# Government Spending Multipliers with the *Real* Cost Channel*

He Nie[1,2]

[1]School of Economics and Management, Wuhan University, China

[2]Department of Economics, National University of Singapore, Singapore

January 30, 2023

**Abstract**

In the benchmark New Keynesian model, I introduce the real cost channel to study government spending multipliers and provide a simple Markov chain closed-form solution. This new model fundamentally departs from previous implications with the nominal cost channel by rotating the New Keynesian Phillips Curve in liquidity traps. When the economy is at the zero lower bound, the nominal interest rate remains unchanged, and government spending can increase inflation. I analytically show that the real cost channel can make inflation rise by less than in a model without this channel. Then this causes a smaller drop in the real interest rate, therefore a lower output gap multiplier. Finally, I confirm the robustness of the real cost channel effect on multipliers using two model extensions.

**Keywords**: Government Spending Multipliers, Real Cost Channel, Zero Lower Bound, Markov Chain, Bounded Rationality

*JEL Classifications*: E52, E58, E62, E63, E70

*I would like to thank my advisor, Jordan Roulleau-Pasdeloup, for his extensive comments. I would also thank Chang Liu, Paul Gabriel Jackson, Denis Tkachenko, Yujie Yang as well as participants in NUS GRS and CEC-NTU Joint Online workshop 2021 for their comments and suggestions.

# 1 Introduction

In late 2008, the Federal Reserve had to lower the interest rate to zero to fight against the Global Financial Crisis (GFC). The conventional monetary policy cannot work with the zero lower bound (ZLB) binding. Therefore the government sought to adopt an alternative effective fiscal policy to stimulate the economy in recessions. In this case, the GFC and recent recessions such as COVID-19 have sparked massive fiscal policy discussions, which usually feature tons of government spending.[1] For example, during the COVID-19 pandemic time in 2020, the United States (US) government spent a total of \$6.55 trillion on a series of programs to ensure the well-being of people.

Understanding the effects of government spending at the ZLB can be a key fiscal analysis. The standard New Keynesian (NK) models show that government spending multipliers can be substantially higher (e.g., above 2) at the ZLB as in Eggertsson and Woodford (2004), Christiano et al. (2011) and Zubairy (2014). This view has been challenged in many recent theoretic studies.[2] In addition, a series of new empirical papers have indicated that multipliers are lower at the ZLB. For instance, Ramey and Zubairy (2018) provide empirical evidence to show not-so-great spending multipliers in liquidity traps. Recently, Auerbach et al. (2021) show that the fiscal multipliers can be lower with the post-COVID supply-side constraints during recessions.

In this paper, I aim to augment the real cost channel in the benchmark NK model to provide *new theoretical insights* and explain, in empirical papers, lower government spending multipliers when the economy is at the ZLB.[3] To be more specific, if firms need to pay bills before production, the interest rate can theoretically influence the borrowing cost and further the firms' marginal cost in the aggregate *supply-side* economy (summarized in the NK Phillips Curve). The critical difference between the model with the cost channel setting and the conventional model is that the interest rate should be included in the firms' marginal cost, and it can, in turn, influence the inflation rate.

The existence of the cost channel is shown in some empirical investigations. For instance, Ravenna and Walsh (2006) estimate and obtain the cost channel parameter

[1] See more discussions on the fiscal tool to resist recessions as in Eggertsson (2011), Kollmann et al. (2012), Bouakez et al. (2020) and House et al. (2020).

[2] See e.g. Kiley (2014), Mertens and Ravn (2014) and Roulleau-Pasdeloup (2021a).

[3] The previous literature mainly employs a standard NK model, given that interest rate can impact aggregate *demand-side* economics only, to theoretically explore the effects of government spending and fiscal policy multipliers (see e.g., Eggertsson and Woodford (2004) and Aruoba and Schorfheide (2013)).

for the US. Similarly, Chowdhury et al. (2006) and Tillmann (2009) show the existence of cost channel in the US and the UK. Recently, Abo-Zaid (2022) employs a structural vector autoregression (SVAR) model to confirm that the cost channel exists in almost all representative industrialized countries.

Compared to most previous literature with the nominal cost channel[4], this paper, however, incorporates the expected *real* interest rate into the firms' marginal cost to study government spending multipliers.[5] Furthermore, I try to conduct an analytical study on the spending multipliers in liquidity traps. To this end, a simple two-state Markov chain as in Eggertsson and Woodford (2003) is utilized to obtain closed-form solutions of spending multipliers.

At the ZLB, the results in this paper stand in stark contrast to most previous implications with the nominal cost channel as in Surico (2008) and Smith (2016). I show that the introduced real cost channel can influence the NK Phillips Curve with the expected disinflation effects during liquidity trap episodes. The threshold of the negative natural rate shock (also the demand shock) to trigger the ZLB constraint binding with the real cost channel is larger than the result in the classical NK model while is less than the model with the nominal cost channel. That means that the economy with the nominal cost channel is the most easily entrapped into fundamental liquidity traps.

Spending multipliers can be effective (larger than one) in liquidity traps. Intuitively, with the ZLB binding, the nominal interest rate remains unchanged, and government spending within a fiscal policy package can increase inflation. This can lower the real interest rate and further stimulate private consumption. However, with the real cost channel, lower borrowing costs due to the expected disinflation effects can make inflation rise by less than in a standard NK model without this real channel. In this way, this channel leads to a smaller drop in the short-run real interest rate. A higher real interest rate can depress people's consumption motivation and further production activity, which, in turn, can reduce the power of government spending on output. Furthermore, the spending multipliers with the nominal cost channel as in Ravenna and Walsh (2006) can

[4]There are a series of papers with the nominal cost channel which means the nominal interest rate is augmented in the firms' marginal cost, such as, Barth III and Ramey (2001), Ravenna and Walsh (2006), Llosa and Tuesta (2009) and Smith (2016).

[5]Compared to the nominal cost channel, as explained at length in Beaudry et al. (2022), the real cost channel can be more empirically relevant to the US data. Furthermore, Nie (2021) formally proves that the real cost channel is more theoretically appealing since it can secure the equilibrium uniqueness/existence with temporary policy shocks.

be invariant with the standard NK model since this nominal channel can not modify the NK Phillips Curve slope at the ZLB.[6] In a nutshell, compared to the standard NK model/the model with the nominal cost channel, the output gap multiplier with the real cost channel can be lower at the ZLB and it decreases in the strength of this channel.

Another result is that the output gap multiplier decreases in the strength of the real cost channel when the economy is outside of liquidity traps. Intuitively, when the nominal interest rate is free to adjust, government spending can increase firms' marginal costs. The real cost channel in the Phillips Curve can further raise inflation since, in normal times, it works as a cost-push shock endogenously along the lines of Ravenna and Walsh (2006). With the Taylor (1993)-type rule followed by the central bank, there will be a rise in the nominal interest rate by more than one-for-one with inflation pressure. A larger real interest rate can stimulate people to save but consume less. As a result, the cost channel can crowd out more private consumption, and thus output gap multiplier is smaller than in the standard model. Consistent with the findings in Abo-Zaid (2022), notice that the output multiplier with the nominal cost channel is less than in the standard model. However, compared to the nominal cost channel, the inflation pressure in the real channel is less prominent due to the additional expected disinflation effects within the expected real interest rate. I, therefore, show that the nominal cost channel can further restrain the output gap multiplier relative to the real cost channel.

This analytical model extends to a more general setting, but the real cost channel still works robustly. As in some recent contributions in Sarin et al. (2021) and Roulleau-Pasdeloup (2021a), the long-run policy effects are usually overlooked or even usually computed numerically in the previous literature. In this case, I further discuss the long-run government spending policy analytically by assuming that the government spending policy is *longer* than economic recessions. Thus, a three-state Markov chain is employed. I follow Bilbiie (2019b) to decompose the long-run period into short and medium-run periods. Therefore, one can capture the effects of prolonged government spending on the short-run spending multipliers. The analytical results show that if the short-run economy is in normal times, it is observed that prolonged government spending can further deflate the output multiplier but increase the inflation multiplier more than the

[6] In the calculation of fiscal multipliers, we need to gain the partial derivative of government spending to inflation or the output gap. If the introduced cost channel can not change the NK Phillips Curve slope, *ceteris paribus*, the partial derivative should be the same as the standard model.

short-run spending policy. Interestingly, if the short-run economy is in liquidity traps, longer government spending after the ZLB has subsided is favored. Intuitively, prolonged government spending can increase inflation more through rational expectations as in Bouakez et al. (2017), and thus, it can further inflate the short-run output gap multiplier. This theoretic result is supported by empirical evidence as in Leduc and Wilson (2013) and Bernardini et al. (2020). Additionally, I show that the real cost channel can still decrease the output gap multiplier in the three-state Markov framework.

At the end of the day, as I have shown the effects of long-run government spending through rational expectations, one may wonder what if the agents can not fully understand the world?[7] Thus, bounded rationality is integrated into our baseline model to see such *behavioral* macroeconomic effects as another extension. I finally develop a similar version of Gabaix (2020)'s model, and the results show that in normal times, the cognitive discounting effects can mitigate the inflation pressure and further increase the output gap multiplier. Interestingly, if we suppose the agents have strong enough bounded rationality, the output gap multiplier can be large in normal times, which is in line with some empirical evidence in Auerbach and Gorodnichenko (2012) and Acconcia et al. (2014). On the other hand, at the ZLB, bounded rationality can attenuate the output gap multipliers. It can echo literature that the agents discounting future wealth in making decisions today can reduce the policy power in recessions as in McKay et al. (2016), Angeletos and Lian (2018), and Campbell et al. (2019). Besides, the real cost channel can still work robustly in this behavioral model, and the output gap multiplier can be further overestimated by ignoring this real channel.

**Related Literature.**—The earliest seminal work focusing on the theoretical estimation of government spending effects in liquidity traps can be traced back to Eggertsson (2001). In that paper, the optimal fiscal policy in the NK economy is characterized, and the real effects of government spending are emphasized. After that, a growing number of literature focuses on the estimation of fiscal effects in theoretical and empirical ways. For example, Blanchard and Perotti (2002) spark the earliest insights on empirically estimating the macroeconomics effects of government spending.[8] Christiano et al. (2011)

[7]There is a similar consideration in the monetary policy. As in Nakata et al. (2019) and Budianto et al. (2020), the favorable effects of Forward Guidance—a promised long-run interest rate binding—on short-run inflation can be much attenuated if the economic agents can not fully understand the world as represented by the NK model with rational expectations.

[8]See Ramey (2011) for a survey on the estimation of government spending multipliers in the literature.

prove that the multiplier is low in normal times in an economy following a Taylor (1993)-type rule but relatively high in liquidity traps. Leeper et al. (2017) theoretically study the fiscal multipliers in a series of models. Two distinct monetary-fiscal policy regimes show that the short-run multipliers are robustly similar across different regimes. More examples among Kraay (2012), Miyamoto et al. (2018), Ramey and Zubairy (2018), etc.

In this paper, I add to the government spending multiplier literature by analytically speaking to the role of the real cost channel on the multipliers. The previous literature as in Barth III and Ramey (2001), Ravenna and Walsh (2006), Llosa and Tuesta (2009) and Smith (2016) always introduces the nominal interest rate into the firms' marginal cost which is called as the nominal cost channel. For example, Ravenna and Walsh (2006) first confirm that the cost-push shock can emerge endogenously in the NK model in a cost channel setting. Further, they discuss the ways for the optimal monetary policy which the new channel can alter. Surico (2008) shows that limiting the economic cyclical with a cost channel can lead to a strong fluctuation of inflation and output. However, this paper heavily builds on Beaudry et al. (2022) and Nie (2021) to augment the expected real interest rate into firms' marginal cost to revise the benchmark NK model.

This paper is in a close spirit to some recent literature—see e.g. Bilbiie (2019a), Bilbiie (2019b), Bilbiie (2020) and Nie and Roulleau-Pasdeloup (2022) which uses three-state Markov Chain to analytically examine the long-run policy on the short-run economy. For example, Bilbiie (2019b) employs a three-state Markov manner to have an in-depth study on the optimal forward guidance policy in the short run and long run. In this paper, this three-state structure can allow us to analytically check the general properties of long-run government spending to echo some empirical evidence in Durevall and Henrekson (2011), Ilzetzki et al. (2013), Leduc and Wilson (2013), Bouakez et al. (2017), and Leff Yaffe (2019).

In addition, recent contributions such as Farhi and Werning (2019), and Gabaix (2020) show that bounded rationality can mitigate the powerful effects of monetary policy. This method can rationalize the so-called "Forward Guidance puzzle" (see i.a. Angeletos and Lian (2018) and Coibion et al. (2020)) compared to the benchmark too forward-looking NK model as in Ganelli and Tervala (2016). This paper, however, is linked with this strand of literature to see the interaction of the cost channel and bounded rationality on the *fiscal* policy.

Finally, this paper is also closely related to Abo-Zaid (2022) who examines government spending multipliers at the ZLB with the nominal cost channel. Abo-Zaid (2022) includes nominal cost channel and differentiates between the policy rate and the loan rate. It turns out that this nominal channel can cause spending multipliers larger in liquidity traps.[9] However, in this paper, I use the real cost channel to explain lower government spending multipliers when the economy is at the ZLB. Simple Markov chain closed-form solutions are computed to compare the government spending multipliers with the real and nominal cost channel. I further clear up the effects of the strength of the real cost channel on the spending multipliers analytically. I also furnish the primary model to confirm the robust role of the real cost channel on multipliers.

**Organization.**—I will specify the prototypical forward-looking New Keynesian(NK) model with the real cost channel in the next Section 2 and provide a transparent analytical analysis using a two-state Markov chain on the government spending multipliers. In Section 3, I furnish the baseline model to explore the general properties of long-run government spending effects. Another extension with bounded rationality is conducted in Section 4. Finally, this paper concludes in Section 5.

# 2 The Baseline Model with the Real Cost Channel

Recent empirical evidence in Abo-Zaid (2022) shows that the existence of the cost channel can influence government spending multipliers. As in Ravenna and Walsh (2006) and Surico (2008), the main insight of the cost channel is that the interest rate can influence the borrowing costs and then the marginal cost function. In this paper, we follow Beaudry et al. (2022) and Nie (2021) to utilize the NK model with the real cost channel, which means the expected real interest rate can impact the firms' marginal costs, to explore government spending multipliers analytically.

## 2.1 Private Sector Behavior

I use a prototypical forward-looking NK model with the real cost channel as in Beaudry et al. (2022) and Nie (2021). The behavior of aggregate demand (AD) side economy can

[9] In the setup in Abo-Zaid (2022), if the two rates (*i.e.* the policy rate and the loan rate) are equal, the cost nominal cost channel would disappear from the Phillips Curve. While in this paper, we simply assume the two rates are the same.

be summarized in the following log-linear Euler condition:

$$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\sigma_c}[R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1} - r_t^n], \tag{1}$$

where $c_t$ is the private consumption, $\sigma_c$ is the risk aversion coefficient, $R_t$ is the nominal interest rate *in level*, $\pi_t$ is inflation, $\mathbb{E}_t$ is the rational expectation operator, and $r_t^n$ is the demand shock (also the natural rate shock).

The resource constraint in this NK economy is:

$$y_t = (1 - s_g)c_t + g_t, \tag{2}$$

where $s_g$ is the fraction of government spending in total production, $g_t$ is the government spending.[10] In this case, one can obtain the path of $y_t$ by substituting the resource constraint into Euler equation to obtain the equation (3) below:

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma}[R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1} - r_t^n] + g_t - \mathbb{E}_t g_{t+1}, \tag{3}$$

where $\sigma = \frac{\sigma_c}{1-s_g}$.

The aggregate supply (AS) side of the economy can be summarized in the following NK Phillips Curve:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \left[\gamma_y y_t + \gamma_g g_t + \gamma_r (R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1})\right], \tag{4}$$

where $\beta$ is the subjective discount factor, and $\kappa$ is the elasticity of inflation with regard to marginal cost. $\gamma_y$, $\gamma_g$, $\gamma_r$ are the elasticity of marginal cost elasticity with regard to the output gap, government spending, and the expected real interest rate, respectively.[11] It is of note that $\gamma_r$ in the equation (4) can be seen as the strength of the cost channel which controls the impact of this channel.[12] This model with the real cost channel can collapse to the conventional one if $\gamma_r = 0$.[13] In addition, it can nest the model with the nominal cost channel (equation (6)) if we assume the nominal interest rate is introduced in the firms' borrowing costs as in Ravenna and Walsh (2006) and then the disinflation

[10] Following Christiano et al. (2011), I define $g_t = (G_t - G)/Y$.

[11] See appendix A for exact expressions for these parameters.

[12] The details for the derivatives of the aggregate supply side of the economy can be seen in appendix A.

[13] See Gertler et al. (1999) and Woodford (2003).

expectation term ($-\mathbb{E}_t\pi_{t+1}$) disappears.

It is assumed that the central bank sets the nominal interest rate following the (truncated) Taylor (1993)-type rule with the ZLB:

$$R_t = \max\{0, -\log(\beta) + \phi_\pi \pi_t\}. \tag{5}$$

### 2.1.1 Real versus Nominal Cost Channel

The Phillips Curve with the nominal cost channel is

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa\left[\gamma_y y_t + \gamma_g g_t + \gamma_r(R_t + \log(\beta))\right]. \tag{6}$$

The equation (6) is used in most previous papers working as the nominal cost channel specifications such as Ravenna and Walsh (2006) and Surico (2008). In these papers, the nominal interest rate is introduced in the marginal cost. Even though the specifications in equations (4) and (6) can be seen as the cost channel, the real cost channel in the equation (4) can highlight an expected disinflation feedback denoted by $-\mathbb{E}_t\pi_{t+1}$ in the expected real interest rate. As in Beaudry et al. (2022), compared to the nominal cost channel, the real cost channel can obtain more support from the US data. This motivates us to use this setting in this paper.

It is of note that the two cost channels can have similar effects in normal times if the central bank follows a simple Taylor rule in the equation (5). In this case, a cost-push shock endogenously emerges in the two cases as in Ravenna and Walsh (2006) and the two channels both increase firms' marginal costs.

The interesting insight at the ZLB with $R_t = 0$ is that the nominal cost channel as in Ravenna and Walsh (2006) and Surico (2008) can not influence the slope of the Phillips Curve. However, it is of note that, at the ZLB, the real cost channel as in Beaudry et al. (2022) can rotate the Phillips Curve with the disinflation expectation feedback. As a result, the Phillips Curve is flatter than the standard NK model and this may explain a declining slope of the empirical Phillips Curve. In this way, the real cost channel in equation (4) can alter the government spending multipliers in liquidity traps. While the nominal cost channel multipliers in equation (6) can be invariant with the standard NK model since the nominal channel in liquidity traps can not be included in the partial derivative of government spending to the output gap/inflation in the calculation of fiscal multipliers.

## 2.2 Quick Tour: Normal Times and Zero Lower Bound

In this section, I employ a two-state static Markov chain as in Eggertsson et al. (2003) to deal with the policy shocks vector $[r_t^n, g_t]$. It is assumed that the the specific policy shock (for example, demand shock $r_n^n$ in this section) stays at the current short-run state with a persistence $p$ and it then reverts to the steady-state *i.e.* $r_t^n = 0$ with a probability $1-p$.[14] Since the NK model with the real cost channel in this paper is perfectly forwarding looking, one can show the expected output gap and inflation as follows:

$$\mathbb{E}_t y_{t+1} = p y_t, \qquad \mathbb{E}_t \pi_{t+1} = p\pi_t. \tag{7}$$

**Assumption 1.** *I assume that the NK Phillips Curve with the real cost channel is always upward sloping in a* $(\pi_t, y_t)$ *graph such that*

$$p < \frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r} = \overline{p}^c. \tag{8}$$

As in much literature in Laubach and Williams (2003), Cochrane (2017), Han et al. (2020) and Nie and Roulleau-Pasdeloup (2022), there is an implicit condition that the NK Phillips Curve is upward sloping in a $(\pi_t, y_t)$ graph as in Assumption 1. Compared with the conventional NK model with the nominal cost channel as in Christiano et al. (2005) and Ravenna and Walsh (2006), this paper utilizes a more empirically relevant real cost channel proposed in Beaudry et al. (2022) and further extended in Nie (2021).

In this section, we discuss two cases which are the economy in normal times without the ZLB binding and in fundamental liquidity traps. There is a threshold of demand shock to trigger the ZLB constraint binding. From the Taylor (1993)-type rule, one can see that if the items $-\log(\beta) + \phi_\pi \pi_S$ are less than or equal to zero, the NK economy can be binding with the ZLB state. If not, the economy is in normal times and the nominal interest rate can be free to adjust with the central bank's monetary policy regulation. If the (negative) natural rate shock (also the demand shock) is too large, the economy can

[14] The duration of the short run state can be calculated as $T = \frac{1}{1-p}$. For instance, if $p = 0.5$, $T = \frac{1}{1-0.5} = 2$ quarters.

be with the ZLB. Thereby there is a boundary condition for the natural rate shock $r^n_S$ in the short run to trigger the economy into a state with the ZLB binding.

**Proposition 1.** *The boundary condition relationship among the three models is*

$$\underline{r^{n,B}_S} < \underline{r^n_S} < \underline{r^{n,N}_S}, \tag{9}$$

*where $\underline{r^{n,B}_S}$ is the boundary condition without the cost channel, $\underline{r^n_S}$ is the boundary condition with the real cost channel and $\underline{r^{n,N}_S}$ is the boundary condition with the nominal cost channel.*

*Proof.* See Appendix B. □

As in Proposition 1, the boundary condition to trigger the ZLB binding with the real cost channel is higher than that in the conventional NK model since the real cost channel can enlarge the effects of the interest rate on inflation and further the output gap through rational expectations. In other words, the economy is more easily into liquidity traps with the real cost channel than in the traditional model. On the other hand, the boundary condition with the nominal cost channel is larger than that in the model with the real cost channel due to disinflation expectations. Therefore, the model with the nominal cost channel is the most easily entrapped into fundamental liquidity traps among the three models.[15]

In the following sections, I will mainly focus on the government spending shock and discuss the issues of both the output and inflation multipliers.

## 2.3 Government Spending: Theoretical Analysis

To have transparent analytical results, I abstract from the demand shock and only focus on the effects of government spending shock with the real cost channel. First, I will compare the three models. Second, I will deliver the general properties of the strength of the cost channel on the multipliers by using a simple Markov chain closed-form solution.

[15]In this paper, the three models refer to the conventional model without the cost channel, the model with the nominal cost channel, and the model with the real cost channel.

### 2.3.1 Government Spending Multiplier in Normal Times

The Markov method is based on Eggertsson and Woodford (2003) and Eggertsson et al. (2003). I assume that the positive government spending shock $g_S > 0$ starts in the short run, stays with the persistence probability $p$, and returns to the steady-state $g_L = 0$ in the long run with a probability $1-p$. If the short-run economy is in normal times, I can rewrite the Euler equation (3) and the Phillips Curve with the real cost channel (equation (4)):

$$y_S = -\frac{1}{\sigma(1-p)}(\phi_\pi - p)\pi_S + g_S \tag{10}$$

$$\pi_S = \kappa\frac{\gamma_y}{1-\beta p-\kappa\gamma_r(\phi_\pi - p)}y_S + \kappa\frac{\gamma_g}{1-\beta p-\kappa\gamma_r(\phi_\pi - p)}g_S. \tag{11}$$

**Fiscal multipliers.**—It can be found that a positive government spending shock can move the Euler equation upward and turn down the Phillips Curve in a $(\pi_t, y_t)$ graph. The solutions with the real cost channel of the output gap $\mathscr{M}^O_{S,N}$ and inflation multipliers $\mathscr{M}^I_{S,N}$ can be generated below:

$$\mathscr{M}^O_{S,N} = \frac{\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi - p)] - \kappa\gamma_g(\phi_\pi - p)}{\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi - p)] + \kappa\gamma_y(\phi_\pi - p)} \tag{12}$$

$$\mathscr{M}^I_{S,N} = \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p)}{\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi - p)] + \kappa\gamma_y(\phi_\pi - p)}. \tag{13}$$

The expected real interest rate can be in the denominator and numerator for the output gap multiplier since the new cost channel here can influence the inflation rate and further impact the output gap through expectations. While for the inflation multiplier, the expected real interest rate is included in the denominator since the cost channel can impact this multiplier directly. In normal times, since the denominator of the multiplier equation is higher than the numerator and the multiplier is less than one. Government spending can crowd out private consumption, which can echo some classical empirical evidence as in Amano and Wirjanto (1997) and Barro and Redlick (2011).

In normal times, the cost channel can reduce the output multiplier, and the intuition is simple. Since the government spending shock can increase firms' marginal costs. Additionally, the (real/nominal) cost channel can further increase the borrowing cost and then inflation. This mechanism is close in spirit to Ravenna and Walsh (2006) such that a cost-push shock can emerge endogenously in the New Keynesian model with the cost channel. The nominal interest rate increases by more than one-for-one with inflation and further a higher nominal interest rate arises due to the cost channel. This results in a higher short-run real interest rate which can cause less private consumption, and thus government spending can crowd out more private consumption. Above all, the output multiplier with the cost channel is less than that without it.

In comparison, as in Appendix C, compared to the nominal cost channel, the inflation multiplier can be smaller with the real cost channel. If we compare the NK Phillips Curve in equations (6) and (4), less influence is triggered by the real cost channel, which can echo the lower inflation fact in the Euro area in normal times (Koester et al. (2021)). In this case, the nominal cost channel can further restrain the output multiplier relative to the real cost channel.

I also explore the effects of the strength of the real cost channel $\gamma_r$ on the spending multipliers. The power of the real cost channel can directly leverage the increment of inflation. On the other hand, we can see that higher inflation can crowd out more private consumption, hence a much lower multiplier with the real cost channel. Therefore, the output gap multiplier decreases in $\gamma_r$ while the inflation multiplier increases in $\gamma_r$. I summarize the above theoretical results in Proposition 2.

**Proposition 2.** *In normal times, the output gap multiplier decreases in the strength of the real cost channel $\gamma_r$ while the inflation multiplier increases in $\gamma_r$. The output gap multiplier relationship among the three models is*

$$\mathcal{M}_{S,N}^{O,N} < \mathcal{M}_{S,N}^{O} < \mathcal{M}_{S,N}^{O,B}, \tag{14}$$

*and the inflation multiplier relationship is*

$$\mathcal{M}_{S,N}^{I,B} < \mathcal{M}_{S,N}^{I} < \mathcal{M}_{S,N}^{I,N}, \tag{15}$$

*where in normal times, $\mathcal{M}_{S,N}^{i,N}$ ($i \in \{O, I\}$) denote the multiplier of the model with the nominal cost channel, and $\mathcal{M}_{S,N}^{i,B}$ denote the multiplier of the model without the cost channel.*

*Proof.* See Appendix C. □

### 2.3.2 Government Spending Multiplier at Zero Lower Bound

In this part, I focus on the case when the economy binds with the ZLB.[16] The Euler equation (3) and Phillips Curve with the real cost channel (equation (4)) can be rewritten as:

$$y_S = -\frac{1}{\sigma(1-p)}[\log(\beta) - p\pi_S] + g_S \tag{16}$$

$$\pi_S = \frac{\kappa\gamma_y}{1-\beta p+\kappa\gamma_r p} y_S + \kappa\frac{\gamma_g}{1-\beta p+\kappa\gamma_r p} g_S + \frac{\kappa\gamma_r \log(\beta)}{1-\beta p+\kappa\gamma_r p}. \tag{17}$$

**Fiscal multipliers.**—One can further use the Euler equation and Phillips Curve to obtain the solutions with the real cost channel of the output gap multipliers $\mathcal{M}^O_{S,Z}$ and the inflation multiplier $\mathcal{M}^I_{S,Z}$ at the ZLB:

$$\mathcal{M}^O_{S,Z} = \frac{\sigma(1-p)[1-\beta p+\kappa\gamma_r p]+\kappa\gamma_g p}{\sigma(1-p)(1-\beta p+\kappa\gamma_r p)-\kappa\gamma_y p} \tag{18}$$

$$\mathcal{M}^I_{S,Z} = \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p)}{\sigma(1-p)(1-\beta p+\kappa\gamma_r p)-\kappa\gamma_y p}. \tag{19}$$

From the solutions, the expected real interest rate can be in the denominator and numerator of the output gap multiplier. The new cost channel here can influence the inflation rate and further impact the output gap through expectations. The real cost channel can impact the inflation multiplier directly through the NK Phillips Curve.

In liquidity traps, since the denominator of the multiplier equation is lower than the numerator and thus the output gap at the ZLB is larger than one. At the ZLB, with no cost channel, the nominal interest rate remains unchanged, and the increase in government spending means that firms will need household members to produce more. This higher real wage makes the firm's marginal cost increase and further generates inflation. Therefore the government spending shock leads to a drop in the short-run real interest rate, which, in turn, gives incentives to the consumers to save less and consume more. This can be seen as the crowding in effects as in Bouakez et al. (2017).

However, the real cost channel can lower the short-run inflation through the disinflation expectation component (equation (4)) in the firms' real borrowing cost following a

[16] In this paper, the zero (effective) lower bound is a state when $R_t = 0$ with the simple assumption that there is no cash storing cost in Galí (2015).

spending shock. Intuitively, a lower borrowing cost arises due to the disinflation expectation during liquidity trap episodes. Further, the disinflation expectation can translate into the realized lower inflation rate through rational expectations and sticky prices. In other words, assuming firms care about the expected real interest rate in liquidity traps, the disinflation expectation in the future can reduce the actual inflation. In this case, the real cost channel can make the inflation rate rise by less than in the conventional model following a spending shock, and there is a smaller drop in the short-run real interest rate. Higher real interest rates due to the real cost channel can depress people's consumption motivation and further production activity, which results in a decline in the effects of government spending on output. Hence compared to the standard NK model/the model with the nominal cost channel[17], there is a lower output gap multiplier with the real cost channel.

I next discuss the effects of the strength of the real cost channel $\gamma_r$ on the spending multipliers. The stronger the power of the real cost channel, the lower the inflation rate due to the disinflation expectations in liquidity traps. In this case, higher real interest rates due to lower inflation can further depress people's consumption motivation, which results in a more decline in the output gap multiplier. In a nutshell, the inflation and the output gap multiplier with the real cost channel decreases in $\gamma_r$. I summarize the main theoretical results in Proposition 3.

**Proposition 3.** *At the ZLB, the output gap and inflation multipliers decrease in the strength of the real cost channel $\gamma_r$. The output gap multiplier relationship among the three models is*

$$\mathcal{M}^{O}_{S,Z} < \mathcal{M}^{O,N}_{S,Z} = \mathcal{M}^{O,B}_{S,Z}, \tag{20}$$

*and the inflation multiplier relationship is*

$$\mathcal{M}^{I}_{S,Z} < \mathcal{M}^{I,N}_{S,Z} = \mathcal{M}^{I,B}_{S,Z}, \tag{21}$$

*where in liquidity traps, $\mathcal{M}^{i,N}_{S,Z}$ $(i \in \{O, I\})$ denote the multiplier of the model with the nominal cost channel, and $\mathcal{M}^{i,B}_{S,Z}$ denote the multiplier of the model without the cost channel.*

*Proof.* See Appendix D. □

[17] As explained in section 2.1.1, it is shown that the nominal cost channel multipliers are invariant with the standard model.

**Comparison with the literature.**—This result is contrary to Abo-Zaid (2022) who shows that the higher borrowing cost of the nominal cost channel can increase inflation to make the output gap multiplier larger at the ZLB. As in Abo-Zaid (2022), the Phillips Curve is steeper in a $(y_t, \pi_t)$ graph than the standard NK model in liquidity traps with the nominal cost channel.[18] In this paper, I stress that inflation in the short run with the real cost channel can be lower due to the disinflation expectation in the real borrowing cost.[19] Thus I can use the real cost channel to explain empirically lower government spending multipliers when the economy is at the ZLB as in Ramey and Zubairy (2018).

**Taking stock on the denominator of the multiplier.**—As discussed in Bilbiie (2008), Mertens and Ravn (2014), Borağan Aruoba et al. (2018) and Lustenhouwer (2020), Nie and Roulleau-Pasdeloup (2022), the denominator of the multiplier in the standard NK model at the ZLB can be negative. In this case, the output gap/inflation multiplier can be negative as in the equations (18)-(19).

However, as described at length in Nie (2021), in the presence of the real cost channel, the denominator is less likely to be negative. It can be always positive if assuming the magnitude of the real cost channel $\gamma_r$ is large enough and meets the below condition:

$$\begin{aligned} &\sigma(1-p)(1-\beta p+\kappa\gamma_r p)-\kappa\gamma_y p>0 \\ &\Leftrightarrow \gamma_r > \Gamma(\gamma_y), \end{aligned} \tag{22}$$

where $\Gamma(\gamma_y) = \frac{\kappa\gamma_y p - \sigma(1-p)(1-\beta p)}{\sigma(1-p)\kappa p}$.

As in the condition (22), $\gamma_r$ should be higher than the composite parameter $\Gamma(\gamma_y)$. Assuming that $\gamma_y$ is small enough (or $\gamma_r$ is relatively large enough), this condition can always hold. In particular, the assumption with a low $\gamma_y$ (or a large $\gamma_r$) is in line with the empirical finding as in Beaudry et al. (2022): the elasticity of real marginal cost w.r.t. output gap $\gamma_y$ is very small while the key parameter $\gamma_r$ leveraging the strength of the real cost channel is significantly much larger than $\gamma_y$.

[18] Abo-Zaid (2022) differentiates between the policy rate and the loan rate, making the Phillips Curve steep in recessions. In that setup, if the two rates are equal, the cost nominal cost channel would disappear from the Phillips Curve. In that way, the nominal cost channel in liquidity traps can not impact the spending multiplier.

[19] The Phillips Curve is locally flat with the real cost channel during recessions as in Beaudry et al. (2022). Besides, Hazell et al. (2020) empirically document that the NK Phillips Curve is flat during the Great Recession.

## 2.4 Numerical Results—Benchmark

In this paper, I follow Budianto et al. (2020) and Roulleau-Pasdeloup (2021a) to set the main baseline parameterization method reported in Table 1. I follow the calibration method in Beaudry et al. (2022) and Nie (2021) to set parameters of the real cost channel.[20]

Table 1: Parameterization

| | |
|---|---|
| Subjective discount factor | $\beta = 0.99$ |
| Inverse of Frisch elasticity | $\eta = 1$ |
| Risk aversion coefficient | $\sigma_c = 1$ |
| Steady-state ratio of government spendin to output | $s_g = 0.2 \times 0.23$ |
| Elasticity of inflation w.r.t. real marginal cost | $\kappa = 0.2$ |
| Elasticity of real marginal cost w.r.t. output gap | $\gamma_y = 0.2$ |
| Elasticity of real marginal cost w.r.t. interest rate | $\gamma_r = 1$ |
| Elasticity of real marginal cost w.r.t. government spending | $\gamma_g = -0.1$ |
| Inflation parameter in Taylor rule | $\phi_\pi = \phi_\pi^q = 1.5$ |
| Persistence specification | $p = 0.7$ |

I first provide numerical AS/AD figures with a contractionary natural rate shock in three NK Phillips Curves (NKPC) shown in Figure 1. If there is a temporary short-term natural rate shock $r_S^n$ (-2%), which exceeds the boundary condition, the economy can be in fundamental liquidity traps.[21] In this case, the conventional monetary policy can not work since the nominal interest rate is bounded at zero. The deflationary pressure can lead to a higher real interest rate and further stimulate people to save but consume less. From Figure 1, it is clear that the real cost channel can change the Phillips Curve slope in liquidity traps and the Phillips Curve is locally flat as in Beaudry et al. (2022). The curve with the nominal cost channel at the ZLB shares the same slope as the conventional model. The numerical result also echoes Proposition 1 that the case with the nominal cost channel is the most easily entrapped into fundamental liquidity traps among the three models. This result can be further confirmed in the impulse response to a contractionary natural rate shock.—See Appendix E.

In normal times, I compare the multipliers for the models without the cost channel ($\gamma_r = 0$) vs. with the cost channel as in Ravenna and Walsh (2006) vs. with the real cost

[20]The calibration method of the real cost channel follows the way in Figure 4 in Beaudry et al. (2022). In addition, these parameters can ensure that the denominator of the multiplier is positive.

[21]This calibration mimics the case of the Great Recession.

Figure 1: AS/AD curves with natural rate shock in three models

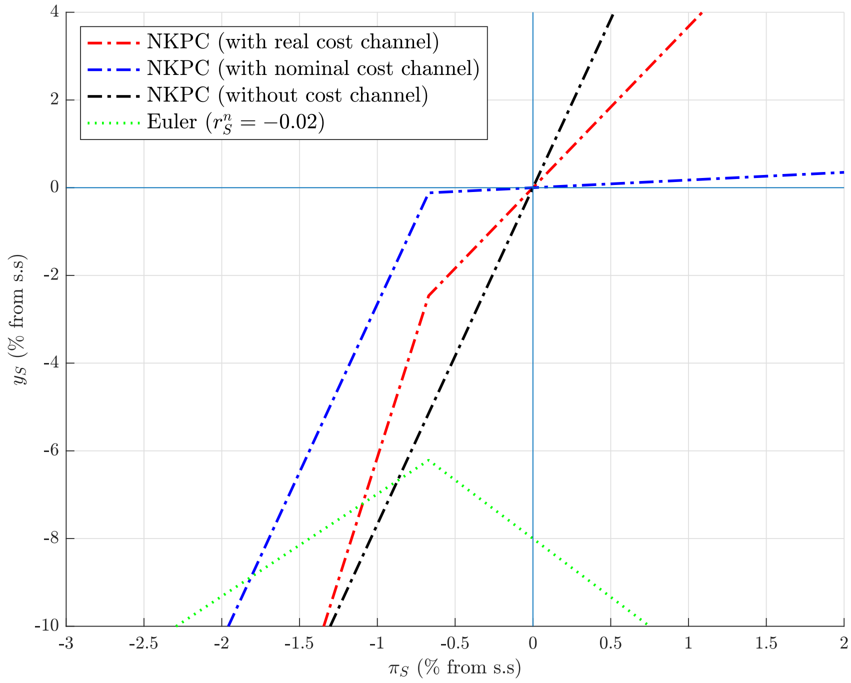
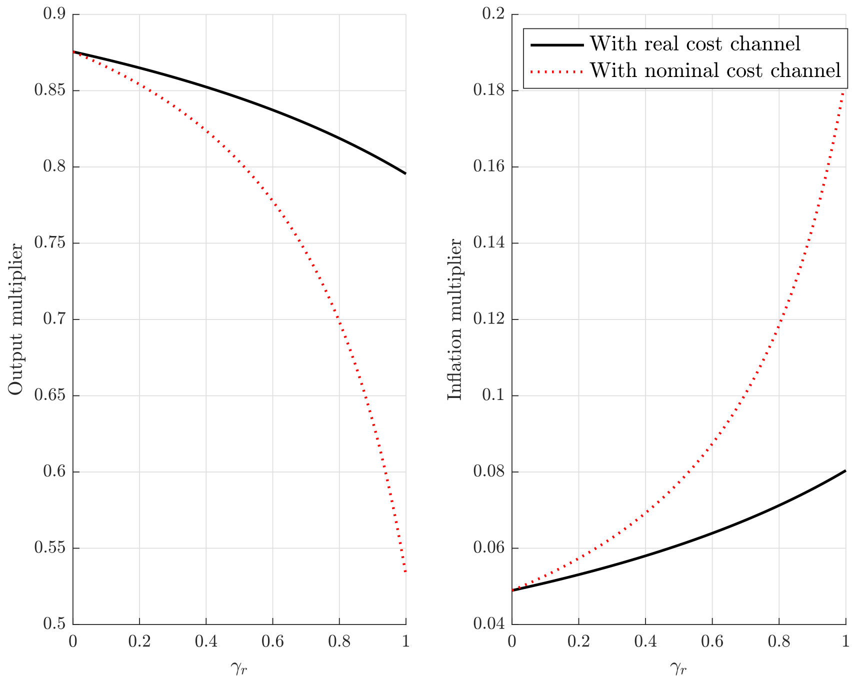



Figure 2: Spending multipliers in normal times




channel in Figure 2. It can be seen that the output multiplier is lower than one as in Lewis (2021) and it decreases in the strength of the cost channel. On the contrary, the inflation multiplier increases with the strength of the cost channel $\gamma_r$. It is observed that the output gap multiplier with the real cost channel is larger than that with the nominal cost channel while less than its counterpart without the cost channel. This can echo the theoretical analysis in Proposition 2. See Appendix F for numerical results w.r.t. the persistence $p$ in the three models. These numerical results robustly echo Proposition 2. I also find that the output gap multiplier decreases with the persistence $p$, while the inflation multiplier is higher with the increment of time duration.

Figure 3: Spending multipliers in liquidity traps

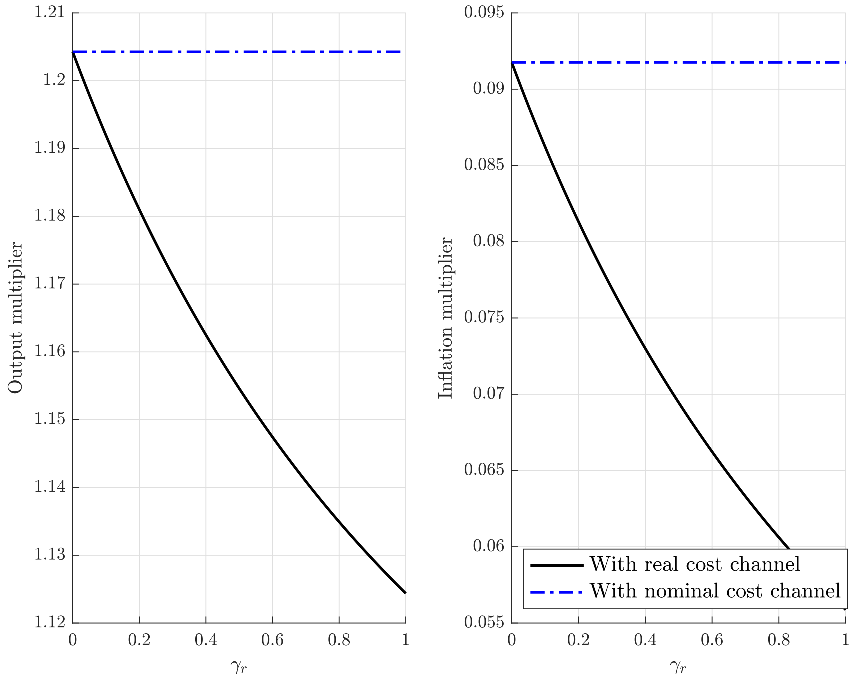



Figure 3 shows spending multipliers with/without the real cost channel in liquidity traps. The simulation results can perfectly match our theoretical analysis in Proposition 3. On the one hand, one can see that the output multiplier is productive as in Christiano et al. (2011) and Schmidt (2017). The real cost channel can attenuate the output gap and inflation multipliers simultaneously. The output gap and inflation multipliers decrease in the strength of the real cost channel $\gamma_r$. The nominal cost channel multipliers are invariant with the standard NK model without the cost channel. See Appendix F for numerical results w.r.t. the persistence $p$ in the three models. It further confirms the theoretical results. In addition, the output gap/inflation multipliers increase in the persistence $p$.

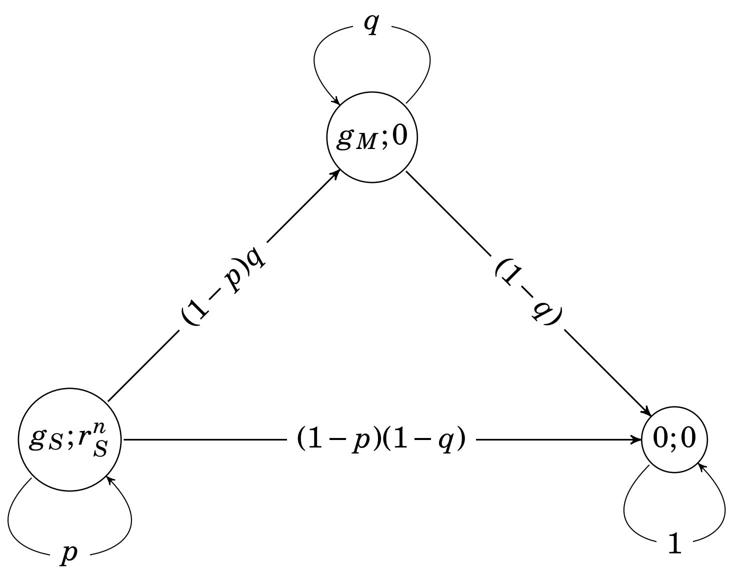
# 3 Government Spending: Long-Run Policy

As in recent contributions in Sarin et al. (2021) and Roulleau-Pasdeloup (2021a), the long-run policy effects are usually overlooked or even usually computed numerically in the previous literature. In this section, I follow Roulleau-Pasdeloup (2021b) to make use of a three-state Markov chain to reassess the long-run government spending effects analytically.

## 3.1 Policy and Shocks

I assume that government spending is longer-lived than the demand shock. To be more specific, the government spending in the short run $g_S$ can step into the medium run government spending $g_M$ with a persistence $q$ and then collapse to the steady-state $g_L = 0$ with a probability $1-q$. For the natural rate shock $r_S^n$, it runs in the short run with a probability $p$ and then returns to the long run $r_L^n = 0$ with a probability $1-p$. The graphical representation of our policy can be seen in Figure 4.

Figure 4: Long run government spending: a graphical representation



With this in mind, for the monetary policy now, I use an adapted Taylor rule:[22]

$$R_t = \begin{cases} \max\left[0; -\log(\beta) + \phi_\pi \pi_S\right] & \text{In the short run} \\ -\log(\beta) + \phi_\pi^q \pi_M & \text{In the medium run} \\ -\log(\beta) & \text{In the long run} \end{cases} \tag{23}$$

One can use the above equation (23) to trace the path of the government spending. In the medium run, the output gap and inflation can be expressed as the product of medium-run multipliers and the medium-run government spending as follows:

$$y_M = y_{M,g} \cdot g_M \;\; \& \;\; \pi_M = \pi_{M,g} \cdot g_M, \tag{24}$$

where $g_M$ is the government spending in the medium run. See appendix G for the medium spending multipliers.[23] In this case, since the model is forward-looking, I can generate the expected output gap below, and one can show the expected inflation using the same manner.[24]

$$\begin{aligned} \mathbb{E}_S y_{t+1} &= p y_S + (1-p) q y_M \\ &= p y_S + (1-p) q y_{M,g} g_M \\ &= p y_S + (1-p) q y_{M,g} \zeta g_S. \end{aligned} \tag{25}$$

## 3.2 Long-run Government Spending: Theoretical Analysis

This section discusses the analytical results regarding the government spending multiplier with a three-state Markov structure.

### 3.2.1 Long-run Government Spending Multiplier in Normal Times

If the *short-run economy is in normal times*, the new Euler equation can be regenerated with the consideration of the long-run government spending using a three-state Markov

[22] In this section, to be in line with Section 2, I consider two economic states in the short run, which are normal times and the ZLB. In the medium run, I simply assume the demand shock reverts to zero and the economy is in normal times.

[23] I follow a simple rule in Nie and Roulleau-Pasdeloup (2022) to deal with the medium-run shock: It is generally assumed that the medium-run spending is contingent on the short-run spending but it is lower than the short-run spending such that $g_M = \zeta g_S$, where $\zeta$ is a discount parameter.

[24] In our simulation results, it is assumed that $q = 0.7$ and $\zeta = 0.5$.

chain:

$$y_S = -\frac{1}{\sigma}\frac{\phi_\pi - p}{1-p}\pi_S + \Theta_{AD} g_s \tag{26}$$

$$\Theta_{AD} = q\zeta(y_{M,g} + \frac{1}{\sigma}\pi_{M,g} - 1) + 1, \tag{27}$$

where $\Theta_{AD}$ is the government spending shock shift in the Euler equation, $\zeta$ is the policy discount parameter, $y_{M,g}$ and $\pi_{M,g}$ are the medium run policy multiplier as in equation (24). For reference, this shift without long-run government spending will collapse to 1, which can nest the case in our baseline model in section 2. The new items in this shift are from *rational expectations* of the output gap, inflation, and medium-run spending shock. Note that the first new term is from the future wealth effects (higher expected output gap in the future) as in Bouakez et al. (2017) and the household has the incentive to increase consumption due to consumption smoothing. The second term comes from the fact that the long-run government can increase the firm's marginal cost and further inflation. The third term is due to the direct demand effect from future government spending. See Appendix H, it turns out that $q\zeta(y_{M,g} + \frac{1}{\sigma}\pi_{M,g} - 1)$ is negative which means the overall expected effects from longer spending policy can crowd out the present output. In addition, the effects of long-run government spending can be controlled by the policy product $q\zeta$.

On the other hand, the new Phillips Curve can be shown below:

$$\pi_S = \kappa\frac{\gamma_y}{1-\beta p - \kappa\gamma_r(\phi_\pi - p)} y_S + \frac{1}{1-\beta p - \kappa\gamma_r(\phi_\pi - p)}\Theta_{AS} g_s \tag{28}$$

$$\Theta_{AS} = (\beta - \kappa\gamma_r)(1-p)q\zeta\pi_{M,g} + \kappa\gamma_g, \tag{29}$$

where $\Theta_{AS}$ is the government spending shock shift in the Phillips Curve, $\zeta$ is the policy discount parameter, $\pi_{M,g}$ is the medium run inflation multiplier as in equation (24). For reference, this shift without long-run government spending will collapse to $\kappa\gamma_g$, which is the same as our baseline model in section 2. The new item in this shirt is from *rational expectations* of inflation and the long-run spending can increase the firm's marginal cost and further inflation due to sticky prices. See appendix H, it turns out the long-run government spending can further increase inflation. Similar to the case in the Euler equation, the effects of long-run policy can be controlled by the policy product $q\zeta$.[25]

One can use the new Euler equation and the Phillips Curve to produce the output gap multiplier $\mathcal{M}_{S,N}^{O,long}$ and the inflation multiplier $\mathcal{M}_{S,N}^{I,long}$:

$$\mathcal{M}_{S,N}^{O,long} = \frac{\Theta_{AD}\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]-\Theta_{AS}(\phi_\pi-p)}{\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]+\kappa\gamma_y(\phi_\pi-p)} \tag{30}$$

$$\mathcal{M}_{S,N}^{I,long} = \frac{\left(\kappa\gamma_y\Theta_{AD}+\Theta_{AS}\right)\sigma(1-p)}{\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]+\kappa\gamma_y(\phi_\pi-p)}. \tag{31}$$

The long-run government spending can further increase the inflation multiplier since the longer government spending can further increase the marginal cost. In normal times, the central bank would increase the nominal interest rate due to a higher price level. In that way, long-run spending policy can crowd out more private consumption due to a larger real interest rate, thus a lower output gap multiplier. Additionally, the real cost channel can increase the marginal cost and further inflation. Thus, the inflation multiplier can be higher in the strength of the cost channel, and this result is the same as the case in Section 2. In turn, the cost channel can lower the output gap. It is summarized that the long-run output gap multiplier reduces in the strength of the cost channel $\gamma_r$ while the long-run inflation multiplier rises in $\gamma_r$.—See a formal proof in Appendix I.

### 3.2.2 Long-run Government Spending Multiplier at Zero Lower Bound

If the *short-run economy is at the ZLB*, the new Euler equation with a long-run spending policy can be given by:

$$y_S = -\frac{1}{\sigma(1-p)}[\log(\beta)-p\pi_S]+\Theta_{AD}g_s \tag{32}$$

$$\Theta_{AD} = q\zeta(y_{M,g}+\frac{1}{\sigma}\pi_{M,g}-1)+1. \tag{33}$$

where $\Theta_{AD}$ is the government spending shock shift in the Euler equation. The long-run government spending terms $q\zeta(y_{M,g}+\frac{1}{\sigma}\pi_{M,g}-1)$ are negative and can be controlled by the policy product $q\zeta$.

[25] In this section, I just assume that $\kappa$ is very small in our theoretical analysis which is in line with e.g. Eggertsson (2011), Gabaix (2020) and Budianto et al. (2020). In this sense, one can assume $\beta-\kappa\gamma_r>0$ such that the long-run government policy can increase inflation here. However, if $\beta-\kappa\gamma_r<0$, this might resolve the fiscal price puzzle (FPP) as in Han et al. (2020) that the long-run fiscal stimulus can lower inflation .

In addition, I move to describe the new Phillips Curve as:

$$\pi_S = \kappa \frac{\gamma_y + \gamma_r \log(\beta)}{1 - \beta p + \kappa \gamma_r p} y_S + \frac{1}{1 - \beta p + \kappa \gamma_r p} \Theta_{AS} g_s \tag{34}$$

$$\Theta_{AS} = (\beta - \kappa \gamma_r)(1 - p) q \zeta \pi_{M,g} + \kappa \gamma_g. \tag{35}$$

where $\Theta_{AS}$ is the government spending shock shift in the Phillips Curve and can be controlled by the policy product $q\zeta$.

With this in mind, I use the Euler equation and the Phillips Curve in liquidity traps to produce the output gap multiplier $\mathcal{M}_{S,Z}^{O,long}$ and the inflation multiplier $\mathcal{M}_{S,Z}^{I,long}$:

$$\mathcal{M}_{S,Z}^{O,long} = \frac{\partial y_S}{\partial g_S} = \frac{\Theta_{AD} \sigma (1-p)(1 - \beta p + \kappa \gamma_r p) + \Theta_{AS} p}{\sigma (1-p)(1 - \beta p + \kappa \gamma_r p) - \kappa \gamma_y p} \tag{36}$$

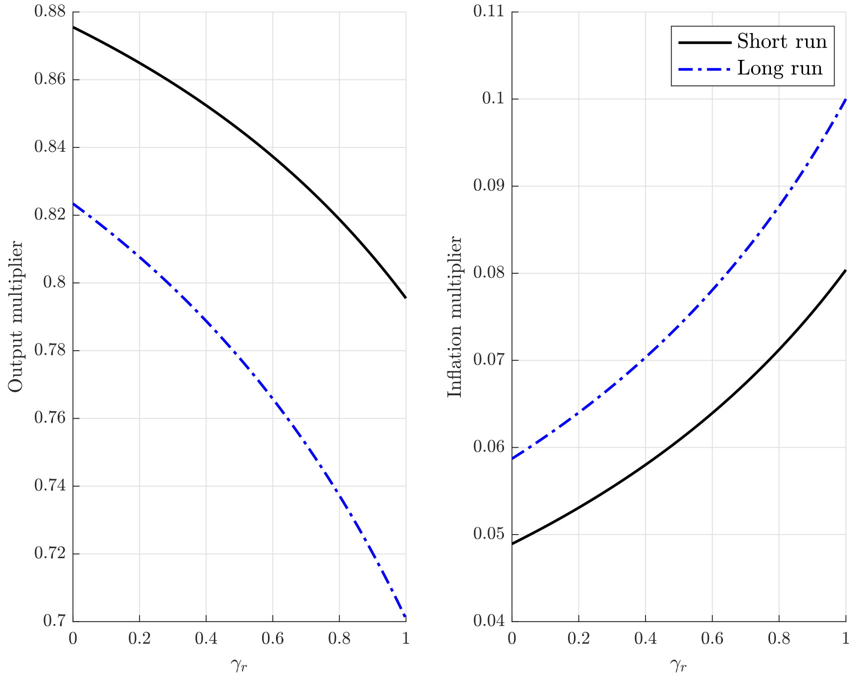
$$\mathcal{M}_{S,Z}^{I,long} = \frac{\partial \pi_S}{\partial g_S} = \frac{\left(\kappa \gamma_y \Theta_{AD} + \Theta_{AS}\right) \sigma (1-p)}{\sigma (1-p)(1 - \beta p + \kappa \gamma_r p) - \kappa \gamma_y p}. \tag{37}$$

The more extended government spending policy after the ZLB has subsided can increase inflation more by increasing firm's marginal costs. The nominal interest rate is zero, and higher inflation can help stimulate our NK economy since it can lower the real interest rate and then higher private consumption. At this time, the spending multiplier is larger, which is in line with Leduc and Wilson (2013) who empirically show similar larger long-term spending effects compared to the short-run policy. However, the real cost channel can reduce the effectiveness. The cost channel can dampen inflation due to the negative inflation expectation in the real borrowing cost. Thus, the output gap and inflation multipliers decrease in the strength of the cost channel.—See the analytical analysis in Appendix J.

## 3.3 Long-run Government Spending: Numerical Results

The numerical result between the strength of the cost channel is revealed in Figure 5. It is observed that long-run government spending can decrease the output multiplier more than short-run government spending. The long-run government spending can further increase the inflation multiplier. The output gap multiplier falls in the strength of the real cost channel from $\gamma_r$ while the inflation multiplier increases in the real cost channel $\gamma_r$. See Appendix K for numerical results w.r.t. the persistence $p$. I find that the output gap multiplier can be negative with the long-run spending policy.

Figure 5: Long run spending multipliers in normal times

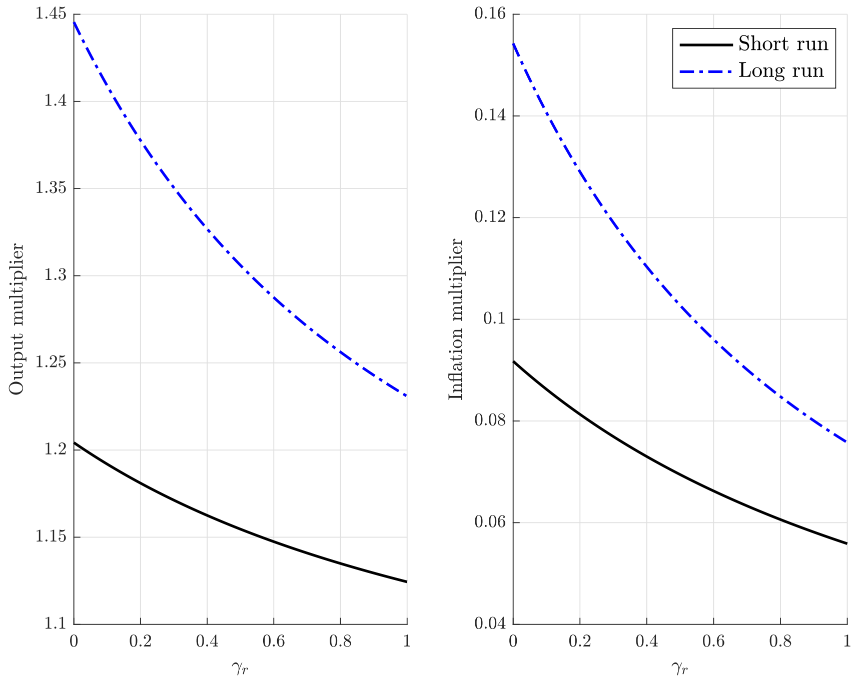



At the ZLB, the numerical results with the cost channel are reported in Figure 6. We find that long-run government spending can further increase the output gap multiplier and the inflation multiplier. The two spending multipliers decrease in the strength of the cost channel $\gamma_r$. It is of note that the output gap multiplier can be more effective for prolonged spending policy in recessions. Similar results can be found in Appendix K for numerical results w.r.t. the persistence $p$.

# 4 A Behavioral Model

As can be seen in section 3, the long-run government spending can drive the multipliers at the ZLB. In our baseline model in section 2, it is common knowledge that the agents have rational expectations. While in this section, I try to extend our baseline model to address the role of rational expectations. I finally develop a similar version of Gabaix (2020)'s model to incorporate bounded rationality where agents can be short-sighted about the world into our simple benchmark setup.

Figure 6: Long run spending multipliers at ZLB




## 4.1 The NK model with Bounded Rationality

I use the model in Gabaix (2020) to show the behavior of the aggregate demand-side economy:

$$c_t = \bar{m}\mathbb{E}_t c_{t+1} - \frac{1}{\sigma_c}(i_t - \mathbb{E}_t\pi_{t+1} - r^n), \tag{38}$$

where $\bar{m} \in [0,1]$ is the cognitive discounting parameter in Gabaix (2020).[26]

In this case, one can obtain the path of $y_t$ by substituting the resource constraint into the Euler equation:

$$y_t = \bar{m}(1-s_g)\mathbb{E}_t y_{t+1} - \frac{1}{\sigma}(R_t + \log(\beta) - \mathbb{E}_t\pi_{t+1} - r^n) + g_t - \bar{m}(1-s_g)\mathbb{E}_t g_{t+1}, \tag{39}$$

where $\sigma = \frac{\sigma_c}{1-s_g}$.

As in Gabaix (2020), I can show the NK Phillips Curve with the real cost channel:

$$\pi_t = \beta\bar{m}\left[\varphi + \frac{1-\beta\varphi}{1-\beta\varphi\bar{m}}(1-\varphi)\right]\mathbb{E}_t\pi_{t+1} + \kappa\left[\gamma_y y_t + \gamma_g g_t + \gamma_r(R_t + \log(\beta) - \mathbb{E}_t\pi_{t+1})\right], \tag{40}$$

where $\kappa = \frac{(1-\varphi)(1-\varphi\beta)}{\varphi(1+\eta\theta)}$, $\eta > 0$ is the standard (inverse) of labor-supply elasticity, $\theta > 1$ is the

[26] As in Gabaix (2020), this parameter can measure the attention to the future, which is a form of global cognitive discounting. Relative to rational expectations ($\bar{m} = 1$), when the behavioral agent contemplates the events in the future, their expectations are geared to the steady state of the economy.

price elasticity of differential goods demand, and $\varphi \in (0,1)$ is the share of firms which can not adjust their prices.

## 4.2 Government Spending with Bounded Rationality: Theoretical Results

This section presents the analytical results regarding the government spending multiplier with bounded rationality.

### 4.2.1 Multiplier with Bounded Rationality in Normal Times

I use the Euler equation and the Phillips Curve in normal times—see appendix L to produce the solutions of the output gap multiplier $\mathcal{M}_{S,N}^{O,BR}$ and the inflation multiplier $\mathcal{M}_{S,N}^{I,BR}$ below:[27]

$$\mathcal{M}_{S,N}^{O,BR} = \frac{\sigma[1-p\bar{m}(1-s_g)]\left\{1-\beta p\bar{m}\left[\varphi+\frac{1-\beta\varphi}{1-\beta\varphi\bar{m}}(1-\varphi)\right]-\kappa\gamma_r(\phi_\pi-p)\right\}-\kappa\gamma_g(\phi_\pi-p)}{\sigma(1-p\bar{m}(1-s_g))\left\{1-\beta p\bar{m}\left[\varphi+\frac{1-\beta\varphi}{1-\beta\varphi\bar{m}}(1-\varphi)\right]-\kappa\gamma_r(\phi_\pi-p)\right\}+\kappa\gamma_y(\phi_\pi-p)}$$

$$\mathcal{M}_{S,N}^{I,BR} = \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma[1-p\bar{m}(1-s_g)]}{\sigma[1-p\bar{m}(1-s_g)]\left\{1-\beta p\bar{m}\left[\varphi+\frac{1-\beta\varphi}{1-\beta\varphi\bar{m}}(1-\varphi)\right]-\kappa\gamma_r(\phi_\pi-p)\right\}+\kappa\gamma_y(\phi_\pi-p)}.$$

In normal times, cognitive discounting can reduce the expectation effects in our baseline model which can lower inflation. It means that there is a lower rise in the real interest rate, hence a higher output gap multiplier compared to the baseline model. In this case, the inflation multiplier with bounded rationality can be lower while the output multiplier can be higher.—See a formal proof in Appendix M.[28] Since the introduced bound rationality is independent of the strength of the cost channel $\gamma_r$, the effects of this real channel on spending multipliers are the same as the baseline model results. Specifically, the output gap multiplier decreases in the strength of the cost channel, and the inflation multiplier increases.

Interestingly, as explained in appendix N, there exists a threshold value, across which the multiplier effects switch from decreasing in the persistence $p$ to increasing.

[27] For simplicity, in this section, the simple two-state Markov chain is used to calculate spending multipliers.

[28] Similar results can also arise with the long-run government spending since the cognitive agent can decrease the expectation effects, which is in spirit with Nakata et al. (2019) and Farhi and Werning (2019).

I also find that the output gap multiplier can be large (near one) in normal times if we suppose the agents have relatively strong bounded rationality which means the expectation effects are extremely weak. Intuitively, agents tend to consume today but not to save since future consumption has less or no impact on the decision today. In this case, the crowding-out effects of government spending should be much attenuated and the output gap multiplier can be near one. This result can echo the previous empirical evidence that the output gap multiplier can be large in normal times as in Auerbach and Gorodnichenko (2012) and Acconcia et al. (2014).

#### 4.2.2 Multiplier with Bounded Rationality at Zero Lower Bound

One can use the Euler equation and the Phillips Curve in liquidity traps—see appendix L to produce the output gap multiplier $\mathscr{M}_{S,Z}^{O,BR}$ and the inflation multiplier $\mathscr{M}_{S,Z}^{I,BR}$:

$$\mathscr{M}_{S,Z}^{O,BR} = \frac{\sigma[1-p\bar{m}(1-s_g)]\left\{1-\beta p\bar{m}\left[\varphi+\frac{1-\beta\varphi}{1-\beta\varphi\bar{m}}(1-\varphi)\right]+\kappa\gamma_r p\right\}+\kappa\gamma_g p}{\sigma[1-p\bar{m}(1-s_g)]\left\{1-\beta p\bar{m}\left[\varphi+\frac{1-\beta\varphi}{1-\beta\varphi\bar{m}}(1-\varphi)\right]+\kappa\gamma_r p\right\}-\kappa\gamma_y p}$$

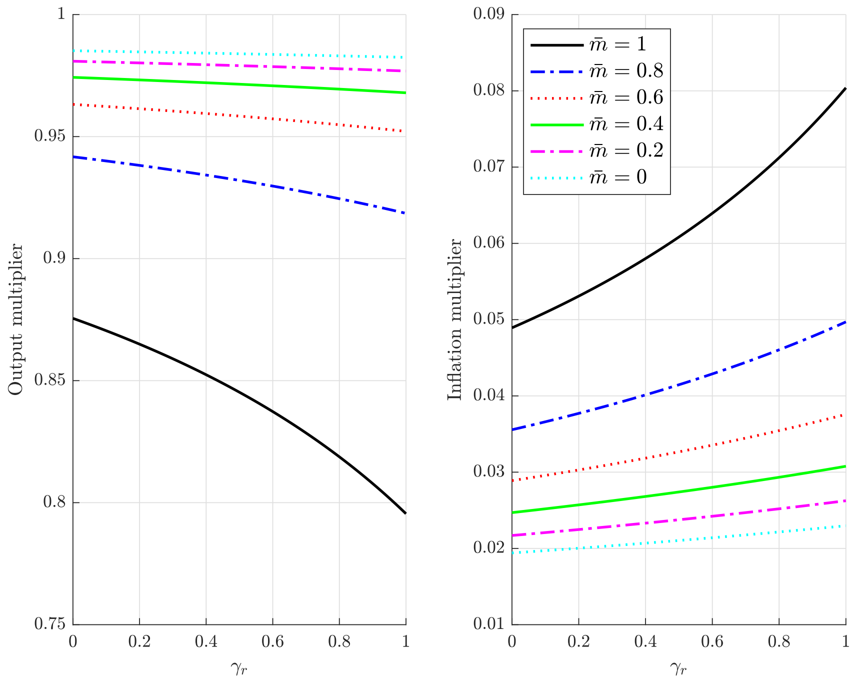
$$\mathscr{M}_{S,Z}^{I,BR} = \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma[1-p\bar{m}(1-s_g)]}{\sigma[1-p\bar{m}(1-s_g)]\left\{1-\beta p\bar{m}\left[\varphi+\frac{1-\beta\varphi}{1-\beta\varphi\bar{m}}(1-\varphi)\right]+\kappa\gamma_r p\right\}-\kappa\gamma_y p}.$$

At the ZLB, inflation caused by government spending can be less due to cognitive discounting. Thus, it can increase the real interest rate, which, in turn, can lower the output gap multiplier.—See the analytical result in Appendix O. The effects on the strength of the real cost channel are the same as the baseline model in liquidity traps. Specifically, the output gap and inflation multipliers decrease in the strength of the real cost channel $\gamma_r$.

### 4.3 Government Spending with Bounded Rationality: Numerical Results

The main takeaway in Figure 7 is that the output multiplier with bounded rationality can be higher in the strength of the cognitive discounting level $\bar{m}$, and the inflation multiplier with bounded rationality can be lower. In addition, the output gap multiplier decreases in the strength of the real cost channel $\gamma_r$ while the inflation multiplier increases in $\gamma_r$. See Appendix P for numerical results w.r.t. the persistence $p$. I find that the output gap multiplier can decrease in the persistence $p$ with rational expectations ($\bar{m} = 1$) but increase in persistence $p$ with a small $\bar{m}$ value.

Figure 7: Spending multipliers with/without bounded rationality in normal times

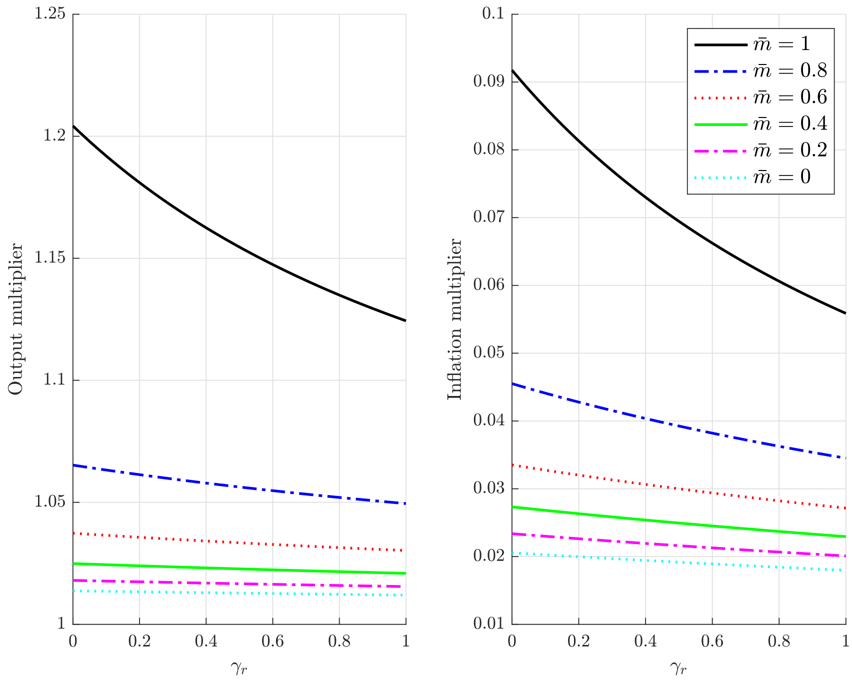



As in our numerical simulation in Figure 8, it is seen that the inflation and output multipliers are lower with bounded rationality. In addition, the role of the real cost channel with bounded rationality is the same as the baseline model. As in Appendix P, the output multiplier increases numerically in the persistence $p$.

# 5 Concluding Remarks

This paper augments the real cost channel in the textbook NK model to explore government spending multipliers. The general properties of spending multipliers are detected using this analytical framework. It is found that the output gap multiplier with the real cost channel decreases in the strength of this channel during liquidity traps episodes. I use this theoretical result to explain the lower government spending multiplier in empirical evidence when the nominal interest rate is fixed at the lower bound. To the extent that firms care about the real interest rate as opposed to the nominal interest rate, this can make government spending less effective in stimulating the economy in times of re-

Figure 8: Spending multipliers with/without bounded rationality at ZLB




cession. In normal times, the output gap multiplier also decreases with the real cost channel.

The robust role of the real cost channel on the spending multipliers is confirmed. To be more specific, our results on the role of the real cost channel, if one considers the long-run government spending policy, are the same as the short-run model. The benchmark model is also modified to include bounded rationality while retaining analytical tractability. It turns out that cognitive behavior can alter spending multipliers, however, the real cost channel can still work robustly.

# References


**Abo-Zaid, Salem**, "The government spending multiplier in a model with the cost channel," *Macroeconomic Dynamics*, 2022, *26* (1), 72–101.

**Acconcia, Antonio, Giancarlo Corsetti, and Saverio Simonelli**, "Mafia and Public Spending: Evidence on the Fiscal Multiplier from a Quasi-experiment," *American Economic Review*, July 2014, *104* (7), 2185–2209.

**Amano, Robert A and Tony S Wirjanto**, "Intratemporal substitution and government spending," *Review of Economics and Statistics*, 1997, *79* (4), 605–609.

**Angeletos, George-Marios and Chen Lian**, "Forward guidance without common knowledge," *American Economic Review*, 2018, *108* (9), 2477–2512.

**Aruoba, S. Boragan and Frank Schorfheide**, "Macroeconomic dynamics near the ZLB: a tale of two equilibria," Working Papers 13-29, Federal Reserve Bank of Philadelphia 2013.

**Aruoba, S Borağan, Pablo Cuba-Borda, and Frank Schorfheide**, "Macroeconomic dynamics near the ZLB: A tale of two countries," *The Review of Economic Studies*, 2018, *85* (1), 87–118.

**Auerbach, Alan and Yuriy Gorodnichenko**, "Fiscal Multipliers in Recession and Expansion," in "Fiscal Policy after the Financial Crisis" NBER Chapters, National Bureau of Economic Research, Inc, 2012.

**Auerbach, Alan J, Yuriy Gorodnichenko, Peter McCrory, and Daniel Murphy**, "Fiscal Multipliers in the COVID19 Recession," Technical Report, National Bureau of Economic Research 2021.

**Barro, Robert J. and Charles J. Redlick**, "Macroeconomic Effects From Government Purchases and Taxes," *The Quarterly Journal of Economics*, 2011, *126* (1), 51–102.

**Beaudry, Paul, Chenyu Hou, and Franck Portier**, "Monetary Policy When the Phillips Curve is Quite Flat," CEPR Discussion Paper DP15184, Working Paper August 2022.

**Bernardini, Marco, Selien De Schryder, and Gert Peersman**, "Heterogeneous government spending multipliers in the era surrounding the great recession," *Review of Economics and Statistics*, 2020, *102* (2), 304–322.

**Bilbiie, Florin O.**, "Limited asset markets participation, monetary policy and (inverted) aggregate demand logic," *Journal of Economic Theory*, May 2008, *140* (1), 162–196.

— , "Monetary Policy and Heterogeneity: An Analytical Framework," CEPR Discussion Papers 12601, C.E.P.R. Discussion Papers January 2019.

— , "Optimal Forward Guidance," *American Economic Journal: Macroeconomics*, October 2019, *11* (4), 310–45.

— , "The new Keynesian cross," *Journal of Monetary Economics*, 2020, *114*, 90–108.

**Blanchard, Olivier and Roberto Perotti**, "An Empirical Characterization Of The Dynamic Effects Of Changes In Government Spending And Taxes On Output," *The Quarterly Journal of Economics*, November 2002, *117* (4), 1329–1368.

**Bouakez, Hafedh, Michel Guillard, and Jordan Roulleau-Pasdeloup**, "Public Investment, Time to Build, and the Zero Lower Bound," *Review of Economic Dynamics*, January 2017, *23*, 60–79.

**__, __, and __** , "The optimal composition of public spending in a deep recession," *Journal of Monetary Economics*, 2020, *114*, 334–349.

**Budianto, Flora, Taisuke Nakata, and Sebastian Schmidt**, "Average inflation targeting and the interest rate lower bound," Working Paper 2020.

**Campbell, Jeffrey R, Filippo Ferroni, Jonas DM Fisher, and Leonardo Melosi**, "The limits of forward guidance," *Journal of Monetary Economics*, 2019, *108*, 118–134.

**Chowdhury, Ibrahim, Mathias Hoffmann, and Andreas Schabert**, "Inflation dynamics and the cost channel of monetary transmission," *European Economic Review*, 2006, *50* (4), 995–1016.

**Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans**, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, February 2005, *113* (1), 1–45.

**__, __, and Sergio Rebelo**, "When Is the Government Spending Multiplier Large?," *Journal of Political Economy*, 2011, *119* (1), 78 – 121.

**Cochrane, John H.**, "The new-Keynesian liquidity trap," *Journal of Monetary Economics*, 2017, *92* (C), 47–63.

**Coibion, Olivier, Dimitris Georgarakos, Yuriy Gorodnichenko, and Michael Weber**, "Forward guidance and household expectations," Technical Report, National Bureau of Economic Research 2020.

**Durevall, Dick and Magnus Henrekson**, "The futile quest for a grand explanation of long-run government expenditure," *Journal of Public Economics*, 2011, *95* (7-8), 708–722.

**Eggertsson, Gauti B**, "Real government spending in a liquidity trap," *Princeton University, unpublished manuscript*, 2001.

__ , "What fiscal policy is effective at zero interest rates?," *NBER Macroeconomics Annual*, 2011, *25* (1), 59–112.

**__ and Michael Woodford**, "Optimal Monetary Policy in a Liquidity Trap," NBER Working Papers 9968, National Bureau of Economic Research, Inc September 2003.

**__ and __** , "Optimal monetary and fiscal policy in a liquidity trap," Working Paper, National Bureau of Economic Research 2004.

— **et al.**, "Zero bound on interest rates and optimal monetary policy," *Brookings papers on economic activity*, 2003, *2003* (1), 139–233.

**Farhi, Emmanuel and Iván Werning**, "Monetary policy, bounded rationality, and incomplete markets," *American Economic Review*, 2019, *109* (11), 3887–3928.

**Gabaix, Xavier**, "A Behavioral New Keynesian Model," *American Economic Review*, August 2020, *110* (8), 2271–2327.

**Galí, Jordi**, *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications*, Princeton University Press, 2015.

**Ganelli, Giovanni and Juha Tervala**, "The welfare multiplier of public infrastructure investment," IMF Working Paper 2016.

**Gertler, Mark, Jordi Gali, and Richard Clarida**, "The Science of Monetary Policy: A New Keynesian Perspective," *Journal of Economic Literature*, December 1999, *37* (4), 1661–1707.

**Han, Zhao, Fei Tan, and Jieran Wu**, "Learning from Monetary and Fiscal Policy," *Available at SSRN 3726905*, 2020.

**Hazell, Jonathon, Juan Herreño, Emi Nakamura, and Jøn Steinsson**, "The Slope of the Phillips Curve: Evidence from U.S. States," Working Paper, National Bureau of Economic Research 2020.

**House, Christopher L, Christian Proebsting, and Linda L Tesar**, "Austerity in the aftermath of the great recession," *Journal of Monetary Economics*, 2020, *115*, 37–63.

**III, Marvin J Barth and Valerie A Ramey**, "The cost channel of monetary transmission," *NBER macroeconomics annual*, 2001, *16*, 199–240.

**Ilzetzki, Ethan, Enrique G. Mendoza, and Carlos A. Végh**, "How big (small?) are fiscal multipliers?," *Journal of Monetary Economics*, 2013, *60* (2), 239–254.

**Kiley, Michael T.**, "Policy Paradoxes in the New Keynesian Model," Finance and Economics Discussion Series 2014-29, Board of Governors of the Federal Reserve System (U.S.) February 2014.

**Koester, Gerrit, Eliza Lis, Christiane Nickel, Chiara Osbat, and Frank Smets**, "Understanding low inflation in the euro area from 2013 to 2019: cyclical and structural drivers," *ECB Occasional Paper*, 2021, (2021280).

**Kollmann, Robert, Werner Roeger et al.**, "Fiscal policy in a financial crisis: standard policy versus bank rescue measures," *American Economic Review*, 2012, *102* (3), 77–81.

**Kraay, Aart**, "How large is the government spending multiplier? Evidence from World Bank lending," *The Quarterly Journal of Economics*, 2012, *127* (2), 829–887.

**Laubach, Thomas and John C Williams**, "Measuring the natural rate of interest," *Review of Economics and Statistics*, 2003, *85* (4), 1063–1070.

**Leduc, Sylvain and Daniel Wilson**, "Roads to Prosperity or Bridges to Nowhere? Theory and Evidence on the Impact of Public Infrastructure Investment," *NBER Macroeconomics Annual*, 2013, *27* (1), 89 – 142.

**Leeper, Eric M, Nora Traum, and Todd B Walker**, "Clearing up the fiscal multiplier morass," *American Economic Review*, 2017, *107* (8), 2409–54.

**Lewis, Daniel J**, "Identifying shocks via time-varying volatility," *The Review of Economic Studies*, 2021, *88* (6), 3086–3124.

**Llosa, Luis-Gonzalo and Vicente Tuesta**, "Learning about monetary policy rules when the cost-channel matters," *Journal of Economic Dynamics and Control*, 2009, *33* (11), 1880–1896.

**Lustenhouwer, Joep**, "Fiscal stimulus in expectations-driven liquidity traps," *Journal of Economic Behavior & Organization*, 2020, *177*, 661–687.

**McKay, Alisdair, Emi Nakamura, and Jón Steinsson**, "The Power of Forward Guidance Revisited," *American Economic Review*, October 2016, *106* (10), 3133–3158.

**Mertens, Karel RSM and Morten O Ravn**, "Fiscal policy in an expectations-driven liquidity trap," *The Review of Economic Studies*, 2014, *81* (4), 1637–1667.

**Miyamoto, Wataru, Thuy Lan Nguyen, and Dmitriy Sergeyev**, "Government Spending Multipliers under the Zero Lower Bound: Evidence from Japan," *American Economic Journal: Macroeconomics*, July 2018, *10* (3), 247–277.

**Nakata, Taisuke, Ryota Ogaki, Sebastian Schmidt, and Paul Yoo**, "Attenuating the forward guidance puzzle: implications for optimal monetary policy," *Journal of Economic Dynamics and Control*, 2019, *105*, 90–106.

**Nie, He**, "The Macroeconomic Effects of Tax Shocks: The Real Cost Channel," *Available at SSRN 3905695*, 2021.

_ **and Jordan Roulleau-Pasdeloup**, "The promises (and perils) of control-contingent forward guidance," *Review of Economic Dynamics*, 2022.

**Ramey, Valerie A.**, "Identifying Government Spending Shocks: It's all in the Timing," *The Quarterly Journal of Economics*, 2011, *126* (1), 1–50.

_ **and Sarah Zubairy**, "Government Spending Multipliers in Good Times and in Bad: Evidence from US Historical Data," *Journal of Political Economy*, 2018, *126* (2), 850–901.

**Ravenna, Federico and Carl E. Walsh**, "Optimal monetary policy with the cost channel," *Journal of Monetary Economics*, 2006, *2* (53), 199–216.

**Roulleau-Pasdeloup, Jordan**, "The Public Investment Multiplier: Insights from a Tractable HANK Framework," Working Paper October 2021.

_ , "Shocks and policies in Linear Rational Expectations models: An expected Markovian Approach," Working Paper October 2021.

**Sarin, Natasha, Lawrence H. Summers, Owen M. Zidar, and Eric Zwick**, "Rethinking How We Score Capital Gains Tax Reform," NBER Working Papers 28362, National Bureau of Economic Research, Inc January 2021.

**Schmidt, Sebastian**, "Fiscal activism and the zero nominal interest rate bound," *Journal of Money, Credit and Banking*, 2017, *49* (4), 695–732.

**Smith, A Lee**, "When does the cost channel pose a challenge to inflation targeting central banks?," *European Economic Review*, 2016, *89*, 471–494.

**Surico, Paolo**, "The cost channel of monetary policy and indeterminacy," *Macroeconomic Dynamics*, 2008, *5* (12), 724–735.

**Taylor, John B.**, "Discretion versus policy rules in practice," *Carnegie-Rochester Conference Series on Public Policy*, December 1993, *39* (1), 195–214.

**Tillmann, Peter**, "Optimal monetary policy with an uncertain cost channel," *Journal of Money, Credit and Banking*, 2009, *41* (5), 885–906.

**Woodford, Michael**, "Optimal interest-rate smoothing," *The Review of Economic Studies*, 2003, *70* (4), 861–886.

**Yaffe, Daniel Leff**, "The Interstate Multiplier," *University of California, San Diego Working paper*, 2019.

**Zubairy, Sarah**, "On fiscal multipliers: Estimates from a medium scale DSGE model," *International Economic Review*, 2014, *55* (1), 169–195.

# Online Appendix

## A Details of the Baseline Model in Section 2

### A.1 Aggregate Supply Side

The behavior of aggregate supply side economics in the standard NK model is shown below as in Tillmann (2009):

$$\pi_t = \kappa mc_t + \beta \mathbb{E}_t \pi_{t+1}.$$

I need to add the government spending ingredient into Beaudry et al. (2022)'s aggregate supply-side economics. Here I assume that there is a large number of firms indexed by $z$ which produce differentiated intermediate goods sold to a final goods producer. The representative firm follows a Leontief production function as shown below

$$Y_t(z) = \min(aN_t(z), bM_t(z)),$$

where $M_t$ is the final good.

The unit price of final goods that enter the production of basic input is $P_t$. As in Beaudry et al. (2022), we assume that the basic input representative should borrow $D_t$ to pay for the input $M_t$ at the risk-free nominal rate $i_t$ for the production. In this case, the firms should produce, sell the product, pay wages, pay back the debt in the previous period, and distributes the dividends. One can show the budget constraint of the firms at the time $t$ by simply assuming zero profits in equilibrium.

$$D_t + P_t^B = W_t P_t L_t + (1 + i_{t-1}) D_t + P_t M_t,$$

where $P_t^B$ is the basic input price, and $D_{t+1} = P_t M_t$. Thus the profit $\Pi_t$ can be shown in the below expressions:

$$\Pi_t = P_t^B - W_t P_t L_t - (1 + i_{t-1}) P_{t-1} M_{t-1}.$$

We further assume that the firms maximize the sum of real profit $\frac{\Pi_t}{P_t}$ with a discount parameter $\beta$. In this case, the first-order condition can be shown:

$$P_t^B = \Big(\frac{1}{a}W_t + \frac{\beta}{b}\mathbb{E}_t\frac{1+i_t}{1+\pi_{t+1}}\Big)P_t.$$

Thus, one can obtain the (real) marginal cost of the basic input:

$$MC_t = \frac{W_t}{a} + \frac{\beta}{b}\mathbb{E}\left[\frac{1+i_t}{1+\pi_{t+1}}\right].$$

With log condition, one can show the linearized equilibrium

$$mc_t = \hat{\gamma}_y(w_t) + \gamma_r(R_t + \log(\beta) - \mathbb{E}\pi_{t+1}),$$

where $\hat{\gamma}_y = \frac{\frac{1}{a}W}{\frac{1}{a}W + \frac{\beta}{b}\frac{1+i}{1+\pi}}$ and $\gamma_r = \frac{\frac{\beta}{b}\frac{1+i}{1+\pi}}{\frac{1}{a}W + \frac{\beta}{b}\frac{1+i}{1+\pi}}$. On the other hand, the optimal labor supply reads

$$\frac{v'(N_t)}{u'(C_t)} = W_t.$$

By using the production function $y_t = n_t$ and resource constraint $y_t = (1-s_g)c_t + g_t$, the marginal cost can be rewritten as

$$\begin{aligned} mc_t &= \hat{\gamma}_y\frac{Nv''(N)}{v'(N)}y_t - \hat{\gamma}_y\frac{Cu''(C)}{u'(C)}(\frac{y_t - g_t}{1-s_g}) + \gamma_r(R_t + \log(\beta) - \mathbb{E}_t\pi_{t+1}) \\ &= \gamma_y y_t + \gamma_g g_t + \gamma_r(R_t + \log(\beta) - \mathbb{E}_t\pi_{t+1}), \end{aligned}$$

where $\gamma_y = \hat{\gamma}_y\left(\frac{Nv''(N)}{v'(N)} - \frac{Cu''(C)}{u'(C)(1-s_g)}\right)$, and $\gamma_g = \hat{\gamma}_y\frac{Cu''(C)}{u'(C)(1-s_g)}$. Therefore, the Phillips Curve with government spending is shown below.[29]

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa\left[\gamma_y y_t + \gamma_g g_t + \gamma_r(R_t + \log(\beta) - \mathbb{E}_t\pi_{t+1})\right].$$

[29]The advantage of this introduced real cost channel method as in Beaudry et al. (2022) is that it allows to set arbitrarily the elasticity of marginal cost rate with regard to wage and interest rate. Please see Beaudry et al. (2022) for a comprehensive comparison between the model with a nominal cost channel and with the real cost channel.

# B Proof for Proposition 1

If I only consider the demand shock and it is assumed that the demand shock $r_S^n$ can put the economy into liquidity traps with one enough (negatively) big number ($r_S^n < \underline{r_S^n}$), one can rewrite the Phillips Curve as

$$y_S = \begin{cases} \frac{1-\beta p+\kappa\gamma_r p-\kappa\gamma_r\phi_\pi}{\kappa\gamma_y}\pi_S & \text{if } r_S^n \geq \underline{r_S^n} \\ \frac{1-\beta p+\kappa\gamma_r p}{\kappa\gamma_y}\pi_S - \frac{\gamma_r}{\gamma_y}\log(\beta) & \text{if } r_S^n < \underline{r_S^n}. \end{cases}$$

Similarly, one can rewrite the Euler equations as follows:

$$y_S = \begin{cases} -\frac{1}{\sigma}\frac{\phi_\pi - p}{1-p}\pi_S + \frac{1}{\sigma}\frac{r_S^n}{1-p} & \text{if } r_S^n \geq \underline{r_S^n} \\ \frac{\frac{1}{\sigma}p}{1-p}\pi_S + \frac{1}{\sigma}\frac{r_S^n - \log(\beta)}{1-p} & \text{if } r_S^n < \underline{r_S^n}. \end{cases}$$

I combine the first questions of Euler equation and Phillips Curve to obtain the exact expression for $\underline{r_S^n}$ which can be written as:

$$\underline{r_S^n} = \left[\frac{(1-\beta p+\kappa\gamma_r p-\kappa\gamma_r\phi_\pi)(1-p)}{\kappa\gamma_y\frac{1}{\sigma}} + (\phi_\pi - p)\right]\frac{\log(\beta)}{\phi_\pi} < 0.$$

One can show the exact boundary condition for $\underline{r_S^n}$ in the standard NK model without the real cost channel:

$$\underline{r_S^{n,B}} = \left[\frac{(1-\beta p)(1-p)}{\kappa\gamma_y\frac{1}{\sigma}} + (\phi_\pi - p)\right]\frac{\log(\beta)}{\phi_\pi} < 0.$$

Likewise, the exact boundary condition for $\underline{r_S^n}$ in the model with the nominal cost channel:

$$\underline{r_S^{n,N}} = \left[\frac{(1-\beta p-\kappa\gamma_r\phi_\pi)(1-p)}{\kappa\gamma_y\frac{1}{\sigma}} + (\phi_\pi - p)\right]\frac{\log(\beta)}{\phi_\pi} < 0.$$

In this case, I have

$$\underline{r_S^n} - \underline{r_S^{n,B}} = \frac{\kappa\gamma_r(p-\phi_\pi)}{\kappa\gamma_y\frac{1}{\sigma}}\frac{\log(\beta)}{\phi_\pi} > 0.$$

And further one can show

$$\underline{r_S^n} - \underline{r_S^{n,N}} < 0.$$

One can use this to obtain the result in the main text.

# C Proof for Proposition 2

I show the solutions for the output gap and inflation multipliers below:

$$\mathcal{M}^{O}_{S,N} = \frac{\partial y_S}{\partial g_S} = \frac{\frac{\sigma(1-p)}{\phi_\pi - p} - \frac{\kappa\gamma_g}{1-\beta p-\kappa\gamma_r(\phi_\pi-p)}}{\frac{\kappa\gamma_y}{1-\beta p-\kappa\gamma_r(\phi_\pi-p)} + \frac{\sigma(1-p)}{\phi_\pi-p}}$$
$$= \frac{\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)] - \kappa\gamma_g(\phi_\pi-p)}{\kappa\gamma_y(\phi_\pi-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]}$$

$$\mathcal{M}^{I}_{S,N} = \frac{\partial \pi_S}{\partial g_S} = \frac{1+\frac{\gamma_g}{\gamma_y}}{\frac{1-\beta p-\kappa\gamma_r(\phi_\pi-p)}{\kappa\gamma_y} + \frac{1}{\sigma(1-p)}(\phi_\pi-p)}$$
$$= \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p)}{\kappa\gamma_y(\phi_\pi-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]}.$$

For the normal cost channel case, the item $[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]$ will switch to $[1-\beta p-\kappa\gamma_r\phi_\pi]$. In this case, one can easily prove that the inflation multiplier $\mathcal{M}^{I,N}_{S,N}$ can be larger with the nominal cost channel due to a smaller denominator. However, the output multiplier with the nominal cost channel $\mathcal{M}^{O,N}_{S,N}$ can be less. For the output multiplier, the numerator is less than the denominator. That is,

$$\mathcal{M}^{O,B}_{S,N} = \sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]-\kappa\gamma_g(\phi_\pi-p) < \kappa\gamma_y(\phi_\pi-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)].$$

Thus, the output gap multiplier is less than one. For the output gap multiplier without the real cost channel:

$$\mathcal{M}^{O,B}_{S,N} = \frac{\partial y_S}{\partial g_S} = \frac{\sigma(1-p)[1-\beta p]-\kappa\gamma_g(\phi_\pi-p)}{\kappa\gamma_y(\phi_\pi-p)+\sigma(1-p)[1-\beta p]}.$$

One can compare this expression to the previous one with the real cost channel and it is easy, after some arrangements, to obtain the output gap multiplier with the real cost channel that is lower.

For the inflation multiplier without the real cost channel:

$$\mathcal{M}^{I,B}_{S,N} = \frac{\partial \pi_S}{\partial g_S} = \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p)}{\kappa\gamma_y(\phi_\pi-p)+\sigma(1-p)[1-\beta p]}.$$

In this case, the denominator of the inflation multiplier with the real cost channel is lower due to a negative item and thus the inflation multiplier is higher. For the output gap multiplier:

$$\frac{\partial \mathscr{M}^O_{S,N}}{\partial \gamma_r} = -\sigma(1-p)(\phi_\pi - p)\kappa \frac{\mathscr{D} - \mathscr{N}}{\mathscr{D}^2} < 0,$$

where $\mathscr{D} = \kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)]$ and $\mathscr{N} = \sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)] - \kappa\gamma_g(\phi_\pi - p)$. Thus, the output gap multiplier in normal times is decreasing in the strength of the real cost channel.

For the inflation multiplier, it is observed with ease that the higher the strength of the real cost channel $\gamma_r$, the higher the denominator of this multiplier. In other words, the inflation multiplier is decreasing in the strength of the real cost channel.

# D Proof for Proposition 3

The output gap and inflation multipliers at the ZLB are reproduced here:

$$\begin{aligned}\mathscr{M}^O_{S,Z} = \frac{\partial y_S}{\partial g_S} &= \frac{\frac{\sigma(1-p)}{-p} - \frac{\kappa\gamma_g}{1-\beta p-\kappa\gamma_r(-p)}}{\frac{\kappa\gamma_y}{1-\beta p-\kappa\gamma_r(-p)} + \frac{\sigma(1-p)}{-p}} \\ &= \frac{\sigma(1-p)[1-\beta p - \kappa\gamma_r(-p)] - \kappa\gamma_g(-p)}{\kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(-p)]}\end{aligned}$$

$$\begin{aligned}\mathscr{M}^I_{S,Z} = \frac{\partial \pi_S}{\partial g_S} &= \frac{1 + \frac{\gamma_g}{\gamma_y}}{\frac{1-\beta p-\kappa\gamma_r(-p)}{\kappa\gamma_y} + \frac{1}{\sigma(1-p)}(-p)} \\ &= \frac{\left[1 + \frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p)}{\kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(-p)]}.\end{aligned}$$

The numerator and denominator of the output and inflation multipliers (we assume $\gamma_r$ is far greater than $\gamma_y$ and the denominator is positive) are both positive here and thus we have the positive spending multipliers. The output gap multiplier can be rewritten as:

$$\mathscr{M}^O_{S,Z} = \frac{\partial y_S}{\partial g_S} = 1 + \frac{\sigma(1-p)[1-\beta p - \kappa\gamma_r(-p)] + \kappa\gamma_g p + \kappa\gamma_y p}{\kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(-p)]}.$$

This output gap multiplier is larger than one. For the output gap multiplier without the real cost channel:

$$\mathcal{M}^{O,B}_{S,Z} = \frac{\partial y_S}{\partial g_S} = \frac{\sigma(1-p)[1-\beta p]-\kappa\gamma_g(-p)}{\kappa\gamma_y(-p)+\sigma(1-p)[1-\beta p]}.$$

Similar with the case in normal times, one can compare this expression to the previous one with the real cost channel, and it is easy, after some arrangements, to obtain the output gap multiplier with the real cost channel that is lower.

For the inflation multiplier without the real cost channel:

$$\mathcal{M}^{I,B}_{S,Z} = \frac{\partial \pi_S}{\partial g_S} = \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p)}{\kappa\gamma_y(-p)+\sigma(1-p)[1-\beta p]}.$$

One can observe that the higher the strength of the real cost channel $\gamma_r$, the higher the denominator of this multiplier. In this case, it can be lower with the real cost channel.

For the output gap multiplier:

$$\frac{\partial \mathcal{M}^{O}_{S,Z}}{\partial \gamma_r} = -\sigma(1-p)(-p)\kappa\frac{\mathcal{D}-\mathcal{N}}{\mathcal{D}^2} < 0,$$

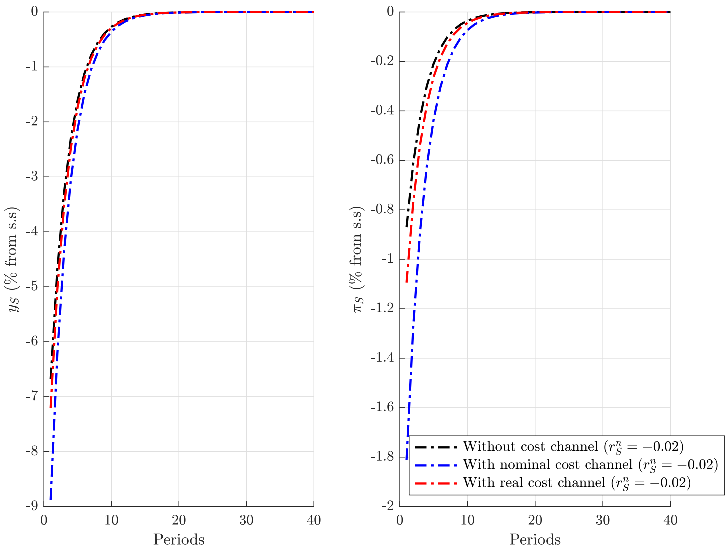
where $\mathcal{D} = \kappa\gamma_y(-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]$ and $\mathcal{N} = \sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]-\kappa\gamma_g(-p)$. Thus, the output gap multiplier in normal times is decreasing in the strength of the real cost channel.

For the inflation multiplier, it is observed with ease that the higher the strength of the real cost channel $\gamma_r$, the lower the denominator of this multiplier. In other words, the inflation multiplier is increasing in the strength of the real cost channel.

While the nominal cost channel multipliers $\mathcal{M}^{O,N}_{S,Z}$ and $\mathcal{M}^{I,N}_{S,Z}$ can be invariant with the standard NK model since the nominal channel in liquidity traps can not be included in the partial derivative of government spending to the output gap/inflation in the calculation of fiscal multipliers.

# E Impulse Response Function

Figure 9: Impulse response to a contractionary natural rate shock



# F Baseline Multiplier Figures w.r.t. Persistent $p$

Figure 10: Spending multipliers in normal times

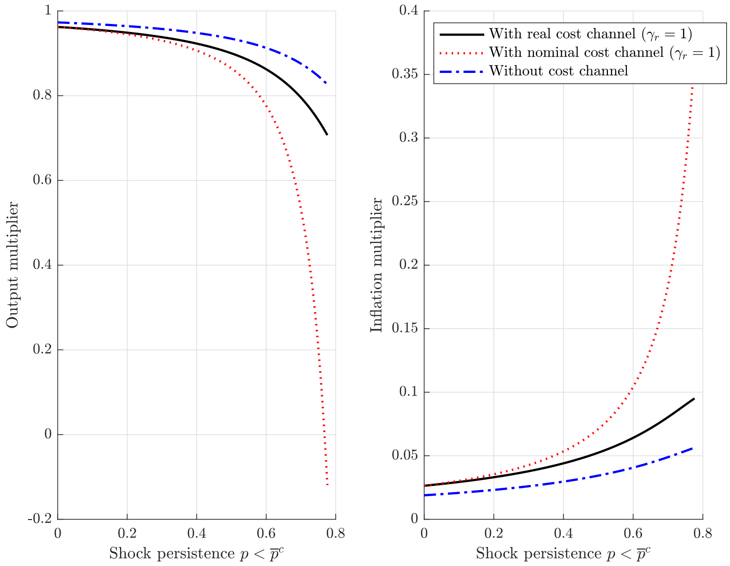
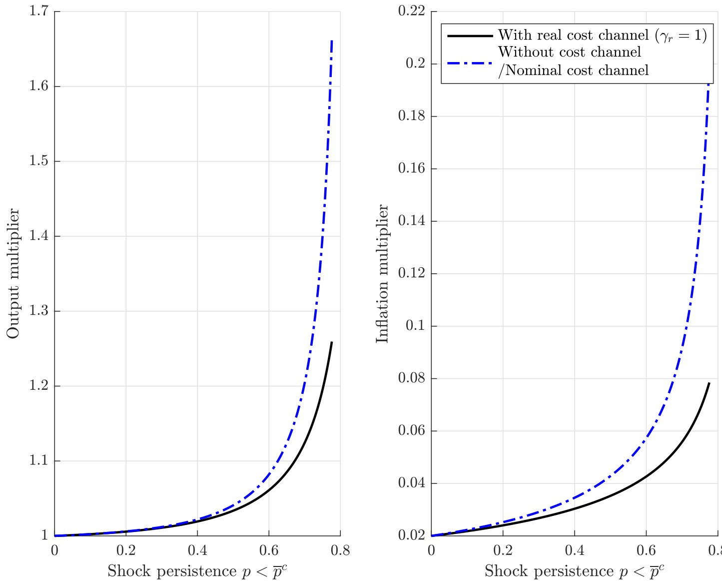



Figure 11: Spending multipliers at ZLB



# G Multipliers in the Medium Run

In the medium run, the economy is absent of natural rate shock, and one can show the medium run government spending with the persistence $q$ as follows

$$
\begin{aligned}
y_M &= y_{M,g} g_M \\
&= \frac{\sigma(1-q)[1-\beta p-\kappa\gamma_r(\phi_\pi^q-q)]-\kappa\gamma_g(\phi_\pi^q-q)}{\kappa\gamma_y(\phi_\pi^q-q)+\sigma(1-q)[1-\beta q-\kappa\gamma_r(\phi_\pi^q-q)]} g_M.
\end{aligned}
$$

$$
\begin{aligned}
\pi_M &= \pi_{M,g} g_M \\
&= \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-q)}{\kappa\gamma_y(\phi_\pi^q-q)+\sigma(1-q)[1-\beta q-\kappa\gamma_r(\phi_\pi^q-q)]} g_M.
\end{aligned}
$$

# H Euler and Phillips shift

The long-run government spending in the Euler equation shift:

$$
q\zeta(y_{M,g}+\frac{1}{\sigma}\pi_{M,g}-1) = q\zeta\frac{\kappa(\gamma_y+\gamma_g)(1-\phi_\pi^q)}{\kappa\gamma_y(\phi_\pi^q-q)+\sigma(1-q)[1-\beta q-\kappa\gamma_r(\phi_\pi^q-q)]} < 0.
$$

The long run government spending in the Phillips Curve shift

$$
(\beta-\kappa\gamma_r)(1-p)q\zeta\pi_{M,g} > 0,
$$

where we assume that the $\beta-\kappa\gamma_r > 0$ since $\kappa$ is very small in our theoretical analysis as in Gabaix (2020) Budianto et al. (2020), and Nie (2021).

# I The long-run government spending effects in normal times

I can use the new Euler equation and the Phillips Curve to reproduce the spending multipliers:

$$\mathcal{M}^{O,long}_{S,N} = \frac{\partial y_S}{\partial g_S} = \frac{\Theta_{AD}\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi - p)] - \Theta_{AS}(\phi_\pi - p)}{\kappa\gamma_y(\phi_\pi - p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi - p)]}$$

$$\mathcal{M}^{I,long}_{S,N} = \frac{\partial \pi_S}{\partial g_S} = \frac{\left[\kappa\gamma_y\Theta_{AD}+\Theta_{AS}\right]\sigma(1-p)}{\kappa\gamma_y(\phi_\pi - p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi - p)]}.$$

For the output gap multipliers, as in appendix H, one can see that the long-run government spending shock can lead to a lower $\Theta_{AD}$ but a higher $\Theta_{AS}$. In this case, the multiplier should be lower.

For inflation multiplier,

$$\kappa\gamma_y q\zeta(y_{M,g}+\frac{1}{\sigma}\pi_{M,g}-1)+(\beta-\kappa\gamma_r)(1-p)q\zeta\pi_{M,g}$$

$$= q\zeta\frac{\kappa\gamma_y\kappa(\gamma_y+\gamma_g)(1-\phi^q_\pi)}{\kappa\gamma_y(\phi^q_\pi - q)+\sigma(1-q)[1-\beta q-\kappa\gamma_r(\phi^q_\pi - q)]} + q\zeta\frac{(\beta-\kappa\gamma_r)(1-p)q\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-q)}{\kappa\gamma_y(\phi^q_\pi - q)+\sigma(1-q)[1-\beta q-\kappa\gamma_r(\phi^q_\pi - q)]} > 0,$$

where we assume $\kappa$ is minor in our theoretical analysis as in e.g. Eggertsson (2011), Gabaix (2020) and Budianto et al. (2020) and the first item has an addition multiplier $\kappa$. One can use this to prove the result in the main text. Since there is no $\gamma_r$ in $\Theta_{AD}$, we only focus on the $\Theta_{AS}$'s effects. For the main proposition result, since the term with $\Theta_{AD}$ is decreasing in $\gamma_r$, we only need to show the other term with $\Theta_{AS}$ is also decreasing in $\gamma_r$ and thus one can prove the output gap multiplier is decreasing in $\gamma_r$. The output multiplier can be reduced below:

$$-\frac{\Theta_{AS}}{\kappa\gamma_y(\phi_\pi - p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi - p)]}.$$

For the main proposition result, since the term with $\Theta_{AD}$ is increasing in $\gamma_r$, we only need to show the other term with $\Theta_{AS}$ is also increasing in $\gamma_r$ and thus one can prove the inflation multiplier is increasing in $\gamma_r$. The inflation multiplier can be reduced below:

$$\frac{\Theta_{AS}}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)]}.$$

One can differentiate this common term with regard to $\gamma_r$ and this common term can be reduced further as:

$$\frac{\beta - \kappa\gamma_r}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)]}\frac{1}{\kappa\gamma_y(\phi_\pi - q) + \sigma(1-q)[1-\beta q - \kappa\gamma_r(\phi_\pi^q - q)]}.$$

I differentiate the above term with regard to $\gamma_r$ to obtain:

$$\frac{\sigma(1-p)\kappa(\beta\phi_\pi - 1)\mathscr{D}_1 + \sigma(1-q)\kappa(\beta\phi_\pi^q - 1)\mathscr{D}_2 - \mathscr{O}(\kappa^2)}{\mathscr{D}_3^2}.$$

where $\mathscr{D}_1 = \kappa\gamma_y(\phi_\pi - q) + \sigma(1-q)[1-\beta q - \kappa\gamma_r(\phi_\pi^q - q)]$, $\mathscr{D}_2 = \kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)]$, $\mathscr{D}_3 = \mathscr{D}_1 \cdot \mathscr{D}_2$ and $\mathscr{O}(\kappa^2)$ is the residual of order two since we assume that $\kappa$ is trivial in our theoretical analysis as in e.g. Eggertsson (2011), Gabaix (2020) and Budianto et al. (2020), it is easy to check that the derivative with regard to $\gamma_r$ is positive. In this case, one can use this to prove the result in the main text.

# J The long-run government spending effects at the ZLB

One can produce the output gap and inflation multipliers below

$$\mathscr{M}_{S,Z}^{O,long} = \frac{\partial y_S}{\partial g_S} = \frac{\Theta_{AD}\sigma(1-p)[1-\beta p - \kappa\gamma_r(-p)] - \Theta_{AS}(-p)}{\kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(-p)]}$$

$$\mathscr{M}_{S,Z}^{I,long} = \frac{\partial \pi_S}{\partial g_S} = \frac{\left[\kappa\gamma_y\Theta_{AD} + \Theta_{AS}\right]\sigma(1-p)}{\kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(-p)]}.$$

For the output gap multipliers, the numerator with medium run spending policy can be decomposed into the following two parts. The first part:

$$\begin{aligned}
&\kappa\gamma_y q\zeta(y_{M,g} + \frac{1}{\sigma}\pi_{M,g} - 1)\sigma(1-p)[1-\beta p + \kappa\gamma_r p] \\
&= q\zeta \frac{\kappa\gamma_y \kappa(\gamma_y + \gamma_g)(1-\phi_\pi^q)}{\kappa\gamma_y(\phi_\pi^q - q) + \sigma(1-q)[1-\beta q - \kappa\gamma_r(\phi_\pi^q - q)]}\sigma(1-p)[1-\beta p + \kappa\gamma_r p].
\end{aligned}$$

The second part:

$$\begin{aligned}
&(\beta - \kappa\gamma_r)(1-p)q\zeta\pi_{M,g}p \\
&= q\zeta \frac{(\beta - \kappa\gamma_r)(1-p)q\left[1 + \frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-q)}{\kappa\gamma_y(\phi_\pi^q - q) + \sigma(1-q)[1-\beta q - \kappa\gamma_r(\phi_\pi^q - q)]}p.
\end{aligned}$$

To simplify the proof, one can plus the two items and show the sum is positive if we assume that $\kappa$ is very small in our theoretical analysis. Similar to the inflation multiplier in normal times, we can have a higher long run inflation multiplier at the ZLB. Since there is no $\gamma_r$ in $\Theta_{AD}$, we only focus on the $\Theta_{AS}$'s effects. For the main result, since the term with $\Theta_{AD}$ is decreasing in $\gamma_r$, we only need to show the the other term with $\Theta_{AS}$ is also decreasing in $\gamma_r$ and thus one can see the output gap multiplier is decreasing in $\gamma_r$. In this case, the output multiplier can be reduced below:

$$\frac{\Theta_{AS}}{\kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(-p)]}.$$

For the main result, since the term with $\Theta_{AD}$ is decreasing om $\gamma_r$, we only need to show the other term with $\Theta_{AS}$ is also decreasing in $\gamma_r$ and thus the inflation multiplier is decreasing in $\gamma_r$. The inflation multiplier can be reduced below:

$$\frac{\Theta_{AS}}{\kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(-p)]}.$$

One can differentiate this common term with regard to $\gamma_r$ and this common term can be reduced further as:

$$\frac{\beta - \kappa\gamma_r}{\kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(-p)]}\frac{1}{\kappa\gamma_y(\phi_\pi^q - q) + \sigma(1-q)[1-\beta q - \kappa\gamma_r(\phi_\pi^q - q)]}.$$

I differentiate the above term with regard to $\gamma_r$ to obtain:

$$\frac{-\sigma(1-p)\kappa\mathscr{D}_1+\sigma(1-q)\kappa(\beta\phi_\pi^q-1)\mathscr{D}_2-\mathscr{O}(\kappa^2)}{\mathscr{D}_3^2}.$$

where $\mathscr{D}_1=\kappa\gamma_y(\phi_\pi^q-q)+\sigma(1-q)[1-\beta q-\kappa\gamma_r(\phi_\pi^q-q)]$, $\mathscr{D}_2=\kappa\gamma_y(-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]$, $\mathscr{D}_3=\mathscr{D}_1\cdot\mathscr{D}_2$ and $\mathscr{O}(\kappa^2)$ is the residual of order two. One can reduce this expression as:

$$-(1-p)\sigma(1-q)(1-\beta q)+(1-q)(\beta\phi_\pi^q-1)\sigma(1-p)(1-\beta p)-\mathscr{O}(\kappa^2)<0,$$

where I use the general condition $\phi_\pi^q\beta-1<1$ and the short run period should be longer or almost equal to the long run period in reality such that $p\geq q$. In this case, one can use this to prove the result in the main text.

# K Long-run Multiplier Figures w.r.t. Persistent $p$

Figure 12: Long-run spending multipliers in normal times

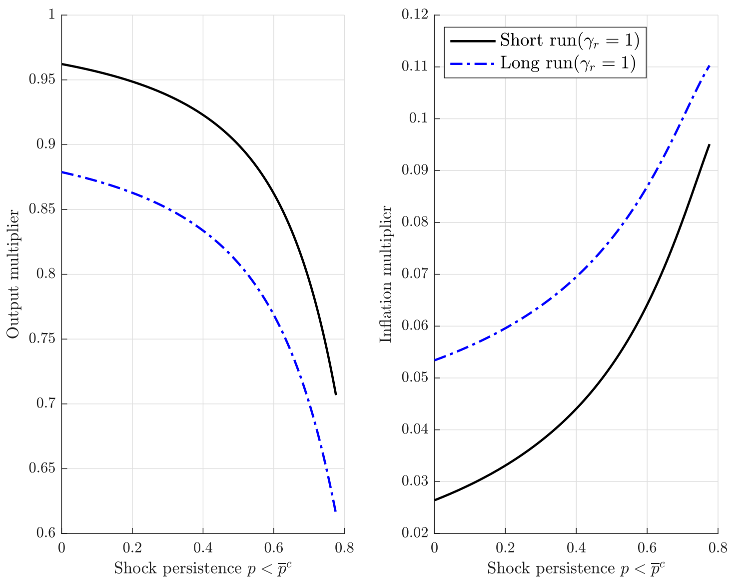
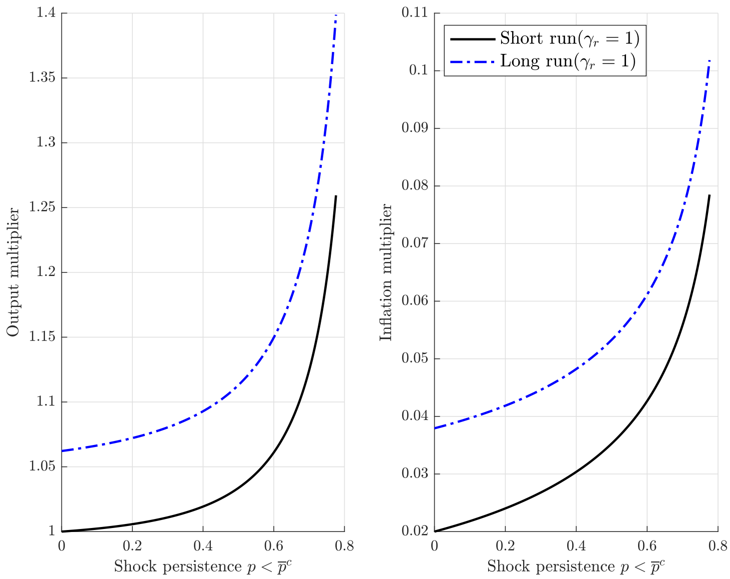


Figure 13: Long-run spending multipliers at ZLB




# L Euler Equation and Phillips Curve with Bounded Rationality

The Euler equation and Phillips Curve in normal times:

$$y_S = -\frac{1}{\sigma(1-\alpha_{EE}p)}(\phi_\pi - p)\pi_S + g_S$$

$$\pi_S = \kappa \frac{\gamma_y}{1-\beta p \alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)} y_S + \kappa \frac{\gamma_g}{1-\beta p \alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)} g_S.$$

The Euler equation and Phillips Curve can be elaborated at the ZLB:

$$y_S = -\frac{1}{\sigma(1-p\alpha_{EE})}[\log(\beta) - p\pi_S] + g_S$$

$$\pi_S = \frac{\kappa\gamma_y y_S + \kappa\gamma_r \log(\beta)}{1-\beta p\alpha_{PC} + \kappa\gamma_r p} + \kappa\frac{\gamma_g}{1-\beta p \alpha_{PC} + \kappa\gamma_r p} g_S.$$

# M The government spending effects with bounded rationality in normal times

I reproduce the multipliers here. For simplicity, we define $\alpha_{EE} = \bar{m}(1-s_g)$ and $\alpha_{PC}(\bar{m}) = \bar{m}[\varphi + \frac{1-\beta\varphi}{1-\beta\varphi\bar{m}}(1-\varphi)]$.

$$\begin{aligned}
\mathscr{M}^{O,BR}_{S,N} = \frac{\partial y_S}{\partial g_S} &= \frac{\frac{\sigma(1-p\alpha_{EE})}{\phi_\pi - p} - \frac{\kappa\gamma_g}{1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi - p)}}{\frac{\kappa\gamma_y}{1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi - p)} + \frac{\sigma(1-p\alpha_{EE})}{\phi_\pi - p}} \\
&= \frac{\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)] - \kappa\gamma_g(\phi_\pi - p)}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)]}
\end{aligned}$$

$$\begin{aligned}
\mathscr{M}^{I,BR}_{S,N} = \frac{\partial \pi_S}{\partial g_S} &= \frac{1+\frac{\gamma_g}{\gamma_y}}{\frac{1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi - p)}{\kappa\gamma_y} + \frac{1}{\sigma(1-p\alpha_{EE})}(\phi_\pi - p)} \\
&= \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p\alpha_{EE})}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)]}.
\end{aligned}$$

where $\alpha_{EE}$ and $\alpha_{PC}$ are increasing in the cognitive discounting parameter $\bar{m}$. One can differentiate output gap multiplier with regard to $\bar{m}$ and after some arrangements we have:

$$\frac{\kappa(\gamma_y + \gamma_g)(\phi_\pi - p)f'_N(\bar{m})}{\mathscr{D}^2_{BN}} > 0,$$

where $\mathscr{D}_{BN} = \kappa\gamma_y(\phi_\pi - p) + \sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)]$ and $f'_N(\bar{m})$ is the derivative of $\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)]$ with regard to $\bar{m}$ which is positive.

One can differentiate inflation multiplier with regard to $\bar{m}$ and after some arrangements we have:

$$\frac{-p\alpha'_{EE}\mathscr{D}_{BN} - f'_N(\bar{m})(1-p\alpha_{EE})}{\mathscr{D}^2_{BN}} < 0,$$

where $\mathscr{D}_{BN} = \kappa\gamma_y(\phi_\pi - p) + \sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)]$ and $f'_N(\bar{m})$ is the derivative of $\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)]$ with regard to $\bar{m}$ which is positive.

Since the strength of the real cost channel $\gamma_r$ is independent of the new ingredient that is bounded rationality. See appendix C, the output gap multiplier is decreasing in the strength of the real cost channel $\gamma_r$ and the inflation multiplier is increasing in the strength of the real cost channel $\gamma_r$. One can use this to prove the main text.

# N Derivatives of Output Gap Multiplier w.r.t. $p$ with Bounded Rationality

One can differentiate the output multiplier w.r.t $p$ and the numerator is[30]:

$$\begin{aligned}&[f'(p)+\kappa\gamma_g][\kappa\gamma_y(\phi_\pi-p)+f(p)]-[f(p)-\kappa\gamma_g(\phi_\pi-p)][-\kappa\gamma_y+f'(p)]\\&=\kappa(\gamma_y+\gamma_g)[f'(p)(\phi_\pi-p)+f(p)],\end{aligned}$$

where $f(p)=\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi-p)]$ and $f'(p)=\sigma(1-p\alpha_{EE})(-\beta\alpha_{PC}+\kappa\gamma_r)-\sigma\alpha_{EE}[1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi-p)]$. One can show:

$$-f'(p)p=-\sigma(1-p\alpha_{EE})(-\beta p\alpha_{PC}+\kappa\gamma_r p)+\sigma\alpha_{EE}p[1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi-p)].$$

In normal times:

$$\begin{aligned}&f'(p)(\phi_\pi-p)+f(p)\\&=f'(p)\phi_\pi+\sigma(1-\kappa\gamma_r\phi_\pi)-\sigma\alpha_{EE}p^2(\beta\alpha_{PC}-\kappa\gamma_r).\end{aligned}$$

When $\bar{m}=1$, one can show that these terms are below zero with ease. When $\bar{m}=0$, one can have the expression below:

$$\sigma\kappa\gamma_r\phi_\pi+\sigma(1-\kappa\gamma_r\phi_\pi)>0.$$

In this case, it seems that there is a threshold value $\underline{\bar{m}}$ across which the multiplier effects switch from decreasing in $p$ to increasing in $p$ or vice versa. One can have this value $\underline{\bar{m}}$ by making the below equation zero:

$$\begin{aligned}&f'(p)\phi_\pi+\sigma(1-\kappa\gamma_r\phi_\pi)-\sigma\alpha_{EE}p^2(\beta\alpha_{PC}-\kappa\gamma_r)=0\\\Longleftrightarrow&\{\sigma(1-p\alpha_{EE})(-\beta\alpha_{PC}+\kappa\gamma_r)-\sigma\alpha_{EE}[1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi-p)]\}\phi_\pi+\sigma(1-\kappa\gamma_r\phi_\pi)-\sigma\alpha_{EE}p^2(\beta\alpha_{PC}-\kappa\gamma_r)=0.\end{aligned}$$

[30] For simplicity, I only compare the numerator of the derivatives to check the monotonicity.

At the ZLB, one can show the derivatives can be reduced below:

$$
\begin{aligned}
&f'(p)(-p)+f(p)\\
&=\sigma-\sigma\alpha_{EE}p^2(\beta\alpha_{PC}-\kappa\gamma_r)>0.
\end{aligned}
$$

In this case, the output gap multiplier is increasing in $p$.

# O The government spending effects with bounded rationality at the ZLB

The spending multipliers at the ZLB are shown below. For simplicity, we define $\alpha_{EE}=\bar{m}(1-s_g)$ and $\alpha_{PC}(\bar{m})=\bar{m}[\varphi+\frac{1-\beta\varphi}{1-\beta\varphi\bar{m}}(1-\varphi)]$.

$$
\begin{aligned}
\mathcal{M}_{S,Z}^{O,BR}=\frac{\partial y_S}{\partial g_S}&=\frac{\frac{\sigma(1-p\alpha_{EE})}{-p}-\frac{\kappa\gamma_g}{1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)}}{\frac{\kappa\gamma_y}{1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)}+\frac{\sigma(1-p\alpha_{EE})}{-p}}\\
&=\frac{\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)]-\kappa\gamma_g(-p)}{\kappa\gamma_y(-p)+\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)]}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_{S,Z}^{I,BR}=\frac{\partial \pi_S}{\partial g_S}&=\frac{1+\frac{\gamma_g}{\gamma_y}}{\frac{1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)}{\kappa\gamma_y}+\frac{1}{\sigma(1-p\alpha_{EE})}(-p)}\\
&=\frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p\alpha_{EE})}{\kappa\gamma_y(-p)+\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)]}.
\end{aligned}
$$

One can differentiate output gap multiplier with regard to $\bar{m}$ and after some arrangements we have:

$$
\frac{\kappa(\gamma_y+\gamma_g)(-p)f_Z'(\bar{m})}{\mathcal{D}_{BZ}^2}<0,
$$

where $\mathcal{D}_{BZ}=\kappa\gamma_y(-p)+\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)]$ and $f_Z'(\bar{m})$ is the derivative of $\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)]$ with regard to $\bar{m}$ which is positive.

One can differentiate inflation multiplier with regard to $\bar{m}$ and after some arrangements we have:

$$
\frac{-p\alpha_{EE}'\mathcal{D}_{BZ}-f_Z'(\bar{m})(1-p\alpha_{EE})}{\mathcal{D}_{BN}^2}<0,
$$

where $\mathscr{D}_{BZ} = \kappa\gamma_y(-p) + \sigma(1 - p\alpha_{EE})[1 - \beta p\alpha_{PC} - \kappa\gamma_r(-p)]$ and $f'_Z(\bar{m})$ is the derivative of $\sigma(1 - p\alpha_{EE})[1 - \beta p\alpha_{PC} - \kappa\gamma_r(-p)]$ with regard to $\bar{m}$ which is positive.

Since the strength of the real cost channel $\gamma_r$ is independent of the new ingredient that is bounded rationality. See appendix D, the output gap and inflation multipliers are decreasing in the strength of the real cost channel $\gamma_r$. One can use this to prove the main text.

# P Figures with Bounded Rationality w.r.t. $p$

Figure 14: Spending multipliers with bounded rationality in normal times

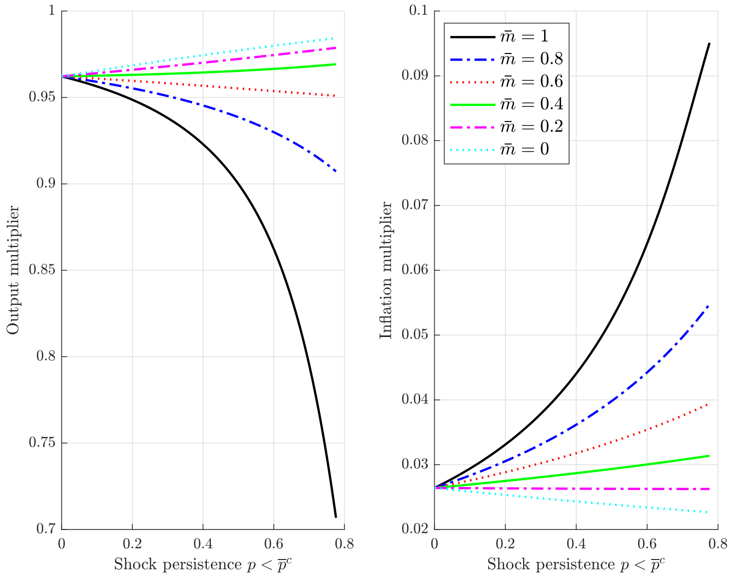



Figure 15: Spending multipliers with bounded rationality at ZLB