Government Spending Multipliers
with the *Real* Cost Channel[*]

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Abstract

In the benchmark New Keynesian model, I introduce the *real* cost channel to study government spending multipliers with a simple Markov chain closed-form solution. This new model fundamentally departs from previous implications with the nominal cost channel by rotating the New Keynesian Phillips Curve in liquidity traps. I analytically show that the real cost channel can make inflation rise by less at the Zero Lower Bound. In turn, it leads to a smaller drop in the short-run real interest rate and thus a lower output gap multiplier during liquidity trap episodes. Finally, I confirm the robustness of the real cost channel effect on multipliers using two model extensions.

Keywords: Government Spending Multipliers, Real Cost Channel, Zero Lower Bound, Markov Chain, Bounded Rationality

JEL Classifications: E52, E58, E62, E63, E70

1 Introduction

In late 2008, the Federal Reserve had to lower the interest rate to zero to fight against the Global Financial Crisis (GFC). The conventional monetary policy cannot work with the zero lower bound (ZLB) binding. Therefore the government sought to adopt an alternative effective fiscal policy to stimulate the economy in recessions. In this case, the GFC and recent recessions such as COVID-19 have sparked massive fiscal policy discussions, which usually feature tons of government spending.[1] For example, during the COVID-19 pandemic time in 2020, the United States (US) government spent a total of $6.55 trillion on a series of programs to ensure the well-being of people.

Understanding the effects of government spending at the ZLB can be a key fiscal analysis. The standard New Keynesian (NK) models show that government spending multipliers can be substantially higher (e.g., above 2) at the ZLB as in Eggertsson and Woodford (2004), Christiano et al. (2011) and Zubairy (2014). This view has been challenged in many recent theoretic studies.[2] In addition, a series of new empirical papers have indicated that multipliers are lower at the ZLB. For instance, Ramey and Zubairy (2018) provide empirical evidence to show not-so-great spending multipliers in liquidity traps. Recently, Auerbach et al. (2021) show that the fiscal multipliers can be lower with the post-COVID supply-side constraints during recessions.

In this paper, I aim to augment the real cost channel in the benchmark NK model to provide *new theoretical insights* to explain empirically lower government spending multipliers when the economy is at the ZLB.[3] To be more specific, if firms need to pay bills before production, the interest rate can theoretically influence the borrowing cost and further the firms' marginal cost in the aggregate *supply-side* economy (summarized in the NK Phillips Curve). The critical difference between the model in the cost channel setting and the conventional model is that the interest rate should be included in the firms' marginal cost, and it can, in turn, influence the inflation rate.

The existence of the cost channel is provided in some empirical investigations. For instance, Ravenna and Walsh (2006) estimate and obtain the cost channel parameter

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[1]See more discussions on the fiscal tool to resist recessions as in Eggertsson (2011), Kollmann et al. (2012), Bouakez et al. (2020) and House et al. (2020).

[2]See e.g. Kiley (2014), Mertens and Ravn (2014) and Roulleau-Pasdeloup (2021a).

[3]The previous literature mainly employs a standard NK model, given that interest rate can impact aggregate *demand-side* economics only, to theoretically explore the effects of government spending and fiscal policy multipliers (see e.g., Eggertsson and Woodford (2004) and Aruoba and Schorfheide (2013)).

for the US. Similarly, Chowdhury et al. (2006) and Tillmann (2009) show the existence of cost channel in the US and the UK. Recently, Abo-Zaid (2022) employs a structural vector autoregression (SVAR) model to confirm that the cost channel exists in almost all representative industrialized countries.

Compared to most previous literature with the nominal cost channel[4], in this paper, I, however, incorporate the expected *real* interest rate into the firms' marginal cost to study government spending multipliers.[5] Furthermore, I try to conduct an analytical study on the spending multipliers in liquidity traps. To this end, a simple two-state Markov chain as in Eggertsson and Woodford (2003) is utilized to mimic policy shocks to obtain closed-form solutions for spending multipliers using pen and paper.

At the ZLB, the results in this paper stand in stark contrast to most previous implications with the nominal cost channel as in Surico (2008) and Smith (2016). I show that the introduced real cost channel can influence the NK Phillips Curve with the expected disinflation effects in the real channel during liquidity trap episodes. The threshold of the negative natural rate shock (also the demand shock) to trigger the ZLB constraint binding with the real cost channel is larger than the result in the classical NK model while is less than the model with the nominal cost channel. That means that the economy with the nominal cost channel is the most easily entrapped into fundamental liquidity traps.

Spending multipliers can be effective (larger than one) in liquidity traps. Intuitively, with the ZLB binding, the nominal interest rate remains unchanged, and the government spending within a fiscal policy package can increase inflation. This can lower the real interest rate and further stimulate private consumption. However, with the real cost channel, the expected real interest rate in firms' marginal cost (borrowing cost) can reduce short-run inflation caused by government spending. In this way, the real cost channel leads to a smaller drop in the short-run real interest rate. A higher real interest rate can depress people's consumption motivation and further production activity, which, in turn, can reduce the power of government spending on output. In a nutshell, compared to the standard NK model, the output gap multiplier with the real cost channel can be

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[4]There are a series of papers with the nominal cost channel which means the nominal interest rate is augmented in the firms' marginal cost, such as, Barth III and Ramey (2001), Ravenna and Walsh (2006), Llosa and Tuesta (2009) and Smith (2016).

[5]Compared to the nominal cost channel, as explained at length in Beaudry et al. (2022), the real cost channel can be more empirically relevant to the US data. Furthermore, Nie (2021) formally proves that the real cost channel is more theoretically appealing since it can secure the equilibrium uniqueness/existence with temporary policy shocks.

lower at the ZLB. Further, this multiplier decreases in the strength of this channel. In contrast, the spending multipliers with the nominal cost channel as in Tillmann (2009) can be invariant with the standard NK model since this channel can not modify the NK Phillips Curve slope at the ZLB.[6]

Another result is that the output gap multiplier decreases in the strength of the real cost channel when the economy is outside of liquidity traps. Intuitively, when the nominal interest rate is free to adjust, government spending can increase firms' marginal cost. The real cost channel in the Phillips Curve can further raise inflation since, in normal times, it works as a cost-push shock endogenously along the lines of Ravenna and Walsh (2006). With the Taylor (1993)-type rule followed by the central bank, there will be a rise in the nominal interest rate by more than one-for-one with inflation pressure. Larger real interest rate can stimulate people to save but to consume *less*. As a result, government spending can crowd out more private consumption. Consistent with the findings in Abo-Zaid (2022), notice that the output multiplier with the nominal cost channel is less than the counterpart without it in normal times. However, compared to the nominal cost channel, the inflation pressure in the real channel is less prominent since the expected inflation within the expected real interest rate can reduce the crowding-out effects. I, therefore, show that the nominal cost channel can further restrain the output gap multiplier in normal times relative to the real cost channel.

This analytical model extends to a more general setting, but the real cost channel still works robustly. As in some recent contributions in Sarin et al. (2021) and Roulleau-Pasdeloup (2021a), the long-run policy effects are usually overlooked or even usually computed numerically in the previous literature. In this case, I further discuss the long-run government spending policy analytically by assuming that the government spending policy is *longer* than economic recessions. Thus, a three-state Markov structure is employed. I follow Bilbiie (2019b) to decompose the long-run period into short and medium-run periods. Therefore, one can capture the effects of prolonged government spending on the short-run spending multipliers. The analytical results show that if the short-run economy is in normal times, it is observed that prolonged government spending can further deflate the output multiplier but increase the inflation multiplier more than the

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[6]In calculation of fiscal multipliers, we need to gain the partial derivative of government spending to inflation or output gap. If the introduced cost channel can not change the NK Phillips Curve slope, *ceteris paribus*, the partial derivative should be the same as the standard model.

short-run spending policy. Interestingly, if the short-run economy is in liquidity traps, longer government spending after the ZLB has subsided is favored. Intuitively, prolonged government spending can increase inflation more through rational expectations as in Bouakez et al. (2017) and thus, it can further inflate the short-run output gap multiplier. This theoretic result is supported by empirical evidence as in Leduc and Wilson (2013) and Bernardini et al. (2020). Additionally, I show that the real cost channel can work robustly in this three-state Markov chain.

At the end of the day, as I have shown the effects of long-run government spending on the short-term spending multipliers through rational expectations, one may wonder what if the agents can not fully understand the world?[7] Thus, bounded rationality is integrated into our baseline model to see such *behavioral* macroeconomic effects as another extension. I finally develop a similar version of Gabaix (2020)'s model, and the results show that in normal times, the cognitive discounting effects can mitigate the inflation pressure and further increase the output gap multiplier. Interestingly, if we suppose the agents have strong enough bounded rationality, the output gap multiplier can be large in normal times without the ZLB binding, which is in line with some empirical evidence in Auerbach and Gorodnichenko (2012) and Acconcia et al. (2014). On the other hand, at the ZLB, bounded rationality can attenuate the output gap multipliers. It can echo literature that the agents discounting future wealth in making decisions today can reduce the policy power in recessions as in McKay et al. (2016), Angeletos and Lian (2018), and Campbell et al. (2019). Besides, the real cost channel can still work robustly in this behavioral model, and the output gap multiplier can be further overestimated by ignoring this channel.

Related Literature.—The earliest seminal work focusing on the theoretical estimation of government spending effects in liquidity traps can be traced back to Eggertsson (2001). In that paper, the optimal fiscal policy in the NK economy is characterized, and the real effects of government spending are emphasized. After that, a growing number of literature focuses on the estimation of fiscal effects in theoretical and empirical ways. For example, Blanchard and Perotti (2002) spark the earliest insights on empirically

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[7]There is a similar consideration in the monetary policy. As in Nakata et al. (2019) and Budianto et al. (2020), the favorable effects of Forward Guidance—a promised long-run interest rate binding—on short-run inflation can be much attenuated if the economic agents can not fully understand the world as represented by the NK model with rational expectations.

estimating the macroeconomics effects of government spending.[8] Christiano et al. (2011) prove that the multiplier is low in normal times in an economy following a Taylor (1993)-type rule but relatively high in liquidity traps. Leeper et al. (2017) theoretically study the fiscal multipliers in a series of models. Two distinct monetary-fiscal policy regimes show that the short-run multipliers are robustly similar across different regimes. More examples among Kraay (2012), Miyamoto et al. (2018), Ramey and Zubairy (2018), etc.

In this paper, I add to the government spending multiplier literature by analytically speaking to the role of the real cost channel on the multipliers. The previous literature as in Barth III and Ramey (2001), Ravenna and Walsh (2006), Llosa and Tuesta (2009) and Smith (2016) always introduces the nominal interest rate into the firms' marginal cost which is called as the nominal cost channel. For example, Ravenna and Walsh (2006) first confirm that the cost-push shock can emerge endogenously in the NK model in a cost channel setting. Further, they discuss the ways for the optimal monetary policy which the new channel can alter. Surico (2008) shows that limiting the economic cyclical with a cost channel can lead to a strong fluctuation of inflation and output. However, this paper heavily builds on Beaudry et al. (2022) and Nie (2021) to augment the expected real interest rate into firms' marginal cost to revise the benchmark NK model.

This paper is in a close spirit to some recent literature—see e.g. Bilbiie (2019a), Bilbiie (2019b), Bilbiie (2020) and Nie and Roulleau-Pasdeloup (2022) which uses three-state Markov Chain to analytically examine the long-run policy on the short-run economy. For example, Bilbiie (2019b) employs a three-state Markov manner to have an in-depth study on the optimal forward guidance policy in the short run and long run. In this paper, this three-state structure can allow us to analytically check the general properties of long-run government spending to echo some empirical evidence in Durevall and Henrekson (2011), Ilzetzki et al. (2013), Leduc and Wilson (2013), Bouakez et al. (2017), and Leff Yaffe (2019).

In addition, recent contributions such as Farhi and Werning (2019), and Gabaix (2020) show that bounded rationality can mitigate the powerful effects of monetary policy. This method can rationalize the so-called "Forward Guidance puzzle" (see i.a. Angeletos and Lian (2018) and Coibion et al. (2020)) compared to the benchmark too forward-looking NK model as in Ganelli and Tervala (2016). This paper, however, is linked with

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[8]See Ramey (2011) for a survey on the estimation of government spending multipliers in the literature.

this strand of literature to see the interaction of the cost channel and bounded rationality on the *fiscal* policy.

Finally, this paper is also closely related to Abo-Zaid (2022) who examines government spending multipliers at the ZLB with the nominal cost channel. Abo-Zaid (2022) includes nominal cost channel and differentiates between the policy rate and the loan rate. It turns out that this nominal channel can cause spending multipliers larger in liquidity traps.[9] However, in this paper, I use the real cost channel to explain empirically lower government spending multipliers when the economy is at the ZLB. Simple Markov chain closed-form solutions are computed to compare the government spending multipliers with the real/nominal cost channel. I further clear up the effects of the strength of the real cost channel on the spending multipliers analytically. I also furnish the primary model to confirm the robust role of the real cost channel on multipliers.

Organization.—I will specify the prototypical forward-looking New Keynesian(NK) model with the real cost channel in the next Section 2 and provide a transparent analytical analysis using a two-state Markov chain on the government spending multipliers with/without the real cost channel. In Section 3, I furnish the baseline model to explore the general properties of long-run government spending effects. Another extension with bounded rationality is conducted in Section 4. Finally, this paper concludes in Section 5.

2 The Baseline Model with the Real Cost Channel

Recent empirical evidence in Abo-Zaid (2022) shows that the existence of the cost channel can influence government spending multipliers. As in Ravenna and Walsh (2006) and Surico (2008), the main insight of the cost channel is that the interest rate can influence the borrowing costs and then the marginal cost function. In this paper, we follow Beaudry et al. (2022) and Nie (2021) to utilize the NK model with the real cost channel, which means the expected real interest rate can impact the firms' marginal costs, is employed to explore government spending multipliers analytically.

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[9]In the setup in Abo-Zaid (2022), if the two rates (*i.e.* the policy rate and the loan rate) are equal, the cost nominal cost channel would disappear from the Phillips Curve. While in this paper, we simply assume the two rates are the same.

2.1 Private Sector Behavior

I follow Beaudry et al. (2022) and Nie (2021) to use a prototypical forward-looking NK model with price rigidity.[10] The behavior of aggregate demand (AD) side economy can be summarized in the following log-linear Euler condition:

$$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\sigma_c}[R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1} - r_t^n],$$ (1)

where c_t is the private consumption, σ_c is the risk aversion coefficient, R_t is the nominal interest rate *in level*, π_t is inflation, $\mathbb{E}_t$ is the rational expectation operator, and r_t^n is the demand shock (also the natural rate shock).

The resource constraint in this NK economy is:

$$y_t = (1 - s_g)c_t + g_t,$$ (2)

where s_g is the fraction of government spending in total production, g_t is the government spending.[11] In this case, one can obtain the path of y_t by substituting the resource constraint into Euler equation to obtain the equation (3) below:

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma}[R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1} - r_t^n] + g_t - \mathbb{E}_t g_{t+1},$$ (3)

where $\sigma = \frac{\sigma_c}{1-s_g}$.

The aggregate supply (AS) side of the economy can be summarized in the following NK Phillips Curve:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \left[\gamma_y y_t + \gamma_g g_t + \gamma_r (R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1})\right],$$ (4)

where β is the subjective discount factor, and κ is the elasticity of inflation with regard to marginal cost. γ_y, γ_g, γ_r are the elasticity of marginal cost elasticity with regard to the output gap, government spending, and the expected real interest rate, respectively.[12] It is of note that γ_r in the equation (4) can be seen as the strength of the cost channel which

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[10]For brevity, I show the aggregate model economy in a log-linear form in the main text. See Galí (2015) for a textbook treatment.

[11]Following Christiano et al. (2011), I define $g_t = (G_t - G)/Y$.

[12]See appendix A for exact expressions for these parameters.

controls the impact of this channel.[13] This model with the real cost channel can collapse to the conventional one if $\gamma_r = 0$.[14] In addition, it can nest the model with the nominal cost channel (equation (6)) if the disinflation expectation term $(-\mathbb{E}_t \pi_{t+1})$ disappears as in Ravenna and Walsh (2006).

Following Nie and Roulleau-Pasdeloup (2022), it is assumed that the central bank sets the nominal interest rate following the (truncated) Taylor (1993)-type rule with the ZLB:

$$R_t = \max\{0, -\log(\beta) + \phi_\pi \pi_t\}. \tag{5}$$

2.1.1 Real versus Nominal Cost Channel

The Phillips Curve with the nominal cost channel:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \left[\gamma_y y_t + \gamma_g g_t + \gamma_r (R_t + \log(\beta)) \right]. \tag{6}$$

The equation (6) is used in most previous papers working as the nominal cost channel specifications such as Ravenna and Walsh (2006) and Surico (2008). In these papers, the nominal interest rate is introduced in the marginal cost. Even though the specifications in equations (4) and (6) can be seen as the cost channel, the real cost channel in the equation (4) can highlight the expected disinflation feedback denoted by $-\mathbb{E}_t \pi_{t+1}$ in the expected real interest rate. As in Beaudry et al. (2022), compared to the nominal cost channel, the real cost channel can obtain more support from the US data. This motivates us to use this setting in this paper.

It is of note that the two cost channels can have similar effects in normal times if the central bank follows a simple Taylor rule in the equation (5). In this case, a cost-push shock endogenously emerges in the two cases as in Ravenna and Walsh (2006) and the two channels both increase inflation.

The interesting insight at the ZLB with $R_t = 0$ is that the conventional nominal cost channel as in Ravenna and Walsh (2006) and Surico (2008) can not influence the slope of the Phillips Curve. However, the real cost channel in this paper with the expected disinflation can still impact the Phillips Curve at the ZLB. It is of note that, at the ZLB, the real cost channel as in Beaudry et al. (2022) can rotate the Phillips Curve with the disin-

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[13] The details for the derivatives of the aggregate supply side of the economy can be seen in appendix A.

[14] See Gertler et al. (1999) and Woodford (2003).

flation expectation feedback. As a result, the Phillips Curve is flatter than the standard NK model and this may explain a declining slope of the empirical Phillips Curve. In this way, the real cost channel in equation (4) can alter the government spending multipliers in liquidity traps. While the nominal cost channel multipliers in equation (6) can be invariant with the standard NK model since the nominal channel in liquidity traps can not be included in the partial derivative of government spending to the output gap/inflation in the calculation of fiscal multipliers.

2.2 Quick Tour: Normal Times and Zero Lower Bound[15]

In this section, I employ a two-state static Markov chain as in Eggertsson et al. (2003) to deal with the policy shocks vector $[r_t^n, g_t]$. It is assumed that the the specific policy shock (for example, demand shock r_n^n in this section) stays at the current short-run state with a persistence p and it then reverts to the steady-state *i.e.* $r_t^n = 0$ with a probability $1 - p$.[16] Since the NK model with the real cost channel in this paper is perfectly forwarding looking, one can show the expected output gap and inflation as follows:

$$\mathbb{E}_t y_{t+1} = p y_t, \qquad \mathbb{E}_t \pi_{t+1} = p \pi_t. \tag{7}$$

Assumption 1. *I assume that the NK Phillips Curve with the real cost channel is always upward sloping in a (π_t, y_t) graph such that*

$$p < \frac{1 - \kappa \gamma_r \phi_\pi}{\beta - \kappa \gamma_r} = \overline{p}^c. \tag{8}$$

As in much literature in Laubach and Williams (2003), Cochrane (2017), Han et al. (2020) and Nie (2021), there is an implicit condition that the NK Phillips Curve is upward sloping and there is a threshold of demand shock to trigger the ZLB constraint binding. Compared with the conventional NK model with the nominal cost channel as in Christiano et al. (2005) and Ravenna and Walsh (2006), this paper utilizes a more empirically relevant setting proposed in Beaudry et al. (2022) and further extended in Nie (2021). In this section, we discuss two cases which are the economy in normal times

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[15]In this paper, normal times refers to a state that the nominal interest rate is flexible to adjust and is not binding with the ZLB. The ZLB state means that the nominal interest rate is fixed at zero.

[16]The duration of the short run state can be calculated as $T = \frac{1}{1-p}$. For instance, if $p = 0.5$, $T = \frac{1}{1-0.5} = 2$ quarters.

without the ZLB binding and in fundamental liquidity traps.

From the Taylor (1993)-type rule, one can see that if the items $-\log(\beta) + \phi_\pi \pi_S$ are less than or equal to zero, the NK economy can be binding with the ZLB state. If not, the economy is in normal times and the nominal interest rate can be free to adjust with the central bank's monetary policy regulation. If the (negative) natural rate shock (also the demand shock) is too large, the economy can be with the ZLB. Thereby there is a boundary condition for the natural rate shock r_S^n in the short run to trigger the economy into a state with the ZLB binding.

As in Proposition 1, the boundary condition to trigger the ZLB binding with the real cost channel is higher than that in the conventional NK model. In other words, the economy is more easily into liquidity traps with the real cost channel than in the traditional model. On the other hand, the boundary condition with the real cost channel is less than that in the model with the nominal cost channel, implying that the economy is more likely to be caught in a lower bound state with the nominal cost channel. Thus the model with the nominal cost channel is the most easily entrapped into fundamental liquidity traps among the three models.[17]

Proposition 1. *The boundary condition relationship among the three models is*

$$\underline{r_S^{n,B}} < \underline{r_S^n} < \underline{r_S^{n,N}}, \tag{9}$$

where $\underline{r_S^{n,B}}$ is the boundary condition without the cost channel, $\underline{r_S^n}$ is the boundary condition with the real cost channel and $\underline{r_S^{n,N}}$ is the boundary condition with the nominal cost channel.

Proof. See Appendix B. □

In the following sections, I will mainly focus on the government spending shock and discuss the issues of both the output and inflation multipliers.

2.3 Government Spending: Theoretical Analysis

To have transparent analytical results, I abstract from the demand shock and only focus on the effects of government spending shock with the real cost channel. First, I will com-

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[17]In this paper, the three models refer to the conventional model without the cost channel, the model with the nominal cost channel, and the model with the real cost channel.

pare the real and the nominal cost channel. Second, I will deliver the general properties of the strength of the cost channel on the multipliers by using a simple Markov chain closed-form solution.

2.3.1 Government Spending Multiplier in Normal Times

The Markov method is based on Eggertsson and Woodford (2003) and Eggertsson et al. (2003). I assume that the positive government spending shock $g_S > 0$ starts in the short run, stays with the persistence probability p, and returns to the steady-state $g_L = 0$ in the long run with a probability $1 - p$. If the short-run economy is in normal times, I can use the Taylor (1993) rule to rewrite the Euler equation (3) and Phillips Curve with the real cost channel (equation (4)):

$$y_S = -\frac{1}{\sigma(1-p)}(\phi_\pi - p)\pi_S + g_S \tag{10}$$

$$\pi_S = \kappa \frac{\gamma_y}{1 - \beta p - \kappa\gamma_r(\phi_\pi - p)} y_S + \kappa \frac{\gamma_g}{1 - \beta p - \kappa\gamma_r(\phi_\pi - p)} g_S. \tag{11}$$

Fiscal multipliers.—It can be found that a positive government spending shock can move the Euler equation upward and turn down the Phillips Curve in a (π_t, y_t) graph. The solutions with the real cost channel of the output gap $\mathcal{M}_{S,N}^O$ and inflation multipliers $\mathcal{M}_{S,N}^I$ can be generated below:

$$\mathcal{M}_{S,N}^O = \frac{\sigma(1-p)[1 - \beta p - \kappa\gamma_r(\phi_\pi - p)] - \kappa\gamma_g(\phi_\pi - p)}{\sigma(1-p)[1 - \beta p - \kappa\gamma_r(\phi_\pi - p)] + \kappa\gamma_y(\phi_\pi - p)} \tag{12}$$

$$\mathcal{M}_{S,N}^I = \frac{\left[1 + \frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p)}{\sigma(1-p)[1 - \beta p - \kappa\gamma_r(\phi_\pi - p)] + \kappa\gamma_y(\phi_\pi - p)}. \tag{13}$$

The expected real interest rate can be in the denominator and numerator for the output gap multiplier since the new cost channel here can influence the inflation rate and further impact the output gap through expectations. While for the inflation multiplier, the expected real interest rate is included in the denominator since the cost channel can impact this multiplier directly.

In normal times, since the denominator of the multiplier equation is higher than the numerator and the multiplier is less than one. Government spending can crowd out private consumption, which can echo some classical empirical evidence as in Amano and

Wirjanto (1997) and Barro and Redlick (2011).

In normal times, the cost channel can reduce the output multiplier, and the intuition is simple. Since the government spending shock can increase firms' marginal costs. Additionally, the (real/nominal) cost channel can further increase the borrowing cost and further inflation. This mechanism is close in spirit to Ravenna and Walsh (2006) such that a cost-push shock can emerge endogenously in the New Keynesian model with the cost channel setting. In this case, the cost channel can trigger a higher increase in inflation. The nominal interest rate increases by more than one-for-one with inflation and further a higher nominal interest rate arise due to the cost channel. This results in a higher short-run real interest rate which can cause less private consumption, and thus government spending can crowd out more private consumption. Above all, the output multiplier with the cost channel is less than that without it.

In comparison, as in Appendix C, compared to the nominal cost channel, the inflation multiplier can be smaller with the real cost channel than the model with the nomial channel. If we compare the NK Phillips Curve in equations (6) and (4), less influence is triggered by the real cost channel, which may echo the lower inflation fact in the Euro area in normal times (Koester et al. (2021)). Lower inflation with the real cost channel can make the output gap multiplier larger. In this case, the nominal cost channel can further restrain the output multiplier relative to the real cost channel.

I further try to discuss the effects of the strength of the cost channel γ_r on the spending multipliers. On the one hand, we can see that the power of the real cost channel can lower the output gap multiplier, which is intuitive since the stronger the cost channel is in normal times, the more inflation is. Thus government spending crowds out more private spending. On the other hand, similarly, the cost channel can directly impact inflation from the expected real interest rate on the aggregate supply side. We summarize the main theoretical results in Proposition 2.

Proposition 2. *In normal times, the output gap multiplier decreases in the strength of the real cost channel γ_r while the inflation multiplier increases in γ_r. The output gap multiplier relationship among the three models is*

$$\mathcal{M}_{S,N}^{O,N} < \mathcal{M}_{S,N}^{O} < \mathcal{M}_{S,N}^{O,B}, \tag{14}$$

and the inflation multiplier relationship is

$$\mathcal{M}_{S,N}^{I,B} < \mathcal{M}_{S,N}^{I} < \mathcal{M}_{S,N}^{I,N}, \tag{15}$$

where in normal times, $\mathcal{M}_{S,N}^{i,N}$ ($i \in \{O,I\}$) denote the multiplier of the model with the nominal cost channel, and $\mathcal{M}_{S,N}^{i,B}$ denote the multiplier of the model without the cost channel.

Proof. See Appendix C. □

2.3.2 Government Spending Multiplier at Zero Lower Bound

In this part, I focus on the case when the economy binds with the ZLB.[18] The Euler equation (3) and Phillips Curve with the real cost channel (equation (4)) can be rewritten as:

$$y_S = -\frac{1}{\sigma(1-p)}[\log(\beta) - p\pi_S] + g_S \tag{16}$$

$$\pi_S = \frac{\kappa\gamma_y}{1-\beta p + \kappa\gamma_r p}y_S + \kappa\frac{\gamma_g}{1-\beta p + \kappa\gamma_r p}g_S + \frac{\kappa\gamma_r\log(\beta)}{1-\beta p + \kappa\gamma_r p}. \tag{17}$$

Fiscal multipliers.—One can further use the Euler equation and Phillips Curve to obtain the solutions with the real cost channel of the output gap multipliers $\mathcal{M}_{S,Z}^{O}$ and the inflation multiplier $\mathcal{M}_{S,Z}^{I}$ at the ZLB:

$$\mathcal{M}_{S,Z}^{O} = \frac{\sigma(1-p)[1-\beta p + \kappa\gamma_r p] + \kappa\gamma_g p}{\sigma(1-p)(1-\beta p + \kappa\gamma_r p) - \kappa\gamma_y p} \tag{18}$$

$$\mathcal{M}_{S,Z}^{I} = \frac{\left[1 + \frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p)}{\sigma(1-p)(1-\beta p + \kappa\gamma_r p) - \kappa\gamma_y p}. \tag{19}$$

I can obtain spending multipliers as the case in normal times if we assume $\phi_\pi = 0$. Likewise, the expected real interest rate can be in the denominator and numerator of the output gap multiplier. The new cost channel here can influence the inflation rate and further impact the output gap through expectations. The real cost channel can impact the inflation multiplier directly through the NK Phillips Curve.

In liquidity traps, since the denominator of the multiplier equation is lower than the numerator and thus the output gap at the ZLB is larger than one. At the ZLB

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[18]In this paper, the zero (effective) lower bound is a state when $R_t = 0$ with the simple assumption that there is no cash storing cost in Galí (2015).

with no cost channel, the nominal interest rate remains unchanged, and the increase in government spending means that firms will need household members to produce more. This higher real wage makes the firm's marginal cost increase and further generates inflation. Therefore the government spending shock leads to a drop in the short-run real interest rate, which, in turn, gives incentives to the consumers to save less and consume more. This can increase in consumption means more demand for the firms, which demand more hours from the household members. This can be seen as "crowding in" effects as in Bouakez et al. (2017).

When the nominal rate is fixes at the lower bound, the government spending shock can boost the wage rate and further increase short-run inflation. However, the real cost channel can lower the short-run inflation through the disinflation expectation component (equation (4)) in the firms' borrowing cost. Intuitively, a lower borrowing cost arises due to the disinflation expectation during liquidity trap episodes. Further, the disinflation expectation can translate into the realized lower inflation rate through rational expectations and sticky prices. In other words, assuming firms care about the expected real interest rate in liquidity traps, the disinflation expectation in the future can reduce the actual inflation. In this case, the real cost channel can make the inflation rate rise by less following a spending shock, and thus a smaller drop in the short-run real interest rate. Therefore, higher real interest rates due to the real cost channel can depress people's consumption motivation and further production activity, which results in a decline in the effects of government spending on output. We summarize the main theoretical results in Proposition 3.

Proposition 3. *At the ZLB, the output gap and inflation multipliers decrease in the strength of the real cost channel* γ_r. *The output gap multiplier relationship among the three models is*

$$\mathcal{M}_{S,Z}^{O} < \mathcal{M}_{S,Z}^{O,N} = \mathcal{M}_{S,Z}^{O,B}, \tag{20}$$

and the inflation multiplier relationship is

$$\mathcal{M}_{S,Z}^{I} < \mathcal{M}_{S,Z}^{I,N} = \mathcal{M}_{S,Z}^{I,B}, \tag{21}$$

where in liquidity traps, $\mathcal{M}_{S,Z}^{i,N}$ ($i \in \{O,I\}$) *denote the multiplier of the model with the nominal cost channel, and* $\mathcal{M}_{S,Z}^{i,B}$ *denote the multiplier of the model without the cost channel.*

Proof. See Appendix D. □

Comparison with the literature.—This result is contrary to Abo-Zaid (2022) who shows that the higher borrowing cost of the nominal cost channel can increase inflation to make the output gap multiplier larger at the ZLB. As in Abo-Zaid (2022), the Phillips Curve is steeper in a (y_t, π_t) graph than the standard NK model in liquidity traps with the nominal cost channel.[19] In this paper, I stress that inflation in the short run with the real cost channel can be lower since the disinflation expectation (in the real borrowing cost) can reduce inflation in the short run caused by government spending.[20] Thus I can use the real cost channel to explain empirically lower government spending multipliers when the economy is at the ZLB as in Ramey and Zubairy (2018).

Taking stock on the denominator of the multiplier—As discussed in Bilbiie (2008), Mertens and Ravn (2014), Borağan Aruoba et al. (2018) and Lustenhouwer (2020), Nie and Roulleau-Pasdeloup (2022), the denominator of the multiplier in the standard NK model at the ZLB can be negative. In this case, the output gap/inflation multiplier can be negative as in the equations (18)-(19).

However, as described at length in Nie (2021), in the presence of the real cost channel, the denominator is less likely to be negative. It can be always positive if assuming the magnitude of the real cost channel γ_r is large enough and meets the below condition:

$$\sigma(1-p)(1-\beta p + \kappa\gamma_r p) - \kappa\gamma_y p > 0$$

$$\Leftrightarrow \gamma_r > \Gamma(\gamma_y), \tag{22}$$

where $\Gamma(\gamma_y) = \frac{\kappa\gamma_y p - \sigma(1-p)(1-\beta p)}{\sigma(1-p)\kappa p}$.

As in the condition (22), γ_r should be higher than the composite parameter $\Gamma(\gamma_y)$. Assuming that γ_y is small enough (or γ_r is relatively large enough), this condition can always hold. In particular, the assumption with a low γ_y (or a large γ_r) is in line with the empirical finding as in Beaudry et al. (2022): the elasticity of real marginal cost w.r.t. output gap γ_y is very small while the key parameter γ_r leveraging the strength of the

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[19]Abo-Zaid (2022) differentiates between the policy rate and the loan rate, making the Phillips Curve steep in recessions. In that setup, if the two rates are equal, the cost nominal cost channel would disappear from the Phillips Curve. In that way, the nominal cost channel in liquidity traps can not impact the spending multiplier.

[20]The Phillips Curve is locally flat with the real cost channel during recessions as in Beaudry et al. (2022). Besides, Hazell et al. (2020) empirically document that the NK Phillips Curve is flat during the Great Recession.

real cost channel is significantly much larger than γ_y.

2.4   Numerical Results—Benchmark

In this paper, following Budianto et al. (2020), Roulleau-Pasdeloup (2021a) and the main baseline parameterization method is reported in Table 1. I follow the calibration method in Beaudry et al. (2022) and Nie (2021) to set parameters of the real cost channel.[21]

Table 1: Parameterization

Subjective discount factor	$\beta = 0.99$
Inverse of Frisch elasticity	$\eta = 1$
Risk aversion coefficient	$\sigma_c = 1$
Steady-state ratio of government spendin to output	$s_g = 0.2 \times 0.23$
Elasticity of inflation w.r.t. real marginal cost	$\kappa = 0.2$
Elasticity of real marginal cost w.r.t. output gap	$\gamma_y = 0.2$
Elasticity of real marginal cost w.r.t. interest rate	$\gamma_r = 1$
Elasticity of real marginal cost w.r.t. government spending	$\gamma_g = -0.1$
Inflation parameter in Taylor rule	$\phi_\pi = \phi_\pi^q = 1.5$
Persistence specification	$p = 0.7$

I first provide numerical AS/AD figures with a contractionary natural rate shock in three Phillips Curves shown in Figure 1. If there is a temporary short-term natural rate shock (-2%), which exceeds the boundary condition, the economy can be in fundamental liquidity traps to mimic the case of the Great Recession. In this case, the conventional monetary policy can not work since the nominal interest rate is bounded at zero. The deflationary pressure can lead to a higher real interest rate and further stimulate people to save but consume less. From Figure 1, it is clear that the real cost channel can change the Phillips Curve slope in liquidity traps and the Phillips Curve is locally flat as in Beaudry et al. (2022). The curve with the nominal cost channel at the ZLB shares the same slope as the conventional model. The numerical result also echoes Proposition 1 that the case with the nominal cost channel is the most easily entrapped into fundamental liquidity traps among the three models. This can be further confirmed in the impulse response to a contractionary natural rate shock.—See Appendix E.

In normal times, I compare the multipliers for the models without the cost channel

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[21]The calibration method of the real cost channel follows the way in Figure 4 in Beaudry et al. (2022). In addition, these parameters are set to ensure that the denominator of the multiplier is positive.

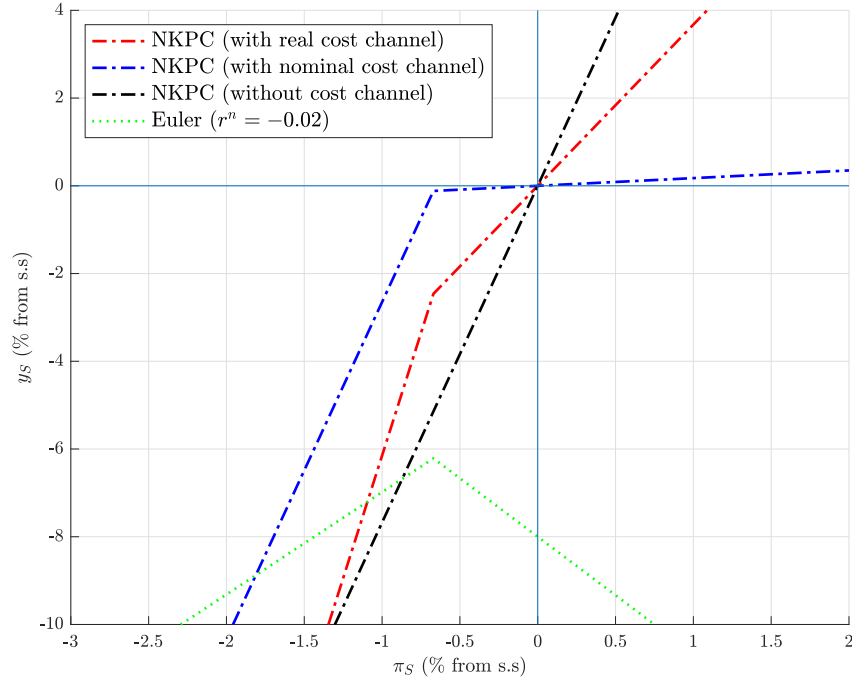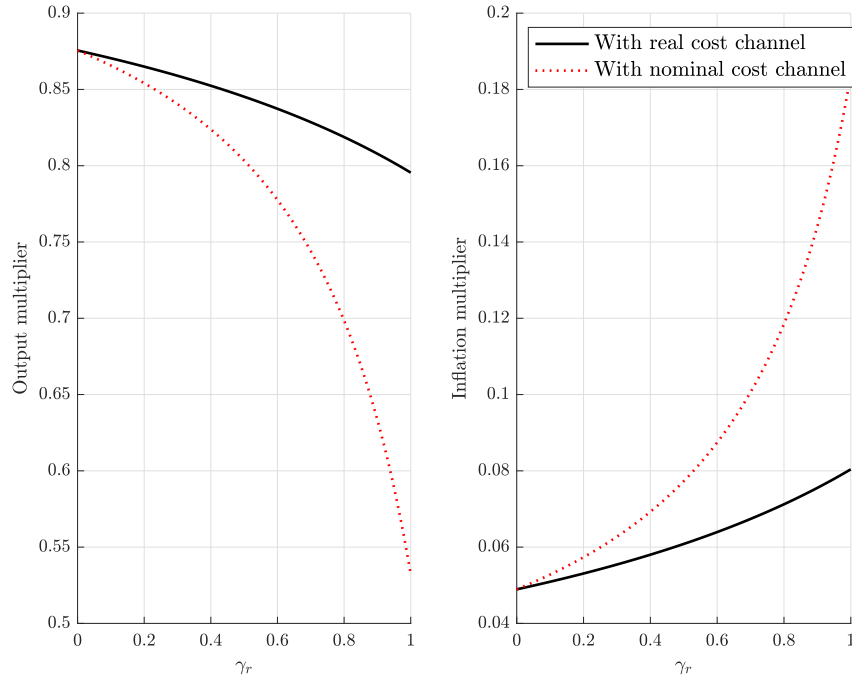Figure 1: AS/AD curves with natural rate shock in three models

Figure 2: Spending multipliers in normal times

($\gamma_r = 0$) vs. with the cost channel as in Ravenna and Walsh (2006) vs. with the real cost channel in Figure 2. It can be seen that the output multiplier is lower than one as in Lewis (2021) and it decreases in the strength of the cost channel. On the contrary, the inflation multiplier increases with the strength of the cost channel γ_r. It is observed

that the output gap multiplier with the real cost channel is larger than that with the nominal cost channel while less than its counterpart without the cost channel. This can echo the theoretical analysis in Proposition 2. See Appendix F for numerical results w.r.t. the persistence p in three models. These numerical results robustly echo Proposition 2. I also find that the output gap multiplier decreases with the persistence p, while the inflation multiplier is higher with the increment of time duration.

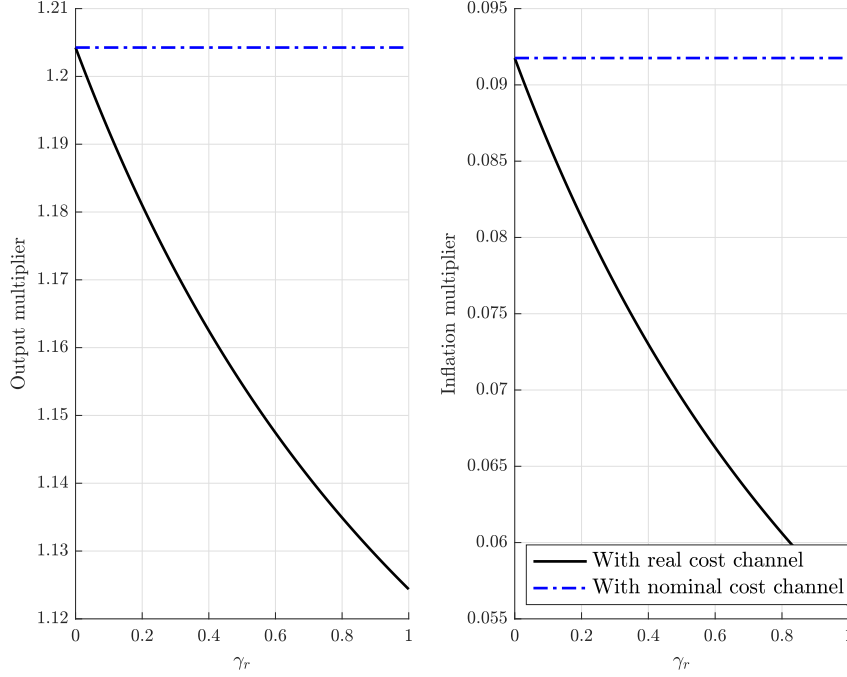Figure 3: Spending multipliers in liquidity traps

Figure 3 shows spending multipliers with/without the real cost channel in liquidity traps. The simulation results can perfectly match our theoretical analysis in Proposition 3; on the one hand, one can see that the output multiplier is productive as in Christiano et al. (2011) and Schmidt (2017). The real cost channel can attenuate the output gap and inflation multipliers simultaneously. The output gap and inflation multipliers decrease in the strength of the real cost channel γ_r. The nominal cost channel multipliers are invariant with the standard NK model without the cost channel. See Appendix F for numerical results w.r.t. the persistence p in three models. It further confirm the theoretical results. In addition, the output gap/inflation multipliers increase in the persistence p.
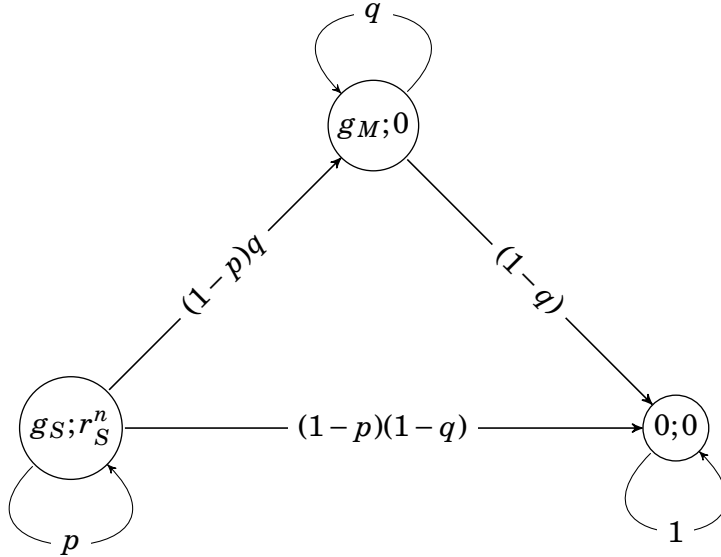
3 Government Spending: Long-Run Policy

As in some recent contributions in Sarin et al. (2021) and Roulleau-Pasdeloup (2021a), the long-run policy effects are usually overlooked or even usually computed numerically in the previous literature. In this section, I follow Roulleau-Pasdeloup (2021b) to make use of a three-state Markov chain to reassess the long-run government spending effects analytically.

3.1 Policy and Shocks

We assume that government spending is longer-lived than the demand shock (economic crisis). To be more specific, the government spending in the short run g_S can step into the medium run government spending g_M with a persistence q and then collapse to the steady-state $g_L = 0$ with a probability $1-q$. For the natural rate shock r_S^n, it runs in the short run with a probability p and then returns to the long run $r_L^n = 0$ with a probability $1-p$. The graphical representation of our policy can be seen in Figure 4.

Figure 4: Long run government spending: a graphical representation

With this in mind, for the monetary policy now, I use an adapted Taylor rule:[22]

$$R_t = \begin{cases} \max\left[0; -\log(\beta) + \phi_\pi \pi_S\right] & \text{In the short run} \\ -\log(\beta) + \phi_\pi^q \pi_M & \text{In the medium run} \\ -\log(\beta) & \text{In the long run} \end{cases} \tag{23}$$

One can use the above equation (23) to trace the path of the government spending. In the medium run, the output gap and inflation can be expressed as the product of medium-run multipliers and the medium-run government spending as follows:

$$y_M = y_{M,g} \cdot g_M \quad \& \quad \pi_M = \pi_{M,g} \cdot g_M, \tag{24}$$

where g_M is the government spending in the medium run. See appendix G for the medium spending multipliers.[23] In this case, since the model is forward-looking and the expected output gap and inflation are contingent on the short-run shock and the medium-run shock. I can generate the expected output gap below, and one can show the expected inflation using the same manner.[24]

$$\mathbb{E}_S y_{t+1} = p y_S + (1-p) q y_M$$
$$= p y_S + (1-p) q y_{M,g} g_M$$
$$= p y_S + (1-p) q y_{M,g} \zeta g_S. \tag{25}$$

3.2 Long-run Government Spending: Theoretical Analysis

This section mainly discuss the analytical results regarding the government spending multiplier with a three-state Markov structure.

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[22]In this section, to be in line with Section 2, I consider two economic states in the short run, which are normal times and the ZLB. In the medium run, I simply assume the demand shock reverts to zero and the economy is in normal times.

[23]I follow a simple rule in Nie and Roulleau-Pasdeloup (2022) to deal with the medium run shock: It is generally assumed that the medium-run spending is contingent on the short-run spending but it is lower than the short-run spending such that $g_M = \zeta g_S$, where ζ is a discount parameter.

[24]In our simulation results, it is assumed that $q = 0.7$ and $\zeta = 0.5$.

3.2.1 Long-run Government Spending Multiplier in Normal Times

If the *short-run economy is in normal times*, the new Euler equation can be regenerated with the consideration of the long-run government spending using a three-state Markov chain:

$$y_S = -\frac{1}{\sigma}\frac{\phi_\pi - p}{1 - p}\pi_S + \Theta_{AD}g_s \tag{26}$$

$$\Theta_{AD} = q\zeta(y_{M,g} + \frac{1}{\sigma}\pi_{M,g} - 1) + 1, \tag{27}$$

where Θ_{AD} is the government spending shock shift in the Euler equation, ζ is the policy discount parameter, $y_{M,g}$ and $\pi_{M,g}$ are the medium run policy multiplier as in equation (24). For reference, this shift without long-run government spending will collapse to 1, which can connect to the case in our baseline model in section 2. The new items in this shift are from *rational expectations* of the output gap, inflation, and medium-run spending shock. Note that the first new term is from the future wealth effects (higher expected output gap in the future) as in Bouakez et al. (2017) and the household has the incentive to increase consumption due to consumption smoothing. The second term comes from the fact that the long-run government can increase the firm's marginal cost and further inflation. The third term is due to the direct demand effect from future government spending. See Appendix H, it turns out that $q\zeta(y_{M,g} + \frac{1}{\sigma}\pi_{M,g} - 1)$ is negative which means the overall expected effects from longer spending policy can crowd out the present output. In this case, long-run government spending can move down the Euler equation in a (π_t, y_t) graph and the effects can be controlled by the policy product $q\zeta$.

On the other hand, the new Phillips Curve can be shown below:

$$\pi_S = \kappa\frac{\gamma_y}{1 - \beta p - \kappa\gamma_r(\phi_\pi - p)}y_S + \frac{1}{1 - \beta p - \kappa\gamma_r(\phi_\pi - p)}\Theta_{AS}g_s \tag{28}$$

$$\Theta_{AS} = (\beta - \kappa\gamma_r)(1 - p)q\zeta\pi_{M,g} + \kappa\gamma_g, \tag{29}$$

where Θ_{AS} is the government spending shock shift in the Phillips Curve, ζ is the policy discount parameter, $\pi_{M,g}$ is the medium run inflation multiplier as in equation (24). For reference, this shift without long-run government spending will collapse to $\kappa\gamma_g$, which is the same as the case in our baseline model in section 2. The new item in this shirt is from *rational expectations* of inflation and the long-run spending can increase the firm's

marginal cost and further inflation due to sticky prices. See appendix H, it turns out the long-run government spending can further increase inflation, moving down the Phillips Curve in a (π_t, y_t) graph. Similar to the case in the Euler equation, the effects of long-run policy can be controlled by the policy product $q\zeta$.[25]

One can use the new Euler equation and the Phillips Curve to produce the output gap multiplier $\mathscr{M}_{S,N}^{O,long}$ and the inflation multiplier $\mathscr{M}_{S,N}^{I,long}$:

$$\mathscr{M}_{S,N}^{O,long} = \frac{\Theta_{AD}\sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)] - \Theta_{AS}(\phi_\pi - p)}{\sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)] + \kappa\gamma_y(\phi_\pi - p)} \tag{30}$$

$$\mathscr{M}_{S,N}^{I,long} = \frac{\left(\kappa\gamma_y\Theta_{AD} + \Theta_{AS}\right)\sigma(1-p)}{\sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)] + \kappa\gamma_y(\phi_\pi - p)}. \tag{31}$$

The long-run government spending can further increase the inflation multiplier since the longer government spending can further increase the marginal cost. Additionally, the cost channel can increase inflation more working as a cost-push shock. In normal times, the central bank would increase the nominal interest rate due to a higher price level. In that way, long-run spending policy can crowd out more private output due to a larger real interest rate. The real cost channel can increase the marginal cost and further inflation. Thus, the inflation multiplier can be higher in the strength of the cost channel, and this result is the same as the case in Section 2. In addition, the cost channel can lower the output gap. It is summarized that the long-run output gap multiplier reduces in the strength of the cost channel γ_r while the long-run inflation multiplier rises in γ_r.—See Appendix I.

3.2.2 Long-run Government Spending Multiplier at Zero Lower Bound

If the *short-run economy is at the ZLB*, the new Euler equation with a long-run spending policy can be given by:

$$y_S = -\frac{1}{\sigma(1-p)}[\log(\beta) - p\pi_S] + \Theta_{AD}g_s \tag{32}$$

$$\Theta_{AD} = q\zeta(y_{M,g} + \frac{1}{\sigma}\pi_{M,g} - 1) + 1. \tag{33}$$

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[25]In this section, I just assume that κ is very small in our theoretical analysis in line with e.g. Eggertsson (2011), Gabaix (2020) and Budianto et al. (2020). In this sense, one can assume that $\beta - \kappa\gamma_r > 0$ and then the long-run government policy can increase inflation here. However, if $\beta - \kappa\gamma_r < 0$, this might resolve the fiscal price puzzle (FPP) as in Han et al. (2020) that the long-run fiscal stimulus can lower inflation .

where Θ_{AD} is the government spending shock shift in the Euler equation. Similar to the case in normal times, the long-run government spending effects $q\zeta(y_{M,g} + \frac{1}{\sigma}\pi_{M,g} - 1)$ which are negative. It indicates that the long-run government spending can move down the Euler equation in a (π_t, y_t) graph and the effects can still be controlled by the policy product $q\zeta$.

In addition, I move to describe the new Phillips Curve as:

$$\pi_S = \kappa \frac{\gamma_y + \gamma_r \log(\beta)}{1 - \beta p + \kappa \gamma_r p} y_S + \frac{1}{1 - \beta p + \kappa \gamma_r p} \Theta_{AS} g_s \tag{34}$$

$$\Theta_{AS} = (\beta - \kappa \gamma_r)(1-p)q\zeta\pi_{M,g} + \kappa\gamma_g. \tag{35}$$

where Θ_{AS} is the government spending shock shift in the Phillips Curve. The long-run government spending can move down the Phillips Curve in a (π_t, y_t) graph, and likewise, the effects can be controlled by the policy product $q\zeta$.

Similar to the case in normal times, I use the Euler equation and the Phillips Curve in liquidity traps to produce the output gap multiplier $\mathcal{M}_{S,Z}^{O,long}$ and the inflation multiplier $\mathcal{M}_{S,Z}^{I,long}$:

$$\mathcal{M}_{S,Z}^{O,long} = \frac{\partial y_S}{\partial g_S} = \frac{\Theta_{AD}\sigma(1-p)(1-\beta p + \kappa\gamma_r p) + \Theta_{AS} p}{\sigma(1-p)(1-\beta p + \kappa\gamma_r p) - \kappa\gamma_y p} \tag{36}$$

$$\mathcal{M}_{S,Z}^{I,long} = \frac{\partial \pi_S}{\partial g_S} = \frac{\left(\kappa\gamma_y\Theta_{AD} + \Theta_{AS}\right)\sigma(1-p)}{\sigma(1-p)(1-\beta p + \kappa\gamma_r p) - \kappa\gamma_y p}. \tag{37}$$

The more extended government spending policy after the ZLB has subsided can increase inflation more by increasing firm's marginal costs. The nominal interest rate is zero, and higher inflation can help stimulate our NK economy since it can lower the real interest rate and then higher private consumption. At this time, the spending multiplier is larger, which is in line with Leduc and Wilson (2013) who empirically show similar larger long-term spending effects compared to the short-run policy. However, the real cost channel can reduce the effectiveness. The cost channel can dampen inflation due to the negative inflation expectation in the real borrowing cost. Thus, the output gap and inflation multipliers decrease in the strength of the cost channel—See Appendix J, and this is the same as the result in the baseline model.

## 3.3 Long-run Government Spending: Numerical Results

The comparison numerical result between the strength of the cost channel is revealed in Figure 5. It is observed that long-run government spending can decrease the output multiplier more than short-run government spending. The long-run government spending can further increase the inflation multiplier. The output gap multiplier falls in the strength of the real cost channel from γ_r while the inflation multiplier increases in the real cost channel γ_r. See Appendix K for numerical results w.r.t. the persistence p. I find that the output gap multiplier can be negative with the long-run spending shock.

Figure 5: Long run spending multipliers in normal times
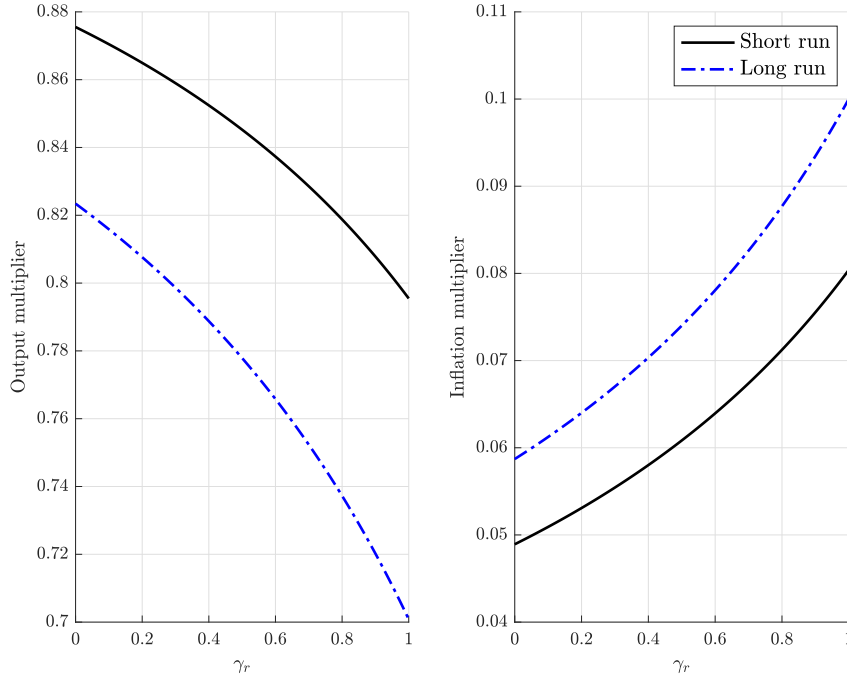
At the ZLB, the numerical comparison result between the strength of the cost channel is reported in Figure 6. We find that long-run government spending can further increase the output gap multiplier and the inflation multiplier. The two spending multipliers decrease in the strength of the cost channel γ_r. It is of note that the output gap multiplier can be more effective for prolonged spending policy in recessions. Similar results can be found in Appendix K for numerical results w.r.t. the persistence p.

Figure 6: Long run spending multipliers at ZLB
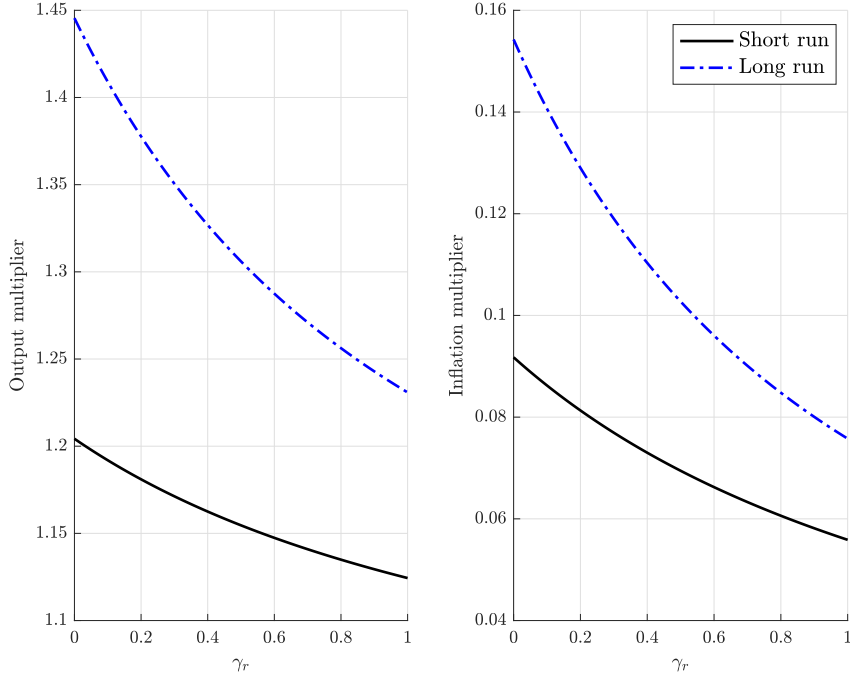
4 A Behavioral Model

As can be seen in section 3, the long-run government spending can drive the multipliers at the ZLB. In our baseline model in section 2, it is common knowledge that the agents have rational expectations. While in this section, I try to extend our baseline model to address the role of rational expectations. I finally develop a similar version of Gabaix (2020)'s model to incorporate bounded rationality where agents can be short-sighted about the world into our simple benchmark setup.

4.1 The NK model with Bounded Rationality

I use the model in Gabaix (2020) to show the behavior of the aggregate demand-side economy:

$$c_t = \bar{m}\mathbb{E}_t c_{t+1} - \frac{1}{\sigma_c}(i_t - \mathbb{E}_t \pi_{t+1} - r^n), \tag{38}$$

where $\bar{m} \in [0,1]$ is the cognitive discounting parameter in Gabaix (2020).[26]

In this case, one can obtain the path of y_t by substituting the resource constraint into

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[26]As in Gabaix (2020), this parameter can measure the attention to the future, which is a form of global cognitive discounting. Relative to the rational expectation ($\bar{m} = 1$), when the behavioral agent contemplates the events in the future, their expectations are geared to the steady state of the economy.

Euler equation:

$$y_t = \bar{m}(1 - s_g)\mathbb{E}_t y_{t+1} - \frac{1}{\sigma}(R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1} - r^n) + g_t - \bar{m}(1 - s_g)\mathbb{E}_t g_{t+1}, \qquad (39)$$

where $\sigma = \frac{\sigma_c}{1 - s_g}$.

As in Gabaix (2020), I can show the NK Phillips Curve with the real cost channel:

$$\pi_t = \beta\bar{m}\left[\varphi + \frac{1 - \beta\varphi}{1 - \beta\varphi\bar{m}}(1 - \varphi)\right]\mathbb{E}_t \pi_{t+1} + \kappa\left[\gamma_y y_t + \gamma_g g_t + \gamma_r(R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1})\right], \qquad (40)$$

where $\kappa = \frac{(1 - \varphi)(1 - \varphi\beta)}{\varphi(1 + \eta\theta)}$, $\eta > 0$ is the standard (inverse) of labor-supply elasticity, $\theta > 1$ is the price elasticity of differential goods demand, $\beta \in (0, 1)$ is the discounted preference parameter and $\varphi \in (0, 1)$ is the share of firms which can not adjust their prices.

4.2 Government Spending with Bounded Rationality: Theoretical Results

This section mainly presents the analytical results regarding the government spending multiplier with bounded rationality.

4.2.1 Multiplier with Bounded Rationality in Normal Times

I use the Euler equation and the Phillips Curve in normal times—see appendix L to produce the solutions of the output gap multiplier $\mathcal{M}_{S,N}^{O,BR}$ and the inflation multiplier $\mathcal{M}_{S,N}^{I,BR}$ below:[27]

$$\mathcal{M}_{S,N}^{O,BR} = \frac{\sigma[1 - p\bar{m}(1 - s_g)]\left\{1 - \beta p\bar{m}\left[\varphi + \frac{1 - \beta\varphi}{1 - \beta\varphi\bar{m}}(1 - \varphi)\right] - \kappa\gamma_r(\phi_\pi - p)\right\} - \kappa\gamma_g(\phi_\pi - p)}{\sigma(1 - p\bar{m}(1 - s_g))\left\{1 - \beta p\bar{m}\left[\varphi + \frac{1 - \beta\varphi}{1 - \beta\varphi\bar{m}}(1 - \varphi)\right] - \kappa\gamma_r(\phi_\pi - p)\right\} + \kappa\gamma_y(\phi_\pi - p)}$$

$$\mathcal{M}_{S,N}^{I,BR} = \frac{\left[1 + \frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma[1 - p\bar{m}(1 - s_g)]}{\sigma[1 - p\bar{m}(1 - s_g)]\left\{1 - \beta p\bar{m}\left[\varphi + \frac{1 - \beta\varphi}{1 - \beta\varphi\bar{m}}(1 - \varphi)\right] - \kappa\gamma_r(\phi_\pi - p)\right\} + \kappa\gamma_y(\phi_\pi - p)}.$$

In normal times, cognitive discounting can reduce the expectation effects in our baseline model which can lower inflation. It means that there is a lower rise in the interest

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[27]For simplicity, in this section, the simple two-state Markov chain is used to calculate spending multipliers.

rate set by the central bank, which can inflate the output gap multiplier accordingly compared to the baseline model. Since the introduced bound rationality is independent on the strength of the cost channel γ_r, the effects of this real channel on spending multipliers are the same as the baseline model results. Specifically, the output gap multiplier decreases in the strength of the cost channel, and the inflation multiplier increases in the strength of the cost channel. Similar results can also arise with the long-run government spending since the cognitive agent can decrease the expectation effects, which is in spirit with Nakata et al. (2019) and Farhi and Werning (2019). The inflation multiplier can be lower while the output multiplier can be higher.—See Appendix M.

Interestingly, as in appendix N, the output gap multiplier can decrease in the persistence p with $\bar{m} = 1$ but increase in persistence p with $\bar{m} = 0$. In this case, there exists a threshold value, across which the multiplier effects switch from decreasing in the persistence p to increasing. I also find that the output gap multiplier can be large (near one) in normal times if we suppose the agents have strong bounded rationality which means the expectation effects are extremely weak. Intuitively, agents tend to consume today but not to save since future consumption has less or no impact on the decision today. In this case, the crowding-out effects of government spending should be much attenuated and the output gap multiplier can be near one. This result can echo the previous empirical evidence that the output gap multiplier can be large in normal times as in Auerbach and Gorodnichenko (2012) and Acconcia et al. (2014).

4.2.2 Multiplier with Bounded Rationality at Zero Lower Bound

One can use the Euler equation and the Phillips Curve in liquidity traps—see appendix L to produce the output gap multiplier $\mathscr{M}_{S,Z}^{O,BR}$ and the inflation multiplier $\mathscr{M}_{S,Z}^{I,BR}$:

$$\mathscr{M}_{S,Z}^{O,BR} = \frac{\sigma[1 - p\bar{m}(1 - s_g)]\left\{1 - \beta p\bar{m}\left[\varphi + \frac{1-\beta\varphi}{1-\beta\varphi\bar{m}}(1-\varphi)\right] + \kappa\gamma_r p\right\} + \kappa\gamma_g p}{\sigma[1 - p\bar{m}(1 - s_g)]\left\{1 - \beta p\bar{m}\left[\varphi + \frac{1-\beta\varphi}{1-\beta\varphi\bar{m}}(1-\varphi)\right] + \kappa\gamma_r p\right\} - \kappa\gamma_y p}$$

$$\mathscr{M}_{S,Z}^{I,BR} = \frac{\left[1 + \frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma[1 - p\bar{m}(1 - s_g)]}{\sigma[1 - p\bar{m}(1 - s_g)]\left\{1 - \beta p\bar{m}\left[\varphi + \frac{1-\beta\varphi}{1-\beta\varphi\bar{m}}(1-\varphi)\right] + \kappa\gamma_r p\right\} - \kappa\gamma_y p}.$$

At the ZLB, inflation caused by government spending can be less due to cognitive discounting. Thus, it can increase the real interest rate, which, in turn, can lower the

output gap multiplier.—See Appendix O. The effects on the strength of the real cost channel are the same as the baseline model in liquidity traps. Specifically, the output gap and inflation multipliers decrease in the strength of the real cost channel γ_r.

4.3 Government Spending with Bounded Rationality: Numerical Results

Figure 7 shows the numerical results, and the main takeaway is that the output multiplier with bounded rationality can be higher in the strength of the cognitive discounting level $\bar{m}$, and the inflation multiplier with bounded rationality can be lower. In addition, the output gap multiplier decreases in the strength of the real cost channel γ_r while the inflation multiplier increases in γ_r. See Appendix P for numerical results w.r.t. the persistence p. I find that the output gap multiplier can decrease in the persistence p with $\bar{m} = 1$ but increase in persistence p with $\bar{m} = 0$.

Figure 7: Spending multipliers with/without bounded rationality in normal times
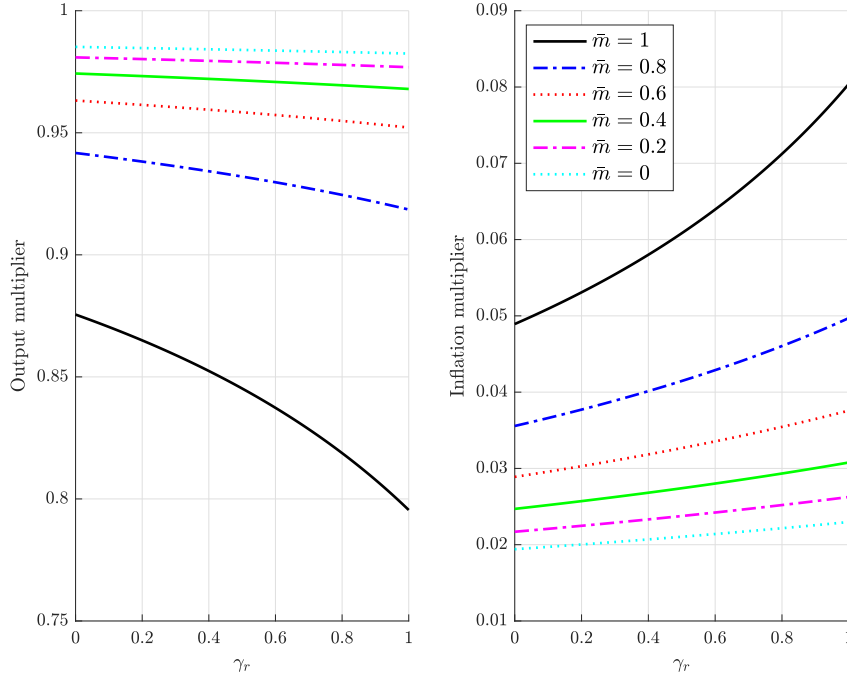
As in our numerical simulation in Figure 8, it is seen that the inflation and output multipliers are lower with bounded rationality. In addition, the spending multipliers in the strength of the real cost channel with bounded rationality are the same as the baseline model. As in Appendix P, the output multiplier increases numerically in the

persistence p.

Figure 8: Spending multipliers with/without bounded rationality at ZLB
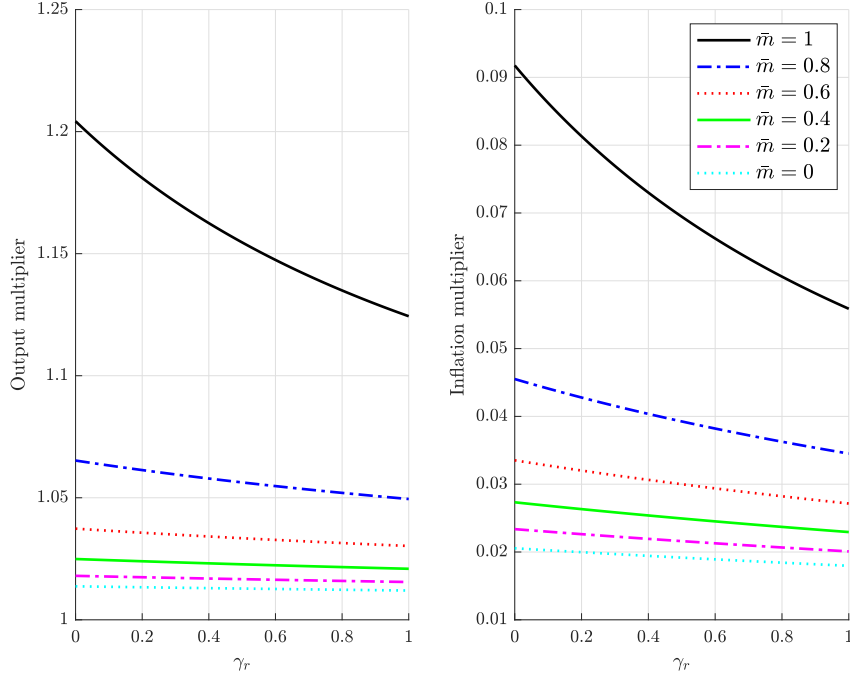
5 Concluding Remarks

This paper augments the real cost channel in the textbook NK model to explore government spending multipliers. The general properties of spending multipliers are detected using this analytical framework. It is found that the government spending multiplier in liquidity traps decreases in the strength of the real cost channel. This result is in contrast to Abo-Zaid (2022). In normal times, the multiplier also decreases with the real cost channel.

The robust role of the real cost channel on the spending multipliers is confirmed. To be more specific, our results on the strength of the real cost channel, if one considers the long-run government spending policy, are similar to the short-run model. I extend the basic model with bounded rationality. It turns out that cognitive behavior can alter spending multipliers, however, the real cost channel can still work robustly.

I add to the literature on government spending multiplier. This paper uses the NK model with the real cost channel to provide new theoretical insights to explain the empirically lower government spending multiplier when the nominal interest rate is fixed

at the lower bound. To the extent that firms care about the real interest rate as opposed to the nominal interest rate, this can make government spending less effective in stimulating the economy in times of recession.

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31

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Online Appendix

A Details of the Baseline Model in Section 2

A.1 Aggregate Supply Side

The behavior of aggregate supply side economics in the standard NK model is shown below as in Tillmann (2009):

$$\pi_t = \kappa m c_t + \beta \mathbb{E}_t \pi_{t+1}.$$

I need to add the government spending ingredient into Beaudry et al. (2022)'s aggregate supply-side economics. Here I assume that there is a large number of firms indexed by z which produce differentiated intermediate goods sold to a final goods producer. The representative firm follows a Leontief production function as shown below

$$Y_t(z) = \min(aN_t(z), bM_t(z)),$$

where M_t is the final good.

The unit price of final goods that enter the production of basic input is P_t. As in Beaudry et al. (2022), we assume that the basic input representative should borrow D_t to pay for the input M_t at the risk-free nominal rate i_t for the production. In this case, the firms should produce, sell the product, pay wages, pay back the debt in the previous period, and distributes the dividends. One can show the budget constraint of the firms at the time t by simply assuming zero profits in equilibrium.

$$D_t + P_t^B = W_t P_t L_t + (1 + i_{t-1}) D_t + P_t M_t,$$

where P_t^B is the basic input price, and $D_{t+1} = P_t M_t$. Thus the profit Π_t can be shown in the below expressions:

$$\Pi_t = P_t^B - W_t P_t L_t - (1 + i_{t-1}) P_{t-1} M_{t-1}.$$

We further assume that the firms maximize the sum of real profit $\frac{\Pi_t}{P_t}$ with a discount parameter β. In this case, the first-order condition can be shown:

$$P_t^B = \left(\frac{1}{a}W_t + \frac{\beta}{b}\mathbb{E}_t\frac{1+i_t}{1+\pi_{t+1}}\right)P_t.$$

Thus, one can obtain the (real) marginal cost of the basic input:

$$MC_t = \frac{W_t}{a} + \frac{\beta}{b}\mathbb{E}\left[\frac{1+i_t}{1+\pi_{t+1}}\right].$$

With log condition, one can show the linearized equilibrium

$$mc_t = \hat{\gamma}_y(w_t) + \gamma_r(R_t + \log(\beta) - \mathbb{E}\pi_{t+1}),$$

where $\hat{\gamma}_y = \frac{\frac{1}{a}W}{\frac{1}{a}W + \frac{\beta}{b}\frac{1+i}{1+\pi}}$ and $\gamma_r = \frac{\frac{\beta}{b}\frac{1+i}{1+\pi}}{\frac{1}{a}W + \frac{\beta}{b}\frac{1+i}{1+\pi}}$. On the other hand, the optimal labor supply reads

$$\frac{v'(N_t)}{u'(C_t)} = W_t.$$

By using the production function $y_t = n_t$ and resource constraint $y_t = (1 - s_g)c_t + g_t$, the marginal cost can be rewritten as

$$mc_t = \hat{\gamma}_y \frac{Nv''(N)}{v'(N)}y_t - \hat{\gamma}_y \frac{Cu''(C)}{u'(C)}\left(\frac{y_t - g_t}{1 - s_g}\right) + \gamma_r(R_t + \log(\beta) - \mathbb{E}_t\pi_{t+1})$$

$$= \gamma_y y_t + \gamma_g g_t + \gamma_r(R_t + \log(\beta) - \mathbb{E}_t\pi_{t+1}),$$

where $\gamma_y = \hat{\gamma}_y\left(\frac{Nv''(N)}{v'(N)} - \frac{Cu''(C)}{u'(C)(1-s_g)}\right)$, and $\gamma_g = \hat{\gamma}_y\frac{Cu''(C)}{u'(C)(1-s_g)}$. Therefore, the Phillips Curve with government spending is shown below.[28]

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa\left[\gamma_y y_t + \gamma_g g_t + \gamma_r(R_t + \log(\beta) - \mathbb{E}_t\pi_{t+1})\right].$$

---

[28]The advantage of this introduced real cost channel method as in Beaudry et al. (2022) is that it allows to set arbitrarily the elasticity of marginal cost rate with regard to wage and interest rate. Please see Beaudry et al. (2022) for a comprehensive comparison between the model with a nominal cost channel and with the real cost channel.

B   Proof for Proposition 1

If I only consider the demand shock and it is assumed that the demand shock r_S^n can put the economy into liquidity traps with one enough (negatively) big number ($r_S^n < \underline{r_S^n}$), one can rewrite the Phillips Curve as

$$
y_S = \begin{cases}
\frac{1 - \beta p + \kappa \gamma_r p - \kappa \gamma_r \phi_\pi}{\kappa \gamma_y} \pi_S & \text{if } r_S^n \geq \underline{r_S^n} \\
\frac{1 - \beta p + \kappa \gamma_r p}{\kappa \gamma_y} \pi_S - \frac{\gamma_r}{\gamma_y} \log(\beta) & \text{if } r_S^n < \underline{r_S^n}.
\end{cases}
$$

Similarly, one can rewrite the Euler equations as follows:

$$
y_S = \begin{cases}
-\frac{1}{\sigma} \frac{\phi_\pi - p}{1 - p} \pi_S + \frac{1}{\sigma} \frac{r_S^n}{1 - p} & \text{if } r_S^n \geq \underline{r_S^n} \\
\frac{\frac{1}{\sigma} p}{1 - p} \pi_S + \frac{1}{\sigma} \frac{r_S^n - \log(\beta)}{1 - p} & \text{if } r_S^n < \underline{r_S^n}.
\end{cases}
$$

I combine the first questions of Euler equation and Phillips Curve to obtain the exact expression for $\underline{r_S^n}$ which can be written as:

$$
\underline{r_S^n} = \left[\frac{(1 - \beta p + \kappa \gamma_r p - \kappa \gamma_r \phi_\pi)(1 - p)}{\kappa \gamma_y \frac{1}{\sigma}} + (\phi_\pi - p) \right] \frac{\log(\beta)}{\phi_\pi} < 0.
$$

One can show the exact boundary condition for $\underline{r_S^n}$ in the standard NK model without the real cost channel:

$$
\underline{r_S^{n,B}} = \left[\frac{(1 - \beta p)(1 - p)}{\kappa \gamma_y \frac{1}{\sigma}} + (\phi_\pi - p) \right] \frac{\log(\beta)}{\phi_\pi} < 0.
$$

Likewise, the exact boundary condition for $\underline{r_S^n}$ in the model with the nominal cost channel:

$$
\underline{r_S^{n,N}} = \left[\frac{(1 - \beta p - \kappa \gamma_r \phi_\pi)(1 - p)}{\kappa \gamma_y \frac{1}{\sigma}} + (\phi_\pi - p) \right] \frac{\log(\beta)}{\phi_\pi} < 0.
$$

In this case, I have

$$
\underline{r_S^n} - \underline{r_S^{n,B}} = \frac{\kappa \gamma_r (p - \phi_\pi)}{\kappa \gamma_y \frac{1}{\sigma}} \frac{\log(\beta)}{\phi_\pi} > 0.
$$

And further one can show

$$
\underline{r_S^n} - \underline{r_S^{n,N}} < 0.
$$

One can use this to obtain the result in the main text.

C Proof for Proposition 2

I show the solutions for the output gap and inflation multipliers below:

$$\mathcal{M}_{S,N}^{O} = \frac{\partial y_S}{\partial g_S} = \frac{\frac{\sigma(1-p)}{\phi_\pi - p} - \frac{\kappa\gamma_g}{1-\beta p - \kappa\gamma_r(\phi_\pi - p)}}{\frac{\kappa\gamma_y}{1-\beta p - \kappa\gamma_r(\phi_\pi - p)} + \frac{\sigma(1-p)}{\phi_\pi - p}}$$

$$= \frac{\sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)] - \kappa\gamma_g(\phi_\pi - p)}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)]}$$

$$\mathcal{M}_{S,N}^{I} = \frac{\partial \pi_S}{\partial g_S} = \frac{1 + \frac{\gamma_g}{\gamma_y}}{\frac{1-\beta p - \kappa\gamma_r(\phi_\pi - p)}{\kappa\gamma_y} + \frac{1}{\sigma(1-p)}(\phi_\pi - p)}$$

$$= \frac{\left[1 + \frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p)}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)]}.$$

For the normal cost channel case, the item $[1-\beta p - \kappa\gamma_r(\phi_\pi - p)]$ will switch to $[1-\beta p - \kappa\gamma_r\phi_\pi]$. In this case, one can easily prove that the inflation multiplier $\mathcal{M}_{S,N}^{I,N}$ can be larger with the nominal cost channel due to a smaller denominator. However, the output multiplier with the nominal cost channel $\mathcal{M}_{S,N}^{O,N}$ can be less. For the output multiplier, the numerator is less than the denominator. That is,

$$\mathcal{M}_{S,N}^{O,B} = \sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)] - \kappa\gamma_g(\phi_\pi - p) < \kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)].$$

Thus, the output gap multiplier is less than one. For the output gap multiplier without the real cost channel:

$$\mathcal{M}_{S,N}^{O,B} = \frac{\partial y_S}{\partial g_S} = \frac{\sigma(1-p)[1-\beta p] - \kappa\gamma_g(\phi_\pi - p)}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1-\beta p]}.$$

One can compare this expression to the previous one with the real cost channel and it is easy, after some arrangements, to obtain the output gap multiplier with the real cost channel that is lower.

For the inflation multiplier without the real cost channel:

$$\mathcal{M}_{S,N}^{I,B} = \frac{\partial \pi_S}{\partial g_S} = \frac{\left[1 + \frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p)}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1-\beta p]}.$$

In this case, the denominator of the inflation multiplier with the real cost channel is lower due to a negative item and thus the inflation multiplier is higher. For the output gap multiplier:

$$\frac{\partial \mathcal{M}^{O}_{S,N}}{\partial \gamma_r} = -\sigma(1-p)(\phi_\pi - p)\kappa \frac{\mathscr{D} - \mathscr{N}}{\mathscr{D}^2} < 0,$$

where $\mathscr{D} = \kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1 - \beta p - \kappa\gamma_r(\phi_\pi - p)]$ and $\mathscr{N} = \sigma(1-p)[1 - \beta p - \kappa\gamma_r(\phi_\pi - p)] - \kappa\gamma_g(\phi_\pi - p)$. Thus, the output gap multiplier in normal times is decreasing in the strength of the real cost channel.

For the inflation multiplier, it is observed with ease that the higher the strength of the real cost channel γ_r, the higher the denominator of this multiplier. In other words, the inflation multiplier is decreasing in the strength of the real cost channel.

D   Proof for Proposition 3

The output gap and inflation multipliers at the ZLB are reproduced here:

$$\mathcal{M}^{O}_{S,Z} = \frac{\partial y_S}{\partial g_S} = \frac{\frac{\sigma(1-p)}{-p} - \frac{\kappa\gamma_g}{1 - \beta p - \kappa\gamma_r(-p)}}{\frac{\kappa\gamma_y}{1 - \beta p - \kappa\gamma_r(-p)} + \frac{\sigma(1-p)}{-p}}$$

$$= \frac{\sigma(1-p)[1 - \beta p - \kappa\gamma_r(-p)] - \kappa\gamma_g(-p)}{\kappa\gamma_y(-p) + \sigma(1-p)[1 - \beta p - \kappa\gamma_r(-p)]}$$

$$\mathcal{M}^{I}_{S,Z} = \frac{\partial \pi_S}{\partial g_S} = \frac{1 + \frac{\gamma_g}{\gamma_y}}{\frac{1 - \beta p - \kappa\gamma_r(-p)}{\kappa\gamma_y} + \frac{1}{\sigma(1-p)}(-p)}$$

$$= \frac{\left[1 + \frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p)}{\kappa\gamma_y(-p) + \sigma(1-p)[1 - \beta p - \kappa\gamma_r(-p)]}.$$

The numerator and denominator of the output and inflation multipliers (we assume γ_r is far greater than γ_y and the denominator is positive) are both positive here and thus we have the positive spending multipliers. The output gap multiplier can be rewritten as:

$$\mathcal{M}^{O}_{S,Z} = \frac{\partial y_S}{\partial g_S} = 1 + \frac{\sigma(1-p)[1 - \beta p - \kappa\gamma_r(-p)] + \kappa\gamma_g p + \kappa\gamma_y p}{\kappa\gamma_y(-p) + \sigma(1-p)[1 - \beta p - \kappa\gamma_r(-p)]}.$$

This output gap multiplier is larger than one. For the output gap multiplier without the real cost channel:

$$\mathscr{M}_{S,Z}^{O,B} = \frac{\partial y_S}{\partial g_S} = \frac{\sigma(1-p)[1-\beta p] - \kappa\gamma_g(-p)}{\kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p]}.$$

Similar with the case in normal times, one can compare this expression to the previous one with the real cost channel, and it is easy, after some arrangements, to obtain the output gap multiplier with the real cost channel that is lower.

For the inflation multiplier without the real cost channel:

$$\mathscr{M}_{S,Z}^{I,B} = \frac{\partial \pi_S}{\partial g_S} = \frac{\left[1 + \frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p)}{\kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p]}.$$

One can observe that the higher the strength of the real cost channel γ_r, the higher the denominator of this multiplier. In this case, it can be lower with the real cost channel.

For the output gap multiplier:

$$\frac{\partial \mathscr{M}_{S,Z}^{O}}{\partial \gamma_r} = -\sigma(1-p)(-p)\kappa\frac{\mathscr{D} - \mathscr{N}}{\mathscr{D}^2} < 0,$$

where $\mathscr{D} = \kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(-p)]$ and $\mathscr{N} = \sigma(1-p)[1-\beta p - \kappa\gamma_r(-p)] - \kappa\gamma_g(-p)$. Thus, the output gap multiplier in normal times is decreasing in the strength of the real cost channel.

For the inflation multiplier, it is observed with ease that the higher the strength of the real cost channel γ_r, the lower the denominator of this multiplier. In other words, the inflation multiplier is increasing in the strength of the real cost channel.

While the nominal cost channel multipliers $\mathscr{M}_{S,Z}^{O,N}$ and $\mathscr{M}_{S,Z}^{I,N}$ can be invariant with the standard NK model since the nominal channel in liquidity traps can not be included in the partial derivative of government spending to the output gap/inflation in the calculation of fiscal multipliers.

E   Impulse Response Function

Figure 9: Impulse response to a contractionary natural rate shock
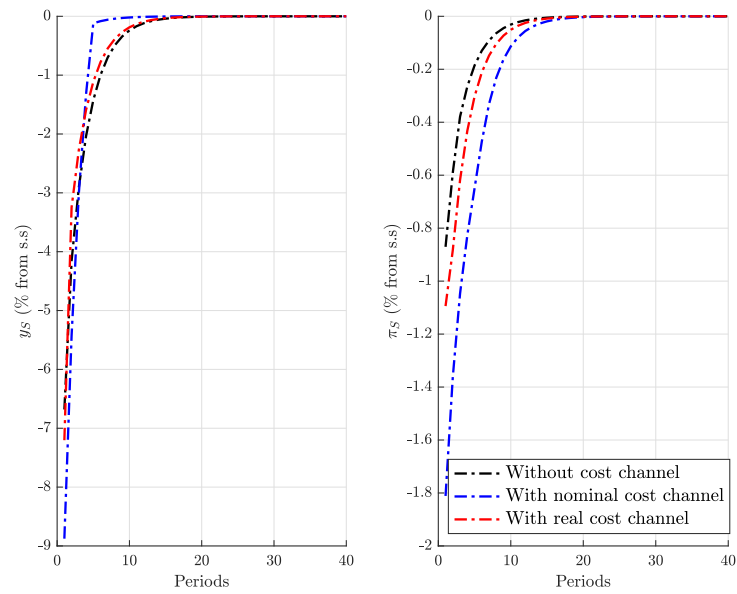
F   Baseline Multiplier Figures w.r.t. Persistent p
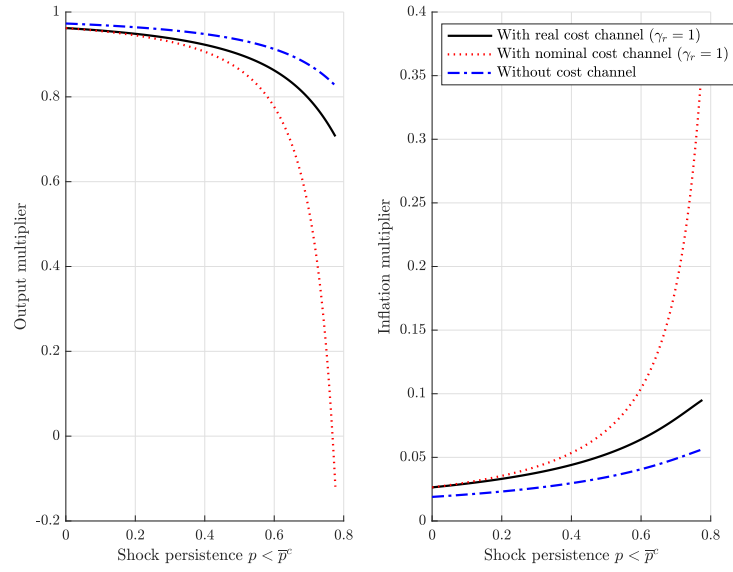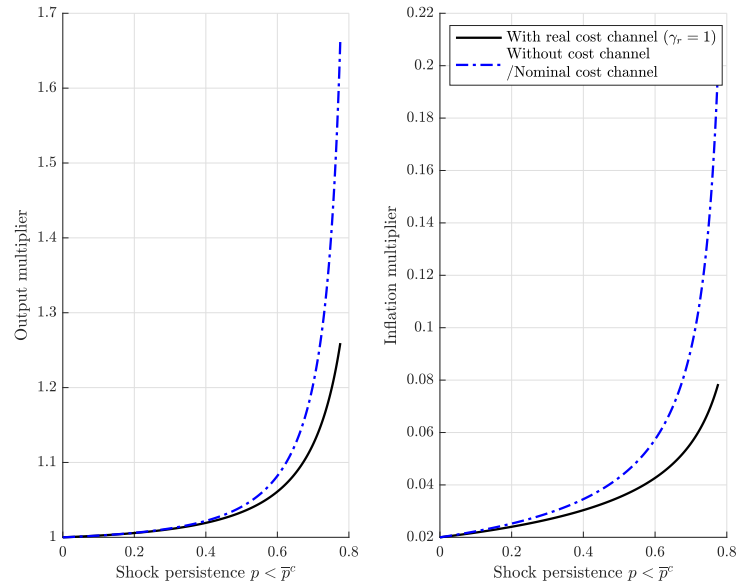
Figure 10: Spending multipliers in normal times

Figure 11: Spending multipliers at ZLB

G Multipliers in the Medium Run

In the medium run, the economy is absent of natural rate shock, and one can show the medium run government spending with the persistence q as follows

$$y_M = y_{M,g} g_M$$

$$= \frac{\sigma(1-q)[1-\beta p - \kappa\gamma_r(\phi_\pi^q - q)] - \kappa\gamma_g(\phi_\pi^q - q)}{\kappa\gamma_y(\phi_\pi^q - q) + \sigma(1-q)[1-\beta q - \kappa\gamma_r(\phi_\pi^q - q)]} g_M.$$

$$\pi_M = \pi_{M,g} g_M$$

$$= \frac{\left[1 + \frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-q)}{\kappa\gamma_y(\phi_\pi^q - q) + \sigma(1-q)[1-\beta q - \kappa\gamma_r(\phi_\pi^q - q)]} g_M.$$

H Euler and Phillips shift

The long-run government spending in the Euler equation shift:

$$q\zeta(y_{M,g} + \frac{1}{\sigma}\pi_{M,g} - 1) = q\zeta\frac{\kappa(\gamma_y + \gamma_g)(1-\phi_\pi^q)}{\kappa\gamma_y(\phi_\pi^q - q) + \sigma(1-q)[1-\beta q - \kappa\gamma_r(\phi_\pi^q - q)]} < 0.$$

The long run government spending in the Phillips Curve shift

$$(\beta - \kappa\gamma_r)(1-p)q\zeta\pi_{M,g} > 0,$$

where we assume that the $\beta - \kappa\gamma_r > 0$ since κ is very small in our theoretical analysis as in Gabaix (2020) Budianto et al. (2020), and Nie (2021).

I The long-run government spending effects in normal times

I can use the new Euler equation and the Phillips Curve to reproduce the spending multipliers:

$$\mathcal{M}_{S,N}^{O,long} = \frac{\partial y_S}{\partial g_S} = \frac{\Theta_{AD}\sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)] - \Theta_{AS}(\phi_\pi - p)}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)]}$$

$$\mathcal{M}_{S,N}^{I,long} = \frac{\partial \pi_S}{\partial g_S} = \frac{\left[\kappa\gamma_y\Theta_{AD} + \Theta_{AS}\right]\sigma(1-p)}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)]}.$$

For the output gap multipliers, as in appendix H, one can see that the long-run government spending shock can lead to a lower Θ_{AD} but a higher Θ_{AS}. In this case, the multiplier should be lower.

For inflation multiplier,

$$\kappa\gamma_y q\zeta(y_{M,g} + \frac{1}{\sigma}\pi_{M,g} - 1) + (\beta - \kappa\gamma_r)(1-p)q\zeta\pi_{M,g}$$

$$= q\zeta\frac{\kappa\gamma_y\kappa(\gamma_y + \gamma_g)(1-\phi_\pi^q)}{\kappa\gamma_y(\phi_\pi^q - q) + \sigma(1-q)[1-\beta q - \kappa\gamma_r(\phi_\pi^q - q)]} + q\zeta\frac{(\beta - \kappa\gamma_r)(1-p)q\left[1 + \frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-q)}{\kappa\gamma_y(\phi_\pi^q - q) + \sigma(1-q)[1-\beta q - \kappa\gamma_r(\phi_\pi^q - q)]} > 0,$$

where we assume κ is minor in our theoretical analysis as in e.g. Eggertsson (2011), Gabaix (2020) and Budianto et al. (2020) and the first item has an addition multiplier κ. One can use this to prove the result in the main text. Since there is no γ_r in Θ_{AD}, we only focus on the Θ_{AS}'s effects. For the main proposition result, since the term with Θ_{AD} is decreasing in γ_r, we only need to show the other term with Θ_{AS} is also decreasing in γ_r and thus one can prove the output gap multiplier is decreasing in γ_r. The output multiplier can be reduced below:

$$-\frac{\Theta_{AS}}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)]}.$$

For the main proposition result, since the term with Θ_{AD} is increasing in γ_r, we only need to show the other term with Θ_{AS} is also increasing in γ_r and thus one can prove the

inflation multiplier is increasing in γ_r. The inflation multiplier can be reduced below:

$$\frac{\Theta_{AS}}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1 - \beta p - \kappa\gamma_r(\phi_\pi - p)]}.$$

One can differentiate this common term with regard to γ_r and this common term can be reduced further as:

$$\frac{\beta - \kappa\gamma_r}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1 - \beta p - \kappa\gamma_r(\phi_\pi - p)]} \frac{1}{\kappa\gamma_y(\phi_\pi - q) + \sigma(1-q)[1 - \beta q - \kappa\gamma_r(\phi_\pi^q - q)]}.$$

I differentiate the above term with regard to γ_r to obtain:

$$\frac{\sigma(1-p)\kappa(\beta\phi_\pi - 1)\mathscr{D}_1 + \sigma(1-q)\kappa(\beta\phi_\pi^q - 1)\mathscr{D}_2 - \mathscr{O}(\kappa^2)}{\mathscr{D}_3^2}.$$

where $\mathscr{D}_1 = \kappa\gamma_y(\phi_\pi - q) + \sigma(1-q)[1 - \beta q - \kappa\gamma_r(\phi_\pi^q - q)]$, $\mathscr{D}_2 = \kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1 - \beta p - \kappa\gamma_r(\phi_\pi - p)]$, $\mathscr{D}_3 = \mathscr{D}_1 \cdot \mathscr{D}_2$ and $\mathscr{O}(\kappa^2)$ is the residual of order two since we assume that κ is trivial in our theoretical analysis as in e.g. Eggertsson (2011), Gabaix (2020) and Budianto et al. (2020), it is easy to check that the derivative with regard to γ_r is positive. In this case, one can use this to prove the result in the main text.

J The long-run government spending effects at the ZLB

One can produce the output gap and inflation multipliers below

$$\mathscr{M}_{S,Z}^{O,long} = \frac{\partial y_S}{\partial g_S} = \frac{\Theta_{AD}\sigma(1-p)[1 - \beta p - \kappa\gamma_r(-p)] - \Theta_{AS}(-p)}{\kappa\gamma_y(-p) + \sigma(1-p)[1 - \beta p - \kappa\gamma_r(-p)]}$$

$$\mathscr{M}_{S,Z}^{I,long} = \frac{\partial \pi_S}{\partial g_S} = \frac{\left[\kappa\gamma_y\Theta_{AD} + \Theta_{AS}\right]\sigma(1-p)}{\kappa\gamma_y(-p) + \sigma(1-p)[1 - \beta p - \kappa\gamma_r(-p)]}.$$

For the output gap multipliers, the numerator with medium run spending policy can be decomposed into the following two parts. The first part:

$$\kappa\gamma_y q\zeta(y_{M,g}+\frac{1}{\sigma}\pi_{M,g}-1)\sigma(1-p)[1-\beta p+\kappa\gamma_r p]$$

$$=q\zeta\frac{\kappa\gamma_y\kappa(\gamma_y+\gamma_g)(1-\phi_\pi^q)}{\kappa\gamma_y(\phi_\pi^q-q)+\sigma(1-q)[1-\beta q-\kappa\gamma_r(\phi_\pi^q-q)]}\sigma(1-p)[1-\beta p+\kappa\gamma_r p].$$

The second part:

$$(\beta-\kappa\gamma_r)(1-p)q\zeta\pi_{M,g}p$$

$$=q\zeta\frac{(\beta-\kappa\gamma_r)(1-p)q\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-q)}{\kappa\gamma_y(\phi_\pi^q-q)+\sigma(1-q)[1-\beta q-\kappa\gamma_r(\phi_\pi^q-q)]}p.$$

To simplify the proof, one can plus the two items and show the sum is positive if we assume that κ is very small in our theoretical analysis. Similar to the inflation multiplier in normal times, we can have a higher long run inflation multiplier at the ZLB. Since there is no γ_r in Θ_{AD}, we only focus on the Θ_{AS}'s effects. For the main result, since the term with Θ_{AD} is decreasing in γ_r, we only need to show the the other term with Θ_{AS} is also decreasing in γ_r and thus one can see the output gap multiplier is decreasing in γ_r. In this case, the output multiplier can be reduced below:

$$\frac{\Theta_{AS}}{\kappa\gamma_y(-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]}.$$

For the main result, since the term with Θ_{AD} is decreasing om γ_r, we only need to show the other term with Θ_{AS} is also decreasing in γ_r and thus the inflation multiplier is decreasing in γ_r. The inflation multiplier can be reduced below:

$$\frac{\Theta_{AS}}{\kappa\gamma_y(-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]}.$$

One can differentiate this common term with regard to γ_r and this common term can be reduced further as:

$$\frac{\beta-\kappa\gamma_r}{\kappa\gamma_y(-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]}\frac{1}{\kappa\gamma_y(\phi_\pi^q-q)+\sigma(1-q)[1-\beta q-\kappa\gamma_r(\phi_\pi^q-q)]}.$$

I differentiate the above term with regard to γ_r to obtain:

$$\frac{-\sigma(1-p)\kappa\mathscr{D}_1 + \sigma(1-q)\kappa(\beta\phi_\pi^q - 1)\mathscr{D}_2 - \mathscr{O}(\kappa^2)}{\mathscr{D}_3^2}.$$

where $\mathscr{D}_1 = \kappa\gamma_y(\phi_\pi^q - q) + \sigma(1-q)[1 - \beta q - \kappa\gamma_r(\phi_\pi^q - q)]$, $\mathscr{D}_2 = \kappa\gamma_y(-p) + \sigma(1-p)[1 - \beta p - \kappa\gamma_r(-p)]$, $\mathscr{D}_3 = \mathscr{D}_1 \cdot \mathscr{D}_2$ and $\mathscr{O}(\kappa^2)$ is the residual of order two. One can reduce this expression as:

$$-(1-p)\sigma(1-q)(1-\beta q) + (1-q)(\beta\phi_\pi^q - 1)\sigma(1-p)(1-\beta p) - \mathscr{O}(\kappa^2) < 0,$$

where I use the general condition $\phi_\pi^q\beta - 1 < 1$ and the short run period should be longer or almost equal to the long run period in reality such that $p \geq q$. In this case, one can use this to prove the result in the main text.

K   Long-run Multiplier Figures w.r.t. Persistent p

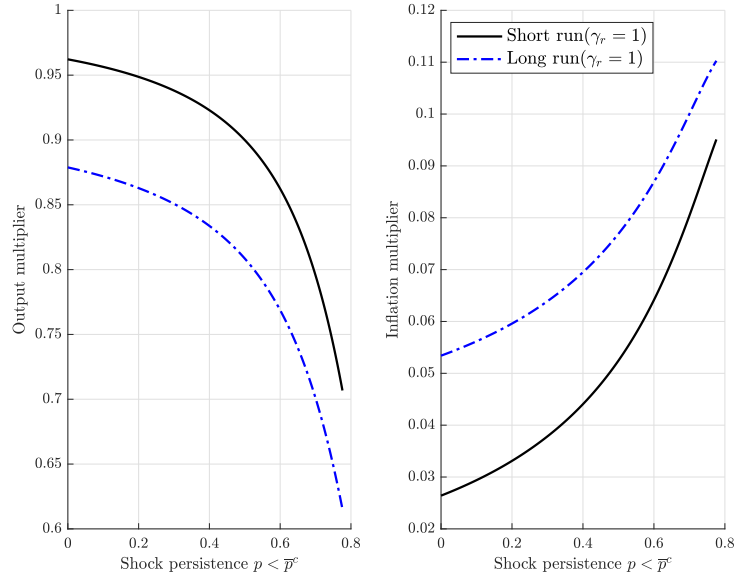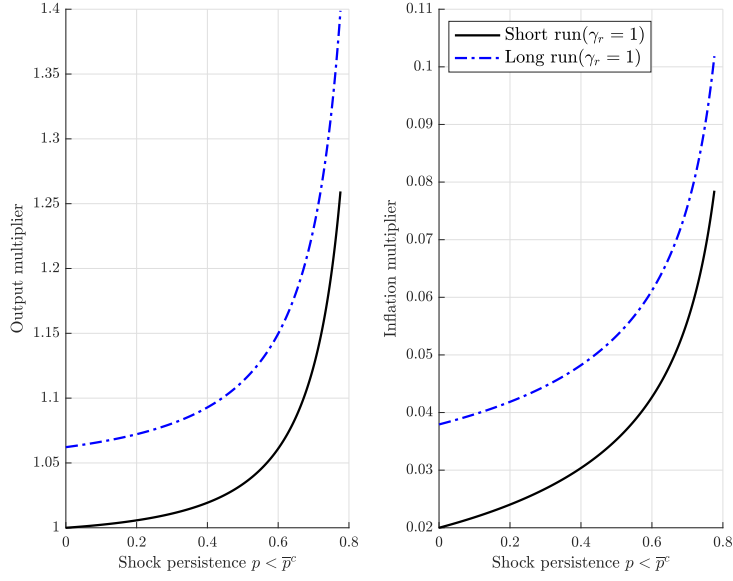Figure 12: Long-run spending multipliers in normal times

Figure 13: Long-run spending multipliers at ZLB

L Euler Equation and Phillips Curve with Bounded Rationality

The Euler equation and Phillips Curve in normal times:

$$y_S = -\frac{1}{\sigma(1-\alpha_{EE}p)}(\phi_\pi - p)\pi_S + g_S$$

$$\pi_S = \kappa \frac{\gamma_y}{1 - \beta p \alpha_{PC} - \kappa \gamma_r(\phi_\pi - p)} y_S + \kappa \frac{\gamma_g}{1 - \beta p \alpha_{PC} - \kappa \gamma_r(\phi_\pi - p)} g_S.$$

The Euler equation and Phillips Curve can be elaborated at the ZLB:

$$y_S = -\frac{1}{\sigma(1 - p\alpha_{EE})}[\log(\beta) - p\pi_S] + g_S$$

$$\pi_S = \frac{\kappa\gamma_y y_S + \kappa\gamma_r \log(\beta)}{1 - \beta p \alpha_{PC} + \kappa\gamma_r p} + \kappa \frac{\gamma_g}{1 - \beta p \alpha_{PC} + \kappa\gamma_r p} g_S.$$

A-14

M   The government spending effects with bounded rationality in normal times

I reproduce the multipliers here. For simplicity, we define $\alpha_{EE} = \bar{m}(1 - s_g)$ and $\alpha_{PC}(\bar{m}) = \bar{m}[\varphi + \frac{1 - \beta\varphi}{1 - \beta\varphi\bar{m}}(1 - \varphi)]$.

$$\mathcal{M}_{S,N}^{O,BR} = \frac{\partial y_S}{\partial g_S} = \frac{\frac{\sigma(1 - p\alpha_{EE})}{\phi_\pi - p} - \frac{\kappa\gamma_g}{1 - \beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)}}{\frac{\kappa\gamma_y}{1 - \beta\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)} + \frac{\sigma(1 - p\alpha_{EE})}{\phi_\pi - p}}$$

$$= \frac{\sigma(1 - p\alpha_{EE})[1 - \beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)] - \kappa\gamma_g(\phi_\pi - p)}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1 - p\alpha_{EE})[1 - \beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)]}$$

$$\mathcal{M}_{S,N}^{I,BR} = \frac{\partial\pi_S}{\partial g_S} = \frac{1 + \frac{\gamma_g}{\gamma_y}}{\frac{1 - \beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)}{\kappa\gamma_y} + \frac{1}{\sigma(1 - p\alpha_{EE})}(\phi_\pi - p)}$$

$$= \frac{\left[1 + \frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1 - p\alpha_{EE})}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1 - p\alpha_{EE})[1 - \beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)]}.$$

where α_{EE} and α_{PC} are increasing in the cognitive discounting parameter $\bar{m}$. One can differentiate output gap multiplier with regard to $\bar{m}$ and after some arrangements we have:

$$\frac{\kappa(\gamma_y + \gamma_g)(\phi_\pi - p)f_N'(\bar{m})}{\mathscr{D}_{BN}^2} > 0,$$

where $\mathscr{D}_{BN} = \kappa\gamma_y(\phi_\pi - p) + \sigma(1 - p\alpha_{EE})[1 - \beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)]$ and $f_N'(\bar{m})$ is the derivative of $\sigma(1 - p\alpha_{EE})[1 - \beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)]$ with regard to $\bar{m}$ which is positive.

One can differentiate inflation multiplier with regard to $\bar{m}$ and after some arrangements we have:

$$\frac{-p\alpha_{EE}'\mathscr{D}_{BN} - f_N'(\bar{m})(1 - p\alpha_{EE})}{\mathscr{D}_{BN}^2} < 0,$$

where $\mathscr{D}_{BN} = \kappa\gamma_y(\phi_\pi - p) + \sigma(1 - p\alpha_{EE})[1 - \beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)]$ and $f_N'(\bar{m})$ is the derivative of $\sigma(1 - p\alpha_{EE})[1 - \beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)]$ with regard to $\bar{m}$ which is positive.

Since the strength of the real cost channel γ_r is independent of the new ingredient that is bounded rationality. See appendix C, the output gap multiplier is decreasing in the strength of the real cost channel γ_r and the inflation multiplier is increasing in the strength of the real cost channel γ_r. One can use this to prove the main text.

N Derivatives of Output Gap Multiplier w.r.t. p with Bounded Rationality

One can differentiate the output multiplier w.r.t p and the numerator is[29]:

$$[f'(p)+\kappa\gamma_g][\kappa\gamma_y(\phi_\pi-p)+f(p)]-[f(p)-\kappa\gamma_g(\phi_\pi-p)][-\kappa\gamma_y+f'(p)]$$

$$=\kappa(\gamma_y+\gamma_g)[f'(p)(\phi_\pi-p)+f(p)],$$

where $f(p)=\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi-p)]$ and $f'(p)=\sigma(1-p\alpha_{EE})(-\beta\alpha_{PC}+\kappa\gamma_r)-\sigma\alpha_{EE}[1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi-p)]$. One can show:

$$-f'(p)p=-\sigma(1-p\alpha_{EE})(-\beta p\alpha_{PC}+\kappa\gamma_r p)+\sigma\alpha_{EE}p[1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi-p)].$$

In normal times:

$$f'(p)(\phi_\pi-p)+f(p)$$

$$=f'(p)\phi_\pi+\sigma(1-\kappa\gamma_r\phi_\pi)-\sigma\alpha_{EE}p^2(\beta\alpha_{PC}-\kappa\gamma_r).$$

When $\bar{m}=1$, one can show that these terms are below zero with ease. When $\bar{m}=0$, one can have the expression below:

$$\sigma\kappa\gamma_r\phi_\pi+\sigma(1-\kappa\gamma_r\phi_\pi)>0.$$

In this case, it seems that there is a threshold value $\underline{m}$ across which the multiplier effects switch from decreasing in p to increasing in p or vice versa. One can have this value $\underline{m}$ by making the below equation zero:

$$f'(p)\phi_\pi+\sigma(1-\kappa\gamma_r\phi_\pi)-\sigma\alpha_{EE}p^2(\beta\alpha_{PC}-\kappa\gamma_r)=0$$

$$\Longleftrightarrow\{\sigma(1-p\alpha_{EE})(-\beta\alpha_{PC}+\kappa\gamma_r)-\sigma\alpha_{EE}[1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi-p)]\}\phi_\pi+\sigma(1-\kappa\gamma_r\phi_\pi)-\sigma\alpha_{EE}p^2(\beta\alpha_{PC}-\kappa\gamma_r)=0.$$

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[29]For simplicity, I only compare the numerator of the derivatives to check the monotonicity.

At the ZLB, one can show the derivatives can be reduced below:

$$f'(p)(-p) + f(p)$$

$$= \sigma - \sigma\alpha_{EE}p^2(\beta\alpha_{PC} - \kappa\gamma_r) > 0.$$

In this case, the output gap multiplier is increasing in p.

O   The government spending effects with bounded rationality at the ZLB

The spending multipliers at the ZLB are shown below. For simplicity, we define $\alpha_{EE} = \bar{m}(1 - s_g)$ and $\alpha_{PC}(\bar{m}) = \bar{m}[\varphi + \frac{1-\beta\varphi}{1-\beta\varphi\bar{m}}(1-\varphi)]$.

$$\mathcal{M}_{S,Z}^{O,BR} = \frac{\partial y_S}{\partial g_S} = \frac{\frac{\sigma(1-p\alpha_{EE})}{-p} - \frac{\kappa\gamma_g}{1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)}}{\frac{\kappa\gamma_y}{1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)} + \frac{\sigma(1-p\alpha_{EE})}{-p}}$$

$$= \frac{\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)] - \kappa\gamma_g(-p)}{\kappa\gamma_y(-p) + \sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)]}$$

$$\mathcal{M}_{S,Z}^{I,BR} = \frac{\partial\pi_S}{\partial g_S} = \frac{1 + \frac{\gamma_g}{\gamma_y}}{\frac{1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)}{\kappa\gamma_y} + \frac{1}{\sigma(1-p\alpha_{EE})}(-p)}$$

$$= \frac{\left[1 + \frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p\alpha_{EE})}{\kappa\gamma_y(-p) + \sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)]}.$$

One can differentiate output gap multiplier with regard to $\bar{m}$ and after some arrangements we have:

$$\frac{\kappa(\gamma_y + \gamma_g)(-p)f_Z'(\bar{m})}{\mathcal{D}_{BZ}^2} < 0,$$

where $\mathcal{D}_{BZ} = \kappa\gamma_y(-p) + \sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)]$ and $f_Z'(\bar{m})$ is the derivative of $\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)]$ with regard to $\bar{m}$ which is positive.

One can differentiate inflation multiplier with regard to $\bar{m}$ and after some arrangements we have:

$$\frac{-p\alpha_{EE}'\mathcal{D}_{BZ} - f_Z'(\bar{m})(1-p\alpha_{EE})}{\mathcal{D}_{BN}^2} < 0,$$

where $\mathscr{D}_{BZ} = \kappa\gamma_y(-p) + \sigma(1 - p\alpha_{EE})[1 - \beta p\alpha_{PC} - \kappa\gamma_r(-p)]$ and $f'_Z(\bar{m})$ is the derivative of $\sigma(1 - p\alpha_{EE})[1 - \beta p\alpha_{PC} - \kappa\gamma_r(-p)]$ with regard to $\bar{m}$ which is positive.

Since the strength of the real cost channel γ_r is independent of the new ingredient that is bounded rationality. See appendix D, the output gap and inflation multipliers are decreasing in the strength of the real cost channel γ_r. One can use this to prove the main text.

P Figures with Bounded Rationality w.r.t. p

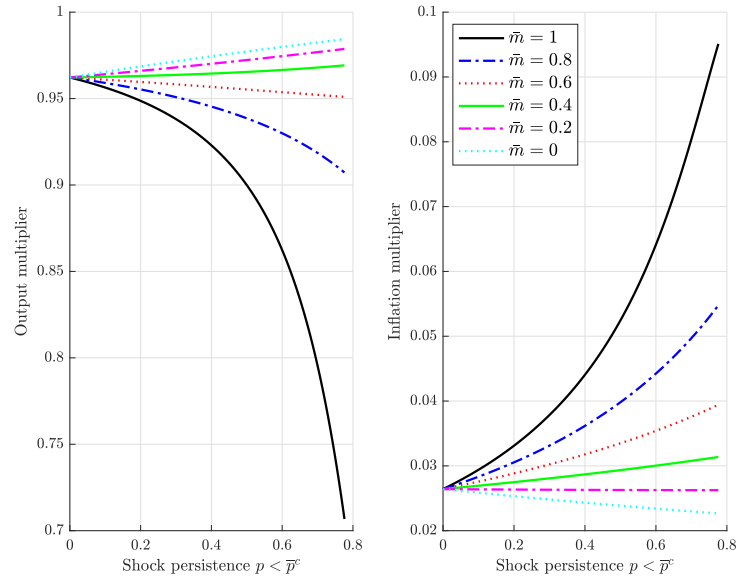Figure 14: Spending multipliers with bounded rationality in normal times

Figure 15: Spending multipliers with bounded rationality at ZLB