MATH-6890 Numerical Solutions of Waves Fall 2016

Due Thursday September 8, 2016.

Problem Set 1

1. Hello From Matlab: Code located on Page 3 under the header ´MATLAB code´

```
HELLO FROM MATLAB
1 1.000000e+00 8.414710e-01
2 5.000000e-01 4.794255e-01
3 3.333333e-01 3.271947e-01
4 2.500000e-01 2.474040e-01
5 2.000000e-01 1.986693e-01
6 1.666667e-01 1.658961e-01
7 1.428571e-01 1.423717e-01
8 1.250000e-01 1.246747e-01
9 1.111111e-01 1.108826e-01
10 1.000000e-01 9.983342e-02
```

2. Hello From C: Code located on Page 3 under the header ´C code´

```
HELLO FROM C
1 1.000000 0.841471
2 0.500000 0.479426
3 0.333333 0.327195
4 0.250000 0.247404
5 0.200000 0.198669
6 0.166667 0.165896
7 0.142857 0.142372
8 0.125000 0.124675
9 0.111111 0.110883
10 0.100000 0.099833
```

3. Hello From Fortran: Code located on Page 3 under the header ´FORTRAN code´

```
 HELLO FROM FORTRAN
           1    1.00000000       0.841470957
           2   0.500000000       0.479425550
           3   0.333333343       0.327194721
           4   0.250000000       0.247403964
           5   0.200000003       0.198669329
           6   0.166666672       0.165896133
           7   0.142857149       0.142371729
           8   0.125000000       0.124674730
           9   0.111111112       0.110882632
          10   0.100000001        9.98334214E-02
```

4. (a) Consider the equation $u_t = i\nu u_{xx}$ for $|x| < \infty$ and $t > 0$, where $\nu > 0$ and u is complex. Determine the exact solution subject to the initial conditions $u(x, 0) = \exp(ikx)$. Discuss the role of the parameters ν and k. Create a surface plot of the real part of the solution with $\nu = 1, k = 2$.
Solution:

We begin by making the ansatz $u(x,t) = A\exp(i(kx - \omega t)$. Substituting this ansatz into the equation results in the dispersion relation

$$\omega = \nu k^2 \tag{1}$$

which implies the wave speed is given

$$\frac{\omega}{k} = \nu k.$$

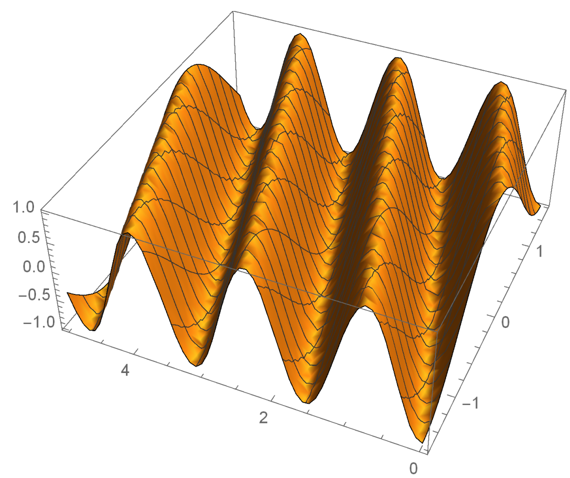
Thus the wave speed is modified by the wave number k. The parameter ν is simply a scaling factor for the wave speed. We then write the solution:

$$u(x,t) = Ae^{ik(x-\nu kt)}.$$

Applying the initial condition simply shows that $A = 1$, thereby giving us the solution:

$$u(x,t) = e^{ik(x-\nu kt)}. \tag{2}$$

A surface plot of the real part of the solution is included here.



(b) Consider the equation $w_{tt} = -\nu^2 w_{xxxx}$ with $\nu > 0, |x| < \infty$, and $t > 0$. Determine the exact solution subject to the initial conditions $w(x,0) = cos(kx)$ and $w_t(x,0) = k^2\nu sin(kx)$. Discuss the role of the parameters ν and k. Create a surface plot of the solution with $\nu = 1,$ and $k = 2$.

Solution:

We begin by making the ansatz $u(x,t) = A\exp(i(kx - \omega t) + B\exp(-i(kx - \omega t)$. Substituting this into the equation results in two dispersion relations

$$\omega = \nu k^2 \quad \text{and} \quad \omega = -\nu k^2 \tag{3}$$

with wave speeds

$$\frac{\omega}{k} = \pm \nu k.$$

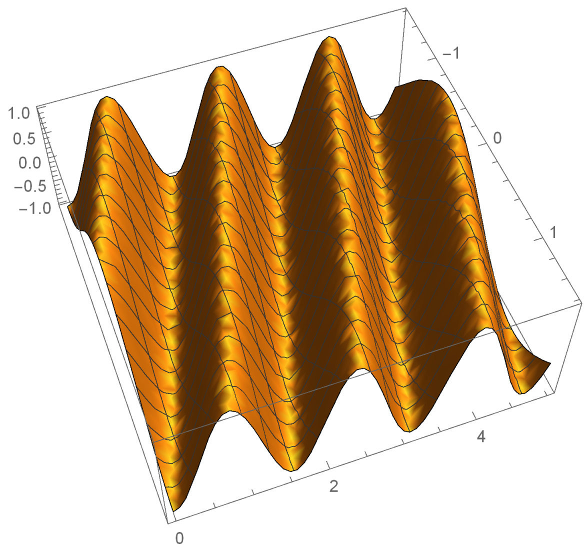
Then the parameters nu and k have the same role as in the previous equation. We can then write our solution:

$$w(x,t) = Ae^{ik(x-\nu kt)} + Be^{ik(x+\nu kt)} + Ce^{-ik(x-\nu kt)} + De^{-ik(x+\nu kt)}.$$

Applying the initial conditions results in the solution:

$$w(x,t) = cos(kx + \nu k^2 t). \tag{4}$$

A surface plot of the real part of the solution is included here:



(c) Discuss the relationship between u and w from parts (a) and (b) above.

Solution:

If we take the time derivative of the equation in part (a), we see

$$u_{tt} = i\nu u_{xxt} = i\nu(i\nu u_{xx})_{xx} = -\nu^2 u_{xxxx}. \tag{5}$$

If we look again at the linear Schrodinger equation, we can see that $u = B\exp(-i(kx - \omega t))$ is an ansatz with dispersion relation $\omega = -\nu k^2$. Thus we can actually write the solution for this equation as

$$u(x,t) = Ae^{ik(x-\nu kt)} + Be^{-ik(x+\nu t)}. \tag{6}$$

Applying the boundary condition clearly requires $B = 0$ along with $A = 1$. Thus we see that part of the solution for w is the trivial solution for u, and the solution for u is part of the trivial solution for w.

5. Consider the equation $u_t = iau_x$ for $a > 0, |x| < \infty, t > 0$. Determine the dispersion relation for this equation and discuss the nature of the solutions.

Solution:

We make the ansatz $u = A\exp(i(kx - \omega t))$ to find the dispersion relation for the equation

$$\omega = -iak. \tag{7}$$

Then the solutions take the form

$$u(x,t) = Ae^{ikx}e^{-akt}. \tag{8}$$

We can see then, that the solutions decay exponentially in time.

The code used to generate the text in problems 1-3 is included here.

- MATLAB code

```
fprintf('HELLO FROM MATLAB \n');

for i=1:10
    fprintf('%1i %4.6e %4.6e \n',i,1/i,sin(1/i));
end
```

- C code

```
#include<stdio.h>
#include<math.h>

double main()
{
     int count;
     double z, y, x;
     printf("HELLO FROM C\n");
     for(count = 1; count<11; count++)
     {
          z=count;
          x=1/z;
          y=sin(x);
          printf("%d %lf %lf\n",count,x,y);
     }
return 0;
}
```

- FORTRAN code

```
      program problemSet1Fortran

     integer i, n
     real y
     write(*,*) 'HELLO FROM FORTRAN'
     do 10 i = 1, 10
          y=i
          write(*,*) i,'', 1/y,'', sin(1/y)
10    continue

      end
```