Chimera States and Seizures in a Mouse Neuronal Model

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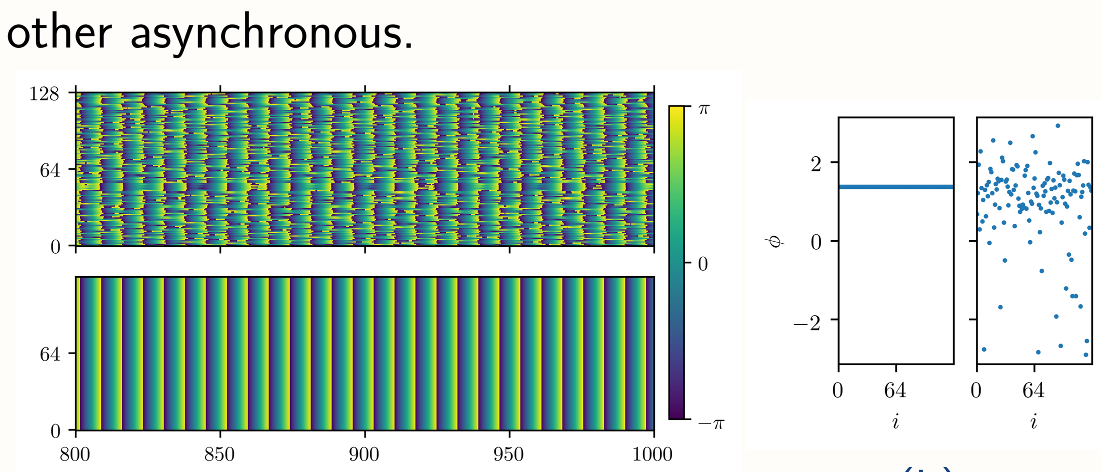
Abstract

527.40332pt**Chimera states**— the coexistence of synchrony and asynchrony in a nonlocally-coupled network of identical oscillators—are often sought as a model for epileptic **seizures**. This work investigates that connection, seeking chimera states in a network of modified Hindmarsh-Rose neurons connected in the graph of the mesoscale mouse connectome. The model was found to be of sufficient quality to produce superficially epileptiform activity. The limitations of the model were investigated, depending on the strength of connections between subcortices within a cortex and between cortices. A wide swath of parameter space revealed persistent chimera states.

Chimera states

The coexistence of syncrhony and asyncrhony in a network of identical nonlocally coupled oscillators is called a **chimera state** [2, 1]. Figure 1 shows an example with a pair of populations, one synchronous and the other asynchronous.



(a) (b)

Figure 1: (a) A time series of a chimeric system, with an asynchronous population (above) and a synchronous population (below). (b) A snapshot of the chimeric system, with the synchronous population (left) and asynchronous population (right) separated.

The time average of the variance between groups of a system's order parameter provides a good measure for how chimeric it is [6]. The **chimera-like index** is given by $\chi = \langle \sigma_{\text{chi}} \rangle_T$ where

$$\sigma_{\text{chi}}(t) = \frac{1}{M-1} \sum_{c \in C} r_c(t) - \langle r_c \rangle_C{}^2 \qquad (1)$$

and $r(t) = \left| \langle e^{i\phi_k(t)} \rangle_{k \in C} \right|$ is a measure of the synchrony of the phases of oscillators within a community C.

Seizures

Seizures are abnormally synchronous electrical activity in the brain [3]. They often present as trances or convulsions, and chronically affect about 3% of the United States population. **Focal seizures** are seizures limited to a small portion of the brain. As they are a subsection of the brain oscillating synchronously while the rest oscillates asynchronously, they often present in models as chimera states.

The Model

This research used a network of modified Hindmarsh-Rose neurons [5]:

$$\begin{aligned} \dot{x}_j &= y_j - x^3 + bx_j^2 + I_j - z_j \\ &\quad - \frac{\alpha}{n'_j} \sum_{k=1}^{N} G'_{jk} \Theta_j(x_k) \\ &\quad - \frac{\beta}{n''_j} \sum_{k=1}^{N} G''_{jk} \Theta_j(x_k) \end{aligned} \qquad (2)$$

$$\dot{y}_j = 1 - 5x_j^2 - y_j \qquad (3)$$

$$\dot{z}_j = \mu(s(x_j - x_{\text{rest}}) - z_j) \qquad (4)$$

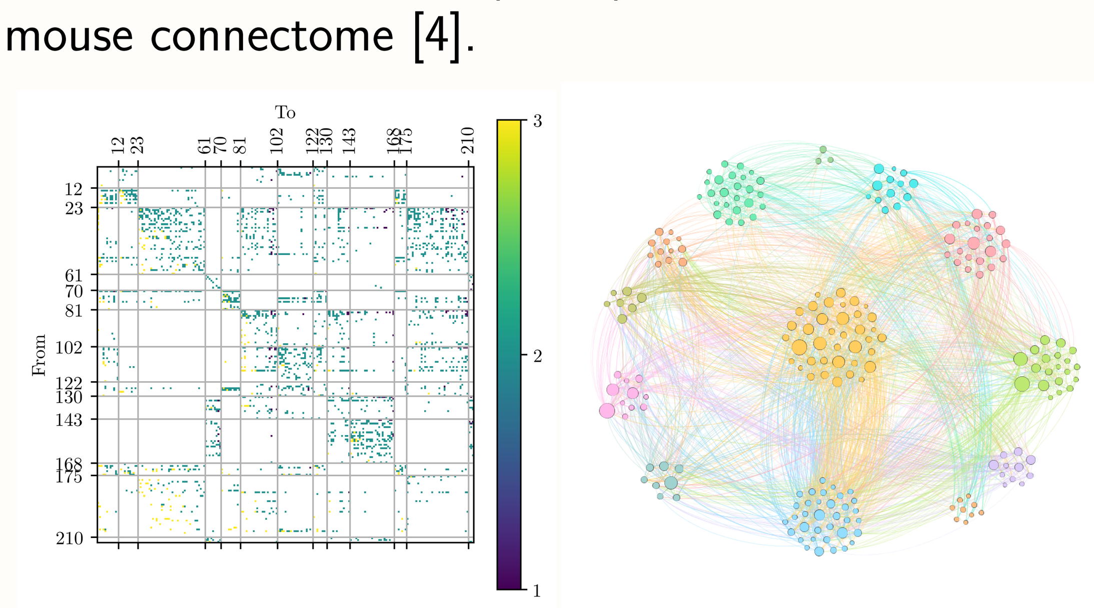
where $\Theta_j(x_k) = \frac{x_j - x_{\text{rev}}}{1+e^{-\lambda(x_k-\theta)}}$. The model was integrated for various values of α and β. The parameter values are shown in table 1. is the sigmoidal activation function, making it a neural mass model. Table 1 shows the values and meanings of the symbols in the model.

Symbol	Value	Meaning
x_j	—	Membrane potential of the jth neural mass
y_j	—	Associated with the fast processes
z_j	—	Associated with slow processes
b	3.2	Tunes the spiking frequency
I_j	4.4	External input current
x_{rev}	2	Ambient reverse potential
λ	10	Sigmoidal activation function parameter
θ	-0.25	Sigmoidal activation function parameter
μ	0.01	Time scale for variation of z
s	4	Governs adaptation
x_{rest}	-1.6	Resting/equilibrium potential
α	Varied	Connection strength within cortices
n'_j	See fig. 2a	Number of connections within a cortex from the jth neuron
G'_{jk}	See fig. 2a	Intra-cortical connection matrix
β	Varied	Connection strength between conrtices
n''_j	See fig. 2a	Number of connections between cortices from the jth neuron
G''_{jk}	See fig. 2a	Inter-cortical connection matrix

Table 1: The list of parameters used in modeling the Hindmarsh-Rose network.

The Network

The oscillator network (fig. 2) was modeled after the mouse connectome [4].



(a) (b)

Figure 2: (a) The mouse connectome represented as (a) a matrix and (b) a graph embedding.

Chimera State Presence

Chimera states are more prevalent in the parts of parameter space where $\alpha > \beta$ (see fig. 4). This is unsurprising, as comparatively strong intra-community coupling is necessary for chimera states. The highly chimeric patch where $(\alpha, \beta) \in (0, 0.1) \times (0, 0.1)$ is worthy of further investigation.

Model Quality

The model produced waveforms which were qualitatively similar to mouse EEG recordings (fig. 3).

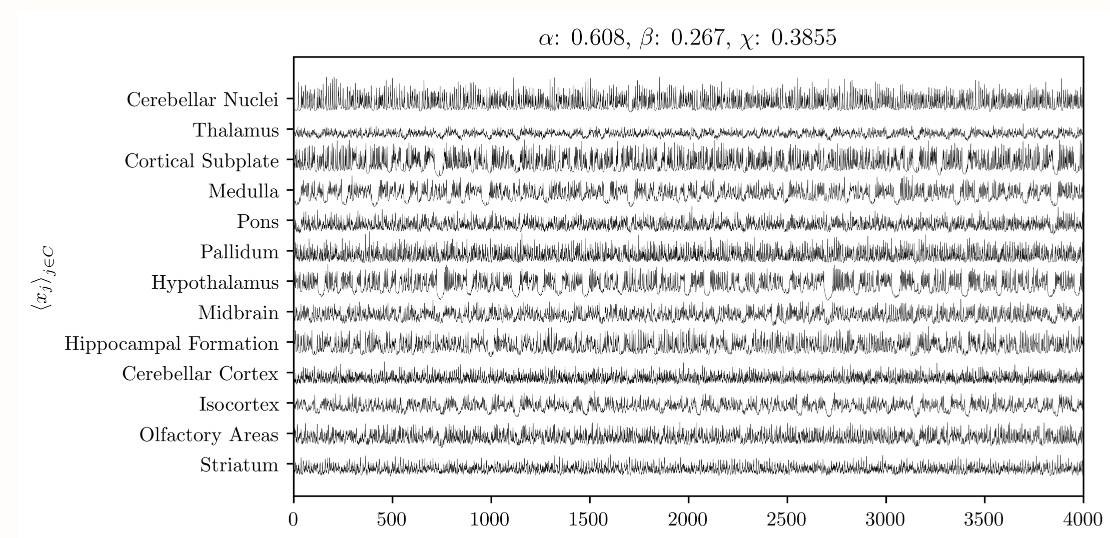


Figure 3: The mean membrane potential within each cortex. χ is normalized to $\frac{1}{7}$.

Physical Region

For many parameter values, the neurons in the model never fired (shown in white in fig. 4).

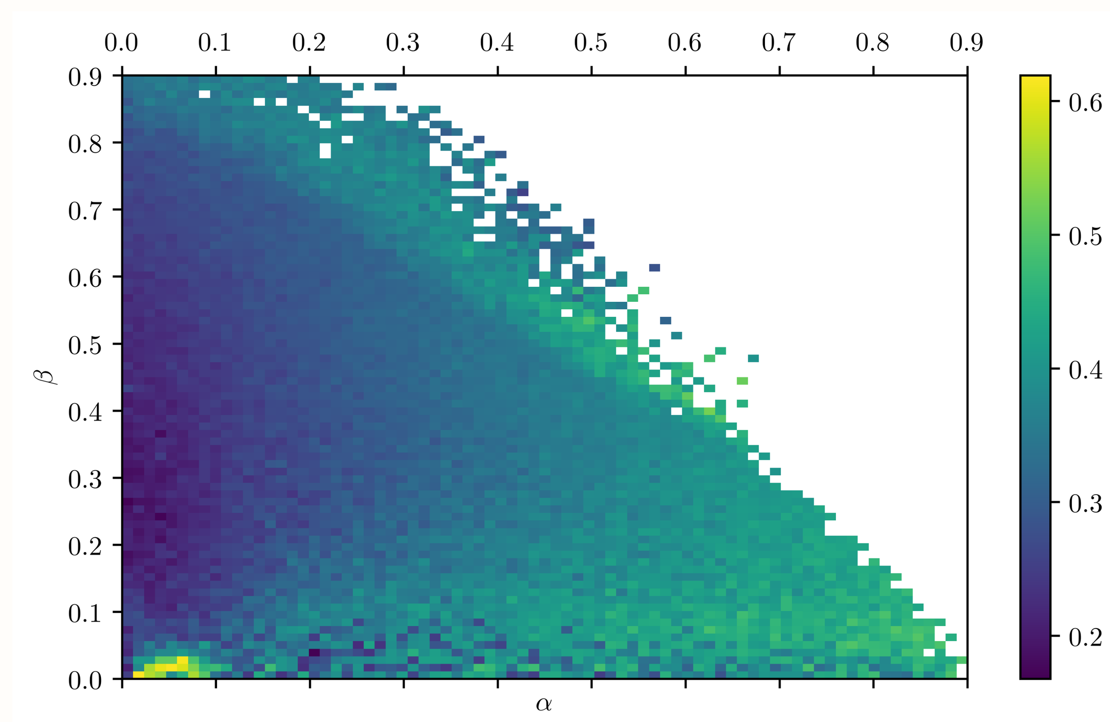


Figure 4: The normalized chimera-like index for the landscape of values of α and β.

The slope of the boundary between the physical and aphysical regions is negative, of a magnitude greater than 1. This indicates that α, the intra-cortical connection strength, has a stronger effect on the physicality of the model than β does.

Future Work

Further investigation of the exact nature of the boundary between the physical and aphysical regions would be beneficial. Additionally, a theoretical basis for the existence of the highly chimeric patch in $(\alpha, \beta) \in (0, 0.1) \times (0, 0.1)$ would provide insight into the nature of these chimera states.

References

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