Avoiding Expectations-driven Liquidity Traps: The Role of the Real Cost Channel*

He Nie†

October 14, 2022

Abstract

New survey evidence suggests that households do not expect deflation in an environment with a high persistence of realized deflation as in Gorodnichenko & Sergeyev (2021), and thus the possibility of expectations-driven liquidity traps is attenuated. To rationalize this fact, I develop a tractable New Keynesian model with the real cost channel: firms' marginal cost depends on the expected real interest rate. I identify this channel that reduces the occurrence of expectations-driven traps by altering the effective slope of the Phillips Curve at the zero lower bound episode. Additionally, this real cost channel can support maintaining model equilibrium.

Keywords: Real Cost channel, Liquidity Traps, New Keynesian Model, Sunspots, Condence-driven ZLB

JEL Codes: E12, E61

*I would like to thank my PhD advisor, Jordan Roulleau-Pasdeloup, for his extensive comments. I would also thank Chenyu Hou, Chang Liu, Oliver Zhen Li, Paul Gabriel Jackson, Denis Tkachenko, Tao Peng as well as participants in the NUS BAA workshop, the 6th PKU-NUS Annual International Conference on Quantitative Finance and Economics, the Asian Meeting of the Econometric Society in China 2022, the 28th International Conference Computing in Economics and Finance (CEF), and the JNU seminar for their comments and suggestions.

†Department of Economics, National University of Singapore. Contact: `heniexka@gmail.com`

1 Introduction

In the standard New Keynesian (NK) models with a lower bound, multiple equilibria can appear as in Benhabib et al. (2001) and Ascari & Mavroeidis (2022). To be more specific, there are generally two short-run equilibria in the standard model. The first equilibrium is when inflation and output gap are stabilized at the targeted steady state. The second is the expectations-driven equilibrium which means inflation and the output gap are both negative.

In theory, people could *expect* deflation for no fundamental reason, and the shift in households' confidence from optimism to pessimism can become a self-fulfilling prophecy (Mertens & Ravn (2014)) in the context of a standard NK model. As a result, sunspots can cause sufficient deflationary pressures to trigger the expectations-driven (or sunspot) liquidity trap without any fundamental shocks hitting the economy (see e.g. Mertens & Ravn (2014), Aruoba et al. (2018), Bilbiie (2019) and Cuba-Borda & Singh (2020)).

However, recently, a new fact about expectations has been documented using survey data: in the US, Europe, and Japan, households do not have deflation expectations even in an environment with a high persistence of realized deflation as explained at length in Gorodnichenko & Sergeyev (2021). This result is in stark contrast with the standard NK model prediction with rational expectations. In this situation, the possibility of expectation-driven (or sunspot) liquidity traps can be reduced because few households anticipate deflation even during recessions. Similarly, Mertens & Williams (2021) use the US options data on future interest rates and find no evidence in favor of the (sunspot) liquidity equilibrium.

In this paper, I develop a standard NK model with the real cost channel where the expected real interest rate appears in the marginal cost of the Phillips Curve. In addition, I contribute to the literature and find that the real cost channel can possibly get rid of the expectations-driven liquidity traps. In particular, this result can provide new theoretical insights for rationalizing the new empirical fact established using survey data.

As in Rabanal (2007) and Beaudry et al. (2022), it is assumed that firms need to borrow for production. As a result, the expected real interest rate can influence borrowing costs and further the marginal cost in the Phillips Curve. This is called the real cost channel.[1] The existence of the cost channel has empirical support in the literature (see e.g. Ravenna & Walsh (2006), Gilchrist & Zakrajšek (2015) and Abo-Zaid (2022)).

I study the possibility of expectations-driven liquidity traps in the canonical NK model with the real cost channel, where inflation and output gap are jointly determined and are affected by expectations of the future output gap and inflation. To be more specific, I develop an NK model based on Rabanal (2007) and Beaudry et al. (2022). I assume a fraction of representative firms must borrow to pay their wage bills (debts) while the rest can produce without paying bills. In that way, the expected interest rate can impact the real marginal cost and further the Phillips Curve, which is in line with Beaudry et al. (2022) and Nie (2022). I solve the model equilibrium analytically and graphically. To this end, I use a (stochastic) two-state Markov structure as in Eggertsson & Woodford (2003), and Eggertsson (2011). In addition, the model equilibrium can be depicted in a (π_S, y_S) diagram, where π_S and y_S denote inflation and the output gap in the short run, respectively.

Following Nie et al. (2022) and Roulleau-Pasdeloup (2022), I derive the *effective slopes* (*i.e.* slopes can feature expectations) of Euler/Phillips Curves in closed form. I further replicate results from Mertens & Ravn (2014), Wieland (2018) and Bilbiie (2021) that the effective slopes of Euler/Phillips Curves at the Zero Lower Bound (ZLB) episode are crucial: the second expectations-driven liquidity trap (sunspot) appears in the standard NK model when the effective

[1]The important difference between the model with the real cost channel and the standard model is that the marginal cost in the former one is a function of both output gap and the expected real interest rate, while the latter one is only a function of the output gap.

slope of the Phillips Curve at the ZLB episode is lower than its Euler counterpart. However, I find that the real cost channel can alter the effective slope of the Phillips Curve at the ZLB to make it higher than its Euler counterpart. This arises because the real cost channel at the ZLB can *counteract* the short-run deflation, implying actual short-run inflation in equilibrium. I then derive simple model restrictions, and show how these can rule out the expectations-driven traps equilibrium.

In the standard NK model, no model solution can appear as in Ascari & Mavroeidis (2022), if the effective slope of the Phillips curve is lower at the ZLB episode than its Euler counterpart. This arises since fundamental shocks can make the Euler curve too much below the Phillips curve. However, even if there exist powerful fundamental shocks, the model can be prone to equilibrium existence with the real cost channel.

Finally, my analytical model clearly displays a caveat to the role of the real cost channel: with a weak real cost channel, it can not rule out sunspots and even worsen the sunspot equilibrium; only a strong enough real cost channel can reduce the occurrence of sunspots.[2] Intuitively, a weak real cost channel can increase the real marginal cost through the counteracting channel while the lessened short-term deflation in equilibrium is insufficient. In this case, households have to save more and obtain the optimal expected return on savings due to expected inflation, which is in line with Nie & Roulleau-Pasdeloup (2022). In contrast, a strong enough real cost channel can make up the short-run deflation, and deflationary expectations can not be an equilibrium outcome.

This paper is closely related to a series of papers using the monetary/fiscal policy to get rid of expectations-driven liquidity traps (Sugo & Ueda (2008), and Nakata & Schmidt (2019)). For example, Schmidt (2016) shows that the fiscal spending policy can rule out the second expectation-driven equilibrium as

[2] In this paper, the weak (or strong) real cost channel means the elasticity of the real marginal cost w.r.t the real interest rate is small (or big enough).

in Schmitt-Grohe et al. (2001). More recently, Nie & Roulleau-Pasdeloup (2022) show that the Forward Guidance could rule out the sunspot ZLB if inflation make-up strategy is bold enough. However, these papers mainly rely on the monetary/fiscal policy specifications, but this paper focuses on the (endogenous) channel in the Phillips Curve.

Relatedly, Gabaix (2020) proves that the expectations-driven ZLB equilibrium can disappear in the NK model with bounded rationality. Similarly, Ono & Yamada (2018), Glover (2019), Michaillat & Saez (2019) and Diba & Loisel (2020) all find prescriptions to avoid the sunspot liquidity traps. To the best of my knowledge, no concurrent work shows that the cost channel can work as a solution to get the economy out of the occurrence of sunspot traps.

This paper also speaks to emerging papers using a standard NK model with the real cost channel. The seminal work of Beaudry et al. (2022) indicates that the real cost channel can match the US data, and they shed light on the relationship between the real cost channel and monetary policy. There are some other fiscal implications with the real cost channel. For example, Nie (2021) uses the NK model with the real cost channel and finds low government spending multipliers in liquidity traps. In this paper, instead of discussing the effects of policies and how they interact with the real cost channel, I document the role of this channel in avoiding expectations-driven liquidity traps.

The rest of this paper is organized as follows. Section 2 presents the model with the real cost channel. Section 3 assumes households' confidence is subject to a sunspot shock which obeys a standard two-stage Markov structure. I show that the sunspot equilibrium can appear in the standard model analytically and graphically. In section 4, I show that the real cost channel can reduce the occurrence of expectations-driven LT and further support maintaining model equilibrium. Finally, I conclude in Section 5.

2 The Model with Real Cost Channel

This section aims to explain the role of the real cost channel in normal times and liquidity traps using a three-equation model with the real cost channel. Normal times is the state when the economy is outside of liquidity traps, and the nominal interest rate is flexible to adjust by the central bank. In contrast, liquidity traps mean that there is a zero lower bound (ZLB) on nominal interest rates. Additionally, I show the short-run model equilibrium with a parsimonious two-stage Markov structure.

2.1 Three-equation Model

I use a standard three-equation New Keynesian Model (NK) model linearized around its (deterministic) targeted steady state, and this steady state is with zero inflation/output gap.[3] The modelling process of the Phillips Curve heavily builds on Rabanal (2007) and Beaudry et al. (2022). As in Rabanal (2007), I assume a fraction of representative firms must borrow wage bills for production while the rest of the firms can produce without paying bills. In this case, the expected real interest rate can impact the real marginal cost and further the Phillips Curve, which is in line with Beaudry et al. (2022) and Nie (2022). The specific model set-up can refer to Appendix A. In the following Definition 1, I show the semi-linear difference equation.

Definition 1. *The semi-linearized New Keynesian Phillips Curve (NKPC) with the real cost channel which represents the aggregate-supply (AS) side of the economy is presented below:*

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \left[\gamma_y y_t + \gamma_r (R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1}) \right]. \tag{1}$$

[3] I focus on the intended steady state with zero inflation in this section. The unintended steady state is a state with the ZLB binding as in Benhabib et al. (2001) and Nie & Roulleau-Pasdeloup (2022). Here I only show the linearized equilibrium condition, and all lower case format variables are the log deviations from the steady state *i.e.* $x_t = \log(X_t) - \log(X)$. Refer to Appendix A for model details.

where π_t is inflation, y_t is the output gap, $\beta < 1$ is the discount rate, κ is the elasticity of inflation with regard to marginal cost, R_t is the nominal interest rate in level. γ_y and γ_r are the elasticity of the real marginal cost with regard to the output gap and the expected real interest rate, respectively.

Eq. (1) is employed in this paper where the expected real interest rate emerges, as in Beaudry et al. (2022) and Nie (2021). The main difference between this model and the standard model is that this model has one additional part to highlight the role of the expected real interest rate on the short-run inflation. In particular, we find that γ_r can be seen as the strength of the real cost channel. In addition, this real cost channel features one additional expected dis-deflation feedback denoted by $-\mathbb{E}_t\pi_{t+1}$ in liquidity traps. This feedback can be seen as the *counteracting channel* as explained in Proposition 1 to mitigate the short-run deflation in equilibrium.

Proposition 1. *The* ***counteracting channel*** *in liquidity traps implies higher expected inflation and counteracts the short-run deflation in equilibrium.*

In liquidity traps, the deflationary pressures are observed in the economy. The dis-deflation feedback (the counteracting channel) in the real cost channel can feature higher expected inflation. As in Angeletos & Lian (2022), due to rational expectations and sticky prices, higher expected inflation can mitigate the short-run deflation in equilibrium at the ZLB.

In addition, this Phillips Curve with the real cost channel can nest the Phillips Curve in the standard NK model if we simply assume $\gamma_r = 0$:

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa\gamma_y y_t. \tag{2}$$

In the short run, we assume that the central bank obeys a standard Taylor (1993)-type rule with a lower bound in the following Definition 2. In this case, sufficient deflationary pressures can trigger a lower bound, and the central bank has to set the nominal interest rate to zero.

Definition 2. *Monetary policy is assumed to follow Taylor (1993)-type rules with a lower bound:*

$$R_t = \max\left[0; -\log(\beta) + \phi_\pi \pi_t\right], \tag{3}$$

where $\phi_\pi > 1$.

Note that the real cost channel can work as a cost-push shock endogenously in normal times if the Central Bank follows a simple Taylor rule as $R_t = \phi_\pi \pi_t - \log(\beta)$. This result in normal times is widely discussed in the literature as in Ravenna & Walsh (2006), Gilchrist & Zakrajšek (2015) and Abo-Zaid (2022).

The zero lower bound policy has plagued the US, Japan and the euro countries for decades. In this paper, I will focus on the ZLB episode. At the ZLB, the nominal interest rate is zero (*i.e.* $R_t = 0$). The real cost channel still works with the expected dis-deflation feedback in the Phillips Curve (the counteracting channel). Following Nie et al. (2022) and Roulleau-Pasdeloup (2022), I derive the effective slope in the NK model where the current inflation and output are jointly affected by expectations of future output and inflation. Therefore, the expected dis-deflation feedback in the real cost channel can alter the effective slope of the Phillips Curve at the ZLB.[4]

I model the aggregate demand side of the economy in a standard way. A representative household consumes, supplies labor elastically and saves in one-period government bonds. The private condition boils down to the Euler equation in Definition 3.[5]

Definition 3. *The following expression represents the equilibrium conditions of the semi-linearized Euler equation, which describes the aggregate demand (AD) side of the*

[4]In this paper, I mainly explore the effective slopes of AS/AD curves at the ZLB. Note that the specific setting of the Taylor (1993)-rule is not critical here since the nominal rate is fixed at zero in liquidity traps.

[5]Refer to Appendix A for model details.

economy:

$$y_t = \mathbb{E}_t y_{t+1} - \sigma_r \left[R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1} - \epsilon_t \right], \tag{4}$$

where σ_r is the elasticity of inter-temporal substitution, and ϵ_t is the demand shock.

To study the dynamics of the economy in normal times and liquidity traps, I assume the central bank can not perfectly track the nominal rate but with a lower bound constraint. As in Aruoba et al. (2018), the fundamental demand shock can impede the central bank from stabilizing the NK economy. To be more specific, if this fundamental shock is potentially large enough, the central bank can not track nominal rate with sufficient deflationary pressures, and the short-run economy can be stuck into liquidity traps. In that way, the nominal interest rate should be fixed at zero. However, if the demand shock is small, the central bank can stabilize the economy by using the standard Taylor (1993) rule. Specifically, the central bank sets a more than one-to-one decrease in nominal interest rate to fight deflationary pressures.

In addition, I assume there exists a sunspot shock in this paper. This shock is arbitrarily small and it is perfectly correlated with the fundamental demand shock. The persistent sunspot shock can shift peoples' confidence, as in Mertens & Ravn (2014) and Nie & Roulleau-Pasdeloup (2022), and cause sufficient deflationary pressures to trigger the expectations-driven (or sunspot) liquidity traps without any fundamental shocks hitting the economy.

2.2 Short-run Equilibrium: A Stochastic Method

This three-equation model above is simple enough for a clear analytical analysis. To this end, I use a parsimonious two-stage Markov structure with an absorbing state to solve the stochastic model analytically as in Eggertsson & Woodford (2003) and Eggertsson (2011). Specifically, the first state of the Markov chain features the short-run economy (where we label it with a subscript S), which can deviate from the steady state with shocks. After a few periods, the economy can be back to the steady state (where we label it with a subscript L), and it is also the second state of the Markov structure which is absorbing.[6]

With this in mind, the short-term economy is hit by the exogenous demand shock ϵ_S which persists with a probability p and recovers to the steady state ($\epsilon_L = 0$) with a probability $1-p$.[7] In addition, the sunspot shock is arbitrarily small and it is perfectly correlated with the demand shock with a persistence p. Since the Phillips Curve and the Euler equation in Eqs. (1) and (4) are both forward-looking, and one can write the expected output gap as

$$\begin{aligned} \mathbb{E}_S y_{t+1} &= p \cdot y_S + (1-p) y_L \\ &= p \cdot y_S, \end{aligned}$$

where the output gap $y_L = 0$ is the steady state, implying no deviations in the long run. Similarly, one can offer $\mathbb{E}_S \pi_{t+1} = p \cdot \pi_S$ with zero long-run inflation for expected inflation next period. In this case, I define the short-run equilibrium with the Markov chain representation below:

Definition 4. *The short-run equilibrium can be expressed as a vector* $[y_S,\ \pi_S,\ R_S]$ *such that, for a given* ϵ_S

$$\pi_S = \beta \mathbb{E}_S \pi_{t+1} + \kappa \left[\gamma_y y_S + \gamma_r (R_S + \log(\beta) - \mathbb{E}_S \pi_{t+1}) \right] \tag{5}$$

$$y_S = \mathbb{E}_S y_{t+1} - \sigma_r \left[R_S + \log(\beta) - \mathbb{E}_S \pi_{t+1} - \epsilon_S \right] \tag{6}$$

$$R_S = \max\left[0; -\log(\beta) + \phi_\pi \pi_S\right] \tag{7}$$

$$\mathbb{E}_S \pi_{t+1} = p \pi_S \tag{8}$$

[6] An absorbing state is a state that, once entered, cannot be left. And this state can be seen as the long-run steady state. See another specification in Armenter (2017) and Nakata & Schmidt (2019) by assuming no absorbing state in a two-state Markov structure.

[7] The transition matrix for the demand shock is:

$$\mathcal{P}_S = \begin{bmatrix} p & 1-p \\ 0 & 1 \end{bmatrix}.$$

The stochastic expected duration of the demand (or sunspot) shock is $\mathcal{T} = 1/(1-p)$.

$$\mathbb{E}_S y_{t+1} = p y_S \tag{9}$$

all hold.

Based on Definition 4, if the economy is in liquidity traps with $R_S = 0$ caused by (strong) negative fundamental shocks, it is in fundamental-driven liquidity traps as in Aruoba et al. (2018). On the flip side, as in Mertens & Ravn (2014), if the economy can feature a ZLB equilibrium ($R_S = \epsilon_S = 0$) with no fundamental reasons, it can be referred as sunspot-driven liquidity traps.

In addition, the short-run equilibrium in Definition 4 can be solved by hand. As in Nie et al. (2022) and Roulleau-Pasdeloup (2022), the short-run Euler/Phillips Curve can be shown in the following systems (Definition 5), which take into account expectations as in Mertens & Williams (2021):

Definition 5. *The short-run New Keynesian Phillips Curve and Euler equation are shown below:*

$$y_S = \begin{cases} \mathcal{S}^c_{PC}\pi_S & \textit{if} \quad \pi_S > \frac{\log(\beta)}{\phi_\pi} \\ \mathcal{S}^{c,z}_{PC}\pi_S + \mathcal{I}_{PC} & \textit{if} \quad \pi_S \leq \frac{\log(\beta)}{\phi_\pi} \end{cases} \tag{10}$$

$$y_S = \begin{cases} \mathcal{S}_{EE}\pi_S + \mathcal{I}_{EE} & \textit{if} \quad \pi_S > \frac{\log(\beta)}{\phi_\pi} \\ \mathcal{S}^{z}_{EE}\pi_S + \mathcal{I}_{EE} - \sigma_r\frac{\log(\beta)}{1-p} & \textit{if} \quad \pi_S \leq \frac{\log(\beta)}{\phi_\pi}, \end{cases} \tag{11}$$

where $\mathcal{S}$ labels the effective slope and $\mathcal{I}$ denotes the intercept. The superscript c and z denote "real cost channel" and "ZLB", respectively. The subscript PC and EE denote "Phillips Curve" and "Euler equation", respectively. The expressions of these effective slopes/intercepts are reported in Appendix D.

I show the Phillips Curve in Eq. (10) and the Euler equation in Eq. (11). The main difference between this model with the standard model is that Eq. (10) in the standard model will collapse to one single equation which is independent of the economic state (*i.e.* either the normal times or the ZLB). In particular, the effective slope can feature expectations of future output gap and inflation.

The effective slope is crucial in determining the type of liquidity traps in this paper, and I simply assume the effective slope of the Phillips Curve is upward sloping in a (π_S, y_S) graph as in Assumption 1, which means $p < \overline{p}^c$—see Appendix B for details. In other words, with the real cost channel, there is a threshold $\overline{p}^c$ such that the Phillips Curve can be upward/downward sloping. Laubach & Williams (2003), Daly & Hobijn (2014) and Nie et al. (2022) assume a similar condition.

Assumption 1. *Assume that the Phillips Curve with the real cost channel is upward sloping in a* (π_S, y_S) *graph such that*

$$p < \frac{1 - \kappa\gamma_r\phi_\pi}{\beta - \kappa\gamma_r} = \overline{p}^c.$$

I have sketched the NK model with the real cost channel and expressed the short-run equilibrium with a two-stage Markov structure. In the next section 3, I will replicate the sunspot equilibria in the standard NK model as in Mertens & Ravn (2014), Wieland (2018), and Bilbiie (2021).

3 Sunspot Equilibria in Standard NK Model

This section aims to show the equilibrium multiplicity property and equilibria solutions analytically and graphically in a textbook NK model without the real cost channel. As in Benhabib et al. (2001) and Nie & Roulleau-Pasdeloup (2022), the standard NK models are prone to equilibrium multiplicity if the central bank follows a Taylor rule with a lower bound constraint. Specifically, there are two short-run equilibria in the standard model. The first one is stabilized at the targeted steady state. The second one is the expectations-driven (or sunspot) liquidity equilibrium with negative inflation and the output gap.

3.1 Equilibrium Multiplicity

Before adding the real cost channel, I first show the two equilibria in the standard model. The modelling is in line with Nie & Roulleau-Pasdeloup (2022), and I assume there exists a sunspot shock.[8] This shock is arbitrarily small, and it remains in the short run with the persistence p. The expectations-driven traps mean that the economy can feature actual deflation and be in liquidity traps with an arbitrarily small sunspot shock in a high persistence of realized deflation environment (*i.e.* the sunspot shock persistence p is large enough)—see Nie & Roulleau-Pasdeloup (2022) for a discussion.

Following the way in Nie et al. (2022) and Roulleau-Pasdeloup (2022), I define the effective slopes in this paper, which can take into account expectations.[9] I first show the effective slopes of AS/AD curves in a (π_S, y_S) graph within the standard model explicitly.

Lemma 1. *In the standard NK model, the effective slope of AD/Euler curve in Eq.(6) at the ZLB is:*

$$\mathcal{S}^z_{EE} = \sigma_r \frac{p}{1-p}.$$

The effective slope of AS/NKPC curve in Eq.(2) at the ZLB is:

$$\mathcal{S}^z_{PC} = \frac{1-\beta p}{\kappa \gamma_y}.$$

Proof. See Appendix E. □

As in the seminal work of Bilbiie (2021), the equilibrium multiplicity can be detected by the probability p in a two-state Markov structure.[10] Based on Lemma 1, increasing p can generate a second crossing in the AS/AD curves at

[8] As also in Mertens & Ravn (2014), sunspots can be seen as exogenous shocks to households' confidence.

[9] In other words, it can represent features that inflation and output are jointly determined and affected by expectations of future output gap and inflation. See also Roulleau-Pasdeloup (2021).

[10] Similar arguments can be found in Mertens & Ravn (2014) and Aruoba et al. (2018).

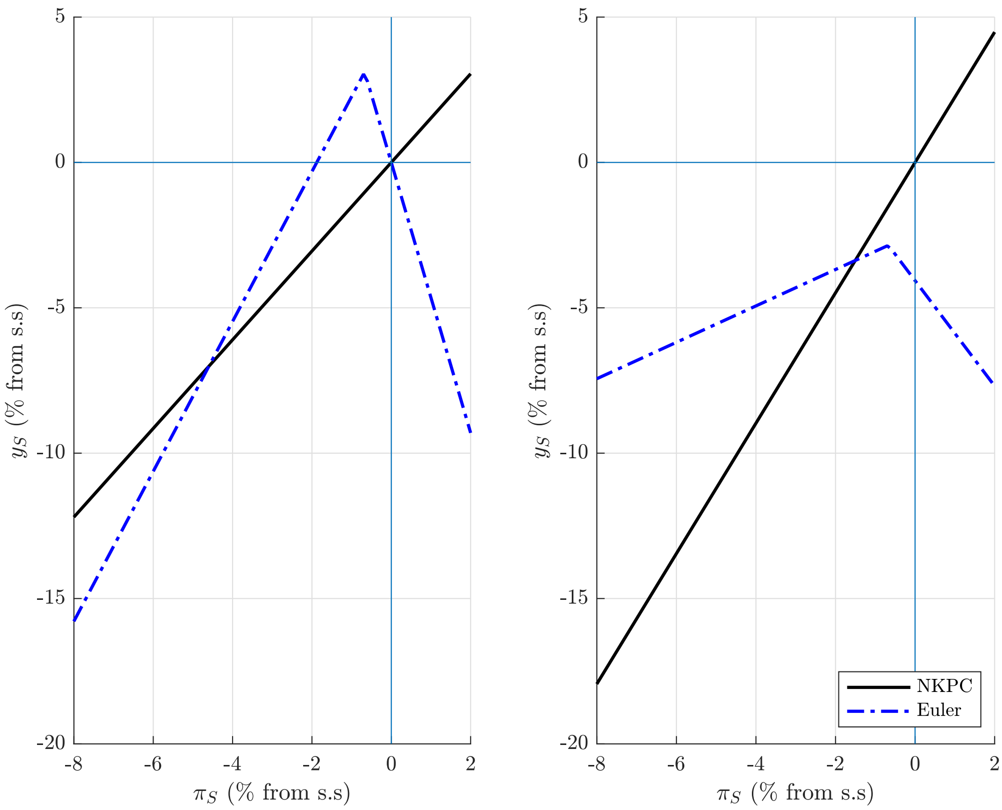
the ZLB episode by (i) increasing the Euler equation slope $\mathcal{S}^z_{EE}$ and (ii) reducing the NKPC slope $\mathcal{S}^z_{PC}$ simultaneously.[11] In this case, there exists a threshold $\overline{p}$ in Lemma 2 such that a second intersection emerges in a (π_S, y_S) graph (*i.e.* expectations-driven LT) in the standard NK model if $p > \overline{p}$.

Lemma 2. *One can use Lemma 1 to calculate the threshold $\overline{p}$ below:*

$$\overline{p} = \frac{(\beta + 1 + \sigma_r \kappa \gamma_y) - \sqrt{(1 + \beta + \sigma_r \kappa \gamma_y)^2 - 4\beta}}{2\beta} < 1.$$

Proof. See Appendix F. □

Figure 1: Expectations-driven LT (left) and fundamental-driven LT (right)



Notes: The black solid line in this figure is the AS curve (aka the New Keynesian Phillips Curve, NKPC) while the blue dashed line is the AD curve (aka the Euler equation). The left panel presents the expectations-driven LT in a standard NK model with $p = \bar{p} + 0.1$ and the right panel shows fundamental-driven LT in the standard model by assuming $p = \bar{p} - 0.1$ with the demand shock $\epsilon_S = -0.02$. Other calibration parameters are shown in Appendix C.

[11] In the standard NK model, we have a first crossing at the origin in the AS/AD curves.

To have a clear observation, I plot the expectations-driven (or sunspot) liquidity traps (LT) and fundamental-driven LT in the AS/AD diagram as in Figure 1, and it is of note that *the effective slopes of the AS/AD curves at the ZLB episode are crucial*. For the fundamental-driven LT case on the right panel, this effective slope of the AS curve at the ZLB is larger than that of the AD curve. The reverse holds for the expectations-driven liquidity traps on the left panel where the effective slope of the AS curve is less than the AD slope. Consequently, the Euler and the NKPC can cross twice, giving rise to the sunspot ZLB.

Characterization of Multiple Equilibria.—According to Lemma 2, the economy can be in expectations-driven liquidity traps with a high p. The intuition is that the expected highly persistent deflationary shock can shift people's confidence. In this case, people could expect deflation for no fundamental reason, and there could be a self-fulfilling prophecy which will result in expectations-driven liquidity traps. To better understand the difference between the fundamental-driven liquidity traps and sunspot traps. I replicate the closed-from solutions for the two liquidity traps in the standard NK model as in Mertens & Ravn (2014), Wieland (2018), and Bilbiie (2021) in Appendix H.

To conclude, there exists sunspot equilibrium in the standard model, and we show that the effective slopes are crucial in determining the liquidity traps, which is in line with Bilbiie (2021) and Nie & Roulleau-Pasdeloup (2022). As in the literature (see e.g. Sugo & Ueda (2008), Nakata & Schmidt (2019) and Schmidt (2016)), many policy prescriptions are proposed to get rid of the sunspot traps. In the following section 4, I will instead show the real cost channel can reduce the occurrence of expectations- driven LT.

4 Getting Rid of Expectations-driven Traps

In this section, I now show that it is possible to get rid of the expectation-driven liquidity traps with a strong real cost channel. To be more specific, the real cost channel in the NK model can rotate the NKPC while the effective slope of the Euler equation is unchanged. Additionally, I show this real cost channel is theoretically appealing since it helps ensure model equilibrium existence.

4.1 Higher effective slope of AS curve with real cost channel

As described at length in Section 3, the effective slopes of AS/AD curves in a (π_S, y_S) graph at the ZLB episode are critical. First, I show the effective slope of the AS curve at the ZLB with the real cost channel explicitly below.

Lemma 3. *Based on Definition 4, the effective slope of the AS/NKPC curve with the real cost channel in Eq.(5) at the ZLB is:*

$$\mathcal{S}_{PC}^{c,z} = \frac{1 - \beta p + \kappa \gamma_r p}{\kappa \gamma_y}.$$

Proof. See Appendix G. □

By comparing Lemma 1 and Lemma 3, the real cost channel can magnify the effective slope of the AS curve at the ZLB episode with term $\kappa\gamma_r p$. Thus, the effective slope of the AS curve at the ZLB episode is higher with the real cost channel as in Proposition 2. In addition, the effective slope of AS curve with this channel can be reduced to the standard one if $\gamma_r = 0$.

Proposition 2. *Relative to the standard NK model, the effective slope of the AS curve at the ZLB episode is higher with the real cost channel.*

Proof. See Appendix J. □

If the AS curve is rotated and the effective slope $\mathcal{S}_{PC}^{c,z}$ is higher than S_{EE}^{z} in the (π_S, y_S) graph at the ZLB episode, the second intersection can disappear, implying that the expectations-driven traps as in Bilbiie (2019) and Cuba-Borda & Singh (2020) is ruled out. In that way, the economy can be in the intended steady state without any fundamental shocks. We summarize this result in Lemma 4.

Lemma 4. *The necessary and sufficient condition to rule out expectations-driven LT is:*

$$\mathcal{S}_{PC}^{c,z} > \mathcal{S}_{EE}^{z}.$$

Proof. See Appendix K. □

It is shown that the real cost channel can increase the effective slope of AS curve at the ZLB episode, however, it can show no influence on the effective slope of the AD curve. In this case, a strong enough real cost channel can help to rule out the sunspot traps if the condition in Lemma 4 is satisfied.[12] We specify the restriction on the real cost channel as in the following Proposition 3.

Proposition 3. *At the ZLB, the effective slope of the AS curve increases in the strength of the real cost channel γ_r. The elasticity of real marginal cost w.r.t output γ_y follows the restriction below:*

$$\gamma_y < \Phi(\gamma_r),$$

where $\Phi(\gamma_r) = \frac{(\beta-\kappa\gamma_r-1+\kappa\gamma_r\phi_\pi)\gamma_r\phi_\pi(\beta-\kappa\gamma_r)}{\sigma_r(1-\kappa\gamma_r\phi_\pi)}$ increases in γ_r. Then the real cost channel can rule out the expectations-driven LT.

Proof. See Appendix L. □

From Lemma 3, the effective slope of AS curve at the ZLB increases in the

[12] In other words, the real cost channel can reduce the occurrence of expectations- driven LT with a big γ_r while a small γ_r can not work.

strength of the real cost channel γ_r[13] while it decreases in the elasticity of real marginal cost w.r.t output γ_y. Regrading the role of the real cost channel in the magnitude of the slope, there is a trade-off between the two elasticises.[14]

With this condition that $\gamma_y < \Phi(\gamma_r)$, the effective slope of the AS curve at the ZLB can be *always* larger than the AD slope in a (π_S, y_S) graph.[15] Interestingly, this theoretic restriction can echo empirical evidence in Beaudry et al. (2022). This seminal paper *empirically* estimates that γ_y in the real cost channel is robustly small (non-significantly). γ_r is significantly positive and is much larger than γ_y. In that way, the sunspot equilibrium is most likely to disappear with such parameters estimations. Moreover, this empirical finding in Beaudry et al. (2022) motivates the restriction in Proposition 3.

On the flip side, this threshold condition increases in γ_r. Therefore, with a higher γ_r, the economy is more likely not in expectations-driven traps. Furthermore, this condition $\gamma_y < \Phi(\gamma_r)$ requires that the strength γ_r should be big enough for a given γ_y, implying most (or all) firms should pay wage bills (debts) for production to magnify the role of real cost channel. As a consequence, no second intersection exists in the AS/AD curves and further the sunspot equilibrium is ruled out.

The potential rationale that the real cost channel can reduce the probability of expectations-driven LT is that the dis-deflation feedback at the ZLB in Eq. (5) can *counteract* deflation in the short run. This counteracting channel can imply short-run inflation in equilibrium due to rational expectations and sticky prices. In this case, for a given level of output gap y_S, the deflation behavior at the ZLB can move less due to the counteracting channel. This gives rise to a higher slope of the AS curve in a (π_S, y_S) graph. Finally, a steep enough AS curve can get rid of the second intersection. In that way, deflationary expectations can not be

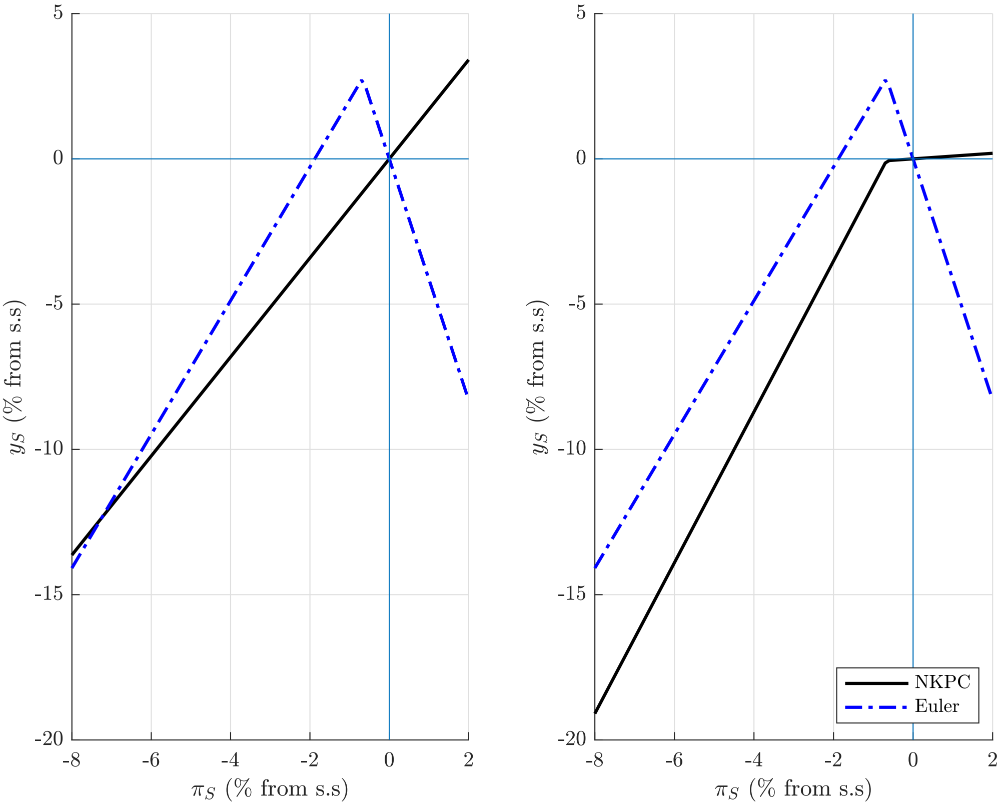
[13] γ_r represents the elasticity of marginal cost w.r.t the interest rate, and it can be seen as the strength of the real cost channel.

[14] In other words, the magnitude of γ_r relative to γ_y reflects the role of the real cost channel.

[15] Note that if the NKPC is upward sloping in a (π_S, y_S) graph, the second intersection can not arise. In addition, I assume $\Phi(\gamma_r) > 0$ in this paper.

an equilibrium outcome, and thus the probability of expectations-driven LT is reduced with this channel.

Figure 2: No expectations-driven LT with the real cost channel



Notes: The black solid line in this figure is the AS curve while the blue dashed line is the AD curve. The left panel presents the expectations-driven LT in a standard NK model without the real cost channel and the right panel shows no expectations-driven LT with the real cost channel, following the calibration method as in Appendix C.

I show the numerical experiment results in Figure 2.[16] On the left panel, in the standard model, when the sunspot shock is persistent enough, there are two equilibria, and the second intersection appears. With the same calibration method, there appears to be no sunspot equilibrium on the right panel of Figure 2: the absence of second intersection in the AS/AD curves due to the steeper AS curve at the ZLB episode.

This result can provide a theoretical justification for why the possibility of expectation-driven liquidity traps is low shown in empirical evidence. For example, there is a survey fact as in Gorodnichenko & Sergeyev (2021) that house-

[16]The calibration method can guarantee that $\overline{p}^c > \overline{p}$.

holds do not expect deflation in an environment with a high persistence of realized deflation. In this case, few households anticipate deflation even during recessions, and the possibility of expectation-driven liquidity traps can be attenuated. In addition, Mertens & Williams (2021) use US financial markets data to show no evidence in favor of the (sunspot) liquidity traps.

In this section, I show that the Phillips Curve can rule out the expectations-driven LT and ensure one unique equilibrium with $\pi_S = 0$. In particular, this Phillips Curve pattern is locally flat in a (y_S, π_S) graph, which can only be observed during the ZLB episode.[17] Interestingly, this locally flat Phillips Curve in this model is consistent with recent empirical evidence. For example, Hazell et al. (2022) use the US cross-sectional data and estimate a flat Phillips curve during the Great Recession.

4.2 Equilibrium Uniqueness/Existence

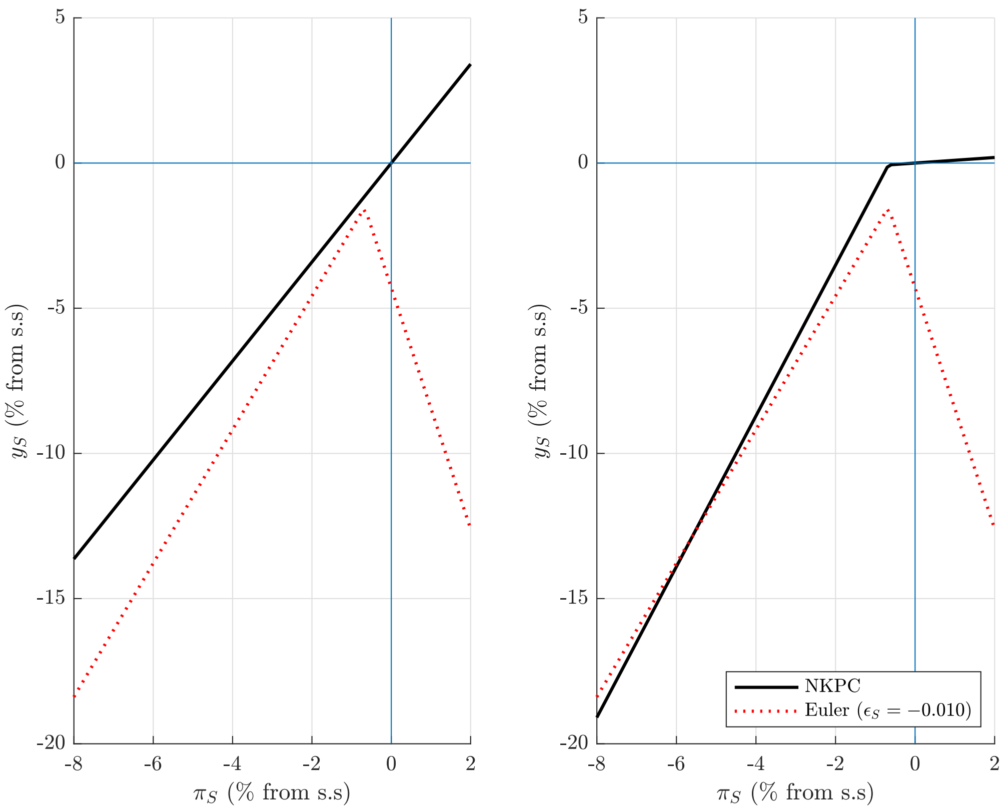
As in Benhabib et al. (2001) and Mertens & Ravn (2014), the NK models can be prone to equilibrium multiplicity. I have shown this occurs since there is a second intersection which can feature the sunspot equilibrium analytically and graphically. Moreover, as in Ascari & Mavroeidis (2022), models with ZLB constraints can have no solution: if there exist supply/demand shocks that make the AD curve shift too much below the AS curve, there can be no equilibrium in the expectations-driven LT case.

To have a clear observation, I plot this situation in Fig 3. It can be seen that, on the left panel, if the effective slope of the AS curve at the ZLB is lower than the AD slope, there can be no equilibrium with an additional strong enough demand shock, as in Ascari & Mavroeidis (2022).[18] This arises since the demand

[17] Note that in Figure 2, the AS/AD curves are shown in a (π_S, y_S) graph for an easier comparison while the textbook Phillips Curve is in a (y_S, π_S) graph. In other words, the Phillips Curve is flat in a (y_S, π_S) graph which means the AS curve is steep in a (π_S, y_S) graph as in Figure 2.

[18] There exists two equilibria with a small demand shock.

Figure 3: Equilibrium uniqueness/existence with demand shock



Notes: The black solid line in this figure is the AS curve while the red dotted line is the AD curve with a demand shock ($\epsilon_S = -0.010$). The left panel presents the no equilibrium in a standard NK model without the real cost channel and the right panel shows a equilibrium with the real cost channel, following the calibration method as in Appendix C.

shock ϵ_S can shift the AD curve too much below the AS curve.[19] However, no solution dilemma can not arise if the effective slope of the AS curve is higher at the ZLB episode.

On the right panel of Fig 3, it can be seen that the real cost channel can increase the effective slope of AS curve at the ZLB. In that way, even if there exist powerful fundamental shocks, there is always a unique intersection at the ZLB episode. Therefore, the real cost channel can help ensure that the AS/AD curves always have a unique equilibrium with fundamental shocks.[20] Overall, this real cost channel is theoretically appealing since it helps ensure model equilibrium existence with fundamental shocks.

[19]The kink of the AD curve is lower than the AS curve.

[20]See Appendix M for a numerical simulative example with supply shocks.

I conclude the main results in Proposition 4. The real cost channel, on the one hand, can help rule out the sunspot equilibrium with a big γ_r, and this can ensure equilibrium uniqueness. On the other hand, the model can be prone to model equilibrium existence with (strong) fundamental shocks.

Proposition 4. *The real cost channel can help ensure model equilibrium uniqueness/existence.*

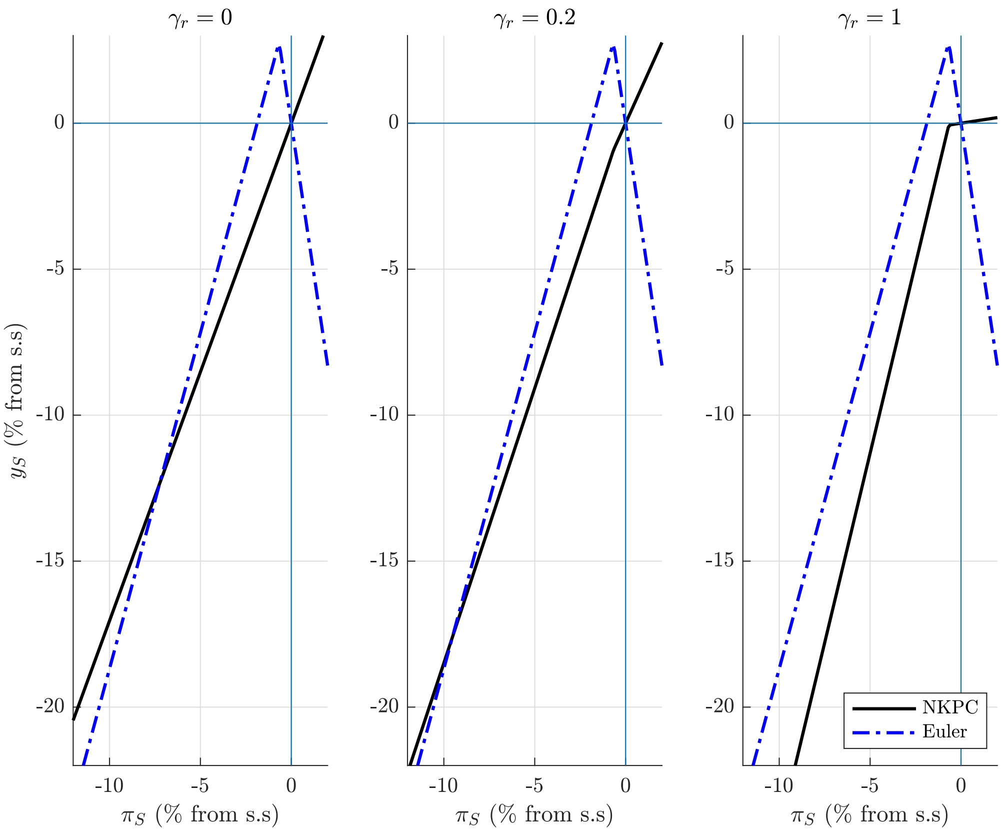
4.3 Strength of Real Cost Channel: A Caveat

As for the discussion outlined above, I have implicitly assumed that the real cost channel is strong enough to rule out the sunspot equilibrium. However, as in Proposition 3, $\Phi(\gamma_r)$ increases in the strength of the real cost channel γ_r, implying a small γ_r may not be able to rule out sunspots. In this case, we aim to illustrate the role of the strength of the real cost channel in this section.

For simplicity, I consider $\gamma_r = \{0, 0.2, 1\}$ in the numerical experiment and plot these results in Figure 4. Remember that $\gamma_r = 0$ means that the model will revert to the standard one. $\gamma_r = 1$ means that all firms in the economy need to pay wage bills for production. $\gamma_r = 0.2$ indicates that only a small portion of firms must pay wage bills. In this case, $\gamma_r = 0.2$ is a case where the role of the real cost channel is marginal. In contrast, $\gamma_r = 1$ shows a strong role of the real cost channel.

The direct takeaway from this Figure 4 is that the real cost channel has various features. On the first panel, it shows that we have two equilibria, and the second intersession can feature the ZLB state with $\pi_S^s < 0$. On the second panel, with a weak cost channel, even if the effective slope of the AS curve in liquidity traps is stepper now, it can not rule out sunspots and even worsen the sunspot equilibrium with $\pi_S^c < \pi_S^s$. On the third panel, this is the situation we have discussed above, and the strong real cost channel can rule out sunspots. Quantitatively, I find $\gamma_r > 0.76$ in the simulation such that the real cost channel can get rid of sunspot equilibrium.

Figure 4: AS/AD with the strength of the real cost channel



Notes: The black solid line in this figure is the AS curve while the blue dotted line is the AD curve. The first panel presents the equilibrium in a standard NK model without the real cost channel, the second panel shows the model with a weak real cost channel, and the third panel displays the model with a potent real cost channel, following the calibration method as in Appendix C.

There is a caveat to the real cost channel since a weak strength can even worsen the sunspot equilibrium. Intuitively, households tend to save instead of consuming in recessions. A weak real cost channel can increase the real marginal cost through the counteracting channel while the lessened short-term deflation in equilibrium is not enough. In this case, households have to save more to obtain the optimal expected return on savings due to expected inflation, which is in line with Nie & Roulleau-Pasdeloup (2022).[21] In contrast, a strong enough real cost channel can make up the short-run deflation fully. In that way, deflationary expectations can not be an equilibrium outcome, and thus the sunspot traps can be ruled out.

[21] As in Nie & Roulleau-Pasdeloup (2022), it explains at length that with not enough inflation make-up in sunspot equilibrium, households have to increase savings.

5 Conclusions

In the presence of the ZLB, the shift in confidence can cause sufficient deflationary pressures to trigger the expectations-driven traps without any fundamental shocks using the standard sticky price New Keynesian model. There is a recent survey fact that households do not expect deflation in an environment with a high persistence of realized deflation, according to European and Japanese surveys as in Gorodnichenko & Sergeyev (2021). This evidence is in stark contrast with the standard NK model with rational expectations, and the possibility of expectation-driven traps can be attenuated.

In this paper, I develop a tractable New Keynesian model via the real cost channel to provide theoretic explanations for rationalizing these new empirical facts. I have shown that the real cost channel *per se* can reduce the occurrence of the expectations-driven liquidity traps by rotating the Phillips Curve. This arises because the strong real cost channel at the lower bound episode can counteract the short-run deflation caused by the drop in confidence, implying actual inflation in equilibrium. In that way, deflationary expectations can not be an equilibrium outcome. Additionally, I show this real cost channel is theoretically appealing since it helps ensure model equilibrium existence.

References

Abo-Zaid, S. (2022). The government spending multiplier in a model with the cost channel. *Macroeconomic Dynamics*, 26(1), 72–101.

Angeletos, G.-M. & Lian, C. (2022). *Dampening General Equilibrium: Incomplete Information and Bounded Rationality*. Technical report, National Bureau of Economic Research.

Armenter, R. (2017). The Perils of Nominal Targets. *Review of Economic Studies*, 85(1), 50–86.

Aruoba, B. S., Cuba-Borda, P., & Schorfheide, F. (2018). Macroeconomic dynamics near the ZLB: A tale of two countries. *Review of Economic Studies*, 85(1), 87–118.

Ascari, G. & Mavroeidis, S. (2022). The unbearable lightness of equilibria in a low interest rate environment. *Journal of Monetary Economics*.

Beaudry, P., Hou, C., & Portier, F. (2022). *Monetary Policy When the Phillips Curve is Quite Flat*. CEPR Discussion Paper DP15184, Working Paper.

Benhabib, J., Schmitt-Grohe, S., & Uribe, M. (2001). The Perils of Taylor Rules. *Journal of Economic Theory*, 96(1-2), 40–69.

Bergholt, D., Furlanetto, F., & Vaccaro-Grange, E. (2020). The death and resurrection of the us price phillips curve. *Working paper*.

Bilbiie, F. O. (2019). Optimal forward guidance. *American Economic Journal: Macroeconomics*, 11(4), 310–45.

Bilbiie, F. O. (2021). Neo-Fisherian Policies and Liquidity Traps. *American Economic Journal: Macroeconomics, Forthcoming*.

Cuba-Borda, P. & Singh, S. R. (2020). Understanding persistent zlb: Theory and assessment. *Available at SSRN 3579765*.

Daly, M. C. & Hobijn, B. (2014). Downward nominal wage rigidities bend the phillips curve. *Journal of Money, Credit and Banking*, 46(S2), 51–93.

Diba, B. & Loisel, O. (2020). *Pegging the Interest Rate on Bank Reserves*. Working papers, Center for Research in Economics and Statistics.

Eggertsson, G. B. (2011). What fiscal policy is effective at zero interest rates? *NBER Macroeconomics Annual*, 25(1), 59–112.

Eggertsson, G. B. & Woodford, M. (2003). *Optimal Monetary Policy in a Liquidity Trap*. NBER Working Papers 9968, National Bureau of Economic Research, Inc.

Gabaix, X. (2020). A behavioral new keynesian model. *American Economic Review*, 110(8), 2271–2327.

Gilchrist, S. & Zakrajšek, E. (2015). Customer markets and financial frictions: Implications for inflation dynamics. In *Prepared for Inflation Dynamics and*

Monetary Policy, 2015 Jackson Hole Symposium, August, volume 11.

Glover, A. (2019). *Avoiding Liquidity Traps With Minimum Wages: Can Stability Justify Distortions?* Technical report, Mimeo, Kansas City Fed.

Gorodnichenko, Y. & Sergeyev, D. (2021). *Zero lower bound on inflation expectations*. Technical report, National Bureau of Economic Research.

Hazell, J., Herreno, J., Nakamura, E., & Steinsson, J. (2022). The slope of the phillips curve: evidence from us states. *The Quarterly Journal of Economics*, 137(3), 1299–1344.

Laubach, T. & Williams, J. C. (2003). Measuring the natural rate of interest. *Review of Economics and Statistics*, 85(4), 1063–1070.

Mertens, K. R. & Ravn, M. O. (2014). Fiscal policy in an expectations-driven liquidity trap. *Review of Economic Studies*, 81(4), 1637–1667.

Mertens, T. M. & Williams, J. C. (2021). What to expect from the lower bound on interest rates: Evidence from derivatives prices. *American Economic Review*, 111(8), 2473–2505.

Michaillat, P. & Saez, E. (2019). Resolving new keynesian anomalies with wealth in the utility function. *Review of Economics and Statistics*, (pp. 1–46).

Nakata, T. & Schmidt, S. (2019). *Expectations-Driven Liquidity Traps: Implications for Monetary and Fiscal Policy*. Technical report.

Nie, H. (2021). Government spending multipliers with the real cost channel. *Available at SSRN 3930472*.

Nie, H. (2022). The macroeconomic effects of tax shocks: The real cost channel. *Available at SSRN 3905695*.

Nie, H. & Roulleau-Pasdeloup, J. (2022). The promises (and perils) of control-contingent forward guidance. *Review of Economic Dynamics, Forthcoming*.

Nie, H., Roulleau-Pasdeloup, J., & Zheng, Z. (2022). Occasionally binding constraints with data-consistent expectations: a new analytical framework. *Working paper.*

Ono, Y. & Yamada, K. (2018). Difference or ratio: Implications of status preference on stagnation. *Australian Economic Papers*, 57(3), 346–362.

Rabanal, P. (2007). Does inflation increase after a monetary policy tightening? answers based on an estimated dsge model. *Journal of Economic Dynamics and control*, 31(3), 906–937.

Ravenna, F. & Walsh, C. E. (2006). Optimal monetary policy with the cost channel. *Journal of Monetary Economics*, 2(53), 199–216.

Roulleau-Pasdeloup, J. (2021). *The Public Investment Multiplier: Insights from a Tractable HANK Framework*. Working paper.

Roulleau-Pasdeloup, J. (2022). *Analyzing Linear Rational Expectations models: the Method of Undetermined Markov States*. Working paper.

Schmidt, S. (2016). Lack of confidence, the zero lower bound, and the virtue of fiscal rules. *Journal of Economic Dynamics and Control*, 70, 36–53.

Schmitt-Grohe, S., Benhabib, J., & Uribe, M. (2001). Monetary policy and multiple equilibria. *American Economic Review*, 91(1), 167–186.

Sugo, T. & Ueda, K. (2008). Eliminating a deflationary trap through superinertial interest rate rules. *Economics Letters*, 100(1), 119–122.

Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39(1), 195–214.

Wieland, J. (2018). *State-dependence of the zero lower bound government spending multiplier*. Working paper.

Online Appendix

A The Model Setup

Time is discrete and there is no government spending.

A.1 Aggregate Demand Side

The representative household has the below preferences:

$$\begin{aligned}\mathcal{U}(C_t, L_t) &= u(C_t) - v(L_t) \\ &= \frac{C_t^{1-\sigma}}{1-\sigma} - \chi\frac{L_t^{1+\eta}}{1+\eta}, \quad \chi, \eta > 0\end{aligned}$$

where households work L_t hours, consume amount C_t, and trade government bonds B_t.

The budget constraint is,

$$C_t + \frac{B_t}{P_t} = W_t L_t + \mathcal{D}_t - \mathcal{T}_t + \exp(\Im_{t-1})\frac{1+R_{t-1}}{P_t}B_{t-1}.$$

where $\Im_t$ is a "risk premium" shock.

The optimal aggregate (individual) labor price is written as:

$$W_t = \frac{L_t^{\eta}\chi}{(C_t)^{-\sigma}},$$

I can obtain the Euler equation with the first order condition (FOC) of the maximization program:

$$(C_t)^{-\sigma} = \beta\exp(\Im_t)\mathbb{E}_t\left\{(C_{t+1})^{-\sigma}\frac{1+R_t}{1+\Pi_{t+1}}\right\}.$$

The semi-linearized equilibrium Euler equation by approximating around the steady state can read. That is, all lower case format variables are the log deviations from steady state ($x_t = \log(X_t) - \log(X)$):

$$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\sigma}\left[R_t + \log(\beta) - \mathbb{E}_t \tau^c_{t+1} - \mathbb{E}_t \pi_{t+1} - \epsilon_t\right].$$

where $\epsilon_t \equiv -\Im_t$ is the natural rate shock (demand shock) and R_t is the nominal interest rate in level.

The following resource constraint is placed in this economy:

$$y_t = c_t,$$

Furthermore the Euler equation is expressed as:

$$y_t = \mathbb{E}_t y_{t+1} - \sigma_r \left[R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1} - \epsilon_t\right],$$

where $\sigma_r \equiv \frac{1}{\sigma}$.

A.2 Aggregate Supply Side

Each monopolist produces a differentiated good using a basic input as the only factor of production, and according to a one to one technology. The marginal cost of production will therefore be the price of that basic input. Basic input Q_t is produced by a representative competitive firm with labor L_t and the labor-like final good M_t with the production function as in Beaudry et al. (2022)) below:

$$Q_t = \min(aL_t, bM_t).$$

The unit price of final good that enters the production of basic input is P_t. I assume that a fraction $\gamma \in [0,1]$ of basic input representative firm must borrow debt at the nominal wage bill $(1+i_t)W_tP_t$ to pay for the input M_t (Rabanal (2007)).[22] The firm sells its production, pays wages, repays the debt contracted the previous period and distributes all the profits D_t as dividends. By using the symmetry condition, the budget constraint for the firm is shown below,

$$\tilde{P}_t Q_t = P_t W_t L_t + (1+i_{t-1})W_{t-1}P_{t-1}M_{t-1},$$

In this case, the basic input representative firm has profits below:

$$D_t = \tilde{P}_t Q_t - P_t W_t L_t - (1+i_{t-1})W_{t-1}P_{t-1}M_{t-1},$$

Firm maximizes the expected discounted sum of real profits D_t/P_t with discount factor β:

$$\tilde{P}_t = \left(\frac{1}{a}W_t + \frac{\beta}{b}\left[\frac{1+i_t}{1+\pi_{t+1}}W_t\right]\right)P_t.$$

The real marginal cost of the γ basic input firm:

$$MC_t = \frac{1}{a}W_t + W_t\frac{\beta}{b}\left[\frac{1+i_t}{1+\pi_{t+1}}\right].$$

The other $1-\gamma$ firms are standard with the real marginal cost below:

$$MC_t = W_t.$$

The optimal household labor supply:

$$\frac{v'(L_t)}{u'(C_t)} = W_t,$$

[22] The labor-like final good can be simply seen as working machines (e.g. robotic support) which is assumed to be rent at the nominal wage bill.

Other parts are standard, and the New Keynesian Phillips curve yields:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \bar{\kappa}\, mc_t.$$

By log condition, I have the semi-linearized equilibrium

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \left[\gamma_y y_t + \gamma_r (R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1}) \right],$$

where $\kappa = \frac{\beta}{b}\bar{\kappa}$, R_t is the nominal interest rate in level, $\gamma_y = \frac{b}{\beta}(\sigma + \eta)$ and $\gamma_r = \gamma$.

In this case, this model can collapse to the standard model if we assume $\gamma = 0$ below:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \gamma_r y_t.$$

B Upward Sloping Assumption

According to Definition 4, in normal times, I can reproduce the solutions for y_S and π_S as follows:

$$\begin{aligned} y_S &= \frac{\sigma_r (1 - \beta p + \kappa \gamma_r p - \kappa \gamma_r \phi_\pi)}{(1-p)(1 - \beta p + \kappa \gamma_r p - \kappa \gamma_r \phi_\pi) + \sigma_r \kappa \gamma_y (\phi_\pi - p)} \epsilon_S \\ \pi_S &= \frac{\sigma_r \kappa \gamma_y}{(1-p)(1 - \beta p + \kappa \gamma_r p - \kappa \gamma_r \phi_\pi) + \sigma_r \kappa \gamma_y (\phi_\pi - p)} \epsilon_S. \end{aligned}$$

If the Phillips Curve is upward sloping in normal times, which means the effective slope of Phillips Curve is positive:

$$\begin{aligned} & 1 - \beta p + \kappa \gamma_r p - \kappa \gamma_r \phi_\pi > 0 \\ \Leftrightarrow\ & p < \frac{1 - \kappa \gamma_r \phi_\pi}{\beta - \kappa \gamma_r}, \end{aligned}$$

where the second line using the assumption $\kappa \gamma_r < \beta$ as in Beaudry et al. (2022)

and Nie (2022).

In this case, there is a threshold $\overline{p}^c = \frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r}$.

C Calibration Parameters

Table 1: The calibrated parameter values

Discount factor	$\beta = 0.99$
Preference parameter	$\sigma_r = 2$
Elasticity of inflation w.r.t marginal cost	$\kappa = 0.4$
Elasticity of marginal cost w.r.t output	$\gamma_y = 0.6$
Elasticity of marginal cost w.r.t interest rate	$\gamma_r = 1$
Taylor rule	$\phi_\pi = 1.5$
Shock persistence	$p = \frac{\overline{p}^c + \overline{p}}{2}$

Notes: I follow Beaudry et al. (2022) to set the value for γ_r and γ_y. We can obtain qualitatively identical results with different sets of γ_r & γ_y and these results can be obtained be request. I follow Bergholt et al. (2020) and Nie & Roulleau-Pasdeloup (2022) to use a standard calibrated method for other parameters. $\bar{p}$ is the threshold such that there exists the expectations-driven LT in the standard model without the real cost channel. $\overline{p}^c$ is the threshold such that the AS curve is upward sloping in the model with the real cost channel.

D The expressions in Definition 5

The NKPC is shown below:

$$y_S = \begin{cases} \frac{1-\beta p+\kappa\gamma_r p-\kappa\gamma_r\phi_\pi}{\kappa\gamma_y}\pi_S & \text{if } \pi_S > \frac{\log(\beta)}{\phi_\pi} \\ \frac{1-\beta p+\kappa\gamma_r p}{\kappa\gamma_y}\pi_S - \frac{\gamma_r}{\gamma_y}\log(\beta) & \text{if } \pi_S \le \frac{\log(\beta)}{\phi_\pi}. \end{cases}$$

One can formally show the Euler equations below:

$$y_S = \begin{cases} -\sigma_r\frac{\phi_\pi-p}{1-p}\pi_S + \sigma_r\frac{\epsilon_S}{1-p} & \text{if } \pi_S > \frac{\log(\beta)}{\phi_\pi} \\ \frac{\sigma_r p}{1-p}\pi_S + \sigma_r\frac{\epsilon_S-\log(\beta)}{1-p} & \text{if } \pi_S \le \frac{\log(\beta)}{\phi_\pi}. \end{cases}$$

E Proofs of Lemma 1

The Euler equation in standard NK model:

$$y_t = \mathbb{E}_t y_{t+1} - \sigma[R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1} - \epsilon_t]$$

The NKPC is below:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \gamma_y y_t$$

Using the simple two-state Markov Chain, we have $\mathbb{E}_S \pi_{t+1} = p\pi_S$ and $\mathbb{E}_S y_{t+1} = p y_S$. We can write the Euler equation at the ZLB below:

$$y_S = -\frac{\sigma_r p}{1-p}\pi_S + \sigma_r \frac{\epsilon_S - \log(\beta)}{1-p}.$$

One can yield the NKPC:

$$y_S = \frac{1-\beta p}{\kappa \gamma_y}\pi_S.$$

Thus, the effective slope of AD/Euler curve is:

$$\mathcal{S}^z_{EE} = \sigma_r \frac{p}{1-p}.$$

And the effective slope of AS/NKPC curve is:

$$\mathcal{S}^z_{PC} = \frac{1-\beta p}{\kappa \gamma_y}.$$

F Proofs of Lemma 2

The standard textbook New Keynesian Phillips Curve without a cost channel can read:

$$\pi_t = \beta \mathbb{E}\pi_{t+1} + \kappa\gamma_y y_t.$$

In this case, the Phillips Curve can be re-written as

$$y_S = \frac{1-\beta p}{\kappa\gamma_y}\pi_s$$

If the Phillips Curve is upward sloping throughout time periods. If there is an absence of demand shock and the effective slope of AS curve is lower than AD curve, *i.e.*:

$$(1-p)(1-\beta p) < \sigma_r p\kappa\gamma_y.$$

We can have the sunspot equilibrium featuring $\pi_S < 0, y_S < 0$: *i.e.* there exists a threshold $\overline{p}$:

$$\begin{aligned}
\overline{p} &= \frac{(\beta + 1 + \sigma_r\kappa\gamma_y) - \sqrt{(1+\beta+\sigma_r\kappa\gamma_y)^2 - 4\beta}}{2\beta} \\
&< \frac{(\beta + 1 + \sigma_r\kappa\gamma_y) - (-\beta + 1 + \sigma_r\kappa\gamma_y)}{2\beta} \\
&= 1
\end{aligned}$$

where there is $\bar{p} \in (0,1)$ to trigger the expectations-driven LT to make $y_S < 0$ in the absence of demand shock. That being said, there is a sunspot equilibrium if $p > \bar{p}$. Note that if the the demand shock is very large, it can shift AD curve down so much that there is no intersection in the AS and AD curves which means no equilibrium in this economy.

G Proofs of Lemma 3

According to Definition 4, under a ZLB, the Phillips Curve is

$$y_S = \frac{1-\beta p + \kappa\gamma_r p}{\kappa\gamma_y}\pi_S - \frac{\gamma_r}{\gamma_y}\log(\beta),$$

The Euler equation at the ZLB is:

$$y_S = -\frac{\sigma_r p}{1-p}\pi_S + \sigma_r\frac{\epsilon_S - \log(\beta)}{1-p}.$$

Thus, the effective slope of AD/Euler curve is:

$$\mathcal{S}^z_{EE} = \sigma_r\frac{p}{1-p}.$$

And the effective slope of AS/NKPC curve is:

$$\mathcal{S}^{c,z}_{PC} = \frac{1-\beta p + \kappa\gamma_r p}{\kappa\gamma_y}.$$

H Characterization of Multiple Equilibria

Proposition 5. *In the standard NK model, the solution of the expectations-driven traps is given:*

$$y_S = \frac{(1-\beta p)\sigma_r}{(1-p)(1-\beta p) - \sigma_r p\kappa}(-\log(\beta))$$
$$\pi_S = \frac{\kappa\gamma_y}{(1-p)(1-\beta p) - \sigma_r p\kappa}(-\log(\beta)),$$

where $(1-p)(1-\beta p)-\sigma_r p\kappa < 0$ *(i.e.* $p > \overline{p}$*).*

The solution of the fundamental-driven traps is shown as:

$$\begin{aligned} y_S &= \frac{(1-\beta p)\sigma_r}{(1-p)(1-\beta p)-\sigma_r p\kappa}(\epsilon_S - \log(\beta)) \\ \pi_S &= \frac{\kappa\gamma_y}{(1-p)(1-\beta p)-\sigma_r p\kappa}(\epsilon_S - \log(\beta)), \end{aligned}$$

where $(1-p)(1-\beta p)-\sigma_r p\kappa > 0$ *(i.e.* $p < \overline{p}$*).*

Proof. See Appendix I. □

In line with Cuba-Borda & Singh (2020) and Nie (2022), I show the two traps in isomorphic expressions with the ZLB binding. It is straightforward to see that the denominator is the same in the two specifications. Here p is crucial, if the fundamental/sunspot shock is large enough (*i.e.* $p > \overline{p}$), the denominator is negative. In this case, the solutions of y_S and π_S are both negative without any fundamental shock hitting the economy (*i.e.* $\epsilon_S = 0$). On the other hand, the fundamental-driven traps are very similar but the shock persistence is small. In that way, the denominator of the solution is positive while the term $(\epsilon_S - \log(\beta))$ is negative with a strong (negative) fundamental shock $\epsilon_S < 0$. Therefore the economy is in liquidity traps with negative y_S and π_S.

The expectations-driven (or sunspot) trap is shown on the left panel of Figure 1 and a second intersection of the AS and AD curves occurs. It indicates that if the sunspot shock persistence is sufficiently large, the economy will feature actual deflation without any fundamental shocks hitting the economy.[23] It is of note that, similar to the results in Nie & Roulleau-Pasdeloup (2022), there are two short-run equilibria on the left panel of Figure 1. One is the targeted steady state which means $y_S = \pi_S = 0$. Another one is the expectations-driven ZLB, implying $y_S < 0$ and $\pi_S < 0$. These experimental results can echo our analytical

[23] In other words, if households do expect deflation for no reason, this can cause sufficient deflationary pressures to trigger the expectations-driven LT with a self-fulfilling state of low confidence.

results in Proposition 5. Therefore the second equilibrium with expectations-driven traps emerges, and there is no stable equilibrium echoing the findings in Aruoba et al. (2018).

On the right panel of Figure 1, there exist fundamental-driven traps where the strong demand shock $\epsilon_S < 0$ can cause sufficient deflation such that the ZLB binds, implying $y_S < 0$ and $\pi_S < 0$. At the same time, the effective slope of the AD curve at the ZLB is lower than its counterpart of AS curve. There is only one unique equilibrium which can feature the ZLB state. For example, the US has been caught in the fundamental-driven ZLB during the global financial crisis (GFC), as in Eggertsson (2011) and Aruoba et al. (2018).

I Proofs of Proposition 5

It is straightforward to use Appendix E and one can combine AS/AD curves to obtain the solution at the ZLB:

$$\begin{aligned} y_S &= \frac{(1-\beta p)\sigma_r}{(1-p)(1-\beta p) - \sigma_r p \kappa \gamma_y}(\epsilon_S - \log(\beta)) \\ \pi_S &= \frac{\kappa \gamma_y}{(1-p)(1-\beta p) - \sigma_r p \kappa \gamma_y}(\epsilon_S - \log(\beta)), \end{aligned}$$

where $p < \overline{p}$.

On the other hand, the sunspot equilibrium emerges without fundamental shocks ϵ_S if $p > \overline{p}$ and the solution can be derived with AS/AD curves:

$$\begin{aligned} y_S &= \frac{(1-\beta p)\sigma_r}{(1-p)(1-\beta p) - \sigma_r p \kappa \gamma_y}(-\log(\beta)) \\ \pi_S &= \frac{\kappa \gamma_y}{(1-p)(1-\beta p) - \sigma_r p \kappa \gamma_y}(-\log(\beta)), \end{aligned}$$

where $p > \overline{p}$.

J Proofs of Proposition 2

One can show that

$$\mathcal{S}_{PC}^{c,z} > \mathcal{S}_{PC}^{z}.$$

Thus, the effective slope of the AS curve at the ZLB episode is higher with the real cost channel.

K Proofs of Lemma 4

This is direct result from the standard model in Appendix F. If there is an absence of demand shock and the effective slope of AS curve is lower than AD curve at the ZLB, we can have sunspots. Otherwise if the the effective slope of the AS curve is higher than the AD curve at the ZLB, sunspots disappear. Thus, the necessary and sufficient condition to rule out expectations-driven traps is:

$$\mathcal{S}_{PC}^{c,z} > \mathcal{S}_{EE}^{z}.$$

L Proofs of Proposition 3

According to Definition 4, under a ZLB, the Phillips Curve is

$$y_S = \frac{1 - \beta p + \kappa \gamma_r p}{\kappa \gamma_y} \pi_S - \frac{\gamma_r}{\gamma_y},$$

where the effective slope is $\frac{1-\beta p+\kappa\gamma_r p}{\kappa\gamma_y}$. It is easy to check this slope is increasing in the elasticity of the marginal cost w.r.t the interest rate γ_r which can be seen as the strength of the real cost channel.

If the flat Phillips Curve is upward sloping throughout time periods, which means that the effective slope of Phillips Curve is always positive:

$$\begin{aligned}&1-\beta p+\kappa\gamma_r p-\kappa\gamma_r\phi_\pi>0\\&\Leftrightarrow p<\frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r}.\end{aligned}$$

In this case, in normal times, it is easy to check that the only equilibrium is the target steady state (*i.e.* $y_S=\pi_S=0$) with no demand shock.

While assuming that the demand shock is large enough to trigger the fundamental-driven ZLB, I reproduce the following solutions for y_S and π_S:

$$\begin{aligned}y_S&=\frac{(1-\beta p+\kappa\gamma_r p)\sigma_r(\epsilon_S-\log(\beta))+\kappa\gamma_r\sigma_r p\log(\beta)}{(1-p)(1-\beta p+\kappa\gamma_r p)-\sigma_r p\kappa\gamma_y}\\\pi_S&=\frac{\kappa\gamma_y\sigma_r(\epsilon_S-\log(\beta))}{(1-p)(1-\beta p+\kappa\gamma_r p)-\sigma_r p\kappa\gamma_y}+\frac{\kappa\gamma_y\kappa\gamma_r\sigma_r p\log(\beta)}{[(1-p)(1-\beta p+\kappa\gamma_r p)-\sigma_r p\kappa\gamma_y](1-\beta p+\kappa\gamma_r p)}\\&\quad+\frac{\kappa\gamma_r\log(\beta)}{1-\beta p+\kappa\gamma_r p}.\end{aligned}$$

If there is no expectations-driven liquidity traps (LT) in the absence of demand shock, the requirement is below:

$$\begin{aligned}&y_S=\frac{(1-\beta p)\sigma_r(-\log(\beta))}{(1-p)(1-\beta p+\kappa\gamma_r p)-\sigma_r p\kappa\gamma_y}>0\\&\Leftrightarrow\mathcal{D}(p)=(1-p)(1-\beta p+\kappa\gamma_r p)-\sigma_r p\kappa\gamma_y>0\end{aligned}$$

One can yield a condition for γ_y to secure $\mathcal{D}(p)>0$:

$$\begin{aligned}\mathcal{D}(p)&=(1-p)(1-\beta p+\kappa\gamma_r p)-\sigma_r p\kappa\gamma_y\\&>\left(1-\frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r}\right)\left(1-\beta\frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r}+\kappa\gamma_r\frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r}\right)-\sigma_r\frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r}\kappa\gamma_y\\&=(\beta-\kappa\gamma_r-1+\kappa\gamma_r\phi_\pi)[\beta\kappa\gamma_r\phi_\pi-\kappa\gamma_r+\kappa\gamma_r(1-\kappa\gamma_r\phi_\pi)]-\sigma_r(1-\kappa\gamma_r\phi_\pi)\kappa\gamma_y>0\\\gamma_y&<\frac{(\beta-\kappa\gamma_r-1+\kappa\gamma_r\phi_\pi)(\beta\gamma_r\phi_\pi-\kappa\gamma_r^2\phi_\pi)}{\sigma_r(1-\kappa\gamma_r\phi_\pi)}=\Phi(\gamma_r),\end{aligned}$$

where the second line we assume $p = \bar{p}^c$ due to monotonicity.

At the ZLB episode, one can compare the effective slope of the AS/AD curves:

$$\frac{1 - \beta p + \kappa \gamma_r p}{\kappa \gamma_y} > \sigma_r \frac{p}{1-p},$$

where we use the condition $\gamma_y < \frac{(\beta - \kappa\gamma_r - 1 + \kappa\gamma_r\phi_\pi)(\beta\gamma_r\phi_\pi - \kappa\gamma_r^2\phi_\pi)}{\sigma_r(1-\kappa\gamma_r\phi_\pi)}$. This means the effective slope of the AS curve is larger than the effective slope of the AD curve at the ZLB.

In addition, one can check the monotonicity of $\Phi(\gamma_r)$ w.r.t. γ_r:

$$\begin{aligned}\frac{\partial\Phi(\gamma_r)}{\partial\gamma_r} &\propto \frac{\partial \frac{\beta - \kappa\gamma_r}{1-\kappa\gamma_r\phi_\pi}}{\partial\gamma_r} \\ &\propto \kappa(\phi_\pi\beta - 1) > 0.\end{aligned}$$

Therefore $\Phi(\gamma_r)$ increases in γ_r.

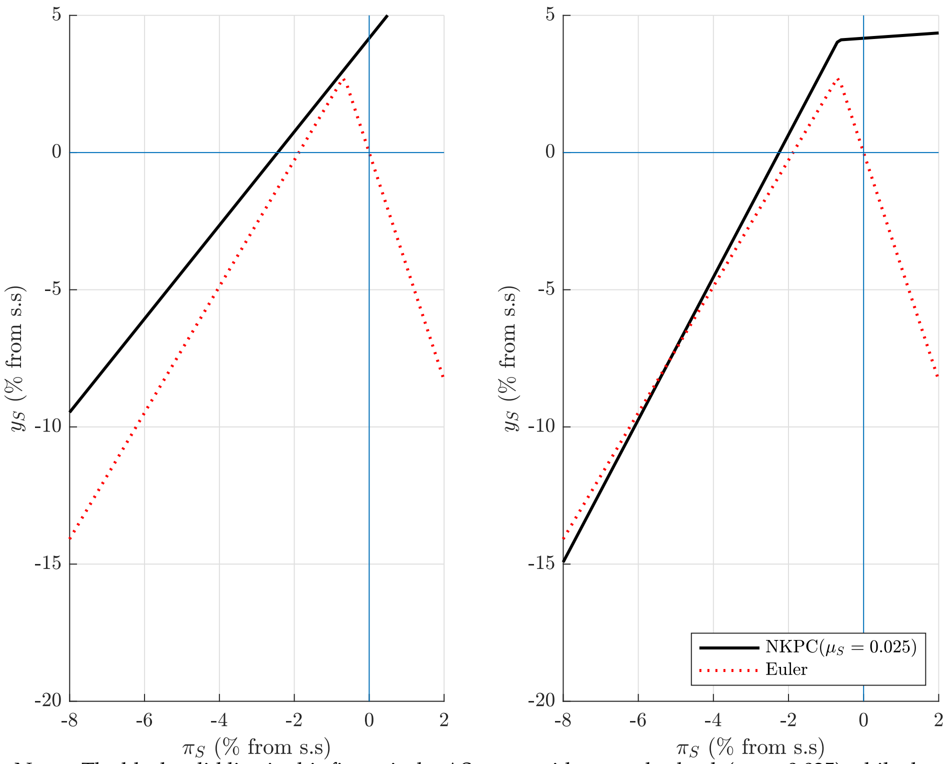
M Additional Figures for Supply Shocks

In this part, we simply assume there is a supply shock in the NKPC as below:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa[\gamma_y y_t + \gamma_r (R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1})] + \mu_t,$$

where μ_t is the temporary supply shock.

Figure 5: Equilibrium uniqueness/existence with the real cost channel (supply shock)



Notes: The black solid line in this figure is the AS curve with a supply shock ($\mu_S = 0.025$) while the red dotted line is the AD curve. The left panel presents the no equilibrium in a standard NK model without the real cost channel and the right panel shows a equilibrium with the real cost channel, following the calibration method as in Appendix C.