

ATIL SAMANCIOGLU

QUANTUM COMPUTING

Bit vs Qubit

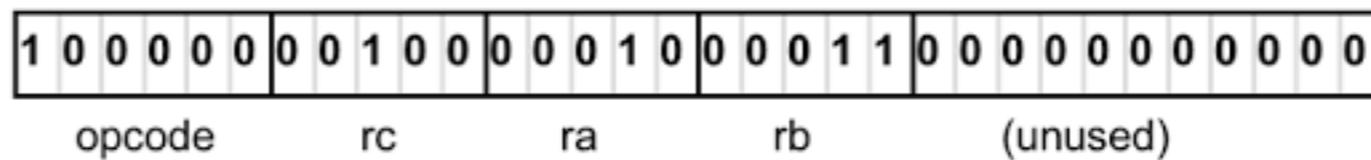
COMPUTER SCIENCE

MACHINE LANGUAGE

ASSEMBLY

HIGH LEVEL LANGUAGE

COMPUTER SCIENCE



ADD (R2, R3, R4)

$A = B + C;$

DECIMAL

$$215 = 5 * 10^0 + 1 * 10^1 + 2 * 10^2$$

5 1 4

$5 * 10^2$

$4 * 10^0$

$1 * 10^1$

BINARY

1 1 0 1 1 0 1

$1 * 2 ^ 6 + 1 * 2 ^ 5 + 0 * 2 ^ 4 + 1 * 2 ^ 3 +$

$1 * 2 ^ 2 + 0 * 2 ^ 1 + 1 * 2 ^ 0$

$64 + 32 + 0 + 8 + 4 + 0 + 1$

109

HEXADECIMAL

0 1 2 3 4 5 6 7 8 9 A B C D E F

10 11 12 13 14 15

HEXADECIMAL

A 4 F 6

$$10 * 16^3 + 4 * 16^2 + 15 * 16^1 + 6 * 16^0$$

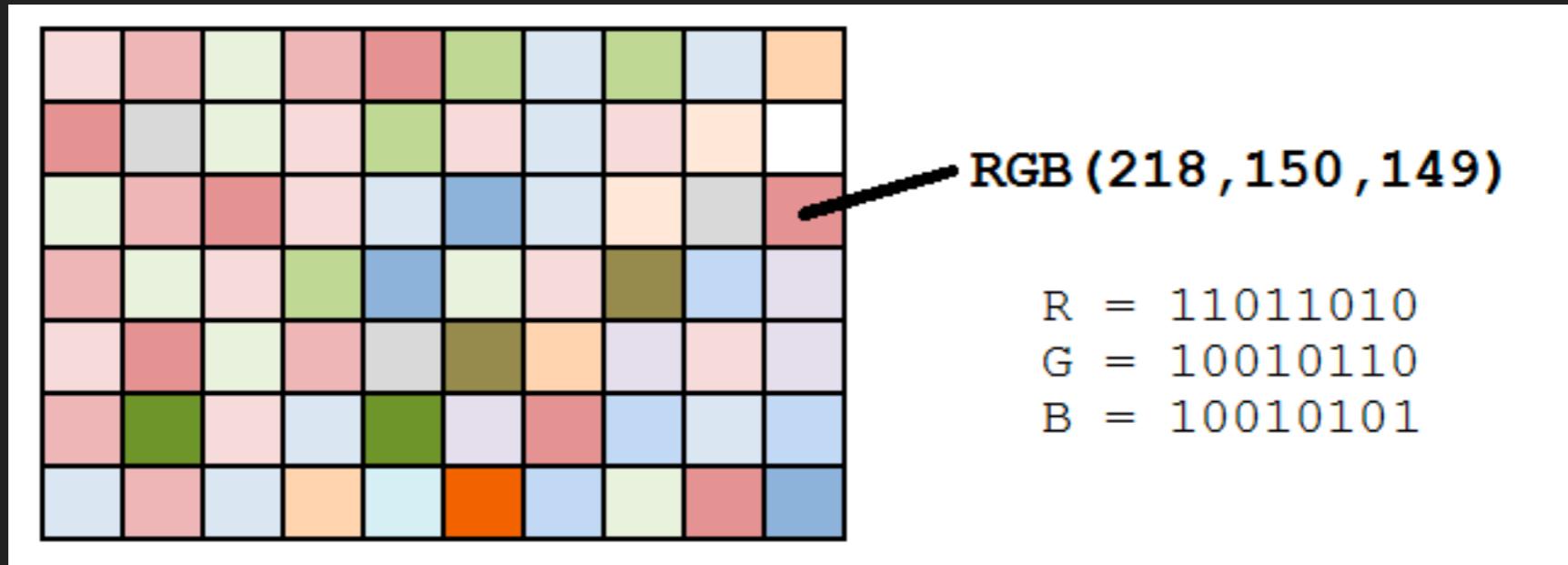
$$40960 + 1024 + 240 + 6$$

42230

Decimal - Binary - Octal - Hex – ASCII Conversion Chart

Decimal	Binary	Octal	Hex	ASCII	Decimal	Binary	Octal	Hex	ASCII	Decimal	Binary	Octal	Hex	ASCII	Decimal	Binary	Octal	Hex	ASCII
0	00000000	000	00	NUL	32	00100000	040	20	SP	64	01000000	100	40	@	96	01100000	140	60	`
1	00000001	001	01	SOH	33	00100001	041	21	!	65	01000001	101	41	A	97	01100001	141	61	a
2	00000010	002	02	STX	34	00100010	042	22	"	66	01000010	102	42	B	98	01100010	142	62	b
3	00000011	003	03	ETX	35	00100011	043	23	#	67	01000011	103	43	C	99	01100011	143	63	c
4	00000100	004	04	EOT	36	00100100	044	24	\$	68	01000100	104	44	D	100	01100100	144	64	d
5	00000101	005	05	ENQ	37	00100101	045	25	%	69	01000101	105	45	E	101	01100101	145	65	e
6	00000110	006	06	ACK	38	00100110	046	26	&	70	01000110	106	46	F	102	01100110	146	66	f
7	00000111	007	07	BEL	39	00100111	047	27	'	71	01000111	107	47	G	103	01100111	147	67	g
8	00001000	010	08	BS	40	00101000	050	28	(72	01001000	110	48	H	104	01101000	150	68	h
9	00001001	011	09	HT	41	00101001	051	29)	73	01001001	111	49	I	105	01101001	151	69	i
10	00001010	012	0A	LF	42	00101010	052	2A	*	74	01001010	112	4A	J	106	01101010	152	6A	j
11	00001011	013	0B	VT	43	00101011	053	2B	+	75	01001011	113	4B	K	107	01101011	153	6B	k
12	00001100	014	0C	FF	44	00101100	054	2C	,	76	01001100	114	4C	L	108	01101100	154	6C	l
13	00001101	015	0D	CR	45	00101101	055	2D	-	77	01001101	115	4D	M	109	01101101	155	6D	m
14	00001110	016	0E	SO	46	00101110	056	2E	.	78	01001110	116	4E	N	110	01101110	156	6E	n
15	00001111	017	0F	SI	47	00101111	057	2F	/	79	01001111	117	4F	O	111	01101111	157	6F	o
16	00010000	020	10	DLE	48	00110000	060	30	0	80	01010000	120	50	P	112	01110000	160	70	p
17	00010001	021	11	DC1	49	00110001	061	31	1	81	01010001	121	51	Q	113	01110001	161	71	q
18	00010010	022	12	DC2	50	00110010	062	32	2	82	01010010	122	52	R	114	01110010	162	72	r
19	00010011	023	13	DC3	51	00110011	063	33	3	83	01010011	123	53	S	115	01110011	163	73	s
20	00010100	024	14	DC4	52	00110100	064	34	4	84	01010100	124	54	T	116	01110100	164	74	t
21	00010101	025	15	NAK	53	00110101	065	35	5	85	01010101	125	55	U	117	01110101	165	75	u
22	00010110	026	16	SYN	54	00110110	066	36	6	86	01010110	126	56	V	118	01110110	166	76	v
23	00010111	027	17	ETB	55	00110111	067	37	7	87	01010111	127	57	W	119	01110111	167	77	w
24	00011000	030	18	CAN	56	00111000	070	38	8	88	01011000	130	58	X	120	01111000	170	78	x
25	00011001	031	19	EM	57	00111001	071	39	9	89	01011001	131	59	Y	121	01111001	171	79	y
26	00011010	032	1A	SUB	58	00111010	072	3A	:	90	01011010	132	5A	Z	122	01111010	172	7A	z
27	00011011	033	1B	ESC	59	00111011	073	3B	;	91	01011011	133	5B	[123	01111011	173	7B	{
28	00011100	034	1C	FS	60	00111100	074	3C	<	92	01011100	134	5C	\	124	01111100	174	7C	
29	00011101	035	1D	GS	61	00111101	075	3D	=	93	01011101	135	5D]	125	01111101	175	7D	}
30	00011110	036	1E	RS	62	00111110	076	3E	>	94	01011110	136	5E	^	126	01111110	176	7E	~
31	00011111	037	1F	US	63	00111111	077	3F	?	95	01011111	137	5F	_	127	01111111	177	7F	DEL

PIXEL



BIT VS QUBIT

0 0 0 0 0 0 0 0 0

1 1 1 1 1 1 1 1 1

PREREQUISITES

|0>

$$v = d / t$$

python

PROBABILITY & BINARY

P(A)

P(A AND B)

P((A AND B) OR C)

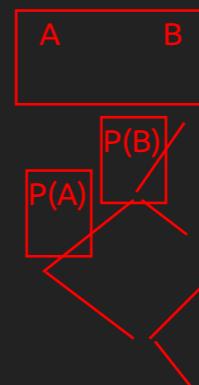
PROBABILITY & BINARY

$P(A \text{ AND } B) = 0$

Mutually Exclusive

$P(A \text{ AND } B) = P(A) \times P(B)$

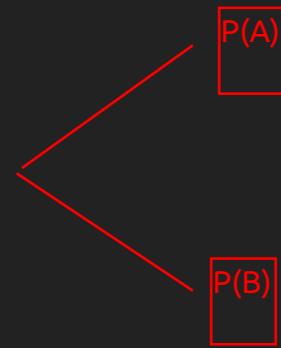
Independent



PROBABILITY & BINARY

$$P(A \text{ OR } B) = P(A) + P(B)$$

Mutually Exclusive



$$P(A \text{ OR } B) = P(A) + P(B) - P(A \text{ AND } B)$$

Independent

Ist ein und/oder -> wichtig für
Entscheidungsbaum

PROBABILITY & BINARY

$P(A) = 30\%$

$P(B) = 40\%$

$$P(A \text{ OR } B) = P(A) + P(B) - P(A \text{ AND } B)$$

$$= 0.3 + 0.4 - (0.3 * 0.4)$$

$$= 0.58$$

STATISTICS

15, 22, 33, 40, 55, 61, 79

Mean -> 43.57~

Median -> 40

Min -> 15

Max -> 79

STATISTICS

15, 22, 33, 40, 55, 61, 79

$$(15 - 43,57)^2$$

$$(22 - 43,57)^2$$

$$(33 - 43,57)^2$$

$$(40 - 43,57)^2$$

$$(55 - 43,57)^2$$

$$(61 - 43,57)^2$$

$$(79 - 43,57)^2$$

$$\sqrt{515,95} = 22,71$$

Variance

Standard
Deviation

COMPLEX NUMBERS

$i \rightarrow \sqrt{-1}$ (Imaginary number)

$a + bi$ (Complex number)

$$\begin{aligned}\sqrt{-9} &= \sqrt{9} * \sqrt{-1} \\ &= 3i\end{aligned}$$

COMPLEX NUMBERS

$$\sqrt{i} = \frac{(1+i)}{\sqrt{2}}$$

COMPLEX NUMBERS

$$(3+2i) + (5+4i) = (8+6i)$$

$$(3-2i) + (-5+4i) = (-2+2i)$$

$$(3+2i) - (5+4i) = (-2-2i)$$

$$5 * (3+2i) = (15+10i)$$

$$(3+2i) / 3 = (1 + \frac{2i}{3})$$

COMPLEX NUMBERS

$$i * (3+2i) = (3i+i^2) = (-1+3i)$$

$$(5 + 3i) * (3+2i)$$

COMPLEX NUMBERS

$$(3+2i) \rightarrow (3-2i)$$

Complex Conjugate

$$(5-4i) \rightarrow (5+4i)$$

$$|3+2i|^2 \rightarrow 3^2 + 2^2$$

Squared Magnitude

COMPLEX NUMBERS

Complex No * Complex Conjugate =
Squared Magnitude

$$(3+2i) * (3-2i)$$

COMPLEX NUMBERS

$$\frac{(6+4i)}{}$$

=

$$\frac{(3+2i)}{}$$

$$\frac{(6+4i)}{}$$

*

$$\frac{(3+2i)}{}$$

$$\frac{(3-2i)}{}$$

$$(6+4i) * (3-2i)$$

=

$$\frac{(3^2+2^2)}$$

$$(18 - 12i + 12i - 8i^2)$$

=

$$\frac{13}{}$$

=

$$2$$

MATRIX

$$\begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & 3 \\ 3 & 1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & 3 \\ 3 & 1 & 4 \\ 9 & 8 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 + 3i \\ 3 - 4i & 2 \end{bmatrix}$$

MATRIX

2 * 2

$$\begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix}$$

+

2 * 2

$$\begin{bmatrix} 2 & 4 \\ 5 & 3 \end{bmatrix}$$

=

2 * 2

$$\begin{bmatrix} 2 & 6 \\ 8 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1+2i & 2 \\ 3 & 2 - 5i \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 3+2i & 6 \\ 8 & 5-5i \end{bmatrix}$$

MATRIX

2 * 2

2 * 2

2 * 2

$$\begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -2 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1+2i & 2 \\ 3 & 2 - 5i \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} -1+2i & -2 \\ -2 & -1-5i \end{bmatrix}$$

MATRIX

$$4 \quad * \quad \begin{bmatrix} 2 & 4 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 16 \\ 20 & 12 \end{bmatrix}$$

$$(1+2i) \quad * \quad \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 2+4i \\ 5+10i \end{bmatrix}$$

MATRIX

Columns der ersten Matrix müssen gleich der Rows der zweitenMatrix sein

2 * 2

2 * 2

2 * 2

$$\begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 4 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 10 & 6 \\ 11 & 15 \end{bmatrix}$$

$$0 * 2 + 2 * 5 = 10$$

$$0 * 4 + 2 * 3 = 6$$

$$3 * 2 + 1 * 5 = 11$$

$$3 * 4 + 1 * 3 = 15$$

MATRIX

$$\begin{bmatrix} 0 & 2 & 3 \\ 3 & 1 & 4 \\ 9 & 8 & 0 \end{bmatrix}$$

*

$$\begin{bmatrix} 1 & -3 & 4 \\ 0 & -1 & 2 \\ 3 & 1 & 0 \end{bmatrix}$$

=

$$\begin{bmatrix} 9 & 1 & 4 \\ 15 & -6 & 14 \\ 9 & -35 & 52 \end{bmatrix}$$

MATRIX

$$3 * 2$$
$$\begin{bmatrix} 0 & 2 \\ 3 & 1 \\ 9 & 8 \end{bmatrix}$$

*

$$2 * 2$$
$$\begin{bmatrix} 2 & 4 \\ 5 & 3 \end{bmatrix}$$

$$3 * 2$$

=

$$\begin{bmatrix} 10 & 6 \\ 11 & 15 \\ 58 & 60 \end{bmatrix}$$

MATRIX

3 * 2

$$\begin{bmatrix} 0 & 2 \\ 3 & 1 \\ 9 & 8 \end{bmatrix}$$

2 * 1

$$\begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

*

3 * 1

$$\begin{bmatrix} 10 \\ 11 \\ 58 \end{bmatrix}$$

=

MATRIX

3 * 2

$$\begin{bmatrix} 0 & 2 \\ 3 & 1 \\ 9 & 8 \end{bmatrix}$$

2 * 1

$$\begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$X * Y \neq Y * X$$

kann nicht invertiert werden

$$(X+Y) * (Q+Z) = XQ + XZ + YQ + YZ$$

MATRIX

$$\begin{bmatrix} x \\ y \end{bmatrix} \circledtimes \begin{bmatrix} z \\ q \end{bmatrix} = \begin{bmatrix} xz \\ xq \\ yz \\ yq \end{bmatrix}$$

Tensor Product

$$\begin{bmatrix} 1 \\ 3 \end{bmatrix} \circledtimes \begin{bmatrix} 2 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 7 \\ 6 \\ 15 \\ 21 \end{bmatrix}$$

MATRIX

$$\begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix}$$

Identity Matrix (I)

$$XI = X$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Inverse (Real Number)

$$A^{-1} = \frac{1}{A} \quad 5^{-1} = \frac{1}{5} \quad 10^{-1} = \frac{1}{10}$$

Inverse (Matrix)

$$AA^{-1} = I$$

$$A^{-1}A = I$$

MATRIX

X

X⁻¹

|

$$\begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix} * \begin{bmatrix} -1/6 & 1/3 \\ 1/2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Matrix Inverse

MATRIX

$$\begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix}^T = \begin{bmatrix} 0 & 3 \\ 2 & 1 \end{bmatrix}$$

Transpose

$$\begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}^T = \begin{bmatrix} 0 & 3 & 6 \\ 1 & 4 & 7 \\ 2 & 5 & 8 \end{bmatrix}$$

MATRIX

$$(XY)^T = Y^T X^T$$

$$\begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 4 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 10 & 6 \\ 11 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 2+5i & i \\ 3 & 3-4i \end{bmatrix}^* = \begin{bmatrix} 2-5i & -i \\ 3 & 3+4i \end{bmatrix}$$

Complex Conjugate

$$\begin{bmatrix} 2+5i & i \\ 3 & 3-4i \end{bmatrix}^+ = \begin{bmatrix} 2-5i & 3 \\ -i & 3+4i \end{bmatrix}$$

Adjoint (Transpose - Complex Conjugate)

$$X^+ = (X^*)^T = (X^T)^*$$

if

$$X^+ = X^{-1}$$

Unitary Matrix

$$XX^+ = XX^{-1} = I$$

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix} * \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Reversible Operation

if

$$X^+ = X$$

Hermitian Matrix

$$\begin{bmatrix} 3 & 2+i \\ 2-i & 5 \end{bmatrix}^+ = \begin{bmatrix} 3 & 2+i \\ 2-i & 5 \end{bmatrix}$$

Irreversible Operation

MATRIX

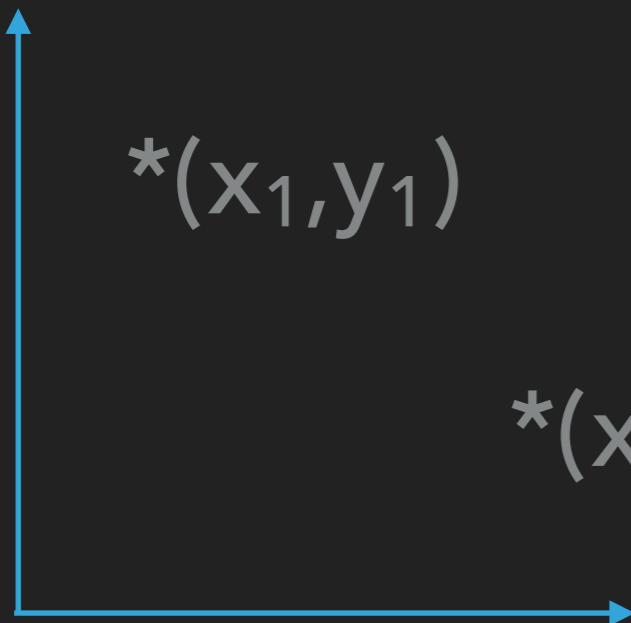
$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -4 \\ 3+i \\ -5i \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix}$$

Vector = Column Matrix (Quantum Computing)

MATRIX



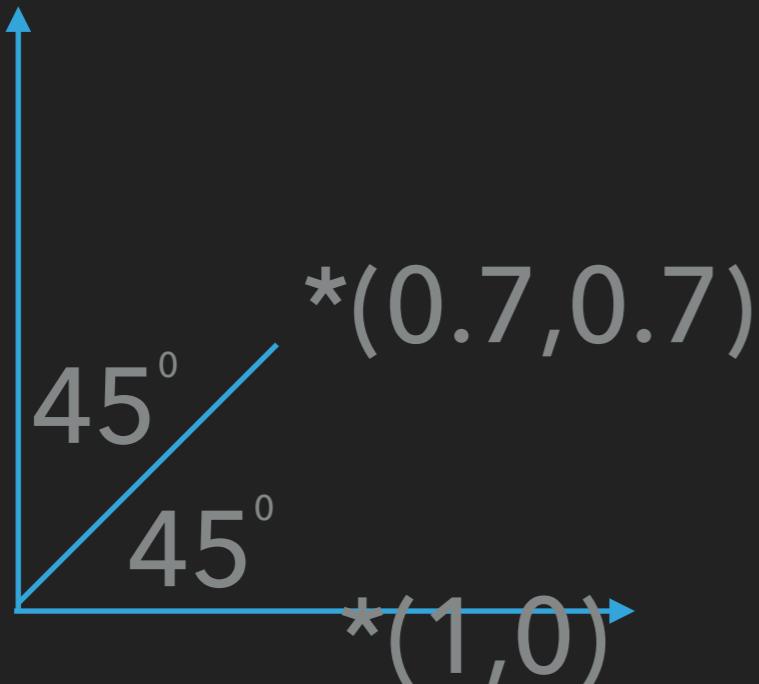
$$x_2 = Ax_1 + By_1$$

$$y_2 = Cx_1 + Dy_1$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} * \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

Linear Transformation

MATRIX

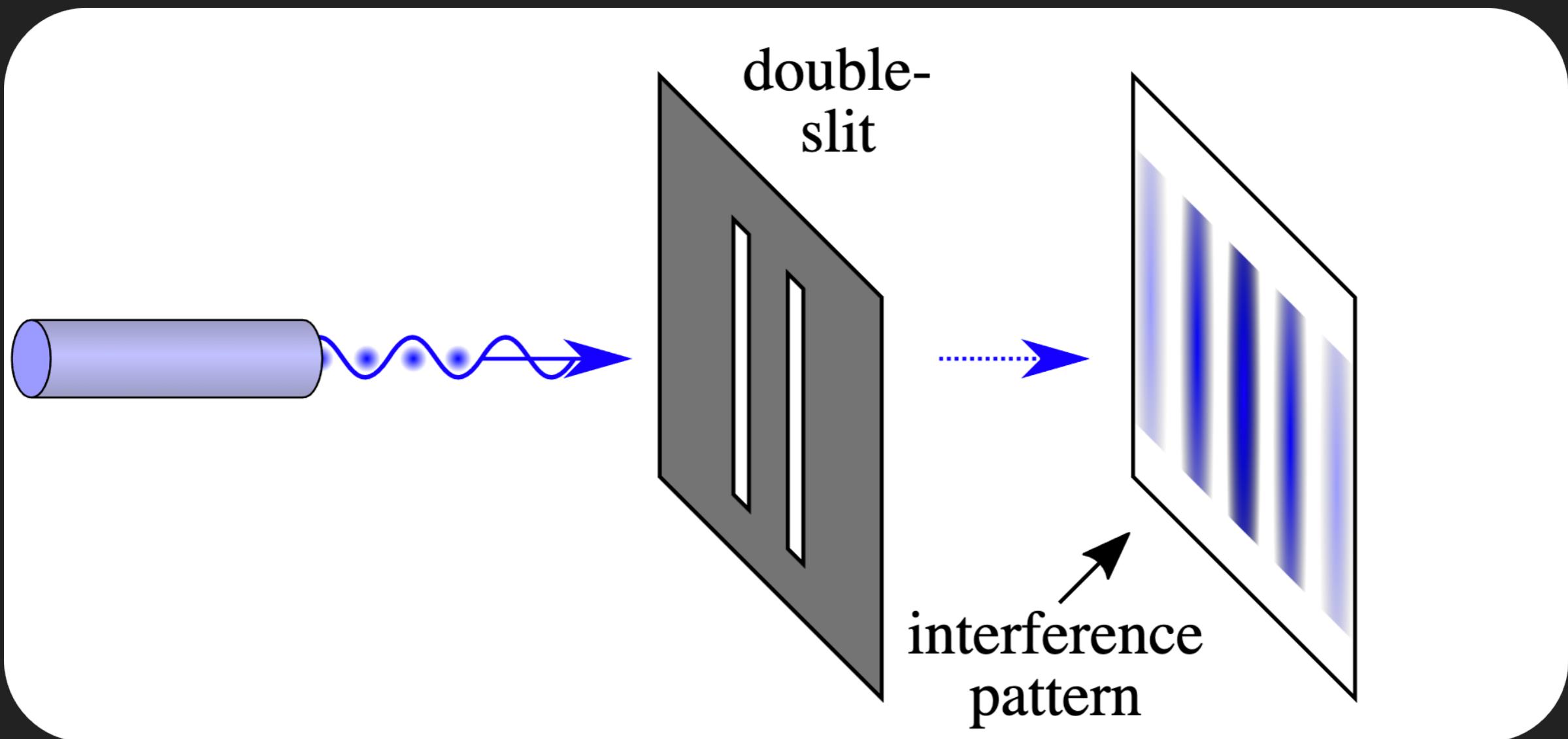


$$x_2 = Ax_1 + By_1$$
$$y_2 = Cx_1 + Dy_1$$

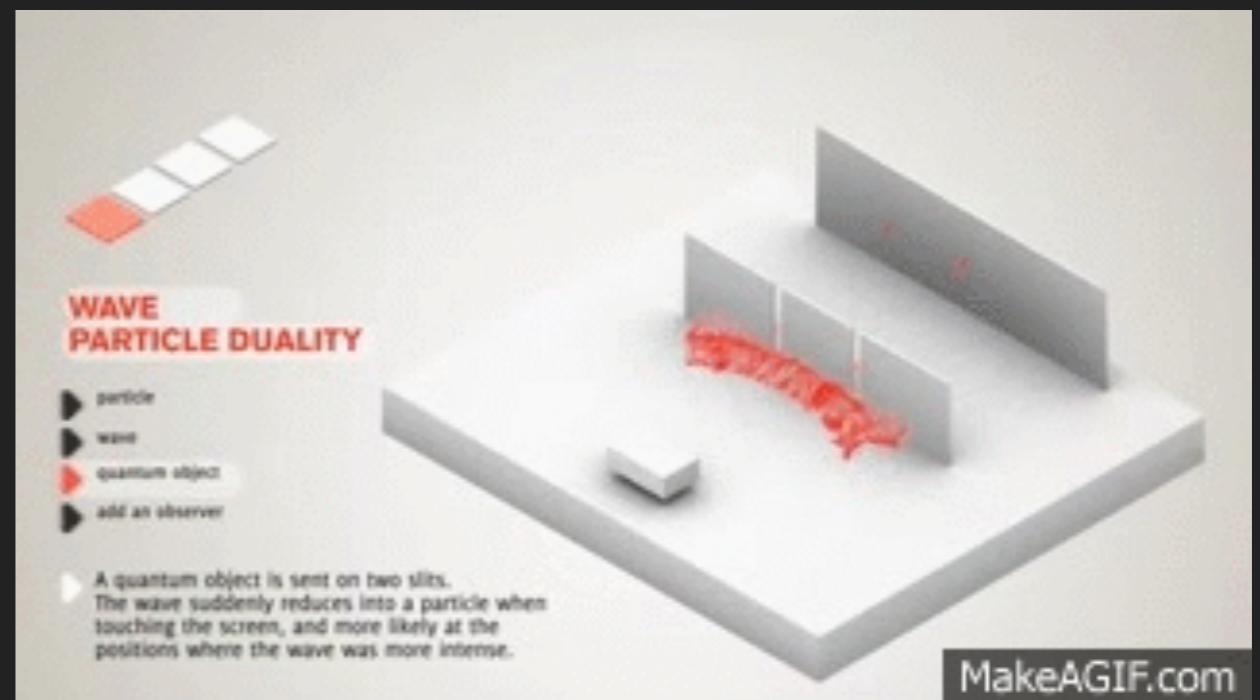
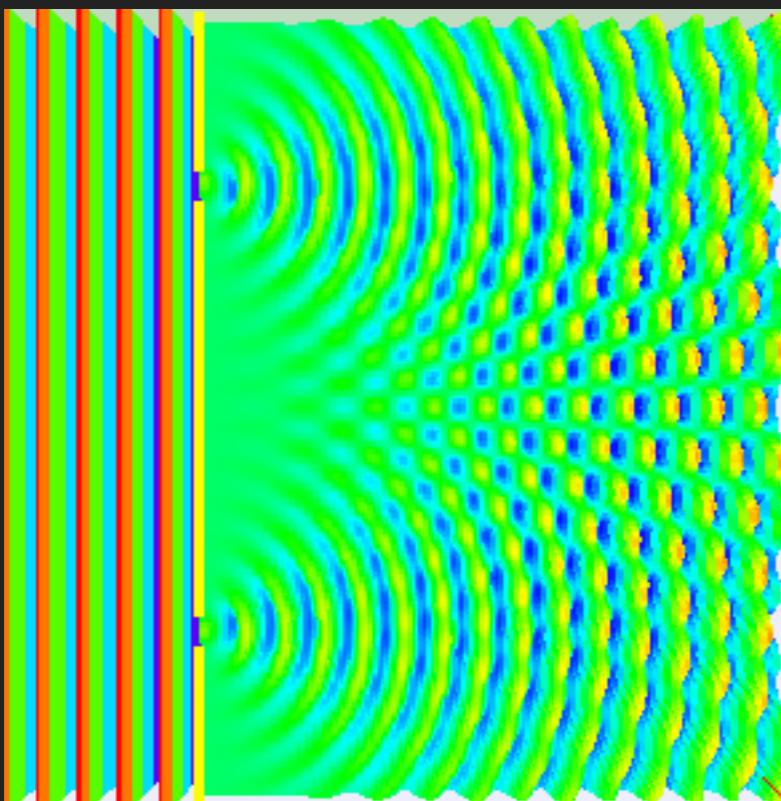
$$\begin{bmatrix} 0.7 \\ 0.7 \end{bmatrix} = \begin{bmatrix} \cos(45) & -\sin(45) \\ +\sin(45) & \cos(45) \end{bmatrix} * \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Rotation

DOUBLE SLIT / YOUNG

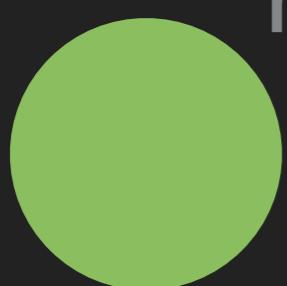


DOUBLE SLIT / YOUNG

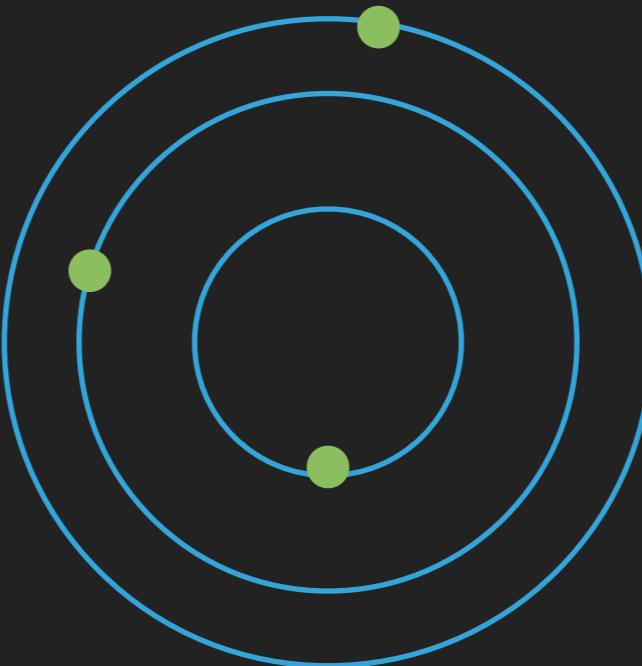


Collapse of the Wave Function

ENTANGLEMENT

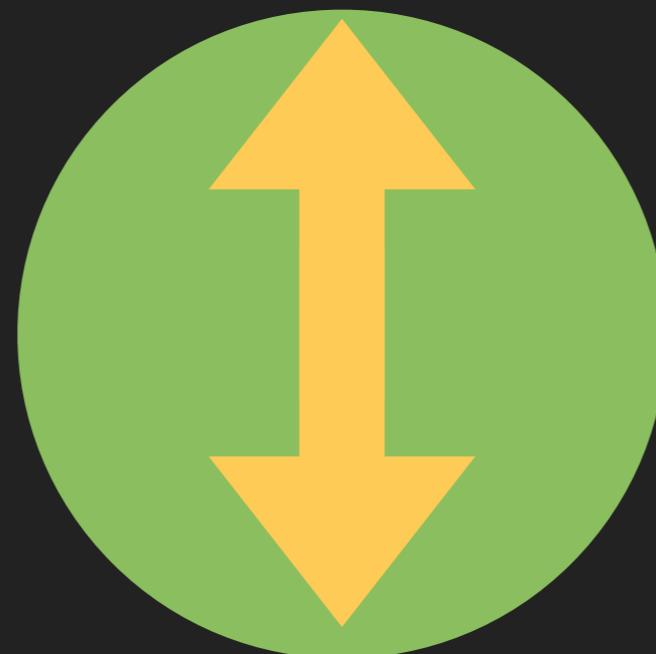


n, l, m_l, m_s



Principal
Orbital Angular Momentum
Magnetic
Electron Spin

ENTANGLEMENT



Superposition

QUBIT

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Spin Down

Qubit Down State

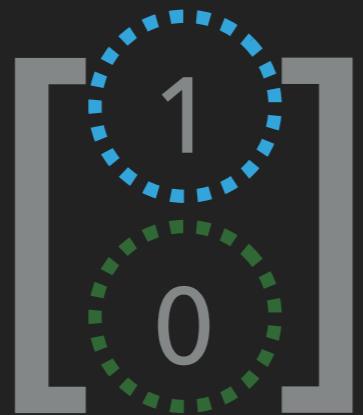
Classical Bit 0

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Spin Up

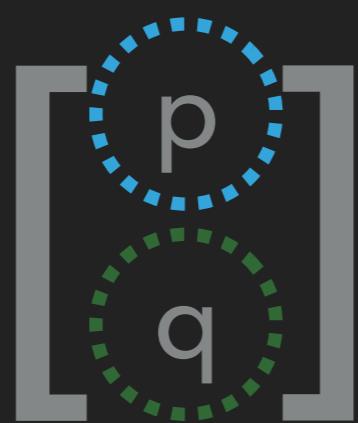
Qubit Up State

Classical Bit 1



$P(\text{Spin Down}) = 1$

$P(\text{Spin Up}) = 0$



$$p = a + bi$$

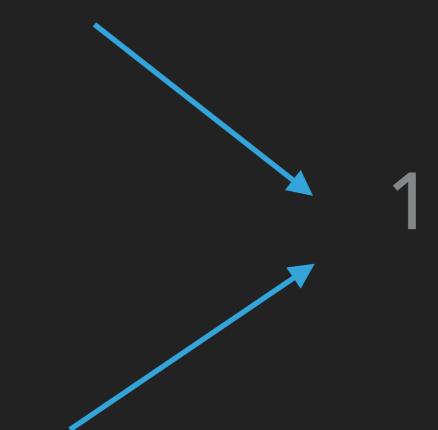
$$|p|^2 = a^2 + b^2$$

$$|p|^2 + |q|^2 = 1$$

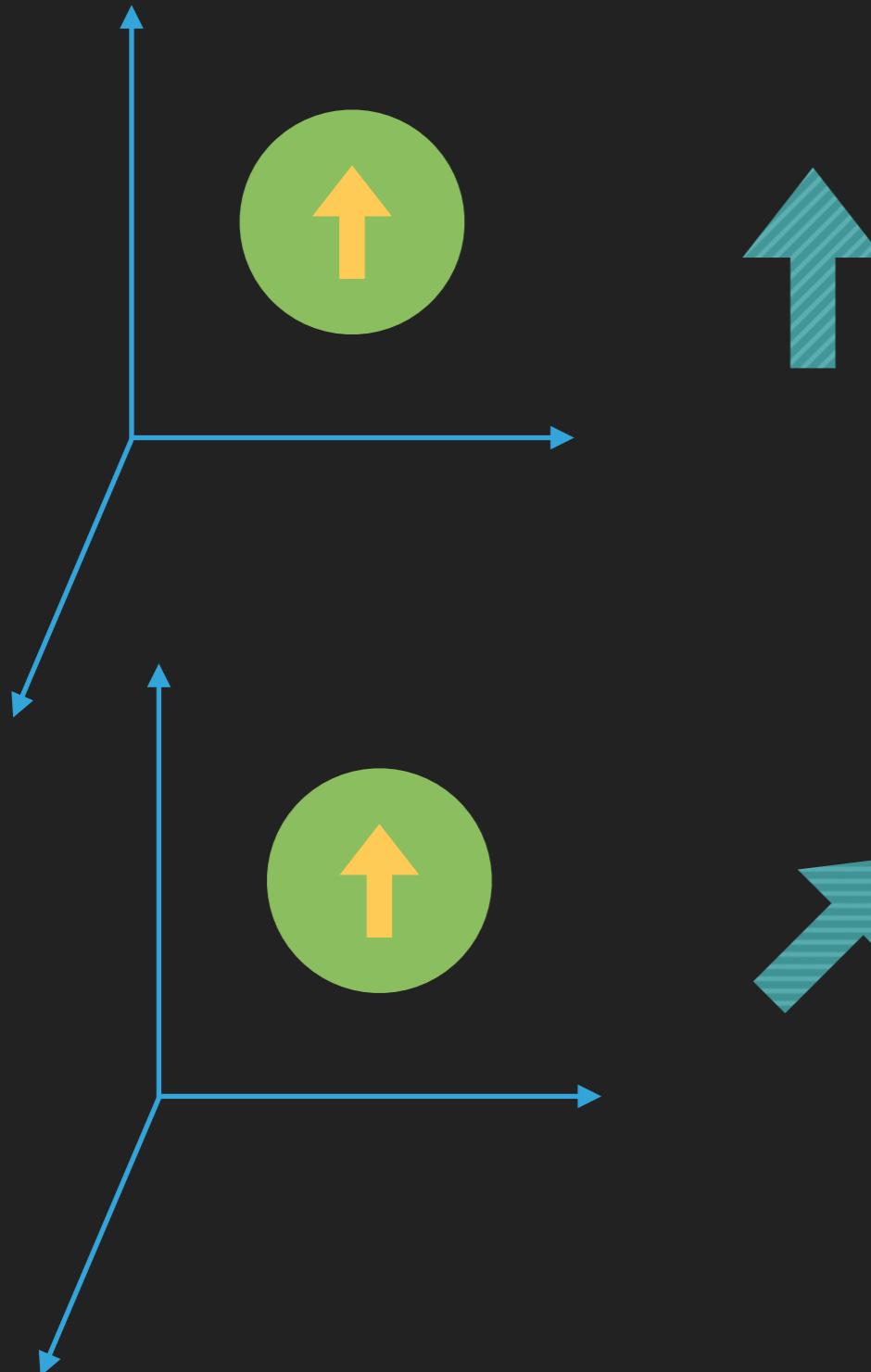
SPIN

$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

Vector in
Superposition

$$P(\text{Spin Down}) = \left(\frac{1}{\sqrt{2}} \right)^2 = \frac{1}{2}$$
$$P(\text{Spin Up}) = \left(\frac{1}{\sqrt{2}} \right)^2 = \frac{1}{2}$$


SPIN



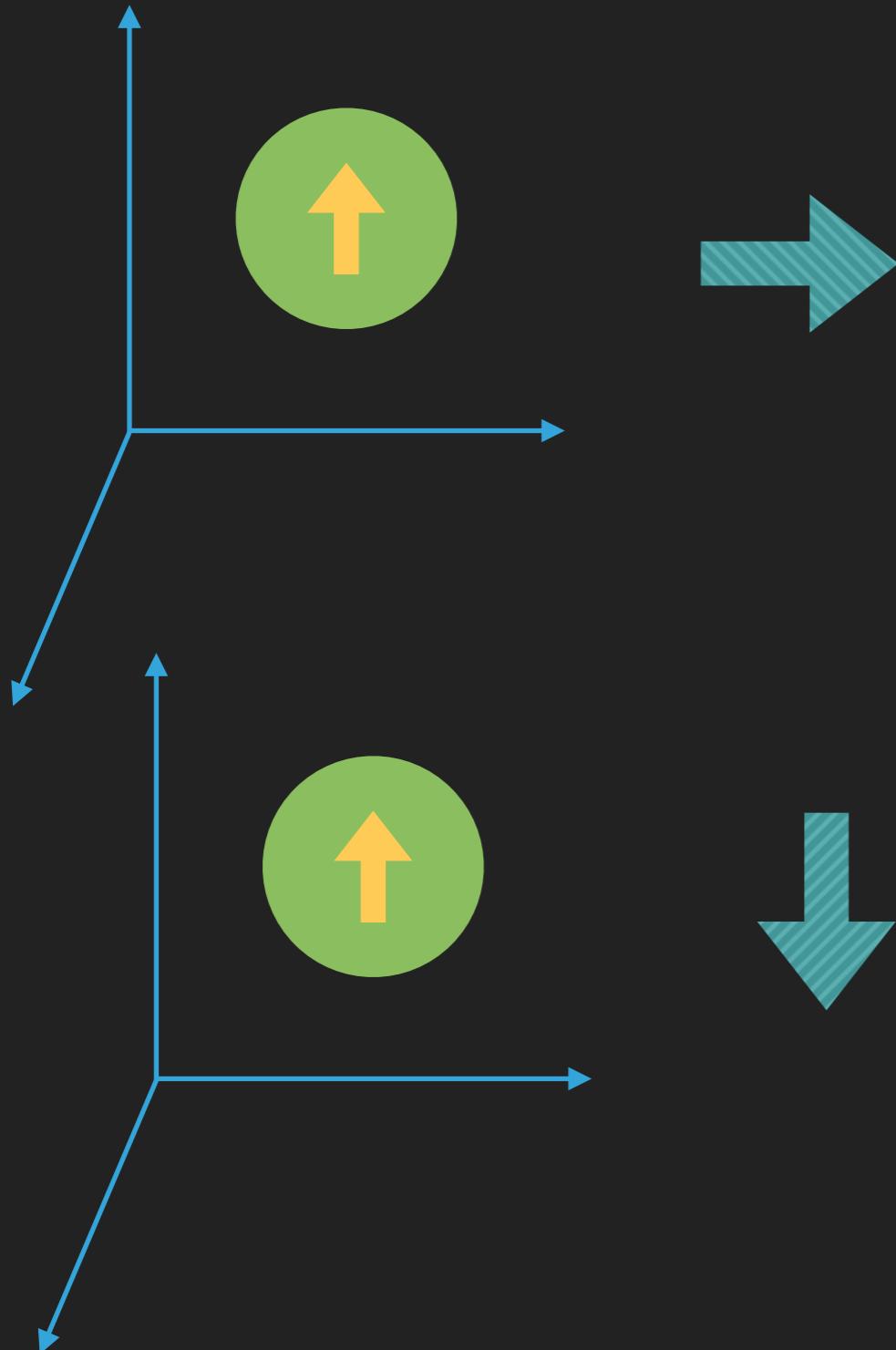
$P(\text{Spin Down}) = 0$

$P(\text{Spin Up}) = 1$

$P(\text{Spin Down}) = 0.2$

$P(\text{Spin Up}) = 0.8$

SPIN



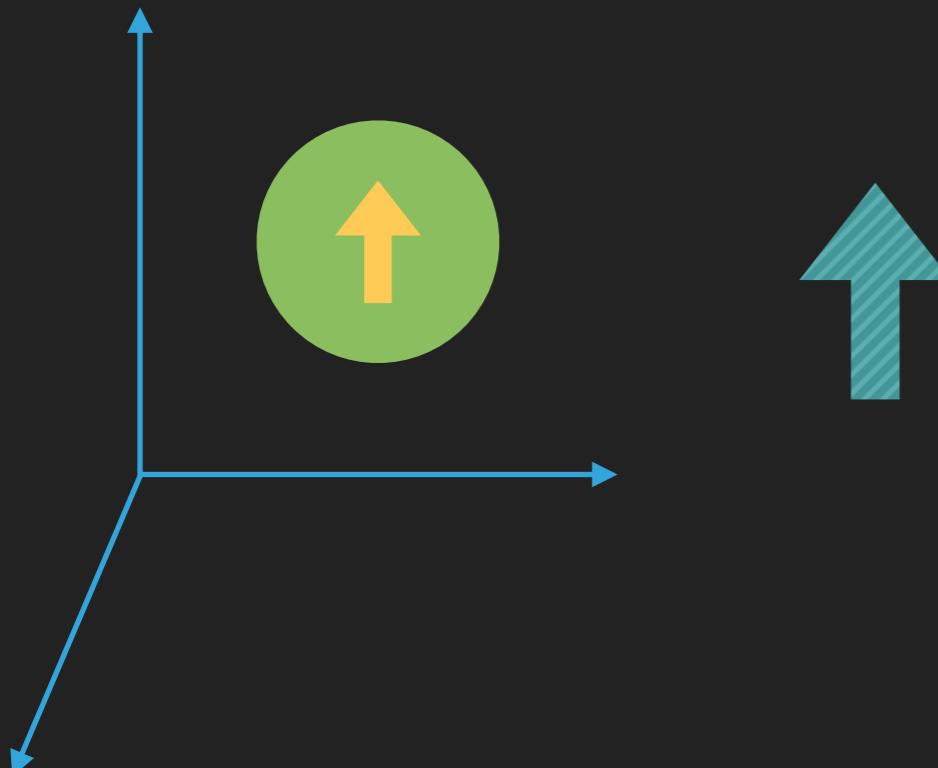
$P(\text{Spin Down}) = 0.5$

$P(\text{Spin Up}) = 0.5$

$P(\text{Spin Down}) = 0$

$P(\text{Spin Up}) = 1$

SPIN



$P(\text{Spin Down}) = 0$

$P(\text{Spin Up}) = 1$

$$A = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad E = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Inner Product

$$A^+ * E$$

$$A = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad E = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$A^+ * E$

$$\begin{bmatrix} 0 & 1 \end{bmatrix} * \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 1$$

$$1^2 = 1$$

BRA-KET

$$A = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad E = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$\langle A | E \rangle$

Bra Ket

$\langle A | * | E \rangle$

$A^+ \quad E$

$|1\rangle$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$|0\rangle$

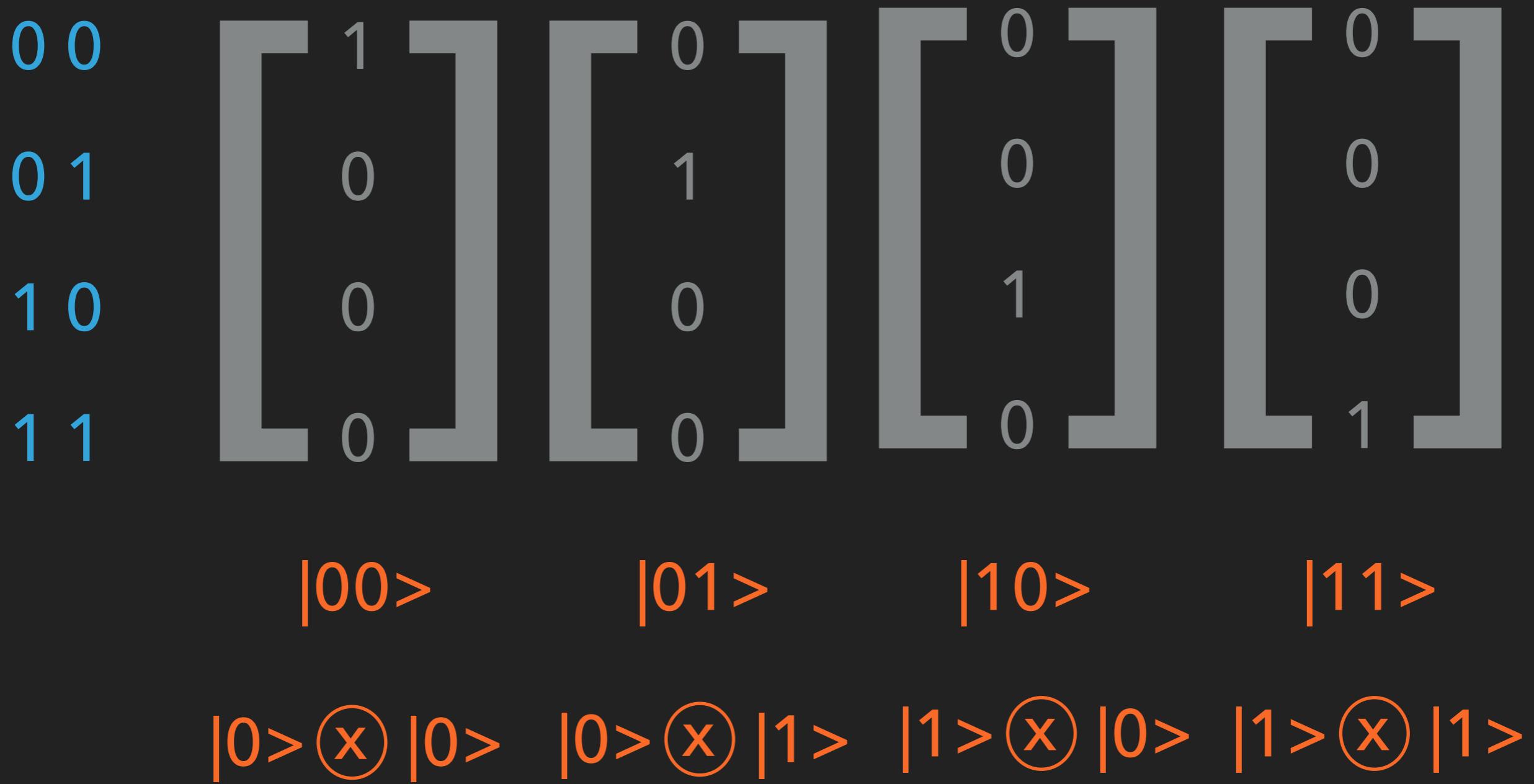
$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

BRA-KET

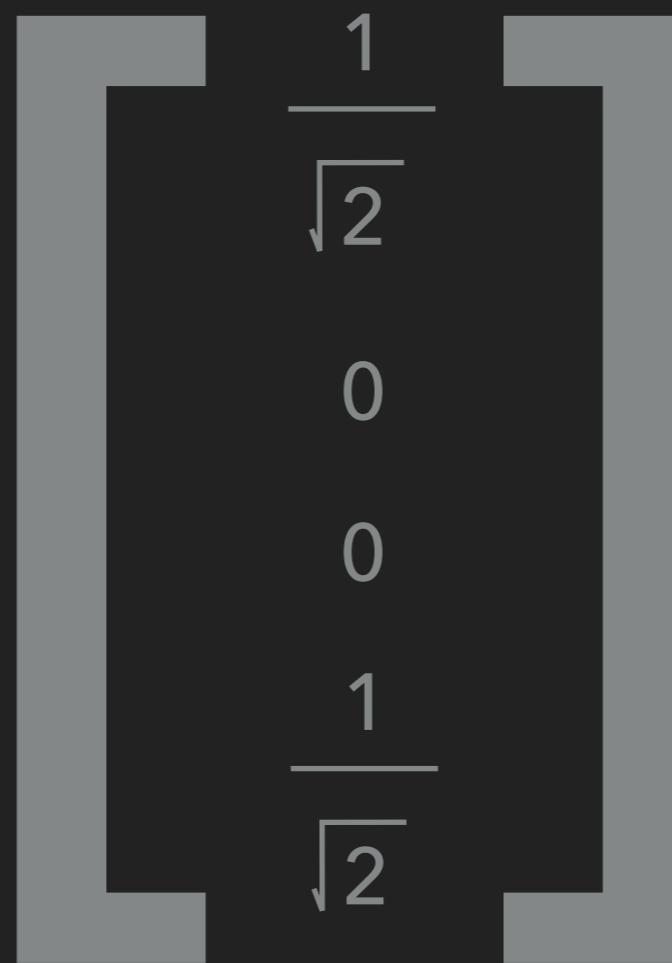
$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$

MULTI QUBIT



MULTI QUBIT

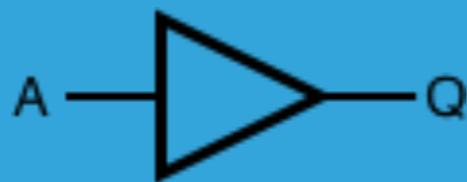


$$\frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$

Superposition + Entanglement

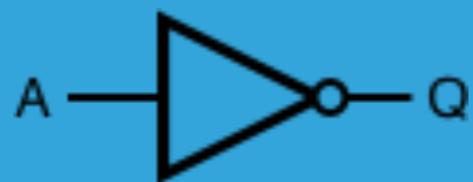
CLASSICAL GATES

Buffer



A	Q
0	0
1	1

NOT



A	Q
0	1
1	0

CLASSICAL GATES

AND



A	B	Q
0	0	0
0	1	0
1	0	0
1	1	1

CLASSICAL GATES

OR



A	B	Q
0	0	0
0	1	1
1	0	1
1	1	1

CLASSICAL GATES

NAND



A	B	Q
0	0	1
0	1	1
1	0	1
1	1	0

CLASSICAL GATES

NOR



A	B	Q
0	0	1
0	1	0
1	0	0
1	1	0

CLASSICAL GATES

XOR



A	B	Q
0	0	0
0	1	1
1	0	1
1	1	0

CLASSICAL GATES

XNOR



A	B	Q
0	0	1
0	1	0
1	0	0
1	1	1

QUANTUM GATES

$$X \text{ (NOT)} \quad \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$X | 1 \rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = | 0 \rangle$$

$$X | 0 \rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = | 1 \rangle$$

QUANTUM GATES

Y (Pauli Y)

$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

Z (Pauli Z)

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

QUANTUM GATES

$$H \text{ (Hadamard)} \quad \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

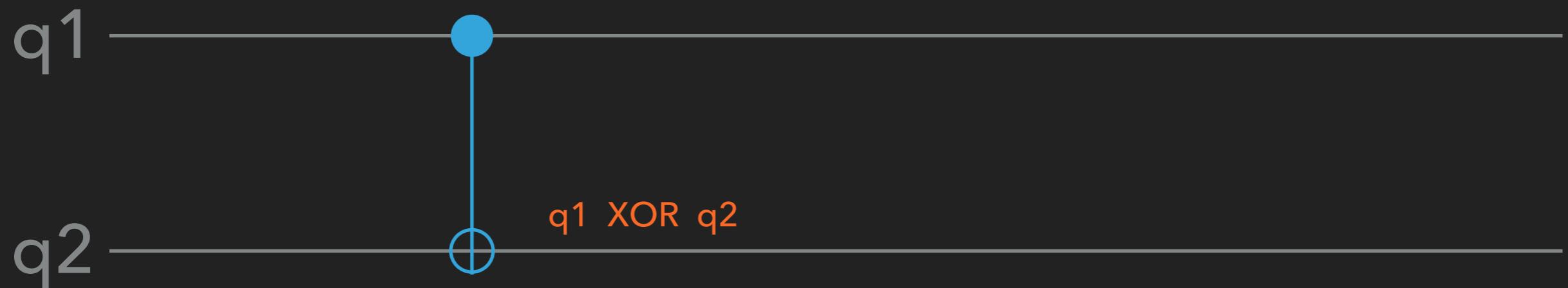
$$H |0\rangle \quad \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$

$$H |1\rangle \quad \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle)$$

QUANTUM GATES

CNOT

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

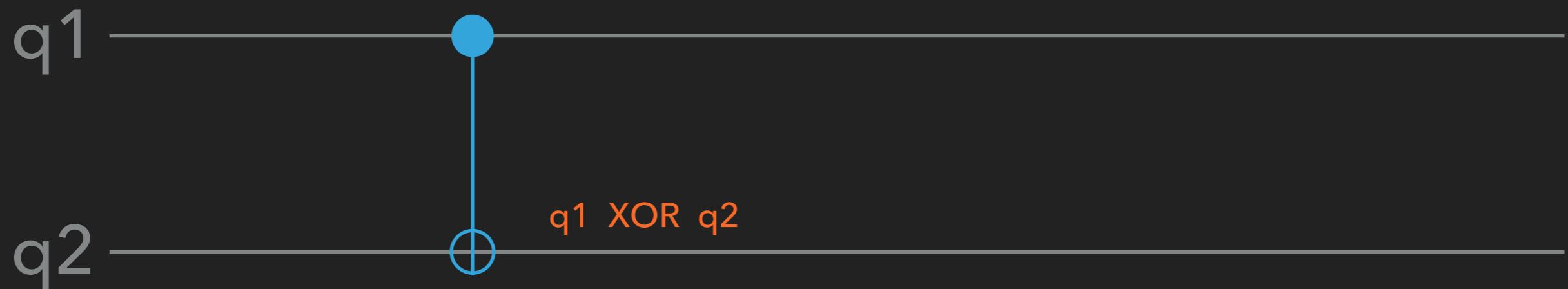


QUANTUM GATES

CNOT

1	0	0	0
0	1	0	0
0	0	0	1
0	0	1	0

Alternative



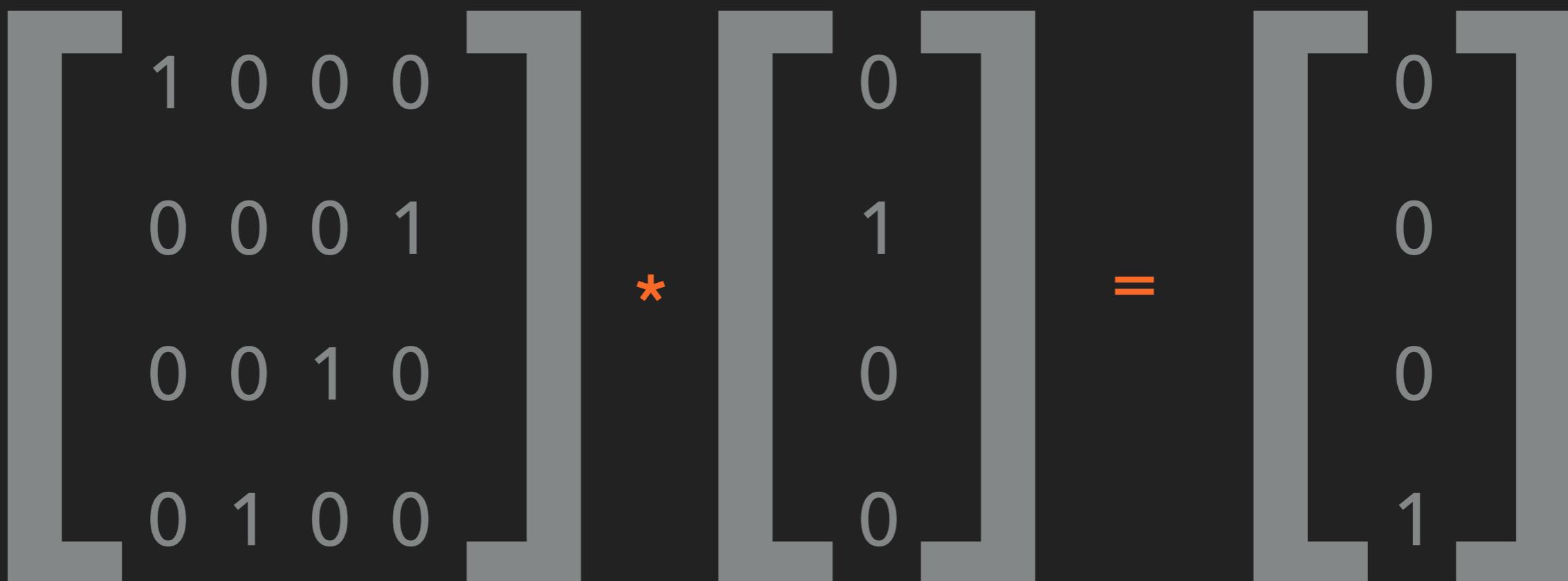
QUANTUM GATES

CNOT $|00\rangle = |00\rangle$

CNOT $|01\rangle = |11\rangle$

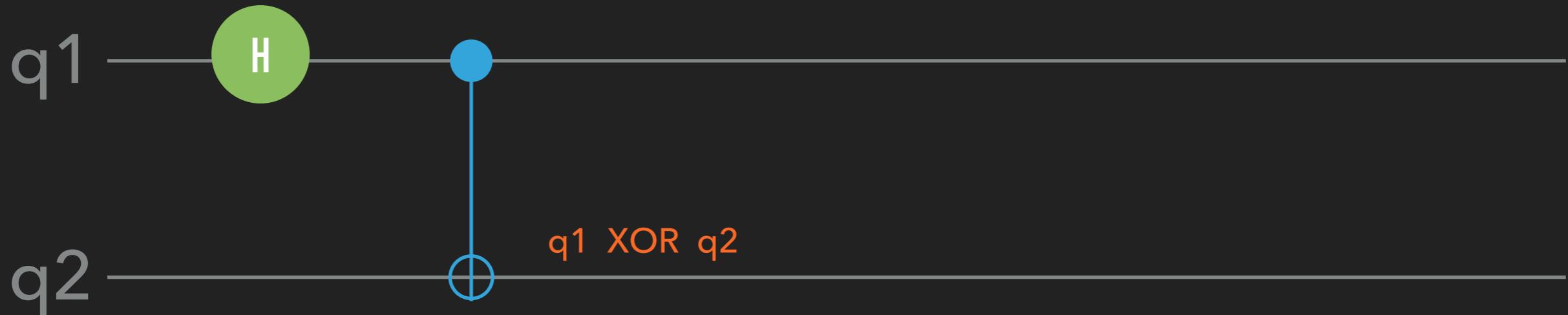
CNOT $|10\rangle = |10\rangle$

CNOT $|11\rangle = |01\rangle$



QUANTUM GATES

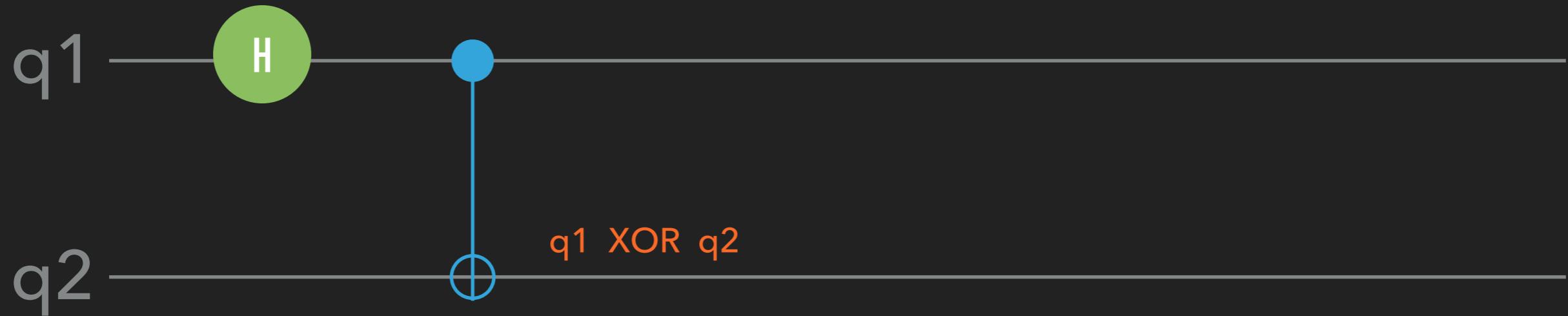
Hadamard & CNOT



$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} * \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

QUANTUM GATES

Hadamard & CNOT



$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

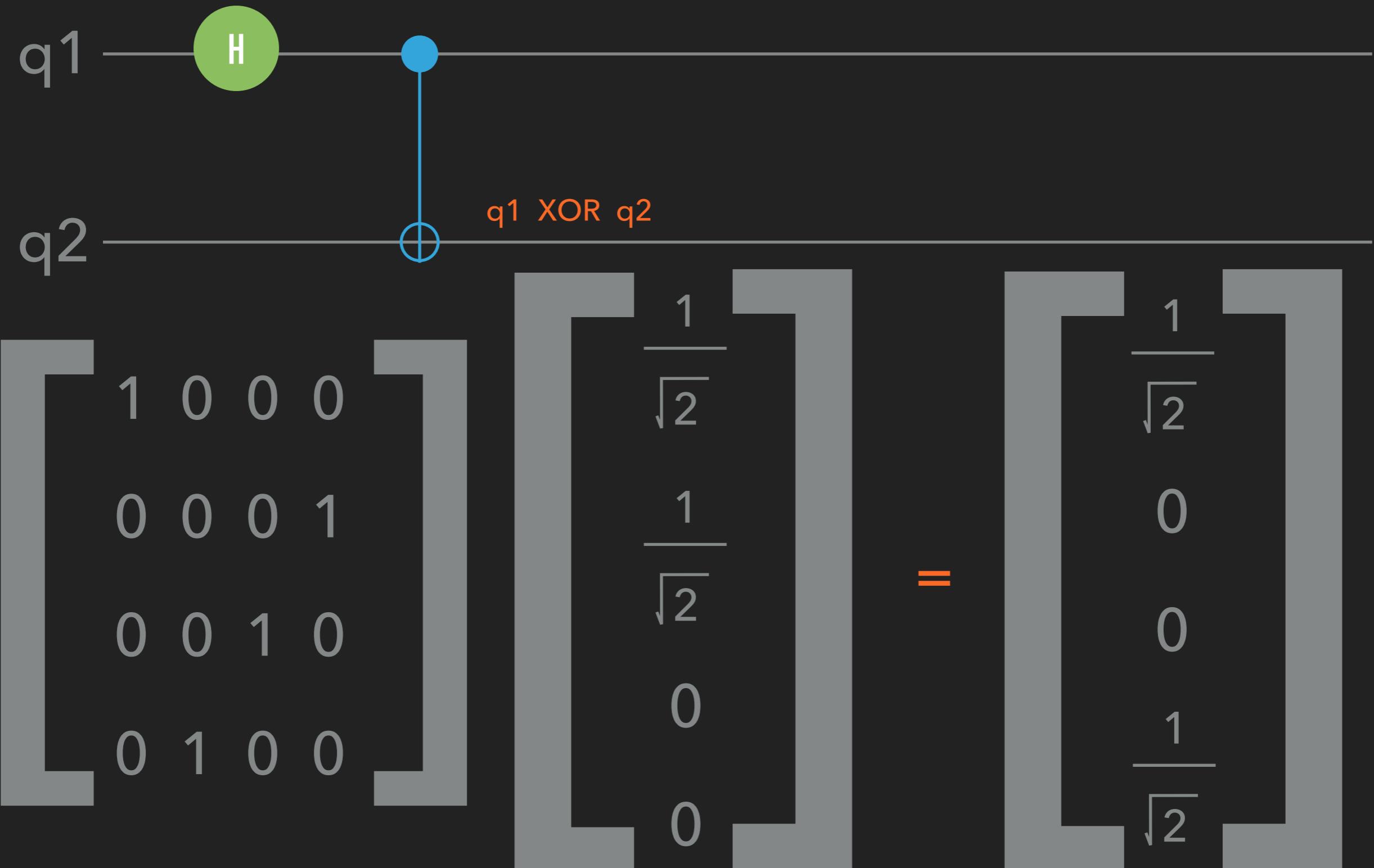
=



$$|0\rangle \otimes |+\rangle = |0+\rangle$$

QUANTUM GATES

Hadamard & CNOT



Z GATE

$$Z |0\rangle = |0\rangle$$

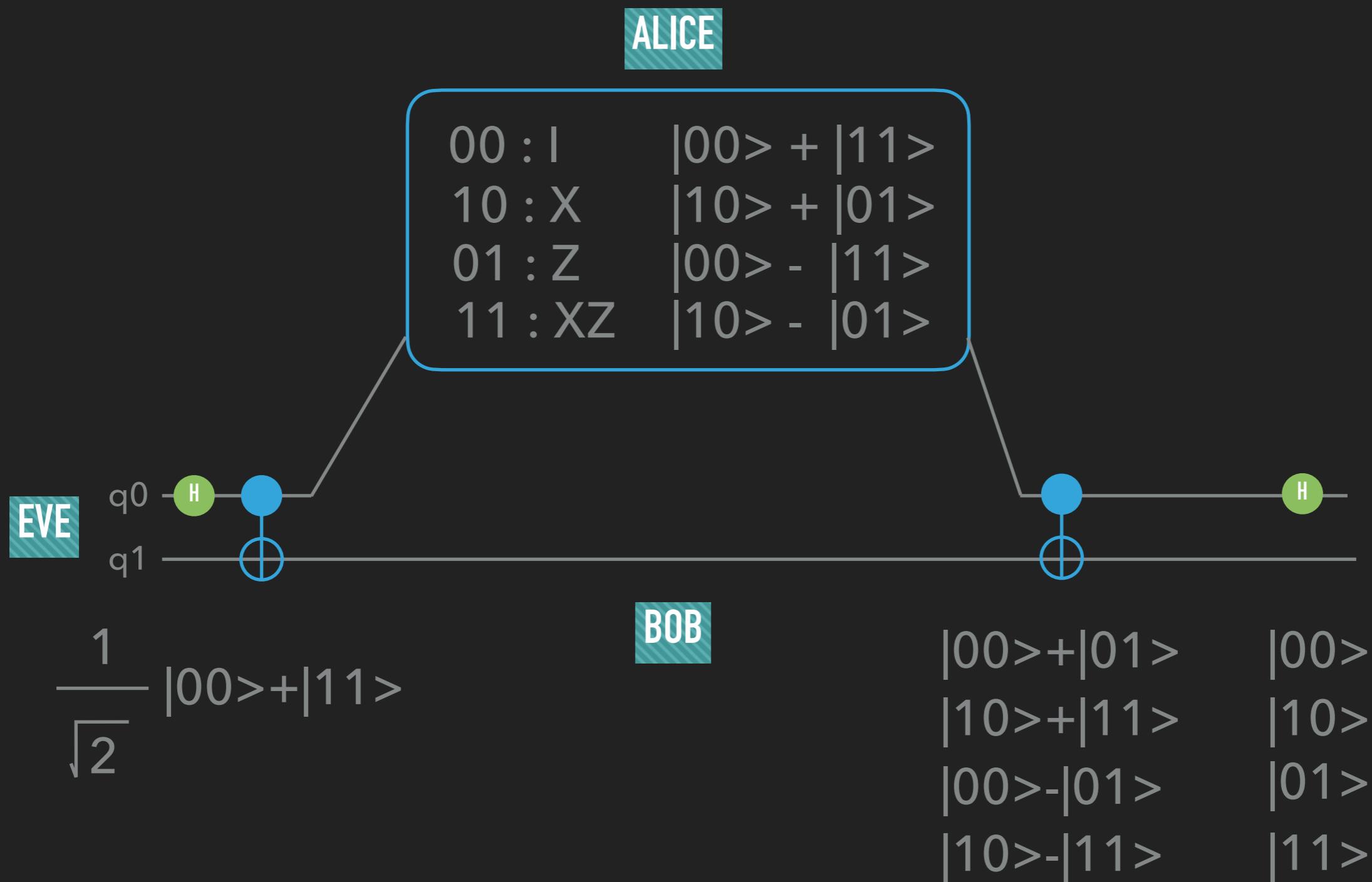
$$Z |1\rangle = -|1\rangle$$

$$Z \left(\frac{1}{\sqrt{2}} |00\rangle + |11\rangle \right) = \frac{1}{\sqrt{2}} |00\rangle - |11\rangle$$

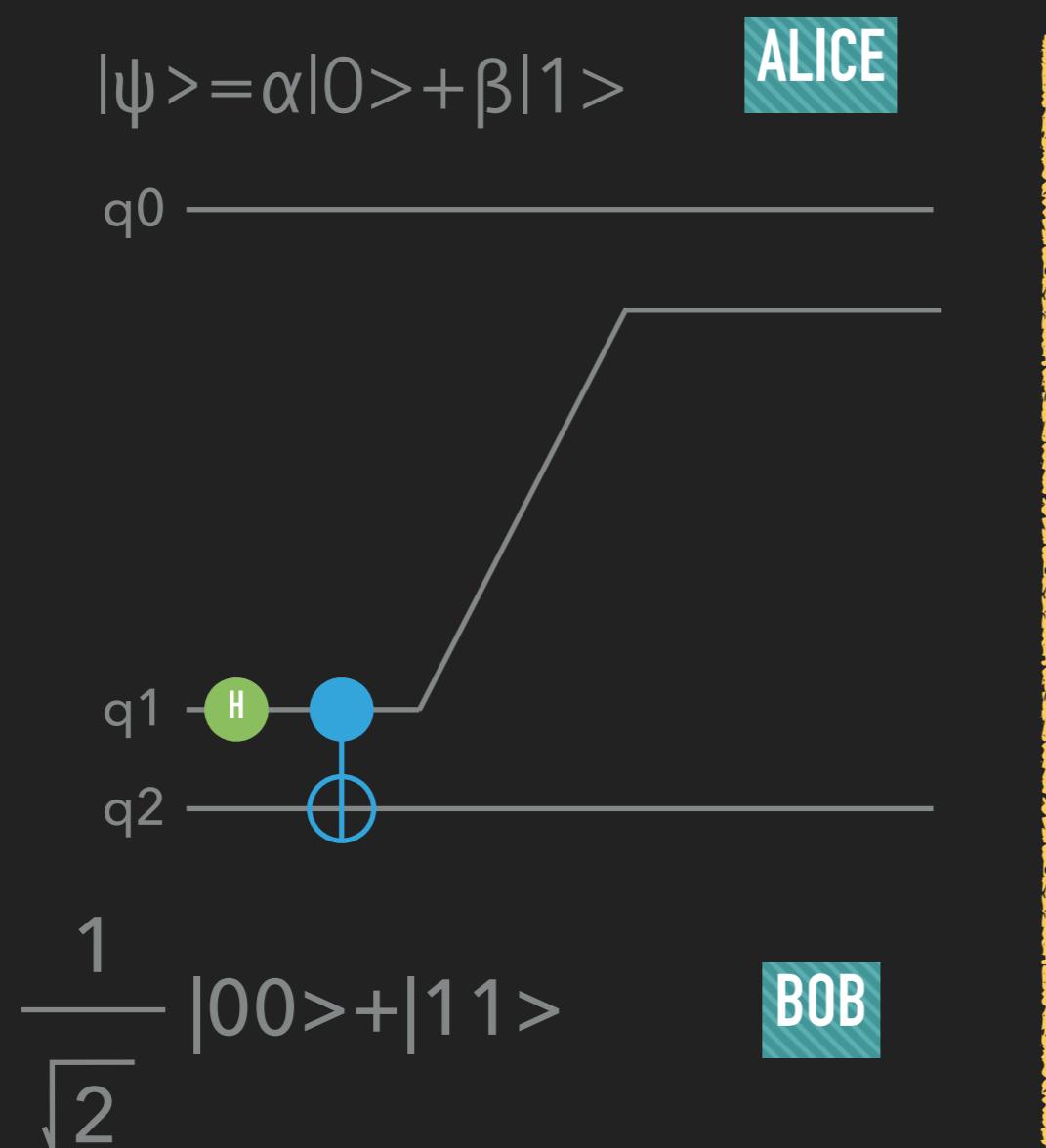
The diagram illustrates the decomposition of a Z gate on two qubits. It consists of three vertical columns representing qubits, separated by horizontal lines.

- Left Column:** Represented by a dark gray rectangle. The matrix elements are:
 - Row 1: 1, 0, 0, 0
 - Row 2: 0, 1, 0, 0
 - Row 3: 0, 0, 1, 0
 - Row 4: 0, 0, 0, -1A red asterisk (*) is placed below the second column of this matrix.
- Middle Column:** Represented by a dark gray rectangle. The matrix elements are:
 - Row 1: $\frac{1}{\sqrt{2}}$, 0, 0, $\frac{1}{\sqrt{2}}$
 - Row 2: 0, 0, 0, 0
 - Row 3: $\frac{1}{\sqrt{2}}$, 0, 0, - $\frac{1}{\sqrt{2}}$A red equals sign (=) is placed below the second column of this matrix.
- Right Column:** Represented by a dark gray rectangle. The matrix elements are:
 - Row 1: $\frac{1}{\sqrt{2}}$, 0, 0, - $\frac{1}{\sqrt{2}}$
 - Row 2: 0, 0, 0, 1
 - Row 3: - $\frac{1}{\sqrt{2}}$, 0, 0, $\frac{1}{\sqrt{2}}$

SUPERDENSE CODING



QUANTUM TELEPORTATION



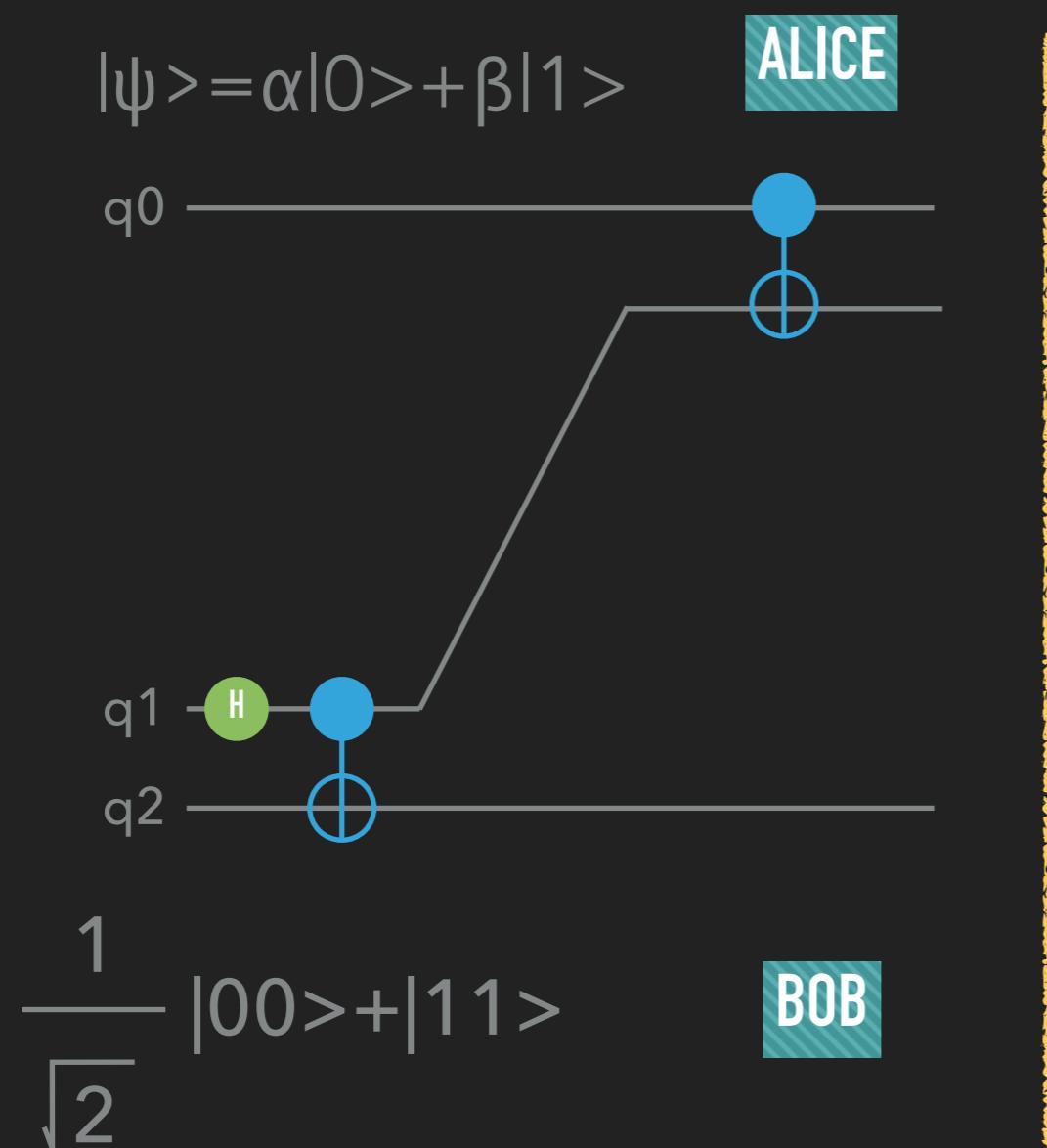
TOTAL QUANTUM STATE

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad \text{X} \quad \frac{1}{\sqrt{2}} |00\rangle + |11\rangle$$

=

$$\frac{1}{\sqrt{2}} [\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle]$$

QUANTUM TELEPORTATION



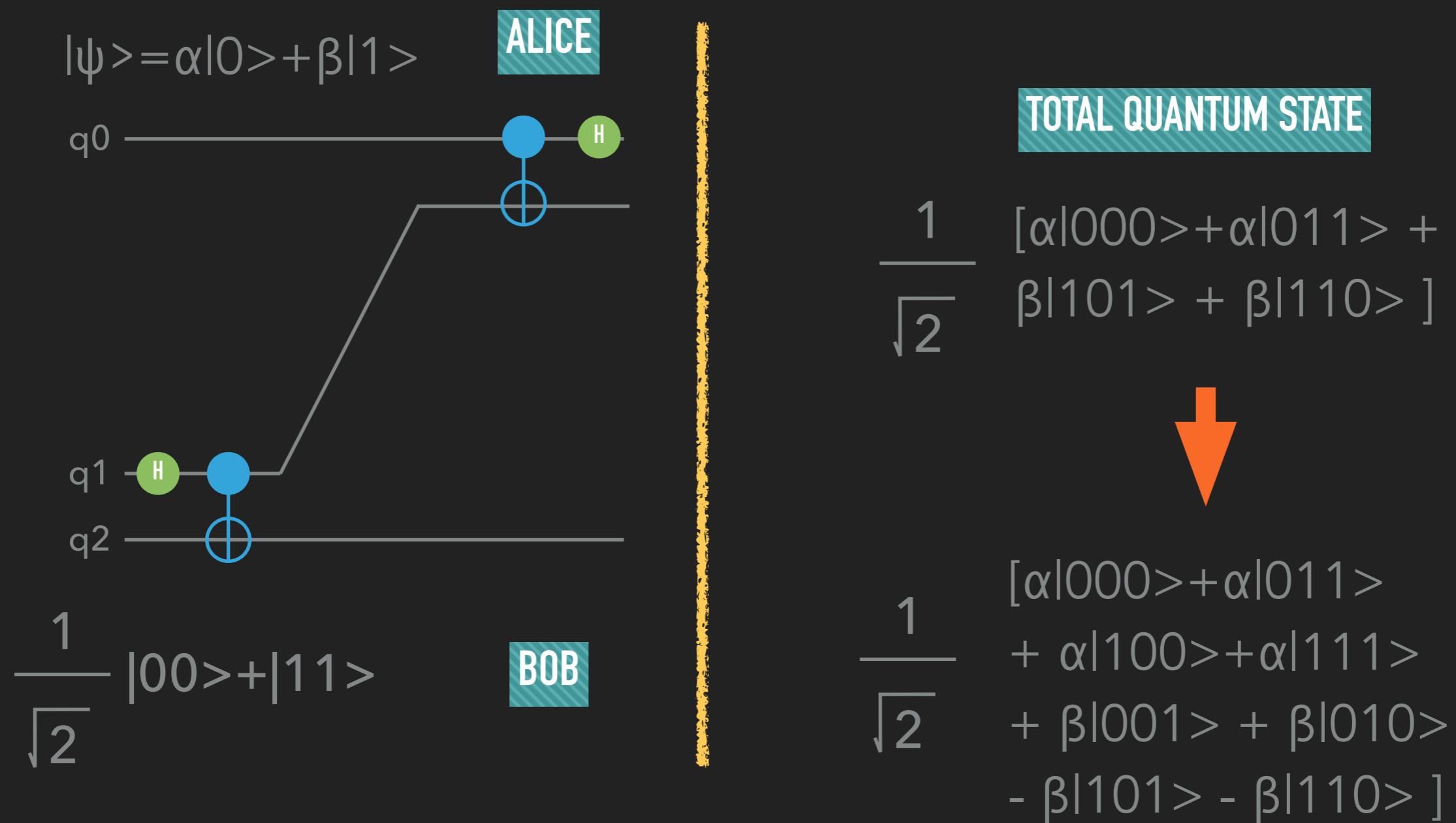
TOTAL QUANTUM STATE

$$\frac{1}{\sqrt{2}} [\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle]$$

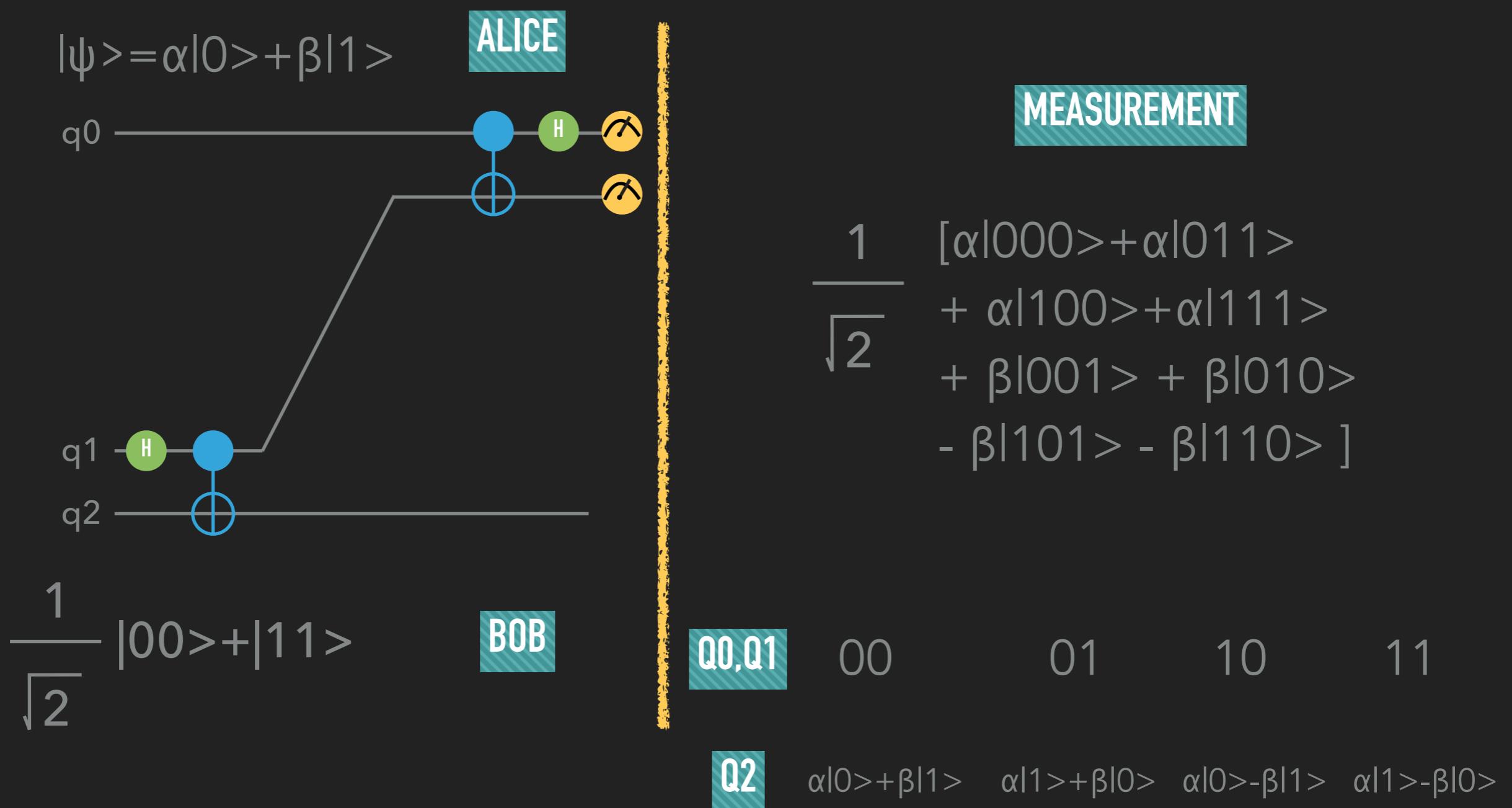


$$\frac{1}{\sqrt{2}} [\alpha|000\rangle + \alpha|011\rangle + \beta|101\rangle + \beta|110\rangle]$$

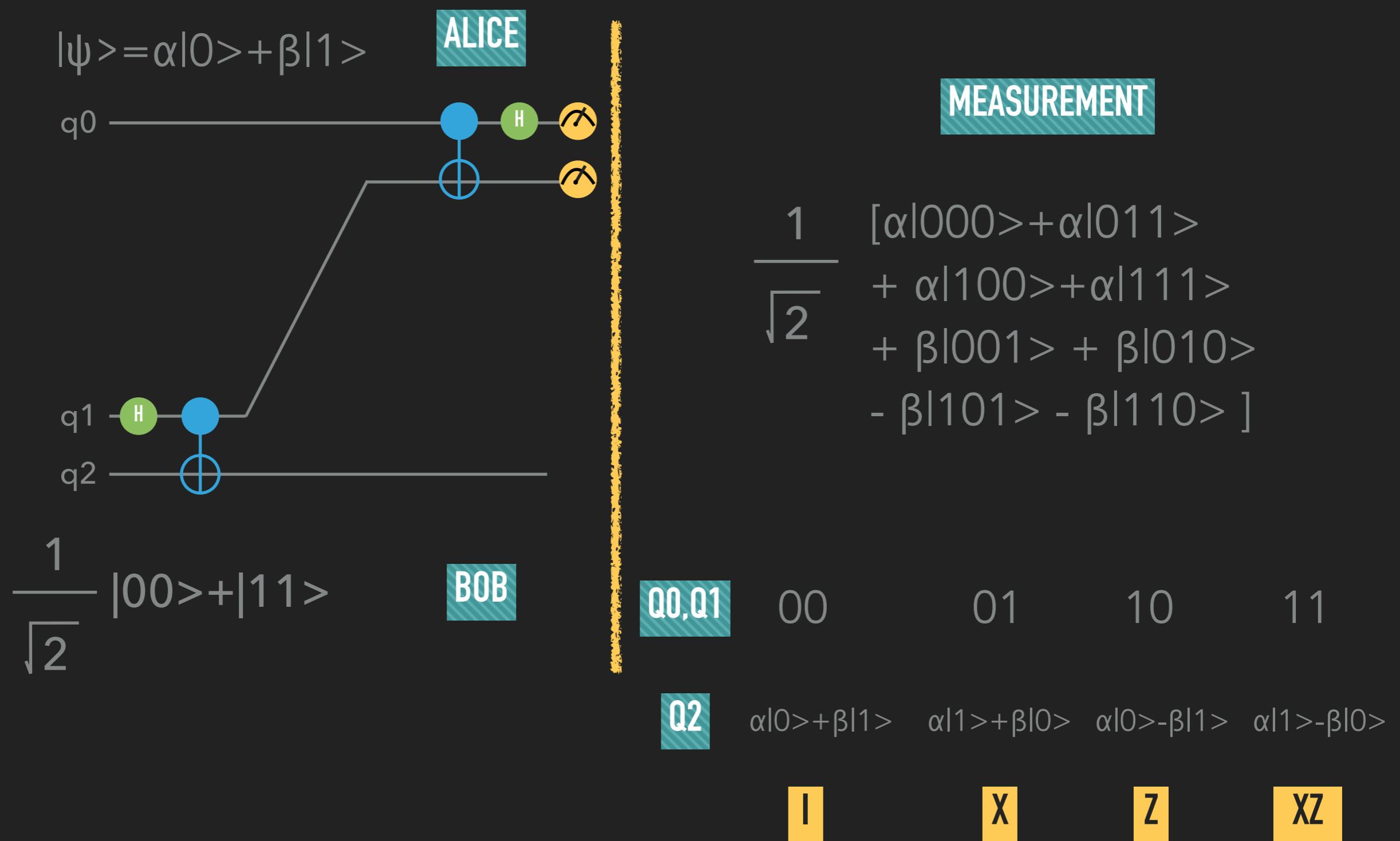
QUANTUM TELEPORTATION



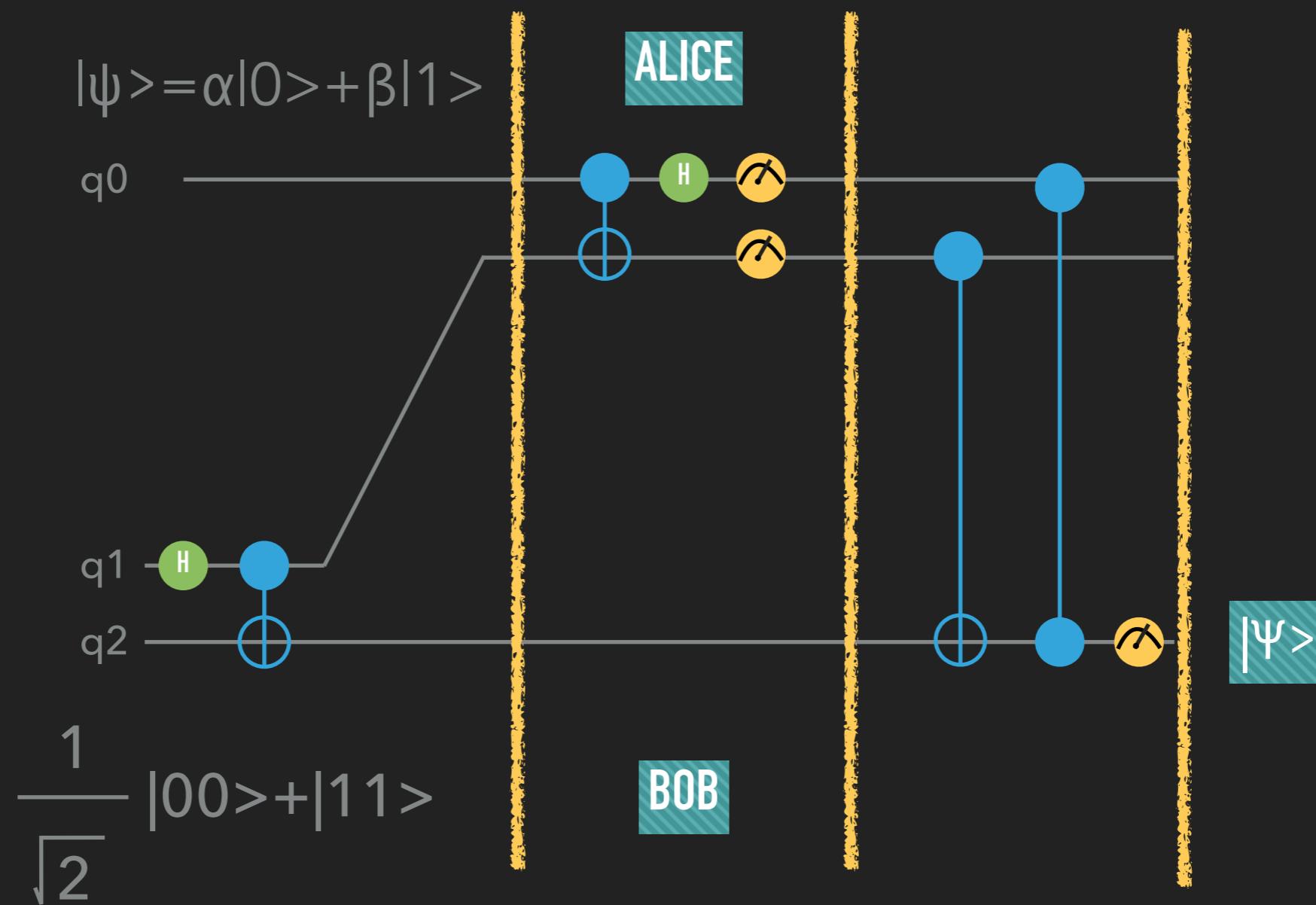
QUANTUM TELEPORTATION



QUANTUM TELEPORTATION



QUANTUM TELEPORTATION



BERNSTEIN - VAZIRANI ALGORITHM

1011001
AND
0000001



1

1011001
AND
0000010



0

N BITS

DEUTSCH ALGORITHM

	$f(0)$	$f(1)$	
1	0	0	Constant
2	0	1	Balanced
3	1	0	Balanced
4	1	1	Constant

$$f : \{0,1\} \rightarrow \{0,1\}$$

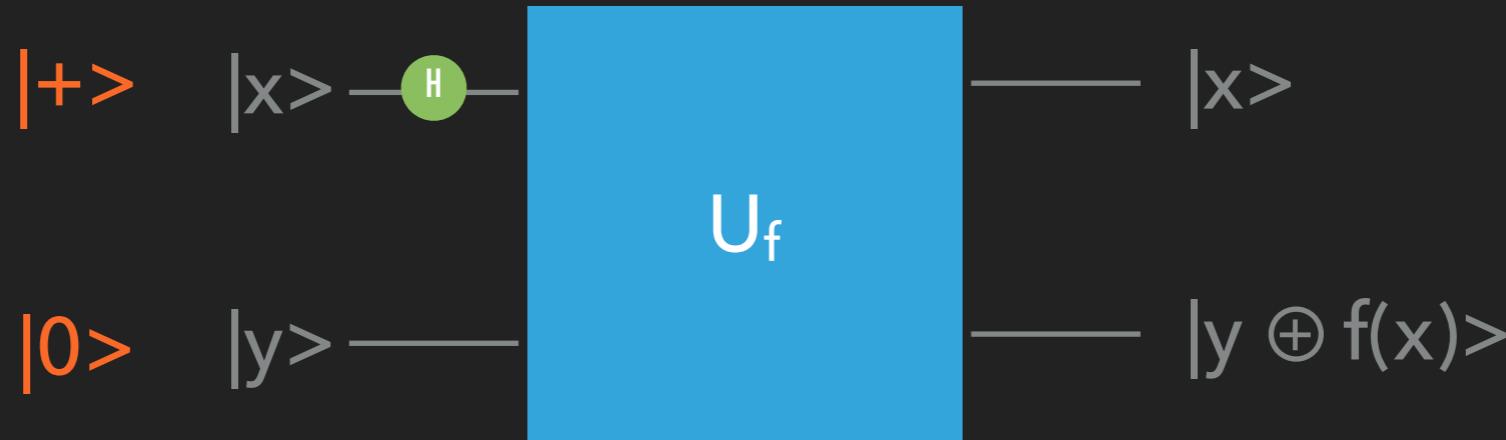
$f(0) = f(1)$ Constant

DEUTSCH ALGORITHM



$$|x\rangle |y\rangle \xrightarrow{U_f} |x\rangle |y \oplus f(x)\rangle \xrightarrow{U_f} |x\rangle |y \oplus f(x)\rangle \oplus f(x)\rangle$$
$$|x\rangle |y \oplus 0\rangle$$
$$|x\rangle |y\rangle$$

DEUTSCH ALGORITHM

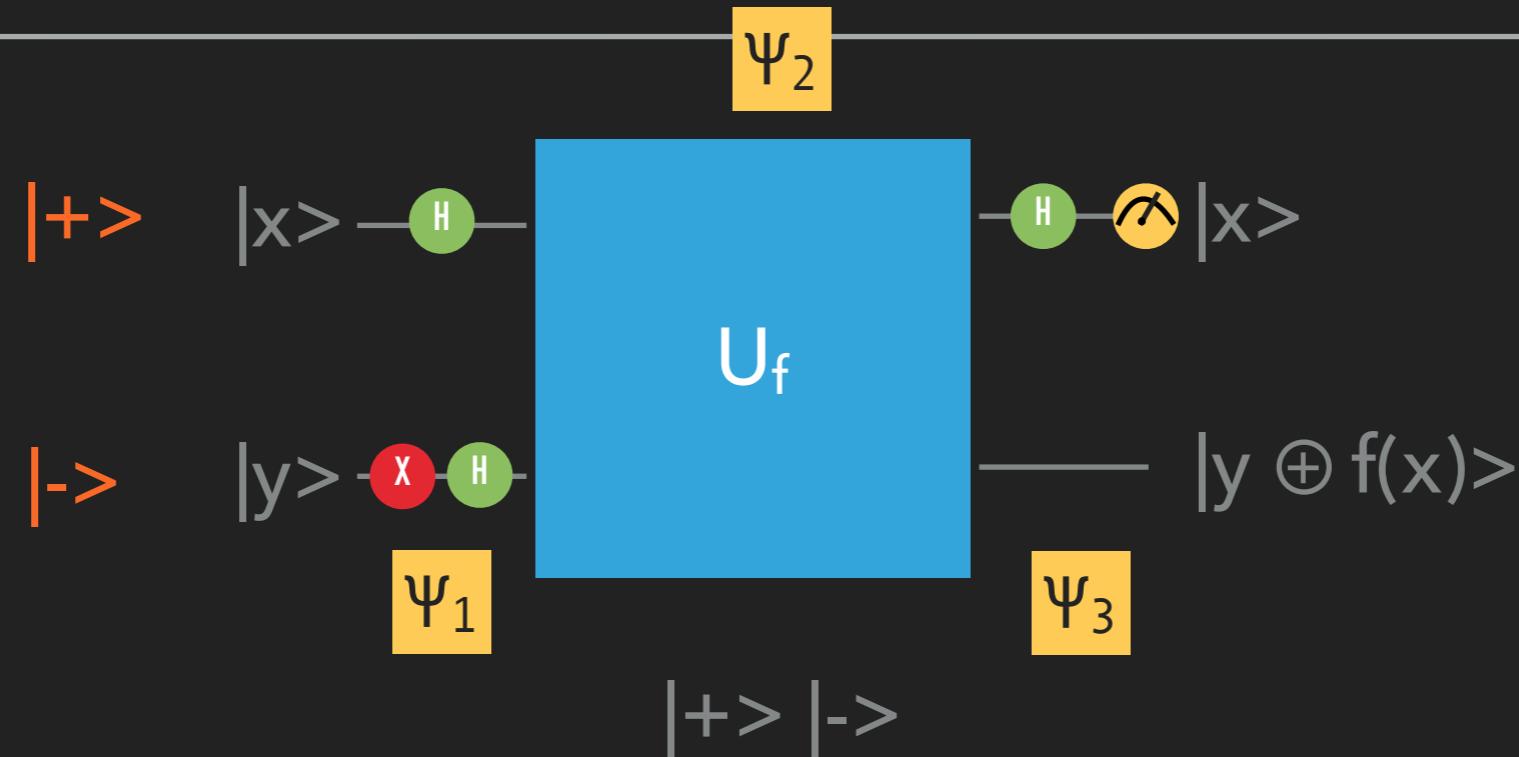


$$U_f(|+\rangle \otimes |0\rangle) \longrightarrow U_f\left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\otimes |0\rangle\right)$$

$$\begin{aligned} &= \frac{1}{\sqrt{2}} U_f |0\rangle \otimes |0\rangle + \frac{1}{\sqrt{2}} U_f |1\rangle \otimes |0\rangle \\ &= \frac{1}{\sqrt{2}} |0\rangle f(0) \oplus |0\rangle + \frac{1}{\sqrt{2}} |1\rangle f(1) \oplus |0\rangle \end{aligned}$$



DEUTSCH ALGORITHM



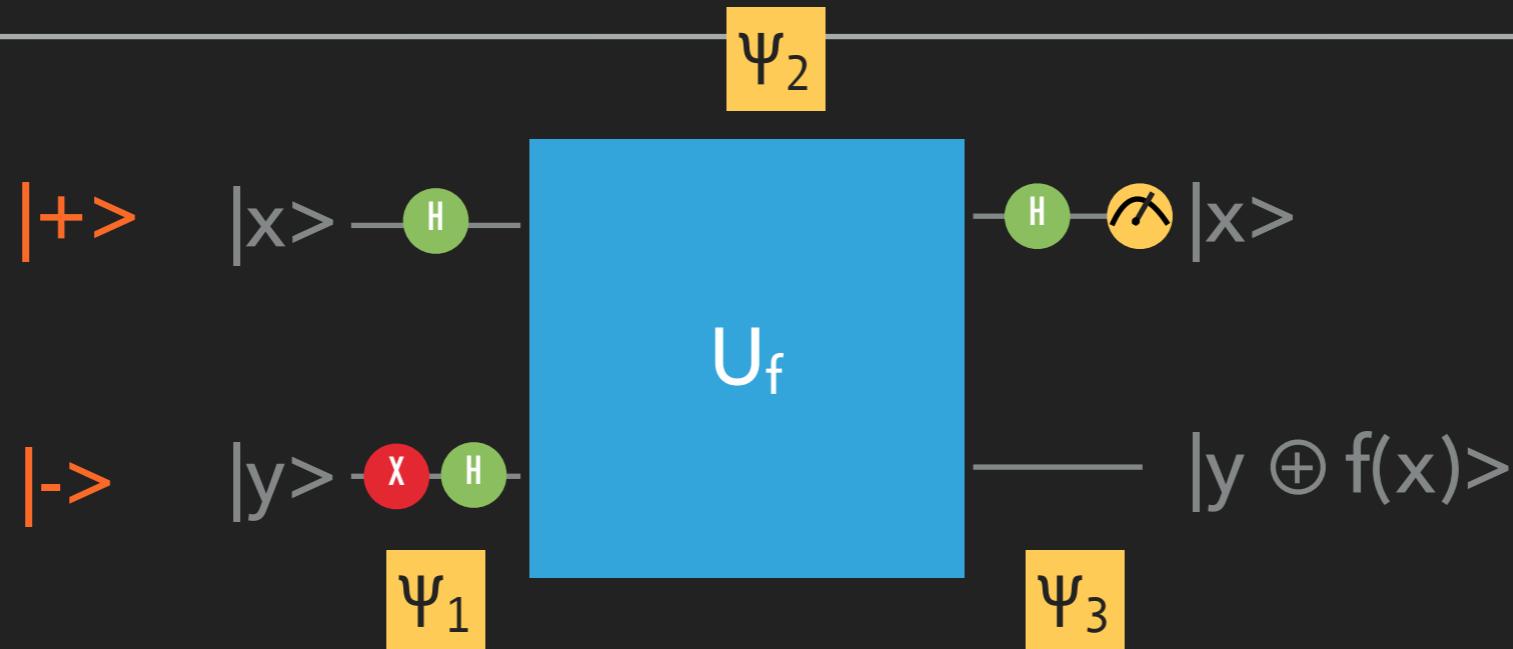
$$\Psi_1 = \frac{1}{2} (|0\rangle + |1\rangle) \otimes (|0\rangle - |1\rangle)$$

$$= \frac{1}{2} (|00\rangle - |01\rangle + |10\rangle - |11\rangle)$$

$$\rightarrow \frac{1}{2} (|0\rangle |0\oplus f(0)\rangle - |0\rangle |1\oplus f(0)\rangle + |1\rangle |0\oplus f(1)\rangle - |1\rangle |1\oplus f(1)\rangle)$$

$$\Psi_2 = \frac{1}{2} (|0\rangle |f(0)\rangle - |0\rangle |1\oplus f(0)\rangle + |1\rangle |f(1)\rangle - |1\rangle |1\oplus f(1)\rangle)$$

DEUTSCH ALGORITHM



$$\Psi_2 = \frac{1}{2} (|0\rangle |f(0)\rangle - |0\rangle |1\oplus f(0)\rangle + |1\rangle |f(1)\rangle - |1\rangle |1\oplus f(1)\rangle)$$

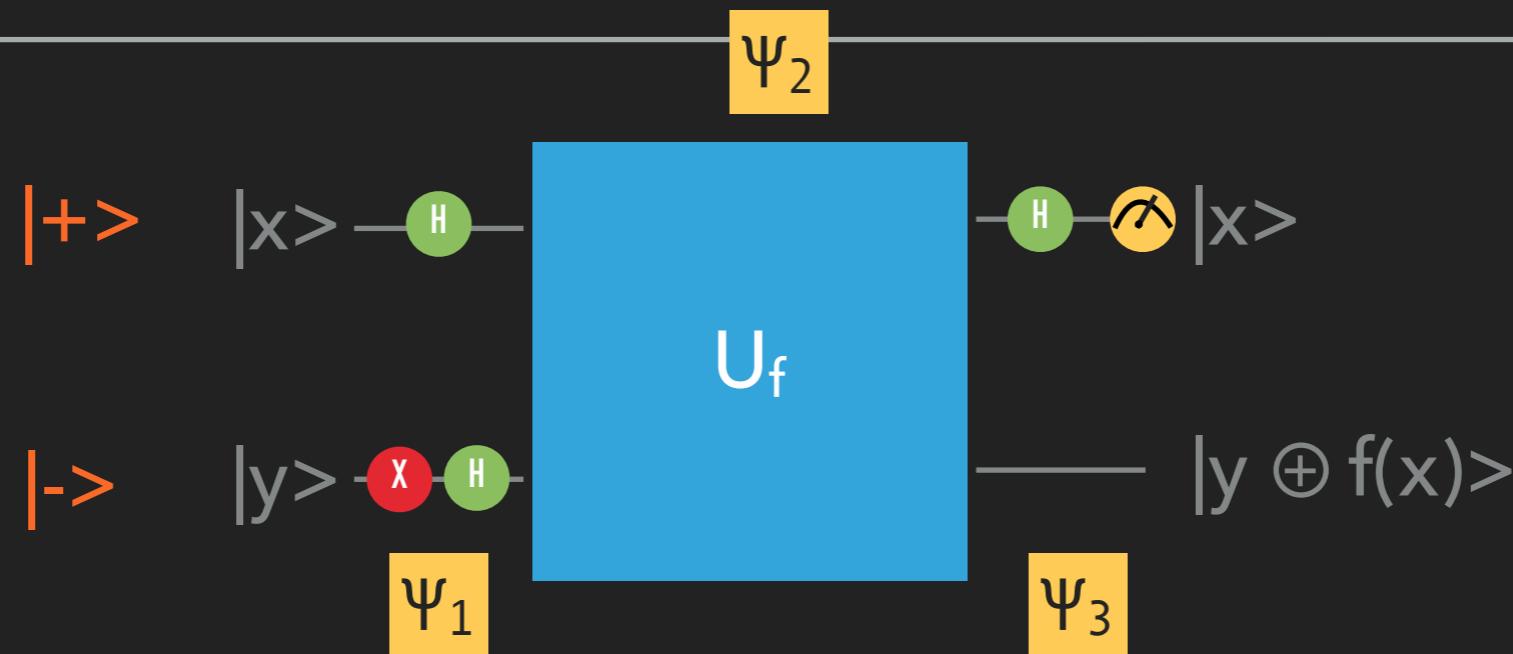
-> IF $f(0) = f(1)$ (CONSTANT)

$$= \frac{1}{2} (|0\rangle |f(0)\rangle - |0\rangle |1\oplus f(0)\rangle + |1\rangle |f(0)\rangle - |1\rangle |1\oplus f(0)\rangle)$$

$$= \frac{1}{2} ((|0\rangle + |1\rangle) |f(0)\rangle) + (-|0\rangle - |1\rangle) |1\oplus f(0)\rangle$$

$$= \frac{1}{2} (|0\rangle + |1\rangle) (|f(0)\rangle - |1\oplus f(0)\rangle)$$

DEUTSCH ALGORITHM

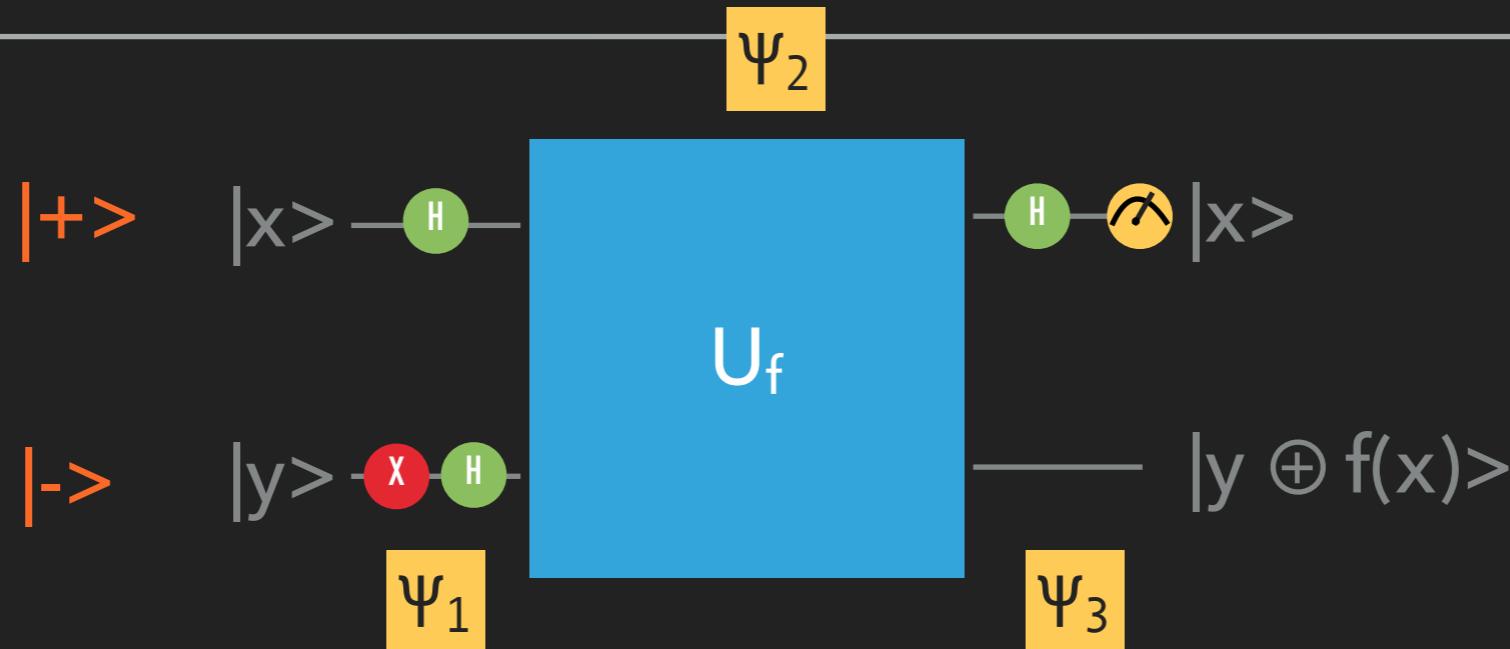


$$\Psi_2 = \frac{1}{2} (|0\rangle + |1\rangle) (|f(0)\rangle - |1 \oplus f(0)\rangle)$$

$$= \frac{1}{\sqrt{2}} (|+\rangle) (|f(0)\rangle - |1 \oplus f(0)\rangle)$$

$$\Psi_3 \quad \text{H} (|+\rangle) = |0\rangle$$

DEUTSCH ALGORITHM



$$\Psi_2 = \frac{1}{2} (|0\rangle |f(0)\rangle - |0\rangle |1\oplus f(0)\rangle + |1\rangle |f(1)\rangle - |1\rangle |1\oplus f(1)\rangle)$$

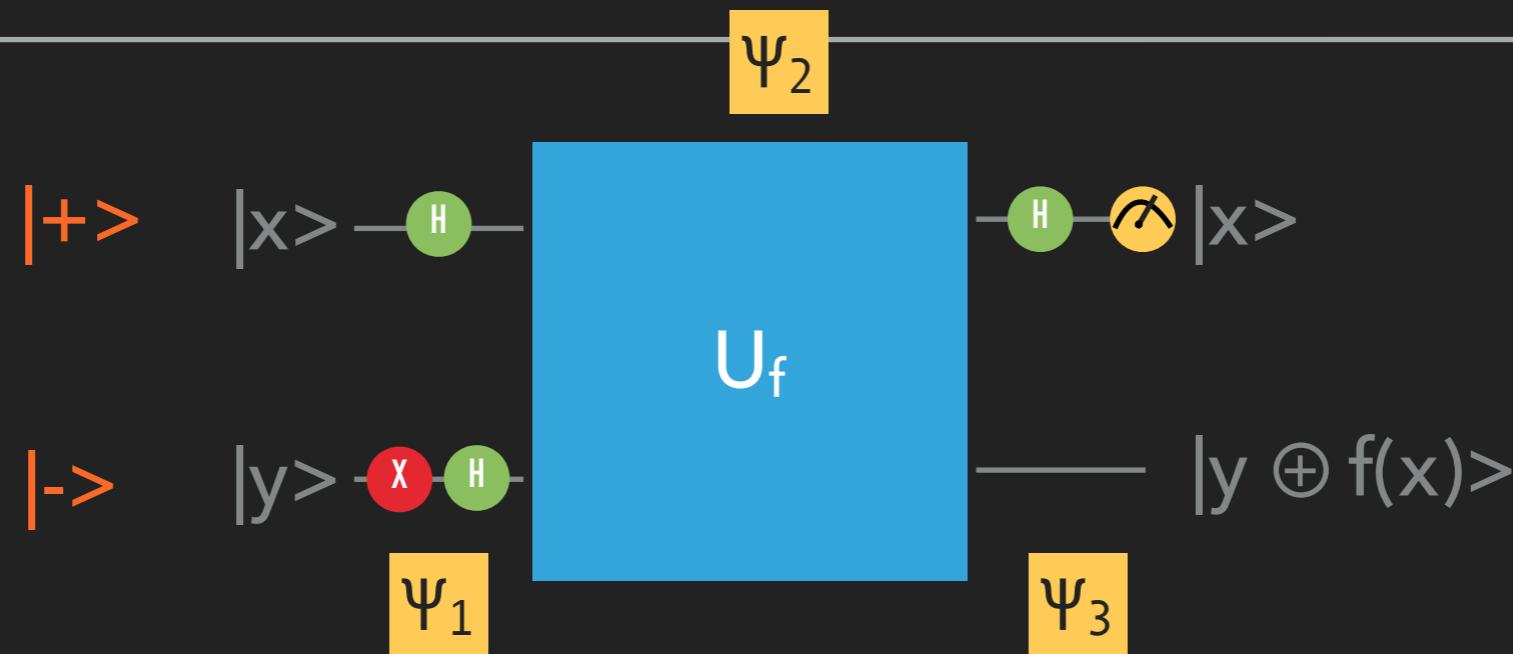
-> IF $f(0) \neq f(1)$ (BALANCED) $f(0) = f(1) \text{ XOR } 1$

$$= \frac{1}{2} (|0\rangle |f(0)\rangle - |0\rangle |f(1)\rangle + |1\rangle |f(1)\rangle - |1\rangle |f(0)\rangle)$$

$$= \frac{1}{2} (|0\rangle \otimes (|f(0)\rangle - |f(1)\rangle) - |1\rangle \otimes (|f(0)\rangle - |f(1)\rangle))$$

$$= \frac{1}{2} (|0\rangle - |1\rangle) \otimes (|f(0)\rangle - |f(1)\rangle)$$

DEUTSCH ALGORITHM

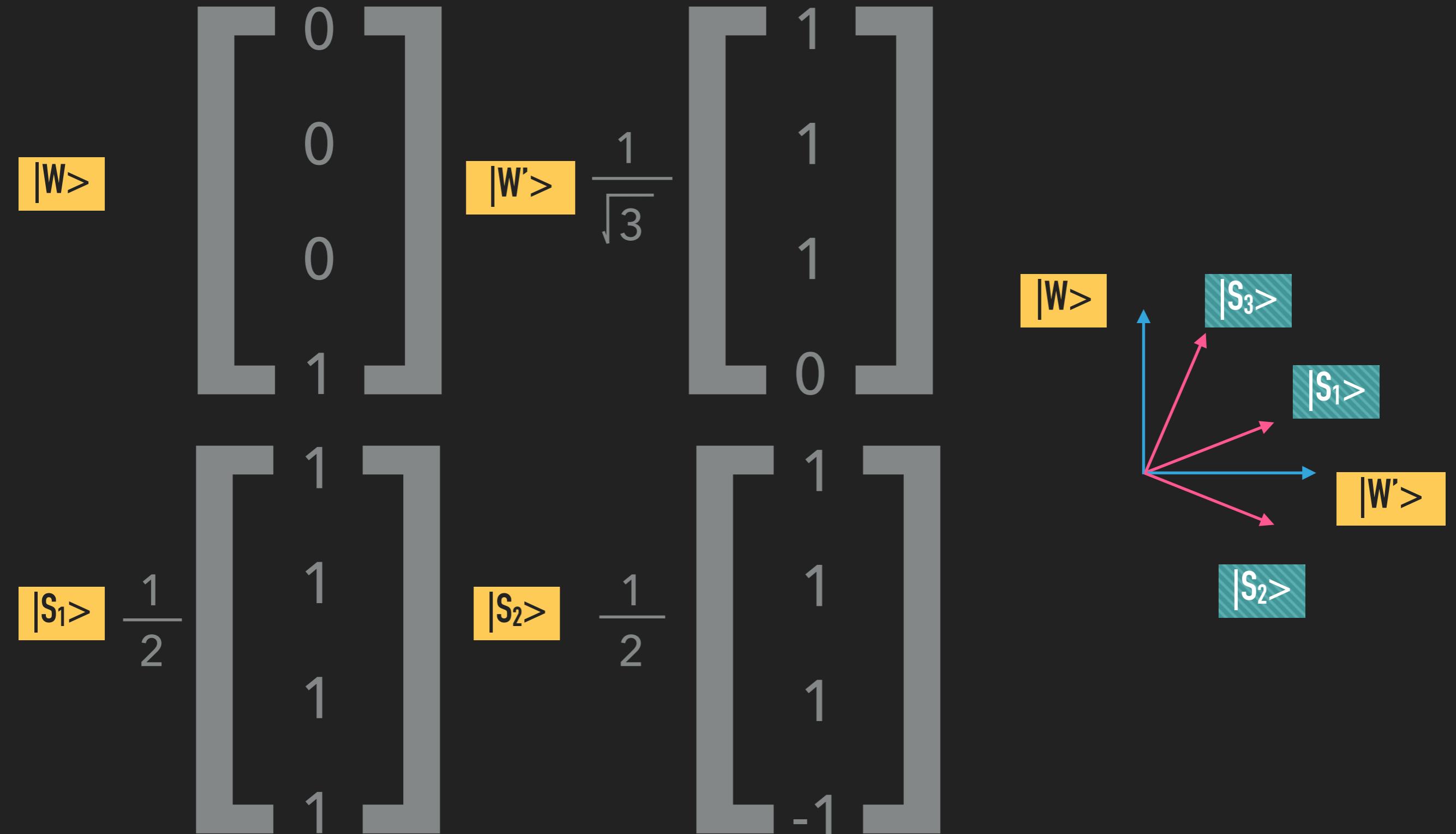


$$\Psi_2 = \frac{1}{2} (|0\rangle - |1\rangle) \otimes (|f(0)\rangle - |f(1)\rangle)$$

$$= \frac{1}{\sqrt{2}} (|-\rangle) \otimes (|f(0)\rangle - |f(1)\rangle)$$

$$\Psi_3 \quad H(|-\rangle) = |1\rangle$$

GROVERS ALGORITHM



DINNER PARTY

JAMES AND LARS

OR

KIRK AND ROB

AND NOT

LARS AND ROB

DINNER PARTY



SHOR'S ALGORITHM

PRIME FACTORS

$$N = 60 = 2^2 * 3 * 5$$

$$N = P * Q$$

$$n = \text{len}(N)$$

$$O(2^n)$$

MODULAR ARITHMETIC

$$3 = 26 \pmod{23}$$

$$20 = 43 \pmod{23}$$

$$1 = 24 \pmod{23}$$

$$22 = -1 \pmod{23}$$

$$46 + 18 \pmod{23}$$

$$= 0 + 18 \pmod{23}$$

$$46 * 18 \pmod{23}$$

$$= 0 * 18 \pmod{23}$$

SHOR'S ALGORITHM

GCD

$$\gcd(15,21) = 3$$

$$3 * 5 \quad 3 * 7$$

FIND PRIME FACTORS OF 21

$$N = 21$$

$$x^2 = 1 \pmod{21}$$

$$x \in \{1, -1, 8, -8, 13, 20, -20\}$$

$$8^2 - 1^2 = 0 \pmod{21}$$

$$(8 - 1) * (8 + 1)$$

SHOR'S ALGORITHM

$$(8 - 1) * (8 + 1)$$

$$\gcd(21, 8+1) = 3$$

$$\gcd(21, 8-1) = 7$$

GENERAL RULE

$$x \neq + - 1 \pmod{N}$$

$$x^2 = 1 \pmod{N}$$

$$\gcd(N, x+1)$$

SHOR'S ALGORITHM

$N = 21$

$$2^1 = 2 \pmod{21}$$

$x = 2$ (random #)

$$2^2 = 4 \pmod{21}$$

$$x^6 = (x^3)^2 = 1 \pmod{21}$$

$$2^3 = 8 \pmod{21}$$

$$2^4 = 16 \pmod{21}$$

50% PROBABILITY

$$2^5 = 11 \pmod{21}$$

$$2^6 = 1 \pmod{21}$$

$$2^7 = 2 \pmod{21}$$

$$2^8 = 4 \pmod{21}$$

SHOR'S ALGORITHM

Quantum Fourier Transform (QFT)

Modular Exponentiation - Quantum Phase Estimation

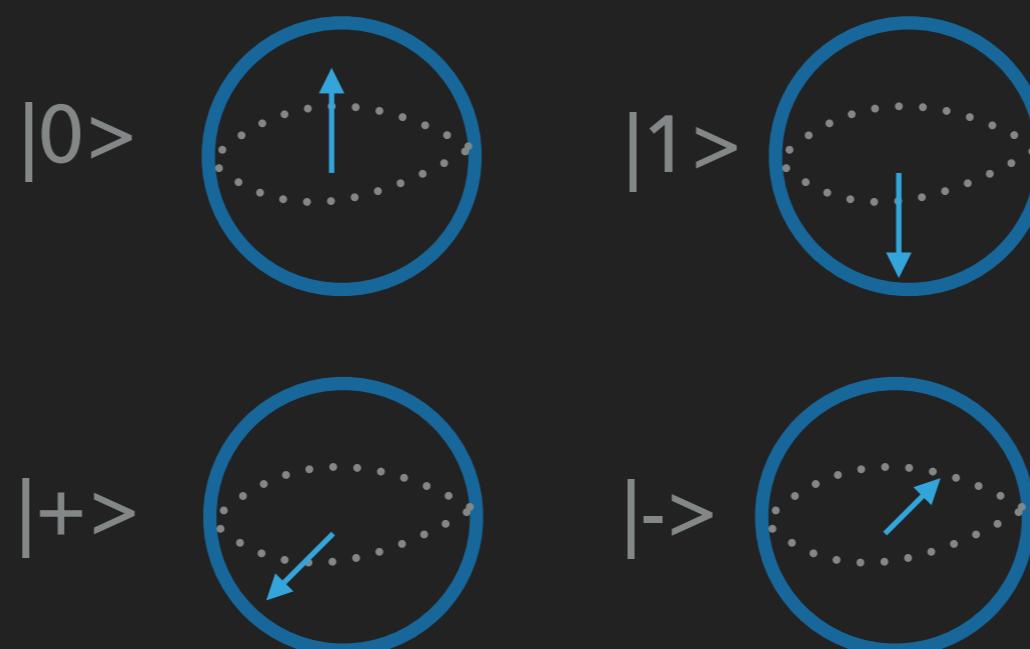
QUANTUM FOURIER TRANSFORM:

1 QUBIT

Computational Basis -> Fourier Basis

$\{|0\rangle, |1\rangle\}$

$\{|+\rangle, |-\rangle\}$



SHOR'S ALGORITHM

QUANTUM FOURIER TRANSFORM:

2 QUBIT

Computational Basis -> Fourier Basis

$\{|00\rangle, |10\rangle, |01\rangle, |11\rangle\}$

$$\text{QFT } |\tilde{x}\rangle = \frac{1}{\sqrt{N}} \sum_{y=0}^{N-1} e^{\frac{2\pi i \tilde{x} y}{N}} |y\rangle$$

Note: $e^{\frac{2\pi i \tilde{x} y}{N}} = -1$

$$N = 2^n$$

$n = \# \text{ Qubits}$

SHOR'S ALGORITHM

$$\text{QFT } |x\rangle = |\tilde{x}\rangle = \frac{1}{\sqrt{N}} \sum_{y=0}^{N-1} e^{\frac{2\pi i x y}{N}} |y\rangle$$

QUANTUM FOURIER TRANSFORM:

$$N = 2^1$$

1 QUBIT

$$= \frac{1}{\sqrt{2}} \sum_{y=0}^{2-1} e^{\frac{2\pi i x y}{2}} |y\rangle$$

$$= \frac{1}{\sqrt{2}} e^{\frac{2\pi i x 0}{2}} |0\rangle + \frac{1}{\sqrt{2}} e^{\frac{2\pi i x 1}{2}} |1\rangle$$

SHOR'S ALGORITHM

$$= \frac{1}{\sqrt{2}} e^{\frac{2\pi i x^0}{2}} |0\rangle + \frac{1}{\sqrt{2}} e^{\frac{2\pi i x^1}{2}} |1\rangle$$

$$= \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} e^{\pi i x} |1\rangle$$

if ($x = 0$):

$$\frac{1}{\sqrt{2}} |0\rangle + |1\rangle$$

|+>

if ($x = 1$):

$$\frac{1}{\sqrt{2}} |0\rangle - |1\rangle$$

|->

SHOR'S ALGORITHM

$$\text{QFT } |\tilde{x}\rangle = \frac{1}{\sqrt{N}} \sum_{y=0}^{N-1} e^{\frac{2\pi i \tilde{x} y}{N}} |y\rangle \quad N = 2^n$$

$$y = [y_1, y_2, y_3, \dots, y_n]$$

$|0\rangle, |1\rangle, |2\rangle, |3\rangle ?$

$$|2\rangle = |10\rangle$$

$$\rightarrow = 2^{n-1} * y_1 + 2^{n-2} * y_2 + \dots + 2^0 * y_n$$

$$y = \sum_{k=0}^n y_k 2^{n-k}$$

SHOR'S ALGORITHM

$$\text{QFT } |x\rangle = |\tilde{x}\rangle = \frac{1}{\sqrt{N}} \sum_{y=0}^{N-1} e^{\frac{2\pi i x^* y}{N}} |y\rangle \quad N = 2^n$$

$$y = \sum_{k=0}^n y_k 2^{n-k}$$

$$\text{QFT } |x\rangle = |\tilde{x}\rangle = \frac{1}{\sqrt{N}} \sum_{y=0}^{N-1} e^{2\pi i x^* \sum_{k=0}^n \frac{y_k}{2^k}} |y_1, y_2, y_3, \dots, y_n\rangle$$

$$= \frac{1}{\sqrt{N}} \sum_{y=0}^{N-1} \prod_{k=1}^n e^{2\pi i \frac{x^* y_k}{2^k}} |y_1, y_2, y_3, \dots, y_n\rangle$$

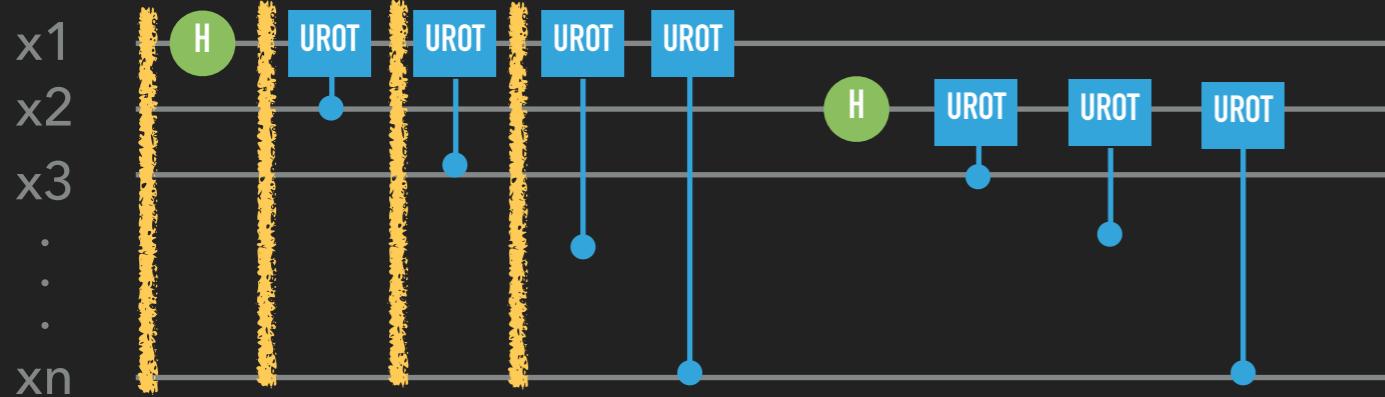
SHOR'S ALGORITHM

$$= \frac{1}{\sqrt{N}} \sum_{y=0}^{N-1} \prod_{k=1}^n e^{2\pi i \frac{y_k}{2^k}} |y_1, y_2, y_3, \dots, y_n\rangle$$

$$= \frac{1}{\sqrt{N}} (|0\rangle + e^{\frac{2\pi i x}{2^1}} |1\rangle) \otimes (|0\rangle + e^{\frac{2\pi i x}{2^2}} |1\rangle) \otimes \\ (|0\rangle + e^{\frac{2\pi i x}{2^3}} |1\rangle) \otimes \dots \otimes (|0\rangle + e^{\frac{2\pi i x}{2^n}} |1\rangle)$$

$$\begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & e^{\frac{2\pi i x}{2^1}} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & e^{\frac{2\pi i x}{2^n}} \end{bmatrix}$$

SHOR'S ALGORITHM



$$= |x_1 x_2 x_3 x_4 x_5 x_6 \dots x_n \rangle$$

$$= (|0\rangle + e^{\frac{2\pi i x_1}{2}} |1\rangle) \otimes |x_2 x_3 x_4 x_5 x_6 \dots x_n \rangle$$

$$= (|0\rangle + e^{\frac{2\pi i x_2}{2^2}} e^{\frac{2\pi i x_1}{2^1}} |1\rangle) \otimes |x_2 x_3 x_4 x_5 x_6 \dots x_n \rangle$$

$$= (|0\rangle + e^{\frac{2\pi i x_3}{2^3}} e^{\frac{2\pi i x_2}{2^2}} e^{\frac{2\pi i x_1}{2^1}} |1\rangle) \otimes |x_2 x_3 x_4 x_5 x_6 \dots x_n \rangle$$



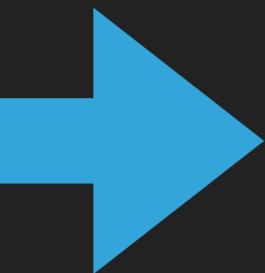
Reverse Order

SHOR'S ALGORITHM

QUANTUM PHASE ESTIMATION:

$$U |\psi\rangle = e^{i\Theta} |\psi\rangle$$

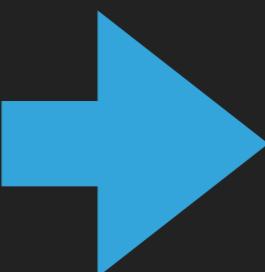
$$\frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$



$$P(1) = 50\%$$

$$P(0) = 50\%$$

$$e^{\frac{i * p_i}{2}} \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$



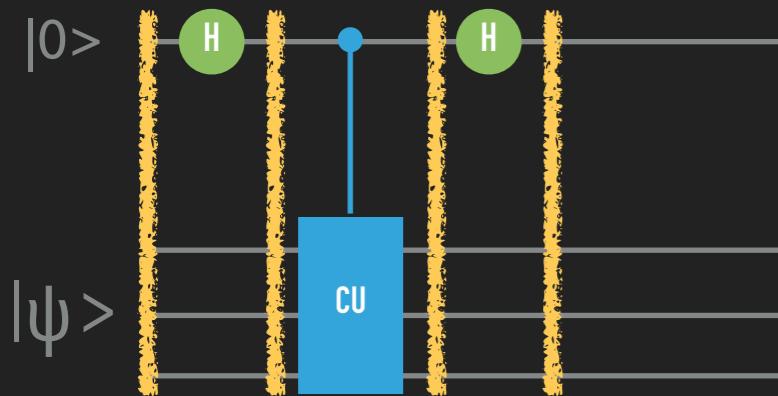
$$P(1) = \left| e^{\frac{i * p_i}{2}} * \frac{1}{\sqrt{2}} \right|^2 = 50\%$$

$$P(0) = 50\%$$

SHOR'S ALGORITHM

QUANTUM PHASE ESTIMATION:

$|0\rangle |\psi\rangle$



$$\frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) |\psi\rangle = \frac{1}{\sqrt{2}} (|0\rangle |\psi\rangle) + (|1\rangle |\psi\rangle)$$

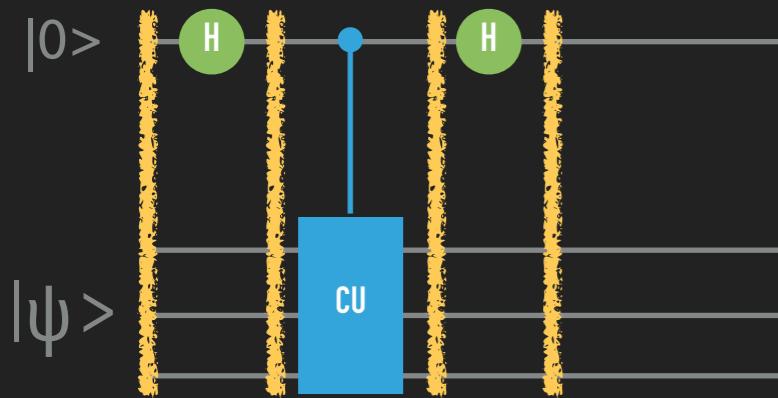
$$\frac{1}{\sqrt{2}} (|0\rangle |\psi\rangle) + (|1\rangle e^{i\Theta} |\psi\rangle)$$

$$\frac{1}{\sqrt{2}} \frac{(|0\rangle + |1\rangle) |\psi\rangle}{\sqrt{2}} + e^{i\Theta} \frac{(|0\rangle - |1\rangle) |\psi\rangle}{\sqrt{2}}$$

$$= \frac{1}{2} [|0\rangle (1 + e^{i\Theta}) + |1\rangle (1 - e^{i\Theta})] |\psi\rangle$$

SHOR'S ALGORITHM

QUANTUM PHASE ESTIMATION:



$$\frac{1}{2} [|0\rangle (1 + e^{i\Theta}) + |1\rangle (1 - e^{i\Theta})] |\psi\rangle$$

$$P(1) = \left| 1 - e^{i\Theta} * \frac{1}{2} \right|^2$$

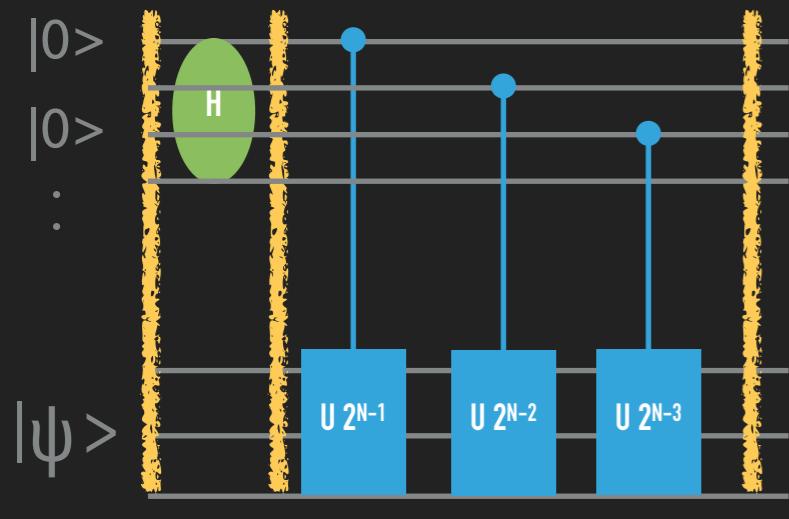
$$P(0) = \left| 1 + e^{i\Theta} * \frac{1}{2} \right|^2$$

$$\Theta = 1 \quad P(0) = 0.9999..$$

$$\Theta = 10 \quad P(0) = 0.9924..$$

SHOR'S ALGORITHM

QUANTUM PHASE ESTIMATION:

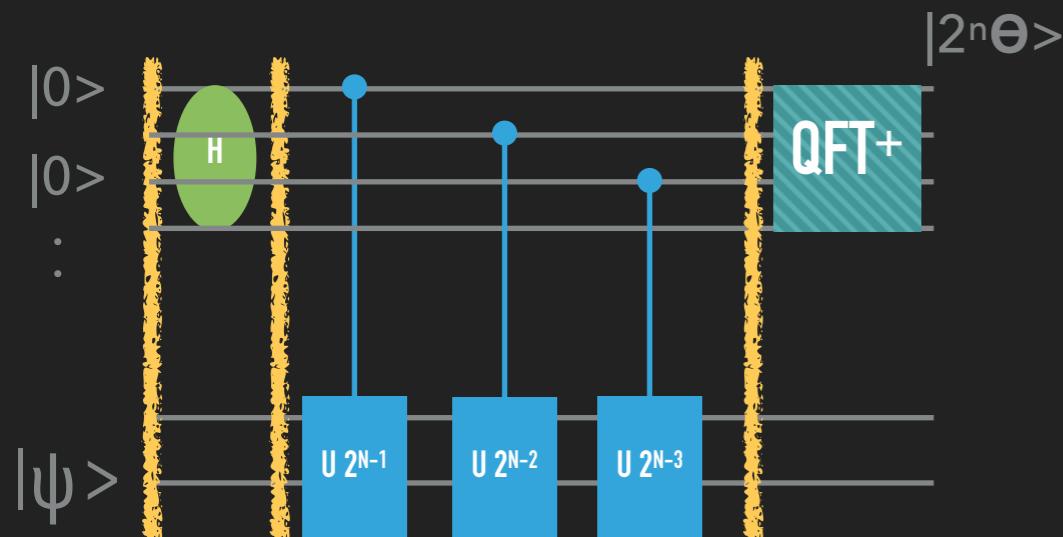


$$|0\rangle^{\star n} |\psi\rangle$$

$$\frac{1}{\sqrt{2}}^n (|0\rangle + |1\rangle)^{\star n} |\psi\rangle$$

$$\frac{1}{\sqrt{2}}^n (|0\rangle + e^{i\Theta 2^{n-1}} |1\rangle) \otimes (|0\rangle + e^{i\Theta 2^{n-2}} |1\rangle)$$

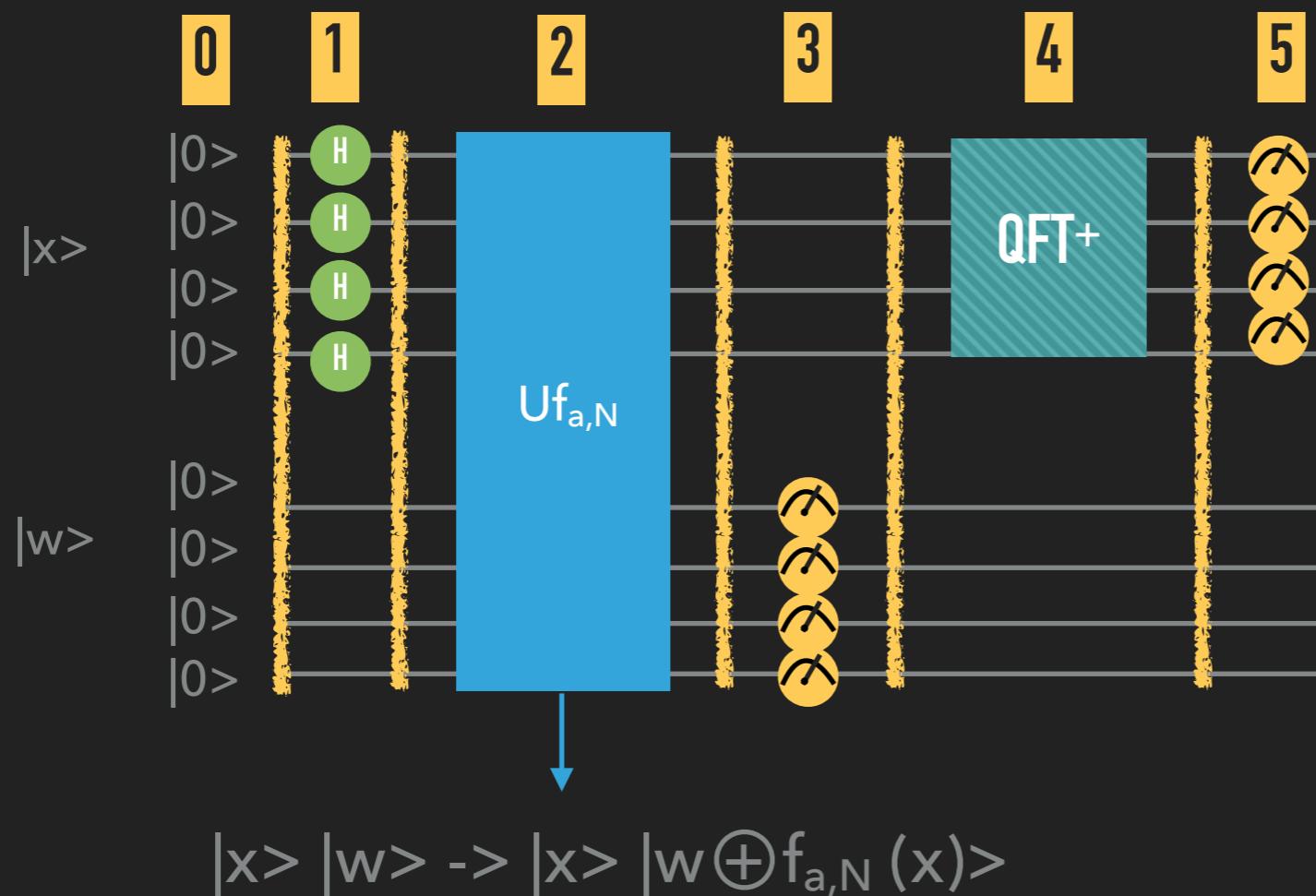
$$\otimes \dots \otimes (|0\rangle + e^{i\Theta 2^0} |1\rangle)$$



QFT VS QPE:

$$\frac{\Theta * 2\pi i}{2^n}$$

SHOR'S ALGORITHM



$$N = pq$$

$$N = 15$$

$$15 = [1111]$$

$$f_{a,N}(x) = a^x \bmod(N)$$

0 $ 0\rangle^{*4} 0\rangle^{*4}$	
1 $\frac{1}{4} [0000\rangle + 0001\rangle + \dots + 1111\rangle] 0\rangle^{*4}$	
2 $\frac{1}{4} [0000\rangle 0 \oplus (13^0 \bmod 15)\rangle + 0001\rangle 0 \oplus (13^1 \bmod 15)\rangle + \dots]$	

SHOR'S ALGORITHM

2 $\frac{1}{4} [|0000\rangle |0 \oplus (13^0 \text{mod} 15)\rangle + |0001\rangle |0 \oplus (13^1 \text{mod} 15)\rangle + \dots$
.....]

$$\frac{1}{4} [|0000\rangle |(13^0 \text{mod} 15)\rangle + |0001\rangle |(13^1 \text{mod} 15)\rangle + \dots]$$

|x> |w>

$$\frac{1}{4} \left[\begin{array}{l} |0\rangle |1\rangle + |1\rangle |13\rangle + |2\rangle |4\rangle + |3\rangle |7\rangle + \\ |4\rangle |1\rangle + |5\rangle |13\rangle + |6\rangle |4\rangle + |7\rangle |7\rangle + \\ |8\rangle |1\rangle + |9\rangle |13\rangle + |10\rangle |4\rangle + |11\rangle |7\rangle + \\ |12\rangle |1\rangle + |13\rangle |13\rangle + |14\rangle |4\rangle + |15\rangle |7\rangle \end{array} \right]$$

if measure (|w>) = 7:

3 $x = \frac{1}{4} [|3\rangle + |7\rangle + |11\rangle + |15\rangle]$

SHOR'S ALGORITHM

3

$$|x\rangle |w\rangle = \frac{1}{2} [|3\rangle + |7\rangle + |11\rangle + |15\rangle] \otimes |7\rangle$$

4

$$\text{QFT } |x\rangle = |\tilde{x}\rangle = \frac{1}{\sqrt{N}} \sum_{y=0}^{N-1} e^{\frac{2\pi i y x}{N}} |y\rangle$$

$$\text{QFT}^+ |\tilde{x}\rangle = |x\rangle = \frac{1}{\sqrt{N}} \sum_{y=0}^{N-1} e^{\frac{-2\pi i y x}{N}} |y\rangle$$

$$\text{QFT } |3\rangle = \frac{1}{\sqrt{16}} \sum_{y=0}^{15} e^{\frac{-2\pi i y 3}{16}} |y\rangle$$

SHOR'S ALGORITHM

4 $\text{QFT}^+ |x\rangle = \frac{1}{8} \sum_{y=0}^{15} [e^{-3\pi i/8 * y} + e^{-7\pi i/8 * y} + e^{-11\pi i/8 * y} + e^{-15\pi i/8 * y}] |y\rangle$

$$= \frac{1}{8} [4|0000\rangle + 4i|0100\rangle - 4|1000\rangle - 4i|1100\rangle]$$

|0> **|4>** **|8>** **|12>**

5 Measure: 0, 4 , 8 , 12

$$x^{r/2} = 13^{4/2} \equiv 4 \pmod{15}$$

$$\begin{array}{ll} x+1, x-1 & \gcd(x+1, N) = \gcd(5, 15) = 5 \\ 5 & \\ 3 & \gcd(x-1, N) = \gcd(3, 15) = 3 \end{array}$$

SHOR'S ALGORITHM

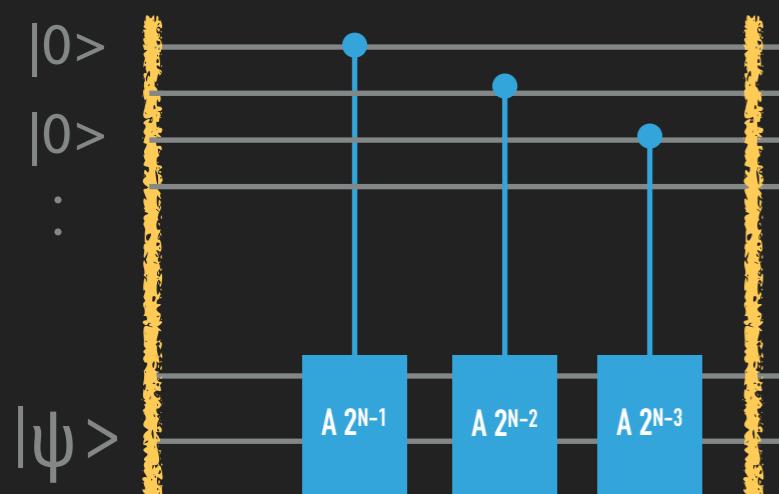
$$f_{a,N}(x) = a^x \bmod(N)$$

$$x = [x_1, x_2, x_3, \dots, x_n] = 2^{n-1}x_1 + 2^{n-2}x_2 + \dots + 2^0 x_n$$

$$f_{a,N}(x) = a^x \bmod(N)$$

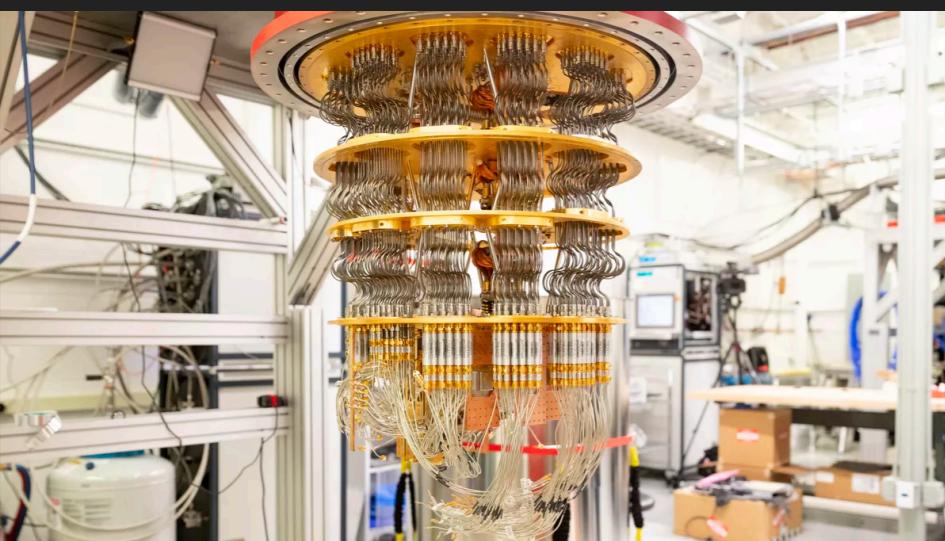
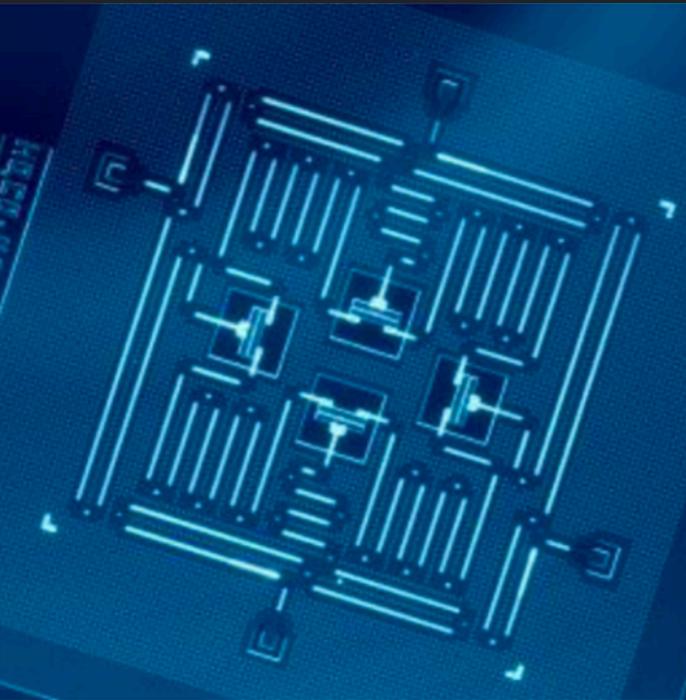
$$= a^{2^{n-1}x_1 + 2^{n-2}x_2 + \dots + 2^0 x_n} \bmod(N)$$

$$= a^{2^{n-1}x_1} \quad a^{2^{n-2}x_2} \quad a^{2^0 x_n}$$
$$= a \quad a \quad \dots \quad a$$



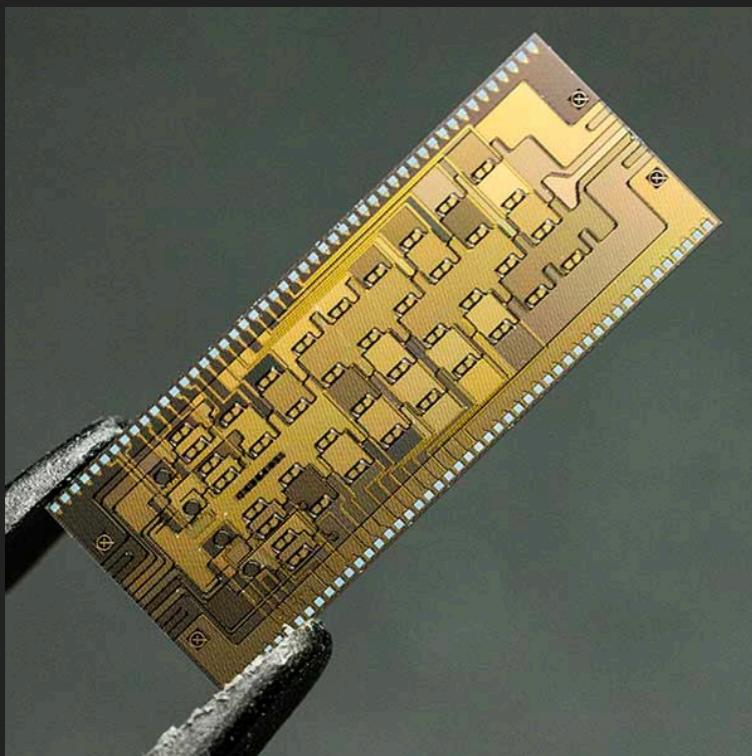
QUANTUM HARDWARE

SUPERCONDUCTING QUBITS



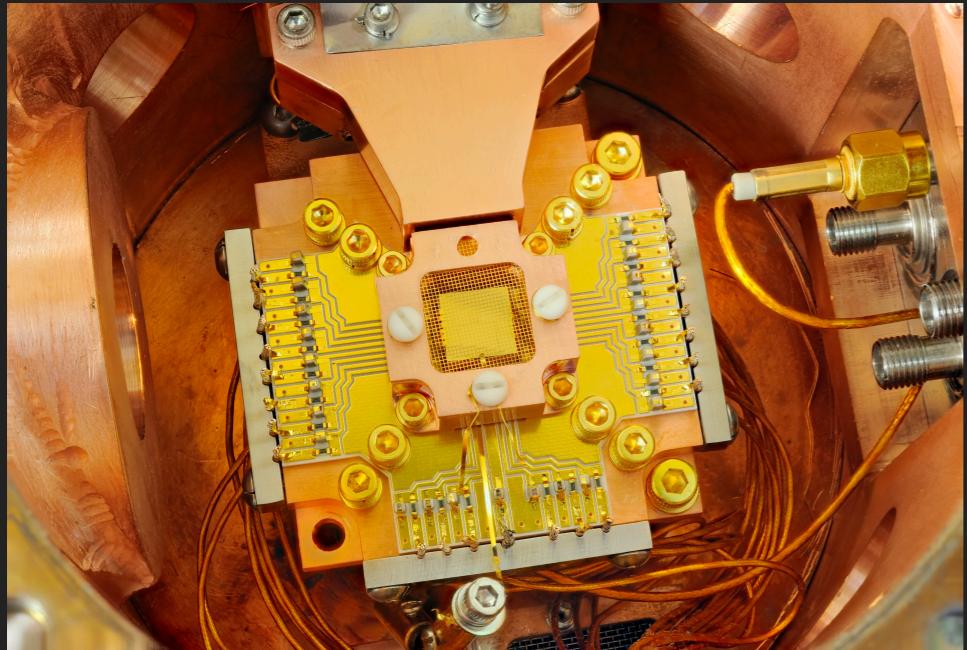
QUANTUM HARDWARE

PHOTONIC QUANTUM COMPUTER



QUANTUM HARDWARE

ION TRAPS



Honeywell
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QUANTUM HARDWARE

DWAVE

