 2019", 'Lexicase selection in learning classifier systems'", *Proceedings of the Genetic and Evolutionary Computation Conference*

[2] Ni, Andrew, Li Ding, and Lee Spector, 2024", 'Dalex: Lexicase-like selection via diverse aggregation'", *European Conference on Genetic Programming (Part of EvoStar)*

[3] Ding, Li, Edward Pantridge, and Lee Spector, 2023", 'Probabilistic lexicase selection'", *Proceedings of the Genetic and Evolutionary Computation Conference*

Thank You!

Questions & Discussion

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