TTK4190 Guidance and Control of Vehicles

LaTeX Template Assignment 1

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Problem 1 - Attitude Control of Satellite

Consider a satellite with inertia matrix $\mathbf{I}_{CG} = mr^2\mathbf{I}_3$, m = 100 kg, r = 2.0 m. The equations of motions are (Fossen, 2011):

$$\dot{\mathbf{q}} = \mathbf{T}_q(\mathbf{q})\boldsymbol{\omega} \tag{1a}$$

$$\mathbf{I}_{CG}\dot{\boldsymbol{\omega}} - \mathbf{S}(\mathbf{I}_{CG}\boldsymbol{\omega})\boldsymbol{\omega} = \boldsymbol{\tau} \tag{1b}$$

Problem 1.1

- What is the equilibrium point x_0 of the closed-loop system $\mathbf{x} = [\boldsymbol{\epsilon}^\top, \boldsymbol{\omega}^\top]^\top$ corresponding to $\mathbf{q} = [\boldsymbol{\eta}, \boldsymbol{\epsilon}_1, \boldsymbol{\epsilon}_2, \boldsymbol{\epsilon}_3] = [1{,}0{,}0{,}0]$ and $\boldsymbol{\tau} = \mathbf{0}$? Linearize the spacecraft model about $\mathbf{x} = \mathbf{x}_0$ and write down the expressions for the $\mathbf{A}$ and $\mathbf{B}$ matrices.

Setting the last row in equation (2.68) in [1] equal to zero gives the following set of equation

$$\dot{\epsilon}_1 = \frac{1}{2}(\eta p - \epsilon_3 q + \epsilon_2 r)$$
$$\dot{\epsilon}_2 = \frac{1}{2}(\epsilon_3 p - nq - \epsilon_1 r)$$
$$\dot{\epsilon}_3 = \frac{1}{2}(-\epsilon_2 p + \epsilon_1 q + \eta r)$$

Solving for $\boldsymbol{\omega} = [p, q, r]^\top$ yields $\boldsymbol{\omega} = [0, 0, 0]^\top$. Thus the equalibrium point is $\boldsymbol{x}_0 = [0, 0, 0, 0, 0, 0]^\top$.

The equations of motions can be written on the form

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) = \begin{bmatrix} \dot{\epsilon} \\ \dot{\boldsymbol{\omega}} \end{bmatrix} = \begin{bmatrix} \mathbf{T}_\epsilon(\epsilon)\boldsymbol{\omega} \\ \mathbf{I}_{CG}^{-1}(\mathbf{S}(\mathbf{I}_{CG}\boldsymbol{\omega})\boldsymbol{\omega} + \boldsymbol{\tau}) \end{bmatrix} \tag{3}$$

The expression $\mathbf{S}(\mathbf{I}_{CG}\boldsymbol{\omega})\boldsymbol{\omega}$ is equal to a zero vector as shown below,

$$\mathbf{S}(\mathbf{I}_{CG}\boldsymbol{\omega})\boldsymbol{\omega} = \mathbf{S}\left(\mathbf{I}_{CG}\begin{bmatrix} p \\ q \\ r \end{bmatrix}\right)\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \frac{1}{mr^2}\begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix}\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \frac{1}{mr^2}\begin{bmatrix} qr - rq \\ rp - pr \\ pq - qp \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{4}$$

and by comparison of (1a) to the second row equation (2.68) in it is seen that

$$\mathbf{T}_\epsilon(\boldsymbol{\epsilon}) = \frac{1}{2}(\eta\mathbf{I}_{3x3} + \mathbf{S}(\epsilon))$$

. Thus (3) can be rewritten as

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) = \begin{bmatrix} \dot{\epsilon} \\ \dot{\boldsymbol{\omega}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_1(\boldsymbol{x}) \\ \boldsymbol{f}_2(\boldsymbol{x}) \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(\eta\mathbf{I}_{3x3} + \mathbf{S}(\epsilon))\boldsymbol{\omega} \\ {\mathbf{I}_{CG}}^{-1}\boldsymbol{\tau} \end{bmatrix} \tag{5}$$

The linearization of a function is the first order of its Taylor expansion around the point of interest, that is the equalibrium point. For the system (3) the linearized system can be written as,

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{\tau} = \left.\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\right|_{\substack{\boldsymbol{x}=\boldsymbol{x}_0 \\ \boldsymbol{\tau}=\boldsymbol{\tau}_0}} \boldsymbol{x} + \left.\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{\tau}}\right|_{\substack{\boldsymbol{x}=\boldsymbol{x}_0 \\ \boldsymbol{\tau}=\boldsymbol{\tau}_0}} \boldsymbol{\tau}$$

where $\boldsymbol{\tau} = [\tau_1, \tau_2, \tau_3]^\top$.

The state matrix is from the Jacobian matrix evaluated to be

$$\mathbf{A} = \mathbf{J}|_{\substack{\boldsymbol{x}=\boldsymbol{x}_0\\ \boldsymbol{\tau}=\boldsymbol{\tau}_0}} = \begin{bmatrix} \frac{\partial f_1(x)}{\partial \boldsymbol{\epsilon}} & \frac{\partial f_1(x)}{\partial \boldsymbol{\omega}} \\ \frac{\partial f_2(x)}{\partial \boldsymbol{\epsilon}} & \frac{\partial f_2(x)}{\partial \boldsymbol{\omega}} \end{bmatrix}\Bigg|_{\substack{\boldsymbol{x}=\boldsymbol{x}_0\\ \boldsymbol{\tau}=\boldsymbol{\tau}_0}} = \begin{bmatrix} \frac{1}{2}\boldsymbol{\omega}^x & \frac{1}{2}(\eta \mathbf{I}_{3x3} + \mathbf{S}(\epsilon)) \\ \mathbf{0}_{3x3} & \mathbf{0}_{3x3} \end{bmatrix}\Bigg|_{\substack{\boldsymbol{x}=\boldsymbol{x}_0\\ \boldsymbol{\tau}=\boldsymbol{\tau}_0}} = \begin{bmatrix} \mathbf{0}_{3x3} & \frac{1}{2}\eta \mathbf{I}_{3x3} \\ \mathbf{0}_{3x3} & \mathbf{0}_{3x3} \end{bmatrix}$$

The input matrix is evaluated to be

$$\mathbf{B} = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{\tau}}\Bigg|_{\substack{\boldsymbol{x}=\boldsymbol{x}_0\\ \boldsymbol{\tau}=\boldsymbol{\tau}_0}} = \frac{\partial}{\partial \boldsymbol{\tau}} \begin{bmatrix} \boldsymbol{f}_1 \\ \boldsymbol{f}_2 \end{bmatrix}\Bigg|_{\substack{\boldsymbol{x}=\boldsymbol{x}_0\\ \boldsymbol{\tau}=\boldsymbol{\tau}_0}} = \begin{bmatrix} \mathbf{0}_{3x3} \\ \mathbf{I}_{CG}^{-1} \end{bmatrix}\Bigg|_{\substack{\boldsymbol{x}=\boldsymbol{x}_0\\ \boldsymbol{\tau}=\boldsymbol{\tau}_0}} = \begin{bmatrix} \mathbf{0}_{3x3} \\ \mathbf{I}_{CG}{}^{-1} \end{bmatrix}$$

Since $\eta = 1$ the linearized system on state-space form $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$ can be written as

$$\dot{\mathbf{x}} = \begin{bmatrix} \mathbf{0}_{3x3} & \frac{1}{2}\eta \mathbf{I}_{3x3} \\ \mathbf{0}_{3x3} & \mathbf{0}_{3x3} \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{0}_{3x3} \\ \mathbf{I}_{CG}{}^{-1} \end{bmatrix} \boldsymbol{\tau} \tag{6}$$

Problem 1.2

- Consider the attitude control law:

$$\boldsymbol{\tau} = -\mathbf{K}_d \boldsymbol{\omega} - k_p \boldsymbol{\epsilon} \tag{7}$$

where $k_p > 0$ is a scalar control gain, $\mathbf{K}_d = k_d \mathbf{I}_3$ is a controller gain matrix with $k_d > 0$ and ϵ is the imaginary part of the unit quaternion. Let $k_p = 1$ and $k_d = 20$ and verify that the closed-loop system is stable. Would you prefer real or complex poles in this particular application? Explain why/why not?

To verify if the closed-loop system is stable, the eigenvalues of the system will be evaluated. The system with feedback control can be written as $\dot{\mathbf{x}} = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{x} = \bar{\mathbf{A}}\mathbf{x}$ where the eigenvalues of $\bar{\mathbf{A}}$ will determine the stability of the system.

$$\begin{aligned} \dot{\boldsymbol{x}} &= \mathbf{A}\boldsymbol{x} + \mathbf{B}(-\mathbf{K}_d\boldsymbol{\omega} - k_p\boldsymbol{\epsilon}) \\ &= \mathbf{A}\boldsymbol{x} + \mathbf{B}(\begin{bmatrix} -k_p\mathbf{I}_{3x3} & -\mathbf{K}_d \end{bmatrix})\boldsymbol{x} \\ &= (\mathbf{A} - \mathbf{B}\mathbf{K})\boldsymbol{x} \\ &= \begin{bmatrix} \mathbf{0}_{3x3} & \frac{1}{2}\eta \mathbf{I}_{3x3} \\ -k_p\mathbf{I}_{CG}{}^{-1} & -k_d\mathbf{I}_{CG}{}^{-1} \end{bmatrix} \boldsymbol{x} \end{aligned}$$

Finding the eigenvalues, that is when the characteristic polynomial is equal to zero

$$\begin{aligned} |\,[\bar{\mathbf{A}} - \lambda \mathbf{I}]\,| &= |\begin{bmatrix} -\lambda \mathbf{I}_{3x3} & \frac{1}{2}\eta \mathbf{I}_{3x3} \\ -k_p\mathbf{I}_{CG}{}^{-1} & -k_d\mathbf{I}_{CG}{}^{-1} - \lambda \mathbf{I}_{3x3} \end{bmatrix}| \\ &= \lambda^2 \mathbf{I}_{3x3} - \lambda k_d \mathbf{I}_{CG}^{-1} + \frac{1}{2}\eta k_p \mathbf{I}_{CG}^{-1} \\ &= (-\lambda^2 - \frac{k_d}{mr^2}\lambda + \frac{\eta k_p}{2mr^2})\mathbf{I}_{3x3} \\ &= 0 \end{aligned}$$

Solving for λ yields

$$\lambda_{1,2} = \frac{k_d}{mr^2} \pm \sqrt{\left(\frac{k_d}{mr^2}\right)^2 - 2\eta \frac{k_p}{mr^2}} = -0.025 \pm 0.025j$$

The system has triple repeated eigenvalues with strictly negative real parts, and is therefor asymptotically stable.

In space there is no naturally present damping because of vacuum, and therefore pure complex poles are undesirable since it will cause the system to become undamped. Coinciding strictly negative real poles will give a critically damped system, and is therefor desired.

Problem 1.3

- Let $k_p = 1$ and $k_d = 20$. Simulate the attitude dynamics of the closed-loop system with the control law given by (7) for initial conditions $\phi(0) = 10°$ $\theta(0) = -5°$ $\psi(0) = 15°$ by modifying attitude.m. The initial angular velocities are zero. Plot the results (convert the resulting q(t) states to Euler angles for easier visualization). Does the behavior of the system match your expectations? Explain why/why not. Include figures of the Euler angles, angular velocities and the control input.

Since the closed-loop system is stable, with a constant desired attitude, it is expected that the system will converge after some finite time. As seen in Figure 1 this is accomplished after approximately 100 seconds. With the given attitude control law, the satellite might encounter problems when following a time-varying trajectory, because of too slow control dynamics.

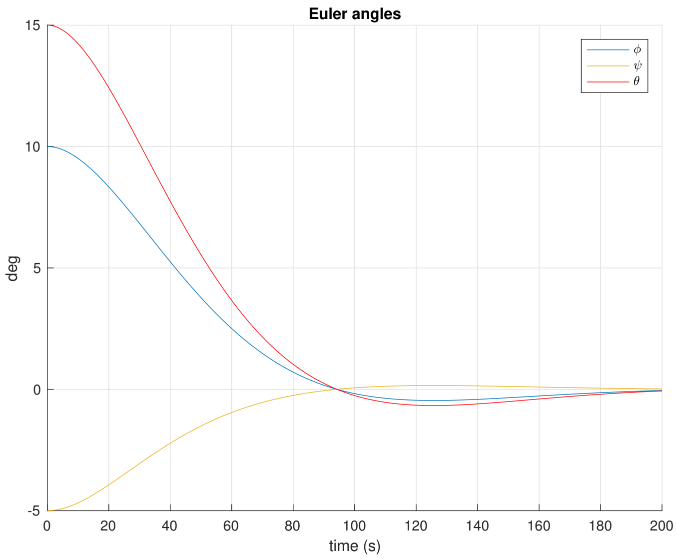

Figure 1: The trajectory of the satellite in problem1.3

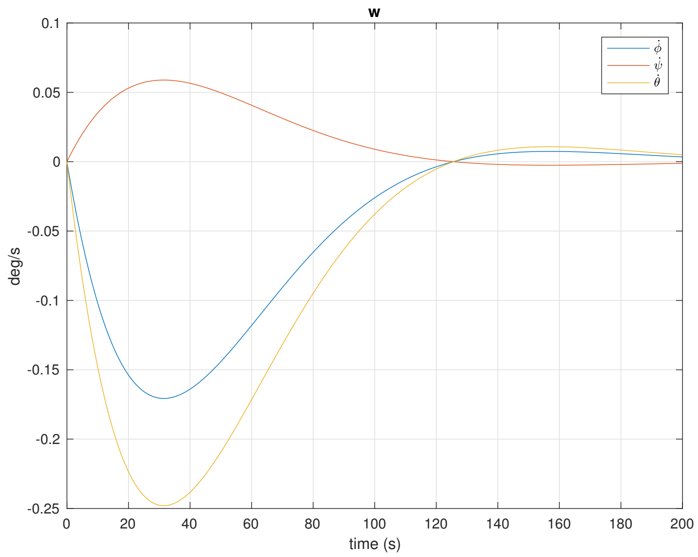

Figure 2: The angular velocities of the satellite in problem1.3

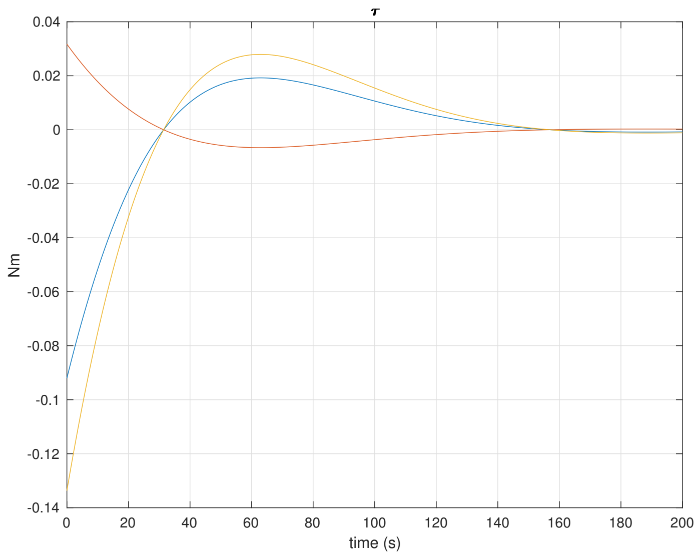

Figure 3: The applied τ in problem1.3

Problem 1.4

- Consider the modified attitude control law:

$$\boldsymbol{\tau} = -\mathbf{K}_d \omega - k_p \tilde{\boldsymbol{\epsilon}} \tag{8}$$

where $\tilde{\boldsymbol{\epsilon}}$ is the error in the imaginary part of the quaternion (between the setpoint and true state). The quaternion error is defined as:

$$\tilde{\mathbf{q}} := \begin{bmatrix} \tilde{\eta} \\ \tilde{\epsilon} \end{bmatrix} = \bar{\mathbf{q}}_d \otimes \mathbf{q} \tag{9}$$

where $\mathbf{q}_d$ is the desired quaternion, $\mathbf{q}$ is the current state and $\bar{\mathbf{q}} = [\boldsymbol{\eta}, -\boldsymbol{\epsilon}^\top]^\top$ denotes the conjugate (sometimes called the inverse) of a quaternion $\mathbf{q}$. Write down the matrix expression for the quaternion error $\tilde{\mathbf{q}}$ on component form. What is $\tilde{\mathbf{q}}$ after convergence, that is $\mathbf{q} = \mathbf{q}_d$?

The matrix expression for the quaterning error $\tilde{\boldsymbol{q}}$ is

$$\tilde{\mathbf{q}} := \begin{bmatrix} \tilde{\eta} \\ \tilde{\epsilon} \end{bmatrix} = \bar{\mathbf{q}}_d \otimes \mathbf{q} = \begin{bmatrix} \eta_d \eta - \boldsymbol{\epsilon}_d^\top \boldsymbol{\epsilon} \\ \eta_d \boldsymbol{\epsilon} + \eta \boldsymbol{\epsilon}_d + \mathbf{S}(\boldsymbol{\epsilon}_d)\boldsymbol{\epsilon} \end{bmatrix} = \begin{bmatrix} \eta_d \eta + \epsilon_{d,1}\epsilon_1 + \epsilon_{d,2}\epsilon_2 + \epsilon_{d,3}\epsilon_3 \\ \eta_d \epsilon_1 - \eta \epsilon_{d,1} + \epsilon_{d,2}\epsilon_3 - \epsilon_{d,3}\epsilon_2 \\ \eta_d \epsilon_2 - \eta \epsilon_{d,2} + \epsilon_{d,3}\epsilon_1 - \epsilon_{d,1}\epsilon_3 \\ \eta_d \epsilon_3 - \eta \epsilon_{d,3} + \epsilon_{d,1}\epsilon_2 - \epsilon_{d,2}\epsilon_1 \end{bmatrix} \tag{10}$$

When $\mathbf{q} = \mathbf{q}_d$, the $\eta_d = \eta$, $\epsilon_{d,1} = \epsilon_1$, $\epsilon_{d,2} = \epsilon_2$, $\epsilon_{d,3} = \epsilon_3$, and thus $\tilde{\boldsymbol{q}} = [1, 0, 0, 0]^\top$. This is reasonable, since after convergence the error $\tilde{\boldsymbol{\epsilon}}$ should be equal to zero.

Problem 1.5

- Let $k_p = 10$ and $k_d = 300$. Simulate the attitude dynamics of the closed-loop system with the control law given by (8). The desired attitude is given by the time-varying reference signal $q_d(t)$ corresponding to $p_d(t) = 10sin(0.1t)$, $q_d(t) = 0$ and $r_d(t) = 15cos(0.05t)$ (all in degrees) and the initial values are equal to the ones in Problem 1.3. Does the behavior of the system match your expectations? Explain why/why not. Include the same set of figures as in Problem 1.3 and the tracking error in the report.

Figure 4 shows the desired and actual attitude of the satellite. The actual attitude is phase shifted, and has a lower amplitude compared with the desired attitude, resulting in a noteworthy tracking error, as shown in Figure 7.
The reason why the tracking error becomes so significant is that $-\mathbf{K}_d\omega$ part restrict the angular velocity. Thus it counteracts the $-k_p\tilde{\epsilon}$ part which increases the angular velocity with the goal of following the reference. This makes the controller unable to follow the reference.
But the $-\mathbf{K}_d\omega$ part of the controller places the poles on the left part of the plane, thus making the system stable. By changing the expression to $\tau = -k_p\epsilon$, the system would behave as seen in Figure 8, that is unstable. This is because the $-\mathbf{K}_d\omega$ part counteract the $-k_p\epsilon$ part, restricting τ increasing to ∞. Even though this expression makes the system behave in a stable manner, the system should punish the deviation in a more appropriate way to achieve an attitude closer to the reference, without restraining the angular velocity too much.

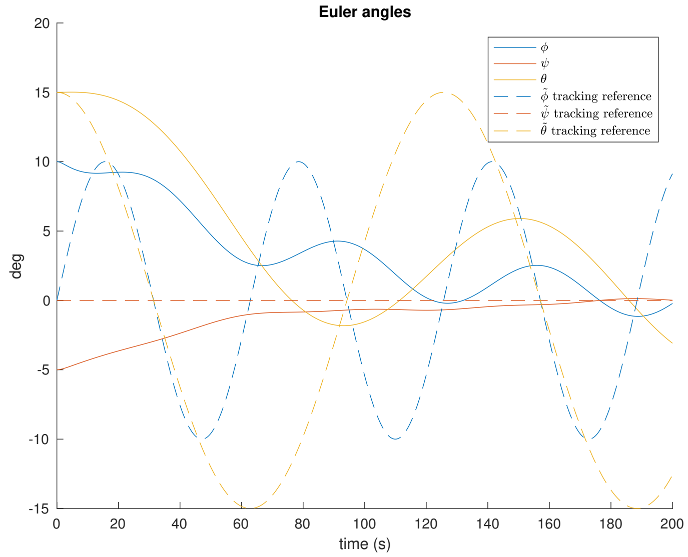

Figure 4: The trajectory of the satellite in problem1.5

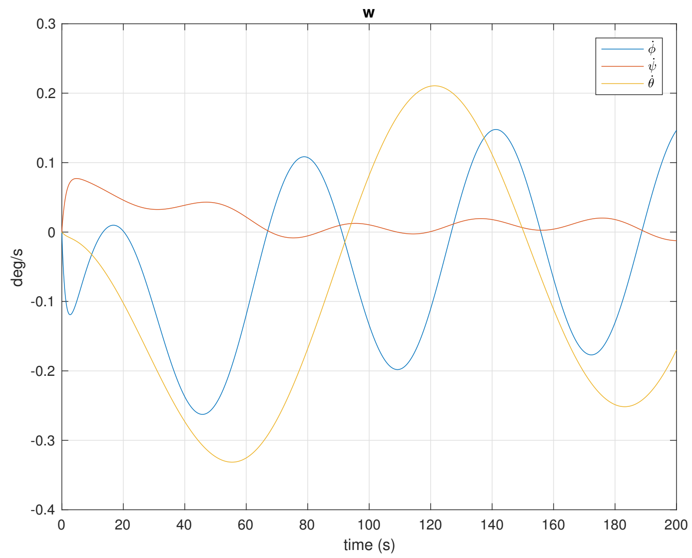

Figure 5: The angular velocities of the satellite in problem1.5

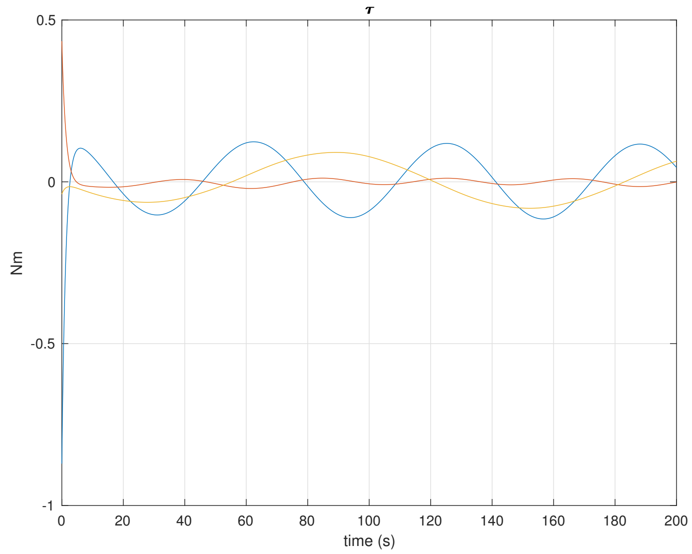

Figure 6: The applied τ in problem1.5

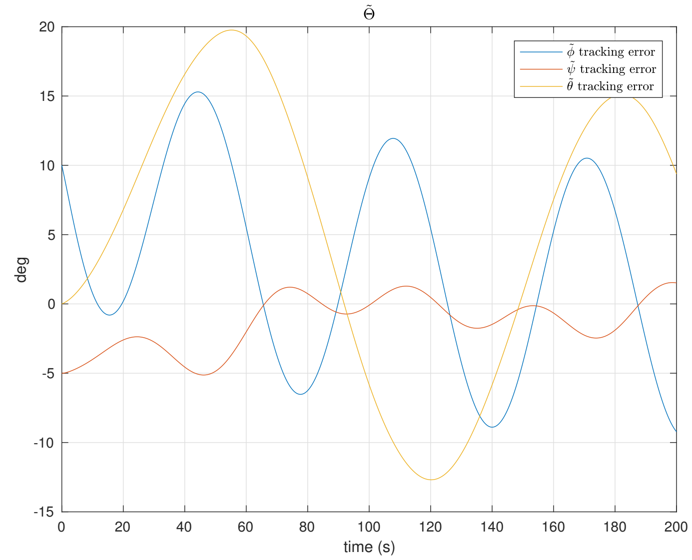

Figure 7: The tracking error in problem1.5

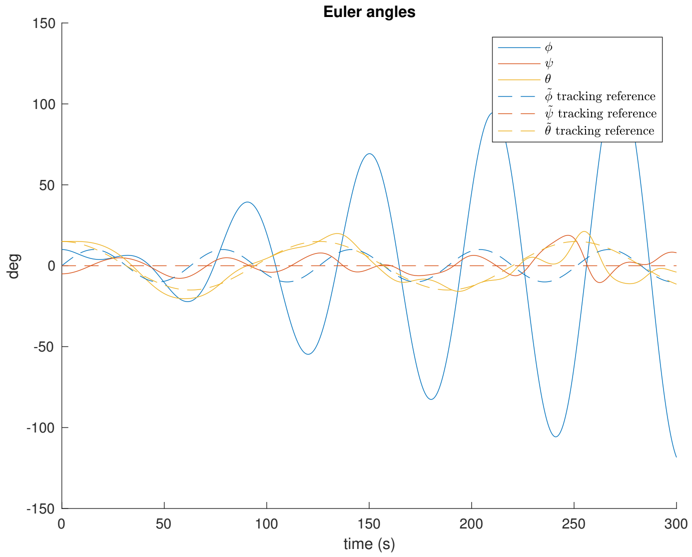

Figure 8: The systems response with $\tau = -k_p \epsilon$ in problem1.5

Problem 1.6

- Consider the modified attitude control law:

$$\tau = -\mathbf{K}_d \tilde{\omega} - k_p \tilde{\epsilon} \tag{11}$$

where $\tilde{\omega} = \omega_d - \omega$ is the difference between the desired and current angular velocity. Let the desired attitude be given by the reference signals from Problem 1.5 and calculate the desired angular velocity using (see equation (2.26) in Fossen).

$$\omega_d = \mathbf{T}_{\Theta_d}^{-1}(\Theta_d)\dot{\Theta}_d \tag{12}$$

where Θ_d is the desired Euler angles. Let $k_p = 10$ and $k_d = 300$ and simulate the controller (11) in Matlab with the same parameters, initial conditions and reference signals as in Problem 1.5. Compare the results. Does the behavior of the system match your expectations? Explain why/why not.

The controller ((11) is now modified to also punish the difference between the desired and current angular velocity. As seen in Figure 9 and Figure 12 the system is now able to follow both the angle and angular velocity reference with a vast decrease in tracking error compared to the controller in problem 1.5. This is natural because of the controllers ability to punish the error in the current angular velocity compared to the desired one, which is computed from the desired euler rate. This will amplify the controller input when the deviation in angular velocity increases, instead of trying to restrain the angular velocity to zero just to achieve stability.

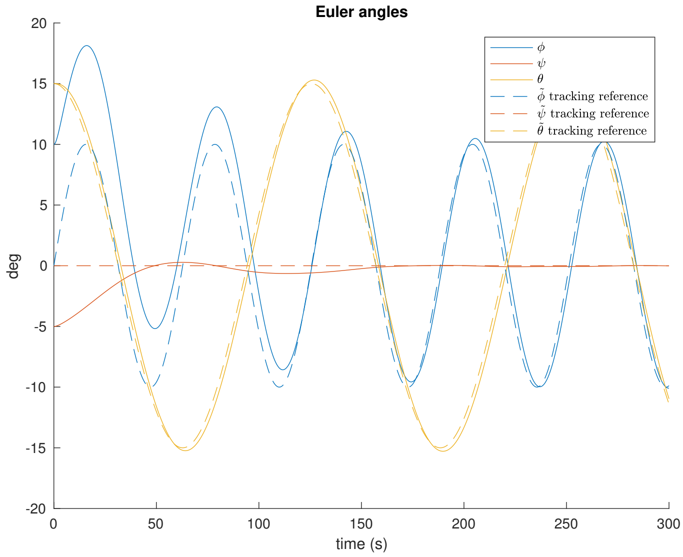

Figure 9: The trajectory of the satellite in problem1.6

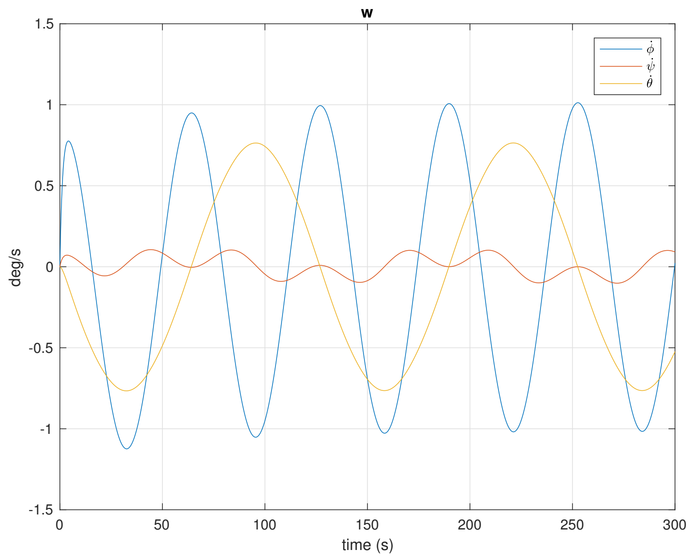

Figure 10: The angular velocities of the satellite in problem1.6

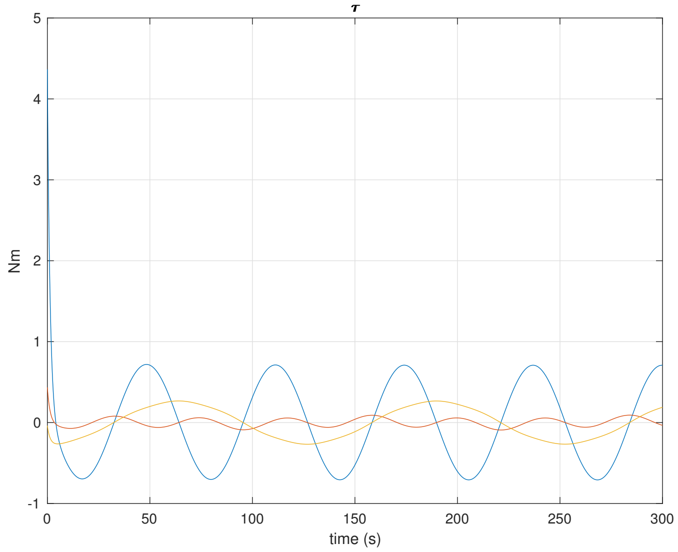

Figure 11: The applied τ in problem1.6

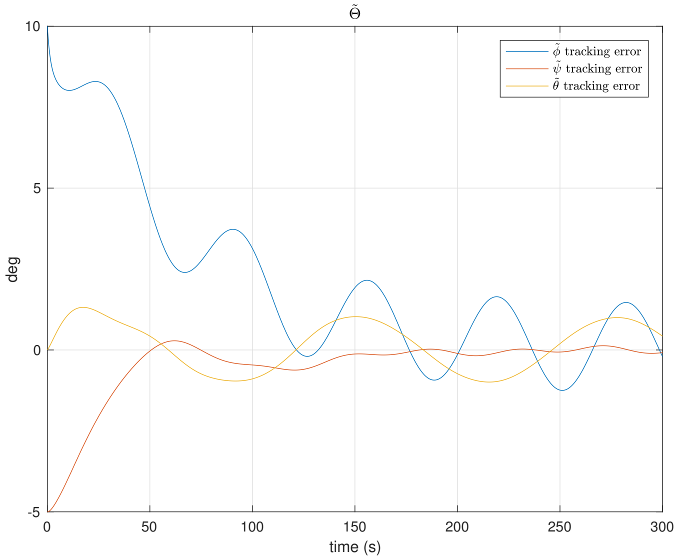

Figure 12: The tracking error in problem1.6

Problem 1.7

- It can be shown that:

$$\dot{\tilde{\eta}} = -\frac{1}{2}\tilde{\epsilon}^\top\tilde{\omega} \tag{13}$$

Assume setpoint regulation, that is $\omega_d = 0$, ϵ_d and η_d is constant, and the control law given by (8). Consider the Lyapunov function candidate (Fjellstad and Fossen, 1994):

$$V = \frac{1}{2}\tilde{\omega}^\top\mathbf{I}_{CG}\tilde{\omega} + 2k_p(1-\tilde{\eta}) \tag{14}$$

Explain why V is positive and radially unbounded. Show that:

$$\dot{V} = -k_d \omega^\top \omega \tag{15}$$

Use Barbalat's lemma to show that ω converges to zero. Is the equilibrium point of the closed-loop system locally or globally convergent? Is the system asymptotically stable?

$\mathbf{I}_{CG}$ is a diagonal matrix with positiv elements, and therefore all its eigenvalues are positive, and thus it is a positive definite matrix, that is $\mathbf{I}_{CG} \succ 0$. This implies that $\tilde{\boldsymbol{\omega}}^\top \mathbf{I}_{CG} \tilde{\boldsymbol{\omega}} \geq 0 \; \forall \; \tilde{\boldsymbol{\omega}}$. Also $|\tilde{\eta}| \leq 1$, and therefore V is always positive, that is $V \geq 0$. Letting $||\tilde{\boldsymbol{\omega}}|| \to \infty \Rightarrow V \to \infty$, and therefor V is also radially unbounded.

Substituting equation (8) for τ into the newton second law balance equation (1b), gives

$$\begin{aligned} \mathbf{I}_{CG}\dot{\omega} &= -\mathbf{K}_d \omega - k_p \tilde{\epsilon} + \mathbf{S}(\mathbf{I}_{CG}\omega)\omega \\ &= -\mathbf{K}_d \omega - k_p \tilde{\epsilon} \end{aligned} \tag{16}$$

The last term is equal to zero according to (4). Then $\dot{V}$ can be shown to be equal to

$$\begin{aligned} \dot{V} &= \tilde{\omega}^\top \mathbf{I}_{CG} \dot{\tilde{\omega}} - 2k_p \dot{\tilde{\eta}} \\ &= \tilde{\omega}^\top \mathbf{I}_{CG} \dot{\tilde{\omega}} + k_p \tilde{\epsilon}^\top \tilde{\omega} \\ &= \tilde{\omega}^\top [-\mathbf{K}_d \omega - k_p \tilde{\boldsymbol{\epsilon}}] + k_p \tilde{\epsilon}^\top \tilde{\omega} \\ &= -k_d \tilde{\omega}^\top \omega \\ &= -k_d \omega^\top \omega \end{aligned} \tag{17}$$

Here (13) and (16) is substituted for $\dot{\tilde{\eta}}$ and $\mathbf{I}_{CG}\dot{\omega}$, and it is used that $\tilde{\boldsymbol{\omega}} = \boldsymbol{\omega}$ because $\boldsymbol{\omega}_d = 0$. $\dot{V} \leq 0 \; \forall \; \omega$, because of the sign and ω squared. Thus the controller in (8) ensures that $\dot{V} \leq 0 \; \forall \; \omega$.

Since $\dot{V} \leq 0$, the energy is always decreasing, and thus ω is also always decreasing and is therefore bounded. Thus $\ddot{V} = -2k_d\omega$ is also bounded, and therefore $\dot{V}$ is uniformly continuous.

Since $V \geq 0$, $\dot{V} \leq 0$ and $\dot{V}$ is shown to be bounded, the system is global convergent according to Barbalat's lemma. The next step is to verify whether or not the system is asymptotically stable.

$$\dot{V} = 0 \Rightarrow -k_d \omega^\top \omega = 0 \Rightarrow \omega = 0$$

By substituting this into (16) the following is obtained

$$\begin{aligned} \mathbf{I}_{CG}\dot{\omega} &= -\mathbf{K}_d \omega - k_p \tilde{\epsilon} && \text{inserting } \omega = 0 \\ \mathbf{I}_{CG}\dot{\omega} &= k_p \tilde{\epsilon} \\ \dot{\omega} &= \mathbf{I}_{CG}^{-1} k_p \tilde{\epsilon} \\ \dot{\omega} &\neq 0 && \forall \tilde{\epsilon} \neq 0 \end{aligned}$$

Since $\dot{\omega} = 0 \iff \tilde{\epsilon} = 0$, the velocity of the satellite will be equal to zero only when the reference is equal to the current state. This means that the satellite won't get 'stuck' unless it has arrived at the desired state. Hence, according to Krazovskii-LaSalle's Theorem, the system is asymptotically stable.

Problem 1.8

One advantage with quaternions compared to Euler angles is the fact that quaternions are globally defined, thus have no singularities. This is not the case with Euler angles. Quaternions are defined by using only one simple rotation around a given axis, while Euler angles consists of three separate rotations. This may also be an advantage by using quaternions. One can argue that Euler angles are easier to understand and more intuitive to read. The mathematical theory behind quaternions are complex, while the three rotations described by Euler angles can easily be explained just by using the theory of rotational matrices. On the other hand, the computational power needed by using quaternions are less than what is needed if Euler angles are used. This can be explained because of the matrix multiplications involved when using Euler angles. Euler angles are described by using three parameters (roll, pitch, jaw or ϕ, θ, ψ), while quaternions are described using four parameters($\eta, \epsilon_1, \epsilon_2, \epsilon_3$). Euler angles are not unique, since the rotations can be done in several ways. This is not the case with quaternions, as the rotation is done about a unique axis.

References

[1] T. Fossen, *Handbook of Marine Craft Hydrodynamics and Motion Control.* John Wiley & Sons, 2011.

[2] O.-E. Fjellstad and T. I. Fossen, "Quaternion feedback regulation of underwater vehicles," *Proceedings of the IEEE Conference on Control Applications*, vol. 2, pp. 857–862, 1994.