Chimera States and Seizures in a Mouse Neuronal Model

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Chimera States

A chimera state is **the coexistence of synchronous and asynchronous populations in a network of nonlocally-coupled oscillators** [1, 2].

A mechanical example of a chimera state

Measures

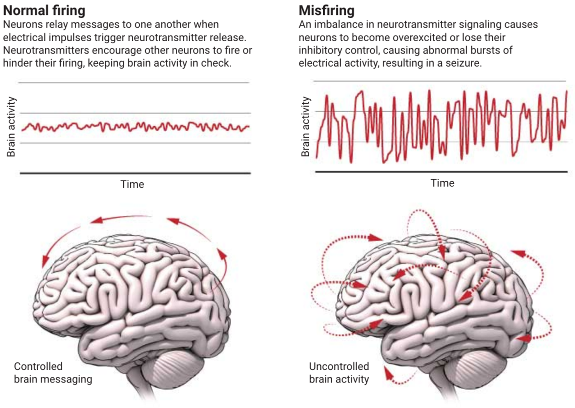
The **chimera-like index** χ is a useful measure for how chimeric a system is.

$$\chi = \langle \sigma_{\text{chi}} \rangle_T \quad \text{where} \quad \sigma_{\text{chi}}(t) = \frac{1}{M-1} \sum_{c \in C} \left(r_c(t) - \langle r_c \rangle_C \right)^2 \qquad (1)$$

and r_c is the order parameter of community c. It is the **time average of the variance between communities of the order parameter**. Its maximum value is $1/7$, and will be normalized as such.

Seizures

Seizures are **excessive(ly) synchronous neural activity** [6]. Focal seizures occur when parts of the brain seize while the rest behaves normally.



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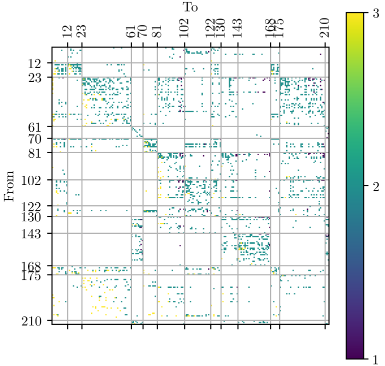
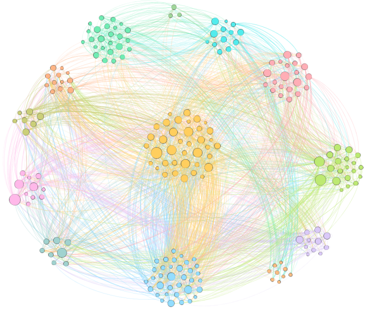
The Mouse Connectome

The oscillator network was modeled after the mouse connectome [4].



(a)



(b)

Model Parameters

Symbol	Value	Meaning
x_j	—	**Membrane potential of the jth neural mass**
y_j	—	**Associated with fast intra-neural processes**
z_j	—	**Associated with slow intra-neural processes**
b	3.2	Tunes the spiking frequency
I_j	4.4	External input current
x_{rev}	2	Ambient reverse potential
λ	10	Sigmoidal activation function parameter
θ	-0.25	Sigmoidal activation function parameter
μ	0.01	Time scale for variation of z
s	4	Governs adaptation
x_{rest}	-1.6	Resting/equilibrium potential
α	Varied	**Connection strength within cortices**
n'_j	Figure 1a	**Number of connections within a cortex from the jth neuron**
G'	Figure 1a	**Intra-cortical connection matrix**
β	Varied	**Connection strength between cortices**
n''_j	Figure 1a	**Number of connections between cortices from the jth neuron**
G''	Figure 1a	**Inter-cortical connection matrix**

Table: The list of parameters used in modeling the Hindmarsh-Rose network.

The Model

We used a network of modified Hindmarsh-Rose neurons [5]:

$$\dot{x}_j = y_j - x_j^3 + bx_j^2 + I_j - z_j - \frac{\alpha}{n'_j}\sum_{k=1}^{N} G'_{jk}\Theta_j(x_k) - \frac{\beta}{n''_j}\sum_{k=1}^{N} G''_{jk}\Theta_j(x_k) \tag{2}$$

$$\dot{y}_j = 1 - 5x_j^2 - y_j \tag{3}$$

$$\dot{z}_j = \mu(s[x_j - x_{\text{rest}}] - z_j) \tag{4}$$

where

$$\Theta_j(x_k) = \frac{x_j - x_{\text{rev}}}{1 + e^{-\lambda(x_k - \theta)}} \tag{5}$$

This sigmoidal activation function helps make the model apply to groups of neurons (subcortices) instead of single neurons.

Simulated EEG Output

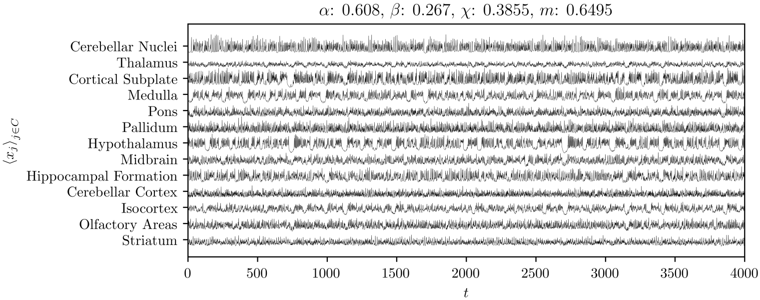


Figure: The mean membrane potential within each cortex.

Thalamus, Pons, Striatum — wild type
Cerebellar cortex — spiking behavior, spike runs
Medulla, Hypothalamus — repetitive spike runs
Cortical subplate — seizure-like behavior

Model Physicality

The neurons only collectively fired for certain values of α (intra-cortical coupling) and β (inter-cortical coupling) [3].

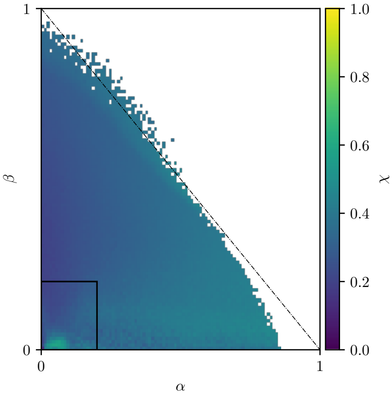


Figure: The chimera-like landscape of parameter space on $(\alpha, \beta) \in (0, 0.9) \times (0, 0.9)$. The aphysical region of the model is shown in white.

That Highly Chimeric Patch

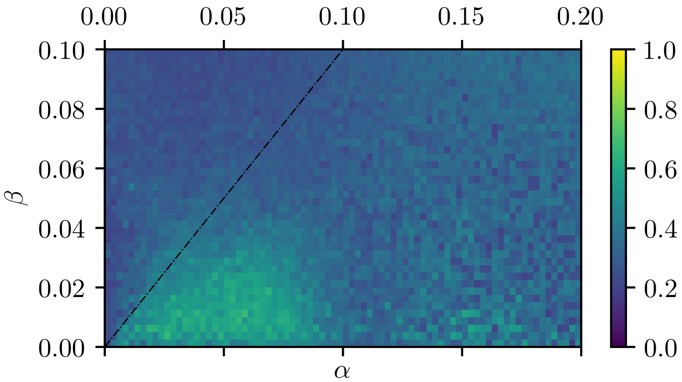


Figure: The chimera index of runs with $(\alpha, \beta) \in (0, 0.2) \times (0, 0.1)$. As before, the chimera-like index is normalized to $\frac{1}{7}$. The dashed line shows $\beta = \alpha$.

Yes, It's Real

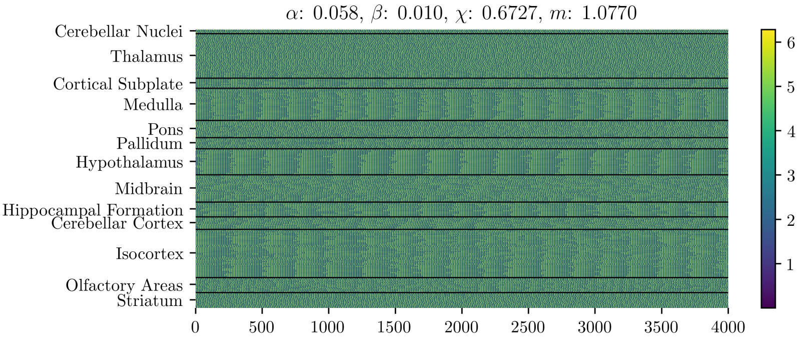


Figure: A run of the Hindmarsh-Rose simulation in the chimeric island. Synchronization is most consistently evident in the medulla, the hypothalamus, and the isocortex.

Another Viewpoint

An animation of the phase of the simulation.

References I

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