TTK4190 Guidance and Control of Vehicles

LaTeX Assignment 1, Part 2

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Problem 2 - Underwater Vehicles

An underwater vehicle moves on a straight line in the horizontal plane at constant depth $z = 10$ m and course angle 30°. The speed is $U = 1.5$ m/s and the vehicle has a pitch angle $\theta = 2.0°$, while the roll angle is $\phi = 0°$.

This report differs from the assignment text in that sense that it is assumed that the underwater vehicle is turning as it moves along the circle, and thus the heading ψ is not constant. This is far more realistic than what the assignment most likely aims for, that is a vehicle that moves on a circle with constant heading. With respect to the assignment text this assumption is not optimal, but because of a somewhat cloudy assignment text with a lot of room for interpretation, this assumption was done. It is therefore in the report done some compromises between the assignments terms and turning assumption.

Another major assumption that was made is that the vehicle moves in a circle on the water surface. When current was added the water surface would move. After a lot of work, it was understood that the vehicle trajectory was not meant to be changed, as it was supposed to still move on the same circle relative to origo. But in terms of time, there was unfortunately no time left to change the simulation and plot, but an extensive discussion in the problems has been made to show understanding.

Problem 2.1

Assuming that there are no currents and that therefor the crab angle is 0°, the velocity vector of the vehicle is parallel to the x_b-axis of BODY. Since the crab-angle is defined to be the difference between these two vectors (Definition 2.6, [1]), the heading ψ is equal to the course angle χ, such that $\psi = 30°$.

The flight path angle γ is defined to be the angle around the y_n-axis between the velocity-vector and the north-vector x^n in NED. Since the vehicle is moving in a straight line in a constant depth, they must be parallel and thus $\gamma = 0°$.

The FLOW-fixed linear velocity is $\boldsymbol{\nu}^{flow} = [U, 0, 0]^\top$, where the x^{flow}-axis is parallel to the free-stream flow. Thus the BODY-fixed linear velocity is given by the following velocity transformation which rotates the velocity vector from FLOW to BODY axis system. Since $\gamma = 0$, the angle of attack α is equal to the pitch θ, and with crab angle $\beta = 0$, the BODY-fixed velocities becomes

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \mathbf{R}_{y,\alpha}^\top \mathbf{R}_{z,-\beta}^\top \boldsymbol{\nu}^{flow} = \begin{bmatrix} U\cos\alpha\cos\beta \\ U\sin\beta \\ U\sin\alpha\cos\beta \end{bmatrix} = \begin{bmatrix} 1.5 \\ 0 \\ 0.05 \end{bmatrix}$$

Problem 2.2

Assuming that the vehicle still has a pitch angle of $= 2.0°$, but now moves on a circle with radius $R = 100$ m and no current, with velocities relative to the current in the water, assumed to be given by

$$\boldsymbol{\nu}_{b/c}^n = \begin{bmatrix} U\cos(\omega t) \\ U\sin(\omega t) \\ 0 \end{bmatrix} \tag{1}$$

the numerical value of ω, that is the frequency of which the vehicle completes one circle, is simply because $\boldsymbol{\nu} = \boldsymbol{\omega} \times \boldsymbol{r}$ given by the absolute value of the velocity vector divided by the radius R. Thus

$$w = \frac{U}{R} = 0.015 \quad \left[\frac{rad}{s}\right]$$

Since the vehicle moves on a circle, it is natural that it also turns horizontally such that the heading ψ always is the same at the course χ. This again means that the turning rate r must be equal to the turning rate of χ. Since χ is tangentially to the circle, it must change with the same angular rate as position vector of the vehicle, that is ω. Thus $r = \dot{\psi} = \omega = 0.015$ rad/s.

The crab angle is defined to be the correction the vehicle must undertake in order for the heading ψ to equal to the course angle χ. Thus the crab angle must be zero on this case, that is $\beta = 0°$. In order to show it mathematically the NED-fixed velocity must the transformed onto BODY-fixed velocity, such that the formula (2) for crab angle β can be used.

$$\beta = \sin^{-1}(\frac{v}{U}) \tag{2}$$

The velocity transformation is given by

$$\boldsymbol{\nu}^b_{b/c} = \mathbf{R}^n_b(\boldsymbol{\Theta}_{nb})^{-1}\boldsymbol{\nu}^n_{b/c} = \mathbf{R}^\top_{x,\phi}\mathbf{R}^\top_{y,\theta}\mathbf{R}^\top_{z,\psi}\boldsymbol{\nu}^n_{b/c}$$

Since $\phi = 0$ and $\psi = \omega t$ the expression is simplified and by the use of rotation matrices on page 22 in [1] gives

$$\boldsymbol{\nu}^b_{b/c} = \mathbf{R}^\top_{y,\theta}\mathbf{R}^\top_{z,\omega t}\boldsymbol{\nu}^n_{b/c} = \begin{bmatrix} C_{\omega t}C_\theta & S_{\omega t}C_\theta & -S_\theta \\ -S_{\omega t} & C_{\omega t} & 0 \\ C_{\omega t}S_\theta & S_{\omega t}S_\theta & C_\theta \end{bmatrix} \begin{bmatrix} UC_{\omega t} \\ US_{\omega t} \\ 0 \end{bmatrix} = \begin{bmatrix} U\cos(\theta) \\ 0 \\ U\sin(\theta) \end{bmatrix}$$

where $s\cdot = \sin(\cdot)$ and $s\cdot = \cos(\cdot)$. This gives

$$\beta = \sin^{-1}\left(\frac{v}{U}\right) = \sin^{-1}\left(\frac{0}{U}\right) = 0$$

which confirms the reasoning above.

Problem 2.3

The underwater vehicle is exposed to a 3-D ocean current with speed $U_c = 0.6$ m/s and directions $\alpha_c = 10°$ and $\beta_c = 45°$ in the vertical and horizontal planes, respectively. The vehicle still moves on a circle as defined by problem 2.2

Since the course book [1] is ambiguous when it comes to the definitions of α_c and β_c in the subsection 8.3.1 compared to intro in 8.3 and figure 2.9, it is a need to clarification. It is beneficial that the current model is independent from the vessel model, and therefore α_c and β_c should be defined to be the relative angels between FLOW and NED as in 8.3.1, not FLOW and BODY as in Figure 2.9 and 8.3 intro.

The current can be described on vector form in FLOW by $\boldsymbol{\nu}^{flow} = [U_c, 0, 0]^\top$, such that the x^{flow}-axis is parallel to the free-stream flow just like in problem 2.1. Thus the NED-fixed current velocity vector $\boldsymbol{\nu}_c = [u, v, w]^\top$ is according to 8.156 in [1], is computed by a simple velocity rotation

$$\boldsymbol{\nu}^n_{c/n} = \mathbf{R}^\top_{y,\alpha_c}\mathbf{R}^\top_{z,-\beta_c} \begin{bmatrix} U_c \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} U_c\cos\alpha_c\cos\beta_c \\ U_c\sin\beta_c \\ U_c\sin\alpha_c\cos\beta_c \end{bmatrix} \tag{3}$$

Then a rotation from NED to BODY is required, where $\psi = \omega t$, $\phi = 0°$, and $\theta = 2°$, and gives

$$
\begin{aligned}
\boldsymbol{\nu}_c &= \mathbf{R}_b^n(\boldsymbol{\Theta})^{-1}\boldsymbol{\nu}_{c/n}^n \\
&= \mathbf{R}_{x,\phi}^\top \mathbf{R}_{y,\theta}^\top \mathbf{R}_{z,\psi}^\top \boldsymbol{\nu}_{c/n}^n \\
&= \mathbf{R}_{y,\theta}^\top \mathbf{R}_{z,\omega t}^\top \boldsymbol{\nu}_{c/n}^n \\
&= \begin{bmatrix} C_{\omega t}C_\theta & S_{\omega t}C_\theta & -S_\theta \\ -S_{\omega t} & C_{\omega t} & 0 \\ C_{\omega t}S_\theta & S_{\omega t}S_\theta & C_\theta \end{bmatrix} \begin{bmatrix} U_c C_{\alpha_c} C_{\beta_c} \\ U_c S_{\beta_c} \\ U_c S_{\alpha_c} C_{\beta_c} \end{bmatrix} \\
&= \begin{bmatrix} C_{\omega t}C_\theta & S_{\omega t}C_\theta & -S_\theta \\ -S_{\omega t} & C_{\omega t} & 0 \\ C_{\omega t}S_\theta & S_{\omega t}S_\theta & C_\theta \end{bmatrix} \begin{bmatrix} 0.4178 \\ 0.4343 \\ 0.0737 \end{bmatrix}
\end{aligned}
$$

The approach in `MATLAB` to compute the numerical values is to use the function `Rxyz()` from the MSS-toolbox to compute the rotation-matrices. Because of the assumption that the boat rotates as it moves on the circle, numerical values for ν_c varies with time. Thus this is the closest one gets to the computation of the numerical values.

The relative velocity is the difference between the vehicle speed over ground and the current speed over ground. But because

$$
\begin{aligned}
\boldsymbol{\nu}_r &= \boldsymbol{\nu} - \boldsymbol{\nu}_c \\
&= \mathbf{R}_b^n(\boldsymbol{\Theta})^{-1}\boldsymbol{\nu}_{b/n}^n - \mathbf{R}_b^n(\boldsymbol{\Theta})^{-1}\boldsymbol{\nu}_{c/n}^n \\
&= \mathbf{R}_b^n(\boldsymbol{\Theta})^{-1}(\boldsymbol{\nu}_{b/c}^n + \boldsymbol{\nu}_{c/n}^n - \boldsymbol{\nu}_{c/n}^n) \\
&= \mathbf{R}_{y,\theta}^\top \mathbf{R}_{z,\omega t}^\top \boldsymbol{\nu}_{b/c}^n \\
&= \begin{bmatrix} C_{\omega t}C_\theta & S_{\omega t}C_\theta & -S_\theta \\ -S_{\omega t} & C_{\omega t} & 0 \\ C_{\omega t}S_\theta & S_{\omega t}S_\theta & C_\theta \end{bmatrix} \begin{bmatrix} UC_{\omega t} \\ US_{\omega t} \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\end{aligned}
\tag{4}
$$

which is a result of the assumption that 1 is defined with respect to the ocean current, not speed over ground. If instead 1 was defined as speed over ground, the expression would become

$$
\begin{aligned}
\boldsymbol{\nu}_r &= \boldsymbol{\nu} - \boldsymbol{\nu}_c \\
&= \mathbf{R}_b^n(\boldsymbol{\Theta})^{-1}\boldsymbol{\nu}_{b/n}^n - \mathbf{R}_b^n(\boldsymbol{\Theta})^{-1}\boldsymbol{\nu}_{c/n}^n \\
&= \mathbf{R}_b^n(\boldsymbol{\Theta})^{-1}(\boldsymbol{\nu}_{b/n}^n - \boldsymbol{\nu}_{c/n}^n) \\
&= \mathbf{R}_{y,\theta}^\top \mathbf{R}_{z,\omega t}^\top (\boldsymbol{\nu}_{b/n}^n - \boldsymbol{\nu}_{c/n}^n) \\
&= \begin{bmatrix} C_{\omega t}C_\theta & S_{\omega t}C_\theta & -S_\theta \\ -S_{\omega t} & C_{\omega t} & 0 \\ C_{\omega t}S_\theta & S_{\omega t}S_\theta & C_\theta \end{bmatrix} \begin{bmatrix} UC_{\omega t} - U_c C_{\alpha_c} C_{\beta_c} \\ US_{\omega t} - U_c S_{\beta_c} \\ -U_c S_{\alpha_c} C_{\beta_c} \end{bmatrix}
\end{aligned}
\tag{5}
$$

This is a far more complicated expression, and since it is dependent on time because of the turning of the vehicle, which is expected. No numerical values can be determined without setting some value for t.

Problem 2.4

The side-slip angle when there is ocean current, denoted β_r, is defined to be the angle between the x_b-axis in BODY and the relative velocity vector given by 2.120-2.122 in [1]. When the relative velocity is BODY-fixed, the side-slip angle is given by

$$\beta_r = \sin^{-1}(\frac{v_r}{U_r}) \tag{6}$$

Thus when vehicle moves on a straight line the relative velocity $\boldsymbol{\nu}_r$ of the vehicle described in BODY is given by (4), with some arbitrary value for $\omega t = \psi$, but because $\boldsymbol{\nu}_r = \mathbf{0} \Rightarrow v_r = 0 \Rightarrow \beta_r = 0$.

If it is assumed that (1) is instead speed over ground (5) could be used. Choosing $\omega t = \psi = 0$ will result in $v_r = -U_c \sin(\beta_c)$ which by the use of (6) results in

$$\beta_r = \sin^{-1}(\frac{v_r}{U_r}) = \sin^{-1}\frac{-U_c \sin(\beta_c}{U - U_c} = -28.12°$$

This is a reasonable aswer considering the ocean current triangle, where U_r becomes a hypotenuse, and $-U_c \sin\beta_c$ becomes the opposite chateaus. Thus simple trigonometry confirms the result.

It is also seen that if $U \to \infty \Rightarrow \beta_r \to 0$, and if $U \to 0 \Rightarrow \beta_r \to \beta_c$. That is reasonable considering the definition of β and β_c, because they should become insignificant if $U >> U_c$, and coincide when U is zero.

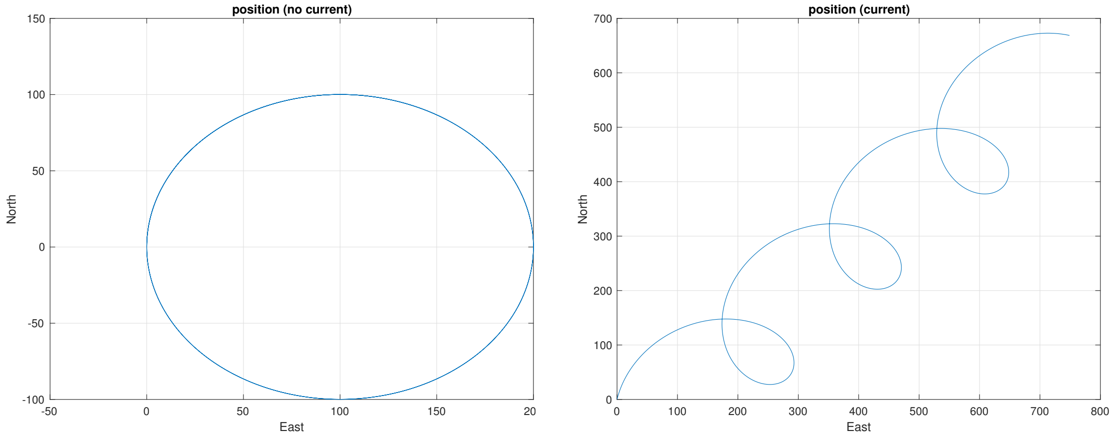
Below the system is simulated by the use of Euler integration for the position given by the velocity $\boldsymbol{\nu}^n_{b/c}$ in (1) with the assumption that it is relative to the ocean current. Thus Euler integration yields

$$\boldsymbol{p}^n_{b/n,i+1} = \boldsymbol{p}^n_{b/n,i} + h(\boldsymbol{\nu}^n_{b/c,i} + \boldsymbol{\nu}^n_{c/n})$$

The absolute-value for the speed over ground changes from constant speed U when there is no current, to an oscillation with amplitude U_c around U with current. This is shown in Figure 2. Together with the movment on the circle this causes the different trajectories shown in Figure 1. This is consistent with the heading in Figure 3, and it is as expected.

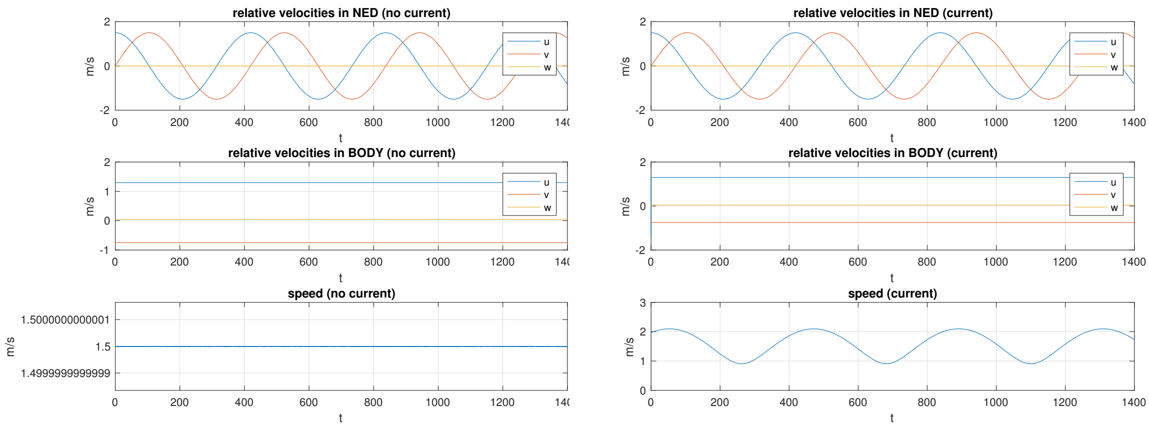
Since it is assumed that the vehicle turn when it moves on the circle, the turning rate $r = w$ is summed from $t_0 = 0$ to t, which for each step, gives $\psi = \omega t$. But because the assignments says that the initial value for $\psi = 30°$, all tough the initial heading χ at $t_0 = 0$, the vehicle actually starts with a crab and side-slip angle equal to $\beta = \beta_r = \chi - \psi = 30°$. This is seen in the Figure 3a where β and β_r is always equal to zero, and in Figure 3b where β oscillates about 30°. The oscillation is due the the time varying effect from the constant current as the vehicle moves on the circle, which only effects the "speed over ground".

Because the Euler angels then is given by $\phi = 0°$, $\theta = 2°$, and $\psi = 30° + \frac{180°}{2\pi}\omega t$ at each time step, the BODY-fixed relative velocity has a speed $u = const \neq 0$, $v = const \neq 0$, and $w = const \neq 0$, and is the same for both with or without current since it is a relative velocity to the current. If ψ was to be constant, then in accordance with what the assignment text may prefer, this would not be the case. The relative BODY-fixed velocities u and v should in that case oscillate.



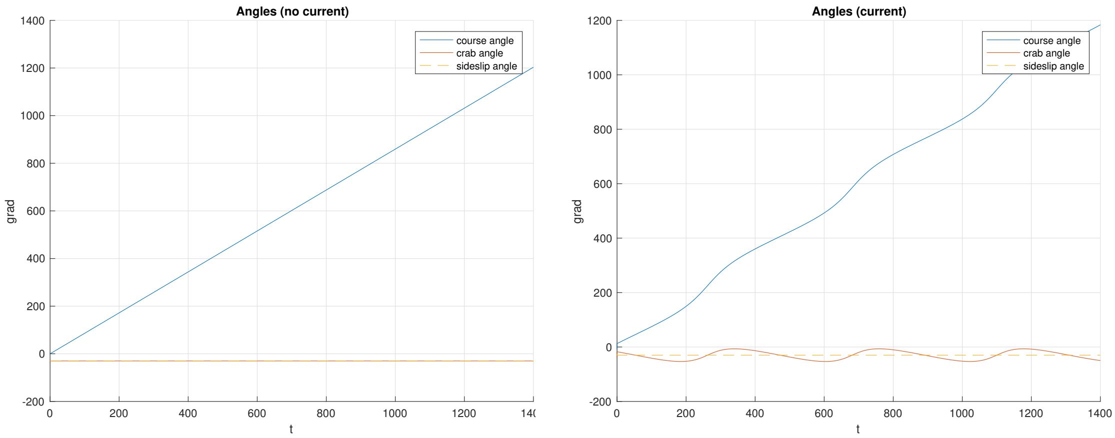
(a) Without current (b) With current

Figure 1: Position of vehicle, and movement with the clock



(a) Without current (b) With current

Figure 2: Relative velocity and speed of the vehicle, relative to current



(a) Without current

(b) With current

Figure 3: Side-slip, crab and course angle of the vehicle

Problem 2.5

The Nomoto model can be written as

$$\frac{r}{\delta}(s) = \frac{K}{Ts+1} \tag{7}$$

with $T = 50s$ and $K = 0.1s^{\{} - 1$, and the equations for the roll and pitch rate as

$$\dot{p} + 2\zeta_p\omega_p p + \omega_p^2\phi = 0 \tag{8a}$$

$$\dot{q} + 2\zeta_q\omega_q q + \omega_q^2\theta = 0 \tag{8b}$$

where damping factor $\zeta_p = 0.1$ and $\zeta_q = 0.2$, and natrual frequencies $\omega_p = 0.1$ and $\omega_q = 0.05$.

The steady-state angular velocity r_s can easily be found by the use of the final value theorem. Since δ is contant it's laplance function is $\delta(s) = \frac{\delta}{s}$, which gives

$$r_s = \lim_{s\to 0} s\frac{K}{Ts+1}\frac{\delta}{s} = K\delta$$

The steady-state angular velocity p_s and q_s is found by setting $\dot{p}$ and $\dot{q}$ equal to zero. This gives

$$2\zeta_p\omega_p p_s + \omega_p^2\phi = 0 \Rightarrow p_s = -\frac{\omega_p^2\phi}{2\zeta_p\omega_p} \Rightarrow \frac{1}{2}\phi$$

$$2\zeta_q\omega_q q_s + \omega_q^2\theta = 0 \Rightarrow q_s = -\frac{\omega_q^2\theta}{2\zeta_q\omega_q} \Rightarrow \frac{1}{8}\theta$$

This gives the steady state angular velocities $p_s = \frac{1}{2}\phi\frac{1}{s}$,$q_s = \frac{1}{8}\theta\frac{1}{s}$, $r_s = 0.1\delta\frac{1}{s}$.

It should be noted that the steady state angular velocities depends on the angles ϕ and θ, and that a possible solution is given as

$$p_s = q_s = \phi = \theta = 0$$

Problem 2.6

In this problem a model of the underwater vehicle, described by the differential equations for the angular velocity $\boldsymbol{\omega}^b_{b/n}$ in (7), (8), the velocity $\boldsymbol{\nu}^n_{b/c}$ given by (1), and the same constant current $\boldsymbol{\nu}^n_{c/n}$ as in problem 2.4, is simulated. After 700 s the rudder input δ is changes from initially $5°$ to $10°$.

The initial values for the Euler angles are $\Theta(0) = [-1°, 2.0°, 0.0°]^\top$, and the angular velocities $p(0) = q(0) = r(0) = 0.0$, and position $p^n_{b/n}(0)$ in origo.

In order to simulate the system the following equation is used. First the position is found by

$$\boldsymbol{p}^n_{b/n,i+1} = \boldsymbol{p}^n_{b/n,i} + h\boldsymbol{\nu}^n_{b/n,i} = \boldsymbol{p}^n_{b/n,i} + h(\boldsymbol{\nu}^n_{b/c,i} + \boldsymbol{\nu}^n_{c/n}) \tag{9}$$

Second the Euler angles is given by equation (2.26) in [1] which gives

$$\boldsymbol{\Theta}_{nb,i+1} = \boldsymbol{\Theta}_{nb,i} + h * \mathbf{T}_\Theta(\boldsymbol{\Theta}_{nb,i})\boldsymbol{\omega}^b_{b/n,i} \tag{10}$$

Third the Euler angels is written on state-space form by first taking the inverse laplace transformation of (7) in order to find $\dot{r}$, such that

$$\dot{r} = -\frac{1}{T}r + \frac{K}{T}\delta \tag{11}$$

and then together with the equation in (8) the angular velocities is given by.

$$\begin{aligned}\dot{\boldsymbol{\omega}}^b_{b/n} = \begin{bmatrix}\dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix} &= \begin{bmatrix}-2\zeta_p\omega_p & 0 & 0\\ 0 & -2\zeta_q\omega_q & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix} + \begin{bmatrix}-\omega_p^2 & 0 & 0\\ 0 & -\omega_p^2 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\phi\\ \theta\\ \psi\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \frac{K}{T}\end{bmatrix}\delta \\ &= \mathbf{A}\boldsymbol{\omega}^b_{b/n} + \mathbf{B}\Theta + \mathbf{C}\delta\end{aligned} \tag{12}$$

which results in

$$\boldsymbol{\omega}^b_{b/n,i+1} = \boldsymbol{\omega}^b_{b/n,i} + h * (\mathbf{A}\boldsymbol{\omega}^b_{b/n,i} + \mathbf{B}\boldsymbol{\Theta}_{nb,i} + \mathbf{C}\delta_i) \tag{13}$$

The implementation is done in the `MATLAB`-script `attitude4.m`.

From Figure 7 it is seen that the steady state is equal to the result in problem 2.5. r behaves like a first order linear model which converges to $K\delta$. Also p and q oscillates in the transient before they converges to 0, witch corresponds to mass-spring-damper systems. The response are as expected.

In Figure 9 the attitude of the vehicle is shown. As expected, ψ is increasing as the vehicle moves in a circle, while $\omega = \phi = 0$ remains unchanged.

As seen in Figure 4a, the vehicle will no longer move in a perfect circle compared to Figure 1a. This is because of the dynamic modelling of the turning rate. It is also notable that the radius of the circle decreases after 700 seconds, due to the increase in rudder angle. When current is applied, the response is somewhat similar to the result in task 2.4, as seen in Figure 4b, except the changed behaviour due to the sudden increase in rudder angle.

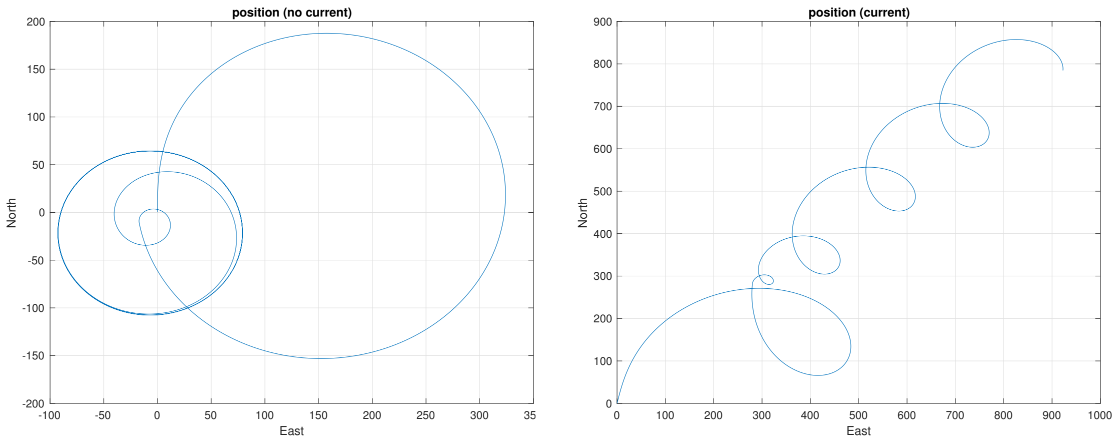
In Figure 5 the course, crab and sideslip angles are plotted with and without current. The crab and sideslip angle both converges to a positive value, and it says that that there is a steady-state constant difference in heading and course. The current creates oscillations in the crab-angle, shown in the figure. This correspond to the same effect the current had on the crab-angle in problem 2.4.

Since it is assumed that the speed given in (1) is "speed over ground", the speed without current will be constant and equal to the one found in task 2.4. This is shown in Figure 6a. When current is applied, this result in oscillations in the speed over ground, due to the ship moving in circles. This is as expected.

As shown in Figure 6b, the frequency of the oscillations will change after 700 seconds, due to the increase in rudder angle.

In Figure 8 the relative velocities of the system is plotted. They are oscillating as expected, with a sudden increase in the frequency after 700 seconds. This is due to the fact that the increase in rudder decreases the radius of the circle, again decreasing the period the vehicle use for one rotation.

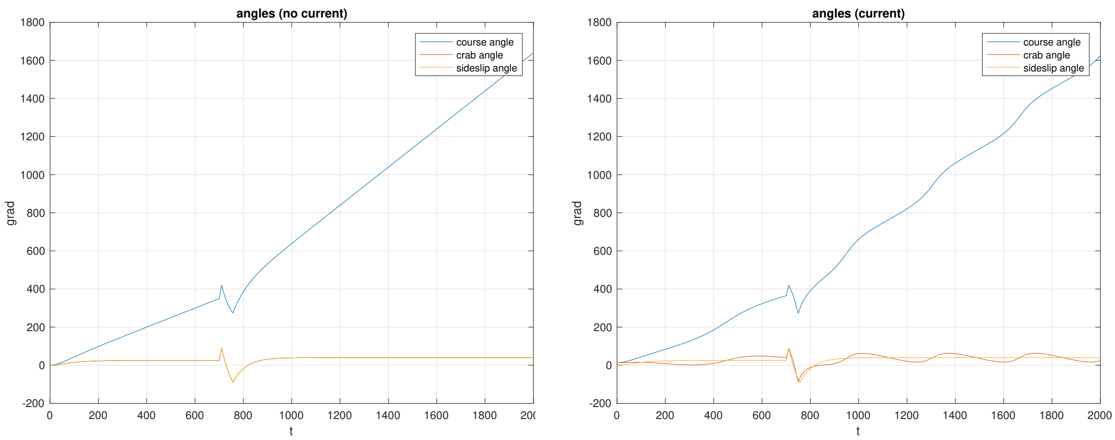
It is interesting that the ship in Figure 4 moves in an expanding circle right after the rudder angle is changed. This happens both at $t = 0s$ and $t = 700s$, although it is much clearer in the figure after 700 s. This can also be seen in the velocity plots where the frequency of the oscillations in velocities and speed is much higher after 700 s than for $t \rightarrow$. This is a result of the change in radius of the trajectory. The reason why the trajectory is like this, is most likely due to the modelling of the speed $\boldsymbol{\nu}_{b/c}^{n}$ which causes dramatic change in radius when r is changing, but as r goes to steady state, so does the radius of the trajectory.



(a) Without current

(b) With current

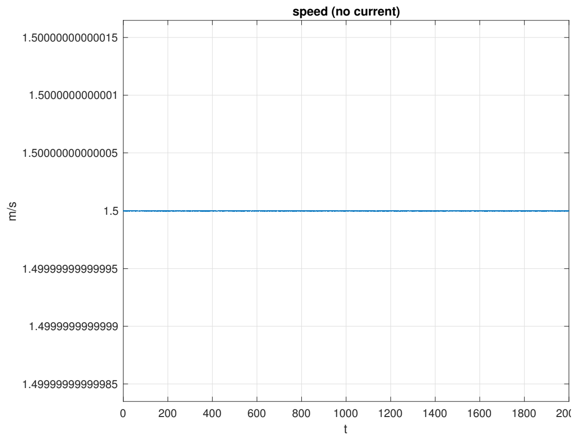
Figure 4: Position of vehicle



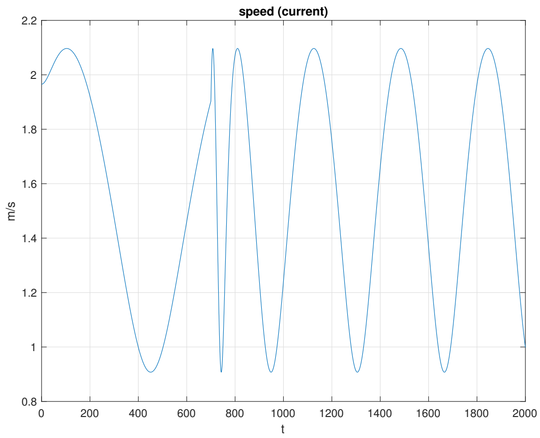
(a) without current

(b) with current

Figure 5: Sideslip, course and crab angle of the vehicle



(a) without current



(b) with current

Figure 6: speed of the vehicle relativ to ground

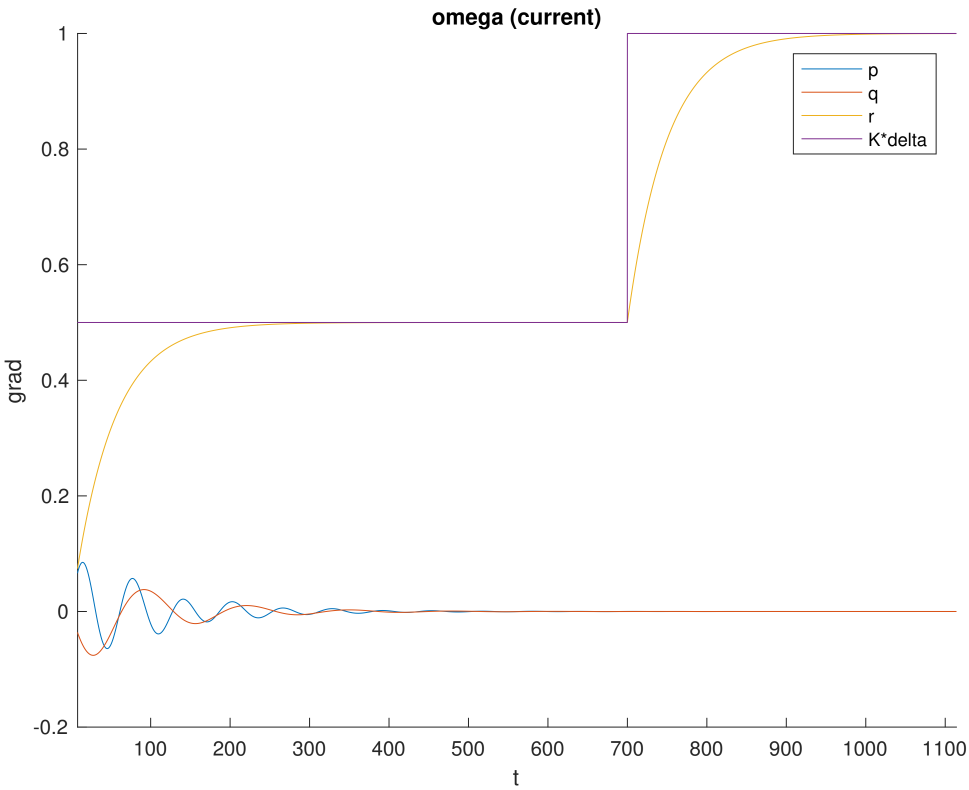


Figure 7: Illustration of p, q and r approaching steady-state

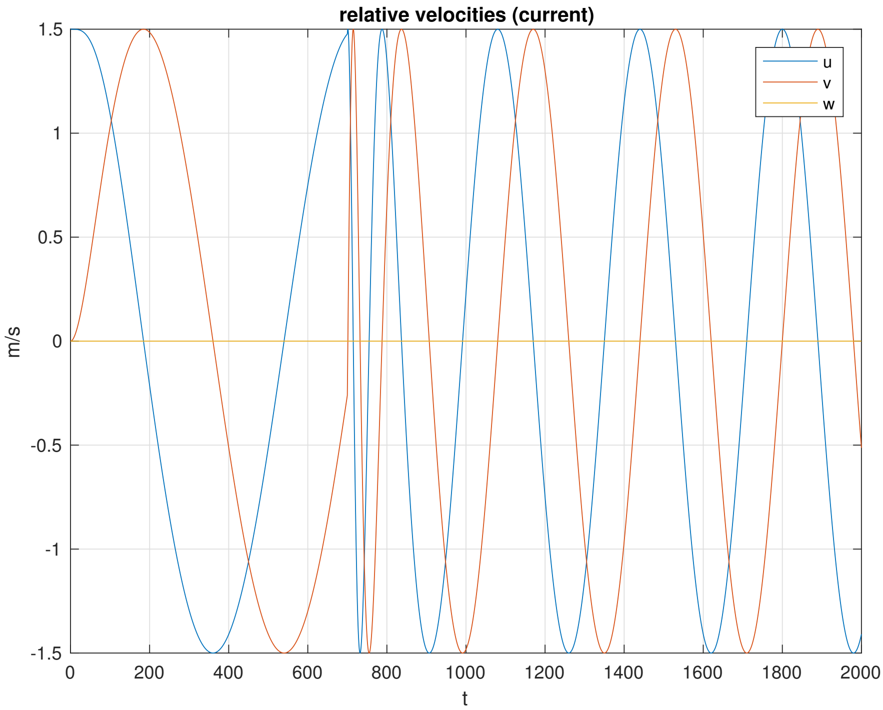


Figure 8: velocities with current

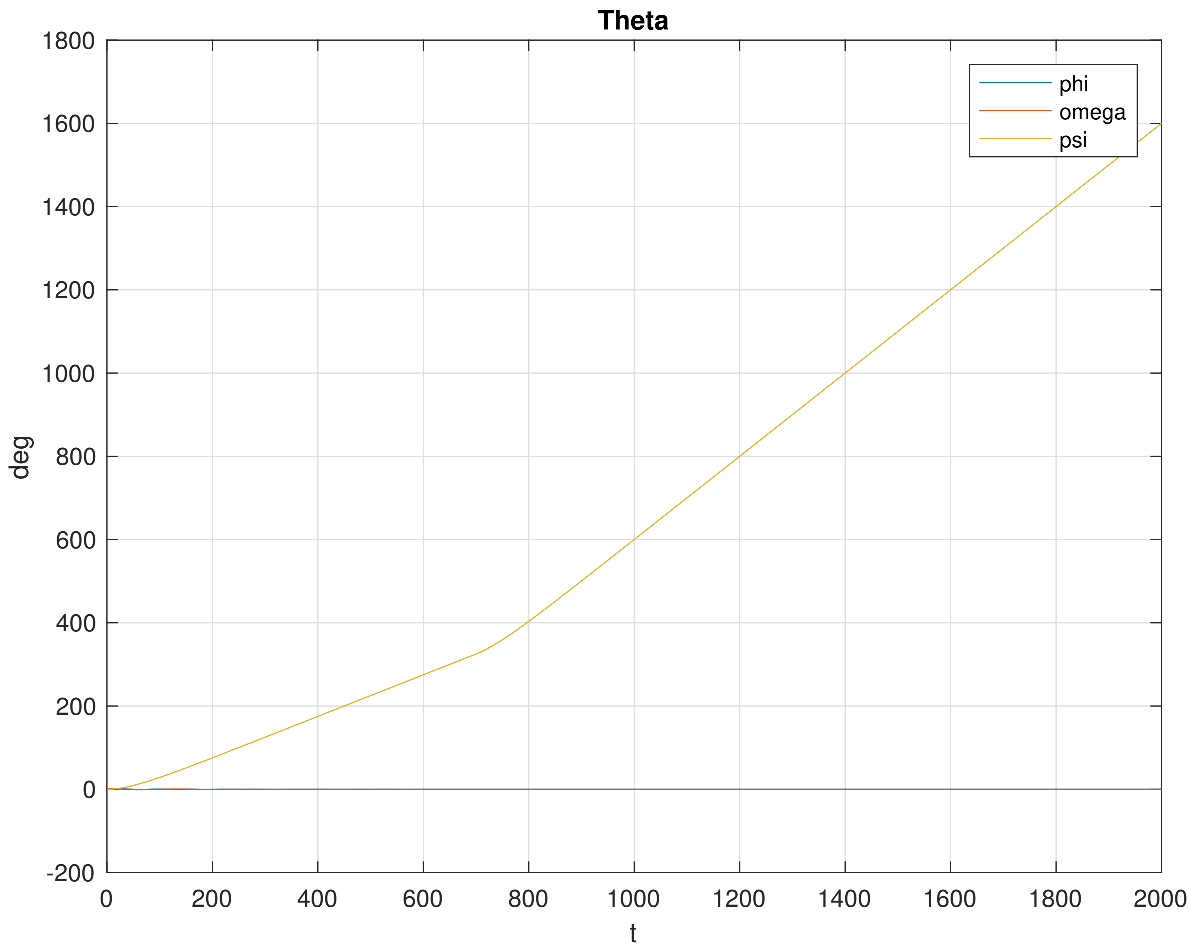


Figure 9: BODY fixed velocities

References

[1] T. Fossen, *Handbook of Marine Craft Hydrodynamics and Motion Control.* John Wiley & Sons, 2011.

[2] O.-E. Fjellstad and T. I. Fossen, "Quaternion feedback regulation of underwater vehicles," *Proceedings of the IEEE Conference on Control Applications*, vol. 2, pp. 857–862, 1994.