GSMMC Michael Hennessey Updated May 31 2016

Single Cell Model

Cell Division $\rightarrow$ Cell Expansion $\rightarrow$ Ripening

Focus on Cell Expansion first, then ripening, then division.

Loosening of cell wall due to auxin stimulated acid pump.

Cell swelling, fluid transport, a few key reactions, and cell wall remodeling.

1 Fruit Growth post-cell division

1.1 Original Model

Variables:

- $w(t)$: amount of water
- $s(t)$ dry matter in fruit pulp

We first model the change in the amount of water in the fruit with time as the sum of the water inflow from the xylem (U_x) and phloem (U_p) and the water outflow due to fruit transpiration (T_f):

$$\frac{dw}{dt} = U_x + U_p - T_f. \tag{1}$$

The rate of change in the amount of dry matter (ds/dt) is the difference between the uptake from the phloem (U_s) and loss through fruit respiration (R_f):

$$\frac{ds}{dt} = U_s - R_f. \tag{2}$$

Fruit transpiration leading to mass loss is assumed to be proportional to the fruit surface area (A_f) and to be driven by the difference in relative humidity between the air-filled space within the fruit (H_f) and the ambient atmosphere (H_a) :

$$T_f = A_f \alpha \rho (H_f - H_a), \tag{3}$$

where ρ is the permeation coefficient of the fruit surface to water vapor and $\alpha = M_W P^*/RT,$ with $M_W = 18g/mol$ being the molecular mass of water, P^* the saturation vapor pressure, $R = 83 cm^3 bar mol^{-1} K^{-1}$ the gas constant. P^* is given as an exponential function

$$P^* = .009048 e^{.0547(T-273.15)} bar.$$

The fruit surface area is related to the fruit total mass (W_T) by the empirical equation

$$A_f = \gamma (W_T)^{\eta}. \tag{4}$$

The fruit total mass is

$$W_T = s + w + W_S,$$

where W_S is the fresh mass of the stone. γ and η depend on the fruit geometry and the density of the fruit.

The flow density (J) through a membrane is described by

$$J = L[P_1 - P_2 - \sigma(\pi_1 - \pi_2)], \tag{5}$$

where the subscripts 1 and 2 indicate the compartments separated by the membrane; L is the hydraulic conductivity coefficient; $P_{1,2}$ are the hydrostatic pressures in the respective compartments; $\pi_{1,2}$ are the respective osmotic pressures, and σ is the reflection coefficient (a measure of the impermeability of the membrane). The osmotic pressure is given as $\pi = RT\Sigma_j C_j^{(m)}$ in which $C_j^{(m)} = n_j(V_W^* n_W)$, where n_j is the number of moles of osmotically active solute, n_W is the number of moles of water, and V_w^* is the partial molal volume of water.
The water potential (Ψ) is expressed as the difference

$$\Psi = P - \pi.$$

The total flow through a membrane of area A is

$$U = AJ. \tag{6}$$

Using subscripts x , p and f for xylem, phloem and fruit variables, respectively, and combining equations 5 and 6 we get

$$U_x = A_x L_x[P_x - P_f - \sigma_x(\pi_x - \pi_f)] \tag{7}$$

and

$$U_p = A_p L_p[P_p - P_f - \sigma_p(\pi_p - \pi_f)]. \tag{8}$$

The vascular network enters the fruit and enlarges as the fruit grows, with A_x and A_p increasing in parallel with fruit growth.

$$A_x(t) = a_x A_f(t),$$
$$A_p(t) = a_p A_f(t).$$

If $\sigma_p < 1$, part of the sugar can be transported from the phloem to the fruit by mass flow (U_p). The contribution of sugar to the mass flow is $(1-\sigma_p)C_s U_p$ ¡ where $C_s \approxeq (C_p + C_f)/2$ is the mean concentration of the solute in the membrane, with C_p and C_f being the sugar concentrations. The total uptake of carbohydrates is

$$U_s = U_a + (1 - \sigma_p)C_s U_p + A_p p_s(C_p - C_f), \tag{9}$$

where p_s is the solute permeability coefficient. The saturating uptake rate, U_a , is assumed dependent on the phloem concentration according to the Michealis-Menten equation for fully non-competitive inhibition with the form

$$U_a = \frac{s(t)\nu_m C_p}{(K_M + C_p)(1 + C_I/K_I)}, \tag{10}$$

where $C_I = C_I^* e^{-t/\tau}$ is the inhibitor concentration and K_I is the equilibrium constant for the formation of an inhibitor-carrier complex. Here, ν_m is the maximum uptake rate per unit of dry mass and K_M is the Michaelis constant. The rate U_a will decline with fruit age if the inhibitor accumulates in the growing fruit.

The dry material loss through fruit respiration (R_f) comprises two components: that due to growth respiration, which is proportional to the rate of dry material intake, and that due to maintenance respiration, which is proportional to the dry mass

$$R_f = q_g\left(\frac{ds}{dt}\right) + q_m(T)s(t), \tag{11}$$

where q_g and $q_m(T) = q_m(293)Q_{10}^{(T-293)/10}$ are the coefficients for growth and maintenance respiration, respectively.