Avoiding Expectations-driven Liquidity Traps*

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Abstract

The linearized standard New Keynesian (NK) models may have equilibrium multiplicity if the central bank follows a Taylor (1993)-type rule with a lower bound. The shift in confidence can cause sufficient deflationary pressures to trigger the expectations-driven liquidity traps (LT) without any fundamental shocks. Using a standard sticky-price NK model, I show that the real cost channel can reduce the occurrence of expectations-driven LT by altering the slope of the New Keynesian Phillips Curve at the zero lower bound.

Keywords: Real Cost channel, Liquidity Traps, New Keynesian Model, Sunspots, Condence-driven ZLB

JEL Codes: E12, E61

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1 Introduction

In the standard New Keynesian (NK) models with a lower bound, multiple equilibria can appear as in Benhabib et al. (2001) and Ascari & Mavroeidis (2022). As a result, sunspots can trigger the expectations-driven liquidity traps in the absence of any fundamental shocks (see e.g. Mertens & Ravn (2014), Aruoba et al. (2018), Bilbiie (2019) and Cuba-Borda & Singh (2020)). However, recently, a new fact about expectations has been documented using survey data: in the US, European, and Japan, households do not have deflation expectations in an environment with persistent high deflation as in Gorodnichenko & Sergeyev (2021). In this case, even during recessions, few households anticipate deflation, and the possibility of expectation-driven liquidity traps can be attenuated.[1] Similarly, Mertens & Williams (2021) use US financial markets data and find no evidence in favor of the (sunspot) liquidity trap equilibrium. In this paper, I use a standard New Keynesian model with the real cost channel where the expected real interest rate appears in the marginal cost of the Phillips Curve to get rid of the occurrence of expectations-driven liquidity traps. This model result can provide new theoretical insights for rationalizing these new empirical facts.

As in Beaudry et al. (2022), it is assumed that firms need to borrow for production. As a result, the real interest rate will influence borrowing costs and further the marginal cost in the Phillips Curve. This is called the real cost channel.[2] In addition, the existence of the cost channel has empirical support in the literature (see e.g. Ravenna & Walsh (2006), Gilchrist & Zakrajšek (2015) and Abo-Zaid (2022)).

I study the expectations-driven liquidity traps in the canonical NK model, where inflation and output are jointly determined and are affected by expectations of future output gap and inflation. I can solve the models in closed form by using a two-state Markov process as in Eggertsson & Woodford (2003), and Eggertsson (2011). In addition, the model equilibrium can be depicted in a (π_S, y_S) diagram. The *slopes* of Euler/Phillips Curves are crucial: the second sunspot equilibrium (the expectations-driven liquidity trap) appears in the standard model when the slope of the Phillips Curve is lower than its Euler counterpart at the Zero Lower Bound (ZLB) episode. However, I show that the real cost channel can alter the slope of the Phillips Curve at the ZLB to make it higher than its Euler counterpart and thus rule out expectations-driven traps equilibrium. This arises because the real cost channel at the ZLB can *counteract* the short-run deflation, implying actual inflation in equilibrium.

There are a series of papers using the monetary/fiscal policy to get rid of expectations-driven liquidity traps (Schmidt (2016), and Nakata & Schmidt (2019)). For example,

[1] People would *expect* deflation for no reason, and it can become a self-fulfilling prophecy. The shift in the expectations can cause sufficient deflationary pressures to trigger the expectations-driven (or sunspot) traps without any fundamental shocks.

[2] The important difference between the model with a cost channel and the standard model is that the marginal cost in the former one is a function of both output gap and interest rate, while the latter one is only a function of the output gap.

Nie & Roulleau-Pasdeloup (2022) show that the Forward Guidance policy can get rid of a sunspot ZLB. In addition, Gabaix (2020) proves that the expectations-driven ZLB state can disappear in the NK model with bounded rationality. To the best of my knowledge, no concurrent work shows that the cost channel can work as a solution to get the economy out of the occurrence of expectations-driven traps.

The rest of this paper is organized as follows. Section 2 presents the model with the real cost channel. Section 3 shows the model equilibrium. Section 4 considers a stochastic set-up and assumes households' confidence is captured by a sunspot shock which obeys a parsimonious two-stage Markov structure. I show that the real cost channel can reduce the occurrence of expectations-driven LT by altering the slope of the NK Phillips Curve at the zero lower bound. I conclude in Section 5.

2 The Model with Real Cost Channel

This section aims to explain the role of the real cost channel in normal times and liquidity traps.[3] I use a standard NK model linearized around its (deterministic) intended steady state with zero inflation.[4]

Definition 1. *The semi-linearized Phillips Curve with the real cost channel which represents the aggregate-supply (AS) side of the economy is presented below:*

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \left[\gamma_y y_t + \gamma_r (R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1}) \right]. \tag{1}$$

where π_t is inflation, y_t is the output gap, $\beta < 1$ is the discount rate, κ is the elasticity of inflation with regard to marginal cost, R_t is the nominal interest rate in level. γ_y and γ_r are the elasticity of marginal cost with regard to the output gap and the interest rate, respectively.[5]

Eq. (1) is employed in this paper where the expected real interest rate emerges as in Beaudry et al. (2022) and Nie (2022). This real cost channel highlights the additional expected disinflation feedback denoted by $-\mathbb{E}_t \pi_{t+1}$ in liquidity traps.

[3] Normal times is the state where the economy is outside of liquidity traps and the nominal interest rate is flexible to adjust by the Central Bank. Liquidity traps means that the nominal interest rate is stuck in the ZLB.

[4] The unintended steady state is a state with the ZLB binding as in Nie & Roulleau-Pasdeloup (2022). In this section, I only show the linearized equilibrium condition and all lower case format variables are the log deviations from the steady state *i.e.* $x_t = \log(X_t) - \log(X)$. Refer to Appendix A for model details.

[5] This Phillips Curve with the real cost channel can collapse to the standard one below if $\gamma_r = 0$:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \gamma_y y_t. \tag{2}$$

Definition 2. *Monetary policy is assumed to follow Taylor (1993)-type rules with a lower bound:*[6]

$$R_t = \max\left[0; -\log(\beta) + \phi_\pi \pi_t\right], \tag{3}$$

where $\phi_\pi > 1$.

Note that the real cost channel can work as a cost-push shock endogenously in normal times if the Central Bank follows a simple Taylor rule as $R_t = \phi_\pi \pi_t - \log(\beta)$. While at the zero lower bound (ZLB), the nominal interest rate is zero (*i.e.* $R_t = 0$). The real cost channel still works with the expected disinflation feedback in the Phillips Curve. I use the NK model where the current inflation and output are jointly affected by expectations of future output and inflation. Therefore, the expected disinflation feedback in the real cost channel can rotate the Phillips Curve at the ZLB.

Definition 3. *The following expression represents the equilibrium conditions of the linearized Euler equation, which describes the aggregate-demand (AD) side of the economy:*[7]

$$y_t = \mathbb{E}_t y_{t+1} - \sigma_r \left[R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1} - \epsilon_t\right], \tag{4}$$

where σ_r is the elasticity of inter-temporal substitution, and ϵ_t is the demand shock.

3 Short-run Equilibrium

I use a parsimonious two-stage Markov structure with an absorbing state to solve the model in close form as in Eggertsson & Woodford (2003) and Eggertsson (2011).[8] The main idea is that the short-term economy is hit by the exogenous demand shock ϵ_S which persists with a probability p and recovers to the steady state with a probability $1-p$.[9] Since the Phillips Curve and the Euler equation in Eqs. (1) and (4) are both forward looking, one can write $\mathbb{E}_S y_{t+1} = p \cdot y_S$ and $\mathbb{E}_S \pi_{t+1} = p \cdot \pi_S$ for the expected output gap and expected inflation next period, respectively. In that way, I define the short run equilibrium with the Markov chain representation as below:

[6]In this paper, I mainly explore the slopes of AS/AD curves at the ZLB. The interest rate rule is not critical here since the nominal rate is fixed at zero in liquidity traps.

[7]Refer to Appendix A for model details.

[8]An absorbing state is a state that, once entered, cannot be left. And this state can be seen as the long-run steady state. See another specification in Armenter (2017) and Nakata & Schmidt (2019) by assuming no absorbing state in a two-state Markov structure.

[9]The transition matrix for the demand shock is:

$$\mathcal{P}_S = \begin{bmatrix} p & 1-p \\ 0 & 1 \end{bmatrix}.$$

Definition 4. *A short run equilibrium in this economy is a vector* $[y_S,\ \pi_S,\ R_S]$ *such that, for a given* ϵ_S

$$\pi_S = \beta \mathbb{E}_S \pi_{t+1} + \kappa \left[\gamma_y y_S + \gamma_r (R_S + \log(\beta) - \mathbb{E}_S \pi_{t+1})\right] \tag{5}$$

$$y_S = \mathbb{E}_S y_{t+1} - \sigma_r \left[R_S + \log(\beta) - \mathbb{E}_S \pi_{t+1} - \epsilon_S\right] \tag{6}$$

$$R_S = \max\left[0; -\log(\beta) + \phi_\pi \pi_S\right] \tag{7}$$

$$\mathbb{E}_S \pi_{t+1} = p \pi_S \tag{8}$$

$$\mathbb{E}_S y_{t+1} = p y_S \tag{9}$$

all hold.

Assumption 1. *Assume that the NK Phillips Curve with the real cost channel is upward sloping in a* (π_S, y_S) *graph such that*

$$p < \frac{1 - \kappa \gamma_r \phi_\pi}{\beta - \kappa \gamma_r} = \overline{p}^c.$$

In this paper, I assume the slope of the Phillips Curve is upward sloping in a (π_S, y_S) graph, which means $p < \overline{p}^c$—see Appendix B for details. Laubach & Williams (2003), Daly & Hobijn (2014) and Nie et al. (2022) assume a similar condition.

4 Sunspot Equilibria

4.1 Expectations-driven (or Sunspot) Traps in Standard NK Model

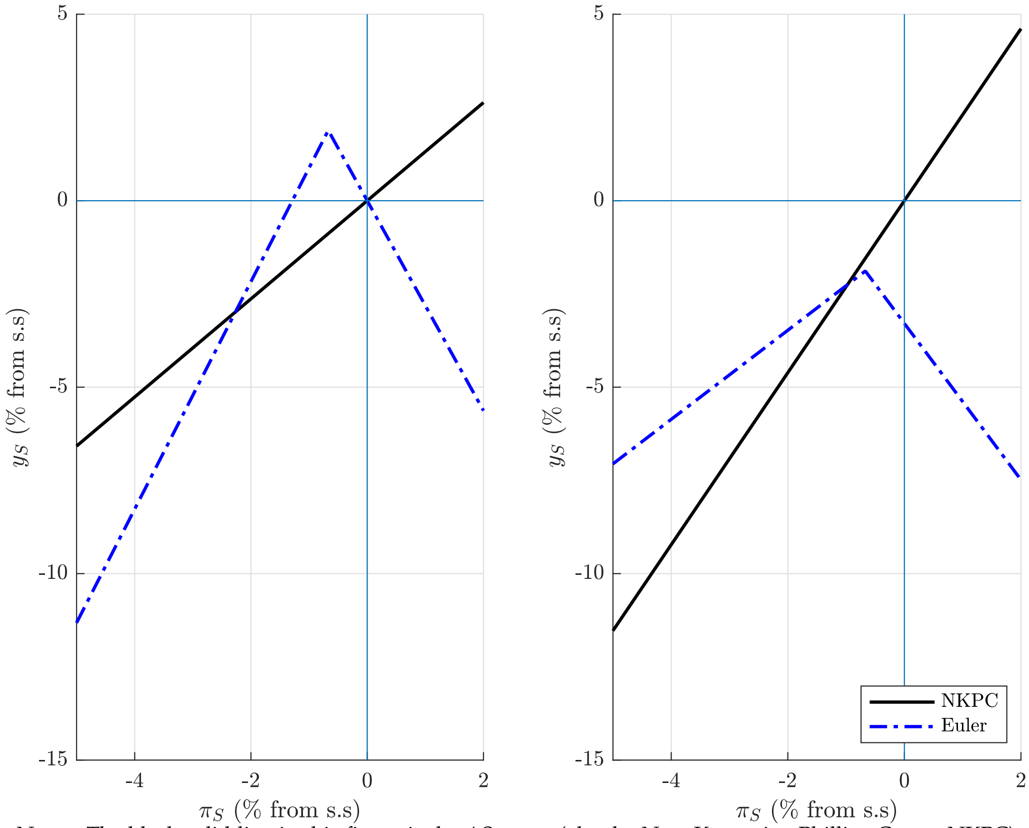
Following Nie & Roulleau-Pasdeloup (2022), I assume there exists a sunspot shock.[10] This shock is arbitrarily small and it is perfectly correlated with the demand shock with a persistence p. The expectations-driven traps means that the economy can feature actual deflation and be in liquidity traps with a arbitrarily small sunspot shock in a high persistent deflation environment (*i.e.* the sunspot shock persistence p is large enough)—see Nie & Roulleau-Pasdeloup (2022) for a discussion.

I plot the expectations-driven (or sunspot) liquidity traps (LT) and fundamental-driven LT in the AS/AD diagram as in Figure 1 and one can see that *the slopes of the AS/AD curves at the ZLB are crucial*. For the fundamental-driven LT case at the right panel, the slope of the AS curve at the ZLB is larger than that of the AD curve. The reverse holds for the expectations-driven liquidity traps on the left panel where the slope of the AS curve is less than the AD slope. As a consequence, the Euler and the NKPC can cross twice which gives rise to the sunspot ZLB. I first show the slopes of AS/AD curves explicitly in a (π_S, y_S) graph.[11]

[10] As also in Mertens & Ravn (2014), sunspots can be seen as exogenous shocks to households' confidence.

[11] These slopes can represent features that inflation and output are jointly determined and affected

Figure 1: Expectations-driven LT and fundamental-driven LT



Notes: The black solid line in this figure is the AS curve (aka the New Keynesian Phillips Curve, NKPC) while the blue dashed line is the AD curve (aka the Euler equation). The left panel presents the expectations-driven LT in a standard NK model with $p = \bar{p} + 0.1$ and the right panel shows fundamental-driven LT in the standard model by assuming $p = \bar{p} - 0.1$ with the demand shock $\epsilon_S = -0.015$. Other calibration parameters are shown in Appendix C.

Lemma 1. *In the standard NK model, the slope of AD/Euler curve in Eq.(6) at the ZLB is:*

$$\mathcal{S}_{EE} = \sigma_r \frac{p}{1-p}.$$

The slope of AS/NKPC curve in Eq.(2) at the ZLB is:

$$\mathcal{S}_{PC} = \frac{1-\beta p}{\kappa \gamma_y}.$$

Proof. See Appendix D. □

As in the seminal work of Bilbiie (2021), the expectation-driven LT can be detected by the probability p in a two-state Markov structure.[12] Based on Lemma 2, increasing p can generate a second crossing in the AS/AD curves by (i) increasing the Euler equation slope $\mathcal{S}_{EE}$ and (ii) reducing the NKPC slope $\mathcal{S}_{PC}$ simultaneously. In this case, there exists a threshold $\bar{p}$ such that the expectations-driven LT in the standard

by expectations of future output gap and inflation.

[12]Similar arguments can be found in Aruoba et al. (2018).

NK model emerges if $p > \bar{p}$.—See Appendix F.3.[13]

The expectations-driven (or sunspot) LT is shown on the left panel of Figure 1 and a second intersection of the AS and AD curves occurs. It indicates that if the sunspot shock persistence is sufficiently large, there could be a self-fulfilling prophecy and the economy will feature actual deflation without any fundamental shocks hitting the economy.[14] Similar to the results in Cuba-Borda & Singh (2020), there are two short-run equilibria on the left panel of Figure 1. One is the Targeted Steady State which means $y_S = \pi_S = 0$. Another one is the Expectations-driven ZLB, implying $y_S < 0$ and $\pi_S < 0$.[15] Therefore the second equilibrium with expectations-driven LT emerges and there is no stable equilibrium echoing the findings in Aruoba et al. (2018). As in the right panel of Figure 1, there exist fundamental-driven traps where the strong demand shock ϵ_S can cause sufficient deflation such that the ZLB binds, implying $y_S < 0$ and $\pi_S < 0$.[16] For example, the US has been caught in the fundamental-driven LT during the global financial crisis (GFC) as in Aruoba et al. (2018).

4.2 Avoiding Expectations-driven Liquidity Traps

The NK model with the real cost channel can rotate *the NKPC at the ZLB* episode while the slope of Euler equation is unchanged. First, I describe the slope of the AS curve with the real cost channel explicitly in a (π_S, y_S) graph.

Lemma 2. *Based on Definition 4, the slope of AS/NKPC curve with the real cost channel in Eq.(5) at the ZLB is:*

$$\mathcal{S}^c_{PC} = \frac{1 - \beta p + \kappa \gamma_r p}{\kappa \gamma_y}.$$

[13]One can use Lemma 1 to show the expression of the threshold $\bar{p}$ below:

$$\bar{p} = \frac{(\beta + 1 + \sigma_r \kappa \gamma_y) - \sqrt{(1 + \beta + \sigma_r \kappa \gamma_y)^2 - 4\beta}}{2\beta}.$$

[14]In other words, if households do expect deflation for no reason, this can cause sufficient deflationary pressures to trigger the expectations-driven LT with a self-fulfilling state of low confidence.

[15]The expression of the expectations-driven traps solution:

$$y_S = \frac{(1 - \beta p)\sigma_r}{(1 - p)(1 - \beta p) - \sigma_r p \kappa}(-\log(\beta))$$
$$\pi_S = \frac{\kappa \gamma_y}{(1 - p)(1 - \beta p) - \sigma_r p \kappa}(-\log(\beta)),$$

where $(1 - p)(1 - \beta p) - \sigma_r p \kappa < 0$.

[16]The expression of the fundamental-driven traps solution:

$$y_S = \frac{(1 - \beta p)\sigma_r}{(1 - p)(1 - \beta p) - \sigma_r p \kappa}(\epsilon_S - \log(\beta))$$
$$\pi_S = \frac{\kappa \gamma_y}{(1 - p)(1 - \beta p) - \sigma_r p \kappa}(\epsilon_S - \log(\beta)),$$

where $(1 - p)(1 - \beta p) - \sigma_r p \kappa > 0$.

Proof. See Appendix E. □

If the AS curve is rotated with the real cost channel and the slope $\mathcal{S}^c_{PC}$ is higher than S_{EE} in the (π_S, y_S) graph, the second intersection can disappear, implying that the expectations-driven traps as in Cuba-Borda & Singh (2020) is ruled out. We describe the main results in the following Proposition 1.

Proposition 1. *At the ZLB, assume that the AS curve is upward sloping ($p < \overline{p}^c$) and γ_y meets the following restriction:*[17]

$$\gamma_y < \frac{(\beta - \kappa\gamma_r - 1 + \kappa\gamma_r\phi_\pi)(\beta\gamma_r\phi_\pi - \kappa\gamma_r^2\phi_\pi)}{\sigma_r(1 - \kappa\gamma_r\phi_\pi)}.$$

Then, in a (π_S, y_S) graph, we have,

1. *The slope of the AS Curve is increasing in the strength of cost channel γ_r;*
2. *The slope of the AS Curve is larger than that of the AD curve;*
3. *The real cost channel can rule out the expectations-driven LT.*

Proof. See Appendix F. □

At the ZLB, the slope of the AS Curve shown in Lemma 2 increases in γ_r.[18] With a condition that $\gamma_y < \frac{(\beta-\kappa\gamma_r-1+\kappa\gamma_r\phi_\pi)(\beta\gamma_r\phi_\pi-\kappa\gamma_r^2\phi_\pi)}{\sigma_r(1-\kappa\gamma_r\phi_\pi)}$, the slope of the AS curve at the ZLB is *always* larger than that of the AD curve in a (π_S, y_S) graph.[19] As a consequence, no second intersection exists in the AS/AD curves and therefore the sunspot equilibrium is ruled out.

The intuition that the real cost channel can rule out sunspot is that the introduced disinflation term at the ZLB in Eq. (5) can *counteract* the short-run deflation due to rational expectations and sticky prices. In equilibrium, this counteraction effect indicates actual short-term inflation, and thus, the inflation behavior in the short run can move less for the given output gap. This explains why the NKPC slope with the real cost channel is steeper at the ZLB in a (π_S, y_S) diagram.

On the right panel of Figure 2, it appears that there is no sunspot equilibrium: the absence of second intersection in the AS/AD curves due to the steeper AS curve at the ZLB episode. This result can provide a theoretical justification for the fact that households do not expect deflation in an environment with high persistent deflation, according to European and Japanese surveys in Gorodnichenko & Sergeyev (2021).[20] In this

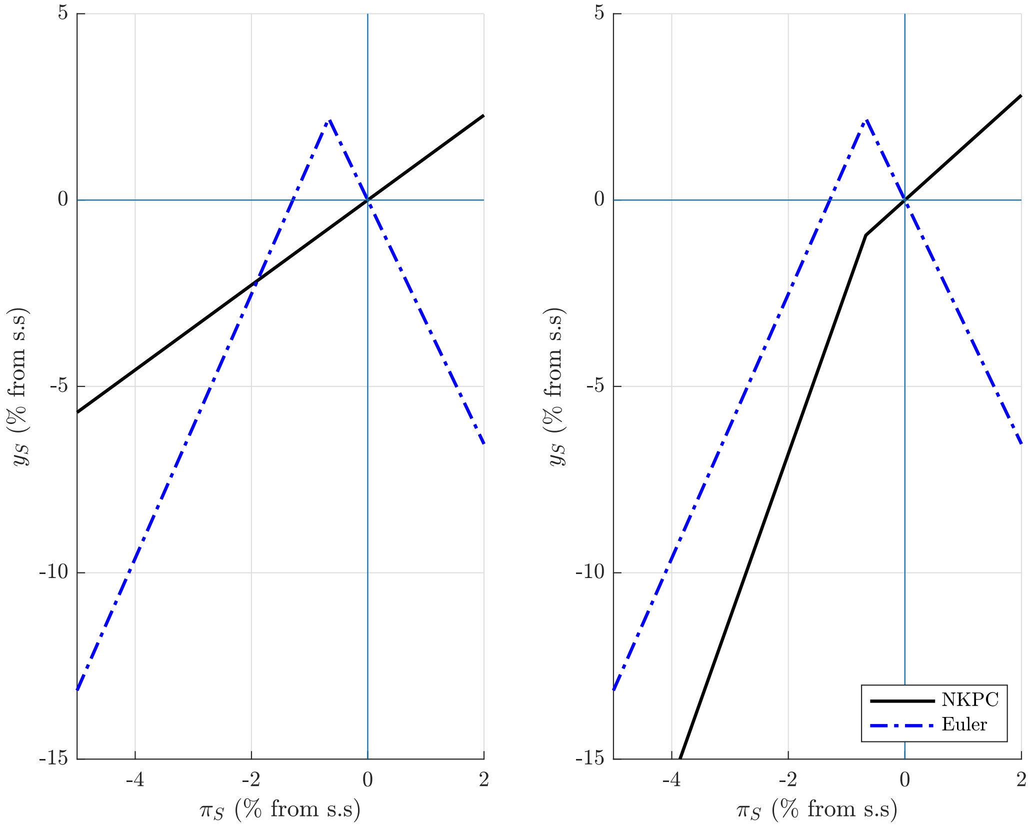
[17] The seminal Beaudry et al. (2022) paper empirically shows that γ_y in the real cost channel is robustly small (non-significantly) and γ_r is significantly positive and is much larger than γ_y. This empirical result motives the restriction for γ_y in this paper.

[18] γ_r represents the elasticity of marginal cost with regard to the interest rate and it can be seen as the strength of the real cost channel.

[19] Note that if the NKPC is upward sloping in a (π_S, y_S) graph, the second intersection can not arise.

[20] Mertens & Williams (2021) use US financial markets data to show no evidence in favor of the (sunspot) liquidity traps.

Figure 2: No expectations-driven LT with the real cost channel



Notes: The black solid line in this figure is the AS curve while the blue dashed line is the AD curve. The left panel presents the expectations-driven LT in a standard NK model without the real cost channel and the right panel shows no expectations-driven LT with the real cost channel, following the calibration method as in Appendix C.

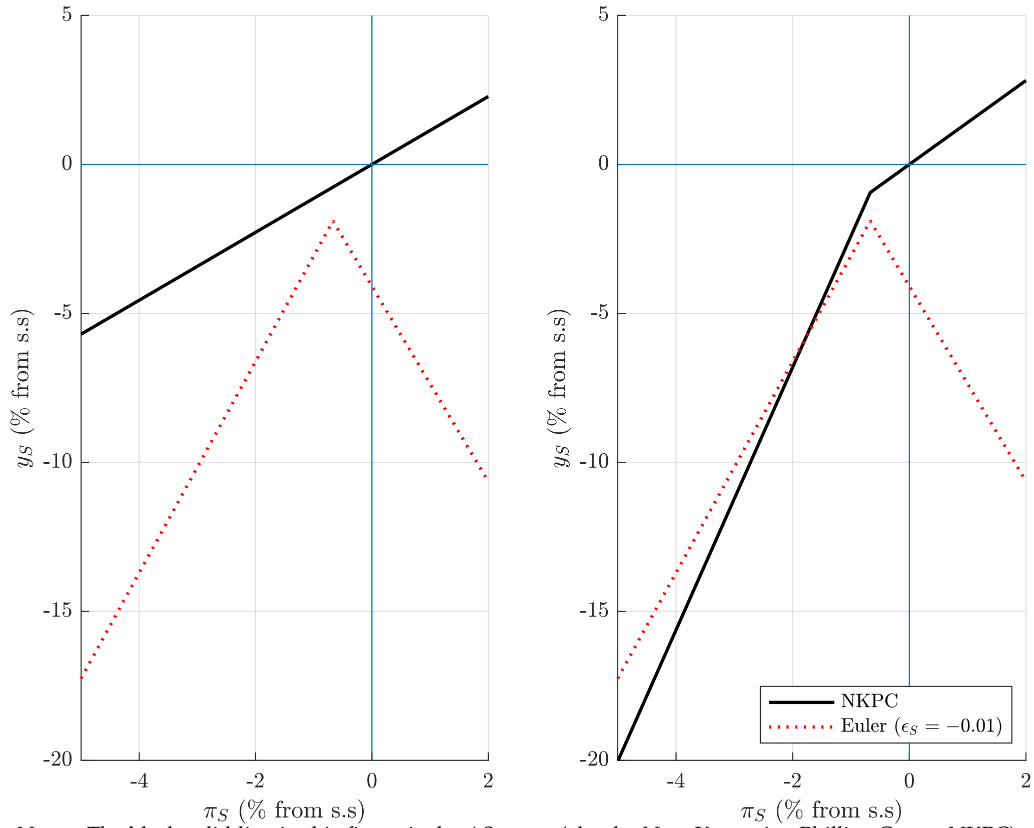
case, the locally flat Phillips Curve in a (y_S, π_S) graph can rule out the expectations-driven LT to ensure one unique equilibrium with $\pi_S = 0$ and this flat pattern can only be observed at the ZLB episode.[21] Interestingly, this locally flat Phillips Curve in this model is consistent with recent empirical evidence. For example, Hazell et al. (2022) use cross-sectional data and estimate a flat Phillips curve during the Great Recession.

4.3 Equilibrium Uniqueness/Existence

Moreover, as in Ascari & Mavroeidis (2022), models with ZLB constraints can have no solution: if there exist supply/demand shocks that make the AD curve shift too much below the AS curve, there can be no equilibrium in the expectations-driven LT case.—See examples in Fig 3. However, this issue can not arise with the real cost channel and AS/AD curves can always have a unique equilibrium with shocks.—See Appendix G. Overall, this real cost channel is theoretically appealing since it helps ensure equilibrium uniqueness/existence with supply/demand shocks.

[21] Note that in Figure 2, the AS/AD curves are shown in a (π_S, y_S) graph for an easier comparison while the standard Phillips Curve is in a (y_S, π_S) graph. In other words, the Phillips Curve is flat in a (y_S, π_S) graph which means the AS curve is steep in a (π_S, y_S) graph as in Figure 2.

Figure 3: Equilibrium uniqueness/existence with the real cost channel with demand shock



Notes: The black solid line in this figure is the AS curve (aka the New Keynesian Phillips Curve, NKPC) while the red dotted line is the AD curve with a demand shock ($\epsilon_S = -0.01$). The left panel presents the no equilibrium in a standard NK model without the real cost channel and the right panel shows a equilibrium with the real cost channel, following the calibration method as in Appendix C.

5 Conclusions

In the presence of the ZLB, the shift in confidence can cause sufficient deflationary pressures to trigger the expectations-driven LT without any fundamental shocks. There is a recent survey fact that households do not expect deflation in an environment with high persistent deflation, according to European and Japanese surveys as in Gorodnichenko & Sergeyev (2021). In this case, even during recessions, few households anticipate deflation, and the possibility of expectation-driven liquidity traps can be attenuated. In this paper, I develop a tractable New Keynesian via the real cost channel to provide theoretic explanations for rationalizing these new empirical facts. I have shown that the real cost channel *per se* can reduce the occurrence of the expectations-driven liquidity traps by rotating the NKPC. This arises because the real cost channel at the ZLB can counteract the short-run deflation caused by the drop in confidence, implying actual inflation in equilibrium. In that way, deflationary expectations can not be an equilibrium outcome.

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Online Appendix

A The Model Setup

Time is discrete and there is no uncertainty and government spending.

A.1 Aggregate Demand Side

The representative household has the below preferences:

$$\mathcal{U}(C_t, N_t) = \frac{C_t^{1-\sigma_c}}{1-\sigma_c} - \chi \frac{N_t^{1+\eta}}{1+\eta}, \quad \chi, \eta > 0$$

where households work N_t hours, consume amount C_t, and trade government bonds B_t.
The budget constraint is,

$$C_t + \frac{B_t}{P_t} = W_t N_t + \mathcal{D}_t - \mathcal{T}_t + \exp(\Im_{t-1}) \frac{1+R_{t-1}}{P_t} B_{t-1}.$$

where $\Im_t$ is a "risk premium" shock.
The optimal aggregate (individual) labor price is written as:

$$W_t = \frac{N_t^\eta \chi}{(C_t)^{-\sigma_c}},$$

I can obtain the Euler equation with the first order condition (FOC) of the maximization program:

$$(C_t)^{-\sigma_c} = \beta \exp(\Im_t) \mathbb{E}_t \left\{ (C_{t+1})^{-\sigma_c} 1 + R_t 1 + \Pi_{t+1} \right\}.$$

The linearized equilibrium Euler equation by approximating around the steady state can read. That is, all lower case format variables are the log deviations from steady state ($x_t = \log(X_t) - \log(X)$):

$$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\sigma_c} \left[R_t + \log(\beta) - \mathbb{E}_t \tau_{t+1}^c - \mathbb{E}_t \pi_{t+1} - \epsilon_t \right].$$

where $\epsilon_t \equiv -\Im_t$ is the natural rate shock (demand shock) and R_t is the nominal interest rate in level. The following resource constraint is placed in this economy:

$$y_t = c_t,$$

Furthermore the Euler equation is expressed as:

$$y_t = \mathbb{E}_t y_{t+1} - \sigma_r \left[R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1} - \epsilon_t \right],$$

where $\sigma_r \equiv \frac{1}{\sigma_c}$.

A.2 Aggregate Supply Side

I follow Beaudry et al. (2022) to incorporate the real interest rate into the marginal cost which allows for the arbitrary elasticity of marginal rate w.r.t wage and interest rate. I assume that the unit final good price is P_t and the basic input firm needs to borrow D^B_{t+1} first with the nominal interest rate r_t to pay for M_t and then produces further to sell the production. Basic input Q_t is produced by a representative competitive firm with labor L_t and the final good M_t:

$$Q_t = \min(aN_t, bM_t).$$

By using the symmetry condition, the budget constraint for the firm is shown below,

$$D^B_{t+1} + P^b_t Y_t = P_t W_t N_t + (1 + r_{t-1})D^B_t + P_t M_t,$$

where P^b_t is the price of basic input and $D^B_{t+1} = P_t M_t$. In this case, the basic input representative firm has the profits:

$$\Pi_t = P^b_t Y_t - P_t W_t N_t - (1 + r_{t-1})P_{t-1}M_{t-1}.$$

By assuming the sum of real profit owns the common discount factor β, I can obtain the (real) marginal cost of the basic input firm:

$$MC_t = \frac{W_t}{a} + \frac{\beta}{b}\mathbb{E}\left[\frac{1 + r_t}{1 + \pi_{t+1}}\right]$$

By log condition, I have the semi-linearized equilibrium

$$mc_t = \gamma_w(w_t) + \gamma_r(R_t + \log(\beta) - \mathbb{E}\pi_{t+1}),$$

where R_t is the nominal interest rate in level, $\gamma_w = \frac{\frac{W}{aP}}{\frac{W}{aP}+\frac{\beta}{b}\frac{1+r}{1+\pi}}$ and $\gamma_r = \frac{\frac{\beta}{b}\frac{1+r}{1+\pi}}{\frac{W}{aP}+\frac{\beta}{b}\frac{1+r}{1+\pi}}$. On the other hand, the optimal labor supply reads

$$\frac{v'(N_t)}{u'(C_t)} = W_t,$$

where $v(N_t) = \chi\frac{N_t^{1+\eta}}{1+\eta}$ and $u(C_t) = \frac{C_t^{1-\sigma_c}}{1-\sigma_c}$. By using the condition $Y_t = C_t = aN_t$, the semi-linearized marginal cost can be rewritten as

$$mc_t = \gamma_y y_t + \gamma_r(R_t + \log(\beta) - \mathbb{E}_t\pi_{t+1}),$$

where $\gamma_y = \gamma_w\left(\frac{Nv''(N)}{v'(N)} - \frac{Cu''(C)}{u'(C)}\right)$. The rest of the model is standard and the New Keynesian Phillips curve:

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa\, mc_t.$$

Therefore the Phillips Curve is shown below:

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa\left[\gamma_y y_t + \gamma_r(R_t + \log(\beta) - \mathbb{E}_t\pi_{t+1})\right].$$

B Upward Sloping Assumption

According to Definition 4, in normal times, I can reproduce the solutions for y_S and π_S as follows:

$$y_S = \frac{\sigma_r(1-\beta p+\kappa\gamma_r p-\kappa\gamma_r\phi_\pi)}{(1-p)(1-\beta p+\kappa\gamma_r p-\kappa\gamma_r\phi_\pi)+\sigma_r\kappa\gamma_y(\phi_\pi-p)}\epsilon_S$$
$$\pi_S = \frac{\sigma_r\kappa\gamma_y}{(1-p)(1-\beta p+\kappa\gamma_r p-\kappa\gamma_r\phi_\pi)+\sigma_r\kappa\gamma_y(\phi_\pi-p)}\epsilon_S.$$

If the Phillips Curve is upward sloping in normal times, which means the slope of Phillips Curve is positive:

$$1-\beta p+\kappa\gamma_r p-\kappa\gamma_r\phi_\pi > 0$$
$$\Leftrightarrow p < \frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r},$$

where the second line using the assumption $\kappa\gamma_r < \beta$ as in Beaudry et al. (2022) and Nie (2022). In this case, there is a threshold $\overline{p}^c = \frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r}$.

C Calibration Parameters

Table 1: The calibrated parameter values

Discount factor	$\beta = 0.99$
Preference parameter	$\eta = 1$
Preference parameter	$\sigma_r = 1$
Elasticity of inflation w.r.t marginal cost	$\kappa = 0.2$
Elasticity of marginal cost w.r.t output	$\gamma_y = 0.4$
Elasticity of marginal cost w.r.t interest rate	$\gamma_r = 0.8$
Taylor rule	$\phi_\pi = 1.5$
Shock persistence	$p = \frac{\overline{p}^c+\overline{p}}{2}$

Notes: I follow Beaudry et al. (2022) to set the value for γ_r and γ_y. We can obtain qualitatively identical results with different sets of γ_r & γ_y and these results can be obtained be request. I follow Mertens & Williams (2019) and Nie & Roulleau-Pasdeloup (2022) to use a standard calibrated method for other parameters. $\bar{p}$ is the threshold such that there exists the expectations-driven LT in the standard model without the real cost channel. $\overline{p}^c$ is the threshold such that the AS curve is upward sloping in the model with the real cost channel.

D Proofs of Lemma 1

The Euler equation in standard NK model:

$$y_t = \mathbb{E}_t y_{t+1} - \sigma[R_t + \log(\beta) - \mathbb{E}_t\pi_{t+1} - \epsilon_t]$$

The NKPC is below:

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa\gamma_y y_t$$

Using the simple two-state Markov Chain, we have $\mathbb{E}_S \pi_{t+1} = p\pi_S$ and $\mathbb{E}_S y_{t+1} = p y_S$. We can write the Euler equation at the ZLB below:

$$y_S = -\frac{\sigma_r p}{1-p}\pi_S + \sigma_r \frac{\epsilon_S - \log(\beta)}{1-p}.$$

One can yield the NKPC:

$$y_S = \frac{1-\beta p}{\kappa \gamma_y}\pi_S.$$

Thus, the slope of AD/Euler curve is:

$$\mathcal{S}_{EE} = \sigma_r \frac{p}{1-p}.$$

The slope of AS/NKPC curve is:

$$\mathcal{S}_{PC} = \frac{1-\beta p}{\kappa \gamma_y}.$$

E Proofs of Lemma 2

According to Definition 4, under a ZLB, the Phillips Curve is

$$y_S = \frac{1-\beta p + \kappa \gamma_r p}{\kappa \gamma_y}\pi_S - \frac{\gamma_r}{\gamma_y},$$

The Euler equation at the ZLB is:

$$y_S = -\frac{\sigma_r p}{1-p}\pi_S + \sigma_r \frac{\epsilon_S - \log(\beta)}{1-p}.$$

Thus, the slope of AD/Euler curve is:

$$\mathcal{S}_{EE} = \sigma_r \frac{p}{1-p}.$$

The slope of AS/NKPC curve is:

$$\mathcal{S}^c_{PC} = \frac{1-\beta p + \kappa \gamma_r p}{\kappa \gamma_y}.$$

F Proofs of Proposition 1

F.1 Part 1

According to Definition 4, under a ZLB, the Phillips Curve is

$$y_S = \frac{1-\beta p + \kappa \gamma_r p}{\kappa \gamma_y}\pi_S - \frac{\gamma_r}{\gamma_y},$$

where the slope is $\frac{1-\beta p+\kappa\gamma_r p}{\kappa\gamma_y}$. It is easy to check this slope is increasing in the elasticity of the marginal cost w.r.t the interest rate γ_r which can be seen as the strength of the real cost channel.

F.2 Part 2

If the flat Phillips Curve is upward sloping throughout time periods, which means that the slope of Phillips Curve is always positive:

$$\begin{aligned} 1-\beta p+\kappa\gamma_r p-\kappa\gamma_r\phi_\pi &> 0 \\ \Leftrightarrow p &< \frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r}. \end{aligned}$$

In this case, in normal times, it is easy to check that the only equilibrium is the target steady state (*i.e.* $y_S=\pi_S=0$) with no demand shock.
While assuming that the demand shock is large enough to trigger the fundamental-driven ZLB, I reproduce the following solutions for y_S and π_S:

$$\begin{aligned} y_S &= \frac{(1-\beta p+\kappa\gamma_r p)\sigma_r(\epsilon_S-\log(\beta))+\kappa\gamma_r\sigma_r p\log(\beta)}{(1-p)(1-\beta p+\kappa\gamma_r p)-\sigma_r p\kappa\gamma_y} \\ \pi_S &= \frac{\kappa\gamma_y\sigma_r(\epsilon_S-\log(\beta))}{(1-p)(1-\beta p+\kappa\gamma_r p)-\sigma_r p\kappa\gamma_y}+\frac{\kappa\gamma_y\kappa\gamma_r\sigma_r p\log(\beta)}{[(1-p)(1-\beta p+\kappa\gamma_r p)-\sigma_r p\kappa\gamma_y](1-\beta p+\kappa\gamma_r p)} \\ &\quad+\frac{\kappa\gamma_r\log(\beta)}{1-\beta p+\kappa\gamma_r p}. \end{aligned}$$

If there is no expectations-driven liquidity traps (LT) in the absence of demand shock, the requirement is below:

$$\begin{aligned} y_S &= \frac{(1-\beta p)\sigma_r(-\log(\beta))}{(1-p)(1-\beta p+\kappa\gamma_r p)-\sigma_r p\kappa\gamma_y} > 0 \\ \Leftrightarrow \mathcal{D}(p) &= (1-p)(1-\beta p+\kappa\gamma_r p)-\sigma_r p\kappa\gamma_y > 0 \end{aligned}$$

One can yield a condition for γ_y to secure $\mathcal{D}(p)>0$:

$$\begin{aligned} \mathcal{D}(p) &= (1-p)(1-\beta p+\kappa\gamma_r p)-\sigma_r p\kappa\gamma_y \\ &> \left(1-\frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r}\right)\left(1-\beta\frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r}+\kappa\gamma_r\frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r}\right)-\sigma_r\frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r}\kappa\gamma_y \\ &= (\beta-\kappa\gamma_r-1+\kappa\gamma_r\phi_\pi)[\beta\kappa\gamma_r\phi_\pi-\kappa\gamma_r+\kappa\gamma_r(1-\kappa\gamma_r\phi_\pi)]-\sigma_r(1-\kappa\gamma_r\phi_\pi)\kappa\gamma_y > 0 \\ \gamma_y &< \frac{(\beta-\kappa\gamma_r-1+\kappa\gamma_r\phi_\pi)(\beta\gamma_r\phi_\pi-\kappa\gamma_r^2\phi_\pi)}{\sigma_r(1-\kappa\gamma_r\phi_\pi)}, \end{aligned}$$

where the second line we assume $p=\bar{p}^c$ due to monotonicity.
At the ZLB episode, one can compare the slope of the AS/AD curves:

$$\frac{1-\beta p+\kappa\gamma_r p}{\kappa\gamma_y} > \sigma_r\frac{p}{1-p},$$

where we use the condition $\gamma_y < \frac{(\beta-\kappa\gamma_r-1+\kappa\gamma_r\phi_\pi)(\beta\gamma_r\phi_\pi-\kappa\gamma_r^2\phi_\pi)}{\sigma_r(1-\kappa\gamma_r\phi_\pi)}$. This means the slope of the AS curve is larger than the slope of the AD curve at the ZLB.

F.3 Part 3

Results in Part 2 can help prove that the real cost channel can rule out the expectations-driven LT. On the other hand, the standard textbook New Keynesian Phillips Curve without a cost channel can read:

$$\pi_t = \beta \mathbb{E}\pi_{t+1} + \kappa\gamma_y y_t.$$

In this case, the Phillips Curve can be re-written as

$$y_S = \frac{1-\beta p}{\kappa\gamma_y}\pi_s$$

Assuming that the demand shock is large enough to trigger a ZLB, I can get the following solutions for y_S and π_s:

$$\begin{aligned}
y_S &= \frac{(1-\beta p)\sigma_r}{(1-p)(1-\beta p) - \sigma_r p\kappa\gamma_y}(\epsilon_S - \log(\beta)) \\
\pi_S &= \frac{\kappa\gamma_y}{(1-p)(1-\beta p) - \sigma_r p\kappa\gamma_y}(\epsilon_S - \log(\beta))
\end{aligned}$$

If the Phillips Curve is upward sloping throughout time periods. If there is an absence of demand shock and the slope of AS curve is lower than AD curve ($(1-p)(1-\beta p) < \sigma_r p\kappa\gamma_y$), we can have the sunspot equilibrium featuring $\pi_S < 0, y_S < 0$: *i.e.* there exists a threshold $\overline{p}$:

$$\begin{aligned}
\overline{p} &= \frac{(\beta+1+\sigma_r\kappa\gamma_y) - \sqrt{(1+\beta+\sigma_r\kappa\gamma_y)^2 - 4\beta}}{2\beta} \\
&< \frac{(\beta+1+\sigma_r\kappa\gamma_y) - (-\beta+1+\sigma_r\kappa\gamma_y)}{2\beta} \\
&= 1
\end{aligned}$$

where there is $\bar{p} \in (0,1)$ to trigger the expectations-driven LT to make $y_S < 0$ in the absence of demand shock. That being said, there is a sunspot equilibrium if $p > \bar{p}$. Note that if the the demand shock is very large, it can shift AD curve down so much that there is no intersection in the AS and AD curves which means no equilibrium in this economy.

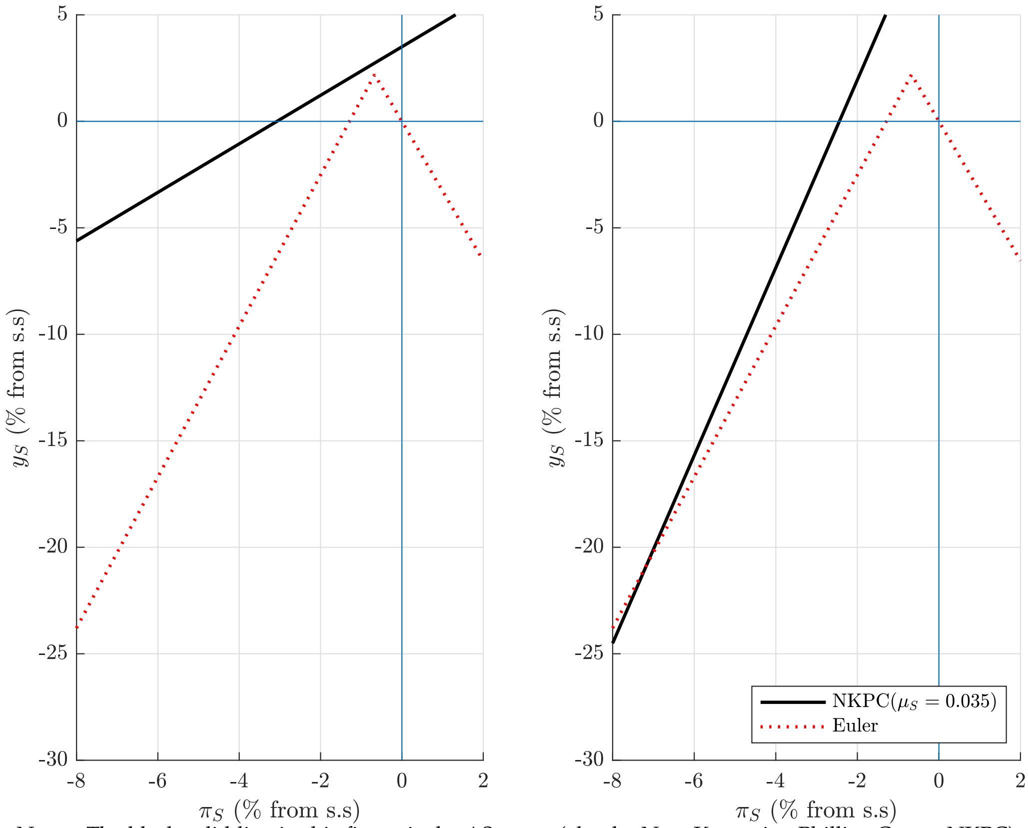
G Additional Figures for Supply Shocks

In this part, we simply assume there is a supply shock in the NKPC as below:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \gamma_y y_t + \gamma_r (R_t + \log(\beta)) + \mu_t,$$

where μ_t is the temporary supply shock.

Figure 4: Equilibrium uniqueness/existence with the real cost channel (supply shock)



Notes: The black solid line in this figure is the AS curve (aka the New Keynesian Phillips Curve, NKPC) with a supply shock ($\mu_S = 0.035$) while the red dotted line is the AD curve. The left panel presents the no equilibrium in a standard NK model without the real cost channel and the right panel shows a equilibrium with the real cost channel, following the calibration method as in Appendix C.