# Government Spending Multipliers with the *Real* Cost Channel*

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**Abstract**

In the benchmark New Keynesian model, I introduce the *real* cost channel to study government spending multipliers with a simple Markov chain closed-form solution. This new model fundamentally departs from previous implications with the nominal cost channel by altering the New Keynesian Phillips Curve slope in liquidity traps. I analytically show that the deflationary pressure is exacerbated in the presence of the real cost channel at the Zero Lower Bound. Then the output gap multiplier will be overestimated by ignoring this channel. Finally, I confirm the robustness of the real cost channel effect on multipliers using two model extensions.

**Keywords**: Government Spending Multipliers, Real Cost Channel, Zero Lower Bound, Markov Chain, Bounded Rationality

*JEL Classifications*: E52, E58, E62, E63, E70

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# 1 Introduction

In late 2008, the Federal Reserve had to lower the interest rate to zero to fight against the Global Financial Crisis (GFC). The conventional monetary policy cannot work with the zero lower bound (ZLB) binding. Therefore the government sought to adopt an alternative effective fiscal policy to stimulate the economy in recessions. In this case, the GFC and recent recessions such as COVID-19 have sparked massive fiscal policy discussions, which usually feature tons of government spending.[1] For example, during the COVID-19 pandemic time in 2020, the United States (US) government spent a total of \$6.55 trillion on a series of programs to ensure the well-being of people.

Understanding the effects of government spending at the ZLB can be a key fiscal analysis. The standard New Keynesian (NK) models show that government spending multipliers can be substantially higher (e.g., above 2) at the ZLB as in Eggertsson and Woodford (2004), Christiano et al. (2011) and Zubairy (2014). This view has been challenged in many recent theoretic studies.[2] In addition, a series of new empirical papers have indicated that multipliers are lower at the ZLB. For instance, Ramey and Zubairy (2018) provide empirical evidence to show not-so-great spending multipliers in liquidity traps. Recently, Auerbach et al. (2021) show that the fiscal multipliers can be lower with the post-COVID supply-side constraints during recessions.

In this paper, I aim to augment the real cost channel in the benchmark NK model to provide *new theoretical insights* to explain empirically lower government spending multipliers when the economy is at the ZLB.[3] To be more specific, if firms need to pay bills before production, the interest rate can theoretically influence the borrowing cost and further the firms' marginal cost in the aggregate *supply-side* economy (summarized in the NK Phillips Curve). The critical difference between the model in the cost channel setting and the conventional model is that the interest rate should be included in the firms' marginal cost, and it can, in turn, influence the inflation rate.

The existence of the cost channel is provided in some empirical investigations. For instance, Ravenna and Walsh (2006) estimate and obtain the cost channel parameter

[1] See more discussions on the fiscal tool to resist recessions as in Eggertsson (2011), Kollmann et al. (2012), Bouakez et al. (2020) and House et al. (2020).

[2] See e.g. Kiley (2014), Mertens and Ravn (2014) and Roulleau-Pasdeloup (2021a).

[3] The previous literature mainly employs a standard NK model, given that interest rate can impact aggregate *demand-side* economics only, to theoretically explore the effects of government spending and fiscal policy multipliers (see e.g., Eggertsson and Woodford (2004) and Aruoba and Schorfheide (2013)).

for the US. Similarly, Chowdhury et al. (2006) and Tillmann (2009) show the existence of cost channel in the US and the UK. Recently, Abo-Zaid (2022) employs a structural vector autoregression (SVAR) model to confirm that the cost channel exists in almost all representative industrialized countries.

Compared to most previous literature with the nominal cost channel[4], in this paper, I, however, incorporate the expected *real* interest rate into the firms' marginal cost to study government spending multipliers.[5] Furthermore, I try to conduct an analytical study on the policy multipliers in liquidity traps.[6] To this end, a simple two-state Markov chain as in Eggertsson and Woodford (2003) is utilized to mimic policy shocks to obtain closed-form solutions for policy multipliers using pen and paper.

At the ZLB, the results in this paper stand in stark contrast to most previous implications with the nominal cost channel as in Surico (2008) and Smith (2016). I show that the introduced real cost channel can alter the NK Phillips Curve slope with the expected inflation in this real channel at the ZLB. The threshold of the negative natural rate shock (also the demand shock) to trigger the ZLB constraint binding with the real cost channel is larger than the result in the classical NK model. That means that the economy with the real cost channel is more easily entrapped into fundamental liquidity traps.

Spending multipliers can be effective (larger than one) in liquidity traps. Intuitively, with the ZLB binding, the nominal interest rate remains unchanged, and the government spending within a fiscal policy package can relax the deflationary pressure. This can lower the real interest rate and further stimulate private consumption. However, with the real cost channel, the expected real interest rate in firms' marginal cost (borrowing cost) can reduce short-run inflation caused by government spending. In this way, the deflationary pressure is exacerbated. A higher real interest rate can depress people's consumption motivation and further production activity, which, in turn, can reduce the power of government spending on output. In a nutshell, compared to the standard NK model, the output gap multiplier with the real cost channel can be lower at the ZLB.

[4]There are a series of papers with the nominal cost channel which means the nominal interest rate is augmented in the firms' marginal cost, such as, Barth III and Ramey (2001), Ravenna and Walsh (2006), Llosa and Tuesta (2009) and Smith (2016).

[5]Compared to the nominal cost channel, as explained at length in Beaudry et al. (2020), the real cost channel can obtain more support from US data; furthermore, Nie (2021) formally proves that the real cost channel is more theoretically appealing since it can secure the equilibrium uniqueness/existence with temporary policy shocks.

[6]In this paper, I use the policy multipliers to denote the inflation and output gap multipliers with the government spending policy.

Further, this multiplier decreases in the strength of this channel. In contrast, the policy multipliers with the nominal cost channel as in Tillmann (2009) can be invariant with the standard NK model since this channel can not modify the NK Phillips Curve slope at the ZLB.[7]

Another result is that the output gap multiplier decreases in the strength of the real cost channel when the economy is outside of liquidity traps. Intuitively, when the nominal interest rate is free to adjust, government spending can increase firms' marginal cost. The real cost channel in the Phillips Curve can further raise inflation since, in normal times, it works as a cost-push shock endogenously along the lines of Ravenna and Walsh (2006). With the Taylor (1993)-type rule followed by Central Bank, there will be a rise in the nominal interest rate by more than one-for-one with inflation pressure. Larger real interest rate can stimulate people to save but to consume *less*. As a result, government spending can crowd out more private consumption. Consistent with the findings in Abo-Zaid (2022), notice that the output multiplier with the nominal cost channel is less than the counterpart without it in normal times. However, compared to the nominal cost channel, the inflation pressure in the real channel is less prominent since the expected inflation within the expected real interest rate can reduce the crowding-out effects. I, therefore, show that the nominal cost channel can further restrain the output gap multiplier in normal times relative to the real cost channel.

This analytical model extends to a more general setting, but the real cost channel still works robustly. As in some recent contributions in Sarin et al. (2021) and Roulleau-Pasdeloup (2021a), the long-run policy effects are usually overlooked or even usually computed numerically in the previous literature. In this case, I further discuss the long-run government spending policy analytically by assuming that the government spending policy is *longer* than economic recessions. Thus, a three-state Markov structure is employed. I follow Bilbiie (2019b) to decompose the long-run period into short and medium-run periods. Therefore, one can capture the effects of prolonged government spending on the short-run policy multipliers. The analytical results show that if the short-run economy is in normal times, it is observed that prolonged government spending can further deflate the output multiplier but increase the inflation multiplier more than the

[7] In calculation of fiscal multipliers, we need to gain the partial derivative of government spending to inflation or output gap. If the introduced cost channel can not change the NK Phillips Curve slope, *ceteris paribus*, the partial derivative should be the same as the standard model.

short-run spending policy. Interestingly, if the short-run economy is in liquidity traps, longer government spending after the ZLB has subsided is favored. Intuitively, prolonged government spending can relax the deflationary pressure more through rational expectations as in Bouakez et al. (2017) and thus, it can further inflate the short-run output gap multiplier. This theoretic result is supported by empirical evidence as in Leduc and Wilson (2013) and Bernardini et al. (2020). Additionally, I show that the real cost channel can work robustly in this three-state Markov chain.

At the end of the day, as I have shown the effects of long-run government spending on the short-term policy multipliers through rational expectations, one may wonder what if the agents can not fully understand the world?[8] Thus, bounded rationality is integrated into our baseline model to see such *behavioral* macroeconomic effects as another extension. I finally develop a similar version of Gabaix (2020)'s model, and the results show that in normal times, the cognitive discounting effects can mitigate the inflation pressure and further increase the output gap multiplier. Interestingly, if we suppose the agents have strong enough bounded rationality, the output gap multiplier can be large in normal times without the ZLB binding, which is in line with some empirical evidence in Auerbach and Gorodnichenko (2012) and Acconcia et al. (2014). On the other hand, at the ZLB, bounded rationality can attenuate the output gap multipliers. It can echo literature that the agents discounting future wealth in making decisions today can reduce the policy power in recessions as in McKay et al. (2016), Angeletos and Lian (2018), and Campbell et al. (2019). Besides, the real cost channel can still work robustly in this behavioral model, and the output gap multiplier can be further overestimated by ignoring this channel.

**Related Literature.**—The earliest seminal work focusing on the theoretical estimation of government spending effects in liquidity traps can be traced back to Eggertsson (2001). In that paper, the optimal fiscal policy in the NK economy is characterized, and the real effects of government spending are emphasized. After that, a growing number of literature focuses on the estimation of fiscal effects in theoretical and empirical ways. For example, Blanchard and Perotti (2002) spark the earliest insights on empirically

[8] There is a similar consideration in the monetary policy. As in Nakata et al. (2019) and Budianto et al. (2020), the favorable effects of Forward Guidance—a promised long-run interest rate binding—on short-run inflation can be much attenuated if the economic agents can not fully understand the world as represented by the NK model with rational expectations.

estimating the macroeconomics effects of government spending.[9] Christiano et al. (2011) prove that the multiplier is low in normal times in an economy following a Taylor (1993)-type rule but relatively high in liquidity traps. Leeper et al. (2017) theoretically study the fiscal multipliers in a series of models. Two distinct monetary-fiscal policy regimes show that the short-run multipliers are robustly similar across different regimes. More examples among Kraay (2012), Miyamoto et al. (2018), Ramey and Zubairy (2018), etc.

In this paper, I add to the government spending multiplier literature by analytically speaking to the role of the real cost channel on the multipliers. The previous literature as in Barth III and Ramey (2001), Ravenna and Walsh (2006), Llosa and Tuesta (2009) and Smith (2016) always introduces the nominal interest rate into the firms' marginal cost which is called as the nominal cost channel. For example, Ravenna and Walsh (2006) first confirm that the cost-push shock can emerge endogenously in the NK model in a cost channel setting. Further, they discuss the ways for the optimal monetary policy which the new channel can alter. Surico (2008) shows that limiting the economic cyclical with a cost channel can lead to a strong fluctuation of inflation and output. However, this paper heavily builds on Beaudry et al. (2020) and Nie (2021) to augment the expected real interest rate into firms' marginal cost to revise the benchmark NK model.

This paper is in a close spirit to some recent literature—see e.g. Bilbiie (2019a), Bilbiie (2019b), Bilbiie (2020) and Nie and Roulleau-Pasdeloup (2021) which uses three-state Markov Chain to analytically examine the long-run policy on the short-run economy. For example, Bilbiie (2019b) employs a three-state Markov manner to have an in-depth study on the optimal forward guidance policy in the short run and long run. In this paper, this three-state structure can allow us to analytically check the general properties of long-run government spending to echo some empirical evidence in Durevall and Henrekson (2011), Ilzetzki et al. (2013), Leduc and Wilson (2013), Bouakez et al. (2017), and Leff Yaffe (2019).

In addition, recent contributions such as Farhi and Werning (2019), and Gabaix (2020) show that bounded rationality can mitigate the powerful effects of monetary policy. This method can rationalize the so-called "Forward Guidance puzzle" (see i.a. Angeletos and Lian (2018) and Coibion et al. (2020)) compared to the benchmark too forward-looking NK model as in Ganelli and Tervala (2016). This paper, however, is linked with

[9] See Ramey (2011) for a survey on the estimation of government spending multipliers in the literature.

this strand of literature to see the interaction of the cost channel and bounded rationality on the *fiscal* policy.

Finally, this paper is also closely related to Abo-Zaid (2022) who examines government spending multipliers at the ZLB with the nominal cost channel. Since Abo-Zaid (2022) includes the labor cost in the nominal cost channel, making the Phillips Curve steep in recessions, this nominal channel can cause spending multipliers larger in liquidity traps. However, in this paper, I use the real cost channel to explain empirically lower government spending multipliers when the economy is at the ZLB. Simple Markov chain closed-form solutions are computed to compare the government spending multipliers with the real/nominal cost channel. In addition, we show that the Phillips Curve is locally flat during recessions which is empirically relevant as in Beaudry et al. (2020). I further clear up the effects of the strength of the real cost channel on the policy multipliers analytically. I also furnish the primary model to confirm the robust role of the real cost channel on multipliers.

**Organization.**—I will specify the prototypical forward-looking New Keynesian(NK) model with the real cost channel in the next Section 2 and provide a transparent analytical analysis using a two-state Markov chain on the government spending multipliers with/without the real cost channel. In Section 3, I furnish the baseline model to explore the general properties of long-run government spending effects. Another extension with bounded rationality is conducted in Section 4. Finally, this paper concludes in Section 5.

# 2 The Baseline Model with the Real Cost Channel

Recent empirical evidence in Abo-Zaid (2022) shows that the existence of the cost channel can influence the government spending multipliers. As in Ravenna and Walsh (2006) and Surico (2008), the main insight of the cost channel is that the interest rate can influence the borrowing costs and then the marginal cost function. In this paper, the NK model in the real cost channel setting, which means the expected real interest rate can impact the firms' marginal costs, is employed to explore government spending multipliers analytically. Beaudry et al. (2020) show that the real cost channel can obtain more support from US data compared to the nominal cost channel. As explained at length in Nie (2021), the real cost channel can be theoretically appealing since it can secure the equilibrium uniqueness/existence with temporary policy shocks.

## 2.1 Private Sector Behavior

I follow Beaudry et al. (2020) and Nie (2021) to use a prototypical forward-looking NK model with price rigidity.[10] The behavior of aggregate demand (AD) side economy can be summarized in the following log-linear Euler condition:

$$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\sigma_c}[R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1} - r_t^n],$$

where $c_t$ is the private consumption, $\sigma_c$ is the risk aversion coefficient, $R_t$ is the nominal interest rate *in level*, $\pi_t$ is inflation, $\mathbb{E}_t$ is the rational expectation operator, and $r_t^n$ is the demand shock (also the natural rate shock).

The resource constraint in this NK economy is:

$$y_t = (1 - s_g)c_t + g_t,$$

where $s_g$ is the fraction of government spending in total production, $g_t$ is the government spending.[11] In this case, one can obtain the path of $y_t$ by substituting the resource constraint into Euler equation to obtain the equation (1) below:

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma}[R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1} - r_t^n] + g_t - \mathbb{E}_t g_{t+1}, \tag{1}$$

where $\sigma = \frac{\sigma_c}{1-s_g}$.

The aggregate supply (AS) side of the economy can be summarized in the following NK Phillips Curve:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \left[\gamma_y y_t + \gamma_g g_t + \gamma_r (R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1})\right], \tag{2}$$

where $\beta$ is the subjective discount factor, and $\kappa$ is the elasticity of inflation with regard to marginal cost. $\gamma_y$, $\gamma_g$, $\gamma_r$ are the elasticity of marginal cost elasticity with regard to the output gap, government spending, interest rate, respectively.[12] It is of note that $\gamma_r$ in

[10] For brevity, I show the aggregate model economy in a log-linear form in the main text. See Galí (2015) for a textbook treatment.

[11] Following Christiano et al. (2011), I define $g_t = (G_t - G)/Y$.

[12] See appendix A for exact expressions for these parameters.

the equation (2) can be seen as the impact of the expected real interest rate on inflation, and also the strength of the cost channel.[13] This model can collapse to the standard NK model if $\gamma_r = 0$.[14]

Following Nie and Roulleau-Pasdeloup (2021), it is assumed that Central Bank sets the nominal interest rate following the (truncated) Taylor (1993)-type rule with the ZLB:

$$R_t = \max\{0, \log(\beta) + \phi_\pi \pi_t\}.$$

## 2.2 Quick Tour: Normal Times and Zero Lower Bound[15]

In this section, I employ a two-state static Markov chain as in Eggertsson et al. (2003) to deal with the policy shocks vector $[r_t^n, g_t]$. It is assumed that the the specific policy shock (for example, demand shock $r_n^n$ in this section) stays at the current short-run state with a persistence $p$ and it then reverts to the steady-state *i.e.* $r_t^n = 0$ with a probability $1-p$.[16] Since the NK model with the real cost channel in this paper is perfectly forwarding looking, one can show the expected output gap and inflation as follows:

$$\mathbb{E}_t y_{t+1} = p y_t, \qquad \mathbb{E}_t \pi_{t+1} = p \pi_t.$$

**Assumption 1.** *I assume that the NK Phillips Curve with the real cost channel is always upward sloping in a $(\pi_t, y_t)$ graph such that*

$$p < \frac{1 - \kappa\gamma_r\phi_\pi}{\beta - \kappa\gamma_r} = \overline{p}^c.$$

As in much literature in Laubach and Williams (2003), Cochrane (2017), Han et al. (2020) and Nie (2021), there is an implicit condition that the NK Phillips Curve is upward sloping and there is a threshold of demand shock to trigger the ZLB constraint binding. Compared with the conventional NK model with the nominal cost channel as in Christiano et al. (2005) and Ravenna and Walsh (2006), this paper utilizes a more empirically relevant setting proposed in Beaudry et al. (2020) and further extended in Nie

[13]The details for the derivatives of the aggregate supply side of the economy can be seen in appendix A.

[14]See Gertler et al. (1999) and Woodford (2003).

[15]In this paper, normal times refers to a state that the nominal interest rate is flexible to adjust and is not binding with the ZLB. The ZLB state means that the nominal interest rate is fixed at zero.

[16]The duration of the short run state can be calculated as $T = \frac{1}{1-p}$. For instance, if $p = 0.5$, $T = \frac{1}{1-0.5} = 2$ quarters.

(2021).[17] In this section, we discuss two cases which are the economy in normal times without the ZLB binding and in fundamental liquidity traps.

**Proposition 1.** *The boundary condition of natural rate shock $\underline{r_S^n}$ to trigger the fundamental liquidity (zero lower bound) with the real cost channel is*

$$\underline{r_S^n} = \left[ \frac{(1-\beta p + \kappa\gamma_r p - \kappa\gamma_r\phi_\pi)(1-p)}{\kappa\gamma_y\frac{1}{\sigma}} + (\phi_\pi - p) \right] \frac{\log(\beta)}{\phi_\pi} < 0.$$

*This boundary condition is larger than the counterpart in the standard NK model.*

*Proof.* See Appendix B. □

From the Taylor (1993)-type rule, one can see that if the items $-\log(\beta) + \phi_\pi \pi_S$ are less than or equal to zero, the NK economy can be binding with the ZLB state. If not, the economy is in normal times and the nominal interest rate can be free to adjust with Central Bank's monetary policy regulation. If the (negative) natural rate shock (also the demand shock) is too large, the economy can be with the ZLB. Thereby there is a boundary condition for the natural rate shock $r_S^n$ in the short run to trigger the economy into a state with the ZLB binding.

As in Proposition 1, the boundary condition to trigger the ZLB binding with the real cost channel is higher than that in the conventional NK model. In other words, the economy is more easily into liquidity traps with the real cost channel than in the traditional model due to the additional monetary transmission in the aggregate demand-side economy (*i.e.* the NK Phillips Curve). The real cost channel can enlarge the effects of the interest rate on inflation and further the output gap through rational expectations.

**Calibration.**—In this paper, following Budianto et al. (2020), Roulleau-Pasdeloup (2021a) and Nie (2021), the main baseline parameterization method is reported in Table 1.[18]

The simulation results with the demand shock are shown in Figure 1. If there is a temporary short-term natural rate shock (-1%), the economy can be in a contractionary

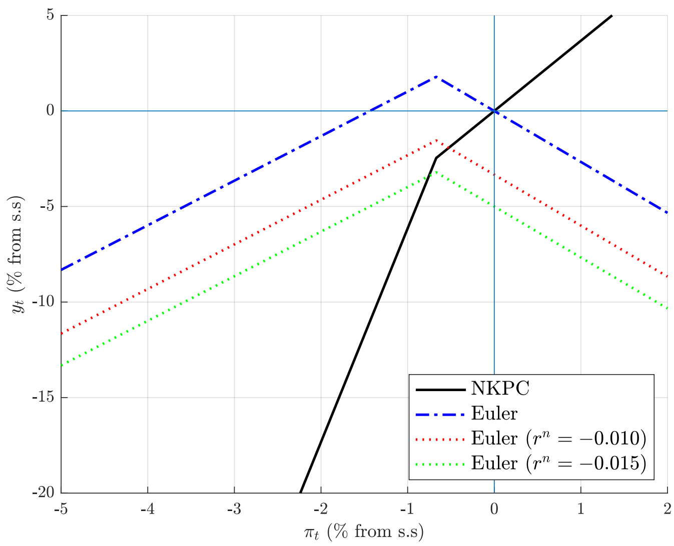
[17] Nie (2021) has proved that the real cost channel itself can rule out the sunspot liquidity traps—see examples in Bilbiie (2008), Boragăn Aruoba et al. (2018) and Lustenhouwer (2020). Agents expect the deflation for no reason will result in sunspot liquidity traps (a.k.a. confidence-driven liquidity traps) without any external shocks. As described at length in Nie (2021), in the presence of the real cost channel, the slope of the Phillips Curve can be larger than that of the Euler equation at the ZLB, and thus, the sunspot equilibrium can be ruled out.

[18] As in Beaudry et al. (2020) and Nie (2021), the empirical evidence shows that the elasticity of real marginal cost w.r.t. output gap $\gamma_y$ is small while the key parameter $\gamma_r$ to control the strength of the real cost channel is much larger than $\gamma_y$.

Table 1: Parameterization

| | |
|---|---|
| Subjective discount factor | $\beta = 0.99$ |
| Inverse of Frisch elasticity | $\eta = 1$ |
| Risk aversion coefficient | $\sigma_c = 1$ |
| Steady-state ratio of government spendin to output | $s_g = 0.2 \times 0.23$ |
| Elasticity of inflation w.r.t. real marginal cost | $\kappa = 0.2$ |
| Elasticity of real marginal cost w.r.t. output gap | $\gamma_y = 0.2$ |
| Elasticity of real marginal cost w.r.t. interest rate | $\gamma_r = 1$ |
| Elasticity of real marginal cost w.r.t. government spending | $\gamma_g = -0.1$ |
| Inflation parameter in Taylor rule | $\phi_\pi = \phi_\pi^q = 1.5$ |
| Persistence specification | $p = 0.7$ |

Figure 1: AS/AD curves with/without demand shock in normal times or at ZLB




state with a negative short-run output gap $y_S$ and inflation $\pi_S$. In this case, Central Bank can reduce nominal interest rates to deal with the deflationary pressure. A lower real interest rate can stimulate private consumption and further stimulate the economy to increase inflation and output gap. However, if, for example, the demand shock is (negatively) strong (-1.5%), which excesses the boundary condition $\underline{r_S^n}$, the economy can be in fundamental liquidity traps. In this case, the conventional monetary policy can not work since the interest rate is bounded at zero, and the deflationary pressure can further stimulate people to save but to consume less. In the following sections, I will mainly focus on the government spending shock and discuss the issues of both the output and inflation multipliers.

### 2.3 Government Spending: Closed-Form Solution

To have transparent analytical results, I abstract from the demand shock and only focus on the effects of government spending shock with the real cost channel. First, I will compare the output and inflation multipliers in the real cost channel with the canonical NK model by using a simple Markov chain closed-form solution. Second, I will deliver the general properties of the strength of the cost channel on the multipliers.

#### 2.3.1 Normal Times

The Markov method is based on Eggertsson and Woodford (2003) and Eggertsson et al. (2003). I assume that the positive government spending shock $g_S > 0$ starts in the short run, stays with the persistence probability $p$, and returns to the steady-state $g_L = 0$ in the long run with a probability $1-p$. If the short-run economy is in normal times, I can use the Taylor (1993) rule to rewrite the Euler equation and Phillips Curve:

$$y_S = -\frac{1}{\sigma(1-p)}(\phi_\pi - p)\pi_S + g_S$$

$$\pi_S = \kappa\frac{\gamma_y}{1-\beta p-\kappa\gamma_r(\phi_\pi - p)}y_S + \kappa\frac{\gamma_g}{1-\beta p-\kappa\gamma_r(\phi_\pi - p)}g_S.$$

**Real versus nominal cost channel.**—In normal times, we can use a simple Taylor rule $R_t = \phi_\pi \pi_t$ to show the Phillips Curve with the nominal cost channel as in Surico (2008):

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa\left[\gamma_y y_t + \gamma_g g_t + \gamma_r(\phi_\pi\pi_t + \log(\beta))\right]. \tag{3}$$

On the other hand, one can show the Phillips Curve with the real cost channel:

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa\left[\gamma_y y_t + \gamma_g g_t + \gamma_r(\phi_\pi\pi_t + \log(\beta) - \mathbb{E}_t\pi_{t+1})\right]. \tag{4}$$

The cost channel can reduce the output multiplier, and the intuition is simple. Since the government spending shock can increase firms' marginal costs. Additionally, the (real/nominal) cost channel can further increase inflation in the aggregate supply-side economy (the NK Phillips Curve). This mechanism is close in spirit to Ravenna and Walsh (2006) such that a cost-push shock can emerge endogenously in the New Keynesian model with the cost channel setting. In that way, there will be a larger increase by more than one-for-one with inflation in the nominal interest rate due to the monetary regulation ($\phi_\pi$ in the Taylor (1993) rule) by Central Bank. Higher interest rate can result in less private consumption and thus government spending can crowd out more private consumption. Above all, the output multiplier with the cost channel is less than that without it.

In comparison, as in Appendix C, compared to the nominal cost channel, the inflation multiplier can be smaller with the real cost channel. If we compare the NK Phillips Curve in equations (3) and (4), less influence caused by the real cost channel *i.e.* $(\phi_\pi - p)\pi$ is observed, which may echo the lower inflation fact in the Euro area in normal times (Koester et al. (2021)). Lower inflation with the real cost channel can make the output gap multiplier larger. In this case, the nominal cost channel can further restrain the output multiplier relative to the real cost channel.

**Fiscal multipliers.**—One can see that a positive government spending shock can move the Euler equation upward and turn down the Phillips Curve in a $(\pi_t, y_t)$ graph. The solutions for the output gap and inflation multipliers can be generated below:

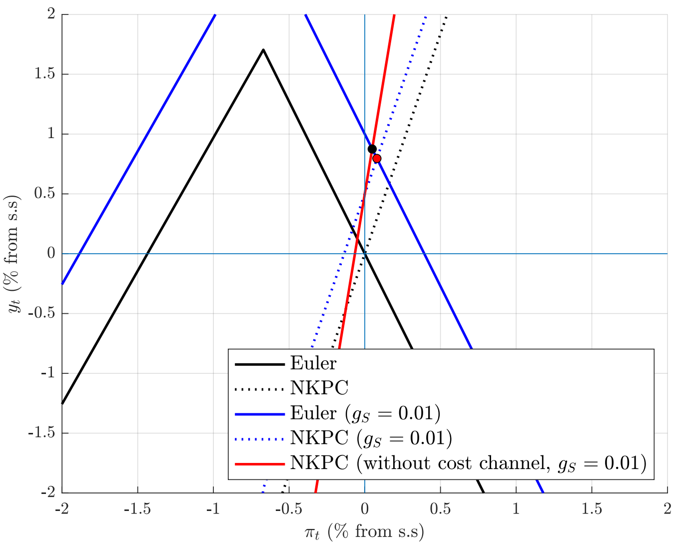
$$\frac{\partial y_S}{\partial g_S} = \frac{\sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)] - \kappa\gamma_g(\phi_\pi - p)}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)]}$$

$$\frac{\partial \pi_S}{\partial g_S} = \frac{\left[1 + \frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p)}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)]}.$$

The expected real interest rate can be in the denominator and numerator for the output gap multiplier since the new cost channel here can influence inflation rate and further impact the output gap through expectations. While for the inflation multiplier, the expected real interest rate is included in the denominator since the cost channel can impact this multiplier directly.

Our numerical results show the whole story as in Figure 2. The temporary spending shock can move up the Euler equation and move down the Phillips Curve, which is the same as our theoretical analysis. In this case, the output gap multiplier (0.796) and inflation multiplier (0.080) with the real cost channel (the red dot) are both positive but the output gap multiplier is not so efficient (less than one) in normal times as in Schmidt (2013). The output gap multiplier is 0.875, and the inflation multiplier is 0.049 without the real cost channel in the black dot. Furthermore, as it can be seen in our simulation and theoretical results, the multiplier is less than one since government spending can crowd out private consumption, which can echo some classical empirical evidence as in Amano and Wirjanto (1997) and Barro and Redlick (2011). In addition, one can see that the slope of the Phillips Curve in a $(\pi_t, y_t)$ graph with the real cost channel is higher than its counterpart, which results in the smaller output multiplier with the real cost channel. In contrast, the inflation multiplier is larger than its counterpart without the cost channel.

Figure 2: AS/AD curves with/without government spending in normal times



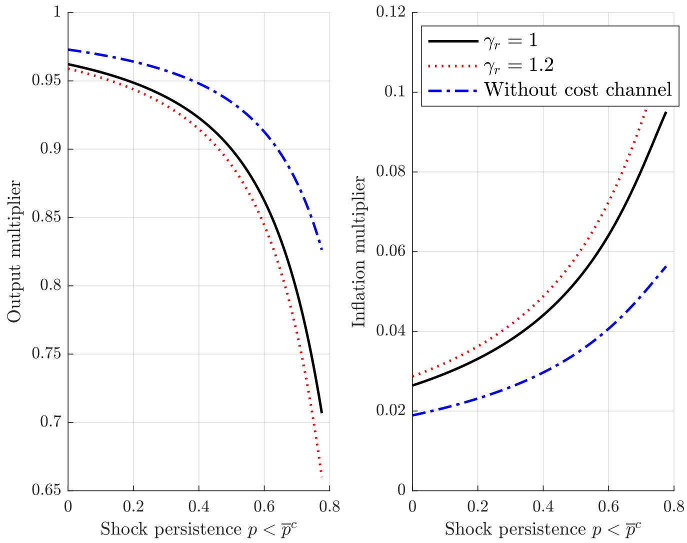

I further try to discuss the effects of the strength of the cost channel $\gamma_r$ on the policy multipliers. On the one hand, we can see that the power of the real cost channel can lower the output gap multiplier, which is intuitive since the stronger the cost channel is in normal times, the more inflation is. Thus government spending crowds out more private spending. On the other hand, similarly, the cost channel can directly impact inflation from the expected real interest rate in the aggregate supply side. Precise numerical results are reported in Figure 3 to echo our theoretical analysis in Proposition 2. It can be seen that the output multiplier is lower than one as in Lewis (2021) and it decreases in the strength of the cost channel $\gamma_r$ from 1 to 1.2. On the contrary, the inflation multiplier is higher than its counterpart, increasing with the strength of the cost channel $\gamma_r$ from 1 to 1.2. Some interesting patterns are observed in this figure: The output multiplier decreases with persistence $p$ (time duration), while the inflation multiplier is higher with the increment of time duration.

**Proposition 2.** *The government spending effects in normal times reveal that*

1. *The output gap and inflation multipliers are both positive.*
2. *The output gap multiplier is less than one.*
3. *The output gap multiplier with the real cost channel are less than its counterpart while the inflation multiplier is larger.*
4. *The output gap multiplier decreases in the strength of the real cost channel $\gamma_r$ while the inflation multiplier increases in $\gamma_r$.*

*Proof.* See Appendix C. □

Figure 3: Policy multipliers with/without real cost channel in normal times



### 2.3.2 Zero Lower Bound

In this part, I focus on the case when the economy binds with the ZLB.[19] The Euler equation and Phillips Curve with the real cost channel can be elaborated as:

$$y_S = -\frac{1}{\sigma(1-p)}[\log(\beta) - p\pi_S] + g_S$$
$$\pi_S = \frac{\kappa\gamma_y y_S + \kappa\gamma_r \log(\beta)}{1-\beta p + \kappa\gamma_r p} + \kappa\frac{\gamma_g}{1-\beta p + \kappa\gamma_r p} g_S.$$

**Real versus nominal cost channel.**—The interesting insight at the ZLB with $R_t = 0$ is that the conventional nominal cost channel as in Ravenna and Walsh (2006) and Surico (2008) can not influence the slope of Phillips Curve below

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa\left[\gamma_y y_t + \gamma_g g_t + \gamma_r(\log(\beta))\right], \tag{5}$$

while the real cost channel in this paper with the additional expected inflation *i.e.* $\mathbb{E}_t\pi_{t+1} = p \times \pi_t$ can do alter the slope of Phillips Curve:

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa\left[\gamma_y y_t + \gamma_g g_t + \gamma_r(\log(\beta) - \mathbb{E}_t\pi_{t+1})\right]. \tag{6}$$

It is of note that, at the ZLB, the real cost channel can title the Phillips Curve with the inflation expectation feedback. As a result, the Phillips Curve is flatter in a $(y_t, \pi_t)$ graph and this may explain a declining slope of the empirical Phillips Curve. In this way, the real cost channel in equation (6) can alter the government spending multipliers in liquidity traps. While the multipliers can be invariant with the standard NK model in the nominal cost channel in equation (5) since the term $\log(\beta)$ in the nominal channel can not be included in the partial derivative of government spending to the output gap/inflation in calculation of fiscal multipliers.

**Fiscal multipliers.**—It can be observed that a positive government shock can turn up the Euler equation and move down the Phillips Curve. One can further use the Euler equation and Phillips Curve to obtain the policy multipliers at the ZLB in the following

[19] In this paper, the zero (effective) lower bound is a state when $R_t = 0$ with the simple assumption that there is no cash storing cost in Galí (2015).

equations:

$$\frac{\partial y_S}{\partial g_S} = \frac{\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]-\kappa\gamma_g(-p)}{\kappa\gamma_y(-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]}$$
$$\frac{\partial \pi_S}{\partial g_S} = \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p)}{\kappa\gamma_y(-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]}.$$

I can obtain similar policy multipliers as the case in normal times if we assume $\phi_\pi = 0$ in the Taylor rule. Likewise, the expected real interest rate can be in the denominator and numerator of the output gap multiplier. The new cost channel here can influence inflation rate and further impact the output gap through expectations. The real cost channel can impact the inflation multiplier directly in the aggregate supply-side economy (the NK Phillips Curve).

In liquidity traps, since the denominator of the multiplier equation is lower than the numerator and thus the output gap at the ZLB is larger than one. At the ZLB, the nominal interest rate remains unchanged, and the increase in government spending means that firms will need household members to produce more. This higher real wage makes the firm's marginal cost increase and further generates inflation. Therefore the government spending shock can relax the deflationary pressure, which, in turn, gives incentives to the consumers to save less and consume more. This can increase consumption means more demand for the firms, which demand more hours from the household members. This can be seen as "crowding in" effects as in Bouakez et al. (2017). In the aggregate supply side of the economy, with the real cost channel—which makes the Phillips Curve locally flat in a $(y_t, \pi_t)$ graph at the ZLB as in Beaudry et al. (2020),—the negative inflation expectation in the real borrowing cost of firm's marginal cost can reduce the short-run inflation caused by government spending. In this way, the deflationary pressure is exacerbated. Higher real interest rates can depress people's consumption motivation and further production activity with the real cost channel, which results in a decline in the effects of government spending on output.

**Discussions.**—This result is contrary to Abo-Zaid (2022) who shows that the higher borrowing cost of the nominal cost channel can increase inflation to make the output gap multiplier larger at the ZLB. As in Abo-Zaid (2022), the Phillips Curve is steeper in a $(y_t, \pi_t)$ graph than the standard NK model in recessions with the inclusion of labor cost in the nominal cost channel.[20] In this paper, I stress that inflation in the short run with the real cost channel can be lower since the inflation expectation (in the real borrowing cost) can reduce inflation in the short run caused by government spending. Thus I can use the real cost channel to explain empirically lower government spending multipliers when the economy is at the ZLB.

**Proposition 3.** *The government spending effects at the ZLB reveal that*

1. *The output gap and inflation multipliers are both positive.*
2. *The output gap multiplier is larger than one.*
3. *The real cost channel can discount the output gap and inflation multipliers.*
4. *The output gap and inflation multipliers decrease in the strength of the real cost channel* $\gamma_r$.

*Proof.* See Appendix D. □

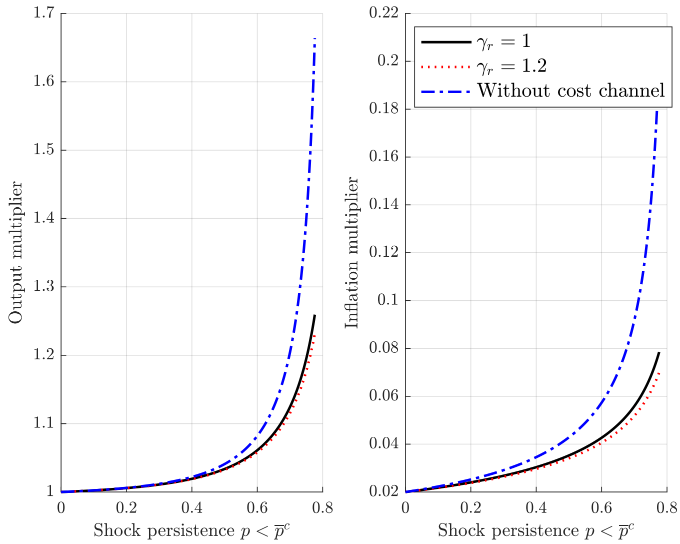
Figure 4 shows policy multipliers with/without the real cost channel in liquidity traps. The simulation results can perfectly match our theoretical analysis in Proposition 3; on the one hand, one can see that the output multiplier is productive here as in Christiano et al. (2011) and Schmidt (2017). The cost channel can attenuate the output gap and inflation multipliers simultaneously. For example, the policy multipliers are lower when $\gamma_r = 1.2$ compared with $\gamma_r = 1$ since the output gap and inflation multipliers decrease in the strength of the real cost channel. In addition, the two policy multipliers increase in the persistence $p$.

# 3 Government Spending: Long-Run Policy

As in some recent contributions in Sarin et al. (2021) and Roulleau-Pasdeloup (2021a), the long-run policy effects are usually overlooked or even usually computed numerically in the previous literature. In this section, I follow Roulleau-Pasdeloup (2021b) to make use of a three-state Markov chain to reassess the long-run government spending effects analytically.

[20] The Phillips Curve is locally flat during recessions as in Beaudry et al. (2020). Besides, Hazell et al. (2020) empirically document that the NK Phillips Curve is flat during the Great Recession.

Figure 4: Policy multipliers with/without real cost channel in liquidity traps




## 3.1 Policy and Shocks

We assume that government spending is longer-lived than the demand shock (economic crisis). To be more specific, the government spending in the short run $g_S$ can step into the medium run government spending $g_M$ with a persistence $q$ and then collapse to the steady-state $g_L = 0$ with a probability $1-q$. For the natural rate shock $r^n_S$, it runs in the short run with a probability $p$ and then returns to the long run $r^n_L = 0$ with a probability $1-p$. The graphical representation of our policy can be seen in Figure 5.

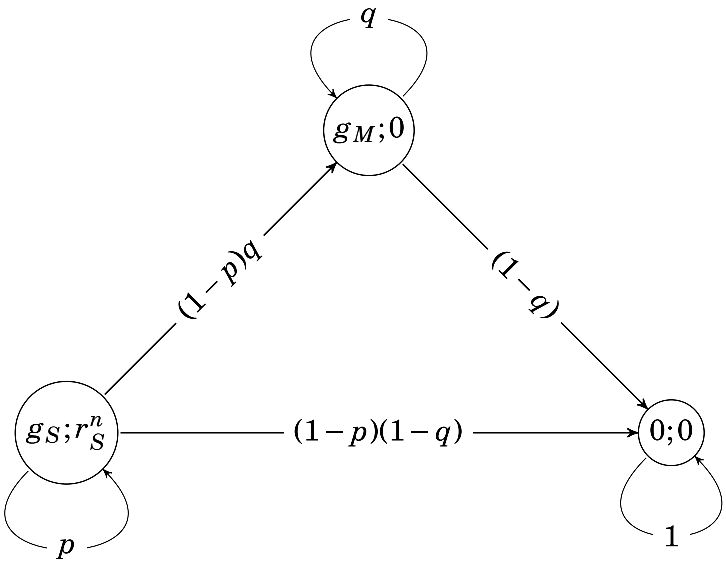
With this in mind, for the monetary policy now, I use an adapted Taylor rule:[21]

$$R_t = \begin{cases} \max\left[0; -\log(\beta) + \phi_\pi \pi_S\right] & \text{In the short run} \\ -\log(\beta) + \phi^q_\pi \pi_M & \text{In the medium run} \\ -\log(\beta) & \text{In the long run} \end{cases} \tag{7}$$

One can use the above equation (7) to trace the path of the government spending. In the medium run, the output gap and inflation can be expressed as the product of

[21]In this section, to be in line with Section 2, I consider two economic states in the short run, which are normal times and the ZLB. In the medium run, I simply assume the demand shock reverts to the steady state and the economy is in normal times.

Figure 5: Long run government spending: a graphical representation




medium-run multipliers and the medium-run government spending as follows:

$$y_M = y_{M,g} \cdot g_M \;\; \& \;\; \pi_M = \pi_{M,g} \cdot g_M, \tag{8}$$

where $g_M$ is the government spending in the medium run. See appendix E for the medium spending multipliers.[22] In this case, since the model is forward-looking and the expected output gap and inflation are contingent on the short-run shock and the medium-run shock. I can generate the expected output gap below, and one can show the expected inflation using the same manner.[23]

$$\begin{aligned}
\mathbb{E}_S y_{t+1} &= p y_S + (1-p) q y_M \\
&= p y_S + (1-p) q y_{M,g} g_M \\
&= p y_S + (1-p) q y_{M,g} \zeta g_S.
\end{aligned}$$

## 3.2 Normal Times

If the *short-run economy is in normal times*, the new Euler equation can be regenerated with the consideration of the long-run government spending using a three-state Markov

[22] I follow a simple rule in Nie and Roulleau-Pasdeloup (2021) to deal with the medium run shock: It is generally assumed that the medium-run spending is contingent on the short-run spending but it is lower than the short-run spending such that $g_M = \zeta g_S$, where $\zeta$ is a discount parameter.

[23] In our simulation results, it is assumed that $q = 0.7$ and $\zeta = 0.5$.

chain:

$$y_S = -\frac{1}{\sigma}\frac{\phi_\pi - p}{1-p}\pi_S + \Theta_{AD} g_s$$
$$\Theta_{AD} = q\zeta(y_{M,g} + \frac{1}{\sigma}\pi_{M,g} - 1) + 1,$$

where $\Theta_{AD}$ is the government spending shock shift in the Euler equation, $\zeta$ is the policy discount parameter, $y_{M,g}$ and $\pi_{M,g}$ are the medium run policy multiplier as in equation (8). For reference, this shift without long-run government spending will collapse to 1, which can connect to the case in our baseline model in section 2. The new items in this shift are from *rational expectations* of output gap, inflation, and medium-run spending shock. Note that the first new term is from the future wealth effects (higher expected output gap in the future) as in Bouakez et al. (2017) and the household has the incentive to increase consumption due to consumption smoothing. The second term comes from the fact that the long-run government can increase the firm's marginal cost and further inflation. The third term is due to the direct demand effect from future government spending. See appendix F, it turns out that $q\zeta(y_{M,g} + \frac{1}{\sigma}\pi_{M,g} - 1)$ is negative which means the overall expected effects from longer spending policy can crowd out the present output. In this case, long-run government spending can move down the Euler equation in a $(\pi_t, y_t)$ graph and the effects can be controlled by the policy product $q\zeta$.

On the other hand, the new Phillips Curve can be shown below:

$$\pi_S = \kappa\frac{\gamma_y}{1-\beta p - \kappa\gamma_r(\phi_\pi - p)} y_S + \frac{1}{1-\beta p - \kappa\gamma_r(\phi_\pi - p)}\Theta_{AS} g_s$$
$$\Theta_{AS} = (\beta - \kappa\gamma_r)(1-p)q\zeta\pi_{M,g} + \kappa\gamma_g,$$

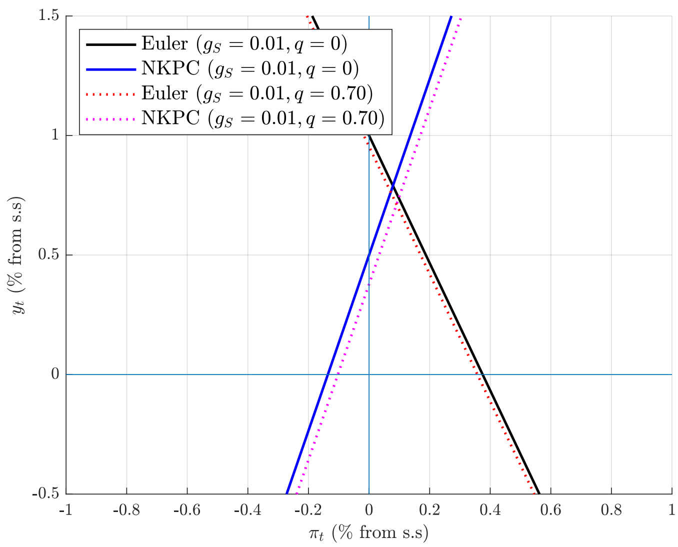
where $\Theta_{AS}$ is the government spending shock shift in the Phillips Curve, $\zeta$ is the policy discount parameter, $\pi_{M,g}$ is the medium run inflation multiplier as in equation (8). For reference, this shift without long-run government spending will collapse to $\kappa\gamma_g$, which is the same as the case in our baseline model in section 2. The new item in this shirt is from *rational expectations* of inflation and the long-run spending can increase the firm's marginal cost and further inflation due to sticky prices. See appendix F, it turns out the long-run government spending can further increase inflation, moving down the Phillips Curve in a $(\pi_t, y_t)$ graph. Similar to the case in the Euler equation, the effects of long-run policy can be controlled by the policy product $q\zeta$.[24]

One can use the new Euler equation and the Phillips Curve to produce the policy multipliers:

$$\frac{\partial y_S}{\partial g_S} = \frac{\Theta_{AD}\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi - p)] - \Theta_{AS}(\phi_\pi - p)}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)]}$$
$$\frac{\partial \pi_S}{\partial g_S} = \frac{\left[\kappa\gamma_y\Theta_{AD} + \Theta_{AS}\right]\sigma(1-p)}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)]}.$$

Figure 6 shows the numerical simulation of the long-run government spending on the AS/AD curves. The main takeaway from this is that the long-run spending can simultaneously move down the Euler equation and Phillips Curve, which is in line with our theoretical analysis. Since the policy product $q\zeta = 0.7 \times 0.5$ is a pretty small quantity, the long-run shift is minimally shown in this figure, and then the multiplier varies little. In addition, the long-run government spending can deflate the output multiplier but increase the inflation multiplier in this experiment.

Figure 6: AS/AD curves with/without long run government spending in normal times




The long-run government spending can further increase the inflation multiplier since the longer government spending can further increase the marginal cost. Additionally, the

[24] In this section, I just assume that $\kappa$ is very small in our theoretical analysis in line with e.g. Eggertsson (2011), Gabaix (2020) and Budianto et al. (2020). In this sense, one can assume that $\beta - \kappa\gamma_r > 0$ and then the long-run government policy can increase inflation here. However, if $\beta - \kappa\gamma_r < 0$, this might resolve the fiscal price puzzle (FPP) as in Han et al. (2020) that the long-run fiscal stimulus can lower inflation .

cost channel can increase inflation more working as a cost-push shock. In normal times, Central Bank would increase the nominal interest rate due to a higher price level. In that way, long-run spending policy can crowd out more private output gap due to a larger real interest rate. The real cost channel can increase the marginal cost and further inflation. Thus, the inflation multiplier can be higher in the strength of the cost channel, and this result is the same as the case in Section 2. In addition, the cost channel can lower the output gap further.

**Proposition 4.** *The long-run government spending effects in normal times reveal that*

1. *The long-run government spending can further deflate the output multiplier but further increase the inflation multiplier.*
2. *The long-run output gap multiplier reduces in the strength of the cost channel $\gamma_r$ while the long run inflation multiplier rises in $\gamma_r$.*

*Proof.* See Appendix G. □

The comparison numerical result between the strength of the cost channel is revealed in Figure 7. This result can echo proposition 4, and it is observed that the long-run government spending can decrease the output multiplier more than the short-run government spending. The long-run government spending can further increase the inflation multiplier. The output gap multiplier falls in the strength of the real cost channel from $\gamma_r = 1$ to $\gamma_r = 1.2$ while the inflation multiplier increases in the strength of the real cost channel from $\gamma_r = 1$ to $\gamma_r = 1.2$. In particular, the output gap multiplier can be negative with the long-run spending shock.

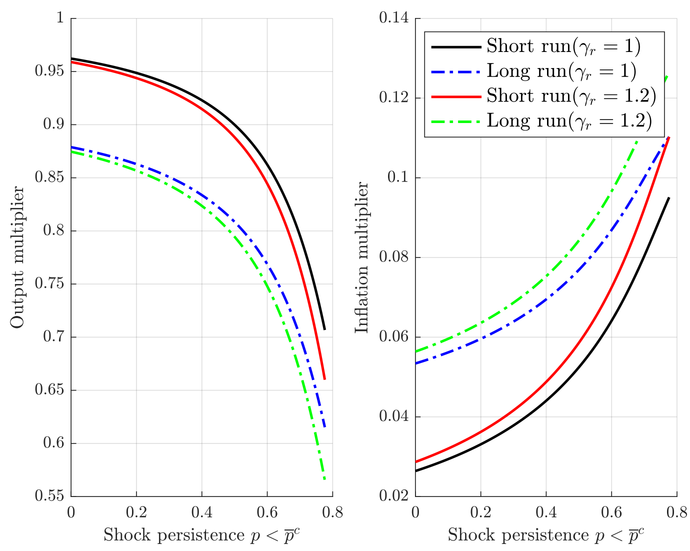
### 3.3 Zero Lower Bound

If the *short-run economy is at the ZLB*, the new Euler equation with a long-run spending policy can be given by:

$$y_S = -\frac{1}{\sigma(1-p)}[\log(\beta) - p\pi_S] + \Theta_{AD} g_s$$
$$\Theta_{AD} = q\zeta(y_{M,g} + \frac{1}{\sigma}\pi_{M,g} - 1) + 1.$$

where $\Theta_{AD}$ is the government spending shock shift in the Euler equation. Similar to the case in normal times, the long-run government spending effects $q\zeta(y_{M,g} + \frac{1}{\sigma}\pi_{M,g} - 1)$

Figure 7: Long run policy multipliers in normal times




which are negative. It indicates that the long-run government spending can move down the Euler equation in a $(\pi_t, y_t)$ graph and the effects can still be controlled by the policy product $q\zeta$.

In addition, I move to describe the new Phillips Curve as:

$$\pi_S = \kappa \frac{\gamma_y + \gamma_r \log(\beta)}{1 - \beta p - \kappa\gamma_r(-p)} y_S + \frac{1}{1 - \beta p - \kappa\gamma_r(-p)} \Theta_{AS} g_s$$
$$\Theta_{AS} = (\beta - \kappa\gamma_r)(1-p) q\zeta \pi_{M,g} + \kappa\gamma_g.$$

where $\Theta_{AS}$ is the government spending shock shift in the Phillips Curve. The long-run government spending can move down the Phillips Curve in a $(\pi_t, y_t)$ graph, and likewise, the effects can be controlled by the policy product $q\zeta$.

Similar to the case in normal times, I use the Euler equation and the Phillips Curve in liquidity traps to produce policy multipliers:

$$\frac{\partial y_S}{\partial g_S} = \frac{\Theta_{AD}\sigma(1-p)[1 - \beta p - \kappa\gamma_r(-p)] - \Theta_{AS}(-p)}{\kappa\gamma_y(-p) + \sigma(1-p)[1 - \beta p - \kappa\gamma_r(-p)]}$$
$$\frac{\partial \pi_S}{\partial g_S} = \frac{\left[\kappa\gamma_y\Theta_{AD} + \Theta_{AS}\right]\sigma(1-p)}{\kappa\gamma_y(-p) + \sigma(1-p)[1 - \beta p - \kappa\gamma_r(-p)]}.$$

The more extended government spending policy after the ZLB has subsided can increase inflation more by increasing firm's marginal costs. The nominal interest rate is zero, and higher inflation can help stimulate our NK economy since it can lower the real interest rate and then higher private consumption. At this time, the spending multiplier is larger, which is in line with Leduc and Wilson (2013) who empirically show similar larger long-term spending effects compared to the short-run policy. However, the real cost channel can reduce the effectiveness. The cost channel can dampen inflation due to the negative inflation expectation in the real borrowing cost. Thus, the output gap and inflation multipliers reduce in the strength of the cost channel, and this is the same as the result in the baseline model.

**Proposition 5.** *The long-run government spending effects at the ZLB reveal that*

1. *The long-run government spending can further raise the output gap and inflation multipliers.*
2. *The long-run output gap and inflation multipliers decrease in the strength of the real cost channel $\gamma_r$.*

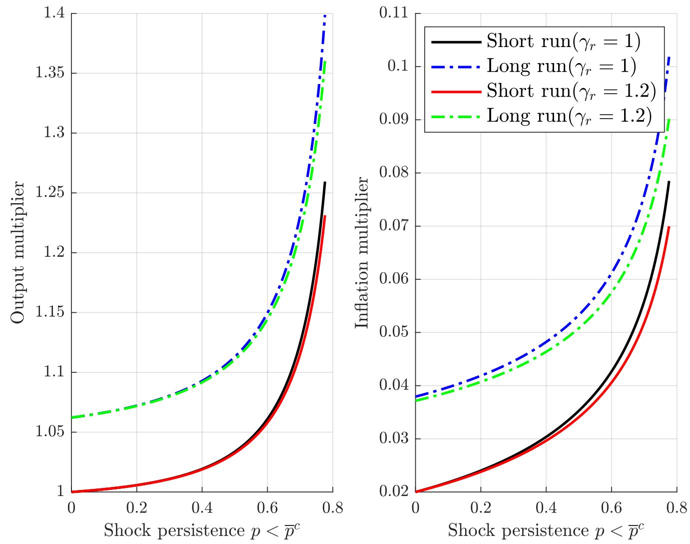
*Proof.* See Appendix H. □

At the ZLB, the numerical comparison result between the strength of the cost channel is reported in Figure 8. This can echo proposition 5, and it is observed that the long-run government spending can further increase the output gap multiplier and the inflation multiplier. The two policy multipliers decrease in the strength of the cost channel from $\gamma_r = 1$ to $\gamma_r = 1.2$. It is of note that the output gap multiplier can be more effective for prolonged spending policy in recessions.

# 4 A Behavioral Model

As can be seen in section 3, the long-run government spending can drive the multipliers at the ZLB. In our baseline model in section 2, it is common knowledge that the agents have rational expectations. While in this section, I try to extend our baseline model to address the role of rational expectations. I finally develop a similar version of Gabaix (2020)'s model to incorporate bounded rationality where agents can be short-sighted about the world into our simple benchmark setup.

Figure 8: Long run policy multipliers at ZLB




## 4.1 Government Spending Multiplier with Bounded Rationality

I use the model in Gabaix (2020) to show the behavior of the aggregate demand-side economy:

$$c_t = \bar{m}\mathbb{E}_t c_{t+1} - \frac{1}{\sigma_c}(i_t - \mathbb{E}_t \pi_{t+1} - r^n),$$

where $\bar{m}$ is the cognitive discounting parameter.

In this case, one can obtain the path of $y_t$ by substituting the resource constraint into Euler equation:

$$y_t = \alpha_{EE}\mathbb{E}_t y_{t+1} - \frac{1}{\sigma}(R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1} - r^n) + g_t - \alpha_{EE}\mathbb{E}_t g_{t+1},$$

where $\sigma = \sigma_c/(1-s_g)$ and $\alpha_{EE} = \bar{m}(1-s_g)$.

As in Gabaix (2020), I can show the NK Phillips Curve with the real cost channel:

$$\pi_t = \beta\alpha_{PC}\mathbb{E}_t \pi_{t+1} + \kappa\left[\gamma_y y_t + \gamma_g g_t + \gamma_r(R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1})\right],$$

where $\alpha_{PC} = \bar{m}[\varphi + \frac{1-\beta\varphi}{1-\beta\varphi\bar{m}}(1-\varphi)]$ which is increasing in $\bar{m}$, $\kappa = \frac{(1-\varphi)(1-\varphi\beta)}{\varphi(1+\eta\theta)}$, $\eta > 0$ is the standard (inverse) of labor-supply elasticity, $\theta > 1$ is the price elasticity of differential goods demand, $\beta \in (0,1)$ is the discounted preference parameter and $\varphi \in (0,1)$ is the share of firms which can not adjust their prices.

#### 4.1.1 Normal Times

I use the Euler equation and the Phillips Curve in normal times—see appendix I to produce the solutions for the output gap and inflation multipliers below:[25]

$$\frac{\partial y_S}{\partial g_S} = \frac{\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi - p)] - \kappa\gamma_g(\phi_\pi - p)}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi - p)]}$$

$$\frac{\partial \pi_S}{\partial g_S} = \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p\alpha_{EE})}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi - p)]}.$$

**Proposition 6.** *The government spending effects with bounded rationality in normal times reveal that*

1. *The output gap multiplier with bounded rationality increases in the strength of the cognitive discounting level $\bar{m}$ while the inflation multiplier reduces in $\bar{m}$.*
2. *The feature of policy multipliers in the strength of the real cost channel with bounded rationality is the same as the baseline model.*

*Proof.* See Appendix J. □

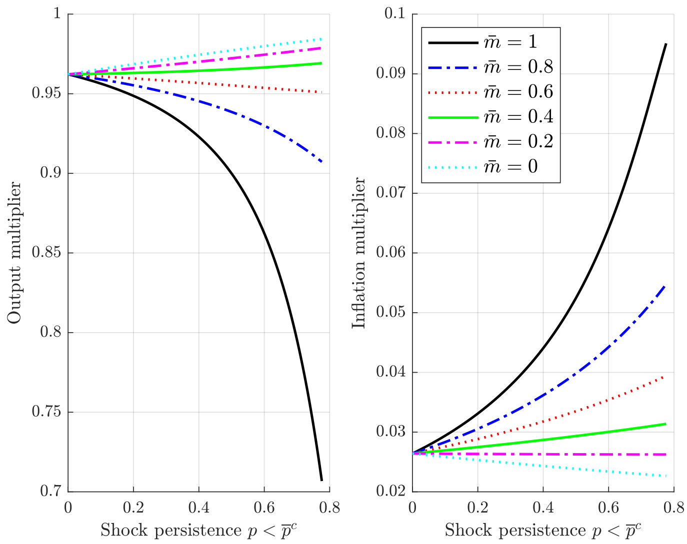
In normal times, the cognitive discounting with bounded rationality can reduce the expectation effects in our baseline model which can lower inflation. It means that there is a lower rise in the interest rate set by Central Bank, which can inflate the output gap multiplier accordingly compared to the baseline model. Since the introduced bound rationality is independent on the strength of the cost channel $\gamma_r$, the effects with this real channel on policy multipliers are the same as the baseline model results. Specifically, the output gap multiplier decreases in the strength of the cost channel, and the inflation multiplier increases in the strength of the cost channel. Similar results can also arise with the long-run government spending since the cognitive agent can decrease the expectation effects, which is in spirit with Nakata et al. (2019) and Farhi and Werning (2019). The inflation multiplier can be lower while the output multiplier can be higher to some extent, compared to the result in section 3.2. Figure 9 shows the numerical results, and the main takeaway is that the output multiplier with bounded rationality can be higher in the strength of the cognitive discounting level, and the inflation multiplier

[25] For simplicity, in this section, the simple two-state Markov chain is used to calculate policy multipliers.

with bounded rationality can be lower. This result can match our theoretical analysis as in Proposition 6.

Interestingly, as in appendix L, the output gap multiplier can decrease in the persistence $p$ with $\bar{m} = 1$ but increase in persistence $p$ with $\bar{m} = 0$. In this case, there exists a threshold value, across which the multiplier effects switch from decreasing in the persistence $p$ to increasing. I also find that the output gap multiplier can be large (near one) in normal times if we suppose the agents have strong bounded rationality which means the expectation effects are extremely weak. Intuitively, agents tend to consume today but not to save since future consumption has less or no impact on the decision today. In this case, the crowding-out effects of government spending should be much attenuated and the output gap multiplier can be near one. This result can echo the previous empirical evidence that the output gap multiplier can be large in normal times as in Auerbach and Gorodnichenko (2012) and Acconcia et al. (2014).

Figure 9: Policy multipliers with/without bounded rationality in normal times



### 4.1.2 Zero Lower Bound

One can use the Euler equation and the Phillips Curve in liquidity traps—see appendix I to produce the inflation and output gap multipliers below:

$$\frac{\partial y_S}{\partial g_S} = \frac{\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)]-\kappa\gamma_g(-p)}{\kappa\gamma_y(-p)+\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)]}$$

$$\frac{\partial \pi_S}{\partial g_S} = \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p\alpha_{EE})}{\kappa\gamma_y(-p)+\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)]}.$$

**Proposition 7.** *The government spending effects with bounded rationality at the ZLB reveal that*

1. *The output gap and inflation multipliers with bounded rationality decrease in the strength of the cognitive discounting level $\bar{m}$.*
2. *The feature of the policy multipliers in the strength of the real cost channel with bounded rationality is the same as the baseline model.*

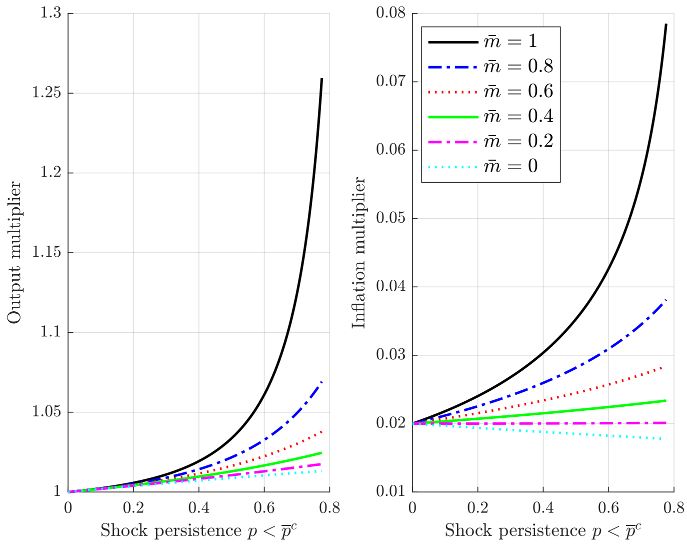
*Proof.* See Appendix K. □

At the ZLB, inflation caused by government spending can be less due to the cognitive discounting. Thus, it can increase the real interest rate, which, in turn, can lower the output gap multiplier. The effects in the strength of the real cost channel are the same as the baseline model in liquidity traps. Specifically, the output gap and inflation multipliers decrease in the strength of the real cost channel $\gamma_r$. Regarding the situation with long-run government spending, a similar result can appear since the cognitive agent can decrease the expectation effects and compare to the result in section 3.3. Overall, the policy multipliers can be reduced. As in our numerical simulation in Figure 10, it is seen that the inflation and output multipliers are lower with bounded rationality, and this result is in line with our theoretical results in Proposition 7. As in appendix L, the output multiplier increases in the persistence $p$.

# 5 Concluding Remarks

This paper augments the real cost channel in the textbook NK model to explore government spending multipliers. The general properties of spending multipliers are detected

Figure 10: Policy multipliers with/without bounded rationality at ZLB




using this analytical framework in liquidity traps. The robust role of the real cost channel on the policy multipliers is confirmed. To be more specific, our results on the strength of the real cost channel, if one considers the long-run government spending policy, are similar to the short-run model. I extend the basic model with bounded rationality. It turns out that cognitive behavior can alter the policy multipliers, however, the real cost channel can still work robustly.

# References


**Abo-Zaid, Salem**, "The government spending multiplier in a model with the cost channel," *Macroeconomic Dynamics*, 2022, *26* (1), 72–101.

**Acconcia, Antonio, Giancarlo Corsetti, and Saverio Simonelli**, "Mafia and Public Spending: Evidence on the Fiscal Multiplier from a Quasi-experiment," *American Economic Review*, July 2014, *104* (7), 2185–2209.

**Amano, Robert A and Tony S Wirjanto**, "Intratemporal substitution and government spending," *Review of Economics and Statistics*, 1997, *79* (4), 605–609.

**Angeletos, George-Marios and Chen Lian**, "Forward guidance without common knowledge," *American Economic Review*, 2018, *108* (9), 2477–2512.

**Aruoba, S. Boragan and Frank Schorfheide**, "Macroeconomic dynamics near the

ZLB: a tale of two equilibria," Working Papers 13-29, Federal Reserve Bank of Philadelphia 2013.

**Aruoba, S Borağan, Pablo Cuba-Borda, and Frank Schorfheide**, "Macroeconomic dynamics near the ZLB: A tale of two countries," *The Review of Economic Studies*, 2018, *85* (1), 87–118.

**Auerbach, Alan and Yuriy Gorodnichenko**, "Fiscal Multipliers in Recession and Expansion," in "Fiscal Policy after the Financial Crisis" NBER Chapters, National Bureau of Economic Research, Inc, 2012.

**Auerbach, Alan J, Yuriy Gorodnichenko, Peter McCrory, and Daniel Murphy**, "Fiscal Multipliers in the COVID19 Recession," Technical Report, National Bureau of Economic Research 2021.

**Barro, Robert J. and Charles J. Redlick**, "Macroeconomic Effects From Government Purchases and Taxes," *The Quarterly Journal of Economics*, 2011, *126* (1), 51–102.

**Beaudry, Paul, Chenyu Hou, and Franck Portier**, "Monetary Policy When the Phillips Curve is Locally Quite Flat," CEPR Discussion Paper DP15184, C.E.P.R. Discussion Paper August 2020.

**Bernardini, Marco, Selien De Schryder, and Gert Peersman**, "Heterogeneous government spending multipliers in the era surrounding the great recession," *Review of Economics and Statistics*, 2020, *102* (2), 304–322.

**Bilbiie, Florin O.**, "Limited asset markets participation, monetary policy and (inverted) aggregate demand logic," *Journal of Economic Theory*, May 2008, *140* (1), 162–196.

_ , "Monetary Policy and Heterogeneity: An Analytical Framework," CEPR Discussion Papers 12601, C.E.P.R. Discussion Papers January 2019.

_ , "Optimal Forward Guidance," *American Economic Journal: Macroeconomics*, October 2019, *11* (4), 310–45.

_ , "The new Keynesian cross," *Journal of Monetary Economics*, 2020, *114*, 90–108.

**Blanchard, Olivier and Roberto Perotti**, "An Empirical Characterization Of The Dynamic Effects Of Changes In Government Spending And Taxes On Output," *The Quarterly Journal of Economics*, November 2002, *117* (4), 1329–1368.

**Bouakez, Hafedh, Michel Guillard, and Jordan Roulleau-Pasdeloup**, "Public Investment, Time to Build, and the Zero Lower Bound," *Review of Economic Dynamics*, January 2017, *23*, 60–79.

__ , __ , **and** __ , "The optimal composition of public spending in a deep recession," *Journal of Monetary Economics*, 2020, *114*, 334–349.

**Budianto, Flora, Taisuke Nakata, and Sebastian Schmidt**, "Average inflation targeting and the interest rate lower bound," Working Paper 2020.

**Campbell, Jeffrey R, Filippo Ferroni, Jonas DM Fisher, and Leonardo Melosi**, "The limits of forward guidance," *Journal of Monetary Economics*, 2019, *108*, 118–134.

**Chowdhury, Ibrahim, Mathias Hoffmann, and Andreas Schabert**, "Inflation dynamics and the cost channel of monetary transmission," *European Economic Review*, 2006, *50* (4), 995–1016.

**Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans**, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, February 2005, *113* (1), 1–45.

__ , __ , **and Sergio Rebelo**, "When Is the Government Spending Multiplier Large?," *Journal of Political Economy*, 2011, *119* (1), 78 – 121.

**Cochrane, John H.**, "The new-Keynesian liquidity trap," *Journal of Monetary Economics*, 2017, *92* (C), 47–63.

**Coibion, Olivier, Dimitris Georgarakos, Yuriy Gorodnichenko, and Michael Weber**, "Forward guidance and household expectations," Technical Report, National Bureau of Economic Research 2020.

**Durevall, Dick and Magnus Henrekson**, "The futile quest for a grand explanation of long-run government expenditure," *Journal of Public Economics*, 2011, *95* (7-8), 708–722.

**Eggertsson, Gauti B**, "Real government spending in a liquidity trap," *Princeton University, unpublished manuscript*, 2001.

__ , "What fiscal policy is effective at zero interest rates?," *NBER Macroeconomics Annual*, 2011, *25* (1), 59–112.

__ **and Michael Woodford**, "Optimal Monetary Policy in a Liquidity Trap," NBER Working Papers 9968, National Bureau of Economic Research, Inc September 2003.

__ **and** __ , "Optimal monetary and fiscal policy in a liquidity trap," Working Paper, National Bureau of Economic Research 2004.

__ **et al.**, "Zero bound on interest rates and optimal monetary policy," *Brookings papers on economic activity*, 2003, *2003* (1), 139–233.

**Farhi, Emmanuel and Iván Werning**, "Monetary policy, bounded rationality, and incomplete markets," *American Economic Review*, 2019, *109* (11), 3887–3928.

**Gabaix, Xavier**, "A Behavioral New Keynesian Model," *American Economic Review*, August 2020, *110* (8), 2271–2327.

**Galí, Jordi**, *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications*, Princeton University Press, 2015.

**Ganelli, Giovanni and Juha Tervala**, "The welfare multiplier of public infrastructure investment," IMF Working Paper 2016.

**Gertler, Mark, Jordi Gali, and Richard Clarida**, "The Science of Monetary Policy: A New Keynesian Perspective," *Journal of Economic Literature*, December 1999, *37* (4), 1661–1707.

**Han, Zhao, Fei Tan, and Jieran Wu**, "Learning from Monetary and Fiscal Policy," *Available at SSRN 3726905*, 2020.

**Hazell, Jonathon, Juan Herreño, Emi Nakamura, and Jøn Steinsson**, "The Slope of the Phillips Curve: Evidence from U.S. States," Working Paper, National Bureau of Economic Research 2020.

**House, Christopher L, Christian Proebsting, and Linda L Tesar**, "Austerity in the aftermath of the great recession," *Journal of Monetary Economics*, 2020, *115*, 37–63.

**III, Marvin J Barth and Valerie A Ramey**, "The cost channel of monetary transmission," *NBER macroeconomics annual*, 2001, *16*, 199–240.

**Ilzetzki, Ethan, Enrique G. Mendoza, and Carlos A. Végh**, "How big (small?) are fiscal multipliers?," *Journal of Monetary Economics*, 2013, *60* (2), 239–254.

**Kiley, Michael T.**, "Policy Paradoxes in the New Keynesian Model," Finance and Economics Discussion Series 2014-29, Board of Governors of the Federal Reserve System (U.S.) February 2014.

**Koester, Gerrit, Eliza Lis, Christiane Nickel, Chiara Osbat, and Frank Smets**, "Understanding low inflation in the euro area from 2013 to 2019: cyclical and structural drivers," *ECB Occasional Paper*, 2021, (2021280).

**Kollmann, Robert, Werner Roeger et al.**, "Fiscal policy in a financial crisis: standard policy versus bank rescue measures," *American Economic Review*, 2012, *102* (3), 77–81.

**Kraay, Aart**, "How large is the government spending multiplier? Evidence from World

Bank lending," *The Quarterly Journal of Economics*, 2012, *127* (2), 829–887.

**Laubach, Thomas and John C Williams**, "Measuring the natural rate of interest," *Review of Economics and Statistics*, 2003, *85* (4), 1063–1070.

**Leduc, Sylvain and Daniel Wilson**, "Roads to Prosperity or Bridges to Nowhere? Theory and Evidence on the Impact of Public Infrastructure Investment," *NBER Macroeconomics Annual*, 2013, *27* (1), 89 – 142.

**Leeper, Eric M, Nora Traum, and Todd B Walker**, "Clearing up the fiscal multiplier morass," *American Economic Review*, 2017, *107* (8), 2409–54.

**Lewis, Daniel J**, "Identifying shocks via time-varying volatility," *The Review of Economic Studies*, 2021, *88* (6), 3086–3124.

**Llosa, Luis-Gonzalo and Vicente Tuesta**, "Learning about monetary policy rules when the cost-channel matters," *Journal of Economic Dynamics and Control*, 2009, *33* (11), 1880–1896.

**Lustenhouwer, Joep**, "Fiscal stimulus in expectations-driven liquidity traps," *Journal of Economic Behavior & Organization*, 2020, *177*, 661–687.

**McKay, Alisdair, Emi Nakamura, and Jón Steinsson**, "The Power of Forward Guidance Revisited," *American Economic Review*, October 2016, *106* (10), 3133–3158.

**Mertens, Karel RSM and Morten O Ravn**, "Fiscal policy in an expectations-driven liquidity trap," *The Review of Economic Studies*, 2014, *81* (4), 1637–1667.

**Miyamoto, Wataru, Thuy Lan Nguyen, and Dmitriy Sergeyev**, "Government Spending Multipliers under the Zero Lower Bound: Evidence from Japan," *American Economic Journal: Macroeconomics*, July 2018, *10* (3), 247–277.

**Nakata, Taisuke, Ryota Ogaki, Sebastian Schmidt, and Paul Yoo**, "Attenuating the forward guidance puzzle: implications for optimal monetary policy," *Journal of Economic Dynamics and Control*, 2019, *105*, 90–106.

**Nie, He**, "The Macroeconomic Effects of Tax Shocks: The Real Cost Channel," *Available at SSRN 3905695*, 2021.

_ **and Jordan Roulleau-Pasdeloup**, "The Promises (and Perils) of Control-Contingent Forward Guidance," Working Paper 2021.

**Ramey, Valerie A.**, "Identifying Government Spending Shocks: It's all in the Timing," *The Quarterly Journal of Economics*, 2011, *126* (1), 1–50.

_ **and Sarah Zubairy**, "Government Spending Multipliers in Good Times and in Bad:

Evidence from US Historical Data," *Journal of Political Economy*, 2018, *126* (2), 850–901.

**Ravenna, Federico and Carl E. Walsh**, "Optimal monetary policy with the cost channel," *Journal of Monetary Economics*, 2006, *2* (53), 199–216.

**Roulleau-Pasdeloup, Jordan**, "The Public Investment Multiplier: Insights from a Tractable HANK Framework," Working Paper October 2021.

_ , "Shocks and policies in Linear Rational Expectations models: An expected Markovian Approach," Working Paper October 2021.

**Sarin, Natasha, Lawrence H. Summers, Owen M. Zidar, and Eric Zwick**, "Rethinking How We Score Capital Gains Tax Reform," NBER Working Papers 28362, National Bureau of Economic Research, Inc January 2021.

**Schmidt, Sebastian**, "Optimal Monetary and Fiscal Policy with a Zero Bound on Nominal Interest Rates," *Journal of Money, Credit and Banking*, October 2013, *45* (7), 1335–1350.

_ , "Fiscal activism and the zero nominal interest rate bound," *Journal of Money, Credit and Banking*, 2017, *49* (4), 695–732.

**Smith, A Lee**, "When does the cost channel pose a challenge to inflation targeting central banks?," *European Economic Review*, 2016, *89*, 471–494.

**Surico, Paolo**, "The cost channel of monetary policy and indeterminacy," *Macroeconomic Dynamics*, 2008, *5* (12), 724–735.

**Taylor, John B.**, "Discretion versus policy rules in practice," *Carnegie-Rochester Conference Series on Public Policy*, December 1993, *39* (1), 195–214.

**Tillmann, Peter**, "Optimal monetary policy with an uncertain cost channel," *Journal of Money, Credit and Banking*, 2009, *41* (5), 885–906.

**Woodford, Michael**, "Optimal interest-rate smoothing," *The Review of Economic Studies*, 2003, *70* (4), 861–886.

**Yaffe, Daniel Leff**, "The Interstate Multiplier," *University of California, San Diego Working paper*, 2019.

**Zubairy, Sarah**, "On fiscal multipliers: Estimates from a medium scale DSGE model," *International Economic Review*, 2014, *55* (1), 169–195.

# Online Appendix

## A Details of the Baseline Model in Section 2

### A.1 Aggregate Supply Side

The behavior of aggregate supply side economics in the standard NK model is shown below as in Tillmann (2009):

$$\pi_t = \kappa mc_t + \beta \mathbb{E}_t \pi_{t+1}.$$

I need to add the government spending ingredient into Beaudry et al. (2020)'s aggregate supply side economics. Here I assume that there is a large number of firms indexed by $z$ which produce a differentiated intermediate goods sold to a final goods producer. The representative firm follows a Leontief production function as shown below

$$Y_t(z) = \min(aN_t(z), bM_t(z)),$$

where $M_t$ is the final good. Similarly, by assuming that the sum of real profit owns the common discount factor $\beta$, we can obtain the (real) marginal cost of the basic input:

$$MC_t = \frac{W_t}{a} + \frac{\beta}{b}\mathbb{E}\left[\frac{1+i_t}{1+\pi_{t+1}}\right].$$

With log condition, one can show the linearized equilibrium

$$mc_t = \hat{\gamma}_y(w_t) + \gamma_r(R_t + \log(\beta) - \mathbb{E}\pi_{t+1}),$$

where $\hat{\gamma}_y = \frac{\frac{1}{a}W}{\frac{1}{a}W + \frac{\beta}{b}\frac{1+i}{1+\pi}}$ and $\gamma_r = \frac{\frac{\beta}{b}\frac{1+i}{1+\pi}}{\frac{1}{a}W + \frac{\beta}{b}\frac{1+i}{1+\pi}}$. On the other hand, the optimal labor supply reads

$$\frac{v'(N_t)}{u'(C_t)} = W_t,$$

where $v(N_t) = \chi\frac{N_t^{1+\eta}}{1+\eta}$ and $u(C_t) = \frac{C_t^{1-\sigma_c}}{1-\sigma_c}$.

By using the production function $y_t = n_t$ and resource constraint $y_t = (1-s_g)c_t + g_t$, the marginal cost can be rewritten as

$$
\begin{aligned}
mc_t &= \hat{\gamma}_y \frac{Nv''(N)}{v'(N)} y_t - \hat{\gamma}_y \frac{Cu''(C)}{u'(C)} (\frac{y_t - g_t}{1 - s_g}) + \gamma_r (R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1}) \\
&= \gamma_y y_t + \gamma_g g_t + \gamma_r (R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1}),
\end{aligned}
$$

where $\gamma_y = \hat{\gamma}_y \left( \frac{Nv''(N)}{v'(N)} - \frac{Cu''(C)}{u'(C)(1-s_g)} \right)$, and $\gamma_g = \hat{\gamma}_y \frac{Cu''(C)}{u'(C)(1-s_g)}$. Therefore, the Phillips Curve with government spending is shown below.[26]

$$
\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \left[ \gamma_y y_t + \gamma_g g_t + \gamma_r (R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1}) \right].
$$

# B Proof for Proposition 1

If I only consider the demand shock and it is assumed that the demand shock $r_S^n$ can put the economy into liquidity traps with one enough (negatively) big number ($r_S^n < \underline{r_S^n}$), one can rewrite the Phillips Curve as

$$
y_S = \begin{cases} \frac{1 - \beta p + \kappa \gamma_r p - \kappa \gamma_r \phi_\pi}{\kappa \gamma_y} \pi_S & \text{if } r_S^n \geq \underline{r_S^n} \\ \frac{1 - \beta p + \kappa \gamma_r p}{\kappa \gamma_y} \pi_S - \frac{\gamma_r}{\gamma_y} \log(\beta) & \text{if } r_S^n < \underline{r_S^n}. \end{cases}
$$

Similarly, one can rewrite the Euler equations as follows:

$$
y_S = \begin{cases} -\frac{1}{\sigma} \frac{\phi_\pi - p}{1-p} \pi_S + \frac{1}{\sigma} \frac{r_S^n}{1-p} & \text{if } r_S^n \geq \underline{r_S^n} \\ \frac{\frac{1}{\sigma} p}{1-p} \pi_S + \frac{1}{\sigma} \frac{r_S^n - \log(\beta)}{1-p} & \text{if } r_S^n < \underline{r_S^n}. \end{cases}
$$

I combine the first questions of Euler equation and Phillips Curve to obtain the exact expression for $\underline{r_S^n}$ which can be written as:

$$
\underline{r_S^n} = \left[ \frac{(1 - \beta p + \kappa \gamma_r p - \kappa \gamma_r \phi_\pi)(1-p)}{\kappa \gamma_y \frac{1}{\sigma}} + (\phi_\pi - p) \right] \frac{\log(\beta)}{\phi_\pi} < 0.
$$

[26] The advantage of this introduced real cost channel method as in Beaudry et al. (2020) is that it allows to set arbitrarily the elasticity of marginal cost rate with regard to wage and interest rate. Please see Beaudry et al. (2020) for a comprehensive comparison between the model with a nominal cost channel and with the real cost channel.

Likewise, one can show the the exact boundary condition for $\underline{r_S^n}$ in the standard NK model without the real cost channel:

$$\underline{r_S^{n,N}} = \left[\frac{(1-\beta p)(1-p)}{\kappa\gamma_y\frac{1}{\sigma}} + (\phi_\pi - p)\right]\frac{\log(\beta)}{\phi_\pi} < 0.$$

In this case, I have

$$\underline{r_S^n} - \underline{r_S^{n,N}} = \frac{\kappa\gamma_r(p-\phi_\pi)}{\kappa\gamma_y\frac{1}{\sigma}}\frac{\log(\beta)}{\phi_\pi} > 0.$$

One can use this to obtain the result in the main text.

# C Proof for Proposition 2

I show the solutions for the output gap and inflation multipliers below:

$$\begin{aligned}\frac{\partial y_S}{\partial g_S} &= \frac{\frac{\sigma(1-p)}{\phi_\pi - p} - \frac{\kappa\gamma_g}{1-\beta p-\kappa\gamma_r(\phi_\pi-p)}}{\frac{\kappa\gamma_y}{1-\beta p-\kappa\gamma_r(\phi_\pi-p)} + \frac{\sigma(1-p)}{\phi_\pi-p}} \\ &= \frac{\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)] - \kappa\gamma_g(\phi_\pi-p)}{\kappa\gamma_y(\phi_\pi-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]}\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi_S}{\partial g_S} &= \frac{1+\frac{\gamma_g}{\gamma_y}}{\frac{1-\beta p-\kappa\gamma_r(\phi_\pi-p)}{\kappa\gamma_y} + \frac{1}{\sigma(1-p)}(\phi_\pi-p)} \\ &= \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p)}{\kappa\gamma_y(\phi_\pi-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]}.\end{aligned}$$

For the normal cost channel case, the item $[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]$ will switch to $[1-\beta p-\kappa\gamma_r\phi_\pi]$. In this case, one can easily prove that the inflation multiplier can be larger with the real cost channel due to larger denominator. However, the output multiplier can be larger with a positive item $\kappa\gamma_r p$ added both in the numerator and denominator.

## C.1 Part 1

I have assumed that $p < \overline{p}^c$ and thus $1-\beta p-\kappa\gamma_r(\phi_\pi-p) > 0$. In this case, one can prove easily that the numerator and denominator of the output gap and inflation multipliers are positive, therefore the two multipliers in normal times are positive.

## C.2 Part 2

For the output multiplier, the numerator is less than the denominator. That is,

$$\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]-\kappa\gamma_g(\phi_\pi-p)<\kappa\gamma_y(\phi_\pi-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)].$$

Thus, the output gap multiplier is less than one.

## C.3 Part 3

For the output gap multiplier without the real cost channel:

$$\frac{\partial y_S}{\partial g_S}=\frac{\sigma(1-p)[1-\beta p]-\kappa\gamma_g(\phi_\pi-p)}{\kappa\gamma_y(\phi_\pi-p)+\sigma(1-p)[1-\beta p]}.$$

One can compare this expression to the previous one with the real cost channel and it is easy, after some arrangements, to obtain the output gap multiplier with the real cost channel that is lower.

For the inflation multiplier without the real cost channel:

$$\frac{\partial \pi_S}{\partial g_S}=\frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p)}{\kappa\gamma_y(\phi_\pi-p)+\sigma(1-p)[1-\beta p]}.$$

In this case, the denominator of the inflation multiplier with the real cost channel is lower due to a negative item and thus the inflation multiplier is higher.

## C.4 Part 4

For the output gap multiplier:

$$\frac{\partial\frac{\partial y_S}{g_S}}{\partial\gamma_r}=-\sigma(1-p)(\phi_\pi-p)\kappa\frac{\mathscr{D}-\mathscr{N}}{\mathscr{D}^2}<0,$$

where $\mathscr{D}=\kappa\gamma_y(\phi_\pi-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]$ and $\mathscr{N}=\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]-\kappa\gamma_g(\phi_\pi-p)$. Thus, the output gap multiplier in normal times is decreasing in the strength of the real cost channel.

For the inflation multiplier, it is observed with ease that the higher the strength of the real cost channel $\gamma_r$, the higher the denominator of this multiplier. In other words, the inflation multiplier is decreasing in the strength of the real cost channel.

# D Proof for Proposition 3

## D.1 Part 1

The output gap and inflation multipliers at the ZLB are reproduced here:

$$\frac{\partial y_S}{\partial g_S} = \frac{\frac{\sigma(1-p)}{-p} - \frac{\kappa\gamma_g}{1-\beta p-\kappa\gamma_r(-p)}}{\frac{\kappa\gamma_y}{1-\beta p-\kappa\gamma_r(-p)} + \frac{\sigma(1-p)}{-p}}$$
$$= \frac{\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)] - \kappa\gamma_g(-p)}{\kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]}$$

$$\frac{\partial \pi_S}{\partial g_S} = \frac{1+\frac{\gamma_g}{\gamma_y}}{\frac{1-\beta p-\kappa\gamma_r(-p)}{\kappa\gamma_y} + \frac{1}{\sigma(1-p)}(-p)}$$
$$= \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p)}{\kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]}.$$

The numerator and denominator of the output and inflation multipliers (we assume $\gamma_r$ is far greater than $\gamma_y$ and the denominator is positive) are both positive here and thus we have the positive policy multipliers.

## D.2 Part 2

The output gap multiplier can be rewritten as:

$$\frac{\partial y_S}{\partial g_S} = 1 + \frac{\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)] + \kappa\gamma_g p + \kappa\gamma_y p}{\kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]}.$$

This output gap multiplier is larger than one.

## D.3 Part 3

For the output gap multiplier without the real cost channel:

$$\frac{\partial y_S}{\partial g_S} = \frac{\sigma(1-p)[1-\beta p] - \kappa\gamma_g(-p)}{\kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p]}.$$

Similar with the case in normal times, one can compare this expression to the previous one with the real cost channel, and it is easy, after some arrangements, to obtain the output gap multiplier with the real cost channel that is lower.

For the inflation multiplier without the real cost channel:

$$\frac{\partial \pi_S}{\partial g_S} = \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p)}{\kappa\gamma_y(-p)+\sigma(1-p)[1-\beta p]}.$$

One can observe that the higher the strength of the real cost channel $\gamma_r$, the higher the denominator of this multiplier. In this case, it can be lower with the real cost channel.

### D.4 Part 4

For the output gap multiplier:

$$\frac{\partial \frac{\partial y_S}{g_S}}{\partial \gamma_r} = -\sigma(1-p)(-p)\kappa\frac{\mathcal{D}-\mathcal{N}}{\mathcal{D}^2} < 0,$$

where $\mathcal{D} = \kappa\gamma_y(-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]$ and $\mathcal{N} = \sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]-\kappa\gamma_g(-p)$. Thus, the output gap multiplier in normal times is decreasing in the strength of the real cost channel.

For the inflation multiplier, it is observed with ease that the higher the strength of the real cost channel $\gamma_r$, the lower the denominator of this multiplier. In other words, the inflation multiplier is increasing in the strength of the real cost channel.

## E Multipliers in the Medium Run

In the medium run, the economy is absent of natural rate shock, and one can show the medium run government spending with the persistence $q$ as follows

$$\begin{aligned} y_M &= y_{M,g} g_M \\ &= \frac{\sigma(1-q)[1-\beta p-\kappa\gamma_r(\phi_\pi^q-q)]-\kappa\gamma_g(\phi_\pi^q-q)}{\kappa\gamma_y(\phi_\pi^q-q)+\sigma(1-q)[1-\beta q-\kappa\gamma_r(\phi_\pi^q-q)]} g_M. \end{aligned}$$

$$\pi_M = \pi_{M,g} g_M$$
$$= \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-q)}{\kappa\gamma_y(\phi_\pi^q - q) + \sigma(1-q)[1-\beta q - \kappa\gamma_r(\phi_\pi^q - q)]} g_M.$$

# F Euler and Phillips shift

The long-run government spending in the Euler equation shift:

$$q\zeta(y_{M,g} + \frac{1}{\sigma}\pi_{M,g} - 1) = q\zeta \frac{\kappa(\gamma_y + \gamma_g)(1-\phi_\pi^q)}{\kappa\gamma_y(\phi_\pi^q - q) + \sigma(1-q)[1-\beta q - \kappa\gamma_r(\phi_\pi^q - q)]} < 0.$$

The long run government spending in the Phillips Curve shift

$$(\beta - \kappa\gamma_r)(1-p)q\zeta\pi_{M,g} > 0,$$

where we assume that the $\beta - \kappa\gamma_r > 0$ since $\kappa$ is very small in our theoretical analysis as in Gabaix (2020) Budianto et al. (2020), and Nie (2021).

# G Proof for Proposition 4

## G.1 Part 1

I can use the new Euler equation and the Phillips Curve to reproduce the policy multipliers:

$$\frac{\partial y_S}{\partial g_S} = \frac{\Theta_{AD}\sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)] - \Theta_{AS}(\phi_\pi - p)}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)]}$$
$$\frac{\partial \pi_S}{\partial g_S} = \frac{\left[\kappa\gamma_y\Theta_{AD} + \Theta_{AS}\right]\sigma(1-p)}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)]}.$$

For the output gap multipliers, as in appendix F, one can see that the long-run government spending shock can lead to a lower $\Theta_{AD}$ but a higher $\Theta_{AS}$. In this case, the multiplier should be lower.

For inflation multiplier,

$$\kappa\gamma_y q\zeta(y_{M,g}+\frac{1}{\sigma}\pi_{M,g}-1)+(\beta-\kappa\gamma_r)(1-p)q\zeta\pi_{M,g}$$
$$=q\zeta\frac{\kappa\gamma_y\kappa(\gamma_y+\gamma_g)(1-\phi_\pi^q)}{\kappa\gamma_y(\phi_\pi^q-q)+\sigma(1-q)[1-\beta q-\kappa\gamma_r(\phi_\pi^q-q)]}+q\zeta\frac{(\beta-\kappa\gamma_r)(1-p)q\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-q)}{\kappa\gamma_y(\phi_\pi^q-q)+\sigma(1-q)[1-\beta q-\kappa\gamma_r(\phi_\pi^q-q)]}>0,$$

where we assume $\kappa$ is minor in our theoretical analysis as in e.g. Eggertsson (2011), Gabaix (2020) and Budianto et al. (2020) and the first item has an addition multiplier $\kappa$. One can use this to prove the result in the main text.

## G.2 Part 2

Since there is no $\gamma_r$ in $\Theta_{AD}$, we only focus on the $\Theta_{AS}$'s effects. For the main proposition result, since the term with $\Theta_{AD}$ is decreasing in $\gamma_r$, we only need to show the other term with $\Theta_{AS}$ is also decreasing in $\gamma_r$ and thus one can prove the output gap multiplier is decreasing in $\gamma_r$. The output multiplier can be reduced below:

$$-\frac{\Theta_{AS}}{\kappa\gamma_y(\phi_\pi-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]}.$$

For the main proposition result, since the term with $\Theta_{AD}$ is increasing in $\gamma_r$, we only need to show the other term with $\Theta_{AS}$ is also increasing in $\gamma_r$ and thus one can prove the inflation multiplier is increasing in $\gamma_r$. The inflation multiplier can be reduced below:

$$\frac{\Theta_{AS}}{\kappa\gamma_y(\phi_\pi-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]}.$$

One can differentiate this common term with regard to $\gamma_r$ and this common term can be reduced further as:

$$\frac{\beta-\kappa\gamma_r}{\kappa\gamma_y(\phi_\pi-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]}\frac{1}{\kappa\gamma_y(\phi_\pi-q)+\sigma(1-q)[1-\beta q-\kappa\gamma_r(\phi_\pi^q-q)]}.$$

I differentiate the above term with regard to $\gamma_r$ to obtain:

$$\frac{\sigma(1-p)\kappa(\beta\phi_\pi-1)\mathscr{D}_1+\sigma(1-q)\kappa(\beta\phi_\pi^q-1)\mathscr{D}_2-\mathscr{O}(\kappa^2)}{\mathscr{D}_3^2}.$$

where $\mathscr{D}_1 = \kappa\gamma_y(\phi_\pi - q) + \sigma(1-q)[1-\beta q - \kappa\gamma_r(\phi_\pi^q - q)]$, $\mathscr{D}_2 = \kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)]$, $\mathscr{D}_3 = \mathscr{D}_1 \cdot \mathscr{D}_2$ and $\mathscr{O}(\kappa^2)$ is the residual of order two since we assume that $\kappa$ is trivial in our theoretical analysis as in e.g. Eggertsson (2011), Gabaix (2020) and Budianto et al. (2020), it is easy to check that the derivative with regard to $\gamma_r$ is positive. In this case, one can use this to prove the result in the main text.

# H Proof for Proposition 5

## H.1 Part 1

One can produce the output gap and inflation multipliers below

$$\frac{\partial y_S}{\partial g_S} = \frac{\Theta_{AD}\sigma(1-p)[1-\beta p - \kappa\gamma_r(-p)] - \Theta_{AS}(-p)}{\kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(-p)]}$$

$$\frac{\partial \pi_S}{\partial g_S} = \frac{\left[\kappa\gamma_y\Theta_{AD} + \Theta_{AS}\right]\sigma(1-p)}{\kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(-p)]}.$$

For the output gap multipliers, the numerator with medium run spending policy can be decomposed into the following two parts. The first part:

$$\begin{aligned}&\kappa\gamma_y q\zeta(y_{M,g} + \frac{1}{\sigma}\pi_{M,g} - 1)\sigma(1-p)[1-\beta p + \kappa\gamma_r p] \\ &= q\zeta\frac{\kappa\gamma_y\kappa(\gamma_y + \gamma_g)(1-\phi_\pi^q)}{\kappa\gamma_y(\phi_\pi^q - q) + \sigma(1-q)[1-\beta q - \kappa\gamma_r(\phi_\pi^q - q)]}\sigma(1-p)[1-\beta p + \kappa\gamma_r p].\end{aligned}$$

The second part:

$$\begin{aligned}&(\beta - \kappa\gamma_r)(1-p)q\zeta\pi_{M,g}p \\ &= q\zeta\frac{(\beta-\kappa\gamma_r)(1-p)q\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-q)}{\kappa\gamma_y(\phi_\pi^q - q) + \sigma(1-q)[1-\beta q - \kappa\gamma_r(\phi_\pi^q - q)]}p.\end{aligned}$$

To simplify the proof, one can plus the two items and show the sum is positive if we assume that $\kappa$ is very small in our theoretical analysis. Similar to the inflation multiplier in normal times, we can have a higher long run inflation multiplier at the ZLB.

## H.2 Part 2

Since there is no $\gamma_r$ in $\Theta_{AD}$, we only focus on the $\Theta_{AS}$'s effects. For the main result, since the term with $\Theta_{AD}$ is decreasing in $\gamma_r$, we only need to show the the other term with $\Theta_{AS}$ is also decreasing in $\gamma_r$ and thus one can see the output gap multiplier is decreasing in $\gamma_r$. In this case, the output multiplier can be reduced below:

$$\frac{\Theta_{AS}}{\kappa\gamma_y(-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]}.$$

For the main result, since the term with $\Theta_{AD}$ is decreasing om $\gamma_r$, we only need to show the other term with $\Theta_{AS}$ is also decreasing in $\gamma_r$ and thus the inflation multiplier is decreasing in $\gamma_r$. The inflation multiplier can be reduced below:

$$\frac{\Theta_{AS}}{\kappa\gamma_y(-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]}.$$

One can differentiate this common term with regard to $\gamma_r$ and this common term can be reduced further as:

$$\frac{\beta-\kappa\gamma_r}{\kappa\gamma_y(-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]}\frac{1}{\kappa\gamma_y(\phi_\pi^q-q)+\sigma(1-q)[1-\beta q-\kappa\gamma_r(\phi_\pi^q-q)]}.$$

I differentiate the above term with regard to $\gamma_r$ to obtain:

$$\frac{-\sigma(1-p)\kappa\mathscr{D}_1+\sigma(1-q)\kappa(\beta\phi_\pi^q-1)\mathscr{D}_2-\mathscr{O}(\kappa^2)}{\mathscr{D}_3^2}.$$

where $\mathscr{D}_1=\kappa\gamma_y(\phi_\pi^q-q)+\sigma(1-q)[1-\beta q-\kappa\gamma_r(\phi_\pi^q-q)]$, $\mathscr{D}_2=\kappa\gamma_y(-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]$, $\mathscr{D}_3=\mathscr{D}_1\cdot\mathscr{D}_2$ and $\mathscr{O}(\kappa^2)$ is the residual of order two. One can reduce this expression as:

$$-(1-p)\sigma(1-q)(1-\beta q)+(1-q)(\beta\phi_\pi^q-1)\sigma(1-p)(1-\beta p)-\mathscr{O}(\kappa^2)<0,$$

where I use the general condition $\phi_\pi^q\beta-1<1$ and the short run period should be longer or almost equal to the long run period in reality such that $p\geq q$. In this case, one can use this to prove the result in the main text.

# I Euler Equation and Phillips Curve with Bounded Rationality

The Euler equation and Phillips Curve in normal times:

$$y_S = -\frac{1}{\sigma(1-\alpha_{EE}p)}(\phi_\pi - p)\pi_S + g_S$$
$$\pi_S = \kappa\frac{\gamma_y}{1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)}y_S + \kappa\frac{\gamma_g}{1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)}g_S.$$

The Euler equation and Phillips Curve can be elaborated at the ZLB:

$$y_S = -\frac{1}{\sigma(1-p\alpha_{EE})}[\log(\beta) - p\pi_S] + g_S$$
$$\pi_S = \frac{\kappa\gamma_y y_S + \kappa\gamma_r\log(\beta)}{1-\beta p\alpha_{PC} + \kappa\gamma_r p} + \kappa\frac{\gamma_g}{1-\beta p\alpha_{PC} + \kappa\gamma_r p}g_S.$$

# J Proof for Proposition 6

I reproduce the multipliers here:

$$\begin{aligned}\frac{\partial y_S}{\partial g_S} &= \frac{\frac{\sigma(1-p\alpha_{EE})}{\phi_\pi - p} - \frac{\kappa\gamma_g}{1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi-p)}}{\frac{\kappa\gamma_y}{1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi-p)} + \frac{\sigma(1-p\alpha_{EE})}{\phi_\pi-p}}\\ &= \frac{\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)] - \kappa\gamma_g(\phi_\pi - p)}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)]}\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi_S}{\partial g_S} &= \frac{1+\frac{\gamma_g}{\gamma_y}}{\frac{1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi-p)}{\kappa\gamma_y} + \frac{1}{\sigma(1-p\alpha_{EE})}(\phi_\pi - p)}\\ &= \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p\alpha_{EE})}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)]}.\end{aligned}$$

where $\alpha_{EE}$ and $\alpha_{PC}$ are increasing in the cognitive discounting parameter $\bar{m}$. One can differentiate output gap multiplier with regard to $\bar{m}$ and after some arrangements we have:

$$\frac{\kappa(\gamma_y + \gamma_g)(\phi_\pi - p)f_N'(\bar{m})}{\mathscr{D}_{BN}^2} > 0,$$

where $\mathscr{D}_{BN} = \kappa\gamma_y(\phi_\pi - p) + \sigma(1 - p\alpha_{EE})[1 - \beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)]$ and $f'_N(\bar{m})$ is the derivative of $\sigma(1 - p\alpha_{EE})[1 - \beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)]$ with regard to $\bar{m}$ which is positive.

One can differentiate inflation multiplier with regard to $\bar{m}$ and after some arrangements we have:

$$\frac{-p\alpha'_{EE}\mathscr{D}_{BN} - f'_N(\bar{m})(1 - p\alpha_{EE})}{\mathscr{D}^2_{BN}} < 0,$$

where $\mathscr{D}_{BN} = \kappa\gamma_y(\phi_\pi - p) + \sigma(1 - p\alpha_{EE})[1 - \beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)]$ and $f'_N(\bar{m})$ is the derivative of $\sigma(1 - p\alpha_{EE})[1 - \beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)]$ with regard to $\bar{m}$ which is positive.

Since the strength of the real cost channel $\gamma_r$ is independent of the new ingredient that is bounded rationality. See appendix C, the output gap multiplier is decreasing in the strength of the real cost channel $\gamma_r$ and the inflation multiplier is increasing in the strength of the real cost channel $\gamma_r$. One can use this to prove the main text.

# K Proof for Proposition 7

The policy multipliers at the ZLB are shown below:

$$\begin{aligned}\frac{\partial y_S}{\partial g_S} &= \frac{\frac{\sigma(1-p\alpha_{EE})}{-p} - \frac{\kappa\gamma_g}{1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)}}{\frac{\kappa\gamma_y}{1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)} + \frac{\sigma(1-p\alpha_{EE})}{-p}}\\ &= \frac{\sigma(1 - p\alpha_{EE})[1 - \beta p\alpha_{PC} - \kappa\gamma_r(-p)] - \kappa\gamma_g(-p)}{\kappa\gamma_y(-p) + \sigma(1 - p\alpha_{EE})[1 - \beta p\alpha_{PC} - \kappa\gamma_r(-p)]}\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi_S}{\partial g_S} &= \frac{1 + \frac{\gamma_g}{\gamma_y}}{\frac{1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)}{\kappa\gamma_y} + \frac{1}{\sigma(1-p\alpha_{EE})}(-p)}\\ &= \frac{\left[1 + \frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1 - p\alpha_{EE})}{\kappa\gamma_y(-p) + \sigma(1 - p\alpha_{EE})[1 - \beta p\alpha_{PC} - \kappa\gamma_r(-p)]}.\end{aligned}$$

One can differentiate output gap multiplier with regard to $\bar{m}$ and after some arrangements we have:

$$\frac{\kappa(\gamma_y + \gamma_g)(-p)f'_Z(\bar{m})}{\mathscr{D}^2_{BZ}} < 0,$$

where $\mathscr{D}_{BZ} = \kappa\gamma_y(-p) + \sigma(1 - p\alpha_{EE})[1 - \beta p\alpha_{PC} - \kappa\gamma_r(-p)]$ and $f'_Z(\bar{m})$ is the derivative of $\sigma(1 - p\alpha_{EE})[1 - \beta p\alpha_{PC} - \kappa\gamma_r(-p)]$ with regard to $\bar{m}$ which is positive.

One can differentiate inflation multiplier with regard to $\bar{m}$ and after some arrangements we have:

$$\frac{-p\alpha'_{EE}\mathscr{D}_{BZ} - f'_Z(\bar{m})(1-p\alpha_{EE})}{\mathscr{D}^2_{BN}} < 0,$$

where $\mathscr{D}_{BZ} = \kappa\gamma_y(-p) + \sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC} - \kappa\gamma_r(-p)]$ and $f'_Z(\bar{m})$ is the derivative of $\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC} - \kappa\gamma_r(-p)]$ with regard to $\bar{m}$ which is positive.

Since the strength of the real cost channel $\gamma_r$ is independent of the new ingredient that is bounded rationality. See appendix D, the output gap and inflation multipliers are decreasing in the strength of the real cost channel $\gamma_r$. One can use this to prove the main text.

# L Derivatives of Output Gap Multiplier w.r.t. $p$ with Bounded Rationality

One can differentiate the output multiplier w.r.t $p$ and the numerator is[27]:

$$\begin{aligned}
&[f'(p) + \kappa\gamma_g][\kappa\gamma_y(\phi_\pi - p) + f(p)] - [f(p) - \kappa\gamma_g(\phi_\pi - p)][-\kappa\gamma_y + f'(p)] \\
&= \kappa(\gamma_y + \gamma_g)[f'(p)(\phi_\pi - p) + f(p)],
\end{aligned}$$

where $f(p) = \sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)]$ and $f'(p) = \sigma(1-p\alpha_{EE})(-\beta\alpha_{PC} + \kappa\gamma_r) - \sigma\alpha_{EE}[1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)]$. One can show:

$$-f'(p)p = -\sigma(1-p\alpha_{EE})(-\beta p\alpha_{PC} + \kappa\gamma_r p) + \sigma\alpha_{EE}p[1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)].$$

In normal times:

$$\begin{aligned}
&f'(p)(\phi_\pi - p) + f(p) \\
&= f'(p)\phi_\pi + \sigma(1-\kappa\gamma_r\phi_\pi) - \sigma\alpha_{EE}p^2(\beta\alpha_{PC} - \kappa\gamma_r).
\end{aligned}$$

When $\bar{m} = 1$, one can show that these terms are below zero with ease. When $\bar{m} = 0$, one can have the expression below:

$$\sigma\kappa\gamma_r\phi_\pi + \sigma(1-\kappa\gamma_r\phi_\pi) > 0.$$

[27] For simplicity, I only compare the numerator of the derivatives to check the monotonicity.

In this case, it seems that there is a threshold value $\underline{\bar{m}}$ across which the multiplier effects switch from decreasing in $p$ to increasing in $p$ or vice versa. One can have this value $\underline{\bar{m}}$ by making the below equation zero:

$$
\begin{aligned}
& f'(p)\phi_\pi + \sigma(1-\kappa\gamma_r\phi_\pi) - \sigma\alpha_{EE}p^2(\beta\alpha_{PC}-\kappa\gamma_r) = 0 \\
\Longleftrightarrow & \{\sigma(1-p\alpha_{EE})(-\beta\alpha_{PC}+\kappa\gamma_r) - \sigma\alpha_{EE}[1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi - p)]\}\phi_\pi + \sigma(1-\kappa\gamma_r\phi_\pi) - \sigma\alpha_{EE}p^2(\beta\alpha_{PC}-\kappa\gamma_r) = 0.
\end{aligned}
$$

at the ZLB, one can show the derivatives can be reduced below:

$$
\begin{aligned}
& f'(p)(-p) + f(p) \\
& = \sigma - \sigma\alpha_{EE}p^2(\beta\alpha_{PC}-\kappa\gamma_r) > 0.
\end{aligned}
$$

In this case, the output gap multiplier is increasing in $p$.