Government Spending Multipliers with the *Real* Cost Channel*

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February 8, 2023

Abstract

In the benchmark New Keynesian (NK) model, I introduce the real cost channel to study government spending multipliers and provide a simple Markov chain closed-form solution. This new model departs fundamentally from most previous interpretations of the nominal cost channel by flattening the NK Phillips Curve in liquidity traps. At the zero lower bound, I analytically show that the real cost channel can make inflation rise less than in a model without this channel following positive government spending shocks. This then causes a smaller drop in real interest rates, resulting in a lower output gap multiplier. Finally, I confirm the robustness of the real cost channel's effect on multipliers using extensions of two models.

Keywords: Government Spending Multipliers, Real Cost Channel, Zero Lower Bound, Markov Chain, Bounded Rationality

JEL Classifications: E52, E58, E62, E63, E70

*I would like to thank my advisor, Jordan Roulleau-Pasdeloup, for his extensive comments. I would also thank Chang Liu, Paul Gabriel Jackson, Denis Tkachenko, Yujie Yang as well as participants in NUS GRS and the CEC-NTU Joint Online workshop 2021 for their comments and suggestions.

1 Introduction

In late 2008, the Federal Reserve had to lower the interest rate to zero to combat the Global Financial Crisis (GFC). Conventional monetary policy cannot work with zero lower bound (ZLB) binding. Therefore the government sought to adopt an alternative effective fiscal policy to stimulate the economy during recessions. In this case, the GFC and other recent recessions resulting from COVID-19 have sparked extensive fiscal policy discussions, which usually feature massive government spending.[1] For example, during the COVID-19 pandemic of 2020, the United States (US) government spent a total of $6.55 trillion on a series of programs to ensure the well-being of its population.

Attempting to understand the effects of government spending at the ZLB warrants a thorough fiscal analysis. Standard New Keynesian (NK) models show that government spending multipliers can be substantially higher (e.g., above 2) at the ZLB as in Eggertsson and Woodford (2004), Christiano et al. (2011) and Zubairy (2014). This view has been challenged in many recent theoretic studies.[2] In addition, a series of new empirical papers have indicated that multipliers are lower at the ZLB. For instance, Ramey and Zubairy (2018) provide empirical evidence to show rather ineffective spending multipliers in liquidity traps. Recently, Auerbach et al. (2021) show that fiscal multipliers can be lower if used along with post-COVID supply-side constraints during recessions.

In this paper, I aim to augment the real cost channel in the benchmark NK model to provide *new theoretical insights* that explain, using empirical papers, lower government spending multipliers when the economy is at the ZLB. More specifically, if firms need to pay bills before production, the interest rate can theoretically influence borrowing costs and firms' marginal costs in the aggregate *supply-side* economy (summarized in the NK Phillips Curve). The critical difference between the model with the cost channel and the conventional model is that the interest rate should be included in firms' marginal costs which can, in turn, influence the inflation rate.

The existence of the cost channel is shown in some empirical investigations. For instance, Ravenna and Walsh (2006) estimate and obtain the cost channel parameter for the US. Similarly, Chowdhury et al. (2006) and Tillmann (2009) show the existence of the cost channel in both the US and the UK. Recently, Abo-Zaid (2022) employs a structural vector autoregression (SVAR) model to confirm that the cost channel exists in almost all

[1]See discussions on the fiscal tool to resist recessions as in Eggertsson (2011), Kollmann et al. (2012), Bouakez et al. (2020) and House et al. (2020).

[2]See e.g. Kiley (2014), Mertens and Ravn (2014) and Roulleau-Pasdeloup (2021b).

representative industrialized countries.

Unlike most previous literature with the nominal cost channel[3], this paper, however, incorporates expected real interest rates into firms' marginal costs to study government spending multipliers.[4] Furthermore, I try to conduct an analytical study on spending multipliers in liquidity traps. To this end, a simple two-state Markov chain as in Eggertsson and Woodford (2003) is utilized to obtain closed-form solutions of spending multipliers in the short run.

At the ZLB, the results in this paper stand in stark contrast to most previous interpretations of the nominal cost channel as in Surico (2008) and Smith (2016). I show that the introduced real cost channel can rotate the NK Phillips Curve with the expected disinflation effects during episodes of liquidity traps. The threshold of the negative natural rate shock to trigger ZLB constraints binding with the real cost channel is larger than in the classical NK model without the cost channel but less than in the model with the nominal cost channel. Therefore, the economy with the nominal channel is the most easily entrapped in liquidity traps.

Spending multipliers can be effective (larger than one) in liquidity traps. Intuitively, with the ZLB binding, nominal interest rates remain unchanged. Government spending within a fiscal policy package can increase inflation in the short run. This can lower real interest rates and stimulate private consumption. However, the expected disinflation effects of the real cost channel result in lower marginal costs, causing inflation to rise by less than in standard models without it. Moreover, marginal costs can decrease even further with the increased strength of this channel. In this way, the real cost channel leads to a smaller drop in real interest rates. Higher real interest rates can depress people's appetite for consumption and production activity, which, in turn, can lower the output gap multiplier. On the other hand, spending multipliers with the nominal cost channel as in Ravenna and Walsh (2006) can be invariant with the standard NK model since this nominal channel cannot modify the NK Phillips Curve slope at the ZLB.[5] In

[3]There are a series of papers with the nominal cost channel which means that nominal interest rates are augmented in firms' marginal costs, such as, Barth III and Ramey (2001), Ravenna and Walsh (2006), Llosa and Tuesta (2009) and Smith (2016).

[4]Compared to the nominal cost channel, as explained at length in Beaudry et al. (2022), the real cost channel can be more empirically relevant to the US data. Furthermore, Nie (2021) formally proves that the real cost channel is theoretically appealing since it can ensure the equilibrium uniqueness/existence with temporary shocks.

[5]In the calculation of fiscal multipliers, we need to gain the partial derivative of government spending to inflation/the output gap. If the introduced cost channel cannot change the NK Phillips Curve slope, *ceteris paribus*, the partial derivative should be the same as for the standard model.

a nutshell, the output gap multiplier with the real cost channel model is smaller at the ZLB and decreases with the increased channel strength, compared to the standard NK model and nominal cost models.

Another result is that the output gap multiplier weakens with the increased strength of the real cost channel when the economy lies outside of liquidity traps. Intuitively, when the nominal interest rate is free to adjust, government spending can increase inflation in the short run. The real cost channel can further increase inflation due to higher borrowing costs and this effect can be amplified with the strengthening of the real channel's impact. With the Taylor (1993)-type rule followed by the central bank, there will be a rise in nominal interest rates by more than one-for-one with inflation pressure. Larger real interest rates due to a rise in inflation triggered by this real channel, can stimulate people to save but consume less. As a result, this real channel can crowd out more private consumption, and thus the output gap multiplier is smaller than in the standard model which abstracts from this channel. Consistent with the findings in Abo-Zaid (2022), notice that the output gap multiplier with the nominal cost channel is less than in the standard model. However, compared to the nominal cost channel, inflation pressure with the real channel is less prominent due to additional expected disinflation effects. I, therefore, show that the nominal cost channel can further restrain the effectiveness of government spending relative to the real channel.

This analytical model extends to a more general setting, but the real cost channel still functions robustly. As in Sarin et al. (2021) and Roulleau-Pasdeloup (2021b), long-run policy effects are usually overlooked or have even been computed numerically in the previous literature. In this case, I discuss the long-run government spending policy analytically by assuming that government spending lasts *longer* than economic recessions. Thus, a three-state Markov chain is employed. I follow Bilbiie (2019b) to decompose the long-run period into short and medium-run periods. Therefore, one can capture the effects of prolonged government spending on short-run spending multipliers. The analytical results show that if the short-run economy is in normal times, it is observed that prolonged government spending can further deflate the output gap multiplier but increase the inflation multiplier. Interestingly, if the short-run economy is in liquidity traps, longer government spending after liquidity traps have subsided is favored. Intuitively, prolonged government spending can increase inflation more through rational expectations as in Bouakez et al. (2017), and thus, it can further inflate the short-run output gap multiplier. This theoretic result is supported by empirical evidence as in Leduc and Wilson (2013) and Bernardini et al. (2020). Additionally, I show that the real cost channel can still decrease the output gap multiplier in the three-state Markov framework.

At the end of the day, I have shown the effects of long-run government spending through rational expectations, but one may wonder what if agents cannot fully understand the world?[6] Thus, bounded rationality is integrated into our baseline model to view such *behavioral* macroeconomic effects as another extension. I finally develop a similar version of the model proposed by Gabaix (2020). The results show that during normal times, cognitive discounting effects can mitigate inflation pressure and increase the output gap multiplier. Interestingly, if we suppose agents have sufficiently strong bounded rationality, the output gap multiplier can be large in normal times, which is in line with some empirical evidence in Auerbach and Gorodnichenko (2012) and Acconcia et al. (2014). On the other hand, at the ZLB, bounded rationality can attenuate the output gap multiplier. This can echo the literature that agents discounting future wealth in making decisions today can reduce policy power in recessions as in McKay et al. (2016), Angeletos and Lian (2018), and Campbell et al. (2019). Besides, the real cost channel can still operate robustly in this behavioral model, and the output gap multiplier can be overestimated by ignoring this real channel.

Related Literature.—Seminal work focusing on the theoretical estimation of government spending effects in liquidity traps can be traced to Eggertsson (2001). In this paper, the optimal fiscal policy in the NK economy is characterized, and the real effects of government spending are emphasized. Since then, an increasing amount of the literature has focused on the estimation of fiscal effects in theoretical and empirical ways. For example, Blanchard and Perotti (2002) spark the earliest insights on empirically estimating the macroeconomic effects of government spending.[7] Christiano et al. (2011) prove that the multiplier is low in normal times in an economy following a Taylor (1993)-type rule but relatively high in liquidity traps. Leeper et al. (2017) theoretically study fiscal multipliers in a series of models. Two distinct monetary-fiscal policy regimes show that the short-run multiplier is robustly similar across different regimes. There are more

[6]There is a similar consideration with monetary policy. As in Nakata et al. (2019) and Budianto et al. (2020), the favorable effects of Forward Guidance—a promised long-run interest rate binding—on short-run inflation can be much attenuated if the economic agents cannot fully comprehend the world as represented by the NK model with rational expectations.

[7]See Ramey (2011) for a survey on the estimation of government spending multipliers in the literature.

examples among Kraay (2012), Miyamoto et al. (2018), Ramey and Zubairy (2018), etc.

In this paper, I add to the government spending multiplier literature by analytically addressing the role of the real cost channel on multipliers. The previous literature as in Barth III and Ramey (2001), Ravenna and Walsh (2006), Llosa and Tuesta (2009) and Smith (2016) introduces the nominal cost channel: Firms' marginal costs augment nominal interest rates. For example, Ravenna and Walsh (2006) first confirm that cost-push shocks can emerge endogenously in the NK model with the nominal cost channel. Furthermore, they discuss the ways by which the new channel can alter the optimal monetary policy. Surico (2008) shows that limiting the economic cycle with the cost channel can lead to strong fluctuations in inflation and output. However, this paper builds heavily on Beaudry et al. (2022) and Nie (2021) and incorporates expected real interest rates into firms' marginal costs to revise the benchmark NK model.

This paper allies closely with some recent literature that uses a three-state Markov chain to analytically examine the long-run policy on the short-run economy (see Bilbiie (2019a), Bilbiie (2019b), Bilbiie (2020) and Nie and Roulleau-Pasdeloup (2022)). For example, Bilbiie (2019b) employs a three-state Markov manner for his in-depth study on the optimal forward guidance policy in both the short and long run. In this paper, a three-state structure can allow us to analytically check the general properties of long-run government spending to echo some empirical evidence in Durevall and Henrekson (2011), Ilzetzki et al. (2013), Leduc and Wilson (2013), Bouakez et al. (2017), and Leff Yaffe (2019).

In addition, recent contributions such as Farhi and Werning (2019) and Gabaix (2020) show that bounded rationality can mitigate the powerful effects of monetary policy. This approach can rationalize the so-called "Forward Guidance puzzle" (see Angeletos and Lian (2018) and Coibion et al. (2020)) compared to the benchmark forward-looking NK model. This paper, however, is linked with this strand of literature and comments on the interaction of the cost channel and bounded rationality on *fiscal* policy.

Finally, this paper is also closely related to Abo-Zaid (2022) who examines government spending multipliers at the ZLB with the nominal cost channel. Abo-Zaid (2022) includes the nominal cost channel and differentiates between the policy rate and the loan rate. It turns out that this nominal channel can cause spending multipliers to be larger in liquidity traps. However, in this paper, I use the real cost channel to explain lower government spending multipliers when the economy is at the ZLB. Simple Markov chain closed-form solutions are computed to study government spending multipliers with the real and nominal cost channels. Analytically, I further clarify the impact of the strength of the real cost channel on spending multipliers. I also furnish a primary model and confirm the robust role of the real cost channel on multipliers.

Organization.—I will specify the prototypical forward-looking NK model with the real cost channel in Section 2 and provide an analytical analysis using a two-state Markov chain on government spending multipliers. In Section 3, I furnish a baseline model to explore the general properties of long-run government spending effects. Another extension with bounded rationality is conducted in Section 4. Finally, this paper concludes in Section 5.

2 The Baseline Model with the Real Cost Channel

Recent empirical evidence in Abo-Zaid (2022) shows that the existence of the cost channel can influence government spending multipliers. As in Ravenna and Walsh (2006) and Surico (2008), the main contribution of the cost channel is that the interest rate can influence borrowing costs and the marginal cost function. In this paper, I follow Beaudry et al. (2022) and Nie (2021) to utilize the NK model with the real cost channel, which means expected real interest rates can impact firms' marginal costs, and explore short-run government spending multipliers analytically.

2.1 Private Sector Behavior

I use a prototypical forward-looking NK model with the real cost channel as in Beaudry et al. (2022) and Nie (2021). The behavior of an aggregate demand (AD) side economy can be summarized in the standard log-linearized Euler equation:

$$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\sigma_c}[R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1} - r_t^n], \tag{1}$$

where c_t is the private consumption, σ_c is the risk aversion coefficient, R_t is the nominal interest rate *in level*, π_t is inflation, $\mathbb{E}_t$ is the rational expectation operator, and r_t^n is the demand shock (also the natural rate shock).

The linear resource constraint in this economy is

$$y_t = (1 - s_g)c_t + g_t, \tag{2}$$

where y_t is the output gap, s_g is the fraction of government spending in total production, and g_t is the government spending.[8] In this case, one can obtain the path of y_t by substituting the resource constraint into the Euler equation below:

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma}[R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1} - r_t^n] + g_t - \mathbb{E}_t g_{t+1}, \tag{3}$$

where $\sigma = \frac{\sigma_c}{1-s_g}$.

The aggregate supply (AS) side of the economy can be summarized in the following log-linearized NK Phillips Curve.[9]

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \left[\gamma_y y_t + \gamma_g g_t + \gamma_r (R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1})\right], \tag{4}$$

where β is the subjective discount factor, and κ is the elasticity of inflation with regard to marginal cost. γ_y, γ_g, γ_r are the elasticity of marginal cost elasticity with regard to the output gap, government spending, and the expected real interest rate, respectively.[10] It is of note that γ_r in equation (4) can be seen as the strength of the cost channel which controls the impact of this channel. This model with the real cost channel can collapse to the conventional one without the cost channel if $\gamma_r = 0$.[11] In addition, it can nest the model with the nominal cost channel (equation (6)) if we assume that nominal interest rates are introduced in firms' borrowing costs as in Ravenna and Walsh (2006) and then the expected disinflation term ($-\mathbb{E}_t \pi_{t+1}$) of equation (4) disappears.

It is assumed that the central bank sets the nominal interest rate following the (truncated) Taylor (1993)-type rule with the ZLB:

$$R_t = \max\{0, -\log(\beta) + \phi_\pi \pi_t\}. \tag{5}$$

2.1.1 Real versus Nominal Cost Channel

The Phillips Curve with the nominal cost channel is

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \left[\gamma_y y_t + \gamma_g g_t + \gamma_r (R_t + \log(\beta))\right]. \tag{6}$$

Equation (6) is used in most previous papers working as the nominal cost channel

[8] Following Christiano et al. (2011), I define $g_t = (G_t - G)/Y$.

[9] The derivation of the NK Phillips Curve with the real cost channel can be seen in Appendix A.

[10] See Appendix A for exact expressions for these parameters.

[11] See Gertler et al. (1999) and Woodford (2003).

specifications such as Ravenna and Walsh (2006) and Surico (2008). In these papers, the nominal interest rate is introduced in borrowing costs. Even though the specifications in equations (4) and (6) can be seen as the cost channel, the real cost channel in equation (4) features expected real interest rates in borrowing costs and highlights one additional episode of expected disinflation denoted by the negative $\mathbb{E}_t\pi_{t+1}$ term. As in Beaudry et al. (2022), compared to the nominal cost channel, the real cost channel can obtain more support from the US data. This motivates us to use this setting in this paper.

It is of note that the two cost channels can have similar effects in normal times if the central bank follows a simple Taylor rule in equation (5). In this case, a cost-push shock endogenously emerges in the two cases as in Ravenna and Walsh (2006), and the two channels both increase firms' marginal costs and inflation.

An interesting insight at the ZLB with $R_t = 0$ is that the nominal cost channel as in Ravenna and Walsh (2006) and Surico (2008) cannot influence the slope of the Phillips Curve. However, it is of note that, at the ZLB, the real cost channel as in Beaudry et al. (2022) can rotate the Phillips Curve with expectations of disinflation. As a result, the Phillips Curve with the real cost channel is flatter than in the standard NK model without this real channel and this may explain a declining slope of the empirical Phillips Curve.

2.2 Quick Tour: Normal Times and ZLB

In this section, I employ a two-state static Markov chain as in Eggertsson et al. (2003) to deal with the policy shocks vector $[r_t^n, g_t]$. It is assumed that the specific policy shock (for example, the demand shock r_n^n in this section) remains at the current short-run state with a persistence p and then reverts to the steady-state *i.e.* $r_t^n = 0$ with a probability $1-p$.[12] Since the NK model with the real cost channel in this paper is forward-looking, one can show the expected output gap and inflation as follows:

$$\mathbb{E}_t y_{t+1} = p y_t, \qquad \mathbb{E}_t \pi_{t+1} = p\pi_t. \tag{7}$$

Assumption 1. *I assume that the NK Phillips Curve with the real cost channel is always*

[12]The duration of the short run state can be calculated as $T = \frac{1}{1-p}$. For instance, if $p = 0.5$, $T = \frac{1}{1-0.5} = 2$ quarters.

upward sloping in a (π_t, y_t) graph such that

$$p < \frac{1 - \kappa\gamma_r\phi_\pi}{\beta - \kappa\gamma_r} = \overline{p}^c. \tag{8}$$

As proposed by Laubach and Williams (2003), Cochrane (2017), Han et al. (2020) and Nie and Roulleau-Pasdeloup (2022), there is an implicit condition that the NK Phillips Curve is upward sloping in a (π_t, y_t) graph as in Assumption 1. Compared with the NK model with the nominal cost channel as in Christiano et al. (2005) and Ravenna and Walsh (2006), this paper utilizes a more empirically relevant real cost channel proposed in Beaudry et al. (2022) and further extended in Nie (2021).

In this section, we discuss two cases which are the economy in normal times without the ZLB binding and also in liquidity traps. There is a threshold of demand shock to trigger the ZLB constraint binding. From the Taylor (1993)-type rule, one can see that if the item $\{-\log(\beta) + \phi_\pi \pi_S\}$ is less than or equal to zero, the NK economy can be binding with the ZLB state. If not, the economy is in normal times and nominal interest rates can be free to adjust with the central bank's monetary policy regulation. If the (negative) natural rate shock is too large, the economy can be caught up in liquidity traps. Thereby there is a boundary condition for the natural rate shock r_S^n in the short run to trigger the economy into a state with the ZLB binding.

Proposition 1. *The boundary condition relationship among the three models is*

$$\underline{r_S^{n,B}} < \underline{r_S^n} < \underline{r_S^{n,N}}, \tag{9}$$

where $\underline{r_S^{n,B}}$ is the boundary condition without the cost channel, $\underline{r_S^n}$ is with the real cost channel, and $\underline{r_S^{n,N}}$ is with the nominal cost channel.

Proof. See Appendix B. □

As in Proposition 1, the boundary condition to trigger the ZLB binding with the real cost channel is larger than in the conventional NK model without the cost channel since the real cost channel features the supply-side effects of interest rates. In other words, the economy can exist more easily in liquidity traps with the real cost channel than in the traditional model. On the other hand, the boundary condition with the nominal cost channel is larger than in the model with the real cost channel due to expectations of disinflation. Therefore, among the three models, the model with the nominal cost channel is the most easily entrapped in liquidity traps.[13]

In the following sections, I will focus on government spending shocks and discuss the issues of the output gap and inflation multipliers.

2.3 Government Spending: Theoretical Analysis

To have transparent results, I abstract from demand shocks and focus only on the effects of government spending shocks with the real cost channel. First, I compare the multiplier relationship among the three models. Second, I deliver the general property of the strength of the cost channel on the multiplier by using a simple Markov chain closed-form solution.

2.3.1 Government Spending Multipliers in Normal Times

I assume that the positive government spending shock $g_S > 0$ follows the Markov process. It starts in the short run, stays with the persistence probability p, and returns to the steady-state $g_L = 0$ in the long run with a probability $1-p$. If the short-run economy is in normal times, I can rewrite the Euler equation (3) and the Phillips Curve with the real cost channel (equation (4)):

$$y_S = -\frac{1}{\sigma(1-p)}(\phi_\pi - p)\pi_S + g_S \tag{10}$$

$$\pi_S = \kappa \frac{\gamma_y}{1-\beta p - \kappa\gamma_r(\phi_\pi - p)} y_S + \kappa \frac{\gamma_g}{1-\beta p - \kappa\gamma_r(\phi_\pi - p)} g_S. \tag{11}$$

Fiscal multipliers.—It can be found that a positive government spending shock can move the Euler equation upward and turn down the Phillips Curve in a (π_t, y_t) graph. The solutions with the real cost channel of the output gap multiplier $\mathcal{M}^O_{S,N}$ and the inflation multiplier $\mathcal{M}^I_{S,N}$ can be generated below:

$$\mathcal{M}^O_{S,N} = \frac{\sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)] - \kappa\gamma_g(\phi_\pi - p)}{\sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)] + \kappa\gamma_y(\phi_\pi - p)} \tag{12}$$

$$\mathcal{M}^I_{S,N} = \frac{\left[1 + \frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p)}{\sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)] + \kappa\gamma_y(\phi_\pi - p)}. \tag{13}$$

[13] In this paper, the three models refer to the conventional model without the cost channel, the model with the nominal cost channel, and the model with the real cost channel.

The expected real interest rate can be in both the denominator and numerator of the output gap multiplier since the new cost channel here can influence the inflation rate and further impact the output gap through expectations. Regarding the inflation multiplier, the expected real interest rate is included in the denominator since the cost channel can impact this multiplier directly. In normal times, since the denominator of the multiplier equation is higher than the numerator, the multiplier is less than one.[14]

In normal times, the real cost channel can decrease the output gap multiplier, and the intuition is simple. In the short run, positive government spending shocks can increase inflation.[15] Additionally, the real cost channel can increase borrowing costs and inflation.[16] In normal times, the nominal interest rate is flexible to adjust and increases by more than one-for-one with inflation; in addition, higher nominal interest rates arise due to a rise in inflation triggered by this real channel. This results in higher real interest rates than in the standard model without the real channel, leading to less private consumption, and thus a lower output gap multiplier. Above all, the output gap multiplier with the real cost channel is less than that without it whereas the inflation multiplier is larger.

As in Appendix C, compared to the nominal cost channel, the inflation multiplier with the real cost channel can be smaller. If we compare the NK Phillips Curve in equations (6) and (4), less influence is triggered by the real cost channel, which can echo the fact of lower inflation in the Euro area in normal times (Koester et al. (2021)). In this scenario, the nominal cost channel can further reduce the output gap multiplier compared to the real cost channel.

I also explore the effects of the strength of the real cost channel γ_r on spending multipliers. The power of the real cost channel can directly leverage the increment of inflation. On the other hand, we can see that higher inflation can crowd out more private consumption, hence leading to a much lower output gap multiplier given a stronger real cost channel. Therefore, the output gap multiplier decreases in γ_r whereas the inflation multiplier increases in γ_r. I summarize the above theoretical results in Proposition 2.

[14] Government spending in normal times can crowd out private consumption, echoing the classical empirical evidence as in Amano and Wirjanto (1997) and Barro and Redlick (2011).

[15] An increase in government spending requires households to produce more, leading to longer working hours. To compensate for this, households demand a higher real wage. This, in turn, increases firms' marginal costs, causing prices, which are set as a markup over the marginal cost, to also increase. This results in inflation.

[16] This mechanism conforms with Ravenna and Walsh (2006) such that cost-push shocks can emerge endogenously in the NK model with the cost channel.

Proposition 2. *In normal times, the output gap multiplier $\mathcal{M}^O_{S,N}$ decreases in γ_r whereas the inflation multiplier $\mathcal{M}^I_{S,N}$ increases in γ_r. The output gap multiplier relationship among the three models is*

$$\mathcal{M}^{O,N}_{S,N} < \mathcal{M}^O_{S,N} < \mathcal{M}^{O,B}_{S,N}, \tag{14}$$

and the inflation multiplier relationship is

$$\mathcal{M}^{I,B}_{S,N} < \mathcal{M}^I_{S,N} < \mathcal{M}^{I,N}_{S,N}, \tag{15}$$

where in normal times, $\mathcal{M}^{i,N}_{S,N}$ $(i \in \{O,I\})$ denotes the multiplier of the model with the nominal cost channel, and $\mathcal{M}^{i,B}_{S,N}$ denotes the multiplier without the cost channel.

Proof. See Appendix C. □

2.3.2 Government Spending Multipliers at ZLB

In this part, I focus on the case when the short-run economy binds with the ZLB.[17] The Euler equation (3) and the Phillips Curve with the real cost channel (equation (4)) can be rewritten as:

$$y_S = -\frac{1}{\sigma(1-p)}[\log(\beta) - p\pi_S] + g_S \tag{16}$$

$$\pi_S = \frac{\kappa\gamma_y}{1-\beta p+\kappa\gamma_r p} y_S + \kappa\frac{\gamma_g}{1-\beta p+\kappa\gamma_r p} g_S + \frac{\kappa\gamma_r \log(\beta)}{1-\beta p+\kappa\gamma_r p}. \tag{17}$$

Fiscal multipliers.—In liquidity traps, one can use the Euler equation and the Phillips Curve to obtain the solutions with the real cost channel of the output gap multiplier $\mathcal{M}^O_{S,Z}$ and the inflation multiplier $\mathcal{M}^I_{S,Z}$:

$$\mathcal{M}^O_{S,Z} = \frac{\sigma(1-p)[1-\beta p+\kappa\gamma_r p] + \kappa\gamma_g p}{\sigma(1-p)(1-\beta p+\kappa\gamma_r p) - \kappa\gamma_y p} \tag{18}$$

$$\mathcal{M}^I_{S,Z} = \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p)}{\sigma(1-p)(1-\beta p+\kappa\gamma_r p) - \kappa\gamma_y p}. \tag{19}$$

From the solutions, the expected real interest rate can be in both the denominator and numerator of the output gap multiplier. The new cost channel here can influence the inflation rate and further impact the output gap through expectations. The real cost

[17] In this paper, the zero (effective) lower bound is a state when $R_t = 0$ with the simple assumption that there is no cash storing cost in Galí (2015).

channel can impact the inflation multiplier directly through the NK Phillips Curve.

In liquidity traps, the denominator of the multiplier equation is lower than the numerator; thus, the output gap at the ZLB is larger than one. At the ZLB, with no cost channel, nominal interest rates remain unchanged, and an increase in government spending can generate inflation. Therefore government spending leads to a drop in real interest rates, which, in turn, gives incentives to households to save less and consume more. This can be seen as the crowding in effects as in Bouakez et al. (2017).

However, following positive spending shocks, the real cost channel can decrease short-run inflation through expectations of disinflation (in equation (4)) in firms' real borrowing costs. Intuitively, lower marginal costs arise due to the expected disinflation during liquidity trap episodes. Further, expectations of disinflation can translate into a realized lower inflation rate through rational expectations and sticky prices.[18] In this case, the real cost channel can make the inflation rate rise by less than in the conventional model following positive spending shocks, and there is a smaller drop in real interest rates. Higher real interest rates due to the real cost channel can depress not only people's appetite for consumption but also decrease production activity, which results in a decline in the effects of government spending on output. Hence compared to the standard NK model without the real channel, both the output gap multiplier and the inflation multiplier with the real cost channel are lower.

On the other hand, as explained in Section 2.1.1, the nominal cost channel cannot modify the NK Phillips Curve slope at the ZLB. Spending multipliers with this nominal channel can be invariant with the standard NK model since this channel in liquidity traps cannot be included in the partial derivative of government spending to the output gap/inflation in the calculation of fiscal multipliers.

I next discuss the effects of the strength of the real cost channel γ_r on spending multipliers. The stronger the power of the real cost channel, the lower the inflation rate due to expectations of disinflation in liquidity traps. In this case, higher real interest rates due to lower inflation can depress people's appetite for consumption, which results in more decline in the output gap multiplier. In a nutshell, the inflation and the output gap multipliers with the real cost channel decrease in γ_r. I summarize the main theoretical results in Proposition 3.

[18] In other words, assuming firms care about expected real interest rates in liquidity traps, expectations of disinflation can reduce actual inflation.

Proposition 3. *At the ZLB, both the output gap multiplier $\mathcal{M}^{O}_{S,Z}$ and the inflation multiplier $\mathcal{M}^{I}_{S,Z}$ decrease in γ_r. The output gap multiplier relationship among the three models is*

$$\mathcal{M}^{O}_{S,Z} < \mathcal{M}^{O,N}_{S,Z} = \mathcal{M}^{O,B}_{S,Z}, \tag{20}$$

and the inflation multiplier relationship is

$$\mathcal{M}^{I}_{S,Z} < \mathcal{M}^{I,N}_{S,Z} = \mathcal{M}^{I,B}_{S,Z}, \tag{21}$$

where in liquidity traps, $\mathcal{M}^{i,N}_{S,Z}$ ($i \in \{O,I\}$) denotes the multiplier of the model with the nominal cost channel, and $\mathcal{M}^{i,B}_{S,Z}$ denotes the multiplier without the cost channel.

Proof. See Appendix D. □

Comparison with the literature.—Our study contrasts with that of Abo-Zaid (2022) who shows that higher borrowing costs with the nominal cost channel can increase inflation which makes the output gap multiplier larger than in the standard model at the ZLB. As in Abo-Zaid (2022), the Phillips Curve with the nominal cost channel at the ZLB is steeper in a (y_t, π_t) graph than in the standard NK model.[19] In this paper, I stress that inflation in the short run with the real cost channel can be lower due to expected disinflation in the real borrowing cost.[20] Thus, I can use the real cost channel to explain lower spending multipliers in liquidity traps as in Ramey and Zubairy (2018).

Taking stock of the denominator of the multiplier.—As discussed extensively in Bilbiie (2008), Mertens and Ravn (2014), Boragan Aruoba et al. (2018), Lustenhouwer (2020), and Nie and Roulleau-Pasdeloup (2022), the denominator of the multiplier in the standard NK model at the ZLB can be negative due to a highly persistent p.[21] In this case, the output gap and inflation multipliers are negative in equations (18)-(19).

However, as described at length in Nie (2022), in the presence of the real cost channel, the denominator with the ZLB binding is less likely to be negative than in the standard model.[22] I borrow the result from Nie (2022), Proposition 3, p. 16: The denominator can

[19] Abo-Zaid (2022) differentiates between the policy rate and the loan rate, making the Phillips Curve steep in recessions. In that setup, if the two rates are equal, the nominal cost channel cannot impact the Phillips Curve slope in liquidity traps and cannot impact spending multipliers.

[20] The Phillips Curve is locally flat with the real cost channel during recessions as in Beaudry et al. (2022). Besides, Hazell et al. (2020) empirically document that the NK Phillips Curve is flat during the Great Recession.

[21] The denominator of the multiplier in the standard model is $\mathcal{D}^{B}_{Z} = \sigma(1-p)(1-\beta p) - \kappa\gamma_y p$ and can be negative with a large p.

[22] The denominator of the multiplier with the real cost channel is $\mathcal{D}_Z = \sigma(1-p)(1-\beta p + \kappa\gamma_r p) - \kappa\gamma_y p$

be always positive if assuming the elasticity of marginal costs w.r.t. the output gap γ_y is sufficiently small and meets the following condition:

$$\gamma_y < \Gamma(\gamma_r), \tag{22}$$

where $\Phi(\gamma_r) = \frac{(\beta-\kappa\gamma_r-1+\kappa\gamma_r\phi_\pi)\gamma_r\phi_\pi(\beta-\kappa\gamma_r)}{\sigma_r(1-\kappa\gamma_r\phi_\pi)}$ increases in γ_r.—See Appendix E.

As in condition (22), γ_y should be lower than the composite parameter $\Gamma(\gamma_r)$. Assuming that γ_y is small enough for a given γ_r, this condition can always hold. In particular, the assumption with a low γ_y is in line with the empirical finding as in Beaudry et al. (2022): γ_y is very small while the key parameter γ_r leveraging the strength of the real cost channel is much larger than γ_y.

Bearing this in mind, in the following numerical exercise, I will consider only the case in which the denominator of the spending multiplier is positive.

2.4 Numerical Results—Benchmark

In this paper, I follow Budianto et al. (2020), Roulleau-Pasdeloup (2021b), and Beaudry et al. (2022) to set the parameterization reported in Table 1.[23]

Table 1: Parameterization

Subjective discount factor	$\beta = 0.99$
Inverse of Frisch elasticity	$\eta = 1$
Risk aversion coefficient	$\sigma_c = 1$
Steady-state ratio of government spending to output	$s_g = 0.2 \times 0.23$
Elasticity of inflation w.r.t. real marginal cost	$\kappa = 0.2$
Elasticity of real marginal cost w.r.t. output gap	$\gamma_y = 0.2$
Elasticity of real marginal cost w.r.t. interest rate	$\gamma_r = 1$
Elasticity of real marginal cost w.r.t. government spending	$\gamma_g = -0.1$
Inflation parameter in Taylor rule	$\phi_\pi = \phi_\pi^q = 1.5$
Persistence specification	$p = 0.7$

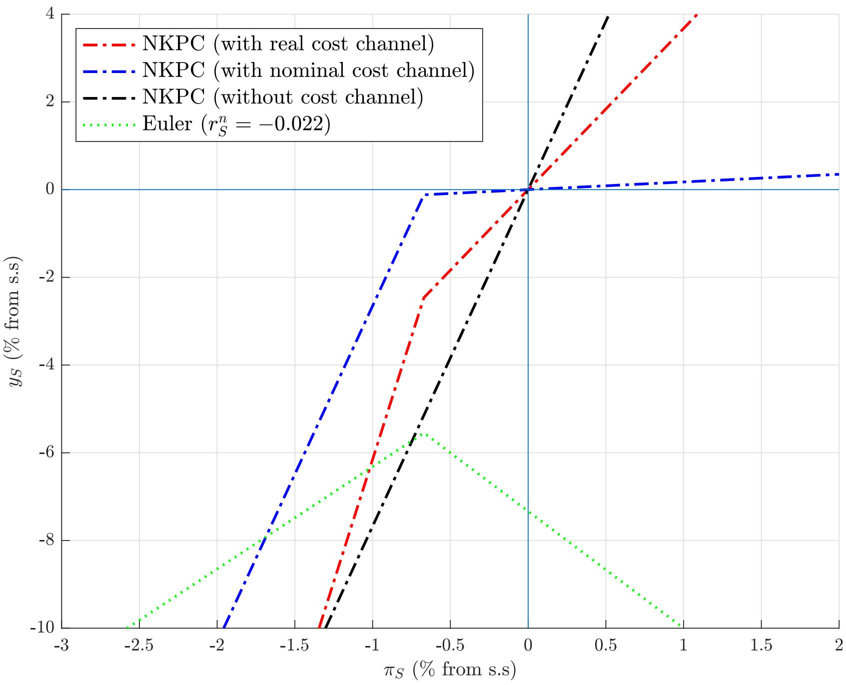
I first provide numerical AS/AD figures with a contractionary natural rate shock in three NK Phillips Curves (NKPC) shown in Figure 1. If there is a temporary short-term

and one can show that $\mathscr{D}_Z = \mathscr{D}_Z^B + \sigma(1-p)\kappa\gamma_r p$. Therefore, $\mathscr{D}_Z$ is larger than $\mathscr{D}_Z^B$ and less likely to be negative.—See Nie (2022) for details.

[23]To be more specific, the calibration method for the real cost channel closely follows Beaudry et al. (2022), Figure 4, p. 13. The other parameters are calibrated in a standard way as in Budianto et al. (2020) and Roulleau-Pasdeloup (2021b). In addition, these parameters can ensure that the denominator of the multiplier is positive.

natural rate shock r^n_S (-2.2%), which exceeds the boundary condition, the economy can be in liquidity traps.[24] In this case, the conventional monetary policy cannot work since the nominal interest rate is bounded at zero. Deflationary pressure can lead to higher real interest rates and further stimulate people to save but consume less. From Figure 1, it is clear that the real cost channel can change the Phillips Curve slope in liquidity traps and the Phillips Curve is locally flat as in Beaudry et al. (2022). The curve with the nominal cost channel at the ZLB shares the same slope as the conventional model. The numerical results also echo Proposition 1 that the case with the nominal cost channel is the most easily entrapped in liquidity traps among the three models. This Proposition can be further confirmed in the impulse response to a contractionary natural rate shock.—See Appendix F.

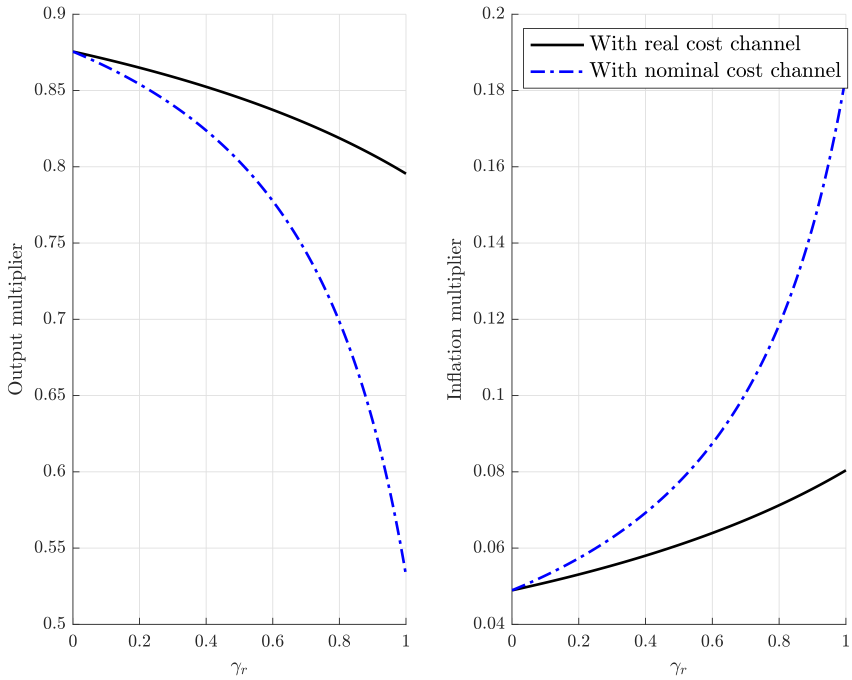
Figure 1: AS/AD curves with natural rate shock in three models



I compare spending multipliers of the models in normal times without the cost channel ($\gamma_r = 0$) vs. with the cost channel as in Ravenna and Walsh (2006) vs. with the real cost channel in Figure 2. It can be seen that the output gap multiplier is lower than one as in Lewis (2021) and it decreases in γ_r. On the contrary, the inflation multiplier increases in γ_r. It is observed that the output gap multiplier with the real cost channel is larger than that with the nominal cost channel while less than its counterpart without

[24] This calibration mimics, for example, the case of the Great Recession.

Figure 2: Spending multipliers in normal times



the cost channel. This can echo the theoretical analysis in Proposition 2. See Appendix G for numerical results w.r.t. the persistence p in the three models. These numerical results robustly echo Proposition 2. I also find that the output gap multiplier decreases in p, whereas the inflation multiplier is higher with the duration of increment of time.

Figure 3: Spending multipliers in liquidity traps

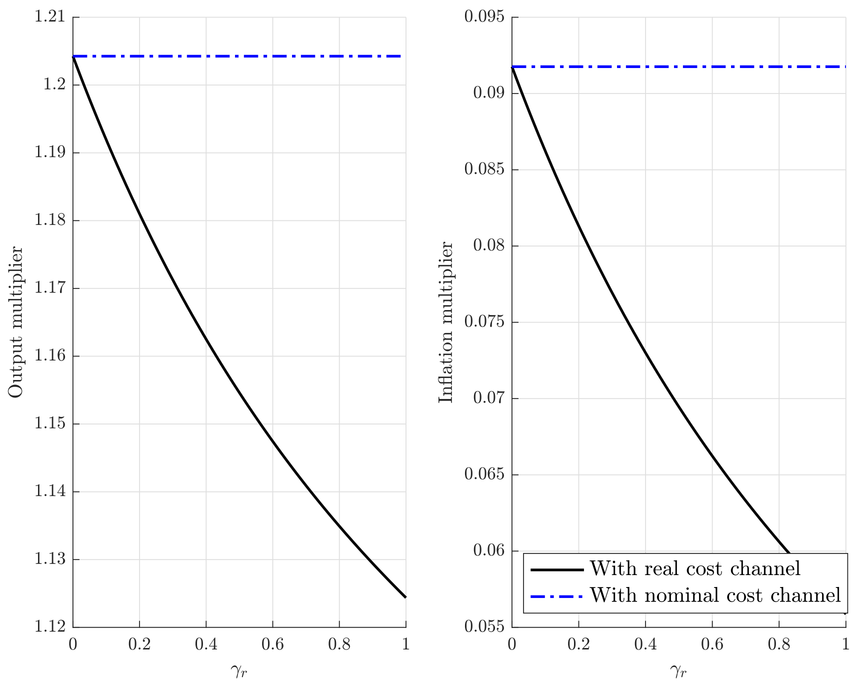


Figure 3 shows spending multipliers in liquidity traps. The simulation results can perfectly match our theoretical analysis in Proposition 3. On the one hand, one can see that the output gap multiplier is productive as in Christiano et al. (2011) and Schmidt (2017). The real cost channel can attenuate the output gap and inflation multipliers simultaneously. The spending multipliers decrease in γ_r. The nominal cost channel multipliers are invariant with the standard NK model without the cost channel. See Appendix G for numerical results w.r.t. p in the three models and these results further confirm our theoretical analysis. In addition, the output gap and inflation multipliers increase in p.

3 Government Spending: Long-Run Policy

As in recent contributions by Sarin et al. (2021) and Roulleau-Pasdeloup (2021b), long-run policy effects are usually overlooked or often even computed numerically in the literature. In this section, I follow Roulleau-Pasdeloup (2021a) to use a three-state Markov chain to assess long-run government spending analytically.

3.1 Policy and Shocks

I assume that government spending is longer-lived than demand shock. To be more specific, government spending in the short run g_S can merge into medium-run government spending g_M with a persistence q and then collapse to the steady-state $g_L = 0$ with a probability $1-q$. The natural rate shock r_S^n operates in the short run with a probability p and then returns to the long run $r_L^n = 0$ with a probability $1-p$. The graphical representation of our policy can be seen in Figure 4.

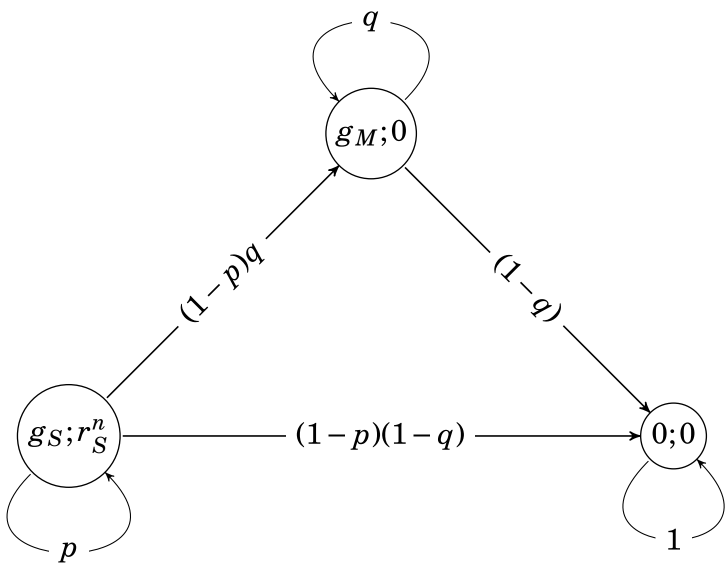
With this in mind, for current monetary policy, I use an adapted Taylor rule:[25]

$$R_t = \begin{cases} \max\left[0; -\log(\beta) + \phi_\pi \pi_S\right] & \text{In the short run} \\ -\log(\beta) + \phi_\pi^q \pi_M & \text{In the medium run} \\ -\log(\beta) & \text{In the long run} \end{cases} \tag{23}$$

One can use the above equation (23) to trace the path of government spending. In the

[25] In this section, to be in line with Section 2, I consider two economic states in the short run, which are normal times and the ZLB. In the medium run, I simply assume the demand shock reverts to zero and the economy is in normal times.

Figure 4: Long run government spending: a graphical representation



medium run, the output gap and inflation can be expressed as the product of medium-run multipliers and medium-run government spending as follows:

$$y_M = \mathcal{M}^O_M \cdot g_M \ \ \& \ \ \pi_M = \mathcal{M}^I_M \cdot g_M, \tag{24}$$

where $\mathcal{M}^O_M$ is the medium-run output gap multiplier and $\mathcal{M}^I_M$ is the medium-run inflation multiplier. See appendix H for the medium–run spending multipliers.[26] In this case, since the model is forward-looking, I can generate the expected output gap below, and one can show the expected inflation using the same manner.

$$\begin{aligned}\mathbb{E}_S y_{t+1} &= p y_S + (1-p) q y_M \\ &= p y_S + (1-p) q \mathcal{M}^O_M g_M \\ &= p y_S + (1-p) q \mathcal{M}^O_M \zeta g_S. \end{aligned} \tag{25}$$

3.2 Long-run Government Spending: Theoretical Analysis

This section discusses the analytical results regarding the government spending multiplier with a three-state Markov structure.

[26] I follow a simple rule in Nie and Roulleau-Pasdeloup (2022) to deal with the medium-run shock: It is generally assumed that medium-run spending is contingent on short-run spending but it is lower than short-run spending such that $g_M = \zeta g_S$, where ζ is a discount parameter.

3.2.1 Multipliers with Long-run Government Spending in Normal Times

If the *short-run economy is in normal times*, the Euler equation can be regenerated with consideration of long-run government spending using a three-state Markov chain:

$$y_S = -\frac{1}{\sigma}\frac{\phi_\pi - p}{1-p}\pi_S + \Theta_{AD} g_s \tag{26}$$

$$\Theta_{AD} = q\zeta(\mathcal{M}_M^O + \frac{1}{\sigma}\mathcal{M}_M^I - 1) + 1, \tag{27}$$

where Θ_{AD} is the government spending shock shift in the Euler equation, ζ is the policy discount parameter, $\mathcal{M}_M^O$ and $\mathcal{M}_M^I$ are the medium-run policy multiplier as in equation (24). For reference, this shift without long-run government spending will collapse to 1, which can nest the case in our baseline model in Section 2. The new items in this shift are from *rational expectations* of the output gap, inflation, and medium-run spending shock. Note that the first new term is from future wealth effects (higher expected output gap in the future) as in Bouakez et al. (2017) and the household has consumption incentives due to consumption smoothing. The second term comes from the fact that government spending, in the long run, can increase firms' marginal costs and inflation. The third term is due to the direct demand effect from future government spending. See Appendix I; it turns out that $q\zeta(\mathcal{M}_M^O + \frac{1}{\sigma}\mathcal{M}_M^I - 1)$ is negative which means the overall expected effects from longer spending can crowd out the present output. In addition, the effects of long-run government spending can be controlled by the policy product $q\zeta$.

On the other hand, the Phillips Curve with long-run government spending can be shown below:

$$\pi_S = \kappa\frac{\gamma_y}{1-\beta p - \kappa\gamma_r(\phi_\pi - p)} y_S + \frac{1}{1-\beta p - \kappa\gamma_r(\phi_\pi - p)}\Theta_{AS} g_s \tag{28}$$

$$\Theta_{AS} = (\beta - \kappa\gamma_r)(1-p)q\zeta\mathcal{M}_M^I + \kappa\gamma_g, \tag{29}$$

where Θ_{AS} is the government spending shock shift in the Phillips Curve, ζ is the policy discount parameter, $\mathcal{M}_M^I$ is the medium run inflation multiplier as in equation (24). For reference, this shift without long-run government spending will collapse to $\kappa\gamma_g$, which is the same as in our baseline model in Section 2. The new items in this shift are from *rational expectations* of inflation and long-run government spending. As seen in Appendix I, it turns out that long-run spending can increase firms' marginal costs and inflation. Similar to the case in the Euler equation, the effects of long-run policy can be controlled by the policy product $q\zeta$.[27]

One can use the Euler equation and the Phillips Curve to produce the output gap multiplier $\mathcal{M}^{O,long}_{S,N}$ and the inflation multiplier $\mathcal{M}^{I,long}_{S,N}$:

$$\mathcal{M}^{O,long}_{S,N} = \frac{\Theta_{AD}\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi - p)] - \Theta_{AS}(\phi_\pi - p)}{\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi - p)] + \kappa\gamma_y(\phi_\pi - p)} \tag{30}$$

$$\mathcal{M}^{I,long}_{S,N} = \frac{\left(\kappa\gamma_y\Theta_{AD} + \Theta_{AS}\right)\sigma(1-p)}{\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi - p)] + \kappa\gamma_y(\phi_\pi - p)}. \tag{31}$$

Long-run government spending can increase short-run inflation through rational expectations and sticky prices. In normal times, the central bank increases nominal interest rates to combat higher prices caused by inflation. In that way, long-run spending policy can crowd out more private consumption due to higher short-run real interest rates. Additionally, the real cost channel can increase marginal costs and inflation. Thus, the inflation multiplier can increase depending on the strength of the real cost channel γ_r, and this result is the same as the case in Section 2. Similarly, the real cost channel can lower the output gap. In summary, the output gap multiplier reduces in γ_r whereas the inflation multiplier grows in γ_r.—See a formal proof in Appendix J.

3.2.2 Multipliers with Long-run Government Spending at ZLB

If the *short-run economy is at the ZLB*, the Euler equation with long-run spending can be given by:

$$y_S = -\frac{1}{\sigma(1-p)}[\log(\beta) - p\pi_S] + \Theta_{AD} g_s \tag{32}$$

$$\Theta_{AD} = q\zeta(\mathcal{M}^O_M + \frac{1}{\sigma}\mathcal{M}^I_M - 1) + 1. \tag{33}$$

where Θ_{AD} is the government spending shock shift in the Euler equation. The long-run government spending terms $q\zeta(\mathcal{M}^O_M + \frac{1}{\sigma}\mathcal{M}^I_M - 1)$ are negative and can be controlled by the policy product $q\zeta$.

In addition, I move to describe the Phillips Curve with long-run government spending

[27] In this section, I assume that κ is very small in our theoretical analysis which is in line with Eggertsson (2011), Gabaix (2020) and Budianto et al. (2020). In this sense, one can assume $\beta - \kappa\gamma_r > 0$ such that long-run government spending can increase inflation here. However, if $\beta - \kappa\gamma_r < 0$, this might resolve the fiscal price puzzle (FPP) as in Han et al. (2020) that a long-run fiscal stimulus can lower inflation .

as:

$$\pi_S = \kappa \frac{\gamma_y + \gamma_r \log(\beta)}{1-\beta p + \kappa\gamma_r p} y_S + \frac{1}{1-\beta p + \kappa\gamma_r p}\Theta_{AS} g_s \tag{34}$$

$$\Theta_{AS} = (\beta - \kappa\gamma_r)(1-p)q\zeta\mathscr{M}^I_M + \kappa\gamma_g. \tag{35}$$

where Θ_{AS} is the government spending shock shift in the Phillips Curve and can be controlled by the policy product $q\zeta$.

With this in mind, I use the Euler equation and the Phillips Curve in liquidity traps to produce the output gap multiplier $\mathscr{M}^{O,long}_{S,Z}$ and the inflation multiplier $\mathscr{M}^{I,long}_{S,Z}$:

$$\mathscr{M}^{O,long}_{S,Z} = \frac{\Theta_{AD}\sigma(1-p)(1-\beta p + \kappa\gamma_r p) + \Theta_{AS} p}{\sigma(1-p)(1-\beta p+\kappa\gamma_r p) - \kappa\gamma_y p} \tag{36}$$

$$\mathscr{M}^{I,long}_{S,Z} = \frac{\left(\kappa\gamma_y\Theta_{AD} + \Theta_{AS}\right)\sigma(1-p)}{\sigma(1-p)(1-\beta p+\kappa\gamma_r p) - \kappa\gamma_y p}. \tag{37}$$

Once the liquidity trap has subsided, the extension of government spending policy can cause inflation to increase. The nominal interest rate is zero, and higher inflation can help stimulate our NK economy since it can lower short-run real interest rates and then increase private consumption. At this time, the spending multiplier is larger, which is in line with Leduc and Wilson (2013) who empirically show that long-term spending effects are larger than in the short-term policy model. However, the real cost channel can reduce the effectiveness of long-run government spending. The cost channel can dampen inflation due to expectations of disinflation in the real borrowing cost. Thus, the output gap and inflation multipliers decrease as the real cost channel becomes stronger.—See the analytical analysis in Appendix K.

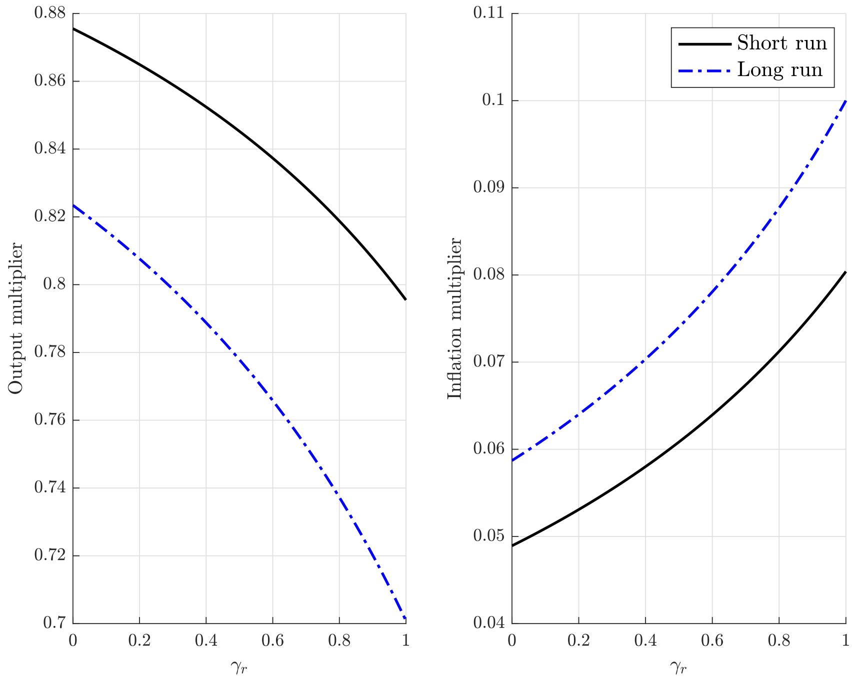
3.3 Numerical Results with Long-run Government Spending

The comparison of the numerical results of short-run and long-run spending policies that vary with the strength of the real cost channel γ_r is presented in Figure 5.[28] In normal times, it is observed that long-run government spending can decrease the output gap multiplier. In addition, long-run government spending can make the inflation multiplier larger than in the short-run policy. The output gap multiplier falls in γ_r while the inflation multiplier increases in γ_r. See Appendix L for numerical results w.r.t. the per-

[28] In our simulation results, it is assumed that $q = 0.7$ and $\zeta = 0.5$.

sistence p. I find that the output gap multiplier can be negative with long-run spending.

Figure 5: Spending multipliers with long-run spending policy in normal times

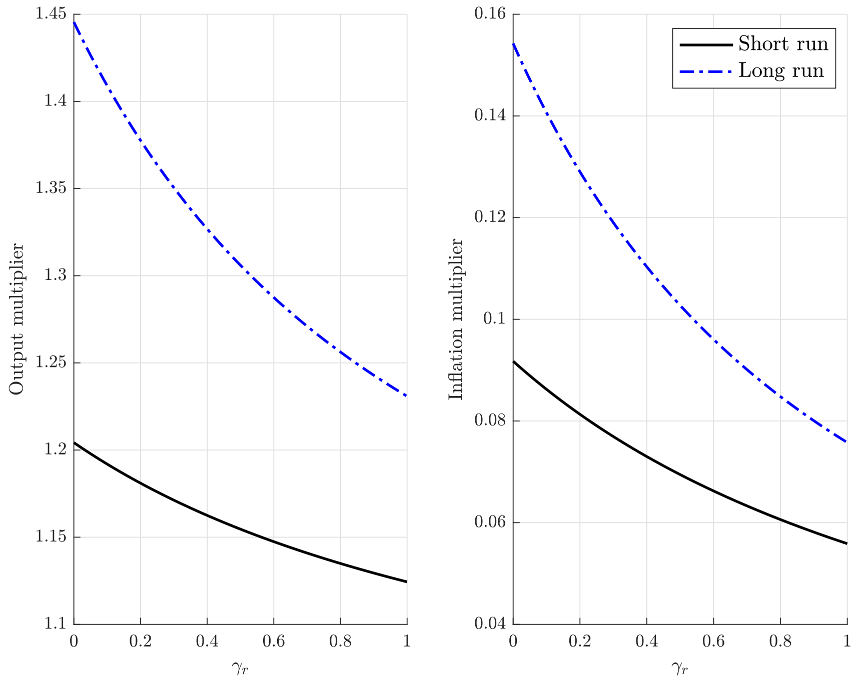


At the ZLB, the numerical results are reported in Figure 6. We find that long-run government spending can further increase the output gap and inflation multipliers. The two spending multipliers decrease in γ_r. Noteworthy is that the output gap multiplier can be more effective for a prolonged spending policy in recessions. Similar results can be found in Appendix L for numerical results w.r.t. p.

4 A Behavioral Model

As can be seen in Section 3, long-run government spending can drive spending multipliers at the ZLB. In our baseline model in Section 2, it is the consensus that agents have rational expectations. While in this section, I try to extend our baseline model to address the role of rational expectations. I finally develop a model similar to Gabaix (2020) and incorporated it into our simple benchmark setup. This model considers bounded rationality, where agents can be short-sighted about the world.

Figure 6: Spending multipliers with long-run spending policy at ZLB



4.1 The NK model with Bounded Rationality

I use the model in Gabaix (2020) to show the behavior of the aggregate demand-side economy:

$$c_t = \bar{m}\mathbb{E}_t c_{t+1} - \frac{1}{\sigma_c}(i_t - \mathbb{E}_t\pi_{t+1} - r^n), \tag{38}$$

where $\bar{m} \in [0,1]$ is the cognitive discounting parameter in Gabaix (2020).[29]

In this case, one can obtain the path of y_t by substituting the resource constraint into the Euler equation:

$$y_t = \bar{m}(1-s_g)\mathbb{E}_t y_{t+1} - \frac{1}{\sigma}(R_t + \log(\beta) - \mathbb{E}_t\pi_{t+1} - r^n) + g_t - \bar{m}(1-s_g)\mathbb{E}_t g_{t+1}, \tag{39}$$

where $\sigma = \frac{\sigma_c}{1-s_g}$.

As in Gabaix (2020), I can show the NK Phillips Curve with the real cost channel:

$$\pi_t = \beta\bar{m}\left[\varphi + \frac{1-\beta\varphi}{1-\beta\varphi\bar{m}}(1-\varphi)\right]\mathbb{E}_t\pi_{t+1} + \kappa\left[\gamma_y y_t + \gamma_g g_t + \gamma_r(R_t + \log(\beta) - \mathbb{E}_t\pi_{t+1})\right], \tag{40}$$

where $\varphi \in (0,1)$ is the share of firms which cannot adjust their prices.

[29] As in Gabaix (2020), this parameter can measure the attention to the future, which is a form of global cognitive discounting. Relative to rational expectations ($\bar{m} = 1$), when the behavioral agents contemplate future events, their expectations are geared to the steady state of the economy.

4.2 Government Spending with Bounded Rationality: Theoretical Results

This section presents the analytical results regarding government spending multipliers with bounded rationality.

4.2.1 Multipliers with Bounded Rationality in Normal Times

In normal times, I use the Euler equation and the Phillips Curve with the consideration of bounded rationality—see appendix M, to produce the solutions of the output gap multiplier $\mathcal{M}_{S,N}^{O,BR}$ and the inflation multiplier $\mathcal{M}_{S,N}^{I,BR}$ below:[30]

$$\mathcal{M}_{S,N}^{O,BR} = \frac{\sigma[1-p\bar{m}(1-s_g)]\left\{1-\beta p\bar{m}\left[\varphi+\frac{1-\beta\varphi}{1-\beta\varphi\bar{m}}(1-\varphi)\right]-\kappa\gamma_r(\phi_\pi-p)\right\}-\kappa\gamma_g(\phi_\pi-p)}{\sigma(1-p\bar{m}(1-s_g))\left\{1-\beta p\bar{m}\left[\varphi+\frac{1-\beta\varphi}{1-\beta\varphi\bar{m}}(1-\varphi)\right]-\kappa\gamma_r(\phi_\pi-p)\right\}+\kappa\gamma_y(\phi_\pi-p)} \tag{41}$$

$$\mathcal{M}_{S,N}^{I,BR} = \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma[1-p\bar{m}(1-s_g)]}{\sigma[1-p\bar{m}(1-s_g)]\left\{1-\beta p\bar{m}\left[\varphi+\frac{1-\beta\varphi}{1-\beta\varphi\bar{m}}(1-\varphi)\right]-\kappa\gamma_r(\phi_\pi-p)\right\}+\kappa\gamma_y(\phi_\pi-p)}. \tag{42}$$

In normal times, cognitive discounting can reduce the expectation effects in our baseline model which can lower inflation following a positive spending shock. This means that there is a lower rise in real interest rates, hence a higher output gap multiplier compared to the baseline model. In this case, the inflation multiplier with bounded rationality can be lower whereas the output gap multiplier can be higher[31]—see Appendix N for a formal proof. Since the introduced bounded rationality is independent of the strength of the real cost channel γ_r, the effects of this real channel on spending multipliers are the same as the baseline model results. Specifically, the output gap multiplier decreases in γ_r while the inflation multiplier increases.

Interestingly, as explained in appendix O, there exists a threshold value, beyond which the multiplier effects switch from decreasing in the persistence p to increasing. I also find that the output gap multiplier can be large (near one) in normal times if we suppose agents have relatively intense bounded rationality which means the expectation effects are extremely weak. Intuitively, agents tend to consume today but not to save since future consumption has less or no impact on today's decisions. In this case,

[30] For simplicity, in this section, a simple two-state Markov chain is used to calculate fiscal multipliers.

[31] Similar results can also arise with long-run government spending since the cognitive agent can decrease the expectation effects, which is in spirit with Nakata et al. (2019) and Farhi and Werning (2019).

the crowding-out effects of government spending should be much attenuated and the output gap multiplier can be near one. This result can echo the previous empirical evidence that the output gap multiplier can be large in normal times as in Auerbach and Gorodnichenko (2012) and Acconcia et al. (2014).

4.2.2 Multipliers with Bounded Rationality at ZLB

At the ZLB, one can use the Euler equation and the Phillips Curve with bounded rationality—see appendix M, to produce the output gap multiplier $\mathcal{M}_{S,Z}^{O,BR}$ and the inflation multiplier $\mathcal{M}_{S,Z}^{I,BR}$:

$$\mathcal{M}_{S,Z}^{O,BR} = \frac{\sigma[1-p\bar{m}(1-s_g)]\left\{1-\beta p\bar{m}\left[\varphi+\frac{1-\beta\varphi}{1-\beta\varphi\bar{m}}(1-\varphi)\right]+\kappa\gamma_r p\right\}+\kappa\gamma_g p}{\sigma[1-p\bar{m}(1-s_g)]\left\{1-\beta p\bar{m}\left[\varphi+\frac{1-\beta\varphi}{1-\beta\varphi\bar{m}}(1-\varphi)\right]+\kappa\gamma_r p\right\}-\kappa\gamma_y p} \tag{43}$$

$$\mathcal{M}_{S,Z}^{I,BR} = \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma[1-p\bar{m}(1-s_g)]}{\sigma[1-p\bar{m}(1-s_g)]\left\{1-\beta p\bar{m}\left[\varphi+\frac{1-\beta\varphi}{1-\beta\varphi\bar{m}}(1-\varphi)\right]+\kappa\gamma_r p\right\}-\kappa\gamma_y p}. \tag{44}$$

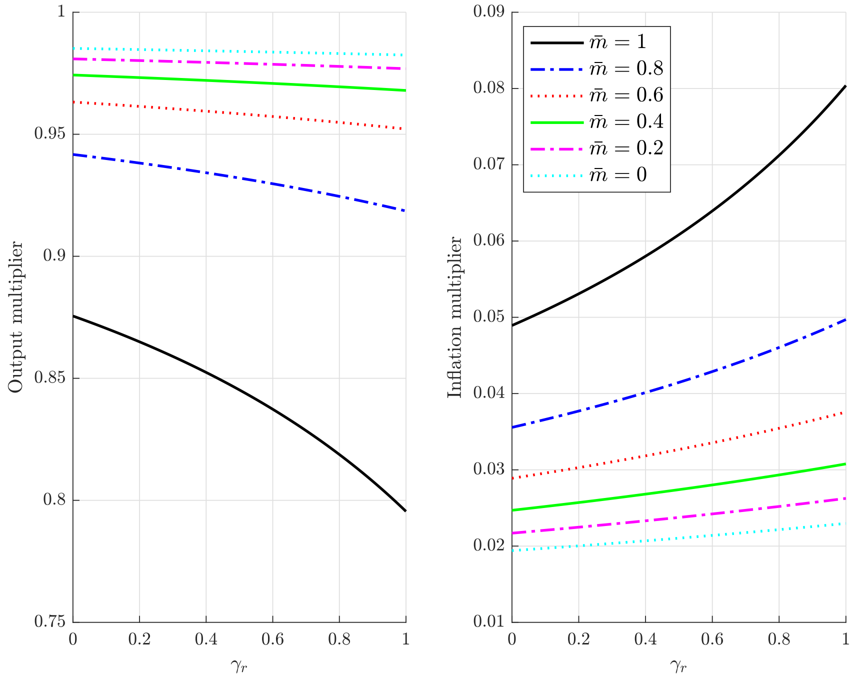
At the ZLB, inflation caused by government spending can be less due to cognitive discounting. Thus, it can increase real interest rates by less than in the standard model, which, in turn, can lower the output gap multiplier. The effects on the strength of the real cost channel are the same as for the baseline model in liquidity traps. Specifically, the output gap and inflation multipliers decrease in γ_r—see Appendix P for a proof.

4.3 Numerical Results with Bounded Rationality

Figure 7 shows how spending multipliers with bounded rationality vary with the value of γ_r for various values of $\bar{m}$ in normal times. The main takeaway is that the output gap multiplier can be larger with a higher cognitive discounting level $\bar{m}$ under bounded rationality while the inflation multiplier can be lower. In addition, the output gap multiplier decreases in γ_r whereas the inflation multiplier increases in γ_r. See Appendix Q for numerical results w.r.t. p. I find that the output gap multiplier can decrease in p with rational expectations ($\bar{m}=1$) but increase in p with a small $\bar{m}$.

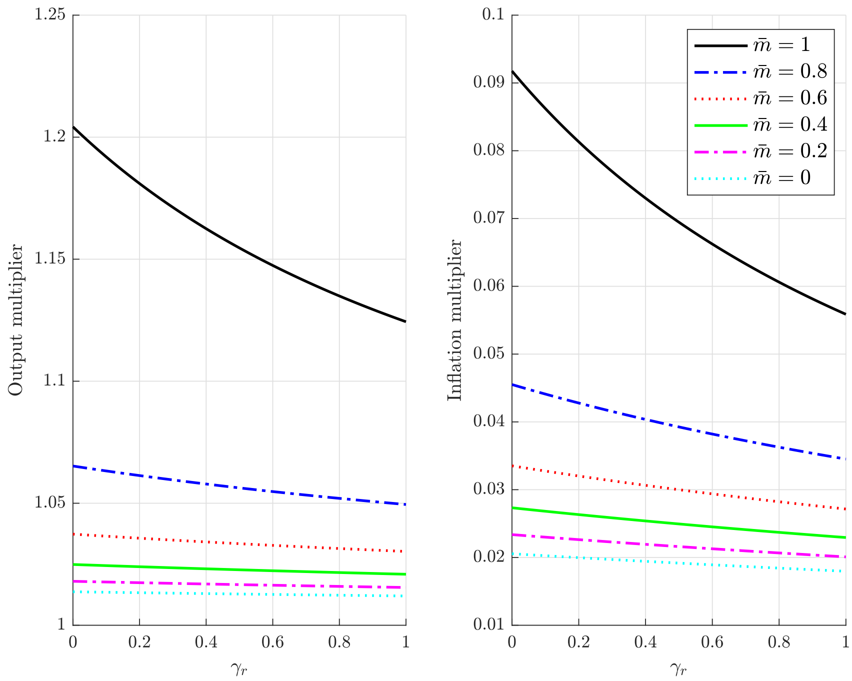
As in our numerical simulation in Figure 8, it is seen that the inflation and output gap multipliers are lower with bounded rationality in liquidity traps. In addition, the role of the real cost channel with bounded rationality is the same as that of the baseline

Figure 7: Spending multipliers with bounded rationality in normal times



model. As in Appendix Q, the output gap multiplier increases numerically in p.

Figure 8: Spending multipliers with bounded rationality at ZLB



5 Concluding Remarks

This paper augments the real cost channel in the standard NK model to explore government spending multipliers. The general properties of spending multipliers are identified using this analytical framework. It is found that the output gap multiplier with the real cost channel is smaller compared to the standard model without this channel, and it decreases with an increase in the strength of the real channel during liquidity trap episodes. I use this theoretical result to explain the lower government spending multiplier in empirical evidence when nominal interest rates are fixed at the lower bound. The extent to which firms care about real borrowing costs can make government spending less effective in stimulating the economy in times of recession. In normal times, the output gap multiplier also decreases with this channel.

The robust role of the real cost channel on spending multipliers is confirmed. More specifically, our results on the role of the real cost channel, if one considers long-run government spending policy, are the same as for the short-run model. The benchmark model is also modified to include bounded rationality while retaining analytical tractability. Cognitive behavior can alter spending multipliers; however, the real cost channel can still operate vigorously. This framework is so flexible that one can introduce heterogeneity along the lines of Bhandari et al. (2021), Cantore and Freund (2021), and Bayer et al. (2022) while retaining analytical tractability. I leave the analyses of other extensions to future work.

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Online Appendix

A Details of the Baseline Model in Section 2

In this section, I add the government spending ingredient to Beaudry et al. (2022)'s aggregate supply-side economics with the real cost channel. To be more specific, each monopolist will use only the basic input Y_t^B for production and follow the one-to-one technology. Therefore, the price of this basic input is the marginal cost. The basic input is produced by representative firms with the following Leontief production function:

$$Y_t^B = \min(aN_t, bM_t),$$

where M_t is the final goods, and N_t is the labor.
The unit price of the final goods attached to the production is P_t. As in Beaudry et al. (2022), we assume that the basic input representative should borrow D_t to pay for the input M_t at the risk-free nominal rate i_t for the production, *i.e.* borrowing costs.[32] In this case, firms should produce, sell the product, pay wages $W_t P_t$, pay back the debt in the previous period D_t, and distribute the dividends Π_t. One can show the budget constraint of firms at time t by simply assuming zero profits in equilibrium below:

$$D_{t+1} + P_t^B Y_t^B = W_t P_t L_t + (1 + i_{t-1})D_t + P_t M_t,$$

where P_t^B is the basic input price, and $D_{t+1} = P_t M_t$.
In that way, the profit Π_t can be shown as:

$$\Pi_t = P_t^B Y_t^B - W_t P_t L_t - (1 + i_{t-1})P_{t-1}M_{t-1}.$$

We further assume that firms maximize the expected discounted sum of real profit $\frac{\Pi_t}{P_t}$ with a discount parameter β. In this case, the first-order condition can be shown:

$$P_t^B = \Big(\frac{1}{a}W_t + \frac{\beta}{b}\mathbb{E}_t\frac{1+i_t}{1+\pi_{t+1}}\Big)P_t,$$

Where π_{t+1} is the next period's inflation rate. Thus, one can obtain the (real) marginal cost of the basic input:

$$MC_t = \frac{W_t}{a} + \frac{\beta}{b}\mathbb{E}\left[\frac{1+i_t}{1+\pi_{t+1}}\right].$$

[32] The borrowing cost is crucial in the modeling since it introduces the real cost channel in the Phillips Curve. The advantage of this introduced real cost channel method as in Beaudry et al. (2022) is that it allows setting arbitrarily the elasticity of marginal cost rate with regard to wage and interest rate. Please see Beaudry et al. (2022) for a comprehensive comparison between the model with the nominal and the real cost channel.

With log condition, one can show the linearized equilibrium

$$mc_t = \hat{\gamma}_y(w_t) + \gamma_r(R_t + \log(\beta) - \mathbb{E}\pi_{t+1}),$$

where $\hat{\gamma}_y = \frac{\frac{1}{a}W}{\frac{1}{a}W + \frac{\beta}{b}\frac{1+i}{1+\pi}}$, $\gamma_r = \frac{\frac{\beta}{b}\frac{1+i}{1+\pi}}{\frac{1}{a}W + \frac{\beta}{b}\frac{1+i}{1+\pi}}$, and R_t is the nominal interest rate in level.
On the other hand, the optimal labor supply reads:

$$\frac{v'(N_t)}{u'(C_t)} = W_t.$$

The linearized resource constraint in this economy is:

$$y_t = (1 - s_g)c_t + g_t,$$

where y_t is the output gap, s_g is the fraction of government spending in total production, and g_t is the government spending.[33]
By using the linearized production function $y_t = n_t$, the marginal cost can be rewritten as

$$\begin{aligned} mc_t &= \hat{\gamma}_y \frac{Nv''(N)}{v'(N)} y_t - \hat{\gamma}_y \frac{Cu''(C)}{u'(C)} (\frac{y_t - g_t}{1 - s_g}) + \gamma_r(R_t + \log(\beta) - \mathbb{E}_t\pi_{t+1}) \\ &= \gamma_y y_t + \gamma_g g_t + \gamma_r(R_t + \log(\beta) - \mathbb{E}_t\pi_{t+1}), \end{aligned}$$

where $\gamma_y = \hat{\gamma}_y \left(\frac{Nv''(N)}{v'(N)} - \frac{Cu''(C)}{u'(C)(1-s_g)} \right)$, and $\gamma_g = \hat{\gamma}_y \frac{Cu''(C)}{u'(C)(1-s_g)}$.
The rest is standard and we have the Phillips Curve as in Tillmann (2009):

$$\pi_t = \kappa mc_t + \beta \mathbb{E}_t \pi_{t+1}.$$

Therefore, the Phillips Curve with the real cost channel and government spending is

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \left[\gamma_y y_t + \gamma_g g_t + \gamma_r(R_t + \log(\beta) - \mathbb{E}_t\pi_{t+1}) \right].$$

B Proof for Proposition 1

If I consider the demand shock and it is assumed that the demand shock r^n_S can put the economy into liquidity traps with one enough (negatively) big shock ($r^n_S < \underline{r^n_S}$), one can rewrite the Phillips Curve as

$$y_S = \begin{cases} \frac{1-\beta p + \kappa\gamma_r p - \kappa\gamma_r\phi_\pi}{\kappa\gamma_y}\pi_S & \text{if } r^n_S \geq \underline{r^n_S} \\ \frac{1-\beta p + \kappa\gamma_r p}{\kappa\gamma_y}\pi_S - \frac{\gamma_r}{\gamma_y}\log(\beta) & \text{if } r^n_S < \underline{r^n_S}. \end{cases}$$

[33] Following Christiano et al. (2011), I define $g_t = (G_t - G)/Y$.

Similarly, one can rewrite the Euler equations as follows:

$$y_S = \begin{cases} -\frac{1}{\sigma}\frac{\phi_\pi - p}{1-p}\pi_S + \frac{1}{\sigma}\frac{r_S^n}{1-p} & \text{if } r_S^n \geq \underline{r_S^n} \\ \frac{\frac{1}{\sigma}p}{1-p}\pi_S + \frac{1}{\sigma}\frac{r_S^n - \log(\beta)}{1-p} & \text{if } r_S^n < \underline{r_S^n}. \end{cases}$$

I combine the first questions of Euler equation and Phillips Curve to obtain the exact expression for $\underline{r_S^n}$ which can be written as:

$$\underline{r_S^n} = \left[\frac{(1-\beta p + \kappa\gamma_r p - \kappa\gamma_r\phi_\pi)(1-p)}{\kappa\gamma_y\frac{1}{\sigma}} + (\phi_\pi - p)\right]\frac{\log(\beta)}{\phi_\pi} < 0.$$

One can show the exact boundary condition for $\underline{r_S^n}$ in the standard NK model without the real cost channel:

$$\underline{r_S^{n,B}} = \left[\frac{(1-\beta p)(1-p)}{\kappa\gamma_y\frac{1}{\sigma}} + (\phi_\pi - p)\right]\frac{\log(\beta)}{\phi_\pi} < 0.$$

Likewise, the exact boundary condition for $\underline{r_S^n}$ in the model with the nominal cost channel:

$$\underline{r_S^{n,N}} = \left[\frac{(1-\beta p - \kappa\gamma_r\phi_\pi)(1-p)}{\kappa\gamma_y\frac{1}{\sigma}} + (\phi_\pi - p)\right]\frac{\log(\beta)}{\phi_\pi} < 0.$$

In this case, I have

$$\underline{r_S^n} - \underline{r_S^{n,B}} = \frac{\kappa\gamma_r(p-\phi_\pi)}{\kappa\gamma_y\frac{1}{\sigma}}\frac{\log(\beta)}{\phi_\pi} > 0.$$

And further one can show

$$\underline{r_S^n} - \underline{r_S^{n,N}} < 0.$$

One can use this to obtain the result in the main text.

C Proof for Proposition 2

I show the solutions for the output gap and inflation multipliers below:

$$\begin{aligned}\mathcal{M}_{S,N}^O = \frac{\partial y_S}{\partial g_S} &= \frac{\frac{\sigma(1-p)}{\phi_\pi - p} - \frac{\kappa\gamma_g}{1-\beta p - \kappa\gamma_r(\phi_\pi - p)}}{\frac{\kappa\gamma_y}{1-\beta p - \kappa\gamma_r(\phi_\pi - p)} + \frac{\sigma(1-p)}{\phi_\pi - p}} \\ &= \frac{\sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)] - \kappa\gamma_g(\phi_\pi - p)}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)]}\end{aligned}$$

$$\mathscr{M}^{I}_{S,N} = \frac{\partial \pi_S}{\partial g_S} = \frac{1+\frac{\gamma_g}{\gamma_y}}{\frac{1-\beta p-\kappa\gamma_r(\phi_\pi-p)}{\kappa\gamma_y}+\frac{1}{\sigma(1-p)}(\phi_\pi-p)}$$
$$= \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p)}{\kappa\gamma_y(\phi_\pi-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]}.$$

For the normal cost channel case, the item $[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]$ will switch to $[1-\beta p-\kappa\gamma_r\phi_\pi]$. In this case, one can easily prove that the inflation multiplier $\mathscr{M}^{I,N}_{S,N}$ can be larger with the nominal cost channel due to a smaller denominator. However, the output gap multiplier with the nominal cost channel $\mathscr{M}^{O,N}_{S,N}$ can be less. For the output gap multiplier, the numerator is less than the denominator. That is,

$$\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]-\kappa\gamma_g(\phi_\pi-p) < \kappa\gamma_y(\phi_\pi-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)].$$

Thus, the output gap multiplier is less than one. For the output gap multiplier without the real cost channel:

$$\mathscr{M}^{O,B}_{S,N} = \frac{\partial y_S}{\partial g_S} = \frac{\sigma(1-p)[1-\beta p]-\kappa\gamma_g(\phi_\pi-p)}{\kappa\gamma_y(\phi_\pi-p)+\sigma(1-p)[1-\beta p]}.$$

One can compare this expression to the previous one with the real cost channel and it is easy, after some arrangements, to obtain the output gap multiplier with the real cost channel that is lower.

For the inflation multiplier without the real cost channel:

$$\mathscr{M}^{I,B}_{S,N} = \frac{\partial \pi_S}{\partial g_S} = \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p)}{\kappa\gamma_y(\phi_\pi-p)+\sigma(1-p)[1-\beta p]}.$$

In this case, the denominator of the inflation multiplier with the real cost channel is lower due to a negative item and thus the inflation multiplier is higher. For the output gap multiplier:

$$\frac{\partial \mathscr{M}^{O}_{S,N}}{\partial \gamma_r} = -\sigma(1-p)(\phi_\pi-p)\kappa\frac{\mathscr{D}-\mathscr{N}}{\mathscr{D}^2} < 0,$$

where $\mathscr{D}_N = \kappa\gamma_y(\phi_\pi-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]$ and $\mathscr{N}_N = \sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]-\kappa\gamma_g(\phi_\pi-p)$. Thus, the output gap multiplier in normal times decreases in the increased strength of the real cost channel.

For the inflation multiplier, it is observed with ease that the higher the strength of the real cost channel γ_r, the lower the denominator of this multiplier. In other words, the inflation multiplier increases in the increased strength of the real cost channel.

D Proof for Proposition 3

The output gap and inflation multipliers at the ZLB are reproduced here:

$$\mathcal{M}^O_{S,Z} = \frac{\partial y_S}{\partial g_S} = \frac{\frac{\sigma(1-p)}{-p} - \frac{\kappa\gamma_g}{1-\beta p-\kappa\gamma_r(-p)}}{\frac{\kappa\gamma_y}{1-\beta p-\kappa\gamma_r(-p)} + \frac{\sigma(1-p)}{-p}}$$
$$= \frac{\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)] - \kappa\gamma_g(-p)}{\kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]}$$

$$\mathcal{M}^I_{S,Z} = \frac{\partial \pi_S}{\partial g_S} = \frac{1+\frac{\gamma_g}{\gamma_y}}{\frac{1-\beta p-\kappa\gamma_r(-p)}{\kappa\gamma_y} + \frac{1}{\sigma(1-p)}(-p)}$$
$$= \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p)}{\kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]}.$$

The numerator and denominator of the output and inflation multipliers (we assume γ_r is far greater than γ_y and the denominator is positive) are both positive here and thus we have the positive spending multipliers. The output gap multiplier can be rewritten as:

$$\mathcal{M}^O_{S,Z} = \frac{\partial y_S}{\partial g_S} = 1 + \frac{\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)] + \kappa\gamma_g p + \kappa\gamma_y p}{\kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]}.$$

This output gap multiplier is larger than one. For the output gap multiplier without the real cost channel:

$$\mathcal{M}^{O,B}_{S,Z} = \frac{\partial y_S}{\partial g_S} = \frac{\sigma(1-p)[1-\beta p] - \kappa\gamma_g(-p)}{\kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p]}.$$

Similar with the case in normal times, one can compare this expression to the previous one with the real cost channel, and it is easy, after some arrangements, to obtain the output gap multiplier with the real cost channel that is lower.
For the inflation multiplier without the real cost channel:

$$\mathcal{M}^{I,B}_{S,Z} = \frac{\partial \pi_S}{\partial g_S} = \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p)}{\kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p]}.$$

One can observe that the higher the strength of the real cost channel γ_r, the higher the denominator of this multiplier. In this case, it can be lower with the real cost channel. For the output gap multiplier:

$$\frac{\partial \mathcal{M}^O_{S,Z}}{\partial \gamma_r} = -\sigma(1-p)(-p)\kappa\frac{\mathcal{D}-\mathcal{N}}{\mathcal{D}^2} < 0,$$

where $\mathcal{D}_Z = \kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]$ and $\mathcal{N}_Z = \sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)] - \kappa\gamma_g(-p)$. Thus, the output gap multiplier at the ZLB reduces in the increased strength of the real cost channel.

For the inflation multiplier, it is observed with ease that the higher the strength of the real cost channel γ_r, the higher the denominator of this multiplier. In other words, the inflation multiplier decreases in the increased strength of the real cost channel.

While the nominal cost channel multipliers $\mathscr{M}_{S,Z}^{O,N}$ and $\mathscr{M}_{S,Z}^{I,N}$ can be invariant with the standard NK model since the nominal channel in liquidity traps cannot be included in the partial derivative of government spending to the output gap/inflation in the calculation of fiscal multipliers.

E The derivation of the condition for γ_y

As in Nie (2022), for brevity, one can yield a condition for γ_y to ensure $\mathscr{D}_Z > 0$:

$$\begin{aligned}
\mathscr{D}_Z &= (1-p)(1-\beta p + \kappa\gamma_r p) - \sigma_r p \kappa \gamma_y \\
&> \left(1 - \frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r}\right)\left(1 - \beta\frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r} + \kappa\gamma_r\frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r}\right) - \sigma_r\frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r}\kappa\gamma_y \\
&= (\beta-\kappa\gamma_r-1+\kappa\gamma_r\phi_\pi)[\beta\kappa\gamma_r\phi_\pi - \kappa\gamma_r + \kappa\gamma_r(1-\kappa\gamma_r\phi_\pi)] - \sigma_r(1-\kappa\gamma_r\phi_\pi)\kappa\gamma_y > 0 \\
\gamma_y &< \frac{(\beta-\kappa\gamma_r-1+\kappa\gamma_r\phi_\pi)(\beta\gamma_r\phi_\pi - \kappa\gamma_r^2\phi_\pi)}{\sigma_r(1-\kappa\gamma_r\phi_\pi)} = \Gamma(\gamma_r),
\end{aligned}$$

where the second line we assume $p = \bar{p}^c$ due to monotonicity.

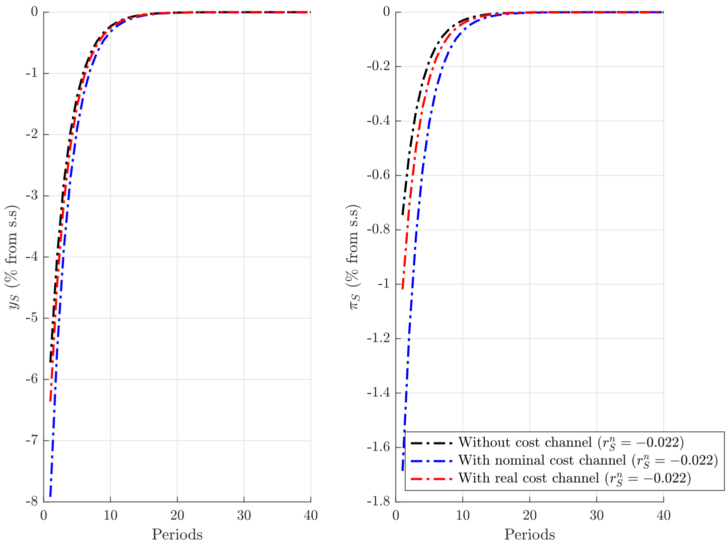
In addition, one can check the monotonicity of $\Gamma(\gamma_r)$ w.r.t. γ_r:

$$\begin{aligned}
\frac{\partial\Gamma(\gamma_r)}{\partial\gamma_r} &\propto \frac{\partial\frac{\beta-\kappa\gamma_r}{1-\kappa\gamma_r\phi_\pi}}{\partial\gamma_r} \\
&\propto \kappa(\phi_\pi\beta - 1) > 0.
\end{aligned}$$

Therefore $\Gamma(\gamma_r)$ increases in γ_r.

F Impulse Response Function

Figure 9: Impulse response to a contractionary natural rate shock



G Baseline Multiplier Figures w.r.t. Persistent p

Figure 10: Spending multipliers in normal times

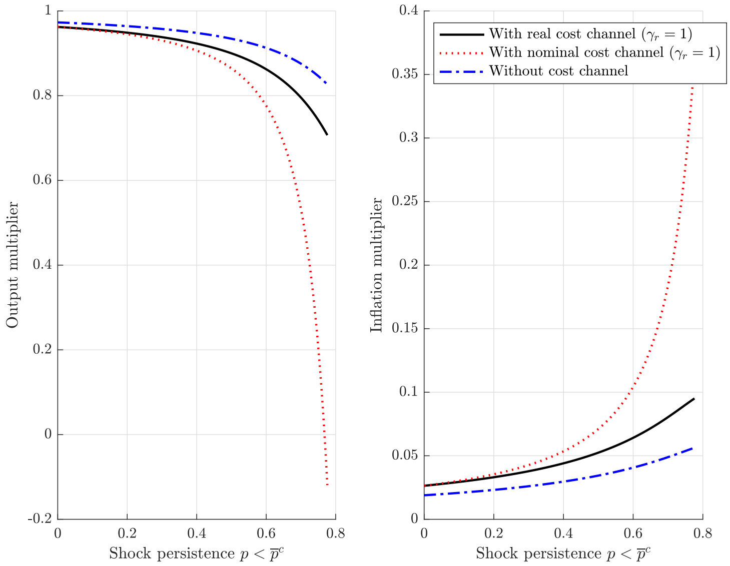
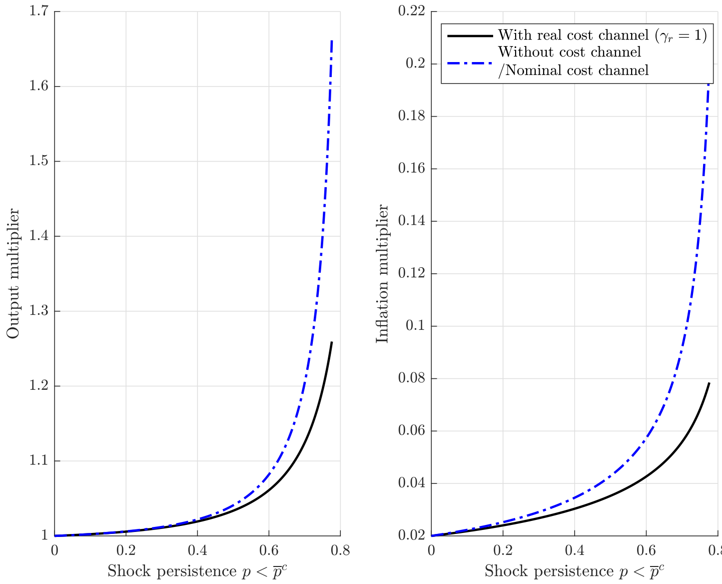


Figure 11: Spending multipliers at ZLB



H Multipliers in the Medium Run

In the medium run, the economy is absent of natural rate shock, and one can show the medium run government spending with the persistence q as follows

$$\begin{aligned} y_M &= \mathcal{M}_M^O \times g_M \\ &= \frac{\sigma(1-q)[1-\beta p-\kappa\gamma_r(\phi_\pi^q-q)]-\kappa\gamma_g(\phi_\pi^q-q)}{\kappa\gamma_y(\phi_\pi^q-q)+\sigma(1-q)[1-\beta q-\kappa\gamma_r(\phi_\pi^q-q)]} g_M. \end{aligned}$$

$$\begin{aligned} \pi_M &= \mathcal{M}_M^I \times g_M \\ &= \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-q)}{\kappa\gamma_y(\phi_\pi^q-q)+\sigma(1-q)[1-\beta q-\kappa\gamma_r(\phi_\pi^q-q)]} g_M. \end{aligned}$$

I Euler and Phillips shift

The long-run government spending in the Euler equation shift:

$$q\zeta(\mathcal{M}_M^O+\frac{1}{\sigma}\mathcal{M}_M^I-1)=q\zeta\frac{\kappa(\gamma_y+\gamma_g)(1-\phi_\pi^q)}{\kappa\gamma_y(\phi_\pi^q-q)+\sigma(1-q)[1-\beta q-\kappa\gamma_r(\phi_\pi^q-q)]}<0.$$

The long run government spending in the Phillips Curve shift

$$(\beta-\kappa\gamma_r)(1-p)q\zeta\mathcal{M}_M^I>0,$$

where we assume that the $\beta-\kappa\gamma_r>0$ since κ is very small in our theoretical analysis as in Gabaix (2020) Budianto et al. (2020), and Nie (2021).

J The long-run government spending effects in normal times

I can use the new Euler equation and the Phillips Curve to reproduce the spending multipliers:

$$\begin{aligned} \mathcal{M}_{S,N}^{O,long} &= \frac{\partial y_S}{\partial g_S} = \frac{\Theta_{AD}\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]-\Theta_{AS}(\phi_\pi-p)}{\kappa\gamma_y(\phi_\pi-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]} \\ \mathcal{M}_{S,N}^{I,long} &= \frac{\partial \pi_S}{\partial g_S} = \frac{\left[\kappa\gamma_y\Theta_{AD}+\Theta_{AS}\right]\sigma(1-p)}{\kappa\gamma_y(\phi_\pi-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]}. \end{aligned}$$

For the output gap multipliers, as in appendix I, one can see that the long-run government spending shock can lead to a lower Θ_{AD} but a higher Θ_{AS}. In this case, the multiplier should be lower.

For inflation multiplier,

$$\kappa\gamma_y q\zeta(\mathcal{M}_M^O + \frac{1}{\sigma}\mathcal{M}_M^I - 1) + (\beta - \kappa\gamma_r)(1-p)q\zeta\mathcal{M}_M^I$$
$$= q\zeta\frac{\kappa\gamma_y\kappa(\gamma_y+\gamma_g)(1-\phi_\pi^q)}{\kappa\gamma_y(\phi_\pi^q - q) + \sigma(1-q)[1-\beta q - \kappa\gamma_r(\phi_\pi^q - q)]} + q\zeta\frac{(\beta-\kappa\gamma_r)(1-p)q\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-q)}{\kappa\gamma_y(\phi_\pi^q - q) + \sigma(1-q)[1-\beta q - \kappa\gamma_r(\phi_\pi^q - q)]} > 0,$$

where we assume κ is minor in our theoretical analysis as in e.g. Eggertsson (2011), Gabaix (2020) and Budianto et al. (2020) and the first item has an addition multiplier κ. One can use this to prove the result in the main text. Since there is no γ_r in Θ_{AD}, we only focus on the Θ_{AS}'s effects. Since the term with Θ_{AD} reduces in γ_r, we only need to show the other terms with Θ_{AS} also decrease in γ_r. Thus one can prove the output gap multiplier decreases in γ_r. The output gap multiplier can be reduced below:

$$-\frac{\Theta_{AS}}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)]}.$$

Since the term with Θ_{AD} increases in γ_r, we only need to show the other terms with Θ_{AS} also increase in γ_r. Thus one can prove the inflation multiplier increases in γ_r. The inflation multiplier can be reduced below:

$$\frac{\Theta_{AS}}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)]}.$$

One can differentiate this common term with regard to γ_r and this common term can be reduced further as:

$$\frac{\beta - \kappa\gamma_r}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)]}\frac{1}{\kappa\gamma_y(\phi_\pi - q) + \sigma(1-q)[1-\beta q - \kappa\gamma_r(\phi_\pi^q - q)]}.$$

I differentiate the above term with regard to γ_r to obtain:

$$\frac{\sigma(1-p)\kappa(\beta\phi_\pi - 1)\mathcal{D}_{1,N} + \sigma(1-q)\kappa(\beta\phi_\pi^q - 1)\mathcal{D}_{2,N} - \mathcal{O}(\kappa^2)}{\mathcal{D}_{3,N}^2}.$$

where $\mathcal{D}_{1,N} = \kappa\gamma_y(\phi_\pi - q) + \sigma(1-q)[1-\beta q - \kappa\gamma_r(\phi_\pi^q - q)]$, $\mathcal{D}_{2,N} = \kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)]$, $\mathcal{D}_{3,N} = \mathcal{D}_{1,N}\cdot\mathcal{D}_{2,N}$ and $\mathcal{O}(\kappa^2)$ is the residual of order two since we assume that κ is trivial in our theoretical analysis as in e.g. Eggertsson (2011), Gabaix (2020) and Budianto et al. (2020), it is easy to check that the derivative with regard to γ_r is positive. In this case, one can use this to prove the result in the main text.

K The long-run government spending effects at ZLB

One can produce the output gap and inflation multipliers below

$$\mathcal{M}_{S,Z}^{O,long} = \frac{\partial y_S}{\partial g_S} = \frac{\Theta_{AD}\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]-\Theta_{AS}(-p)}{\kappa\gamma_y(-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]}$$

$$\mathcal{M}_{S,Z}^{I,long} = \frac{\partial \pi_S}{\partial g_S} = \frac{\left[\kappa\gamma_y\Theta_{AD}+\Theta_{AS}\right]\sigma(1-p)}{\kappa\gamma_y(-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]}.$$

For the output gap multipliers, the numerator with medium run spending policy can be decomposed into the following two parts. The first part:

$$\begin{aligned}&\kappa\gamma_y q\zeta(\mathcal{M}_M^O+\frac{1}{\sigma}\mathcal{M}_M^I-1)\sigma(1-p)[1-\beta p+\kappa\gamma_r p]\\ &= q\zeta\frac{\kappa\gamma_y\kappa(\gamma_y+\gamma_g)(1-\phi_\pi^q)}{\kappa\gamma_y(\phi_\pi^q-q)+\sigma(1-q)[1-\beta q-\kappa\gamma_r(\phi_\pi^q-q)]}\sigma(1-p)[1-\beta p+\kappa\gamma_r p].\end{aligned}$$

The second part:

$$\begin{aligned}&(\beta-\kappa\gamma_r)(1-p)q\zeta\mathcal{M}_M^I p\\ &= q\zeta\frac{(\beta-\kappa\gamma_r)(1-p)q\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-q)}{\kappa\gamma_y(\phi_\pi^q-q)+\sigma(1-q)[1-\beta q-\kappa\gamma_r(\phi_\pi^q-q)]}p.\end{aligned}$$

To simplify the proof, one can plus the two items and show the sum is positive if we assume that κ is very small in our theoretical analysis. Similar to the inflation multiplier in normal times, we can have a higher long-run inflation multiplier at the ZLB. Since there is no γ_r in Θ_{AD}, we only focus on the Θ_{AS}'s effects. For the main result, since the term with Θ_{AD} reduces in γ_r, we only need to show the other terms with Θ_{AS} also decrease in γ_r. Thus one can see the output gap multiplier decreases in γ_r. In this case, the output gap multiplier can be reduced below:

$$\frac{\Theta_{AS}}{\kappa\gamma_y(-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]}.$$

For the main result, since the term with Θ_{AD} decreases om γ_r, we only need to show the other terms with Θ_{AS} also decrease in γ_r. Thus the inflation multiplier will decrease in γ_r. The inflation multiplier can be reduced below:

$$\frac{\Theta_{AS}}{\kappa\gamma_y(-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]}.$$

One can differentiate this common term with regard to γ_r and this common term can be reduced further as:

$$\frac{\beta-\kappa\gamma_r}{\kappa\gamma_y(-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]}\frac{1}{\kappa\gamma_y(\phi_\pi^q-q)+\sigma(1-q)[1-\beta q-\kappa\gamma_r(\phi_\pi^q-q)]}.$$

I differentiate the above term with regard to γ_r to obtain:

$$\frac{-\sigma(1-p)\kappa\mathscr{D}_{1,N}+\sigma(1-q)\kappa(\beta\phi_\pi^q-1)\mathscr{D}_{2,Z}-\mathscr{O}(\kappa^2)}{\mathscr{D}_{3,Z}^2}.$$

where $\mathscr{D}_{2,Z}=\kappa\gamma_y(-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]$, $\mathscr{D}_{3,Z}=\mathscr{D}_{1,N}\cdot\mathscr{D}_{2,Z}$ and $\mathscr{O}(\kappa^2)$ is the residual of order two. One can reduce this expression as:

$$-(1-p)\sigma(1-q)(1-\beta q)+(1-q)(\beta\phi_\pi^q-1)\sigma(1-p)(1-\beta p)-\mathscr{O}(\kappa^2)<0,$$

where I use the general condition $\phi_\pi^q\beta-1<1$ and the short run period should be longer or almost equal to the long run period in reality such that $p\geq q$. In this case, one can use this to prove the result in the main text.

L Multiplier with Long-run Spending Policy Figures w.r.t. Persistent p

Figure 12: Spending multipliers with long-run spending policy in normal times

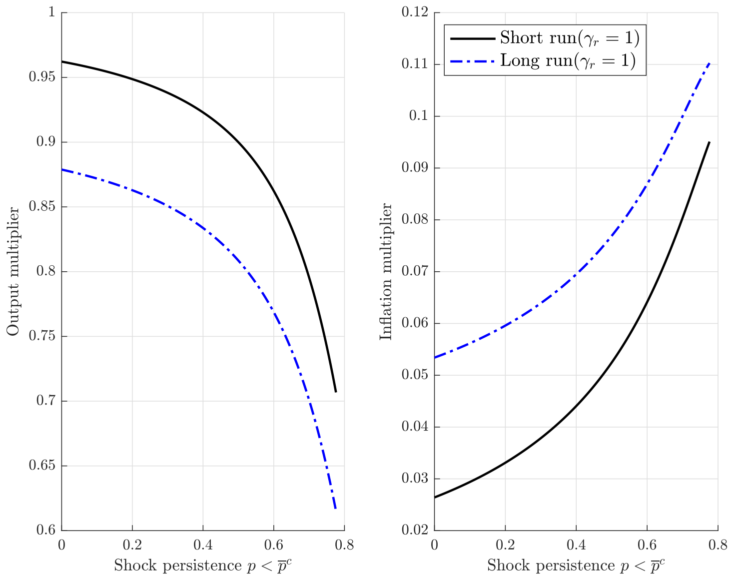
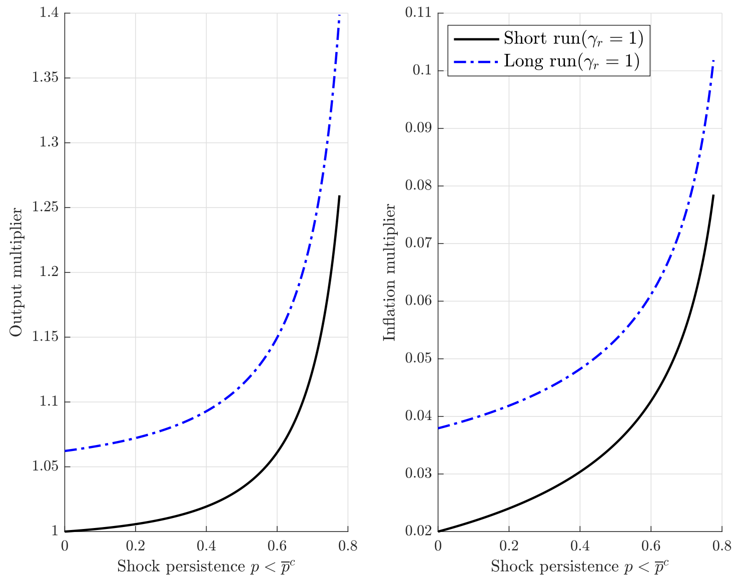


Figure 13: Spending multipliers with long-run spending policy at ZLB



M Euler Equation and Phillips Curve with Bounded Rationality

The Euler equation and Phillips Curve with bounded rationality in normal times:

$$y_S = -\frac{1}{\sigma(1-\alpha_{EE}p)}(\phi_\pi - p)\pi_S + g_S$$
$$\pi_S = \kappa\frac{\gamma_y}{1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)}y_S + \kappa\frac{\gamma_g}{1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)}g_S.$$

The Euler equation and Phillips Curve with bounded rationality can be elaborated at the ZLB:

$$y_S = -\frac{1}{\sigma(1-p\alpha_{EE})}[\log(\beta) - p\pi_S] + g_S$$
$$\pi_S = \frac{\kappa\gamma_y y_S + \kappa\gamma_r\log(\beta)}{1-\beta p\alpha_{PC} + \kappa\gamma_r p} + \kappa\frac{\gamma_g}{1-\beta p\alpha_{PC} + \kappa\gamma_r p}g_S.$$

N The government spending effects with bounded rationality in normal times

I reproduce the multipliers here. For simplicity, we define $\alpha_{EE} = \bar{m}(1-s_g)$ and $\alpha_{PC}(\bar{m}) = \bar{m}[\varphi + \frac{1-\beta\varphi}{1-\beta\varphi\bar{m}}(1-\varphi)]$.

$$\mathcal{M}^{O,BR}_{S,N} = \frac{\partial y_S}{\partial g_S} = \frac{\frac{\sigma(1-p\alpha_{EE})}{\phi_\pi - p} - \frac{\kappa\gamma_g}{1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi - p)}}{\frac{\kappa\gamma_y}{1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi - p)} + \frac{\sigma(1-p\alpha_{EE})}{\phi_\pi - p}}$$
$$= \frac{\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)] - \kappa\gamma_g(\phi_\pi - p)}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)]}$$

$$\mathcal{M}^{I,BR}_{S,N} = \frac{\partial \pi_S}{\partial g_S} = \frac{1+\frac{\gamma_g}{\gamma_y}}{\frac{1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi - p)}{\kappa\gamma_y} + \frac{1}{\sigma(1-p\alpha_{EE})}(\phi_\pi - p)}$$
$$= \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p\alpha_{EE})}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)]}.$$

where α_{EE} and α_{PC} increase in the cognitive discounting parameter $\bar{m}$. One can differentiate the output gap multiplier with regard to $\bar{m}$ and after some arrangements we have:

$$\frac{\kappa(\gamma_y + \gamma_g)(\phi_\pi - p)f'_N(\bar{m})}{\mathcal{D}^2_{BN}} > 0,$$

where $\mathcal{D}_{BN} = \kappa\gamma_y(\phi_\pi - p) + \sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)]$ and $f'_N(\bar{m})$ is the derivative of $\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)]$ with regard to $\bar{m}$ which is positive.
One can differentiate inflation multiplier with regard to $\bar{m}$ and after some arrangements we have:

$$\frac{-p\alpha'_{EE}\mathcal{D}_{BN} - f'_N(\bar{m})(1-p\alpha_{EE})}{\mathcal{D}^2_{BN}} < 0,$$

where $\mathcal{D}_{BN} = \kappa\gamma_y(\phi_\pi - p) + \sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)]$ and $f'_N(\bar{m})$ is the derivative of $\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)]$ with regard to $\bar{m}$ which is positive.
Since the strength of the real cost channel γ_r is independent of the new ingredient (bounded rationality). See appendix C, the output gap multiplier decreases in the increased strength of the real cost channel γ_r, and the inflation multiplier increases in the increased strength of the real cost channel γ_r. One can use this to prove the main text.

O Derivatives of Output Gap Multiplier w.r.t. p with Bounded Rationality

One can differentiate the output gap multiplier w.r.t p and the numerator is[34]:

$$\begin{aligned}&[f'(p)+\kappa\gamma_g][\kappa\gamma_y(\phi_\pi-p)+f(p)]-[f(p)-\kappa\gamma_g(\phi_\pi-p)][-\kappa\gamma_y+f'(p)]\\&=\kappa(\gamma_y+\gamma_g)[f'(p)(\phi_\pi-p)+f(p)],\end{aligned}$$

where $f(p)=\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi-p)]$ and $f'(p)=\sigma(1-p\alpha_{EE})(-\beta\alpha_{PC}+\kappa\gamma_r)-\sigma\alpha_{EE}[1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi-p)]$. One can show:

$$-f'(p)p=-\sigma(1-p\alpha_{EE})(-\beta p\alpha_{PC}+\kappa\gamma_r p)+\sigma\alpha_{EE}p[1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi-p)].$$

In normal times:

$$\begin{aligned}&f'(p)(\phi_\pi-p)+f(p)\\&=f'(p)\phi_\pi+\sigma(1-\kappa\gamma_r\phi_\pi)-\sigma\alpha_{EE}p^2(\beta\alpha_{PC}-\kappa\gamma_r).\end{aligned}$$

When $\bar{m}=1$, one can show that these terms are below zero with ease. When $\bar{m}=0$, one can have the expression below:

$$\sigma\kappa\gamma_r\phi_\pi+\sigma(1-\kappa\gamma_r\phi_\pi)>0.$$

In this case, since this derivative of the output gap multiplier w.r.t. p is continuous in $\bar{m}$, it shows that there is a threshold value $\underline{\bar{m}}$ across which the multiplier effects switch from decreasing in p to increasing in p or vice versa. One can have this value $\underline{\bar{m}}$ by making the below equation zero:

$$\begin{aligned}&f'(p)\phi_\pi+\sigma(1-\kappa\gamma_r\phi_\pi)-\sigma\alpha_{EE}p^2(\beta\alpha_{PC}-\kappa\gamma_r)=0\\\Longleftrightarrow&\{\sigma(1-p\alpha_{EE})(-\beta\alpha_{PC}+\kappa\gamma_r)-\sigma\alpha_{EE}[1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi-p)]\}\phi_\pi+\sigma(1-\kappa\gamma_r\phi_\pi)-\sigma\alpha_{EE}p^2(\beta\alpha_{PC}-\kappa\gamma_r)=0.\end{aligned}$$

At the ZLB, one can show the derivatives can be reduced below:

$$\begin{aligned}&f'(p)(-p)+f(p)\\&=\sigma-\sigma\alpha_{EE}p^2(\beta\alpha_{PC}-\kappa\gamma_r)>0.\end{aligned}$$

In this case, the output gap multiplier increases in p.

[34]For simplicity, I only compare the numerator of the derivation to check the monotonicity.

P The government spending effects with bounded rationality at ZLB

The spending multipliers at the ZLB are shown below. For simplicity, we define $\alpha_{EE} = \bar{m}(1-s_g)$ and $\alpha_{PC}(\bar{m}) = \bar{m}[\varphi + \frac{1-\beta\varphi}{1-\beta\varphi\bar{m}}(1-\varphi)]$.

$$
\begin{aligned}
\mathcal{M}_{S,Z}^{O,BR} = \frac{\partial y_S}{\partial g_S} &= \frac{\frac{\sigma(1-p\alpha_{EE})}{-p} - \frac{\kappa\gamma_g}{1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)}}{\frac{\kappa\gamma_y}{1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)} + \frac{\sigma(1-p\alpha_{EE})}{-p}} \\
&= \frac{\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)] - \kappa\gamma_g(-p)}{\kappa\gamma_y(-p) + \sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)]}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_{S,Z}^{I,BR} = \frac{\partial \pi_S}{\partial g_S} &= \frac{1+\frac{\gamma_g}{\gamma_y}}{\frac{1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)}{\kappa\gamma_y} + \frac{1}{\sigma(1-p\alpha_{EE})}(-p)} \\
&= \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p\alpha_{EE})}{\kappa\gamma_y(-p) + \sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)]}.
\end{aligned}
$$

One can differentiate the output gap multiplier with regard to $\bar{m}$ and after some arrangements we have:

$$
\frac{\kappa(\gamma_y+\gamma_g)(-p)f_Z'(\bar{m})}{\mathcal{D}_{BZ}^2} < 0,
$$

where $\mathcal{D}_{BZ} = \kappa\gamma_y(-p) + \sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)]$ and $f_Z'(\bar{m})$ is the derivative of $\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)]$ with regard to $\bar{m}$ which is positive.

One can differentiate the inflation multiplier with regard to $\bar{m}$ and after some arrangements we have:

$$
\frac{-p\alpha_{EE}'\mathcal{D}_{BZ} - f_Z'(\bar{m})(1-p\alpha_{EE})}{\mathcal{D}_{BN}^2} < 0,
$$

where $\mathcal{D}_{BZ} = \kappa\gamma_y(-p) + \sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)]$ and $f_Z'(\bar{m})$ is the derivative of $\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)]$ with regard to $\bar{m}$ which is positive.

Since the strength of the real cost channel γ_r is independent of the new ingredient that is bounded rationality. See appendix D, the output gap and inflation multipliers decrease in the increased strength of the real cost channel γ_r. One can use this to prove the main text.

Q Figures with Bounded Rationality w.r.t. p

Figure 14: Spending multipliers with bounded rationality in normal times

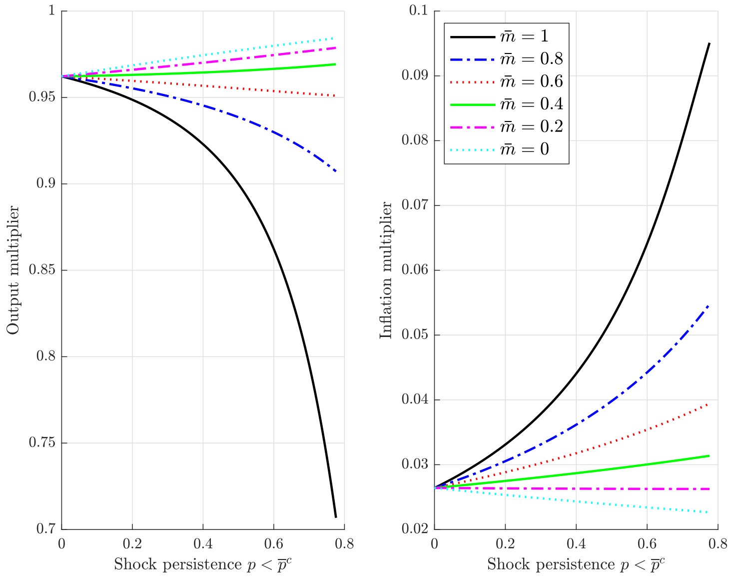


Figure 15: Spending multipliers with bounded rationality at ZLB