DEBRE MARKOS UNIVERSITY

INSTITUTE OF TECHNOLOGY

Department of Software Engineering

Course Name: computer graphics
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Topic: Perspective Projection with Mathematical Description

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NO.	NAME	ID No.
1	Akrem Nasir	1302906
2	Henok Ayalew	1304800
3	Marishet Begashew	1304173
4	Tadesse Belachew	1308576

Instructor: Yohannes D.
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1. Introduction

In the realm of computer graphics, the representation of three-dimensional (3D) objects on two-dimensional (2D) screens is not only a fundamental requirement but also a complex challenge. Devices such as monitors, smartphones, and virtual reality headsets are inherently two-dimensional display systems. Yet, the real world that we perceive and interact with exists in three dimensions. Bridging this gap between 3D data and 2D display is one of the core objectives of computer graphics. This is where the concept of projection becomes crucial.

Projection in computer graphics refers to the method used to map 3D points from a scene onto a 2D viewing surface, such as a computer screen or a printed image. This transformation is necessary because, while 3D models are created and manipulated in three-dimensional space, they must ultimately be rendered in two dimensions for human observation.

Among the various types of projections used in computer graphics, perspective projection is particularly significant. This is because it closely mimics the way the human visual system perceives the world. When we look at objects around us, we observe that they appear smaller as they move farther away and larger as they come closer. This natural behavior is captured effectively through perspective projection, making it essential for creating realistic and immersive visual experiences in digital environments.

Furthermore, perspective projection plays a critical role in enhancing the believability of 3D scenes, whether in movies, virtual simulations, architectural visualizations, or video games. Without perspective projection, scenes would appear flat and unrealistic, stripping away the depth that gives users a sense of space and position.

This document aims to provide a comprehensive overview of perspective projection, including its theoretical foundation, mathematical formulation, and practical applications. By understanding the underlying principles and mathematical operations involved in perspective projection, we can better appreciate its role in producing visually compelling and accurate computer-generated imagery. Additionally, this knowledge helps computer graphics practitioners make informed decisions about when and how to use different types of projections depending on the nature of the scene and the desired visual outcome.

2.Overview of Projection in Computer Graphics

Projection in computer graphics refers to the process of transforming 3D objects into a 2D view. There are two main types of projections:

- **Orthographic Projection:** This is a type of parallel projection where the projection lines are orthogonal to the projection plane. It does not account for depth, meaning objects appear the same size regardless of their distance from the camera.

- **Perspective Projection:** This projection mimics the human eye. Objects appear smaller as they get farther from the viewer, creating a sense of depth and realism.

Orthographic projection is commonly used in engineering drawings and CAD applications where accurate measurements and true scale representations are more important than realism. Since all projection lines are parallel and perpendicular to the projection plane, there is no foreshortening, and objects retain their actual dimensions regardless of their position in space.

On the other hand, perspective projection introduces depth cues by using converging lines. This technique produces images where distant objects appear smaller, effectively simulating how humans perceive the environment. It is used extensively in gaming, animation, virtual reality, and any application where the goal is to produce a lifelike representation of a 3D scene.

Understanding the fundamental differences between these two projection types is essential for selecting the appropriate method based on the specific requirements of a graphical application. While orthographic projection offers precision and simplicity, perspective projection delivers visual realism and immersion.

3. What is Perspective Projection?

Perspective projection is a technique in computer graphics that transforms 3D points into a 2D representation in such a way that it simulates the appearance of depth and distance as perceived by the human eye. Unlike orthographic projection, where objects retain their size regardless of position, perspective projection makes distant objects appear smaller and closer objects appear larger, giving a realistic illusion of depth.

This type of projection is inspired by the principles of linear perspective, a method used by artists since the Renaissance to depict spatial depth on flat surfaces. It relies on the concept of a "vanishing point" a point in the image where parallel lines appear to converge. For example, railway tracks seem to meet at a point on the horizon when viewed from a distance. Perspective projection mathematically reproduces this effect in a virtual 3D scene.

The main idea behind perspective projection is to define a "camera" or "eye" point (also called the center of projection) and a projection plane (like the screen). Rays are drawn from the 3D object points to the camera, and where these rays intersect the projection plane, we get the 2D coordinates that are used to render the image.

In a simple form, consider a point P in 3D space with coordinates (X, Y, Z). The projection of this point onto a 2D plane (usually the Z = 1 plane) from a viewpoint at the origin (0, 0, 0) is given by:

$$x = X / Z$$
$$y = Y / Z$$

Here, (x, y) are the coordinates of the projected point on the 2D screen. This formula demonstrates that as Z increases (i.e., the point moves further from the camera), the values of x and y decrease, making the point appear smaller — just as in human vision.

Perspective projection is fundamental to 3D graphics rendering and is used in every major graphics engine and API (like OpenGL, DirectX, and Unity). Without it, scenes would lack the visual realism necessary for immersive user experiences.

In summary, perspective projection is the cornerstone of realistic rendering in computer graphics, making virtual worlds look natural and believable to the human eye. It plays a vital role in various domains such as entertainment, education, architecture, and simulation technologies.

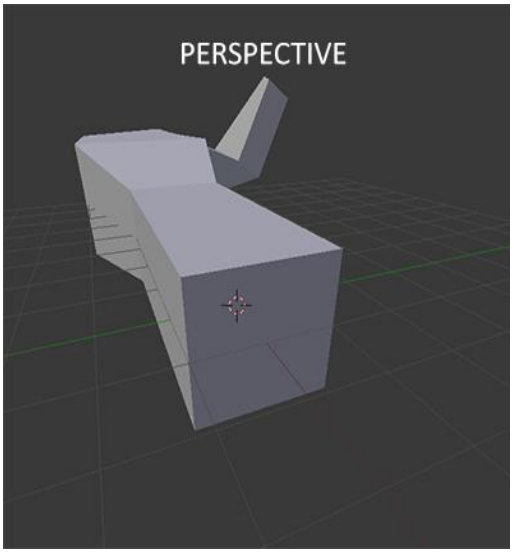

Figure 1 Perspective projection

4. Mathematical Description of Perspective Projection

To understand perspective projection thoroughly, we need to describe it both geometrically and algebraically using linear algebra and transformation matrices.

4.1 Geometric Derivation

Assume a 3D point **P** with coordinates (X, Y, Z) in camera space, and we want to project it onto a 2D screen (the projection plane) located at a distance **d** along the Z-axis from the origin (camera).then the 2D coordinates (x' , y') on the screen are calculated as:

$$x' = \frac{d.X}{Z} \text{ and } y' = \frac{d.Y}{Z}$$

Example:

Let's assume a point P(10,20,50) and the projection plane is at z=10.

$$x' = \frac{d.X}{Z} \text{ and } y' = \frac{d.Y}{Z}$$

$$x' = \frac{10.10}{50} = 2 \text{ and } y' = \frac{10.20}{50} = 4$$

So, the projected point on the 2D view plane will be(2, 4).

4.2 Homogeneous Coordinates and Matrix Representation

To enable transformations using matrix multiplication, we use **homogeneous coordinates**. A 3D point (X, Y, Z) becomes (X, Y, Z, 1). The perspective projection matrix for a simple pinhole camera model is:

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{d} & 0 \end{bmatrix}$$

Multiplying this with the homogeneous coordinate vector $\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$ gives $\begin{bmatrix} X \\ Y \\ Z \\ \frac{Z}{d} \end{bmatrix}$

To get the final 2D coordinates, we divide each of the first three components by the fourth (homogeneous) component:

$$x' = \frac{X}{Z/d} = \frac{d.X}{Z} \text{ and } y' = \frac{Y}{Z/d} = \frac{d.Y}{Z}$$

Example:

Use the same point P(10,20,50) and d=10:

$$x' = \frac{X}{Z/d} = \frac{d.X}{Z} \text{ and } y' = \frac{Y}{Z/d} = \frac{d.Y}{Z}$$

$$x' = \frac{10.10}{50} = 2 \text{ and } y' = \frac{10.20}{50} = 4$$

Again, the result is (2,4)

This confirms that the matrix-based derivation yields the same result as the geometric method.

4.3 General Perspective Projection Matrix

For real-world applications such as OpenGL or DirectX, the perspective projection matrix also accounts for the **field of view (FOV)**, **aspect ratio**, and the **near and far clipping planes**. A generalized 4×4 perspective projection matrix looks like:

$$\text{PerspectiveMatrix} = \begin{bmatrix} \frac{1}{\alpha.\tan(\theta/2)} & 0 & 0 & 0 \\ 0 & \frac{1}{\tan(\theta/2)} & 0 & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & -\frac{2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Where:

- ✓ θ is the vertical field of view (in radians)
- ✓ α is the aspect ratio (width / height)
- ✓ n is the near clipping plane
- ✓ f is the far clipping plane

This matrix is crucial for rendering scenes in real-time 3D engines. It transforms a view frustum (a pyramid-shaped viewing volume) into a canonical cube that graphics hardware can efficiently rasterize.

Example:

Given:

- ✓ $\theta = \pi/2 \rightarrow \tan(\theta/2) = \tan(\pi/4) = 1$
- ✓ $\alpha = 16/9$
- ✓ $n = 1.0$
- ✓ $f = 100.0$

We use the standard perspective projection matrix formula:

$$P = \begin{bmatrix} \frac{1}{\alpha.\tan(\theta/2)} & 0 & 0 & 0 \\ 0 & \frac{1}{\tan(\theta/2)} & 0 & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & -\frac{2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Now plug in the values:

- ✓ $\tan(\theta/2)=\tan(\pi/4) = 1$
- ✓ $\alpha = \frac{16}{9} \Rightarrow \frac{1}{\alpha.1} = \frac{9}{16}$
- ✓ $\frac{1}{tan(\theta/2)} = 1$
- ✓ $-\frac{f+n}{f-n} = -\frac{101}{99}$
- ✓ $-\frac{2fn}{f-n} = -\frac{200}{99}$

So the matrix becomes:

$$P=\begin{bmatrix} \frac{9}{16} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{101}{99} & -\frac{200}{99} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

4.4 Key Insights

- Perspective projection causes parallel lines to converge to a vanishing point.
- Objects farther from the camera appear smaller.
- It uses depth (Z value) to scale the x and y coordinates.
- Homogeneous coordinates and 4×4 matrices make complex transformations like rotation, scaling, translation, and projection possible in a unified framework.

4.5 Practical Use in Graphics Pipelines

In modern graphics pipelines, perspective projection is applied as part of the **model-view-projection (MVP)** transformation chain:

Clip Coordinates = Projection × View × Model × Vertex Position

Here, the projection matrix (like the one described above) is responsible for converting 3D geometry into a form suitable for 2D display while preserving depth cues.

5. Types of Perspective Projection

Classification of perspective projection is on basis of vanishing points (It is a point in image where a parallel line through center of projection intersects view plane.). We can say that a vanishing point is a point where projection line intersects view plane. The classification is as follows :

5.1 One-Point Perspective

One point perspective projection occurs when any of principal axes intersects with projection plane or we can say when projection plane is perpendicular to principal axis. Common in architectural drawings and interior designs, one-point perspective is simple and effective for frontal views.

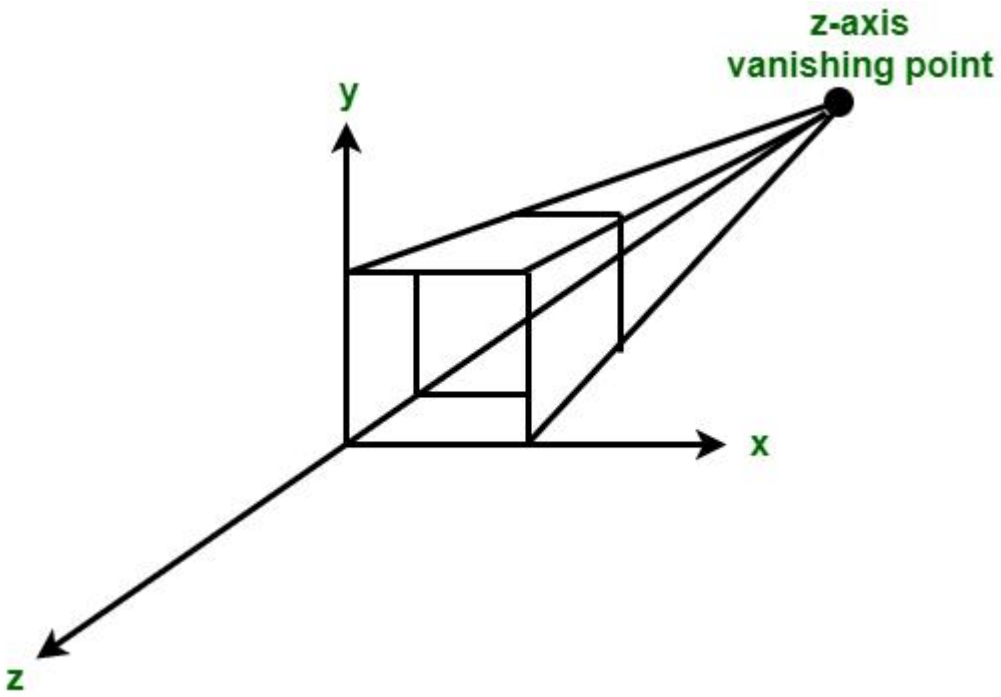

Figure 2 one point perspective

5.2 Two-Point Perspective

In this type, two vanishing points are placed on the horizon line. It is ideal for visualizing objects at an angle rather than directly facing the viewer. This type of projection is widely used in 3D modeling and rendering to depict realistic structures and environments.

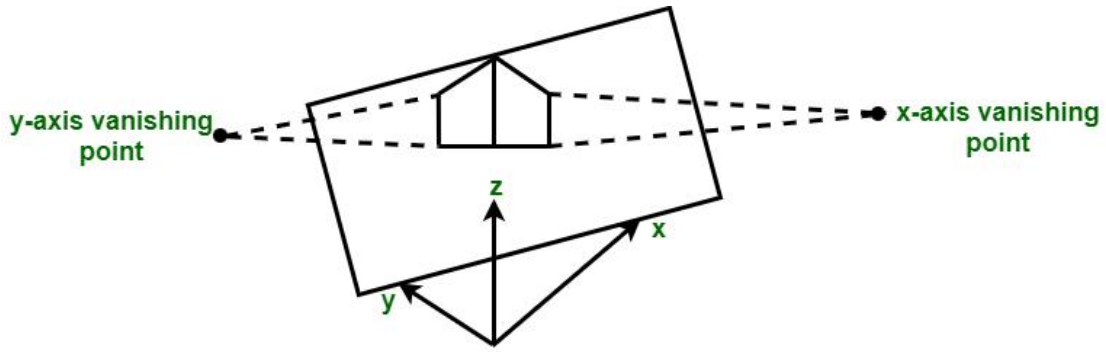

Figure 3 two point perspective

5.3 Three-Point Perspective

Three-point perspective introduces a third vanishing point, either above or below the horizon line. This type is used to depict scenes viewed from high above or far below (such as tall buildings). It adds dramatic depth and is common in dynamic visualizations like animations and cinematics.

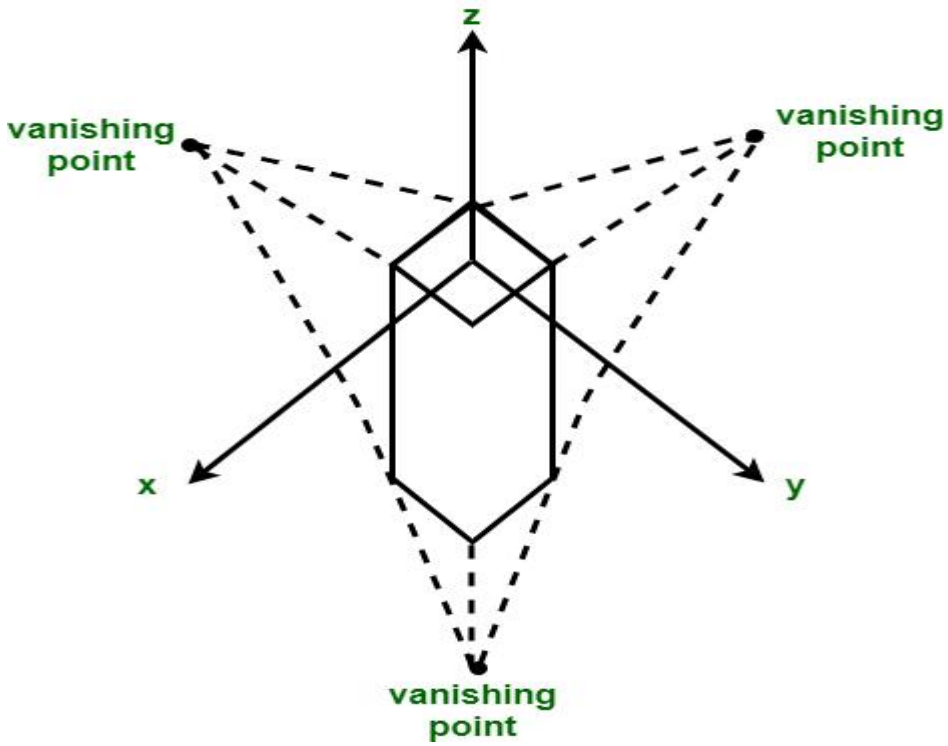

Figure 4 three point perspective

6. Applications of Perspective Projection

Perspective projection is widely applied in various fields where the visualization of 3D scenes on 2D media is essential. Here are some of the key applications:

6.1 Video Games and Simulations

Most modern video games rely on perspective projection to create immersive environments. It helps simulate how objects shrink with distance, improving realism in 3D environments. Flight simulators, driving games, and VR-based training systems all use perspective projection extensively.

6.2 Architectural Visualization

Architects use perspective projections to represent how buildings and spaces will appear in reality. Renderings using three-point or two-point perspectives give clients a realistic preview of future construction.

6.3 Animation and Film

In film production and animation, perspective projection is vital for producing believable scenes. It ensures that characters, objects, and environments interact in a spatially consistent way.

6.4 Computer-Aided Design (CAD)

Though CAD often uses orthographic views for technical precision, perspective projection is used to present 3D designs to clients or stakeholders in a more realistic and understandable manner.

6.5 Augmented and Virtual Reality

AR and VR systems depend heavily on accurate perspective projection to align virtual objects with the real world and maintain immersion by simulating real-world depth.

7. Conclusion

Perspective projection stands at the heart of realistic computer graphics rendering. It bridges the gap between how we perceive the world and how computers display it on a screen. By using mathematical models and transformation matrices, perspective projection effectively simulates depth and distance, making 3D scenes appear lifelike on 2D displays.

Understanding its various types, mathematical formulations, and applications provides valuable insight into its importance across fields such as gaming, architecture, and virtual reality. As technologies continue to evolve, the use of perspective projection will remain a cornerstone in the creation of visually rich and spatially accurate digital experiences.

8. References

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