Monetary policy, real cost channel, and expectations-driven liquidity traps*

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Abstract

This paper analyzes the implications for the expectations-driven liquidity trap (LT) in a New Keynesian model with the cost channel. I find that the nominal cost channel alone cannot preclude the expectations-driven LT. However, when the real cost channel is considered, the expectations-driven LT is no longer relevant under possible assumptions by making the effective slope of the Phillips Curve steeper than its counterpart of the Euler equation during periods of zero lower bound. Finally, we establish that, under the real cost channel, the neo-Fisherian effects would vanish if the expectations-driven LT is irrelevant. When forward guidance is incorporated with the real cost channel, the economy is susceptible to falling into low-inflation traps.

Keywords: Real Cost channel, Liquidity Traps, New Keynesian Model, Sunspots, Expectations-driven Liquidity Traps, Monetary Policy

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1 Introduction

Since the global financial crisis, the zero lower bound (ZLB) on policy rates has become a central issue in macroeconomics, especially as central banks have struggled with it for over a decade. Despite global recovery, many central banks keep policy rates near zero, leading to widespread liquidity traps (LTs) where rates hit their lower bound, highlighting the need to understand the ZLB's economic impact. Furthermore, the ZLB's role in shaping economic policy and performance in OECD countries, particularly given its persistent nature as evidenced in Japan and its more recent emergence in the Eurozone and the U.S., underscores the importance of examining the ZLB's existence and consequences in exploring its implications for monetary policy.

The seminal work of Benhabib et al. (2002) highlights a crucial issue stemming from the presence of the ZLB: the emergence of multiple equilibria within the framework of standard New Keynesian (NK) models. This phenomenon, further examined by Bilbiie (2021), Ascari & Mavroeidis (2022), and Nakata & Schmidt (2023), presents a complex scenario in economic modeling. Specifically, in the standard NK model, two distinct short-run equilibria are generally observed. The first is characterized by stabilized inflation and output gap at the targeted steady state. The second, known as the expectations-driven equilibrium, is marked by both inflation and the output gap being below the target.

In theory, people could *expect* deflation for no fundamental reason, and the shift in households' confidence from optimism to pessimism can become a self-fulfilling prophecy (Mertens & Ravn (2014)). As a result, sunspots can cause sufficient deflationary pressures to trigger expectations-driven LTs (or sunspot LTs) without any fundamental shocks hitting the economy (see, e.g. Mertens & Ravn (2014), Aruoba et al. (2018), Bilbiie (2019) and Cuba-Borda & Singh (2020)). The phenomena of multiple equilibria and expectations-driven LTs have captivated both academics and policymakers. However, recent empirical studies suggest that expectations-driven LTs may be absent. Gorodnichenko & Sergeyev (2021) use survey data from the US, Europe, and Japan, demonstrating that the likelihood of expectation-driven liquidity traps could be minimal. Similarly, Mertens & Williams (2021) analyze US options data on future interest rates, finding no evidence in favor of the (sunspot) liquidity equilibrium.

In this context, current research indicates that expectations-driven LTs can be averted through effective (exogenous) fiscal or monetary policy interventions, as discussed in theoretical studies such as Sugo & Ueda (2008), Nakata & Schmidt (2023), and Nie & Roulleau-Pasdeloup (2023). Building on this knowledge, this paper contributes by examining how an endogenous real cost channel, in the absence of policy intervention, might render expectations-driven LTs irrelevant under certain assumptions. Specifically, I show that a sufficiently strong real cost channel can effectively neutralize sunspot LTs, while a weaker one might worsen the equilibrium. Furthermore, incorporating this channel into our analysis offers fresh perspectives on monetary policy effects, including neo-Fisherian and forward guidance effects.

Prior to COVID-19, advanced economies frequently encountered inflation rates below target, despite expansive monetary policies and low unemployment rates. This scenario, marked by ineffective inflation-boosting monetary policies, highlights the importance of integrating the real cost channel into NK models, a point extensively discussed in Beaudry et al. (2022) since this channel offers a coherent explanation for "missing inflation". Building on the work of Rabanal (2007) and Beaudry et al. (2022), we consider firms' borrowing needs for production, emphasizing how the expected real interest rate impacts borrowing costs and the Phillips Curve's marginal cost. This concept is known as the real cost channel. Notably, the real cost channel model differs from standard models in that its marginal cost depends on both the output gap and the expected real interest rate, whereas the latter relies solely on the output gap. Empirical support for the cost channel's existence is well-documented in studies as in Ravenna & Walsh (2006), Gilchrist & Zakrajšek (2015), and Beaudry et al. (2022).

I study the possibility of expectations-driven LTs in the canonical NK model with the real cost channel as in Beaudry et al. (2022) and Nie (2023), where inflation and the output gap are jointly determined and are affected by expectations of the future output gap and inflation. I solve the model equilibrium analytically and graphically. To this end, I use a (stochastic) two-state Markov structure as in Eggertsson & Woodford (2003), and Eggertsson (2011). In addition, the model equilibrium can be depicted in a (π_S, y_S) diagram, where π_S and y_S denote inflation and the output gap in the short run, respectively.

Following Nie et al. (2022) and Roulleau-Pasdeloup (2023), I derive the *effective slopes* (*i.e.* slopes can feature expectations) of Euler/Phillips Curves in closed form. I further replicate results from Mertens & Ravn (2014), Wieland (2018) and Bilbiie (2021) that the effective slopes of Euler/Phillips Curves at the Zero Lower Bound (ZLB) episode are crucial: The second expectations-driven LT (sunspot) appears in the standard NK model when the effective slope of the Phillips Curve at the ZLB episode is lower than its Euler counterpart. However, I find that the real cost channel can alter the effective slope of the Phillips Curve at the ZLB to make it higher than its Euler counterpart. This arises because the real cost channel at the ZLB can *counteract* the short-run deflation, implying actual short-run inflation in equilibrium. I proceed to derive simple model restrictions and show how these can make the expectations-driven LT lose its relevance when taking into account the real cost channel.

In the standard NK model, no model solution can appear as in Ascari & Mavroeidis (2022), if the effective slope of the Phillips curve is lower at the ZLB episode than its Euler counterpart. This arises since fundamental shocks can make the Euler curve too much below the Phillips curve. However, even if there exist powerful fundamental shocks, the model can be prone to equilibrium existence with the real cost channel.

How robust are the primary findings of this paper if we consider the nominal rather than the real cost channel, as typically modeled in Ravenna & Walsh (2006)? I show that the nominal cost channel can not alter the effective slope of the Phillips curve during recessions, although it can shift the Phillips curve. In that way, the nominal cost channel can not reduce the possibility of expectations-driven trap dynamics.

Additionally, my analytical model clearly displays a caveat to the role of the real cost channel: With a weak real cost channel, it can not reduce the occurrence of sunspots and even worsen the sunspot equilibrium; only a strong enough real cost channel can make the expectations-driven LT lose its relevance.[1] Intuitively, a weak real cost channel can increase the real marginal cost, while the lessened short-term deflation in equilibrium is insufficient. In this case, households have to save more and obtain the optimal expected return on savings due to expected inflation, which is in line with Nie & Roulleau-Pasdeloup (2023). In contrast, a strong enough real cost channel can make up short-run deflation caused by a drop in confidence, and deflationary expectations can not be an equilibrium outcome.

Finally, the paper examines the effects of monetary policy with the inclusion of the real cost channel within a tractable framework, following the approaches utilized in studies such as Bilbiie (2019) and Bilbiie (2021). Firstly, the analysis focuses on investigating the presence of neo-Fisherian effects when incorporating the real cost channel. It is found that if the possibility of expectations-driven LTs is no longer relevant in the NK model, the neo-Fisherian effects can indeed disappear. Additionally, the study models the effects of forward guidance (FG) in the context of the real cost channel. Interestingly, the findings demonstrate that such a policy can lead to future deflation, which contrasts with the conclusions drawn in Bilbiie (2021). Consequently, the implementation of FG in the presence of the real cost channel could potentially steer the economy into a

[1] In this paper, the weak (or strong) real cost channel means the elasticity of the real marginal cost w.r.t the real interest rate is small (or big enough).

low inflation trap, characterized by a persistent state of subdued inflation rates. This discovery provides a notable deviation from the conventional understanding of FG and underscores the importance of incorporating the real cost channel when assessing its impact on the macroeconomy.

This paper is closely related to a series of papers using the monetary/fiscal policy to get rid of expectations-driven LTs (Sugo & Ueda (2008), and Nakata & Schmidt (2023)). In a Ramsey-type model incorporating flexible pricing mechanisms, Benhabib et al. (2002) elucidates that employing a "non-Ricardian" fiscal approach, which contravenes the agents' transversality stipulation amidst deflationary trends leading to LTs, can decisively avert the emergence of an LT equilibrium. In a similar vein, within the framework of NK models, Schmidt (2016) explores this concept. Additionally, Piergallini (2023) examines the efficacy of "Ricardian" fiscal policies in an overlapping generations model, also characterized by flexible pricing. These policies, inherently aligned with the agents' transversality condition, are shown to safeguard the economy from succumbing to expectations-driven LTs, particularly those driven by expectation dynamics. On the other hand, Nie & Roulleau-Pasdeloup (2023) show that the Forward Guidance could rule out the sunspot ZLB if the inflation make-up strategy is bold enough. While these papers primarily emphasize the role of monetary and fiscal policy specifications in eliminating sunspot equilibria, this paper shifts focus to an endogenous channel within the Phillips Curve. This approach aims to render the expectations-driven LT irrelevant under plausible assumptions.

Relatedly, Gabaix (2020) proves that the expectations-driven ZLB equilibrium can disappear in the NK model with bounded rationality. Similarly, Ono & Yamada (2018), Glover (2019), Michaillat & Saez (2019) and Diba & Loisel (2020) all find prescriptions to avoid the sunspot LT. To the best of my knowledge, no concurrent work shows that the cost channel can work as a solution to get the economy out of the occurrence of sunspot traps.

This paper also speaks to emerging papers using a standard NK model with the real cost channel. The seminal work of Beaudry et al. (2022) indicates that the real cost channel can match the US data, and they shed light on the relationship between the real cost channel and monetary policy. There are some other fiscal implications with the real cost channel. For example, Nie (2023) uses the NK model with the real cost channel and finds low government spending multipliers in LTs. In this paper, instead of discussing the effects of policies and how they interact with the real cost channel, I document the role of this channel in expectations-driven LTs.

The rest of this paper is organized as follows. Section 2 presents the model with the real cost channel. I assume households' confidence is subject to a sunspot shock which obeys a standard two-stage Markov structure. I show that the sunspot equilibrium can appear in the standard model analytically and graphically. In section 3, I show that the real cost channel can reduce the occurrence of expectations-driven LTs and further support maintaining model equilibrium. Section 4 examines the effects of monetary policy with the real cost channel. Finally, I conclude in Section 5.

2 The model with real cost channel

This section aims to explain the role of the real cost channel in normal times and a liquidity trap (LT) using a three-equation model with the real cost channel. Normal times is the state when the economy is outside of an LT, and the nominal interest rate is flexible to adjust by the central bank. In contrast, LTs mean that there is a zero lower bound (ZLB) on nominal interest rates. Additionally, I show the short-run model equilibrium with a parsimonious two-stage Markov structure.

2.1 Three-equation model

I use a standard three-equation New Keynesian (NK) model linearized around its (deterministic) targeted steady state, and this steady state is with zero inflation/output gap.[2] I model the aggregate demand side of the economy in a standard way. A representative household consumes, supplies labor elastically and saves in one-period government bonds. The private condition boils down to the Euler equation in Definition 1.[3]

Definition 1. *The following expression represents the equilibrium conditions of the semi-linearized Euler equation, which describes the aggregate demand (AD) side of the economy:*

$$y_t = \mathbb{E}_t y_{t+1} - \sigma_r \left[R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1} - \epsilon_t \right], \tag{1}$$

where σ_r is the elasticity of inter-temporal substitution, and ϵ_t is the demand shock.

The modeling process of the Phillips Curve heavily builds on Beaudry et al. (2022) and Nie (2023). Firms have to finance for production and in this case, the expected real interest rate can impact the real marginal cost and the Phillips Curve. The specific model set-up can refer to Appendix A. In the following Definition 2, I show the semi-linear difference equation.

Definition 2. *The semi-linearized New Keynesian Phillips Curve (NKPC) with the real cost channel which represents the aggregate-supply (AS) side of the economy is shown below:*

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \left[\gamma_y y_t + \gamma_r (R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1}) \right]. \tag{2}$$

[2] I focus on the intended steady state with zero inflation in this section. The unintended steady state is a state with the ZLB binding as in Benhabib et al. (2001) and Nie & Roulleau-Pasdeloup (2023). Here I only show the linearized equilibrium condition, and all lower case format variables are the log deviations from the steady state *i.e.* $x_t = \log(X_t) - \log(X)$. Refer to Appendix A for model details.

[3] Refer to Appendix A for model details.

where π_t is inflation, y_t is the output gap, $\beta < 1$ is the discount rate, κ is the elasticity of inflation with regard to marginal cost, R_t is the nominal interest rate in level. γ_y and γ_r are the elasticity of the real marginal cost with regard to the output gap and the expected real interest rate, respectively.

Eq. (2) is employed in this paper where the expected real interest rate emerges, as in Beaudry et al. (2022) and Nie (2023). The main difference between this model and the standard model is that this model has one additional part to highlight the role of the expected real interest rate on short-run inflation. In particular, γ_r can be seen as the strength of the real cost channel.[4] In addition, this real cost channel features one additional expected inflation feedback denoted by $-\mathbb{E}_t \pi_{t+1}$ in LTs, and in Proposition 1, we show the real cost channel can mitigate the short-run deflation in equilibrium.

Proposition 1. *The real cost channel in liquidity traps implies higher expected inflation and counteracts the short-run deflation in equilibrium.*

Proof. See Appendix B. □

Without the real cost channel (that is, $\gamma_r = 0$), sufficient deflationary pressures can trigger a ZLB state. Since nominal interest rates are zero at ZLB, deflation leads to higher ex-post real interest rates, implying lower aggregate demand through the AD curve. The decline in demand in turn causes further deflation via the AS curve, creating a deflationary spiral.

However, the real cost channel can imply higher expected marginal costs and inflation expectations through the AS curve, eventually leading to short-

[4] As noted in Rabanal (2007), the parameter γ_r represents the fraction of representative firms that need to borrow funds to cover their wage bills for production. Its value typically lies within the range of [0, 1], indicating the proportion of firms dependent on borrowing for wage payments. Similarly, studies such as Beaudry et al. (2022) and Nie (2023) have estimated the range of γ_r to also fall within [0, 1]. When γ_r approaches 0, it indicates a weak real cost channel. In contrast, when γ_r is closer to 1, it signifies a relatively strong real cost channel. The strength of the real cost channel, determined by the value of γ_r, plays a crucial role in shaping the dynamics of the model and the transmission mechanisms of monetary policy.

run inflation in equilibrium. The higher marginal costs stemming from the real cost channel can counteract the deflationary effects, thereby stabilizing the economy and preventing prolonged recession.

While ZLB causes monetary policy to lose its potency as a stabilization tool, the real cost channel creates inflation expectations that can be self-fulfilling. When firms anticipate higher borrowing costs and inflation, they increase their prices preemptively. This change in inflation expectations and actual inflation counteracts lower demand and breaks the deflationary spiral even when nominal rates are constrained at ZLB.

In addition, this Phillips Curve with the real cost channel can nest the Phillips Curve in the standard NK model below if we simply assume $\gamma_r = 0$:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \gamma_y y_t. \tag{3}$$

In the short run, we assume that the central bank obeys a standard Taylor (1993)-type rule with a lower bound in the following Definition 3. In this case, sufficient deflationary pressures can trigger a lower bound, and the central bank has to set the nominal interest rate to zero.

Definition 3. *Monetary policy is assumed to follow Taylor (1993)-type rules with a lower bound:*

$$R_t = \max\left[0; -\log(\beta) + \phi_\pi \pi_t\right]. \tag{4}$$

To study the dynamics of the economy in normal times and LTs, I assume the central bank can not perfectly track the nominal rate but with a lower bound constraint. As in Aruoba et al. (2018), the fundamental demand shock can impede the central bank from stabilizing the NK economy. To be more specific, if this fundamental shock is potentially large enough, the central bank can not track nominal rates with sufficient deflationary pressures, and the short-run economy can be stuck into LTs. In that way, the nominal interest rate should be fixed at zero. However, if the demand shock is small, the central bank can stabilize the economy by using the standard Taylor (1993) rule. Specifically, the central bank sets a more than one-to-one decrease in nominal interest rate to fight deflationary pressures.

In addition, I assume there exists a sunspot shock in this paper. The persistent sunspot shock can shift peoples' confidence, as in Mertens & Ravn (2014) and Nie & Roulleau-Pasdeloup (2023), and cause sufficient deflationary pressures to trigger the expectations-driven (or sunspot) LTwithout any fundamental shocks hitting the economy.

Note that the real cost channel can work as a cost-push shock endogenously in normal times if the Central Bank follows a simple Taylor rule as $R_t = \phi_\pi \pi_t - \log(\beta)$. This result in normal times is widely discussed in the literature as in Ravenna & Walsh (2006), Gilchrist & Zakrajšek (2015), and Nie (2023).

The zero lower bound policy has plagued the US, Japan, and the euro countries for decades. In this paper, I will focus on the ZLB episode. At the ZLB, the nominal interest rate is zero (*i.e.* $R_t = 0$). The real cost channel still works with the expected inflation feedback in the Phillips Curve. Following Nie et al. (2022) and Roulleau-Pasdeloup (2023), I derive the effective slope in the NK model where the current inflation and output are jointly affected by expectations of future output and inflation. Therefore, the expected inflation feedback in the real cost channel can alter the effective slope of the Phillips Curve at the ZLB.[5]

[5] In this paper, I mainly explore the effective slopes of AS/AD curves at the ZLB. Note that the specific setting of the Taylor (1993)-rule is not critical here since the nominal rate is fixed at zero in LTs.

2.2 Equilibrium determinacy

In this subsection, I begin by deriving the analytical condition necessary for ensuring that our NK model, incorporating a real cost channel, possesses a (locally) unique equilibrium when subjected to a standard Taylor rule, while staying away from the zero lower bound. Proposition 2 succinctly summarizes the condition required for equilibrium determinacy.

Proposition 2. *With the real cost channel, the NK model has equilibrium determinacy if and only if:*

$$1 < \phi_\pi < \frac{3(\beta - \kappa\gamma_r) + \sigma_r\kappa\gamma_y + 1}{\kappa\gamma_r}. \tag{5}$$

Proof. See Appendix C. □

This equilibrium determinacy can be demonstrated by directly analyzing the eigenvalues of the system described by Definitions (1)-(3), a rational expectation equilibrium provided that condition (5) is met. It is important to note that the aforementioned condition has the flexibility to accommodate the standard model (*i.e.* $\phi_\pi > 1$), where $\gamma_r = 0$ (as outlined in Woodford (2001)). This indicates that our framework can encompass the traditional model as a special case. By considering the real cost channel, which imposes an upper bound on the variable ϕ_π, our analysis aligns with the findings of Surico (2008).[6]

As discussed in Surico (2008), the upper bound for the real cost channel arises due to the interaction between nominal interest rates and inflation. When the response of nominal rates to inflation is excessively aggressive, higher interest rates lead to increased borrowing costs for firms. Consequently, the benefits derived from lower wages are outweighed by the increased costs of borrowing, prompting firms to prefer raising prices instead.[7]

[6]See Appendix C, the condition for equilibrium determinacy of the NK model with the real cost channel can nest the one with the nominal cost channel.

[7]It is worth noting that in the standard simulation case, the upper bound can become binding for values of ϕ_π as large as 190. This implies that when the responsiveness of nominal

2.3 Short-run equilibrium: A stochastic method

This three-equation model above is simple enough for a clear analytical analysis. To this end, I use a parsimonious two-stage Markov structure with an absorbing state to solve the stochastic model analytically as in Eggertsson & Woodford (2003) and Eggertsson (2011). Specifically, the initial *recurrent* state of the Markov chain features the short-run economy (where we label it with a subscript S), which can deviate from the steady state with shocks. After a few periods, the economy can be back to the steady state (where we label it with a subscript L), and it is also the second state of the Markov structure.

In our study, we adopt the approach delineated by Bilbiie (2019), positing the existence of a steady state, which is characterized as an absorbing state. An absorbing state is uniquely defined as a state that, once reached, cannot be exited. This particular state can be regarded as a long-term steady state. The primary merit of employing an absorbing steady-state assumption lies in its facilitation of *a graphical depiction* of the interaction between the NKPC and the Euler equation. Moreover, this approach facilitates elucidation of the mechanism underpinning the efficacy of the real cost channel through effective slopes, both analytically and graphically, as delineated in Roulleau-Pasdeloup (2023).[8]

With this in mind, the short-term economy is hit by the exogenous demand shock ϵ_S which persists with a probability p and recovers to the steady state ($\epsilon_L = 0$) with a probability $1 - p$.[9] In addition, the sunspot shock is arbitrarily small with a persistence p. Since the Phillips Curve and the Euler equation

interest rates to inflation exceeds a certain threshold, the real cost channel becomes a crucial factor influencing the behavior of firms and their pricing decisions.

[8] It should be noted that, as explored in Appendix Q, the relaxation of this absorbing state assumption as in Coyle & Nakata (2019) can also reveal our main results that, with the real cost channel's contribution, the possibility of expectation-driven liquidity traps could be reduced.

[9] The transition matrix for the demand shock is:

$$\mathcal{P}_S = \begin{bmatrix} p & 1-p \\ 0 & 1 \end{bmatrix}.$$

The stochastic expected duration of the demand (or sunspot) shock is $\mathcal{T} = 1/(1-p)$.

in Eqs. (2) and (1) are both forward-looking, and one can write the expected output gap as

$$\mathbb{E}_S y_{t+1} = p \cdot y_S + (1-p) y_L$$
$$= p \cdot y_S,$$

where the output gap $y_L = 0$ is the steady state, implying no deviations in the long run. Similarly, one can offer $\mathbb{E}_S \pi_{t+1} = p \cdot \pi_S$ with zero long-run inflation for expected inflation next period. In this case, I define the short-run equilibrium with the Markov chain representation below:

Definition 4. *The short-run equilibrium can be expressed as a vector* $[y_S, \pi_S, R_S]$ *such that, for a given* ϵ_S

$$\pi_S = \beta \mathbb{E}_S \pi_{t+1} + \kappa \left[\gamma_y y_S + \gamma_r (R_S + \log(\beta) - \mathbb{E}_S \pi_{t+1}) \right] \tag{6}$$

$$y_S = \mathbb{E}_S y_{t+1} - \sigma_r \left[R_S + \log(\beta) - \mathbb{E}_S \pi_{t+1} - \epsilon_S \right] \tag{7}$$

$$R_S = \max \left[0; -\log(\beta) + \phi_\pi \pi_S \right] \tag{8}$$

$$\mathbb{E}_S \pi_{t+1} = p \pi_S \tag{9}$$

$$\mathbb{E}_S y_{t+1} = p y_S \tag{10}$$

all hold.

Based on Definition 4, if the economy is in LTs with $R_S = 0$ caused by (strong) negative fundamental shocks, it is in fundamental-driven LTs as in Aruoba et al. (2018). On the flip side, as in Mertens & Ravn (2014), if the economy can feature a ZLB equilibrium ($R_S = \epsilon_S = 0$) with no fundamental reasons, it can be referred to as sunspot-driven traps.

In addition, the short-run equilibrium in Definition 4 can be solved by hand. As in Nie et al. (2022) and Roulleau-Pasdeloup (2023), the short-run Euler/Phillips Curves can be shown in the following systems (Definition 5), which take into account expectations as in Mertens & Williams (2021):

Definition 5. *The short-run New Keynesian Phillips Curve and Euler equation are shown below:*

$$y_S = \begin{cases} \mathcal{S}^c_{PC}\pi_S & \textit{if} \quad \pi_S > \frac{\log(\beta)}{\phi_\pi} \\ \mathcal{S}^{c,z}_{PC}\pi_S + \mathcal{I}^c_{PC} & \textit{if} \quad \pi_S \leq \frac{\log(\beta)}{\phi_\pi} \end{cases} \tag{11}$$

$$y_S = \begin{cases} \mathcal{S}_{EE}\pi_S + \mathcal{I}_{EE} & \textit{if} \quad \pi_S > \frac{\log(\beta)}{\phi_\pi} \\ \mathcal{S}^z_{EE}\pi_S + \mathcal{I}_{EE} - \sigma_r\frac{\log(\beta)}{1-p} & \textit{if} \quad \pi_S \leq \frac{\log(\beta)}{\phi_\pi}, \end{cases} \tag{12}$$

where $\mathcal{S}$ labels the effective slope and $\mathcal{I}$ denotes the intercept. The superscript c and z denote "real cost channel" and "ZLB", respectively. The subscript PC and EE denote "Phillips Curve" and "Euler equation", respectively. The expressions of these effective slopes/intercepts are reported in Appendix F.

I show the Phillips Curve in Eq. (11) and the Euler equation in Eq. (12). The main difference between this model with the standard model is that Eq. (11) in the standard model will collapse to one single equation which is independent of the economic state (*i.e.* either the normal times or the ZLB). In particular, the effective slope can feature expectations of the future output gap and inflation.

The effective slope is crucial in determining the type of LTs in this paper, and I simply assume the effective slope of the Phillips Curve is upward sloping in a (π_S, y_S) graph as in Assumption 1, which means $p < \overline{p}^u$—see Appendix D for details. In other words, with the real cost channel, there is a threshold $\overline{p}^u$ such that the Phillips Curve can be upward/downward sloping. Laubach & Williams (2003), Daly & Hobijn (2014) and Nie (2023) assume a similar condition.

Assumption 1. *Assume that the Phillips Curve with the real cost channel is upward sloping in a (π_S, y_S) graph such that*

$$p < \frac{1-\kappa\gamma_r\phi_\pi}{\beta - \kappa\gamma_r} = \overline{p}^u.$$

I have sketched the NK model with the real cost channel and expressed the short-run equilibrium with a two-stage Markov structure. In the next section 2.4, I will replicate the sunspot equilibria in the standard NK model as in Mertens & Ravn (2014), Wieland (2018), Bilbiie (2021), and Nie & Roulleau-Pasdeloup (2023).

2.4 Sunspot equilibria in standard NK model

This subsection aims to show the equilibrium multiplicity property and equilibria solutions analytically and graphically in a textbook NK model without the real cost channel. As in Benhabib et al. (2001), Bilbiie (2019), Ascari & Mavroeidis (2022), and Nakata & Schmidt (2023), the standard NK models are prone to equilibrium multiplicity if the central bank follows a Taylor rule with a lower bound constraint. Specifically, there are two short-run equilibria in the standard model. The first one is stabilized at the targeted steady state. The second one is the expectations-driven (or sunspot) liquidity equilibrium with negative inflation and the output gap.

2.4.1 Equilibrium Multiplicity

Before adding the real cost channel, I first show the two equilibria in the standard model. The modelling is in line with Nie & Roulleau-Pasdeloup (2023), and I assume there exists a sunspot shock.[10] This shock is arbitrarily small, and it remains in the short run with the persistence p. The expectations-driven traps mean that the economy can feature actual deflation and be in LTs with an arbitrarily small sunspot shock in a high persistence of realized deflation environment (*i.e.* the sunspot shock persistence p is large enough)—see Nie & Roulleau-Pasdeloup (2023) for a discussion.

[10] As also in Mertens & Ravn (2014), sunspots can be seen as exogenous shocks to households' confidence.

Following the way in Nie et al. (2022) and Roulleau-Pasdeloup (2023), I define the effective slopes in this paper, which can take into account expectations.[11] I first show the effective slopes of AS/AD curves in a (π_S, y_S) graph within the standard model explicitly.

Lemma 1. *In the standard NK model, the effective slope of AD/Euler curve in Eq.(7) at the ZLB is:*

$$\mathcal{S}^z_{EE} = \sigma_r \frac{p}{1-p}.$$

The effective slope of AS/NKPC curve in Eq.(3) at the ZLB is:

$$\mathcal{S}^z_{PC} = \frac{1-\beta p}{\kappa \gamma_y}.$$

Proof. See Appendix G. □

As in the seminal work of Bilbiie (2021), the equilibrium multiplicity can be detected by the probability p in a two-state Markov structure.[12] Based on Lemma 1, increasing p can generate a second crossing in the AS/AD curves at the ZLB episode by (i) increasing the Euler equation slope $\mathcal{S}^z_{EE}$ and (ii) reducing the NKPC slope $\mathcal{S}^z_{PC}$ simultaneously.[13] In this case, there exists a threshold $\overline{p}$ in Lemma 2 such that a second intersection emerges in a (π_S, y_S) graph (*i.e.* the expectations-driven LT) in the standard NK model if $p > \overline{p}$.

Lemma 2. *One can use Lemma 1 to calculate the threshold $\overline{p}$ below:*

$$\overline{p} = \frac{(\beta + 1 + \sigma_r \kappa \gamma_y) - \sqrt{(1 + \beta + \sigma_r \kappa \gamma_y)^2 - 4\beta}}{2\beta} < 1.$$

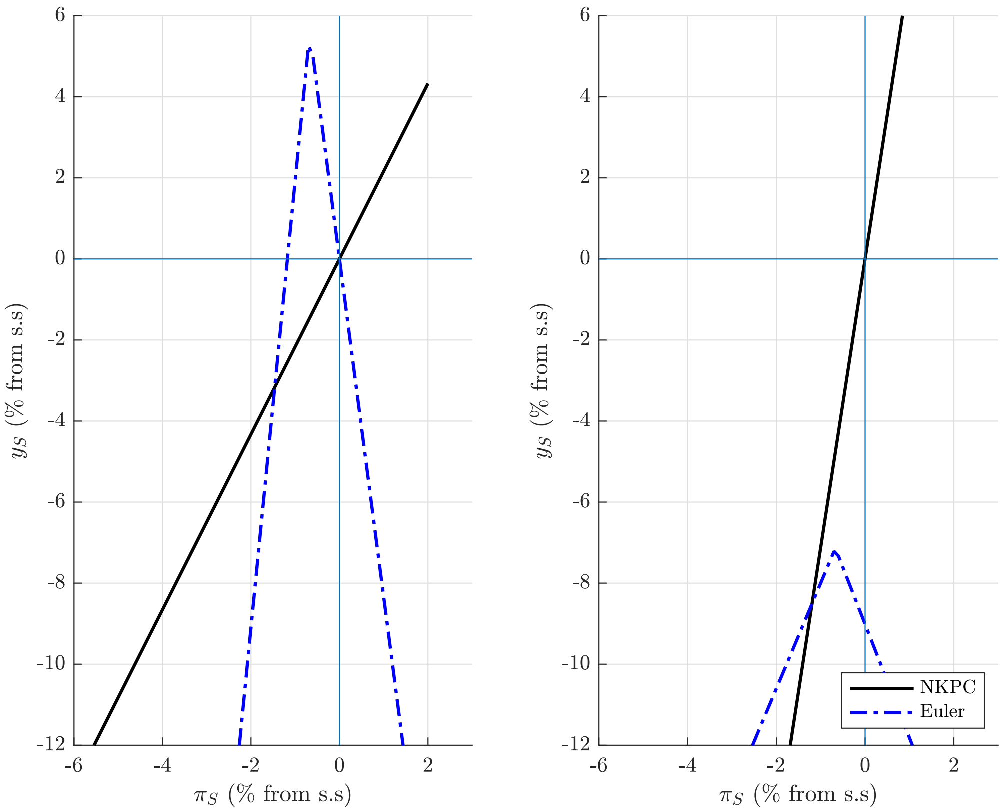
Proof. See Appendix H. □

[11] In other words, it can represent features that inflation and output are jointly determined and affected by expectations of the future output gap and inflation. See also Roulleau-Pasdeloup (2021).

[12] Similar arguments can be found in Mertens & Ravn (2014) and Aruoba et al. (2018).

[13] In the standard NK model, we have a first crossing at the origin in the AS/AD curves.

Figure 1: Expectations-driven LT (left) and fundamental-driven LT (right)



Notes: The black solid line in this figure is the AS curve (aka the New Keynesian Phillips Curve, NKPC) while the blue dashed line is the AD curve (aka the Euler equation). The left panel presents the expectations-driven LT in a standard NK model with $p = \bar{p} + 0.1$ and the right panel shows the fundamental-driven LT in the standard model by assuming $p = \bar{p} - 0.1$ with the demand shock $\epsilon_S = -0.025$. Other calibration parameters are shown in Appendix E.

As mentioned in Lemma 2, this threshold is highly dependent on the slope of the NKPC, which represents the degree of price stickiness, as well as the intertemporal substitution of the Euler equation. Furthermore, as discussed in Bilbiie (2021), a higher overall elasticity, denoted as $\kappa\gamma_y\sigma_r$, can increase the likelihood of sunspot occurrences.

To have a clear observation, I plot the expectations-driven (or sunspot) LT and the fundamental-driven LT in the AS/AD diagram as in Figure 1. It is of note that *the effective slopes of the AS/AD curves at the ZLB episode are crucial*. For the fundamental-driven LT case on the right panel, this effective slope of the AS curve at the ZLB is larger than that of the AD curve. The reverse holds for the expectations-driven liquidity traps on the left panel where the effective slope of the AS curve is less than the AD slope. Consequently, the Euler and the NKPC can cross twice, giving rise to the sunspot ZLB.

2.4.2 Characterization of multiple equilibria

According to Lemma 2, the economy can be in expectations-driven LTs with a high p. The intuition is that the expected highly persistent deflationary shock can shift people's confidence. In this case, people could expect deflation for no fundamental reason, and there could be a self-fulfilling prophecy that will result in expectations-driven LTs. To better understand the difference between fundamental-driven LTs and sunspot traps. I replicate the closed-from solutions for the two LTs in the standard NK model as in Mertens & Ravn (2014), Wieland (2018), and Bilbiie (2021) in Lemma 3.

Lemma 3. *In the standard NK model, the solution of the expectations-driven traps is given:*

$$\begin{aligned} y_S &= \frac{(1-\beta p)\sigma_r}{(1-p)(1-\beta p) - \sigma_r p \kappa \gamma_y}(-\log(\beta)) \\ \pi_S &= \frac{\kappa \gamma_y}{(1-p)(1-\beta p) - \sigma_r p \kappa \gamma_y}(-\log(\beta)), \end{aligned}$$

where $(1-p)(1-\beta p) - \sigma_r p \kappa < 0$ *(i.e.* $p > \overline{p}$*).*
The solution of the fundamental-driven traps is shown as:

$$\begin{aligned} y_S &= \frac{(1-\beta p)\sigma_r}{(1-p)(1-\beta p) - \sigma_r p \kappa \gamma_y}(\epsilon_S - \log(\beta)) \\ \pi_S &= \frac{\kappa \gamma_y}{(1-p)(1-\beta p) - \sigma_r p \kappa \gamma_y}(\epsilon_S - \log(\beta)), \end{aligned}$$

where $(1-p)(1-\beta p) - \sigma_r p \kappa > 0$ *(i.e.* $p < \overline{p}$*).*

Proof. See Appendix I. □

In line with Cuba-Borda & Singh (2020), I show the two traps in isomorphic expressions with the ZLB binding. It is straightforward to see that the denominator is the same in the two specifications. Here p is crucial, if the fundamental/sunspot shock is large enough (*i.e.* $p > \overline{p}$), the denominator is negative. In this case, the solutions of y_S and π_S are both negative without any fundamental shock hitting the economy (*i.e.* $\epsilon_S = 0$). On the other hand, the fundamental-driven traps are very similar but the shock persistence is small. In that way, the denominator of the solution is positive while the term $(\epsilon_S - \log(\beta))$ is negative with a strong (negative) fundamental shock $\epsilon_S < 0$. Therefore the economy is in LTs with negative y_S and π_S.

The expectations-driven (or sunspot) trap is shown on the left panel of Figure 1 and a second intersection of the AS and AD curves occurs. It indicates that if the sunspot shock persistence is sufficiently large, the economy will feature actual deflation without any fundamental shocks hitting the economy. In other words, if households do expect deflation for no reason, this can cause sufficient deflationary pressures to trigger the expectations-driven LT with a self-fulfilling state of low confidence. It is of note that, similar to the results in Bilbiie (2019) and Nie & Roulleau-Pasdeloup (2023), there are two short-run equilibria on the left panel of Figure 1. One is the targeted (intended) steady state which means $y_S = \pi_S = 0$. Another one is the expectations-driven ZLB, implying $y_S < 0$ and $\pi_S < 0$. These experimental results can echo our analytical results in Lemma 3. Therefore the second equilibrium with expectations-driven traps emerges, and there is no stable equilibrium echoing the findings in Aruoba et al. (2018).

On the right panel of Figure 1, there exist fundamental-driven traps where the strong demand shock $\epsilon_S < 0$ can cause sufficient deflation such that the ZLB binds, implying $y_S < 0$ and $\pi_S < 0$. At the same time, the effective slope of the AD curve at the ZLB is lower than its counterpart of AS curve. There is only one unique equilibrium that can feature the ZLB state. For example, the US has been caught in the fundamental-driven ZLB during the global financial crisis (GFC), as in Eggertsson (2011) and Aruoba et al. (2018).

To conclude, there exists sunspot equilibrium in the standard model, and we show that the effective slopes are crucial in determining the LTs, which is in line with Bilbiie (2021) and Nie & Roulleau-Pasdeloup (2023). As in the literature (see e.g. Sugo & Ueda (2008), Nakata & Schmidt (2023) and Schmidt (2016)), many policy prescriptions are proposed to get rid of the sunspot traps. In the following section 3, I will instead show the real cost channel that can reduce the occurrence of the expectations-driven LT.

3 Expectations-Driven LT: Losing relevance

In this section, I now show that when the real cost channel is considered, the expectations- driven LT is no longer relevant under possible assumptions. To be more specific, the real cost channel in the NK model can rotate the NKPC while the effective slope of the Euler equation is unchanged. Additionally, I show this real cost channel is theoretically appealing since it helps ensure model equilibrium existence. I finally show that the nominal cost channel alone cannot preclude the expectations-driven LT.

3.1 Higher effective slope of AS curve with real cost channel

As described at length in Section 2.4, the effective slopes of AS/AD curves in a (π_S, y_S) graph at the ZLB episode are critical. First, I show the effective slope of the AS curve at the ZLB with the real cost channel explicitly below.

Lemma 4. *Based on Definition 4, the effective slope of the AS/NKPC curve with the real cost channel in Eq.(6) at the ZLB is:*

$$\mathcal{S}_{PC}^{c,z} = \frac{1 - \beta p + \kappa \gamma_r p}{\kappa \gamma_y}.$$

Proof. See Appendix J. □

By comparing Lemma 1 and Lemma 4, the real cost channel can magnify the effective slope of the AS curve at the ZLB episode with the term $\kappa\gamma_r p$. Thus, the effective slope of the AS curve at the ZLB episode is higher with the real cost channel as in Proposition 3. In addition, the effective slope of AS curve with this channel can be reduced to the standard one if $\gamma_r = 0$.

Proposition 3. *Relative to the standard NK model, the effective slope of the AS curve at the ZLB episode is higher with the real cost channel. Furthermore, the slope increases with the intensity of the real cost channel, as represented by γ_r.*

Proof. See Appendix K. □

If the AS curve is rotated and the effective slope $\mathcal{S}_{PC}^{c,z}$ is higher than S_{EE}^{z} in the (π_S, y_S) graph at the ZLB episode, the second intersection can disappear, implying that the expectations-driven traps as in Bilbiie (2019) and Cuba-Borda & Singh (2020) is no longer relevant. In that way, the economy can be in the intended steady state without any fundamental shocks.

On the other hand, Lemma 4 can be employed to compute the threshold $\overline{p}^c$ that triggers the sunspot equilibrium when considering the real cost channel, as shown in Lemma 5. This expression is isomorphic to the one in Lemma 2. It is observed that this threshold value is higher than that in the standard model. In other words, the presence of the real cost channel reduces the likelihood of the economy being in a sunspot equilibrium.

Lemma 5. *The threshold $\overline{p}^c$ with the real cost channel below:*

$$\overline{p}^c = \frac{(\beta + 1 + \sigma_r\kappa\gamma_y - \kappa\gamma_r) - \sqrt{(1 + \beta + \sigma_r\kappa\gamma_y - \kappa\gamma_r)^2 - 4(\beta - \kappa\gamma_r)}}{2(\beta - \kappa\gamma_r)} > \overline{p}.$$

Proof. See Appendix L. □

It is shown that the real cost channel can increase the effective slope of the AS curve at the ZLB episode, however, it can show no influence on the effective slope of the AD curve. In this case, a strong enough real cost channel can help to make the sunspot traps irrelevant if the effective slope of the AS curve is steeper than its counterpart of the AD curve at the ZLB.[14] We specify the restriction on the real cost channel as in the following Proposition 4.

Proposition 4. *The elasticity of real marginal cost w.r.t output γ_y follows the restriction below:*

$$\gamma_y < \Phi(\gamma_r),$$

where $\Phi(\gamma_r) = \frac{(\beta-\kappa\gamma_r-1+\kappa\gamma_r\phi_\pi)\gamma_r\phi_\pi(\beta-\kappa\gamma_r)}{\sigma_r(1-\kappa\gamma_r\phi_\pi)}$ *increases in γ_r. Then the real cost channel can make the expectations-driven LT irrelevant.*

Proof. See Appendix M. □

From Lemma 4, the effective slope of AS curve at the ZLB increases in the strength of the real cost channel γ_r[15] while it decreases in the elasticity of real marginal cost w.r.t output γ_y. For a given value of γ_y, the effectiveness of the real cost channel can be enhanced with a higher γ_r.[16]

With this condition that $\gamma_y < \Phi(\gamma_r)$, the effective slope of the AS curve at the ZLB can be *always* larger than the AD slope in a (π_S, y_S) graph.[17] On the flip side, this threshold condition increases in γ_r. Therefore, with a higher γ_r, the economy is more likely not in expectations-driven traps. Furthermore, this condition of $\gamma_y < \Phi(\gamma_r)$ requires that the strength γ_r should be big enough for a given γ_y. As a consequence, no second intersection exists in the AS/AD curves and further, the sunspot equilibrium is no longer relevant.

Interestingly, this theoretic restriction can echo empirical evidence in Beaudry et al. (2022). This seminal paper *empirically* estimates that γ_y in the real cost

[14] In other words, the real cost channel can reduce the occurrence of the expectations- driven LT with a big γ_r while a small γ_r can not work.

[15] γ_r represents the elasticity of marginal cost w.r.t the interest rate, and it can be seen as the strength of the real cost channel.

[16] In other words, the magnitude of γ_r relative to γ_y reflects the role of the real cost channel.

[17] Note that if the NKPC is upward sloping in a (π_S, y_S) graph, the second intersection can not arise. In addition, I assume $\Phi(\gamma_r) > 0$ in this paper.

channel is robustly small (non-significantly). γ_r is significantly positive and is much larger than γ_y. In that way, the sunspot equilibrium is most likely to disappear with such parameter estimations. Moreover, this empirical finding in Beaudry et al. (2022) motivates the restriction in Proposition 4.

The potential rationale that the real cost channel can reduce the probability of the expectations-driven LT is that the inflation feedback at the ZLB in Eq. (6) can *counteract* deflation in the short run. The counteracting effects can imply short-run inflation in equilibrium due to rational expectations and sticky prices. In this case, for a given level of output gap y_S, the deflation behavior at the ZLB can move less due to the counteracting effects. This gives rise to a higher slope of the AS curve in a (π_S, y_S) graph. Finally, a steep enough AS curve can get rid of the second intersection. In that way, deflationary expectations can not be an equilibrium outcome, and thus the probability of the expectations-driven LT is reduced with this channel.

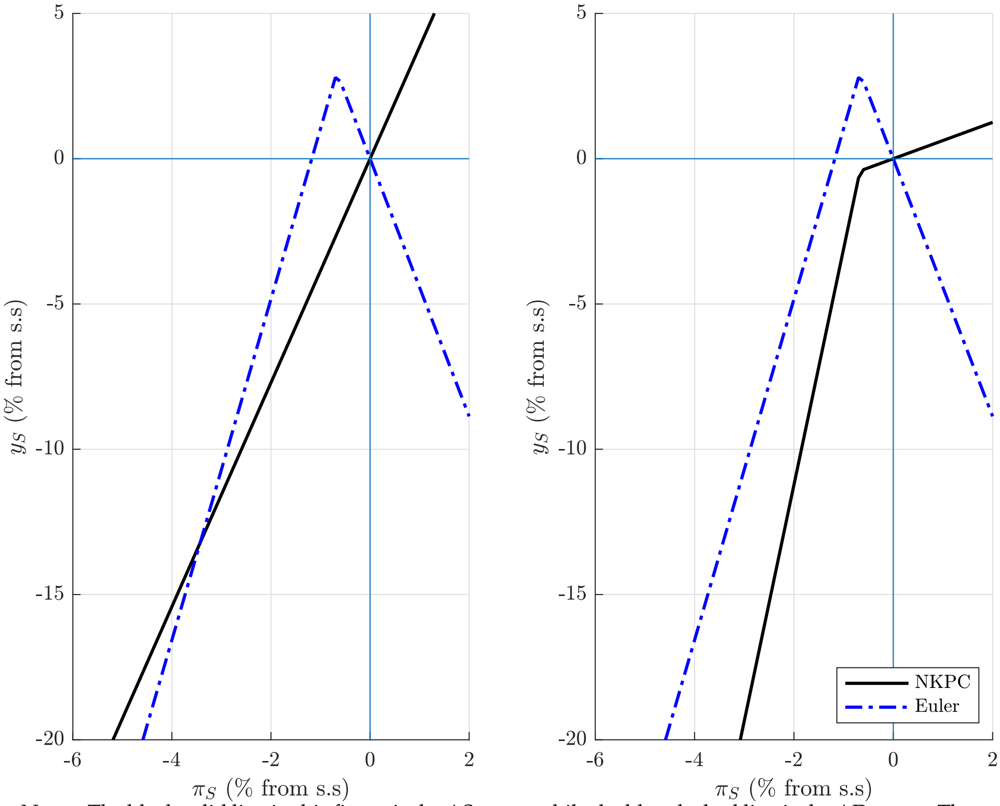
I show the numerical experiment results in Figure 2.[18] On the left panel, in the standard model, when the sunspot shock is persistent enough, there are two equilibria, and the second intersection appears. With the same calibration method, there appears to be no sunspot equilibrium on the right panel of Figure 2: The absence of a second intersection in the AS/AD curves due to the steeper AS curve at the ZLB episode. This result can provide a theoretical justification for why the possibility of expectation-driven LTs, as shown in survey evidence in Gorodnichenko & Sergeyev (2021).

In this section, we introduce the real cost channel, which results in the NKPC exhibiting a locally flat characteristic in a (y_S, π_S) graph, observable primarily during ZLB episodes.[19] This study demonstrates that the locally flat NKPC can effectively negate the relevance of expectations-driven LTs and ensure a

[18] The calibration method can guarantee that $\overline{p}^u > \overline{p}$.

[19] It should be noted that in Figure 2, the AS/AD curves are depicted in a (π_S, y_S) graph for ease of comparison, whereas the flatness of the Phillips Curve is represented in a (y_S, π_S) graph.

Figure 2: No expectations-driven LT with the real cost channel



Notes: The black solid line in this figure is the AS curve while the blue dashed line is the AD curve. The left panel presents the expectations-driven LT in a standard NK model without the real cost channel and the right panel shows no expectations-driven LT with the real cost channel, following the calibration method as in Appendix E.

unique equilibrium with $\pi_S = 0$. Notably, this model's portrayal of a locally flat NKPC during ZLB episodes is corroborated by recent empirical evidence. For instance, Hazell et al. (2022) uses cross-sectional data from the United States to estimate a flattened Phillips Curve during the period of the Great Recession.

3.1.1 Economic intuitions

To gain a better understanding of how the real cost channel can reduce the occurrence of the sunspot equilibrium, following Nie & Roulleau-Pasdeloup (2023), we can rewrite the Euler equation in the following way:

$$y_S = \Gamma_y(p, \gamma_r, \gamma_y)\mathbb{E}_S y_{t+1} + \Gamma_\beta(p, \gamma_r)\log(\beta), \tag{13}$$

where the elasticity Γ is a function of the model parameters, which are listed in Appendix O. We can show $\mathbb{E}_S y_{t+1} = p y_S$ and assuming that the sunspot shock is persistent ($p > \bar{p}$) in the standard model, the coefficient that multiplies y_S on the right-hand side of equation (13) is greater than 1. Initially, we assume that the output gap is in a steady state in the short run. However, using equation (13), we can see that this cannot be equilibrium as the marginal benefit of consuming today, represented on the left-hand side, is zero. On the other hand, the right-hand side of equation (13) shows a positive marginal benefit of saving today, as $\Gamma_\beta(p, \gamma_r) \log(\beta) > 0$. To restore equilibrium, households will reduce consumption and increase savings, leading to a decrease in aggregate demand and ultimately resulting in y_S becoming negative.

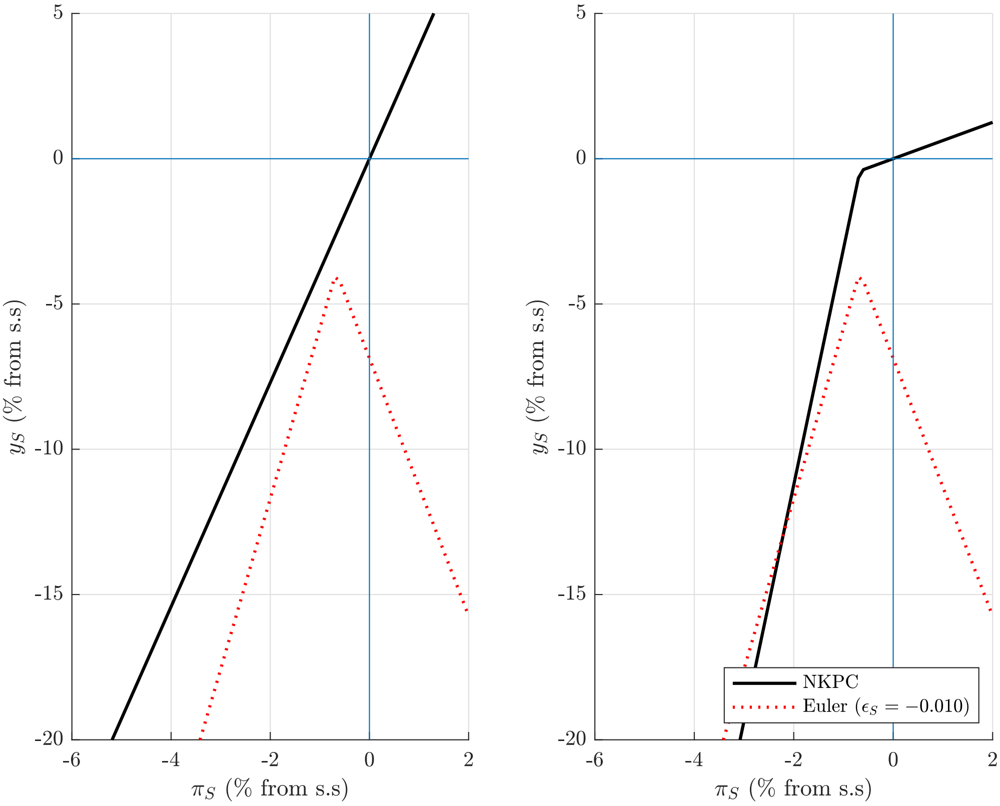
Due to the presence of the real cost channel, the elasticity Γ_y is lower compared to the standard model. Consequently, the coefficient that multiplies y_S on the right-hand side of equation (13) is less likely to be greater than 1. As a result, the impact of increased savings on the expected return to savings will be reduced.

3.2 Equilibrium uniqueness/existence

As in Benhabib et al. (2001) and Mertens & Ravn (2014), the NK models can be prone to equilibrium multiplicity. I have shown this occurs since there is a second intersection that can feature the sunspot equilibrium analytically and graphically. Moreover, as in Ascari & Mavroeidis (2022), models with ZLB constraints can have no solution: if there exist supply/demand shocks that make the AD curve shift too much below the AS curve, there can be no equilibrium in the expectations-driven LT case.

To have a clear observation, I plot this situation in Figure 5. It can be seen that, on the left panel, if the effective slope of the AS curve at the ZLB is lower than the AD slope, there can be no equilibrium with an additional strong enough

Figure 3: Equilibrium uniqueness/existence with demand shock



Notes: The black solid line in this figure is the AS curve while the red dotted line is the AD curve with a demand shock ($\epsilon_S = -0.010$). The left panel presents the no equilibrium in a standard NK model without the real cost channel and the right panel shows an equilibrium with the real cost channel, following the calibration method as in Appendix E.

demand shock, as in Ascari & Mavroeidis (2022).[20] This arises since the demand shock ϵ_S can shift the AD curve too much below the AS curve.[21] However, no solution dilemma can not arise if the effective slope of the AS curve is higher at the ZLB episode.

On the right panel of Figure 5, it can be seen that the real cost channel can increase the effective slope of AS curve at the ZLB. In that way, even if there exist powerful fundamental shocks, there is always a unique intersection at the ZLB episode. Therefore, the real cost channel can help ensure that the AS/AD curves always have a unique equilibrium with fundamental shocks.[22] Over-

[20] There exists two equilibria with a small demand shock.
[21] The kink of the AD curve is lower than the AS curve.
[22] See Appendix N for a numerical example with supply shocks.

all, this real cost channel is theoretically appealing since it helps ensure model equilibrium existence with fundamental shocks.

The key findings of this subsection can be summarized as follows: The real cost channel plays a pivotal role in ensuring the uniqueness and existence of model equilibrium. Firstly, with a sufficiently large γ_r, the real cost channel can render the sunspot equilibrium irrelevant. In this scenario, there is no second point of intersection between the AS and AD curves, and the real cost channel ensures a unique targeted steady state in the absence of fundamental shocks. This phenomenon is referred to as the real cost channel's contribution to model equilibrium uniqueness.

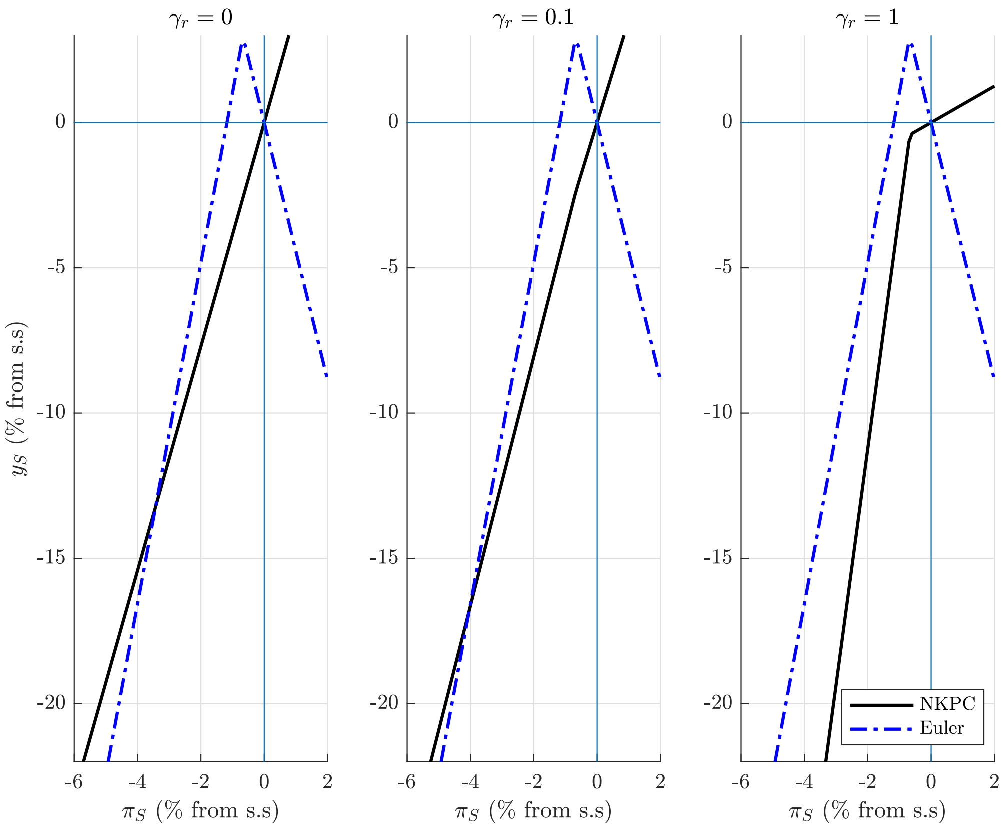
Secondly, as previously discussed, demand or supply shocks might cause the AD curve too much below the AS curve, potentially leading to a lack of model equilibrium in an AS/AD framework. However, incorporating the real cost channel allows the model to maintain equilibrium existence, even in the presence of strong fundamental shocks. This demonstrates the channel's effectiveness in stabilizing the model under various economic disturbances.

3.3 Strength of real cost channel: A caveat

As for the discussion outlined above, I have implicitly assumed that the real cost channel is strong enough to make the sunspot equilibrium irrelevant. However, as in Proposition 4, $\Phi(\gamma_r)$ increases in the strength of the real cost channel γ_r, implying a small γ_r may not be able to reduce the occurrence of sunspots. In this case, we aim to illustrate the role of the strength of the real cost channel in this section.

In the numerical experiment, I consider three values for γ_r: $\gamma_r = \{0, 0.1, 1\}$, and the corresponding results are plotted in Figure 4. It is important to note that when $\gamma_r = 0$, the model reverts to the standard one. When $\gamma_r = 1$, it represents a sufficiently strong cost channel, while $\gamma_r = 0.1$ indicates a weak cost channel.[23]

Figure 4: AS/AD with the strength of the real cost channel



Notes: The black solid line in this figure is the AS curve while the blue dotted line is the AD curve. The first panel presents the equilibrium in a standard NK model without the real cost channel, the second panel shows the model with a weak real cost channel, and the third panel displays the model with a potent real cost channel, following the calibration method as in Appendix E.

The direct takeaway from this Figure 4 is that the real cost channel has various features. On the first panel, it shows that we have two equilibria, and the second intersession can feature the ZLB state with inflation $\pi_S^s < 0$. On the second panel, with a weak cost channel, even if the effective slope of the AS curve in LTs now, it can not rid the possibility of sunspots and even worsen

[23]In the study by Rabanal (2007), an explanation for the strength of the real cost channel is provided. It is assumed that a fraction of representative firms need to borrow to cover their wage bills for production, while the remaining firms can produce without incurring any payment obligations. When γ_r approaches 0, it signifies a weak real cost channel, as only a small fraction of firms rely on borrowing for their wage obligations. On the other hand, when γ_r is closer to 1, it indicates a relatively strong real cost channel, as a larger proportion of firms depend on borrowing to meet their wage payments.

the sunspot equilibrium with inflation $\pi_S^c < \pi_S^s$. On the third panel, this is the situation we have discussed above, and the strong real cost channel can make sunspots irrelevant. For a given sunspot persistence p^s, I find that $\gamma_r > \overline{\gamma_r}$ in the simulation such that the expectations- driven LT is no longer relevant under possible assumptions.[24]

There is a caveat to the real cost channel since a weak strength can even worsen the sunspot equilibrium. Intuitively, households tend to save instead of consuming in recessions. A weak real cost channel can increase the real marginal cost through the expected inflation while the lessened short-term deflation in equilibrium is not enough. In this case, households have to save more to obtain the optimal expected return on savings due to expected inflation by examining Eq. (13).[25] In contrast, a strong enough real cost channel can make up the short-run deflation fully. In that way, deflationary expectations can not be an equilibrium outcome, and thus the sunspot traps can disappear.

3.4 Comparison with the nominal cost channel

How robust are the primary findings of this paper if we consider the nominal rather than the real cost channel, as typically modeled in Ravenna & Walsh (2006)?[26] I follow Beaudry et al. (2022) and Nie (2023) to show the semi-linearized NKPC with the nominal cost channel:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \left[\gamma_y y_t + \gamma_r (R_t + \log(\beta)) \right]. \tag{14}$$

One important observation is that during the ZLB, nominal interest rates

[24] The threshold strength of the real cost channel is $\overline{\gamma_r} = \frac{\sigma_r p^s \kappa \gamma_y - (1-p^s)(1-\beta p^s)}{(1-p^s)\kappa p^s}$, with which the NKPC and the Euler equation become parallel and the sunspot equilibrium is no longer supported.

[25] As in Nie & Roulleau-Pasdeloup (2023), it explains at length that with not enough inflation make-up in sunspot equilibrium, households have to increase savings.

[26] See also Surico (2008) and Gilchrist & Zakrajšek (2015).

are constrained to be fixed at zero, rendering the nominal cost channel ineffective in marginally influencing the inflation rate. In that way, the nominal cost channel does not have the ability to alter the effective slope of the NKPC during recessions, although it can still lead to shifts in the NKPC. As previously highlighted, the effective slopes at the ZLB are crucial in ensuring the irrelevance of the expectations-driven LT under certain assumptions. In that case, this nominal cost channel can not decrease the possibility of expectations-driven LT dynamics.

4 Monetary policy with the real cost channel

In this section, the real cost channel is incorporated into the standard NK model to examine the effects of monetary policy. Specifically, the focus is on discussing the neo-Fisherian effects and the impact of forward guidance in the presence of the real cost channel. It is noteworthy that the real cost channel has the potential to make the neo-Fisherian effects irrelevant. Additionally, forward guidance can lead to the economy falling into low-inflation traps.

4.1 Neo-Fisherian effects: short-run expansionary inflationary interest rate increases

How does the real cost channel affect neo-Fisherian effects, which are defined as short-run expansionary-inflationary interest rate increases? Following a tractable way in Bilbiie (2021), I assume the central bank sets the interest rate according to an exogenous process r_t^n which follows a two-state Markov process with persistence p. More specifically, the exogenous interest rate process r_t^n starts above the steady state at $r^n > 0$ but converges back to the steady state $r^n = 0$ with persistence p. With this in mind, we have the solutions of inflation with Definition

5:

$$\pi_S = \frac{\mathcal{I}_{EE}(-r^n) - \sigma_r \frac{\log(\beta)}{1-p} - \mathcal{I}^c_{PC}}{\mathcal{S}^{c,z}_{PC} - \mathcal{S}^z_{EE}}. \tag{15}$$

In Bilbiie (2021), the condition that determines the possibility of both neo-Fisherian effects and expectations-driven LT dynamics in the standard NK model is that the slope of the NKPC at the ZLB is lower than its counterpart of the Euler equation. This condition can be straightforwardly verified by examining Eq. (15): If this condition is satisfied, it implies that short-run inflationary interest rate increases can lead to expansionary effects.

As detailed in Appendix P, the irrelevance of expectations-driven LTs implies a specific relationship between the effective slopes of the AS and AD curves:

$$\mathcal{S}^z_{EE} < \mathcal{S}^{c,z}_{PC}.$$

In this scenario, if the expectations-driven LT is no longer relevant, it implies that the condition stated in Bilbiie (2021) is no longer valid. This inference becomes clear upon analyzing Equation (15). It can be observed that an increase in interest rates might actually lead to a decrease in inflation, thereby diminishing the relevance of neo-Fisherian effects, a concept explored in studies such as Cochrane (2016), Garín et al. (2018), and Bilbiie (2021). I summarize the main result in Proposition 5.

Proposition 5. *Under the real cost channel, if the expectations-driven liquidity trap is irrelevant, the neo-Fisherian effects can disappear.*

Proof. See Appendix R. □

How robust are the primary findings of this paper if we consider a nominal rather than real cost channel? With a nominal cost channel, the possibility of the expectations-driven LT is not reduced, and neo-Fisherian effects could still exist. As Ali & Qureshi (2022) note, neo-Fisherian effects are much more pronounced with a nominal cost channel for a given persistence of the shock. We can provide an intuitive justification using our tractable model. Eq. (15) incorporates the term $\mathcal{I}^{c}_{PC}$, which is specific to the cost channel and significantly amplifies the effects compared to the standard NK model.

4.2 Forward guidance

What are the effects of forward guidance (FG) in the model with the real cost channel? In this analysis, I adopt a similar approach to Bilbiie (2019) to incorporate FG into the model. Specifically, I assume that the central bank commits to maintaining a zero interest rate policy with a probability q in the medium run (where we label it with the subscript F) after the short-run LT ends. This assumption allows us to derive the equilibrium condition in the medium run:

$$y_F = \frac{(1-\beta q)\sigma_r}{(1-q)(1-\beta q+\kappa\gamma_r q)-\sigma_r q\kappa\gamma_y}[-\log(\beta)] \tag{16}$$

$$\pi_F = \frac{\kappa\gamma_r \log(\beta)+\kappa\gamma_y y_F}{1-\beta q+\kappa\gamma_r q} \tag{17}$$

This result contrasts sharply with Bilbiie (2021), where announcing zero interest rates after encountering the ZLB generates future economic expansion and inflation in the expectations-driven LT. However, as Eq. (17) demonstrates, the actual inflation π_F should remain deflationary considering the real cost channel.[27] Although Eq. (16) indicates the economy continues expanding.

Our findings are consistent with the simulation results presented in Beaudry et al. (2022), which demonstrate that maintaining interest rates below standard policy levels following a period of being at the ZLB with inflation below the target could have adverse consequences. In the standard model, FG can create

[27] In this context, one can observe that terms involving γ_r dominate over terms involving γ_y in Eq. (17) if the condition $\gamma_y < \frac{\gamma_r(1-q)}{\sigma_r}$ is satisfied. This condition implies that the real cost channel is significantly strong for a given value of γ_y.

expectations of future inflation that help offset the short-run deflation and bring inflation back to target levels as in Nie & Roulleau-Pasdeloup (2023). However, with the incorporation of the real cost channel, the effects of FG can become deflationary. In this case, FG is unable to effectively offset the short-run deflationary pressures and may even exacerbate the deflationary dynamics. In this scenario, such a policy could potentially lead the economy into a low inflation trap, characterized by persistently low inflation rates.

4.3 Welfare analysis

If a weak real cost channel does not make expectations-driven LT dynamics irrelevant, it is necessary to set $\pi_S = y_S = 0$ in the expectations-driven LT and the interest rate can be determined which is the same as in the standard NK model (Bilbiie (2021)) given the loss function as shown in Appendix S. However, if the real cost channel is sufficiently strong, it can make the expectations-driven LT irrelevant. In this case, the economy has a unique equilibrium, which is the intended steady state with $\pi_S = y_S = 0$, and welfare is maximized.

5 Conclusions

In the presence of the zero lower bound, even in the absence of fundamental shocks, a shift in confidence can lead to sufficient deflationary pressures, triggering expectations-driven traps in the standard sticky-price New Keynesian model. To address this issue, this paper introduces a tractable New Keynesian model that incorporates the real cost channel. The findings reveal that the real cost channel can effectively reduce the occurrence of expectations-driven liquidity traps by rotating the Phillips Curve. The mechanism behind this phenomenon is attributed to the strong influence of the real cost channel during episodes of the lower bound, which counteracts the short-run deflation resulting from a drop in confidence. Consequently, equilibrium conditions entail actual inflation, making deflationary expectations can not be an equilibrium outcome.

Additionally, I show that a weak real cost channel may even worsen the sunspot equilibrium. I also show this real cost channel is theoretically appealing since it helps ensure model equilibrium existence. Moreover, I investigate the impact of monetary policy in the presence of the real cost channel, demonstrating its potential to make the neo-Fisherian effects irrelevant. When forward guidance is incorporated with the real cost channel, the economy is susceptible to falling into low-inflation traps.

This study, anchored in a New Keynesian framework and employing log-linearized aggregate demand and supply equations, provides substantive insights in the realm of a targeted steady state characterized by zero inflation and zero output gap. Nevertheless, the scope of our analysis is primarily 'local', concentrating on linearized conditions within a narrowly defined vicinity of this steady state, under the specific condition of a zero nominal interest rate typical of liquidity trap scenarios. Consequently, the incorporation of a 'global' analytical framework, as exemplified in recent works such as Piergallini (2023), which encompasses nonlinearities and the potential for multiple steady states, represents a compelling trajectory for future research.

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Online Appendix

A The Model Setup

Time is discrete and there is no government spending.

A.1 Aggregate Demand Side

The representative household has the below preferences:

$$\begin{aligned}\mathcal{U}(C_t, L_t) &= u(C_t) - v(L_t) \\ &= \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{L_t^{1+\eta}}{1+\eta}, \quad \chi, \eta > 0\end{aligned}$$

where households work L_t hours, consume amount C_t, and trade government bonds B_t.

The budget constraint is,

$$C_t + \frac{B_t}{P_t} = W_t L_t + \mathcal{D}_t - \mathcal{T}_t + \exp(\Im_{t-1}) \frac{1 + R_{t-1}}{P_t} B_{t-1}.$$

where $\Im_t$ is a "risk premium" shock.

The optimal aggregate (individual) labor price is written as:

$$W_t = \frac{L_t^{\eta} \chi}{(C_t)^{-\sigma}},$$

I can obtain the Euler equation with the first-order condition (FOC) of the maximization program:

$$(C_t)^{-\sigma} = \beta \exp(\Im_t) \mathbb{E}_t \left\{ (C_{t+1})^{-\sigma} \frac{1 + R_t}{1 + \Pi_{t+1}} \right\}.$$

The semi-linearized equilibrium Euler equation by approximating around the steady state can be read. That is, all lowercase format variables are the log deviations from steady state ($x_t = \log(X_t) - \log(X)$):

$$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\sigma}\left[R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1} - \epsilon_t\right].$$

where $\epsilon_t \equiv -\Im_t$ is the demand shock (also can be seen as interest rate shock) and R_t is the nominal interest rate in level.
The following resource constraint is placed in this economy:

$$y_t = c_t,$$

Furthermore the Euler equation is expressed as:

$$y_t = \mathbb{E}_t y_{t+1} - \sigma_r\left[R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1} - \epsilon_t\right],$$

where $\sigma_r \equiv \frac{1}{\sigma}$.

A.2 Aggregate Supply Side

Each monopolist will use only the basic input Y_t^B for production and follow the one-to-one technology. Therefore, the price of this basic input is the marginal cost. The basic input is produced by representative firms with the following Leontief production function:

$$Y_t^B = \min(aN_t, bM_t),$$

where M_t is the final goods, and N_t is the labor.
The unit price of the final goods attached to the production is P_t. As in Beaudry et al. (2022), we assume that the basic input representative should borrow D_{t+1} to pay for the input M_t at the risk-free nominal rate i_t for the production, *i.e.*

borrowing costs.[28] In this case, firms should produce, sell the product, pay wages $W_t P_t$, pay back the debt in the previous period, and distribute the dividends Π_t. One can show the budget constraint of firms at time t by simply assuming zero profits in equilibrium below:

$$D_{t+1} + P_t^B Y_t^B = W_t P_t N_t + (1 + i_{t-1}) D_t + P_t M_t,$$

where P_t^B is the basic input price, and $D_{t+1} = P_t M_t$.
In that way, the profit Π_t can be shown as:

$$\Pi_t = P_t^B Y_t^B - W_t P_t N_t - (1 + i_{t-1}) P_{t-1} M_{t-1}.$$

We further assume that firms maximize the expected discounted sum of real profit $\frac{\Pi_t}{P_t}$ with a discount parameter β. In this case, the first-order condition can be shown:

$$P_t^B = \left(\frac{1}{a} W_t + \frac{\beta}{b} \mathbb{E}_t \frac{1 + i_t}{1 + \pi_{t+1}} \right) P_t,$$

Where π_{t+1} is the next period's inflation rate. Thus, one can obtain the (real) marginal cost of the basic input:

$$MC_t = \frac{W_t}{a} + \frac{\beta}{b} \mathbb{E} \left[\frac{1 + i_t}{1 + \pi_{t+1}} \right].$$

In logs, one can show the linearized equilibrium

$$mc_t = \gamma_w (w_t) + \gamma_r (R_t + \log(\beta) - \mathbb{E} \pi_{t+1}),$$

where $\gamma_w = \frac{\frac{1}{a} W}{\frac{1}{a} W + \frac{\beta}{b} \frac{1+i}{1+\pi}}$, $\gamma_r = \frac{\frac{\beta}{b} \frac{1+i}{1+\pi}}{\frac{1}{a} W + \frac{\beta}{b} \frac{1+i}{1+\pi}}$, and R_t is the nominal interest rate in level.

[28] The borrowing cost is crucial in modeling since it introduces the real cost channel in the Phillips Curve. The advantage of this introduced real cost channel method as in Beaudry et al. (2022) is that it allows setting arbitrarily the elasticity of marginal cost rate with regard to wage and interest rate. Please see Beaudry et al. (2022) for a comprehensive comparison between the model with the nominal and the real cost channel.

On the other hand, the optimal labor supply reads:

$$\frac{v'(N_t)}{u'(C_t)} = W_t.$$

Other parts are standard, and the New Keynesian Phillips curve yields:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \; mc_t.$$

By log condition, I have the semi-linearized equilibrium

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \left[\gamma_y y_t + \gamma_r (R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1}) \right],$$

where $\gamma_y = \gamma_w \left(\frac{N v''(N)}{v'(N)} - \frac{C u''(C)}{u'(C)} \right)$.
In this case, this model can collapse to the standard model if we assume $\gamma_r = 0$ below:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \gamma_y y_t.$$

B Proof for proposition 1

In liquidity traps, the Phillips Curve with the real cost channel can be shown as

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \left[\gamma_y y_t + \underbrace{\gamma_r (-\mathbb{E}_t \pi_{t+1})}_{\text{real cost channel}} \right] + tip$$

Without the real cost channel, it is assumed that $\gamma_r = 0$, deflationary pressures can trigger a ZLB state. Since nominal interest rates are zero, deflation can result in higher real rates which can imply lower demand via the AD curve, which in turn leads to deflation via the AS curve.
However, the real cost channel can imply higher expected marginal costs (higher expected inflation) via the AS curve, in equilibrium, which can imply short-run

inflation through rational expectation and sticky prices. The higher marginal costs due to the real cost channel can counteract short-run deflation.

C Proof for proposition 2

Using Definitions (1)-(3), we can show the model in the canonical form representation below:

$$\mathbb{E}_t \mathbf{X_{t+1}} = \mathbf{A}\mathbf{X_t} + \mathbf{B}Z_t,$$

where $\mathbf{X_t} = [y_t \ \pi_t]^{\mathrm{T}}$, $Z_t = [\epsilon_t]^{\mathrm{T}}$ and $\mathbf{A}$ and $\mathbf{B}$ are conformable matrices. Since the shocks have no impact on whether the equilibrium is unique or not, we will assume $\epsilon_t = 0$ for convenience.

Using equations above, the matrix $\mathbf{A}$ can be written as:

$$\begin{bmatrix} 1 + \frac{\sigma_r \kappa \gamma_y}{\beta - \kappa\gamma_r} & \sigma_r \phi_\pi - \frac{\sigma_r(1-\kappa\gamma_r\phi_\pi)}{\beta-\kappa\gamma_r} \\ -\frac{\kappa\gamma_y}{\beta-\kappa\gamma_r} & \frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r} \end{bmatrix}.$$

Whether we get a unique equilibrium or not depends on the values taken by the eigenvalues of matrix $\mathbf{A}$. The NK model has equilibrium determinacy if the matrix $\mathbf{A}$ has both eigenvalues which are outside the unit circle. A standard result from linear algebra is that the two eigenvalues of matrix $\mathbf{A}$ are the solution to the following second-order polynomial:

$$\mathbf{P}(\lambda) = \lambda^2 - tr(\mathbf{A})\lambda + \det(\mathbf{A}),$$

where the trace and determinant are given by

$$tr(\mathbf{A}) = 1 + \frac{\sigma_r \kappa \gamma_y}{\beta - \kappa \gamma_r} + \frac{1 - \kappa\gamma_r\phi_\pi}{\beta - \kappa\gamma_r}, \quad \det(\mathbf{A}) = \frac{1 - \kappa\gamma_r\phi_\pi + \kappa\gamma_y\sigma_r\phi_\pi}{\beta - \kappa\gamma_r}.$$

In this paper, we simply assume $\beta - \kappa\gamma_r > 0$ as in Beaudry et al. (2022). By assuming both roots are lower (or higher) than a unit, we know $\det(\mathbf{A}) > 1$

and

$$\begin{aligned}
1-\kappa\gamma_r\phi_\pi+\kappa\gamma_y\sigma_r\phi_\pi &> \beta-\kappa\gamma_r \\
\phi_\pi(\kappa\gamma_y\sigma_r-\kappa\gamma_r) &> \beta-\kappa\gamma_r-1 \\
\phi_\pi &> \frac{\beta-\kappa\gamma_r-1}{\kappa\gamma_y\sigma_r-\kappa\gamma_r}.
\end{aligned}$$

From the definition of the polynomial, both roots are outside the unit circle if

$$\mathbf{P}(-1)>0 \quad \& \quad \mathbf{P}(1)>0.$$

In this case, one can re-write it as

$$\begin{aligned}
\det(\mathbf{A})+tr(\mathbf{A}) &> -1 \\
\det(\mathbf{A})-tr(\mathbf{A}) &> -1.
\end{aligned}$$

The first condition can hold if

$$\begin{aligned}
1+\frac{\sigma_r\kappa\gamma_y+1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r}+\det(\mathbf{A}) &> -1 \\
\frac{\sigma_r\kappa\gamma_y+1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r} &> -3 \\
\kappa\gamma_r\phi_\pi &< 3(\beta-\kappa\gamma_r)+\sigma_r\kappa\gamma_y+1 \\
\phi_\pi &< \frac{3(\beta-\kappa\gamma_r)+\sigma_r\kappa\gamma_y+1}{\kappa\gamma_r}.
\end{aligned}$$

The second condition can be satisfied if

$$\begin{aligned}
\frac{1-\kappa\gamma_r\phi_\pi+\kappa\gamma_y\sigma_r\phi_\pi}{\beta-\kappa\gamma_r}-1-\frac{\sigma_r\kappa\gamma_y}{\beta-\kappa\gamma_r}-\frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r} &> -1 \\
\frac{\kappa\gamma_y\sigma_r\phi_\pi}{\beta-\kappa\gamma_r} &> \frac{\sigma_r\kappa\gamma_y}{\beta-\kappa\gamma_r} \\
\phi_\pi &> 1.
\end{aligned}$$

Thus we can conclude the equilibrium determinacy condition for ϕ_π:

$$\max\left(1, \frac{\beta - \kappa\gamma_r - 1}{\kappa\gamma_y\sigma_r - \kappa\gamma_r}\right) < \phi_\pi < \frac{3(\beta - \kappa\gamma_r) + \sigma_r\kappa\gamma_y + 1}{\kappa\gamma_r}.$$

It is easy to prove that $\beta - \kappa\gamma_r - 1 < \kappa\gamma_y\sigma_r - \kappa\gamma_r$, and the equilibrium determinacy condition for ϕ_π can be rewritten as

$$1 < \phi_\pi < \frac{3(\beta - \kappa\gamma_r) + \sigma_r\kappa\gamma_y + 1}{\kappa\gamma_r}.$$

On the other hand, if one root is lower than a unit and one root is higher than a unit, we have the condition for the polynomial.

$$\begin{aligned} \det(\mathbf{A}) + tr(\mathbf{A}) &< -1 \\ \det(\mathbf{A}) - tr(\mathbf{A}) &< -1, \end{aligned}$$

where the second condition tells $\phi_\pi < 1$. Further, one can show $\det(\mathbf{A}) < -1$:

$$\det(\mathbf{A}) = \frac{1 - \kappa\gamma_r\phi_\pi + \kappa\gamma_y\sigma_r\phi_\pi}{\beta - \kappa\gamma_r} < -1,$$

where $\gamma_r \in [0,1]$ and κ is a small number in general. If $\phi_\pi < 1$, there is a contradiction since $1 - \kappa\gamma_r\phi_\pi$ should be positive.
For the nominal cost channel case, the the matrix $\mathbf{A}$ can be written as:

$$\begin{bmatrix} 1 + \frac{\sigma_r\kappa\gamma_y}{\beta} & \sigma_r\phi_\pi - \frac{\sigma_r(1-\kappa\gamma_r\phi_\pi)}{\beta} \\ -\frac{\kappa\gamma_y}{\beta} & \frac{1-\kappa\gamma_r\phi_\pi}{\beta} \end{bmatrix}.$$

Similarly, one can show the equilibrium determinacy condition for ϕ_π:

$$\max\left(1, \frac{\beta - 1}{\kappa\gamma_y\sigma_r - \kappa\gamma_r}\right) < \phi_\pi < \frac{3\beta + \sigma_r\kappa\gamma_y + 1}{\kappa\gamma_r}.$$

It is easy to prove that $\beta - 1 < \kappa\gamma_y\sigma_r - \kappa\gamma_r$, and the equilibrium determinacy condition for ϕ_π can be rewritten as

$$1 < \phi_\pi < \frac{3\beta + \sigma_r\kappa\gamma_y + 1}{\kappa\gamma_r}.$$

D Upward Sloping Assumption

According to Definition 4, in normal times, I can reproduce the solutions for y_S and π_S as follows:

$$\begin{aligned} y_S &= \frac{\sigma_r(1 - \beta p + \kappa\gamma_r p - \kappa\gamma_r\phi_\pi)}{(1-p)(1 - \beta p + \kappa\gamma_r p - \kappa\gamma_r\phi_\pi) + \sigma_r\kappa\gamma_y(\phi_\pi - p)}\epsilon_S \\ \pi_S &= \frac{\sigma_r\kappa\gamma_y}{(1-p)(1 - \beta p + \kappa\gamma_r p - \kappa\gamma_r\phi_\pi) + \sigma_r\kappa\gamma_y(\phi_\pi - p)}\epsilon_S. \end{aligned}$$

If the Phillips Curve is upward sloping in normal times, which means the effective slope of Phillips Curve is positive:

$$\begin{aligned} & 1 - \beta p + \kappa\gamma_r p - \kappa\gamma_r\phi_\pi > 0 \\ \Leftrightarrow\ & p < \frac{1 - \kappa\gamma_r\phi_\pi}{\beta - \kappa\gamma_r}, \end{aligned}$$

where the second line using the assumption $\kappa\gamma_r < \beta$ as in Beaudry et al. (2022). In this case, there is a threshold $\overline{p}^u = \frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r}$.

E Calibration Parameters

In Table 1, the parameterization of the cost channel is grounded in the calibration framework established by Beaudry et al. (2022). Beaudry et al. (2022) fundamentally presupposes $\kappa = 1$, a premise that suggests an elasticity of inflation relative to the real interest rate of 0.2. This inference is in harmony with the estimates derived from my research. It is pertinent to note that deploying alternative parameter configurations of γ_r and γ_y generates results that are qualitatively analogous. These supplementary outcomes are accessible upon request. Additionally, my methodology aligns with the calibrated approaches advocated by Mertens & Williams (2019), Bergholt et al. (2020), and Nie & Roulleau-Pasdeloup (2023) for other standard model parameters. Concerning the shock persistence, the model adopts $p = \frac{\overline{p}^u + \overline{p}}{2}$. This specification is meticulously chosen to ensure that the model not only facilitates a sunspot equilibrium within the conventional New Keynesian framework but also adheres to the upward-sloping prerequisite of the New Keynesian Phillips Curve.

Table 1: The calibrated parameter values

Discount factor	$\beta = 0.99$
Preference parameter	$\eta = 1$
Preference parameter	$\sigma_r = 1$
Elasticity of inflation w.r.t. output gap	$\kappa \times \gamma_y = 0.04$
Elasticity of inflation w.r.t. interest rate	$\kappa \times \gamma_r = 0.2$
Inflation feedback parameter	$\phi_\pi = 1.5$
Persistence	$p = \frac{\overline{p}^u + \overline{p}}{2}$

Notes: I follow Beaudry et al. (2022) to set the value for elasticity of inflation w.r.t. output gap/inflation. We can obtain qualitatively identical results with different sets of γ_r & γ_y. I follow Mertens & Williams (2019),Bergholt et al. (2020), and Nie & Roulleau-Pasdeloup (2023) to use a standard calibrated method for other parameters. $\bar{p}$ is the threshold such that there exists the expectations-driven LT in the standard model without the real cost channel. $\overline{p}^u$ is the threshold such that the AS curve is upward-sloping in the model with the real cost channel.

F The expressions in Definition 5

The NKPC is shown below:

$$y_S = \begin{cases} \frac{1-\beta p+\kappa\gamma_r p-\kappa\gamma_r\phi_\pi}{\kappa\gamma_y}\pi_S & \text{if } \pi_S > \frac{\log(\beta)}{\phi_\pi} \\ \frac{1-\beta p+\kappa\gamma_r p}{\kappa\gamma_y}\pi_S - \frac{\gamma_r}{\gamma_y}\log(\beta) & \text{if } \pi_S \leq \frac{\log(\beta)}{\phi_\pi}. \end{cases}$$

One can formally show the Euler equations below:

$$y_S = \begin{cases} -\sigma_r \frac{\phi_\pi - p}{1-p} \pi_S + \sigma_r \frac{\epsilon_S}{1-p} & \text{if } \pi_S > \frac{\log(\beta)}{\phi_\pi} \\ \frac{\sigma_r p}{1-p} \pi_S + \sigma_r \frac{\epsilon_S - \log(\beta)}{1-p} & \text{if } \pi_S \leq \frac{\log(\beta)}{\phi_\pi}. \end{cases}$$

G Proofs of Lemma 1

The Euler equation in standard NK model:

$$y_t = \mathbb{E}_t y_{t+1} - \sigma[R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1} - \epsilon_t]$$

The NKPC is below:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \gamma_y y_t$$

Using the simple two-state Markov Chain, we have $\mathbb{E}_S \pi_{t+1} = p\pi_S$ and $\mathbb{E}_S y_{t+1} = p y_S$. We can write the Euler equation at the ZLB below:

$$y_S = -\frac{\sigma_r p}{1-p} \pi_S + \sigma_r \frac{\epsilon_S - \log(\beta)}{1-p}.$$

One can yield the NKPC:

$$y_S = \frac{1 - \beta p}{\kappa \gamma_y} \pi_S.$$

Thus, the effective slope of AD/Euler curve is:

$$\mathcal{S}^z_{EE} = \sigma_r \frac{p}{1-p}.$$

And the effective slope of AS/NKPC curve is:

$$\mathcal{S}^z_{PC} = \frac{1 - \beta p}{\kappa \gamma_y}.$$

H Proofs of Lemma 2

The standard textbook New Keynesian Phillips Curve without a cost channel can read:

$$\pi_t = \beta \mathbb{E} \pi_{t+1} + \kappa \gamma_y y_t.$$

In this case, the Phillips Curve can be re-written as

$$y_S = \frac{1 - \beta p}{\kappa \gamma_y} \pi_s$$

If the Phillips Curve is upward-sloping throughout time periods. If there is an absence of demand shock and the effective slope of AS curve is lower than AD curve, *i.e.*:

$$(1 - p)(1 - \beta p) < \sigma_r p \kappa \gamma_y.$$

We can have the sunspot equilibrium featuring $\pi_S < 0, y_S < 0$: *i.e.* there exists a threshold $\overline{p}$:

$$\begin{aligned}
\overline{p} &= \frac{(\beta + 1 + \sigma_r \kappa \gamma_y) - \sqrt{(1 + \beta + \sigma_r \kappa \gamma_y)^2 - 4\beta}}{2\beta} \\
&< \frac{(\beta + 1 + \sigma_r \kappa \gamma_y) - (-\beta + 1 + \sigma_r \kappa \gamma_y)}{2\beta} \\
&= 1
\end{aligned}$$

where there is $\bar{p} \in (0, 1)$ to trigger the expectations-driven LT to make $y_S < 0$ in the absence of demand shock. That being said, there is a sunspot equilibrium if $p > \bar{p}$. Note that if the the demand shock is very large, it can shift AD curve down so much that there is no intersection in the AS and AD curves which means no equilibrium in this economy.

I Proofs of Lemma 3

It is straightforward to use Appendix G and one can combine AS/AD curves to obtain the solution at the ZLB:

$$y_S = \frac{(1-\beta p)\sigma_r}{(1-p)(1-\beta p) - \sigma_r p\kappa\gamma_y}(\epsilon_S - \log(\beta))$$
$$\pi_S = \frac{\kappa\gamma_y}{(1-p)(1-\beta p) - \sigma_r p\kappa\gamma_y}(\epsilon_S - \log(\beta)),$$

where $p < \overline{p}$.

On the other hand, the sunspot equilibrium emerges without fundamental shocks ϵ_S if $p > \overline{p}$ and the solution can be derived with AS/AD curves:

$$y_S = \frac{(1-\beta p)\sigma_r}{(1-p)(1-\beta p) - \sigma_r p\kappa\gamma_y}(-\log(\beta))$$
$$\pi_S = \frac{\kappa\gamma_y}{(1-p)(1-\beta p) - \sigma_r p\kappa\gamma_y}(-\log(\beta)),$$

where $p > \overline{p}$.

J Proofs of Lemma 4

According to Definition 4, under a ZLB, the Phillips Curve is

$$y_S = \frac{1-\beta p + \kappa\gamma_r p}{\kappa\gamma_y}\pi_S - \frac{\gamma_r}{\gamma_y}\log(\beta),$$

The Euler equation at the ZLB is:

$$y_S = -\frac{\sigma_r p}{1-p}\pi_S + \sigma_r\frac{\epsilon_S - \log(\beta)}{1-p}.$$

Thus, the effective slope of AD/Euler curve is:

$$\mathcal{S}^z_{EE} = \sigma_r\frac{p}{1-p}.$$

And the effective slope of AS/NKPC curve is:

$$\mathcal{S}_{PC}^{c,z} = \frac{1-\beta p + \kappa\gamma_r p}{\kappa\gamma_y}.$$

K Proofs of Proposition 3

One can show that

$$\mathcal{S}_{PC}^{c,z} > \mathcal{S}_{PC}^{z}.$$

Thus, the effective slope of the AS curve at the ZLB episode is higher with the real cost channel.

L Proofs of Lemma 5

According to Definition 4, under a ZLB, the Phillips Curve is

$$y_S = \frac{1-\beta p + \kappa\gamma_r p}{\kappa\gamma_y}\pi_S - \frac{\gamma_r}{\gamma_y},$$

where the effective slope is $\frac{1-\beta p+\kappa\gamma_r p}{\kappa\gamma_y}$. It is easy to check this slope is increasing in the elasticity of the marginal cost w.r.t the interest rate γ_r which can be seen as the strength of the real cost channel.

If the flat Phillips Curve is upward-sloping throughout time periods, which means that the effective slope of the Phillips Curve is always positive:

$$\begin{aligned} 1 - \beta p + \kappa\gamma_r p - \kappa\gamma_r\phi_\pi &> 0 \\ \Leftrightarrow p &< \frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r}. \end{aligned}$$

In this case, in normal times, it is easy to check that the only equilibrium is the target steady state (*i.e.* $y_S = \pi_S = 0$) with no demand shock.

While assuming that the demand shock is large enough to trigger the fundamental-driven ZLB, I reproduce the following solutions for y_S and π_S:

$$
\begin{aligned}
y_S &= \frac{(1-\beta p+\kappa\gamma_r p)\sigma_r(\epsilon_S-\log(\beta))+\kappa\gamma_r\sigma_r p\log(\beta)}{(1-p)(1-\beta p+\kappa\gamma_r p)-\sigma_r p\kappa\gamma_y} \\
\pi_S &= \frac{\kappa\gamma_y\sigma_r(\epsilon_S-\log(\beta))}{(1-p)(1-\beta p+\kappa\gamma_r p)-\sigma_r p\kappa\gamma_y}+\frac{\kappa\gamma_y\kappa\gamma_r\sigma_r p\log(\beta)}{[(1-p)(1-\beta p+\kappa\gamma_r p)-\sigma_r p\kappa\gamma_y](1-\beta p+\kappa\gamma_r p)} \\
&\quad +\frac{\kappa\gamma_r\log(\beta)}{1-\beta p+\kappa\gamma_r p}.
\end{aligned}
$$

If there is no expectations-driven liquidity trap (LT) in the absence of demand shock, the requirement is below:

$$
\begin{aligned}
y_S &= \frac{(1-\beta p)\sigma_r(-\log(\beta))}{(1-p)(1-\beta p+\kappa\gamma_r p)-\sigma_r p\kappa\gamma_y} > 0 \\
&\Leftrightarrow \mathcal{D}(p) = (1-p)(1-\beta p+\kappa\gamma_r p)-\sigma_r p\kappa\gamma_y > 0
\end{aligned}
$$

Similar to the result in Appendix H, one can show the threshold $\overline{p}^c$ by making $\mathcal{D}(p)=0$:

$$
\overline{p}^c = \frac{(\beta+1+\sigma_r\kappa\gamma_y-\kappa\gamma_r)-\sqrt{(1+\beta+\sigma_r\kappa\gamma_y-\kappa\gamma_r)^2-4(\beta-\kappa\gamma_r)}}{2(\beta-\kappa\gamma_r)}.
$$

This expression is isomorphic to the expression of $\overline{p}$ and to have a study on the monotonicity, we can have a general expression $\overline{p}(x)$:

$$
\overline{p}(x) = \frac{(1+\sigma_r\kappa\gamma_y+x)-\sqrt{(1+\sigma_r\kappa\gamma_y+x)^2-4x}}{2x}.
$$

We then show the derivative of $\overline{p}(x)$ w.r.t. x:

$$
\frac{\partial\overline{p}(x)}{\partial x} \propto \frac{(1+\sigma_r\kappa\gamma_y)\left[(1+\sigma_r\kappa\gamma_y+x)-\sqrt{(1+\sigma_r\kappa\gamma_y+x)^2-4x}\right]-2x}{\sqrt{(1+\sigma_r\kappa\gamma_y+x)^2-4x}}\frac{1}{x^2}.
$$

We first assume $\frac{\partial \overline{p}(x)}{\partial x} < 0$ and it should meet the below

$$
\begin{aligned}
(1+\sigma_r\kappa\gamma_y)\left[(1+\sigma_r\kappa\gamma_y+x)-\sqrt{(1+\sigma_r\kappa\gamma_y+x)^2-4x}\right]-2x<0& \\
(1+\sigma_r\kappa\gamma_y)(1+\sigma_r\kappa\gamma_y+x)-2x &< (1+\sigma_r\kappa\gamma_y)\sqrt{(1+\sigma_r\kappa\gamma_y+x)^2-4x} \\
4x^2-4x(1+\sigma_r\kappa\gamma_y)(1+\sigma_r\kappa\gamma_y+x) &< (1+\sigma_r\kappa\gamma_y)^2 4x \\
4x^2 &< 4x^2(1+\sigma_r\kappa\gamma_y).
\end{aligned}
$$

In this case, it is true that $\frac{\partial \overline{p}(x)}{\partial x} < 0$. Since $\beta > (\beta-\kappa\gamma_r)$, $\overline{p} < \overline{p}^c$.

M Proofs of Proposition 4

Using the result in Appendix L, One can yield a condition for γ_y to secure $\mathcal{D}(p) > 0$:

$$
\begin{aligned}
\mathcal{D}(p) &= (1-p)(1-\beta p+\kappa\gamma_r p)-\sigma_r p\kappa\gamma_y \\
&> \left(1-\frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r}\right)\left(1-\beta\frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r}+\kappa\gamma_r\frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r}\right)-\sigma_r\frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r}\kappa\gamma_y \\
&= (\beta-\kappa\gamma_r-1+\kappa\gamma_r\phi_\pi)[\beta\kappa\gamma_r\phi_\pi-\kappa\gamma_r+\kappa\gamma_r(1-\kappa\gamma_r\phi_\pi)]-\sigma_r(1-\kappa\gamma_r\phi_\pi)\kappa\gamma_y > 0 \\
\gamma_y &< \frac{(\beta-\kappa\gamma_r-1+\kappa\gamma_r\phi_\pi)(\beta\gamma_r\phi_\pi-\kappa\gamma_r^2\phi_\pi)}{\sigma_r(1-\kappa\gamma_r\phi_\pi)} = \Phi(\gamma_r),
\end{aligned}
$$

where the second line we assume $p = \bar{p}^c$ due to monotonicity.

At the ZLB episode, one can compare the effective slope of the AS/AD curves:

$$
\frac{1-\beta p+\kappa\gamma_r p}{\kappa\gamma_y} > \sigma_r\frac{p}{1-p},
$$

where we use the condition $\gamma_y < \frac{(\beta-\kappa\gamma_r-1+\kappa\gamma_r\phi_\pi)(\beta\gamma_r\phi_\pi-\kappa\gamma_r^2\phi_\pi)}{\sigma_r(1-\kappa\gamma_r\phi_\pi)}$. This means the effective slope of the AS curve is larger than the effective slope of the AD curve at the ZLB.

In addition, one can check the monotonicity of $\Phi(\gamma_r)$ w.r.t. γ_r:

$$\frac{\partial \Phi(\gamma_r)}{\partial \gamma_r} \propto \frac{\partial \frac{\beta-\kappa\gamma_r}{1-\kappa\gamma_r\phi_\pi}}{\partial \gamma_r}$$
$$\propto \kappa(\phi_\pi \beta - 1) > 0.$$

Therefore $\Phi(\gamma_r)$ increases in γ_r.

N Additional Figures for Supply Shocks

In this part, we simply assume there is a supply shock in the NKPC as in Figure 5:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa[\gamma_y y_t + \gamma_r (R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1})] + \mu_t,$$

where μ_t is the temporary supply shock.

O Derivation for equation (13)

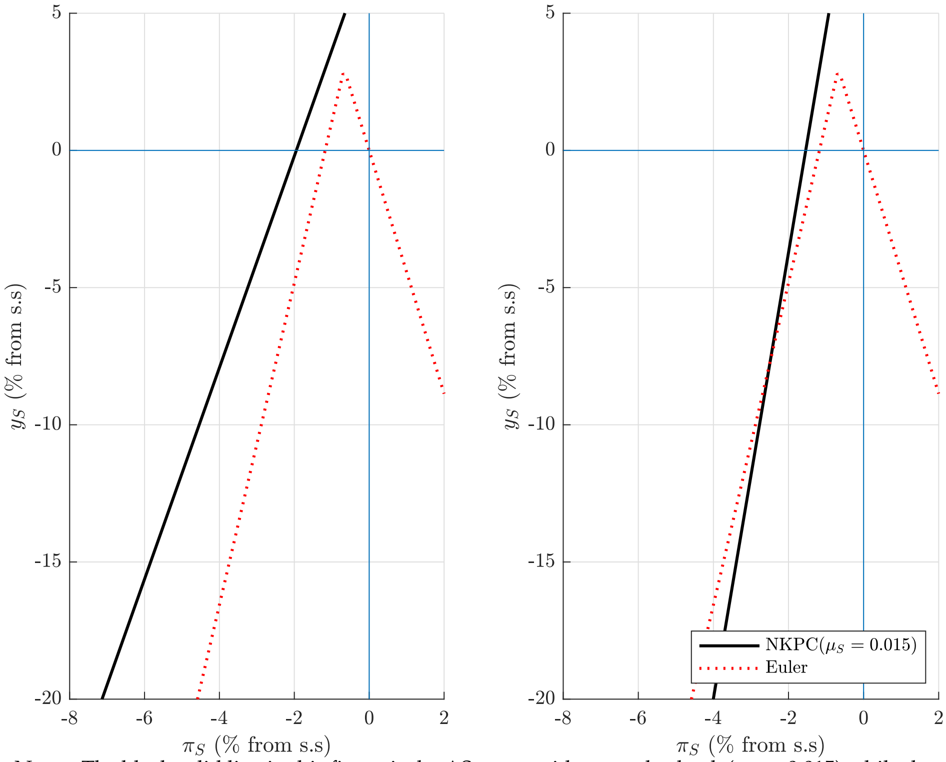
In sunspot-driven recessions, the Euler equation can be shown as:

$$y_S = \mathbb{E}_S y_{t+1} + \sigma_r \mathbb{E}_t \pi_{t+1} - \sigma_r(\log(\beta)).$$

Note that we can show

$$\begin{aligned}
\mathbb{E}_S \pi_{t+1} &= p\pi_S \\
&= \frac{p}{1-(\beta-\kappa\gamma_t)p}\kappa\gamma_y y_S + \frac{p}{1-(\beta-\kappa\gamma_t)p}\kappa\gamma_y\kappa\gamma_r \log(\beta) \\
&= \frac{1}{1-(\beta-\kappa\gamma_r)p}\kappa\gamma_y \mathbb{E}_S y_{t+1} + \frac{p}{1-(\beta-\kappa\gamma_r)p}\kappa\gamma_r \log(\beta),
\end{aligned}$$

Figure 5: Equilibrium uniqueness/existence with the real cost channel (supply shock)



Notes: The black solid line in this figure is the AS curve with a supply shock ($\mu_S = 0.015$) while the red dotted line is the AD curve. The left panel presents the no equilibrium in a standard NK model without the real cost channel and the right panel shows an equilibrium with the real cost channel, following the calibration method as in Appendix E.

where the last line we use the fact $\mathbb{E}_S y_{t+1} = p y_S$. With this in mind, one can rewrite the Euler equation as

$$\begin{aligned} y_S &= \mathbb{E}_S y_{t+1} + \sigma_r \frac{1}{1-(\beta-\kappa\gamma_r)p}\kappa\gamma_y \mathbb{E}_S y_{t+1} + \sigma_r \frac{p}{1-(\beta-\kappa\gamma_r)p}\kappa\gamma_r \log(\beta) - \sigma_r \log(\beta) \\ &= \Gamma_y(p,\gamma_r,\gamma_y)\mathbb{E}_S y_{t+1} + \Gamma_\beta(p,\gamma_r)\log(\beta), \end{aligned}$$

where

$$\Gamma_y(p,\gamma_r,\gamma_y) = 1 + \sigma_r \frac{1}{1-(\beta-\kappa\gamma_r)p}\kappa\gamma_y$$

and

$$\Gamma_\beta(p,\gamma_r) = \sigma_r \frac{p}{1-(\beta-\kappa\gamma_r)p}\kappa\gamma_r - 1 < 0.$$

One can show the composite parameter $\Gamma_\beta(p, \gamma_r, \gamma_y)$ in the standard model with $\gamma_r = 0$:

$$\Gamma_\beta(p, 0, \gamma_y) = 1 + \sigma_r \frac{1}{1 - \beta p} \kappa \gamma_y.$$

In that way, with the real cost channel, for a given level p, it can lower the composite parameter $\Gamma_y(p, \gamma_r, \gamma_y)$ to make sunspot liquidity less likely to occur.

P Irrelevance Condition

This is a direct result of the standard model in Appendix H. If there is an absence of demand shock and the effective slope of AS curve is lower than the AD curve at the ZLB, we can have sunspots. Otherwise, if the effective slope of the AS curve is higher than the AD curve at the ZLB, sunspots disappear. Thus, the necessary and sufficient condition to make expectations-driven traps irrelevant which is:

$$\mathcal{S}_{PC}^{c,z} > \mathcal{S}_{EE}^{z}.$$

Q Robustness check: No absorbing state

In this part, we relax the previously held assumption that the target steady state is absorbing. Following the approach in Coyle & Nakata (2019), we model expectations-driven LTs. Our primary objective is to explore how the real cost channel can influence the relevance of expectations-driven LTs. Accordingly, we assume the presence of two distinct equilibria in the economy: The first equilibrium is characterized by inflation and the output gap stabilizing at the targeted steady state, while the second, the expectations-driven equilibrium, is marked by both inflation and the output gap being negative, and nominal interest rates constrained at the ZLB.

The model incorporates a sunspot shock s_t that follows a two-state Markov process. The economy resides in the targeted steady state when $s_t = N$, representing a normal state, and shifts to the unintended equilibrium when $s_t = C$, indicative of a crisis state. The sunspot shock s_t is revealed at the beginning of each period and is observable by households and firms. This information plays a crucial role in the coordination of decision-making among private agents, as their expectation formation integrates the current realization of s_t. The transition probabilities are defined as follows:

$$\text{Prob}(s_t = N | s_{t-1} = N) = p_N,$$
$$\text{Prob}(s_t = C | s_{t-1} = C) = p_C.$$

In the normal state, the inflation rate closely aligns with the target steady state, and the ZLB constraint is not active. Conversely, the crisis state is characterized by low inflation with the interest rate at the ZLB. Utilizing the defined equilibrium transitions, when the economy is in the normal state, the equilibrium conditions can be shown as:

$$\pi_N = \frac{(\beta - \kappa\gamma_r)(1 - p_N)}{1 - \beta p_N - \kappa\gamma_r(\phi_\pi - p_N)}\pi_C + \frac{\kappa\gamma_y}{1 - \beta p_N - \kappa\gamma_r(\phi_\pi - p_N)}y_N,$$
$$y_N = y_C - \frac{\sigma_r(\phi_\pi - p_N)}{1 - p_N}\pi_N + \sigma_r\pi_C.$$

When the economy is in crisis state, we have the equilibrium conditions below:

$$\pi_C = \frac{(\beta - \kappa\gamma_r)(1 - p_C)}{1 - \beta p_C + \kappa\gamma_r p_C}\pi_N + \frac{\kappa\gamma_y}{1 - \beta p_C + \kappa\gamma_r p_C}y_C + \frac{\kappa\gamma_r}{1 - \beta p_C + \kappa\gamma_r p_C}\log(\beta),$$
$$y_C = y_N - \frac{\sigma_r}{1 - p_C}\log(\beta) + \frac{\sigma_r p_C}{1 - p_C}\pi_C + \sigma_r\pi_N.$$

In the normal state, the inflation rate closely aligns with the target steady state, and the ZLB constraint is not active. Conversely, the crisis state is characterized by low inflation with the interest rate at the ZLB. We can define these formally:

$$\pi_N \geq \frac{\log(\beta)}{\phi_\pi},$$

$$\pi_C < \frac{\log(\beta)}{\phi_\pi}.$$

We can solve the above equations and derive the short-run inflation for each state:

$$\pi_C = \frac{N_1 \frac{\sigma_r}{1-p_C} + N_2}{D_1 + N_1 \frac{\sigma_r}{1-p_C}} \log(\beta),$$
$$\pi_N = \frac{\frac{\sigma_r}{1-p_C} \log(\beta) - (\sigma_r + \frac{\sigma_r p_C}{1-p_C})\pi_C}{\sigma_r - \frac{\phi_\pi - p_N}{1-p_N}},$$

where

$$N_1 = \frac{\frac{\kappa\gamma_y}{1-\beta p_N - \kappa\gamma_r(\phi_\pi - p_N)} \left[\frac{\frac{(\beta-\kappa\gamma_r)(1-p_C)}{1-\beta p_C+\kappa\gamma_r p_C}}{\frac{\kappa\gamma_y}{1-\beta p_C+\kappa\gamma_r p_C}} + \frac{\sigma_r(\phi_\pi - p_N)}{1-p_N} \right] + 1}{\sigma_r - \frac{\sigma_r(\phi_\pi - p_N)}{1-p_N}}$$
$$= \frac{\frac{\kappa\gamma_y}{1-\beta p_N - \kappa\gamma_r(\phi_\pi - p_N)} \left[\frac{(\beta-\kappa\gamma_r)(1-p_C)}{\kappa\gamma_y} + \frac{\sigma_r(\phi_\pi - p_N)}{1-p_N} \right] + 1}{\sigma_r - \frac{\sigma_r(\phi_\pi - p_N)}{1-p_N}}.$$

$$N_2 = \frac{\frac{\kappa\gamma_y}{1-\beta p_N - \kappa\gamma_r(\phi_\pi - p_N)}}{\frac{\kappa\gamma_y}{1-\beta p_C+\kappa\gamma_r p_C}} \frac{\kappa\gamma_r}{1-\beta p_C+\kappa\gamma_r p_C}$$
$$= \frac{\kappa\gamma_r}{1-\beta p_N - \kappa\gamma_r(\phi_\pi - p_N)}.$$

$$D_1 = \frac{(\beta-\kappa\gamma_r)(1-p_N)}{1-\beta p_N - \kappa\gamma_r(\phi_\pi - p_N)} + \frac{\frac{\kappa\gamma_y}{1-\beta p_N - \kappa\gamma_r(\phi_\pi - p_N)}}{\frac{\kappa\gamma_y}{1-\beta p_C+\kappa\gamma_r p_C}} + \sigma_r \frac{\kappa\gamma_y}{1-\beta p_N - \kappa\gamma_r(\phi_\pi - p_N)}$$
$$= \frac{(\beta-\kappa\gamma_r)(1-p_N)}{1-\beta p_N - \kappa\gamma_r(\phi_\pi - p_N)} + \frac{1-\beta p_C+\kappa\gamma_r p_C}{1-\beta p_N - \kappa\gamma_r(\phi_\pi - p_N)} + \sigma_r \frac{\kappa\gamma_y}{1-\beta p_N - \kappa\gamma_r(\phi_\pi - p_N)}.$$

The expectations-driven LTs do exist if $\pi_C < \frac{\log(\beta)}{\phi_\pi}$:

$$\frac{N_1 \frac{\sigma_r}{1-p_C} + N_2}{D_1 + N_1 \frac{\sigma_r}{1-p_C}} \log(\beta) < \frac{\log(\beta)}{\phi_\pi}$$

$$
\begin{aligned}
&\Leftrightarrow \frac{N_1 \frac{\sigma_r}{1-p_C} + N_2}{D_1 + N_1 \frac{\sigma_r}{1-p_C}} > \frac{1}{\phi_\pi} \\
&\Leftrightarrow \Psi = 1 + \frac{N_2 - D_1}{D_1 + N_1 \frac{\sigma_r}{1-p_C}} > \frac{1}{\phi_\pi}.
\end{aligned}
$$

We first extract the terms with γ_r in $N_2 - D_1$ and it says:

$$
\frac{\kappa \gamma_r (p_N - p_C)}{1 - \beta p_N - \kappa \gamma_r (\phi_\pi - p_N)} > 0,
$$

where it is generally assumed that $p_N > p_C$ since the period of the normal state should be much longer than that in the crisis state as in Coyle & Nakata (2019). Then the terms with γ_r should be positive. In addition, since the left-hand side of the above inequality is positive, $D_1 + N_1 \frac{\sigma_r}{1-p_C}$ should be negative since $N_1 \frac{\sigma_r}{1-p_C} + N_2$ is not positive from our assumption. For simplicity, $D_1 + N_1 \frac{\sigma_r}{1-p_C}$ can be similar with/without the real cost channel. In that case, with the real cost channel, the terms in Ψ should be lower compared to ones without this channel. In other words, with the real cost channel, the economy can be less likely to get stuck into the expectations-driven LT. With this result, we can say that the results with no absorbing state: when the real cost channel is considered, the possibility of expectations-driven LT can be less.

Adopting the method without an absorbing state can effectively capture the dynamic relationship of Markov Chains in different state transitions. However, in the main text, we have not primarily used this method for analysis, for three dimensions. First, this method involves a considerable number of parameters, such as the persistence probabilities for two states, which complicates the derivation of analytical solutions. Second, and more importantly, this method cannot be implemented in the context of presenting results within the AS/AD framework, which is the focus of this paper. Third, the concept of a sunspot shock having a long-term (absorbing) steady state is widely used in the literature, especially in recent studies as in Bilbiie (2021), Nie & Roulleau-Pasdeloup (2023) and Nie (2023).

R Proof for Proposition 5

Following Bilbiie (2021), we assume the central bank sets the interest rate according to an exogenous process r_t^n which follows a two-state Markov process with persistence p. More specifically, the exogenous interest rate process r_t^n starts above the steady state at $r^n > 0$ but converges back to the steady state $r^n = 0$ with persistence p. One can assume an isomorphic Euler equation below:

$$y_t = \mathbb{E}_t y_{t+1} - \sigma_r \left[R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1} + r_t^n \right],$$

where r_t^n is the interest rate shock. With this in mind, we have the solution below:

$$y_S = \frac{\mathcal{I}_{EE}(-r^n) - \sigma_r \frac{\log(\beta)}{1-p} - \frac{\mathcal{I}_{PC}^c}{\mathcal{S}_{PC}^{c,z}}}{1 - \frac{\mathcal{S}_{EE}^z}{\mathcal{S}_{PC}^{c,z}}}.$$

$$\pi_S = \frac{\mathcal{I}_{PC}^c - \mathcal{I}_{EE}(-r^n) + \sigma_r \frac{\log(\beta)}{1-p}}{\mathcal{S}_{PC}^{c,z} - \mathcal{S}_{EE}^z}.$$

From Appendix P, we know that the condition to make expectations-driven LT irrelevant is that

$$\frac{\mathcal{S}_{EE}^z}{\mathcal{S}_{PC}^{c,z}} < 1.$$

In this case, if the expectations-driven liquidity trap is irrelevant, the increased interest can reduce inflation and the neo-fisherian effects as in Bilbiie (2021) can disappear.

S Welfare analysis

The welfare objective can be illustrated given a two-state Markov process:

$$\min_{r^n} \frac{1}{1-\beta p} [\pi_S^2 + \omega_y y_S^2].$$

In expectations-driven LT, we need to make $\pi_S = y_S = 0$ and the interest rate can be set in the standard model which is the same as in Bilbiie (2021):

$$r^n = \begin{cases} \log(\beta), 0 \leqslant t < T \\ 0, t \geqslant T. \end{cases}$$