Avoiding Expectations-driven Liquidity Traps*

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Abstract

The shift in confidence can cause sufficient deflationary pressures to trigger expectations-driven liquidity traps (LT) without fundamental shocks in the context of a standard New Keynesian (NK) model. However, new survey evidence shows that households do not expect deflation in an environment with high persistent deflation as in Gorodnichenko & Sergeyev (2021), and thus the possibility of expectation-driven LT is attenuated. In this paper, I develop a tractable NK model with the real cost channel. I identify the real cost channel that reduces the occurrence of expectations-driven LT by altering the effective slope of the Phillips Curve at the zero lower bound episode. Furthermore, this channel can help ensure model equilibrium uniqueness/existence.

Keywords: Real Cost channel, Liquidity Traps, New Keynesian Model, Sunspots, Condence-driven ZLB

JEL Codes: E12, E61

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1 Introduction

In the standard New Keynesian (NK) models with a lower bound, multiple equilibria can appear as in Benhabib et al. (2001) and Ascari & Mavroeidis (2022). To be more specific, there are generally two short-run equilibria in the standard model. The first one is the intended steady state which means inflation and output gap are stabilized at the targeted steady state. The second one is the expectations-driven equilibrium which features a state where inflation and output gap are both negative.

In theory, people would *expect* deflation for no reason, and the shift in households' confidence can become a self-fulfilling prophecy (Mertens & Ravn (2014)). As a result, sunspots can cause sufficient deflationary pressures to trigger the expectations-driven (or sunspots) liquidity traps without any fundamental shocks hitting the economy (see e.g. Mertens & Ravn (2014), Aruoba et al. (2018), Bilbiie (2019) and Cuba-Borda & Singh (2020)). However, recently, a new fact about expectations has been documented using survey data: in the US, European, and Japan, households do not have deflation expectations even in an environment with persistent high deflation as explained at length in Gorodnichenko & Sergeyev (2021). In this case, even during recessions, few households anticipate deflation, and the possibility of expectation-driven liquidity traps can be attenuated. Similarly, Mertens & Williams (2021) use US financial markets data and find no evidence in favor of the (sunspot) liquidity equilibrium.

In this paper, the key contribution is that I develop a standard New Keynesian model with the real cost channel where the expected real interest rate appears in the marginal cost of the Phillips Curve. Moreover, I find that a bold real cost channel can get rid of the expectations-driven liquidity traps. In particular, this model result can provide new theoretical insights for rationalizing these new empirical (survey) facts.

As in Beaudry et al. (2022), it is assumed that firms need to borrow for production. As a result, the real interest rate will influence borrowing costs and further the marginal cost in the Phillips Curve. This is called the real cost channel.[1] In addition, the existence of the cost channel has empirical support in the literature (see e.g. Ravenna & Walsh (2006), Gilchrist & Zakrajšek (2015) and Abo-Zaid (2022)).

I study the expectations-driven liquidity traps in the canonical NK model, where inflation and output gap are jointly determined and are affected by expectations of future output gap and inflation. To be more specific, I develop a NK model based on Rabanal (2007) and Beaudry et al. (2022). I assume a fraction of representative firms must borrow to pay for their wage bill while the rest of firms can produce without paying bills. In that way, the real interest can impact the real marginal cost and further the Phillips Curve which is in line with Beaudry et al. (2022) and Nie (2022). This paper aims to display the model equilibrium analytically and graphically. To this end, I can solve the models in closed form by using a (stochastic) two-state Markov structure as in Eggertsson & Woodford (2003), and Eggertsson (2011). In addition, the model equilibrium can be depicted in a (π_S, y_S) diagram.

Following Nie et al. (2022) and Roulleau-Pasdeloup (2022), I show the effective *slopes* (*i.e.* slopes can feature expectations) of Euler/Phillips Curves in closed form. I further show the effective slopes of Euler/Phillips Curves at the Zero Lower Bound (ZLB) episode are crucial: the second sunspot equilibrium (*i.e.* the expectations-driven liquidity traps) appears in the standard NK model when the effective slope of the Phillips Curve at the ZLB episode is lower than its Euler counterpart. However, I find that the real cost channel can alter the effective slope of the Phillips Curve at the ZLB episode to make it higher than its Euler counterpart. This arises because the real cost channel at the ZLB

[1] The important difference between the model with the real cost channel and the standard model is that the marginal cost in the former one is a function of both output gap and the expected real interest rate, while the latter one is only a function of the output gap.

episode can *counteract* the short-run deflation, implying actual short-run inflation at equilibrium. I then derive simple model restrictions and it can rule out the expectations-driven traps equilibrium.

Our model clearly shows that the real cost channel used appears a potential "go big or go home" behavior: with a weak real cost channel, it can not rule out sunspots and even worsen the sunspot equilibrium; only a bold real cost channel can reduce the occurrence of sunspots.[2] Intuitively, a timid real cost channel can increase the real marginal cost through the counteracting channel while the increased short-term inflation at equilibrium is not enough. In this case, households have to save more working as precautionary savings and obtain the optimal expected return on savings in future due to the expected inflation, as in Nie & Roulleau-Pasdeloup (2022). In contrast, a bold real cost channel can make-up the short-run deflation and deflationary expectations can not be an equilibrium outcome.

This paper is closely related to a series of papers using the monetary/fiscal policy to get rid of expectations-driven liquidity traps (Sugo & Ueda (2008), and Nakata & Schmidt (2019)). For example, Schmidt (2016) shows that the fiscal spending policy can rule out the second expectation-driven equilibrium as in Schmitt-Grohe et al. (2001). More recently, Nie & Roulleau-Pasdeloup (2022) show that the Forward Guidance can get rid of the sunspot ZLB if the inflation make-up strategy is bold enough. However, these papers mainly rely on the monetary/fiscal policy specifications but this paper focuses on the (endogenous) channel in the Phillips Curve.

In addition, Gabaix (2020) proves that the expectations-driven ZLB state can disappear in the NK model with bounded rationality. Similarly, Ono & Yamada (2018), Glover (2019), Michaillat & Saez (2019) and Diba & Loisel (2020) all find prescriptions to avoid the expectations-driven liquidity traps. To the best of

[2] In this paper, the weak (or bold) real cost channel means the elasticity of the real marginal cost w.r.t the real interest rate is small (or big).

my knowledge, no concurrent work shows that the cost channel can work as a solution to get the economy out of the occurrence of sunspot traps.

This paper is also closed related to emerging papers using the standard NK model with the real cost channel. The seminal work in Beaudry et al. (2022) indicates that the real cost channel can better match the US data and they shed light on the relationship between real cost channel and monetary policy. There are some other fiscal implications with the real cost channel. For example, Nie (2021) uses the NK model with the real cost channel to provide low government spending multipliers in liquidity traps. In this paper, instead of policy discussions with the real cost channel, I document the role of this channel, on avoiding expectations-driven liquidity traps.

The rest of this paper is organized as follows. Section 2 presents the model with the real cost channel. I further take into account a stochastic set-up and show the model equilibrium. Section 3 assumes households' confidence is captured by a sunspot shock which obeys a parsimonious two-stage Markov structure. I show the sunspot equilibria can appear in the standard model analytically and graphically. In section 4, I show that the real cost channel can reduce the occurrence of expectations-driven LT. Finally, I conclude in Section 5.

2 The Model with Real Cost Channel

This section aims to explain the role of the real cost channel in normal times and liquidity traps. Normal times is the state where the economy is outside of liquidity traps and the nominal interest rate is flexible to adjust by the central bank. In contrast, liquidity traps means that the nominal interest rate is stuck in the zero lower bound (ZLB).

I use a standard NK model linearized around its (deterministic) targeted steady state and this state is with zero inflation/output gap.[3] The modelling process of Phillips Curve (PC) heavily builds on Rabanal (2007) and Beaudry et al. (2022). As in Rabanal (2007), I assume a fraction of representative firm must borrow the wage bill for production while the rest of firms can produce without paying bills. In this case, the real interest can impact the real marginal cost and further the NKPC which is in line with Beaudry et al. (2022). The specific model set-up can refer to Appendix A. In the following Definition 1, I show the semi-linear difference equation.

Definition 1. *The semi-linearized Phillips Curve with the real cost channel which represents the aggregate-supply (AS) side of the economy is presented below:*

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \left[\gamma_y y_t + \gamma_r (R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1}) \right]. \tag{1}$$

where π_t is inflation, y_t is the output gap, $\beta < 1$ is the discount rate, κ is the elasticity of inflation with regard to marginal cost, R_t is the nominal interest rate in level. γ_y and γ_r are the elasticity of marginal cost with regard to the output gap and the interest rate, respectively.[4]

Eq. (1) is employed in this paper where the expected real interest rate emerges as in Beaudry et al. (2022) and Nie (2021). The main difference between this model and the standard model is that this model has one additional part to highlight the role of the expected real interest rate on the short-run inflation. In particular, we find that γ_r can be seen the strength of the real cost channel. In addition, this real cost channel features one additional expected dis-deflation feedback denoted by $-\mathbb{E}_t \pi_{t+1}$ in liquidity traps. This feedback can be seen

[3]The unintended steady state is a state with the ZLB binding as in Benhabib et al. (2001) and Nie & Roulleau-Pasdeloup (2022). In this section, I only show the linearized equilibrium condition and all lower case format variables are the log deviations from the steady state *i.e.* $x_t = \log(X_t) - \log(X)$. Refer to Appendix A for model details.

[4]This Phillips Curve with the real cost channel can collapse to the standard one below if $\gamma_r = 0$:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \gamma_y y_t. \tag{2}$$

as the counteracting channel to reduce the short-run deflation in recessions at equilibrium.

In the short run, we assume that the central bank obeys a standard Taylor (1993)-type rule with a lower bound in the following Definition 2. In this case, a sufficient deflationary pressure can trigger a lower bound and the central bank has to set the nominal interest rate to zero.

Definition 2. *Monetary policy is assumed to follow Taylor (1993)-type rules with a lower bound:*[5]

$$R_t = \max\left[0; -\log(\beta) + \phi_\pi \pi_t\right], \tag{3}$$

where $\phi_\pi > 1$.

Note that the real cost channel can work as a cost-push shock endogenously in normal times if the Central Bank follows a simple Taylor rule as $R_t = \phi_\pi \pi_t - \log(\beta)$. While at the zero lower bound (ZLB), the nominal interest rate is zero (*i.e.* $R_t = 0$). The real cost channel still works with the expected dis-deflation feedback in the Phillips Curve. Following Nie et al. (2022) and Roulleau-Pasdeloup (2022), I use the effective slope in the NK model where the current inflation and output are jointly affected by expectations of future output and inflation. Therefore, the expected dis-deflation feedback in the real cost channel can alter the effective slope of the Phillips Curve at the ZLB.

I model the aggregate demand-side of the economy in a standard way. A representative household consumes, supplies labor elastically and saves in one period government bonds. The private condition boils down to the Euler equation in the Definition 3.

Definition 3. *The following expression represents the equilibrium conditions of the linearized Euler equation, which describes the aggregate demand (AD) side of the econ-*

[5] In this paper, I mainly explore the effective slopes of AS/AD curves at the ZLB. The interest rate rule is not critical here since the nominal rate is fixed at zero in liquidity traps.

omy:[6]

$$y_t = \mathbb{E}_t y_{t+1} - \sigma_r \left[R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1} - \epsilon_t \right], \tag{4}$$

where σ_r is the elasticity of inter-temporal substitution, and ϵ_t is the demand shock.

To study the dynamics of the economy in normal times and liquidity traps, I assume the central bank can not perfectly track the natural rate of interest rate but with a lower bound constraint. As in Aruoba et al. (2018), the fundamental demand shock can impede the central bank to stabilize the economy. To be more specific, if this fundamental shock is potentially large enough, the central bank can not track the natural rate with sufficient deflationary pressures and the short-run economy can be stuck into liquidity traps. In that way, the nominal interest rate should be fixed at zero. However, if the demand shock is small, the central bank can stabilize the economy to the normal state using the Taylor (1993) rule and the central bank set a more than one-to-one decrease in nominal interest rate to fight with the deflationary pressure.

In addition, I assume there exists a sunspot shock in this paper. This shock is arbitrarily small and it is perfectly correlated with the demand shock. The persistent sunspot shock can shift peoples' confidence as in Mertens & Ravn (2014) and cause sufficient deflationary pressures to trigger the expectations-driven (or sunspot) liquidity traps without any fundamental shocks.

2.1 Short-run Equilibrium: A Stochastic Method

This three-equation model above is simple enough for us to have a clear analytical analysis. To this end, I use a parsimonious two-stage Markov structure with an absorbing state to solve the stochastic model in closed form as in Eggertsson & Woodford (2003) and Eggertsson (2011). As in Eggertsson &

[6]Refer to Appendix A for model details.

Woodford (2003), the first state of the Markov chain features the short-run economy (where we label it with a subscript S) which can deviate from the steady state with shocks. After a few periods, the economy can be back to steady state (where we label it with a subscript L) and it is also the second state of the Markov structure which is absorbing.[7]

With this in mind, the short-term economy is hit by the exogenous demand shock ϵ_S which persists with a probability p and recovers to the steady state ($\epsilon_L = 0$) with a probability $1 - p$.[8] In addition, the sunspot shock is arbitrarily small and it is perfectly correlated with the demand shock with a persistence p. Since the Phillips Curve and the Euler equation in Eqs. (1) and (4) are both forward looking, one can write the expected output gap as

$$\begin{aligned}\mathbb{E}_S y_{t+1} &= p \cdot y_S + (1-p) y_L \\ &= p \cdot y_S,\end{aligned}$$

where the output gap $y_L = 0$ is the steady state, implying no deviations in the long run. Similarly, one can offer $\mathbb{E}_S \pi_{t+1} = p \cdot \pi_S$ with zero long-run inflation for expected inflation next period. In this case, I define the short run equilibrium with the Markov chain representation below:

Definition 4. *A short run equilibrium in this economy is a vector* $[y_S,\ \pi_S,\ R_S]$ *such that, for a given* ϵ_S

$$\pi_S = \beta \mathbb{E}_S \pi_{t+1} + \kappa \left[\gamma_y y_S + \gamma_r (R_S + \log(\beta) - \mathbb{E}_S \pi_{t+1})\right] \tag{5}$$

$$y_S = \mathbb{E}_S y_{t+1} - \sigma_r \left[R_S + \log(\beta) - \mathbb{E}_S \pi_{t+1} - \epsilon_S\right] \tag{6}$$

$$R_S = \max\left[0; -\log(\beta) + \phi_\pi \pi_S\right] \tag{7}$$

[7] An absorbing state is a state that, once entered, cannot be left. And this state can be seen as the long-run steady state. See another specification in Armenter (2017) and Nakata & Schmidt (2019) by assuming no absorbing state in a two-state Markov structure.

[8] The transition matrix for the demand shock is:

$$\mathcal{P}_S = \begin{bmatrix} p & 1-p \\ 0 & 1 \end{bmatrix}.$$

The stochastic expected duration of the demand (or sunspot) shock is $\mathcal{T} = 1/(1-p)$.

$$\mathbb{E}_S \pi_{t+1} = p\pi_S \tag{8}$$

$$\mathbb{E}_S y_{t+1} = p y_S \tag{9}$$

all hold.

Based on Definition 4, if the economy is in liquidity traps with $R_S = 0$ caused by (strong) negative demand shocks, it is in fundamental-driven liquidity traps as in Aruoba et al. (2018). On the flip side, as in Mertens & Ravn (2014), if the economy can feature a ZLB equilibrium with $R_S = \epsilon_S = 0$, it can be referred as the sunspot-driven liquidity traps.

In addition, the short run equilibrium in Definition 4 can be solved by hand. As in Nie et al. (2022) and Roulleau-Pasdeloup (2022), the short-run Euler/Phillips Curve can be shown in the following systems (Definition 5) which take into account expectations as in Mertens & Williams (2021):

Definition 5. *The short-run Euler equation and Phillips Curve are shown below:*

$$y_S = \begin{cases} \mathcal{S}^c_{PC}\pi_S & \text{if} \quad \pi_S > \frac{\log(\beta)}{\phi_\pi} \\ \mathcal{S}^{c,z}_{PC}\pi_S + \mathcal{I}_{PC} & \text{if} \quad \pi_S \leq \frac{\log(\beta)}{\phi_\pi} \end{cases} \tag{10}$$

$$y_S = \begin{cases} \mathcal{S}_{EE}\pi_S + \mathcal{I}_{EE} & \text{if} \quad \pi_S > \frac{\log(\beta)}{\phi_\pi} \\ \mathcal{S}^{z}_{EE}\pi_S + \mathcal{I}_{EE} - \sigma_r\frac{\log(\beta)}{1-p} & \text{if} \quad \pi_S \leq \frac{\log(\beta)}{\phi_\pi}, \end{cases} \tag{11}$$

where $\mathcal{S}$ labels the effective slope and $\mathcal{I}$ denotes the intercept. The expressions of these effective slopes/intercepts are reported in Appendix D.

I show the Phillips Curve in Eq. (10) and the Euler equation in Eq. (11). The main difference between this model with the standard model is that Eq. (10) in the standard model will collapse to one single equation which is independent of the economic state (*i.e.* either the normal times or the ZLB). In particular, the effective slope can feature inflation and output gap are jointly determined and affected by expectations of future output gap and inflation.

The effective slope is crucial in determining the liquidity traps in this paper and I simply assume the effective slope of the Phillips Curve is upward sloping in a (π_S, y_S) graph as in Assumption 1, which means $p < \overline{p}^c$—see Appendix B for details. In other words, with the real cost channel, there is a threshold $\overline{p}^c$ such that the Phillips Curve can be upward/downward sloping. Laubach & Williams (2003), Daly & Hobijn (2014) and Nie et al. (2022) assume a similar condition.

Assumption 1. *Assume that the Phillips Curve with the real cost channel is upward sloping in a (π_S, y_S) graph such that*

$$p < \frac{1 - \kappa \gamma_r \phi_\pi}{\beta - \kappa \gamma_r} = \overline{p}^c.$$

3 Sunspot Equilibria in Standard NK Model

As in Benhabib et al. (2001) and Nie & Roulleau-Pasdeloup (2022), the standard New Keynesian (NK) models are prone to appear equilibrium multiplicity if the central bank follows a Taylor rule with a lower bound constraint. To be more specific, there are general two short-run equilibria in the standard model. The first one is the intended steady state which means inflation and output gap are stabilized at the targeted steady state. The second one is the expectations-driven (sunspot) equilibrium which features a state where inflation and output gap are both negative.

Before adding the real cost channel, I first show the two equilibria in the standard model. The modelling is in line with Nie & Roulleau-Pasdeloup (2022) and I assume there exists a sunspot shock.[9] This shock is arbitrarily small and it remains in the short run with the persistence p. The expectations-driven traps means that the economy can feature actual deflation and be in liquid-

[9] As also in Mertens & Ravn (2014), sunspots can be seen as exogenous shocks to households' confidence.

ity traps with a arbitrarily small sunspot shock in a high persistent deflation environment (*i.e.* the sunspot shock persistence p is large enough)—see Nie & Roulleau-Pasdeloup (2022) for a discussion.

To have a clear observation, I plot the expectations-driven (or sunspot) liquidity traps (LT) and fundamental-driven LT in the AS/AD diagram as in Figure 1 and one can see that *the effective slopes of the AS/AD curves at the ZLB episode are crucial*. For the fundamental-driven LT case on the right panel, the effective slope of the AS curve at the ZLB is larger than that of the AD curve. The reverse holds for the expectations-driven liquidity traps on the left panel where the effective slope of the AS curve is less than the AD slope. As a consequence, the Euler and the NKPC can cross twice which gives rise to the sunspot ZLB.

Follow the way in Nie et al. (2022) and Roulleau-Pasdeloup (2022), I define the effective slopes in this paper which can take into account expectations.[10] I first show the effective slopes of AS/AD curves in a (π_S, y_S) graph within the standard model explicitly.

Lemma 1. *In the standard NK model, the effective slope of AD/Euler curve in Eq.(6) at the ZLB is:*

$$\mathcal{S}^{z}_{EE} = \sigma_r \frac{p}{1-p}.$$

The effective slope of AS/NKPC curve in Eq.(2) at the ZLB is:

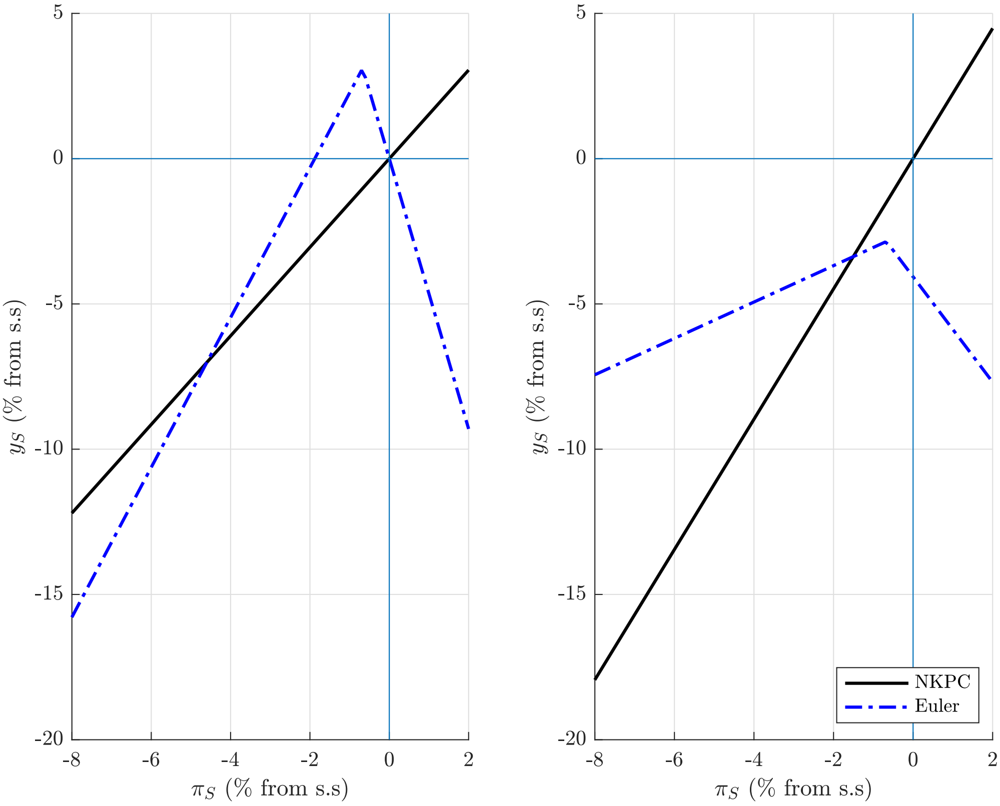
$$\mathcal{S}^{z}_{PC} = \frac{1-\beta p}{\kappa \gamma_y}.$$

Proof. See Appendix E. □

As in the seminal work of Bilbiie (2021), the expectation-driven LT can be detected by the probability p in a two-state Markov structure.[11] Based on Lemma

[10]In other words, it can represent features that inflation and output are jointly determined and affected by expectations of future output gap and inflation.
[11]Similar arguments can be found in Aruoba et al. (2018).

Figure 1: Expectations-driven LT and fundamental-driven LT



Notes: The black solid line in this figure is the AS curve (aka the New Keynesian Phillips Curve, NKPC) while the blue dashed line is the AD curve (aka the Euler equation). The left panel presents the expectations-driven LT in a standard NK model with $p = \bar{p} + 0.1$ and the right panel shows fundamental-driven LT in the standard model by assuming $p = \bar{p} - 0.1$ with the demand shock $\epsilon_S = -0.02$. Other calibration parameters are shown in Appendix C.

3, increasing p can generate a second crossing in the AS/AD curves by (i) increasing the Euler equation slope $\mathcal{S}_{EE}$ and (ii) reducing the NKPC slope $\mathcal{S}_{PC}$ simultaneously. In this case, there exists a threshold $\overline{p}$ in Lemma 2 such that the expectations-driven LT in the standard NK model emerges if $p > \overline{p}$.

Lemma 2. *One can use Lemma 1 to calculate the threshold $\overline{p}$ below:*

$$\overline{p} = \frac{(\beta + 1 + \sigma_r \kappa \gamma_y) - \sqrt{(1 + \beta + \sigma_r \kappa \gamma_y)^2 - 4\beta}}{2\beta} < 1.$$

Proof. See Appendix F. □

3.1 Equilibrium Multiplicity

According to Lemma 2, the economy can be in expectations-driven liquidity traps with a high p. The intuition is that the expected highly persistent deflationary shock can shift people's confidence. In this case, people would expect deflation for no reason and there could be a self-fulfilling prophecy which will result in expectations-driven liquidity traps. To better understand the difference between the fundamental-driven liquidity traps and sunspot traps. I show the closed-from solutions for the two liquidity traps in Proposition 1.

Proposition 1. *In the standard NK model, the solution of the expectations-driven traps is given:*

$$
\begin{aligned}
y_S &= \frac{(1-\beta p)\sigma_r}{(1-p)(1-\beta p)-\sigma_r p\kappa}(-\log(\beta)) \\
\pi_S &= \frac{\kappa\gamma_y}{(1-p)(1-\beta p)-\sigma_r p\kappa}(-\log(\beta)),
\end{aligned}
$$

where $(1-p)(1-\beta p)-\sigma_r p\kappa < 0$ *(i.e.* $p > \overline{p}$*).*
The solution of the fundamental-driven traps is shown as:

$$
\begin{aligned}
y_S &= \frac{(1-\beta p)\sigma_r}{(1-p)(1-\beta p)-\sigma_r p\kappa}(\epsilon_S-\log(\beta)) \\
\pi_S &= \frac{\kappa\gamma_y}{(1-p)(1-\beta p)-\sigma_r p\kappa}(\epsilon_S-\log(\beta)),
\end{aligned}
$$

where $(1-p)(1-\beta p)-\sigma_r p\kappa > 0$ *(i.e.* $p < \overline{p}$*).*

Proof. See Appendix G. □

In line with Cuba-Borda & Singh (2020) and Nie (2022), I show the two traps in isomorphic expressions with the ZLB binding. It is straightforward to see the denominator is the same in the two specifications. Here p is crucial here, if the fundamental/sunspot shock is large enough (*i.e.* $p > \overline{p}$), the denominator is negative. In this case, the solutions of y_S and π_S are both negative without any fundamental shock hitting the economy (*i.e.* $\epsilon_S = 0$). On the other hand, the fundamental-driven traps are very similar but the shock persistence is small. In that way, the denominator of the solution is positive while the term $(\epsilon_S - \log(\beta))$ is negative with a strong (negative) fundamental shock $\epsilon_S < 0$. Therefore the economy is in liquidity traps with negative y_S and π_S.

The expectations-driven (or sunspot) traps is shown on the left panel of Figure 1 and a second intersection of the AS and AD curves occurs. It indicates that if the sunspot shock persistence is sufficiently large, the economy will feature actual deflation without any fundamental shocks hitting the economy.[12] It is of note that, similar to the results in Nie & Roulleau-Pasdeloup (2022), there are two short-run equilibria on the left panel of Figure 1. One is the targeted steady state which means $y_S = \pi_S = 0$. Another one is the expectations-driven ZLB, implying $y_S < 0$ and $\pi_S < 0$. These experimental results can echo our analytical results in Proposition 1. Therefore the second equilibrium with expectations-driven traps emerges and there is no stable equilibrium echoing the findings in Aruoba et al. (2018).

On the right panel of Figure 1, there exist fundamental-driven traps where the strong demand shock $\epsilon_S < 0$ can cause sufficient deflation such that the ZLB binds, implying $y_S < 0$ and $\pi_S < 0$. At the same time, the effective slope of the AD curve at the ZLB is lower than its counterpart of AS curve. There is only one unique equilibrium which can feature the ZLB state. For example, the US has been caught in the fundamental-driven ZLB during the global financial crisis (GFC) as in Eggertsson (2011) and Aruoba et al. (2018).

To conclude, there exists sunspot equilibrium in the standard model and we show the effective slopes are crucial in determining the liquidity traps as in Bilbiie (2021) and Nie & Roulleau-Pasdeloup (2022). As in the literature (see e.g. Sugo & Ueda (2008), Nakata & Schmidt (2019) and Schmidt (2016)), many pol-

[12] In other words, if households do expect deflation for no reason, this can cause sufficient deflationary pressures to trigger the expectations-driven LT with a self-fulfilling state of low confidence.

icy prescriptions are proposed to get rid of the sunspot traps. In the following section 4, I will instead introduce the real cost channel into the standard model to reduce the occurrence of expectations- driven LT.

4 Getting Rid of Expectations-driven Liquidity Traps

I now show that it is possible to use the real cost channel to get rid of the expectation-driven liquidity traps. To be more specific, the NK model with the real cost channel can rotate the NKPC while the effective slope of Euler equation is unchanged. As described at length in Section 3, the effective slope of AS/AD curves in a (π_S, y_S) graph at the ZLB episode are critical. First, I show the effective slope of the AS curve at the ZLB with the real cost channel explicitly below.

Lemma 3. *Based on Definition 4, the effective slope of AS/NKPC curve with the real cost channel in Eq.(5) at the ZLB is:*

$$\mathcal{S}_{PC}^{c,z} = \frac{1 - \beta p + \kappa \gamma_r p}{\kappa \gamma_y}.$$

Proof. See Appendix H. □

From Lemma 3, the effective slope of AS curve increases in the strength of the real cost channel γ_r[13] while it decreases in the elasticity of real marginal cost w.r.t output γ_y. In addition, the effective slope of AS curve can be reduced to the standard one if $\gamma_r = 0$.

If the AS curve is rotated and the effective slope $\mathcal{S}_{PC}^{c,z}$ is higher than S_{EE}^{z} in the (π_S, y_S) graph, the second intersection can disappear, implying that the expectations-driven traps as in Bilbiie (2019) and Cuba-Borda & Singh (2020) is

[13] γ_r represents the elasticity of marginal cost with regard to the interest rate and it can be seen as the strength of the real cost channel.

ruled out. In that way, the economy can be in the intended steady state with inflation $\pi_S = 0$ without fundamental shocks. We summarize this result in Lemma 4.

Lemma 4. *The necessary and sufficient condition to rule out expectations-driven LT is:*

$$\mathcal{S}_{PC}^{c,z} > \mathcal{S}_{EE}^{z}.$$

Proof. See Appendix I. □

The real cost channel can increase the effective slope of AS curve at the ZLB episode and it can show no influence on the effective slope of AD curve. In this case, a strong enough real cost channel can help to rule out the sunspot traps if the condition in Lemma 4 is satisfied.[14] We describe the main results in the following Proposition 2.

Proposition 2. *At the ZLB, the effective slope of the AS curve increase in the strength of the real cost channel γ_r. The elasticity of real marginal cost w.r.t output γ_y follows the restriction below:*

$$\gamma_y < \Phi(\gamma_r),$$

where $\Phi(\gamma_r) = \frac{(\beta-\kappa\gamma_r-1+\kappa\gamma_r\phi_\pi)\gamma_r\phi_\pi(\beta-\kappa\gamma_r)}{\sigma_r(1-\kappa\gamma_r\phi_\pi)}$ increases in γ_r. Then the real cost channel can rule out the expectations-driven LT.

Proof. See Appendix J. □

At the ZLB, the effective slope of the AS Curve shown in Lemma 3 increases in γ_r. With a condition that $\gamma_y < \Phi(\gamma_r)$, the effective slope of the AS curve at the ZLB is *always* larger than its AD counterpart in a (π_S, y_S) graph.[15] Interestingly, this theoretic restriction can echo empirical evidence in Beaudry et al. (2022).

[14] In other words, the real cost channel can reduce the occurrence of expectations- driven LT with a big γ_r while a small γ_r can not work.

[15] Note that if the NKPC is upward sloping in a (π_S, y_S) graph, the second intersection can not arise. In addition, I assume $\Phi(\gamma_r) > 0$ in this paper.

This seminal paper empirically shows that γ_y in the real cost channel is robustly small (non-significantly) and γ_r is significantly positive and is much larger than γ_y. This empirical result motives this restriction on γ_y.

On the flip side, this threshold condition increases in γ_r and therefore with a higher γ_r, the economy are more likely not in expectations-driven traps. Furthermore, this condition $\gamma_y < \Phi(\gamma_r)$ requires that the strength γ_r should be big enough for a given γ_y, implying most (or all) firms should pay wage bills for production to magnify the role of real cost channel. As a consequence, no second intersection exists in the AS/AD curves and therefore the sunspot equilibrium is ruled out.

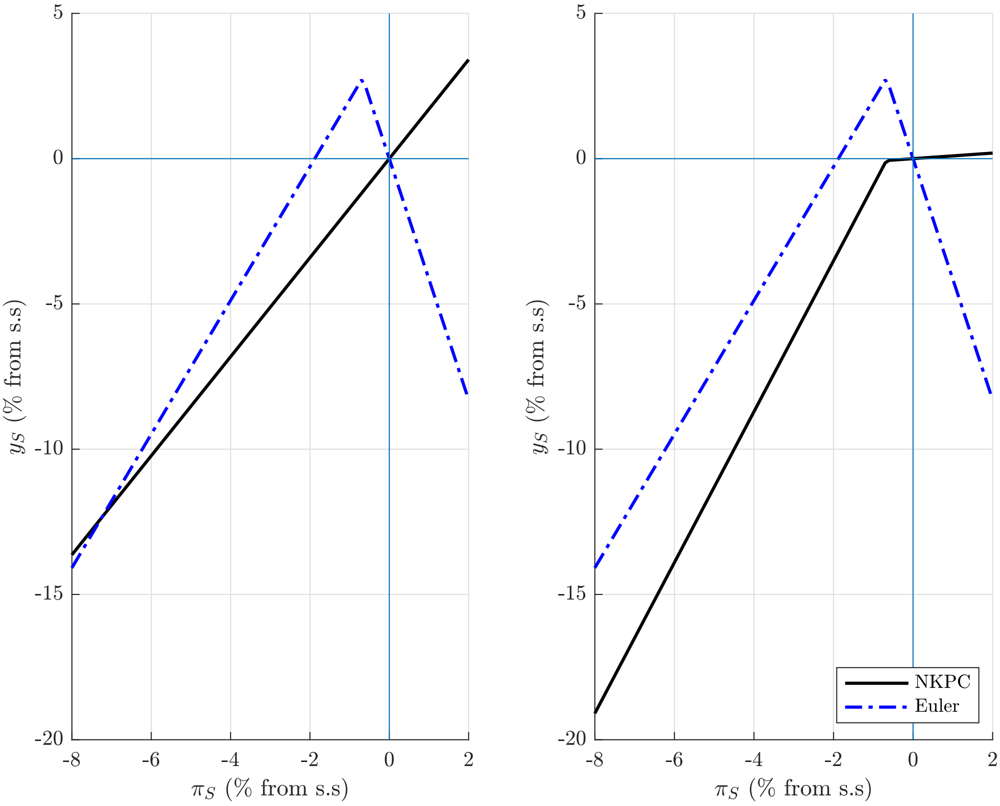
The potential rationale that the real cost channel can reduce the probability of expectations-driven LT is that the dis-deflation feedback at the ZLB in Eq. (5) can *counteract* deflation in the short run. This feedback effect can imply short-run inflation at equilibrium due to rational expectations and sticky prices. We call this as the counteracting channel in this paper. In this case, for a given level of output gap y_S, the deflation behavior at the ZLB can move less. This gives rise to a higher slope of the AS curve in a (π_S, y_S) graph. Finally, a steep enough AS curve can get rid of sunspot equilibrium.

I show the numerical experiment result in Figure 2.[16] On the left panel, in the standard model, the sunspot shock is persistent enough, there are two equilibria and the second intersection appears. With the same calibration method, it appears that there is no sunspot equilibrium on the right panel of Figure 2: the absence of second intersection in the AS/AD curves due to the steeper AS curve at the ZLB episode.

This result can provide a theoretical justification for the fact that households do not expect deflation in an environment with high persistent deflation. In this case, even during recessions, few households anticipate deflation, and the

[16]The calibration method can guarantee that $\overline{p}^c > \overline{p}$.

Figure 2: No expectations-driven LT with the real cost channel



Notes: The black solid line in this figure is the AS curve while the blue dashed line is the AD curve. The left panel presents the expectations-driven LT in a standard NK model without the real cost channel and the right panel shows no expectations-driven LT with the real cost channel, following the calibration method as in Appendix C.

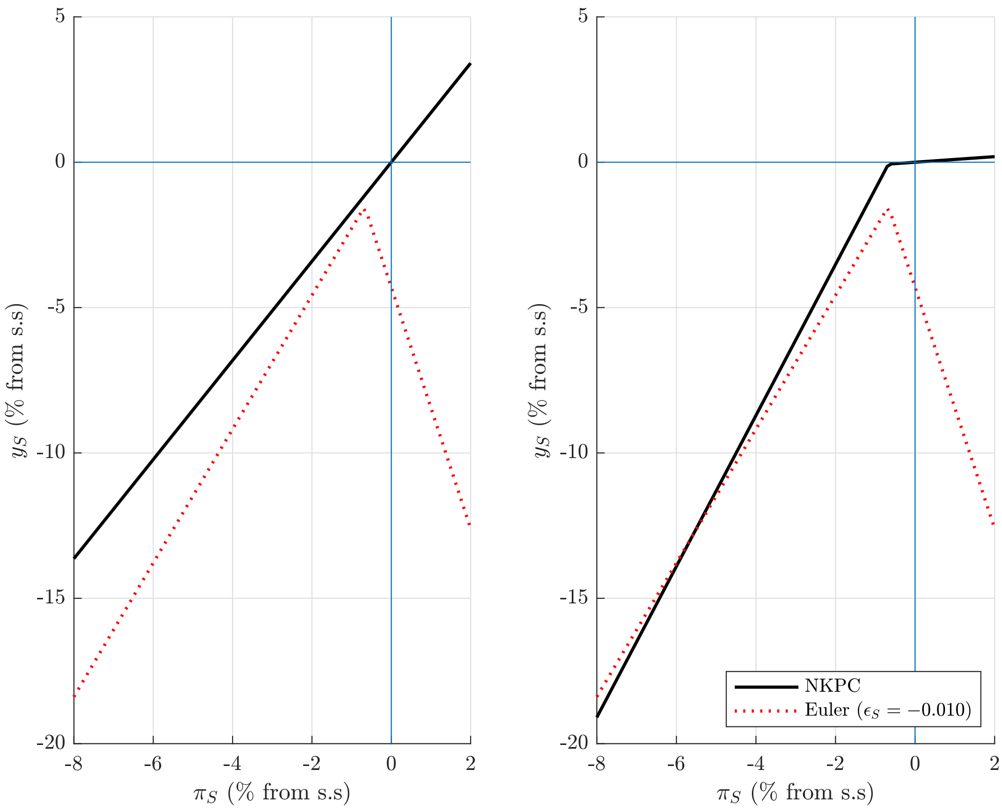
possibility of expectation-driven liquidity traps can be attenuated, according to European and Japanese surveys in Gorodnichenko & Sergeyev (2021). In addition, Mertens & Williams (2021) use US financial markets data to show no evidence in favor of the (sunspot) liquidity traps. In this case, the locally flat Phillips Curve in a (y_S, π_S) graph can rule out the expectations-driven LT and it can ensure one unique equilibrium with $\pi_S = 0$. In particular, this locally flat pattern can only be observed at the ZLB episode.[17] Interestingly, this locally flat Phillips Curve in this model is consistent with recent empirical evidence. For example, Hazell et al. (2022) use the US cross-sectional data and estimate a flat Phillips curve during the Great Recession.

[17] Note that in Figure 2, the AS/AD curves are shown in a (π_S, y_S) graph for an easier comparison while the standard Phillips Curve is in a (y_S, π_S) graph. In other words, the Phillips Curve is flat in a (y_S, π_S) graph which means the AS curve is steep in a (π_S, y_S) graph as in Figure 2.

4.1 Equilibrium Uniqueness/Existence

As in Benhabib et al. (2001) and Mertens & Ravn (2014), the NK models can be prone to equilibrium multiplicity. I have shown this occurs since there is a second intersection which can feature the sunspot equilibrium analytically and graphically. Moreover, as in Ascari & Mavroeidis (2022), models with ZLB constraints can have no solution: if there exist supply/demand shocks that make the AD curve shift too much below the AS curve, there can be no equilibrium in the expectations-driven LT case.

Figure 3: Equilibrium uniqueness/existence with demand shock



Notes: The black solid line in this figure is the AS curve while the red dotted line is the AD curve with a demand shock ($\epsilon_S = -0.010$). The left panel presents the no equilibrium in a standard NK model without the real cost channel and the right panel shows a equilibrium with the real cost channel, following the calibration method as in Appendix C.

To have a clear observation, I plot this situation in Fig 3. It can be seen that, on the left panel, if the effective slope of the AS curve at the ZLB is lower than its AD counterpart, there is no equilibrium with additional strong enough demand shock.[18] This arises since the demand shock ϵ_S shift the AD curve too much below the AS curve.[19] However, no solution dilemma can not arise if the effective slope of AS curve higher at the ZLB than its counterpart of the AD curve.

On the right panel of Fig 3, it can be seen that the real cost channel can increase the effective slope of AS curve at the ZLB. In this case, even if there is a powerful demand shock, it can always have a unique intersection at the ZLB. Therefore, the real cost channel can help to ensure that the AS/AD curves can always have a unique equilibrium with shocks.[20] Overall, this real cost channel is theoretically appealing since it helps ensure equilibrium existence with supply/demand shocks.

We summarize our main results in Proposition 3. The real cost channel, on the one hand, can help rule out the sunspot equilibrium with a big γ_r and this can ensure equilibrium uniqueness. On the other hand, if there exists a unique equilibrium without fundamental shocks, the model can be prone to have equilibrium existence with (strong) supply/demand shocks.

Proposition 3. *The real cost channel can help ensure model equilibrium uniqueness/existence.*

4.2 Strength of Real Cost Channel: "Go Big or Go Home"

As for the discussion outlined above, I have implicitly assume that the real cost channel is strong enough to rule out the sunspot equilibrium. However, as in Proposition 2, $\Phi(\gamma_r)$ increases in the strength of the real cost channel γ_r, implying a small γ_r may not able to rule out sunspots. In this case, we aim to illustrate the role of the strength of the real cost channel in this section.

For simplicity, I consider $\gamma_r = \{0, 0.2, 1\}$ in the numerical experiment and

[18] There exists two equilibria with a small demand shock.

[19] The kink of the AD curve is below than the AS curve.

[20] See Appendix K for a numerical simulative example with supply shocks.

I plot these results in Figure 4. Remember that $\gamma_r = 0$ means the model will revert to the standard NK model. $\gamma_r = 1$ means that all firms in the economy needs to pay wage bills for production. $\gamma_r = 0.2$ indicates only a small portion of firms must pay wage bills. In this case, $\gamma_r = 0.2$ is a case where the role of the real cost channel is marginal. In contrast, $\gamma_r = 1$ shows a bold role of the real cost channel.

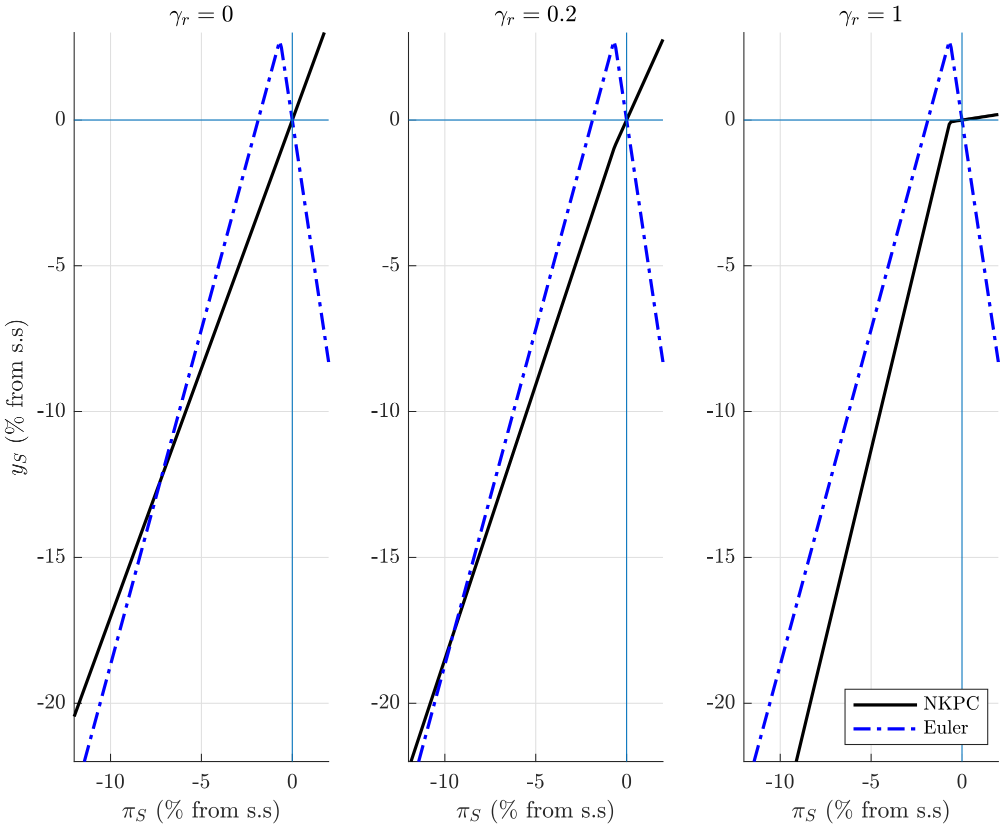
The direct takeaway from this Figure 4 is that the real cost channel has a potential "go big or go home" behavior. On the first panel, it shows that we have two equilibria and the second intersession can feature the ZLB state with $\pi_S^s < 0$. On the second panel, with a timid cost channel, even if the effective slope of the AS curve at the ZLB is stepper now, it can not rule out sunspots and even worsen the sunspot equilibrium with $\pi_S^c < \pi_S^s$. On the third panel, this is the situation we have discussed above and the bold real cost channel can rule out sunspots. Quantitatively, I find $\gamma_r > 0.76$ in the simulation such that the real cost channel can get rid of sunspot equilibrium.

There is a caveat for the real cost channel since a weak strength can even worsen the sunspot equilibrium. Intuitively, households tend to save instead of consuming in recessions. A timid real cost channel can increase the real marginal cost through the counteracting channel while the increased short-term inflation at equilibrium is not enough. In this case, households have to save more[21] to obtain the optimal expected return on savings in future due to the expected inflation as in Nie & Roulleau-Pasdeloup (2022).[22] In contrast, a bold real cost channel can make-up the short-run deflation fully. Besides, the real interest rate in short run is higher and it can stimulate households to consume. In that way, deflationary expectations can not be an equilibrium outcome and thus the sunspot traps can be ruled out.

[21] This extra saving can be seen as precautionary saving.

[22] As in Nie & Roulleau-Pasdeloup (2022), it explains at length that the expected return of saving is less elastic to savings with not enough inflation make-up in sunspot equilibrium and households has to increase savings.

Figure 4: AS/AD with the strength of the real cost channel



Notes: The black solid line in this figure is the AS curve while the blue dotted line is the AD curve. The first panel presents the equilibrium in a standard NK model without the real cost channel, the second panel shows the model with a weak real cost channel, and the third panel displays the model with a bold real cost channel, following the calibration method as in Appendix C.

5 Conclusions

In the presence of the ZLB, the shift in confidence can cause sufficient deflationary pressures to trigger the expectations-driven LT without any fundamental shocks. There is a recent survey fact that households do not expect deflation in an environment with high persistent deflation, according to European and Japanese surveys as in Gorodnichenko & Sergeyev (2021). In this case, even during recessions, few households anticipate deflation, and the possibility of expectation-driven liquidity traps can be attenuated.

In this paper, I develop a tractable New Keynesian model via the real cost channel to provide theoretic explanations for rationalizing these new empirical facts. I have shown that the real cost channel *per se* can reduce the occurrence of the expectations-driven liquidity traps by rotating the Phillips Curve. This arises because the bold real cost channel at the ZLB can counteract the short-run deflation caused by the drop in confidence, implying actual inflation on equilibrium. In that way, deflationary expectations can not be an equilibrium outcome. Additionally, I show this real cost channel is theoretically appealing since it helps ensure model equilibrium uniqueness/existence with supply and demand shocks.

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Online Appendix

A The Model Setup

Time is discrete and there is no government spending.

A.1 Aggregate Demand Side

The representative household has the below preferences:

$$
\begin{aligned}
\mathcal{U}(C_t, L_t) &= u(C_t) - v(L_t) \\
&= \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{L_t^{1+\eta}}{1+\eta}, \quad \chi, \eta > 0
\end{aligned}
$$

where households work L_t hours, consume amount C_t, and trade government bonds B_t.

The budget constraint is,

$$
C_t + \frac{B_t}{P_t} = W_t L_t + \mathcal{D}_t - \mathcal{T}_t + \exp(\Im_{t-1}) \frac{1+R_{t-1}}{P_t} B_{t-1}.
$$

where $\Im_t$ is a "risk premium" shock.

The optimal aggregate (individual) labor price is written as:

$$
W_t = \frac{L_t^{\eta} \chi}{(C_t)^{-\sigma}},
$$

I can obtain the Euler equation with the first order condition (FOC) of the maximization program:

$$
(C_t)^{-\sigma} = \beta \exp(\Im_t) \mathbb{E}_t \left\{ (C_{t+1})^{-\sigma} \frac{1+R_t}{1+\Pi_{t+1}} \right\}.
$$

The semi-linearized equilibrium Euler equation by approximating around the steady state can read. That is, all lower case format variables are the log deviations from steady state ($x_t = \log(X_t) - \log(X)$):

$$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\sigma}\left[R_t + \log(\beta) - \mathbb{E}_t \tau^c_{t+1} - \mathbb{E}_t \pi_{t+1} - \epsilon_t\right].$$

where $\epsilon_t \equiv -\Im_t$ is the natural rate shock (demand shock) and R_t is the nominal interest rate in level. The following resource constraint is placed in this economy:

$$y_t = c_t,$$

Furthermore the Euler equation is expressed as:

$$y_t = \mathbb{E}_t y_{t+1} - \sigma_r \left[R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1} - \epsilon_t\right],$$

where $\sigma_r \equiv \frac{1}{\sigma}$.

A.2 Aggregate Supply Side

Each monopolist produces a differentiated good using a basic input as the only factor of production, and according to a one to one technology. The marginal cost of production will therefore be the price of that basic input. Basic input Q_t is produced by a representative competitive firm with labor L_t and the labor-like final good M_t with the production function as in Beaudry et al. (2022)) below:

$$Q_t = \min(aL_t, bM_t).$$

The unit price of final good that enters the production of basic input is P_t. I assume that a fraction $\gamma \in [0, 1]$ of basic input representative firm must borrow debt at the nominal wage bill $(1 + i_t)W_t P_t$ to pay for the input M_t (Rabanal

(2007)).[23] The firm sells its production, pays wages, repays the debt contracted the previous period and distributes all the profits D_t as dividends. By using the symmetry condition, the budget constraint for the firm is shown below,

$$\tilde{P}_t Q_t = P_t W_t L_t + (1 + i_{t-1}) W_{t-1} P_{t-1} M_{t-1},$$

In this case, the basic input representative firm has profits below:

$$D_t = \tilde{P}_t Q_t - P_t W_t L_t - (1 + i_{t-1}) W_{t-1} P_{t-1} M_{t-1},$$

Firm maximizes the expected discounted sum of real profits D_t/P_t with discount factor β:

$$\tilde{P}_t = \left(\frac{1}{a} W_t + \frac{\beta}{b} \left[\frac{1 + i_t}{1 + \pi_{t+1}} W_t \right] \right) P_t.$$

The real marginal cost of the γ basic input firm:

$$MC_t = \frac{1}{a} W_t + W_t \frac{\beta}{b} \left[\frac{1 + i_t}{1 + \pi_{t+1}} \right].$$

The other $1 - \gamma$ firms are standard with the real marginal cost below:

$$MC_t = W_t.$$

The optimal household labor supply:

$$\frac{v'(L_t)}{u'(C_t)} = W_t,$$

Other parts are standard, and the New Keynesian Phillips curve yields:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \bar{\kappa}\, mc_t.$$

[23]The labor-like final good can be simply seen as working machines (e.g. robotic support) which should be rent at the nominal wage bill.

By log condition, I have the semi-linearized equilibrium

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \left[\gamma_y y_t + \gamma_r (R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1}) \right],$$

where $\kappa = \frac{\beta}{b}\bar{\kappa}$, R_t is the nominal interest rate in level, $\gamma_y = \frac{b}{\beta}(\sigma + \eta)$ and $\gamma_r = \gamma$. In this case, this model can collapse to the standard model if we assume $\gamma = 0$ below:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \gamma_r y_t.$$

B Upward Sloping Assumption

According to Definition 4, in normal times, I can reproduce the solutions for y_S and π_S as follows:

$$\begin{aligned} y_S &= \frac{\sigma_r(1 - \beta p + \kappa\gamma_r p - \kappa\gamma_r\phi_\pi)}{(1-p)(1 - \beta p + \kappa\gamma_r p - \kappa\gamma_r\phi_\pi) + \sigma_r\kappa\gamma_y(\phi_\pi - p)}\epsilon_S \\ \pi_S &= \frac{\sigma_r\kappa\gamma_y}{(1-p)(1 - \beta p + \kappa\gamma_r p - \kappa\gamma_r\phi_\pi) + \sigma_r\kappa\gamma_y(\phi_\pi - p)}\epsilon_S. \end{aligned}$$

If the Phillips Curve is upward sloping in normal times, which means the effective slope of Phillips Curve is positive:

$$\begin{aligned} 1 - \beta p + \kappa\gamma_r p - \kappa\gamma_r\phi_\pi &> 0 \\ \Leftrightarrow p &< \frac{1 - \kappa\gamma_r\phi_\pi}{\beta - \kappa\gamma_r}, \end{aligned}$$

where the second line using the assumption $\kappa\gamma_r < \beta$ as in Beaudry et al. (2022) and Nie (2022). In this case, there is a threshold $\overline{p}^c = \frac{1-\kappa\gamma_r\phi_\pi}{\beta - \kappa\gamma_r}$.

C Calibration Parameters

Table 1: The calibrated parameter values

Discount factor	$\beta = 0.99$
Preference parameter	$\sigma_r = 2$
Elasticity of inflation w.r.t marginal cost	$\kappa = 0.4$
Elasticity of marginal cost w.r.t output	$\gamma_y = 0.6$
Elasticity of marginal cost w.r.t interest rate	$\gamma_r = 1$
Taylor rule	$\phi_\pi = 1.5$
Shock persistence	$p = \frac{\overline{p}^c + \overline{p}}{2}$

Notes: I follow Beaudry et al. (2022) to set the value for γ_r and γ_y. We can obtain qualitatively identical results with different sets of γ_r & γ_y and these results can be obtained be request. I follow Bergholt et al. (2020) and Nie & Roulleau-Pasdeloup (2022) to use a standard calibrated method for other parameters. $\bar{p}$ is the threshold such that there exists the expectations-driven LT in the standard model without the real cost channel. $\overline{p}^c$ is the threshold such that the AS curve is upward sloping in the model with the real cost channel.

D The expressions in Definition 5

The NKPC is shown below:

$$y_S = \begin{cases} \frac{1-\beta p+\kappa\gamma_r p-\kappa\gamma_r\phi_\pi}{\kappa\gamma_y}\pi_S & \text{if } \pi_S > \frac{\log(\beta)}{\phi_\pi} \\ \frac{1-\beta p+\kappa\gamma_r p}{\kappa\gamma_y}\pi_S - \frac{\gamma_r}{\gamma_y}\log(\beta) & \text{if } \pi_S \leq \frac{\log(\beta)}{\phi_\pi}. \end{cases}$$

One can formally show the Euler equations below:

$$y_S = \begin{cases} -\sigma_r\frac{\phi_\pi - p}{1-p}\pi_S + \sigma_r\frac{\epsilon_S}{1-p} & \text{if } \pi_S > \frac{\log(\beta)}{\phi_\pi} \\ \frac{\sigma_r p}{1-p}\pi_S + \sigma_r\frac{\epsilon_S - \log(\beta)}{1-p} & \text{if } \pi_S \leq \frac{\log(\beta)}{\phi_\pi}. \end{cases}$$

E Proofs of Lemma 1

The Euler equation in standard NK model:

$$y_t = \mathbb{E}_t y_{t+1} - \sigma[R_t + \log(\beta) - \mathbb{E}_t\pi_{t+1} - \epsilon_t]$$

The NKPC is below:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \gamma_y y_t$$

Using the simple two-state Markov Chain, we have $\mathbb{E}_S \pi_{t+1} = p\pi_S$ and $\mathbb{E}_S y_{t+1} = py_S$. We can write the Euler equation at the ZLB below:

$$y_S = -\frac{\sigma_r p}{1-p}\pi_S + \sigma_r \frac{\epsilon_S - \log(\beta)}{1-p}.$$

One can yield the NKPC:

$$y_S = \frac{1-\beta p}{\kappa \gamma_y}\pi_S.$$

Thus, the effective slope of AD/Euler curve is:

$$\mathcal{S}^z_{EE} = \sigma_r \frac{p}{1-p}.$$

the effective slope of AS/NKPC curve is:

$$\mathcal{S}^z_{PC} = \frac{1-\beta p}{\kappa \gamma_y}.$$

F Proofs of Lemma 2

The standard textbook New Keynesian Phillips Curve without a cost channel can read:

$$\pi_t = \beta \mathbb{E} \pi_{t+1} + \kappa \gamma_y y_t.$$

In this case, the Phillips Curve can be re-written as

$$y_S = \frac{1-\beta p}{\kappa \gamma_y}\pi_s$$

If the Phillips Curve is upward sloping throughout time periods. If there is an absence of demand shock and the effective slope of AS curve is lower than AD curve, *i.e.*:

$$(1-p)(1-\beta p) < \sigma_r p \kappa \gamma_y.$$

We can have the sunspot equilibrium featuring $\pi_S < 0, y_S < 0$: *i.e.* there exists a threshold $\overline{p}$:

$$\begin{aligned}
\overline{p} &= \frac{(\beta + 1 + \sigma_r \kappa \gamma_y) - \sqrt{(1 + \beta + \sigma_r \kappa \gamma_y)^2 - 4\beta}}{2\beta} \\
&< \frac{(\beta + 1 + \sigma_r \kappa \gamma_y) - (-\beta + 1 + \sigma_r \kappa \gamma_y)}{2\beta} \\
&= 1
\end{aligned}$$

where there is $\bar{p} \in (0,1)$ to trigger the expectations-driven LT to make $y_S < 0$ in the absence of demand shock. That being said, there is a sunspot equilibrium if $p > \bar{p}$. Note that if the the demand shock is very large, it can shift AD curve down so much that there is no intersection in the AS and AD curves which means no equilibrium in this economy.

G Proofs of Proposition 1

It is straightforward to use Appendix E and one can combine AS/AD curves to obtain the solution at the ZLB:

$$\begin{aligned}
y_S &= \frac{(1-\beta p)\sigma_r}{(1-p)(1-\beta p) - \sigma_r p \kappa \gamma_y}(\epsilon_S - \log(\beta)) \\
\pi_S &= \frac{\kappa \gamma_y}{(1-p)(1-\beta p) - \sigma_r p \kappa \gamma_y}(\epsilon_S - \log(\beta)),
\end{aligned}$$

where $p < \overline{p}$. On the other hand, the sunspot equilibrium emerges without fundamental shocks ϵ_S if $p > \overline{p}$ and the solution can be derived with AS/AD curves:

$$y_S = \frac{(1-\beta p)\sigma_r}{(1-p)(1-\beta p) - \sigma_r p \kappa \gamma_y}(-\log(\beta))$$
$$\pi_S = \frac{\kappa \gamma_y}{(1-p)(1-\beta p) - \sigma_r p \kappa \gamma_y}(-\log(\beta)),$$

where $p > \overline{p}$.

H Proofs of Lemma 3

According to Definition 4, under a ZLB, the Phillips Curve is

$$y_S = \frac{1 - \beta p + \kappa \gamma_r p}{\kappa \gamma_y} \pi_S - \frac{\gamma_r}{\gamma_y} \log(\beta),$$

The Euler equation at the ZLB is:

$$y_S = -\frac{\sigma_r p}{1-p} \pi_S + \sigma_r \frac{\epsilon_S - \log(\beta)}{1-p}.$$

Thus, the effective slope of AD/Euler curve is:

$$\mathcal{S}^z_{EE} = \sigma_r \frac{p}{1-p}.$$

the effective slope of AS/NKPC curve is:

$$\mathcal{S}^{c,z}_{PC} = \frac{1 - \beta p + \kappa \gamma_r p}{\kappa \gamma_y}.$$

I Proofs of Lemma 4

This is direct result from the standard model in Appendix F. If there is an absence of demand shock and the effective slope of AS curve is lower than AD curve at the ZLB, we can have sunspots. Otherwise if the the effective slope of the AS curve is higher than the AD curve at the ZLB, sunspots disappear. Thus, the necessary and sufficient condition to rule out expectations-driven traps is:

$$\mathcal{S}_{PC}^{c,z} > \mathcal{S}_{EE}^{z}.$$

J Proofs of Proposition 2

According to Definition 4, under a ZLB, the Phillips Curve is

$$y_S = \frac{1 - \beta p + \kappa\gamma_r p}{\kappa\gamma_y}\pi_S - \frac{\gamma_r}{\gamma_y},$$

where the effective slope is $\frac{1-\beta p+\kappa\gamma_r p}{\kappa\gamma_y}$. It is easy to check this slope is increasing in the elasticity of the marginal cost w.r.t the interest rate γ_r which can be seen as the strength of the real cost channel.

If the flat Phillips Curve is upward sloping throughout time periods, which means that the effective slope of Phillips Curve is always positive:

$$\begin{aligned} 1 - \beta p + \kappa\gamma_r p - \kappa\gamma_r\phi_\pi > 0 \\ \Leftrightarrow p < \frac{1 - \kappa\gamma_r\phi_\pi}{\beta - \kappa\gamma_r}. \end{aligned}$$

In this case, in normal times, it is easy to check that the only equilibrium is the target steady state (*i.e.* $y_S = \pi_S = 0$) with no demand shock.

While assuming that the demand shock is large enough to trigger the fundamental-driven ZLB, I reproduce the following solutions for y_S and π_S:

$$\begin{aligned} y_S &= \frac{(1 - \beta p + \kappa\gamma_r p)\sigma_r(\epsilon_S - \log(\beta)) + \kappa\gamma_r\sigma_r p \log(\beta)}{(1-p)(1 - \beta p + \kappa\gamma_r p) - \sigma_r p\kappa\gamma_y} \\ \pi_S &= \frac{\kappa\gamma_y\sigma_r(\epsilon_S - \log(\beta))}{(1-p)(1 - \beta p + \kappa\gamma_r p) - \sigma_r p\kappa\gamma_y} + \frac{\kappa\gamma_y\kappa\gamma_r\sigma_r p \log(\beta)}{[(1-p)(1 - \beta p + \kappa\gamma_r p) - \sigma_r p\kappa\gamma_y](1 - \beta p + \kappa\gamma_r p)} \\ &\quad + \frac{\kappa\gamma_r \log(\beta)}{1 - \beta p + \kappa\gamma_r p}. \end{aligned}$$

If there is no expectations-driven liquidity traps (LT) in the absence of demand shock, the requirement is below:

$$
\begin{aligned}
y_S = \frac{(1-\beta p)\sigma_r(-\log(\beta))}{(1-p)(1-\beta p+\kappa\gamma_r p)-\sigma_r p\kappa\gamma_y} > 0 \\
\Leftrightarrow \mathcal{D}(p) = (1-p)(1-\beta p+\kappa\gamma_r p)-\sigma_r p\kappa\gamma_y > 0
\end{aligned}
$$

One can yield a condition for γ_y to secure $\mathcal{D}(p) > 0$:

$$
\begin{aligned}
\mathcal{D}(p) &= (1-p)(1-\beta p+\kappa\gamma_r p)-\sigma_r p\kappa\gamma_y \\
&> \left(1-\frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r}\right)\left(1-\beta\frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r}+\kappa\gamma_r\frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r}\right)-\sigma_r\frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r}\kappa\gamma_y \\
&= (\beta-\kappa\gamma_r-1+\kappa\gamma_r\phi_\pi)[\beta\kappa\gamma_r\phi_\pi-\kappa\gamma_r+\kappa\gamma_r(1-\kappa\gamma_r\phi_\pi)]-\sigma_r(1-\kappa\gamma_r\phi_\pi)\kappa\gamma_y > 0 \\
\gamma_y &< \frac{(\beta-\kappa\gamma_r-1+\kappa\gamma_r\phi_\pi)(\beta\gamma_r\phi_\pi-\kappa\gamma_r^2\phi_\pi)}{\sigma_r(1-\kappa\gamma_r\phi_\pi)} = \Phi(\gamma_r),
\end{aligned}
$$

where the second line we assume $p = \bar{p}^c$ due to monotonicity.
At the ZLB episode, one can compare the effective slope of the AS/AD curves:

$$
\frac{1-\beta p+\kappa\gamma_r p}{\kappa\gamma_y} > \sigma_r\frac{p}{1-p},
$$

where we use the condition $\gamma_y < \frac{(\beta-\kappa\gamma_r-1+\kappa\gamma_r\phi_\pi)(\beta\gamma_r\phi_\pi-\kappa\gamma_r^2\phi_\pi)}{\sigma_r(1-\kappa\gamma_r\phi_\pi)}$. This means the effective slope of the AS curve is larger than the effective slope of the AD curve at the ZLB.
In addition, one can check the monotonicity of $\Phi(\gamma_r)$ w.r.t. γ_r:

$$
\begin{aligned}
\frac{\partial\Phi(\gamma_r)}{\partial\gamma_r} &\propto \frac{\partial\frac{\beta-\kappa\gamma_r}{1-\kappa\gamma_r\phi_\pi}}{\partial\gamma_r} \\
&\propto \kappa(\phi_\pi\beta-1) > 0.
\end{aligned}
$$

Therefore $\Phi(\gamma_r)$ increases in γ_r.

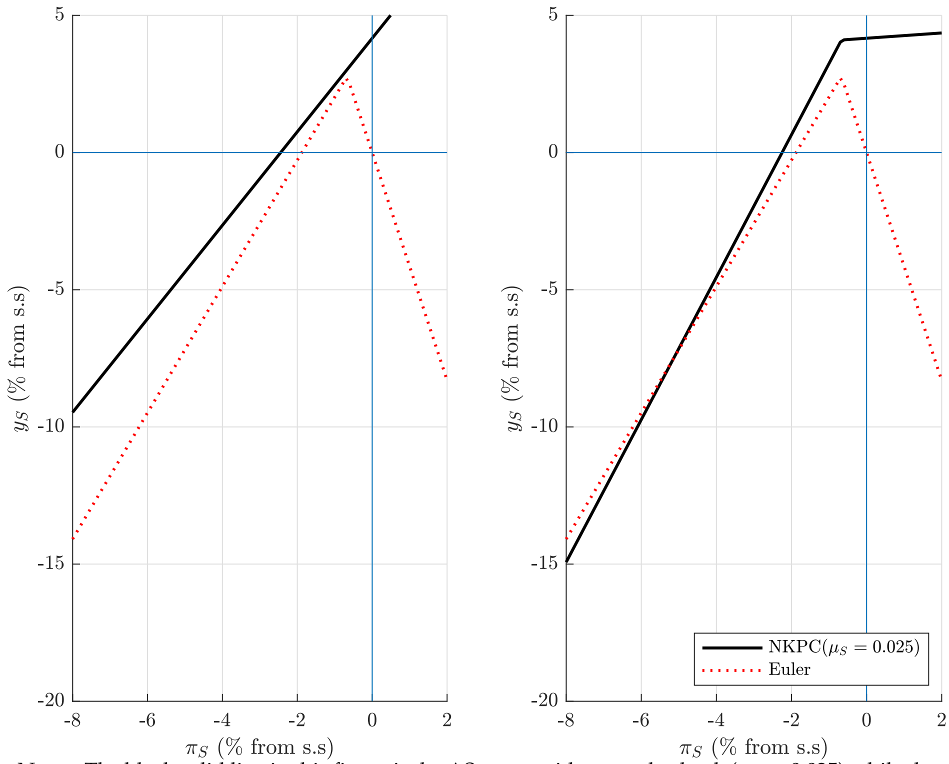
K Additional Figures for Supply Shocks

In this part, we simply assume there is a supply shock in the NKPC as below:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa[\gamma_y y_t + \gamma_r (R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1})] + \mu_t,$$

where μ_t is the temporary supply shock.

Figure 5: Equilibrium uniqueness/existence with the real cost channel (supply shock)



Notes: The black solid line in this figure is the AS curve with a supply shock ($\mu_S = 0.025$) while the red dotted line is the AD curve. The left panel presents the no equilibrium in a standard NK model without the real cost channel and the right panel shows a equilibrium with the real cost channel, following the calibration method as in Appendix C.