ANOVA:
Two-way ANOVA and more

Research Methods for Human Inquiry

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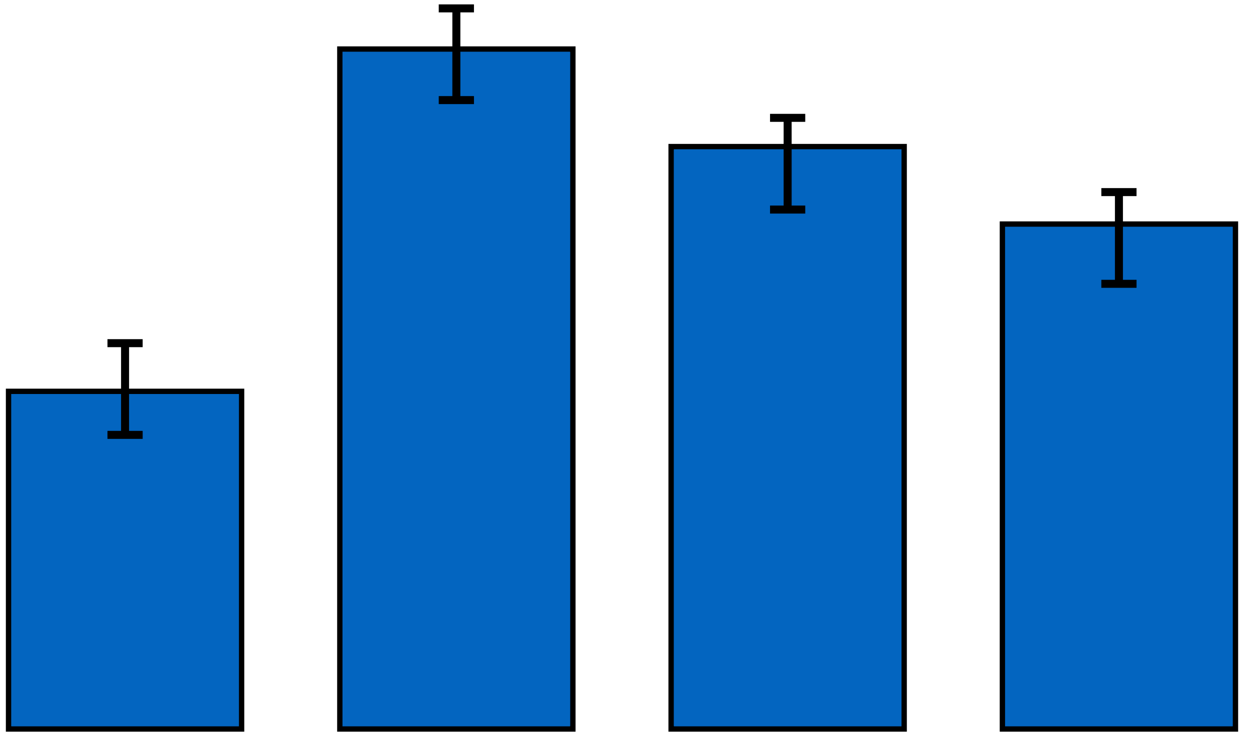
So far

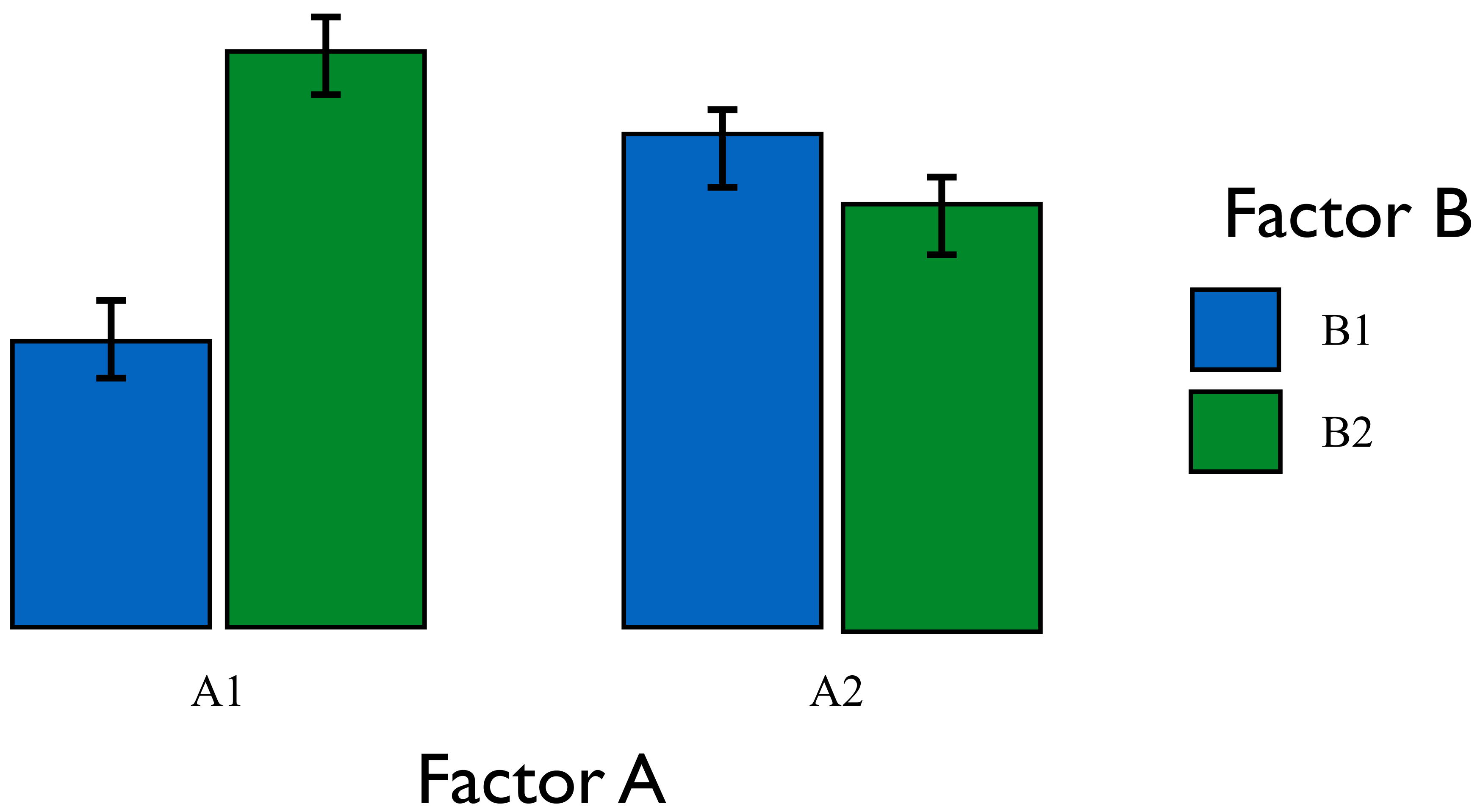
What if we have more than two means?

Answer: Analysis of Variance (ANOVA)



But what if there is more than one grouping variable (factor)?

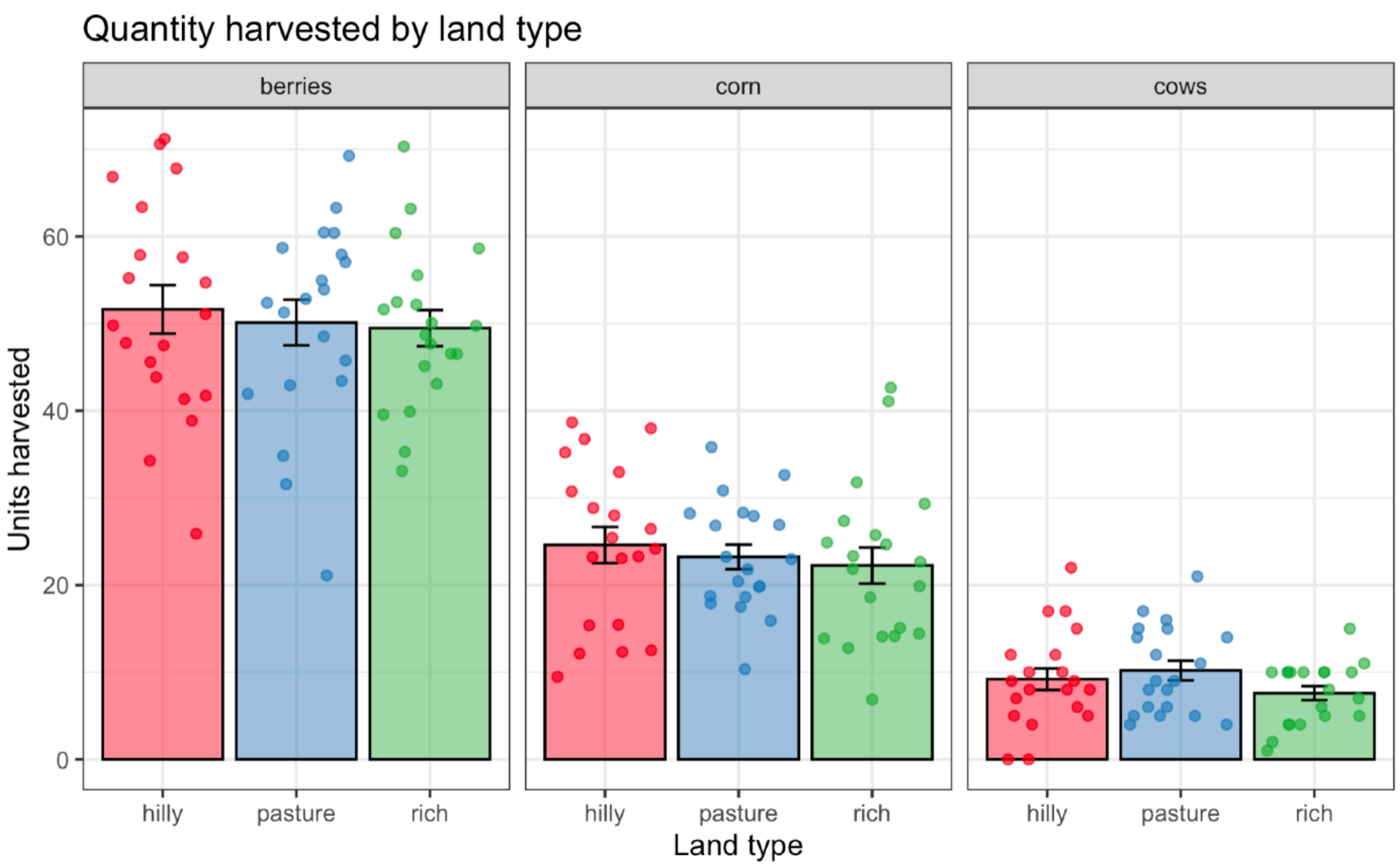
Answer: Two-way ANOVA



Note: Only applies to balanced designs that have the same # of datapoint in each cell

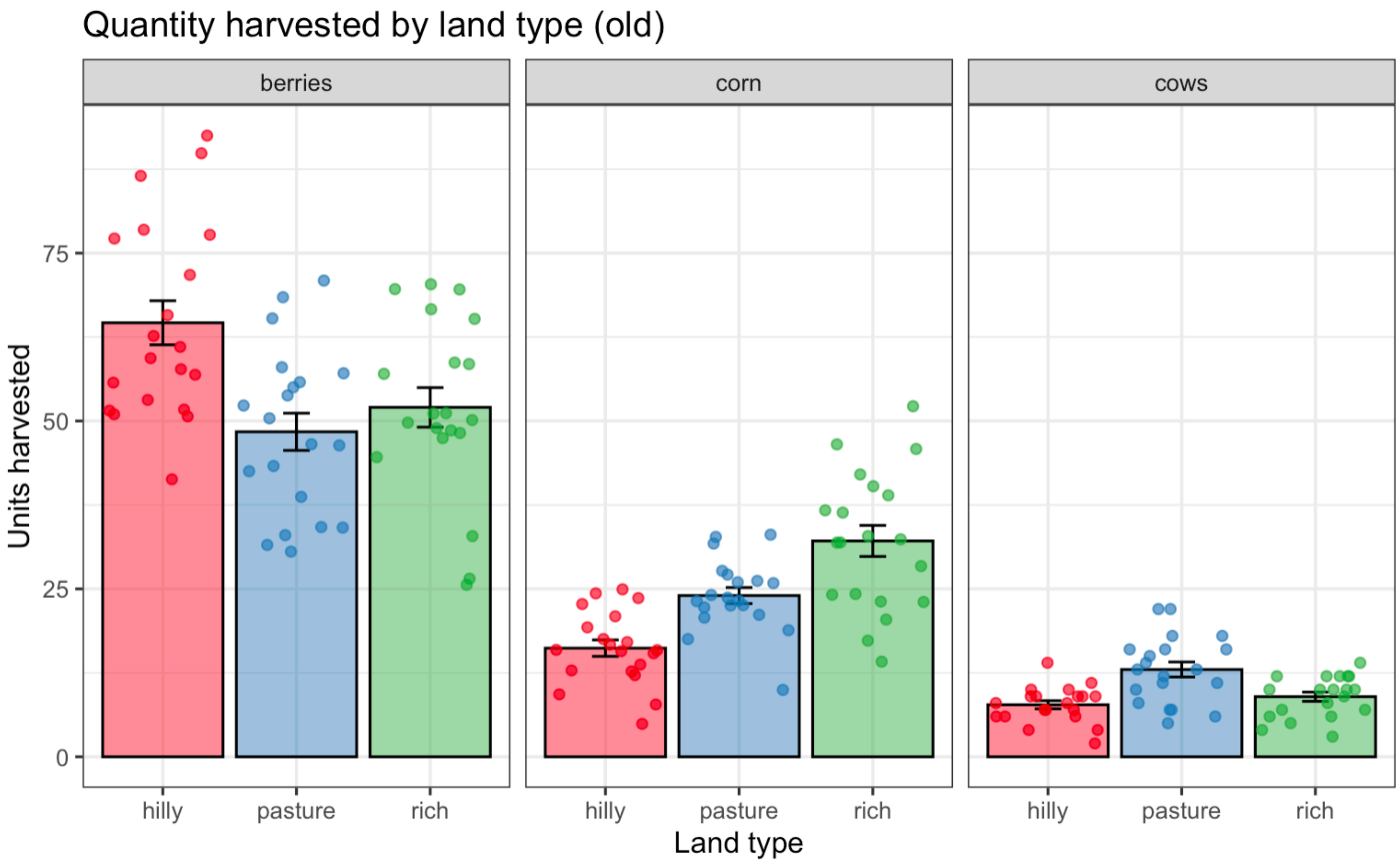
But what if there is more than one grouping variable (factor)?

Factors: food *and* land (new)



But what if there is more than one grouping variable (factor)?

Factors: food *and* land (old)



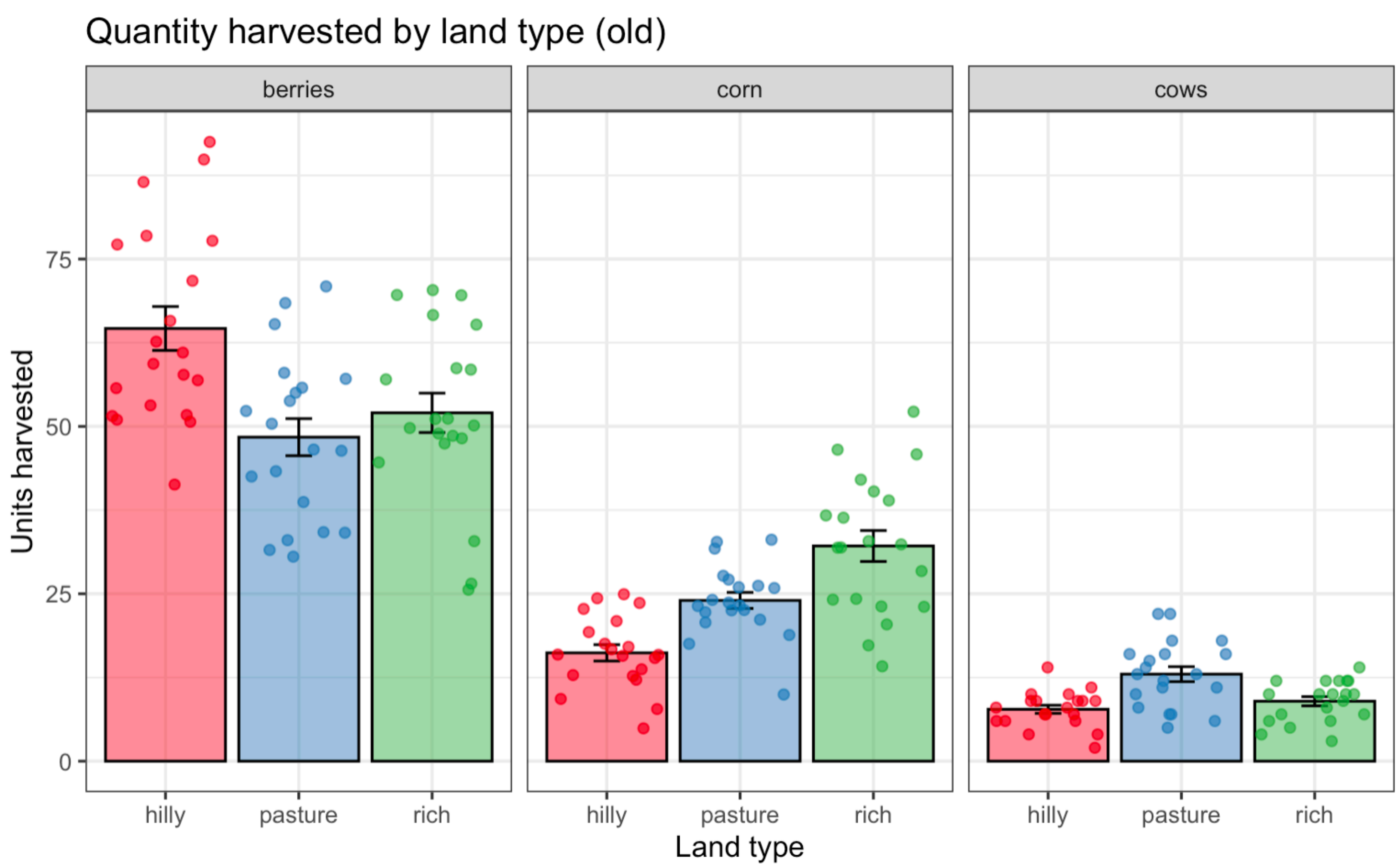
As before, let's have two separate tibbles - but they need to be in long form

- Called `do_long` and `dn_long`
- Each has five variables:
  - `plot`... code uniquely identifying each plot of land
  - `type`... type of land: pasture, rich, or hilly
  - `food`... which food it is (corn, cows, berries)
  - `quantity`... units of that food

```
  plot     type    food    quantity
1 9Jx21zaa pasture cows        6
2 9Jx21zaa pasture berries    58
3 9Jx21zaa pasture corn        9.96
4 Qp72PepB pasture cows       12
5 Qp72PepB pasture berries    33.0
6 Qp72PepB pasture corn       26.2
```

Let's focus just on the old data first

- Were there significantly different kinds of food on different kinds of land 15 years ago, as you'd expect if people were allocating land sensibly?

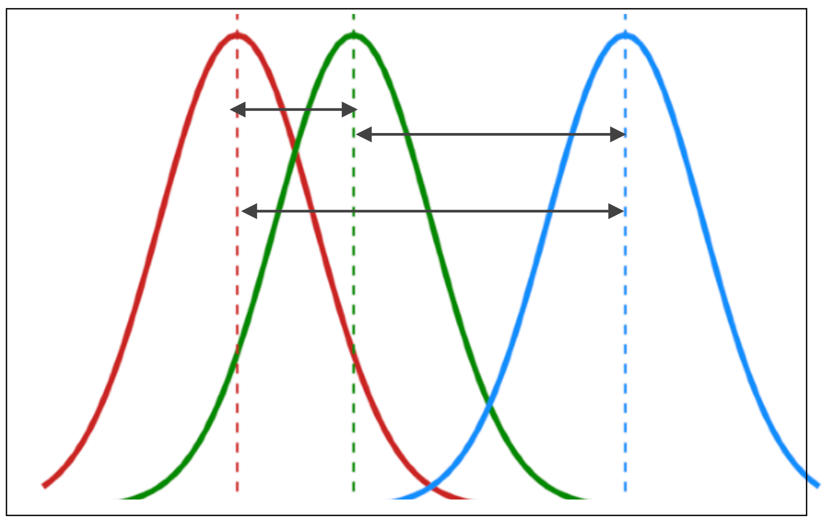


The test statistic for one-way ANOVA focused on two kinds of variability

Total variability: $SS_{tot} = SS_b + SS_w$

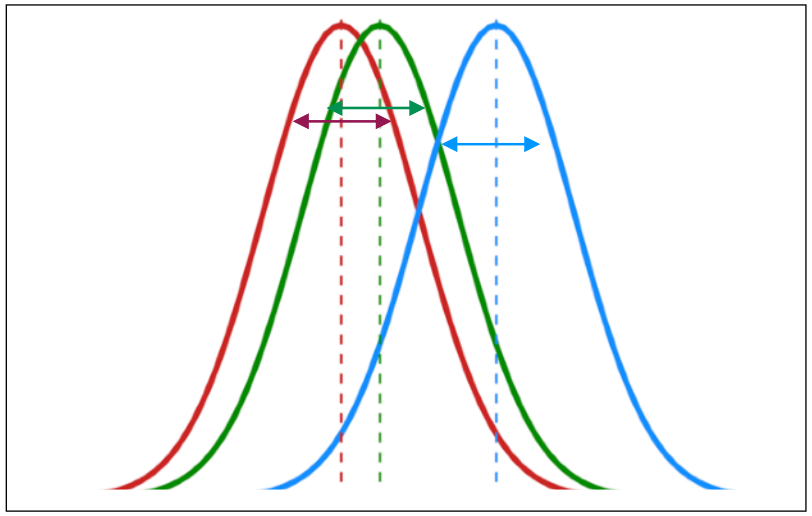
Between groups (SS_b): how different are the group means from one another?

Land type



Within groups (SS_w): how much do individuals within a group differ from the group mean?

Residuals

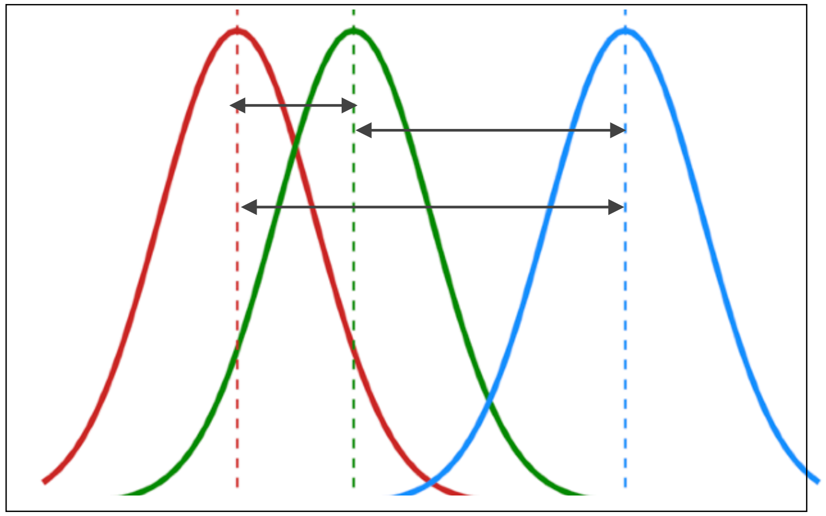


Two-way ANOVA is very similar!

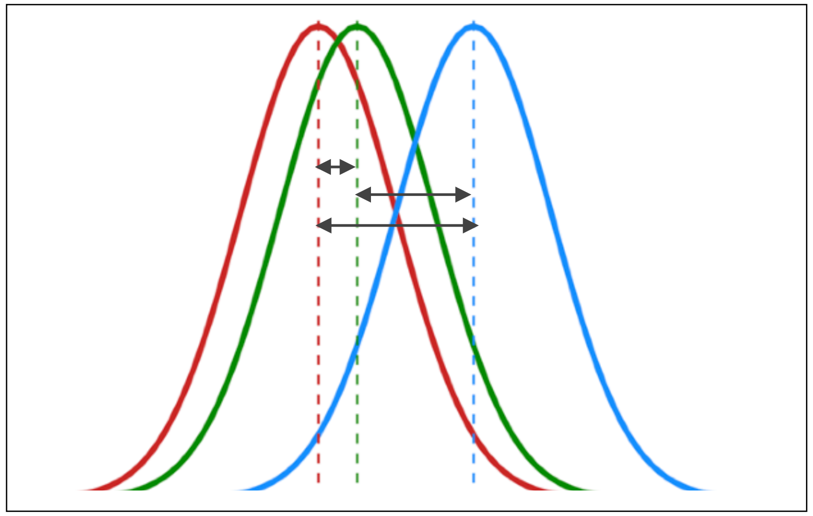
Each factor has its own between-group sum of squares

Between groups SS (SS_A, SS_B): for each factor A and B, how different are the group means from one another?

Land type

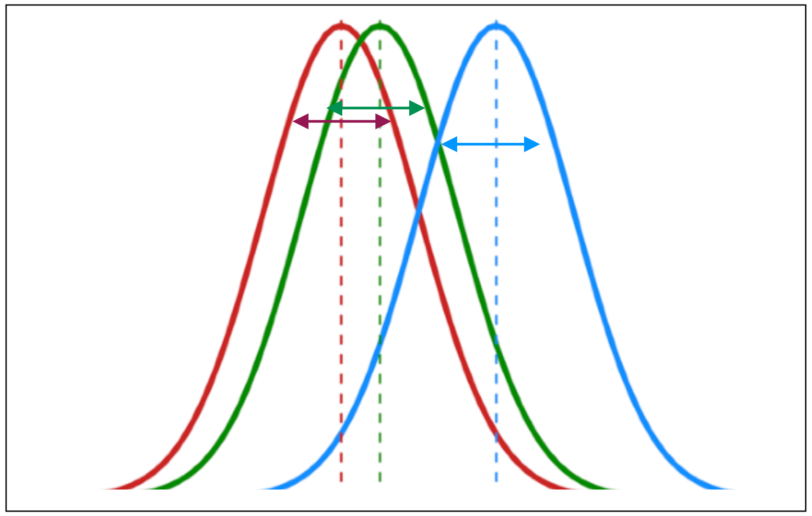


Food type



Residual SS (SS_R): how much variation in the outcome is there after taking into account the variation associated with *both* factors?

Residuals



Two-way ANOVA is very similar!

- We calculate an F-statistic for each factor in the same way:

$$MS_b = SS_b/(G\text{-}1)$$
$$MS_R = SS_R/(N\text{-}R\text{-}C+1)$$
$$F = MS_b/MS_R$$

- The rest is the same! We compare that F-statistic to the F distribution we would expect under the null, and can thus conclude for each factor whether it is significant

Running a two-way ANOVA in R

```
> foodtypeoldModel <- aov(quantity ~ type + food, data = do_long)
> foodtypeoldModel
Call:
   aov(formula = quantity ~ type + food, data = do_long)

Terms:
                    type     food Residuals
Sum of Squares    200.53 63851.67  19782.56
Deg. of Freedom        2        2       175
```

This is SS_b for land

This is $G\text{-}1$ for land

This is SS_b for food

This is $G\text{-}1$ for food

This is SS_R

This is $N\text{-}R\text{-}C\text{+}1$

Running a two-way ANOVA in R

```
> foodtypeoldModel <- aov(quantity ~ type + food, data = do_long)
> summary(foodtypeoldModel)

              Df Sum Sq Mean Sq F value Pr(>F)
type           2    201     100   0.887  0.414
food           2  63852   31926 282.422 <2e-16 ***
Residuals    175  19783     113
```

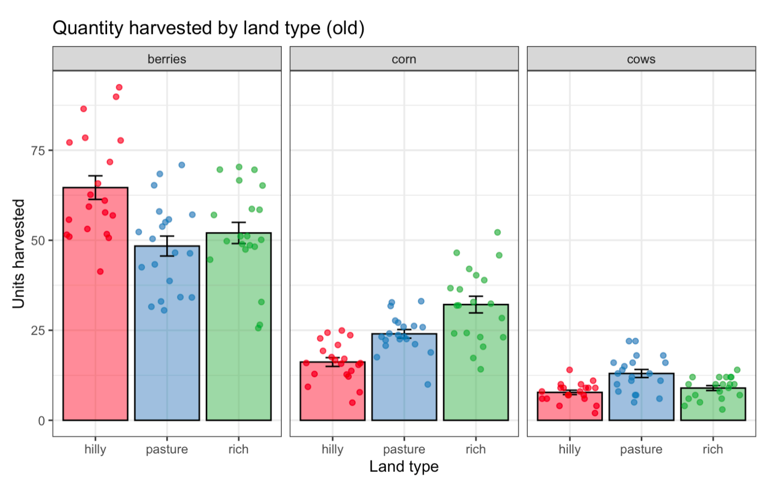
This indicates that food is significant, but land type is not

We often say that this means there is a single main effect of food

Note that this is different from what you'd get if you tested for each variable separately

Two-way ANOVA: food x land

`quantity ~ type + food`

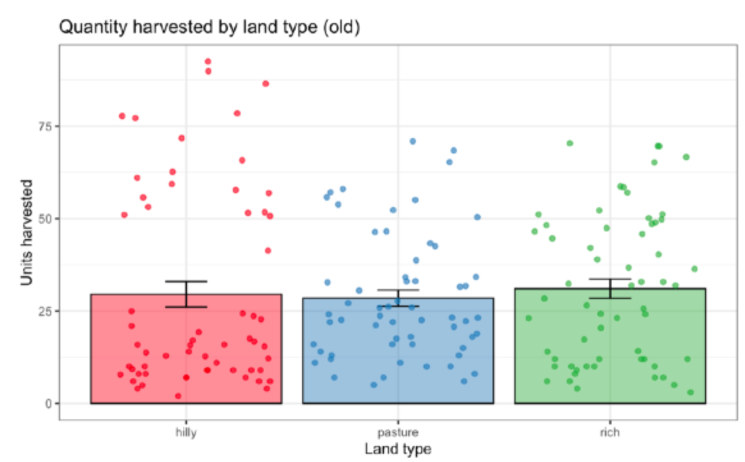


Main effect of food, $F(2,175)=282.42$, $p<.0001$

No effect of land type, $F(2,175)=0.89$, $p=.414$

One-way ANOVA: land

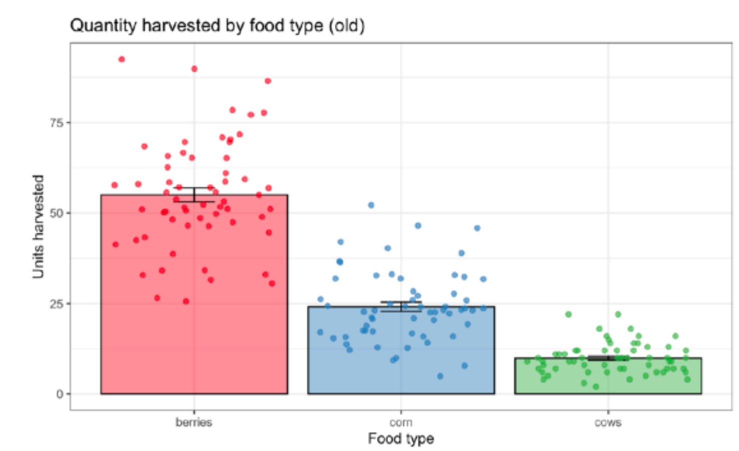
`quantity ~ type`



No effect of land type, $F(2,177)=0.21$, $p=.809$

One-way ANOVA: food

`quantity ~ food`



Significant effect of food, $F(2,177)=282.8$, $p<.0001$

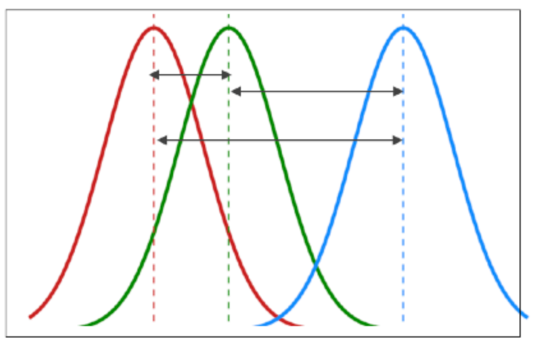
Why are the F-values different?

Note that this is different from what you'd get if you tested for each variable separately

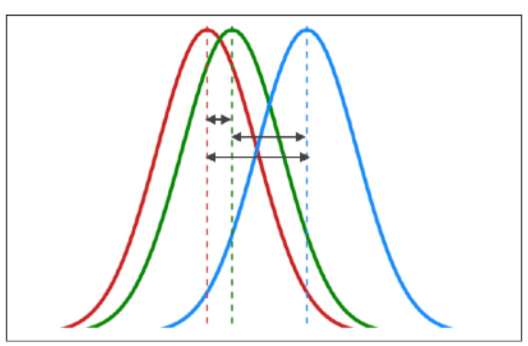
Two-way ANOVAs are different from two separate one-way ANOVAs because of the residuals!

Between groups SS (SS_A, SS_B): for each factor A and B, how different are the group means from one another?

Land type

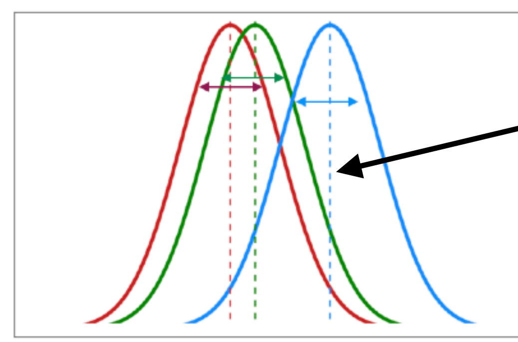


Food type



Residual SS (SS_R): how much variation in the outcome is there after taking into account the variation associated with *both* factors?

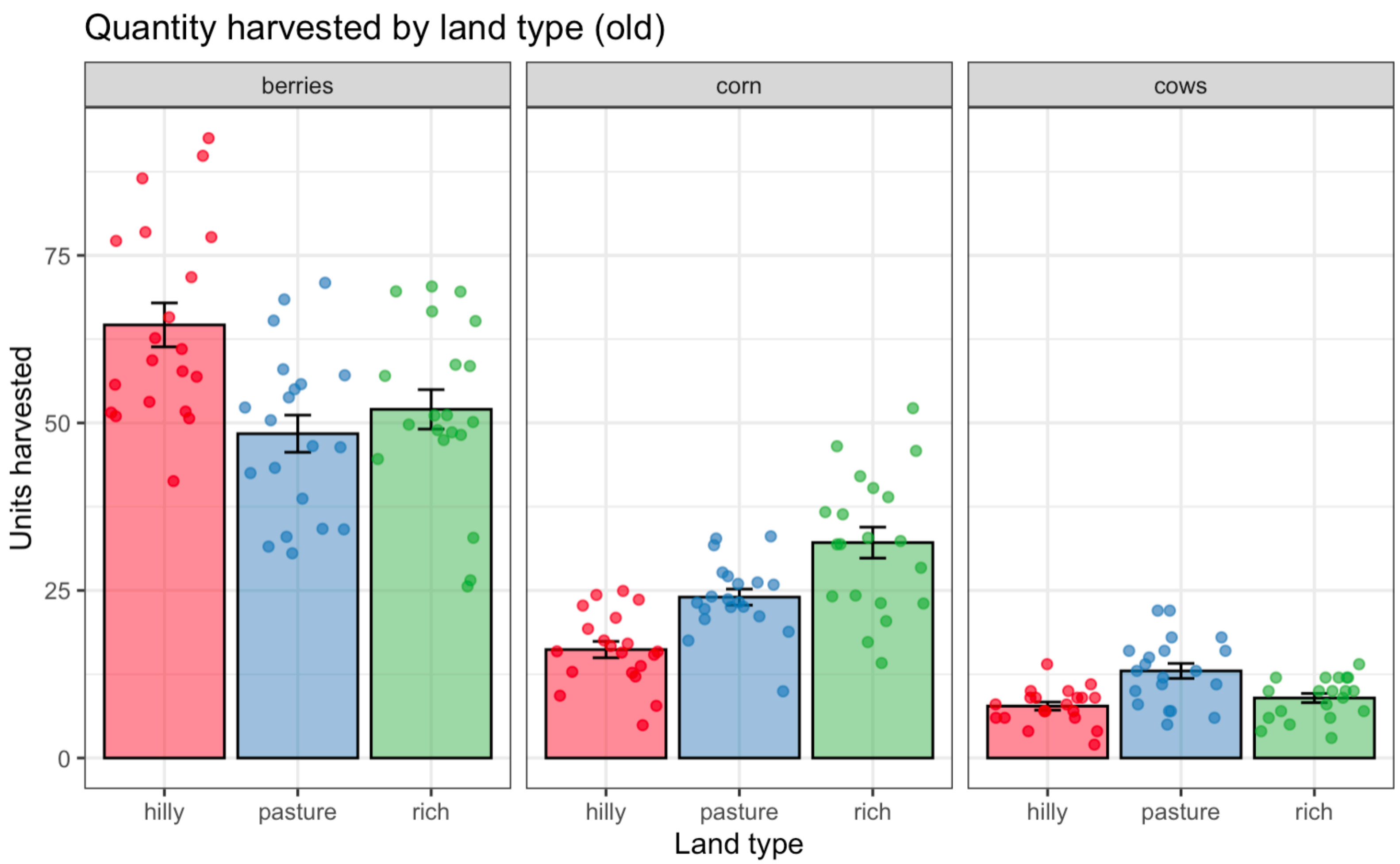
Residuals



Between-groups sum of squares are the same, but the residuals are different. They capture the leftover variance after taking into account the variation from the factors. If you change the factors — especially if you add a factor that captures meaningful variation, you change what is left over!

Moral: if you have multiple factors with meaningful variation, you want to do a two-way ANOVA and not separate one-way ANOVAs

A weird thing you might have noticed…



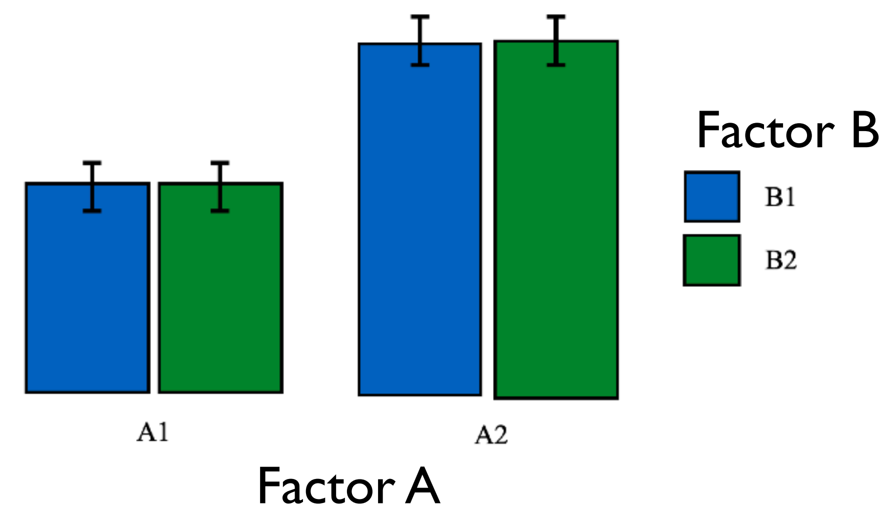
If the data look like this, isn't it odd that there's only a main effect of food and no effect of land? Especially since we found a significant effect of land when we did the one-way ANOVA on cows only?

Yes. We neglected to look for *interaction* effects. Interactions occur when the effect of one variable/factor depends on the level of another variable/factor

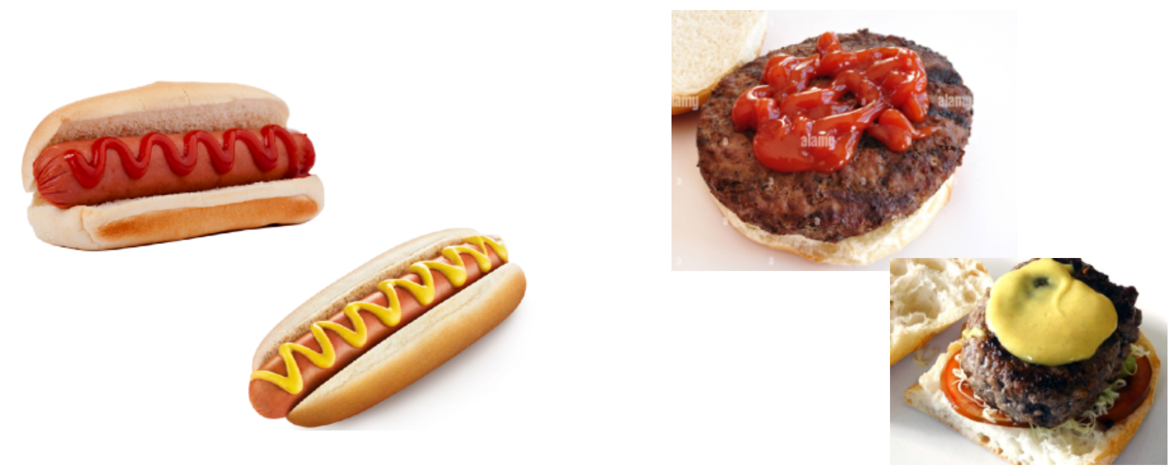
No interaction

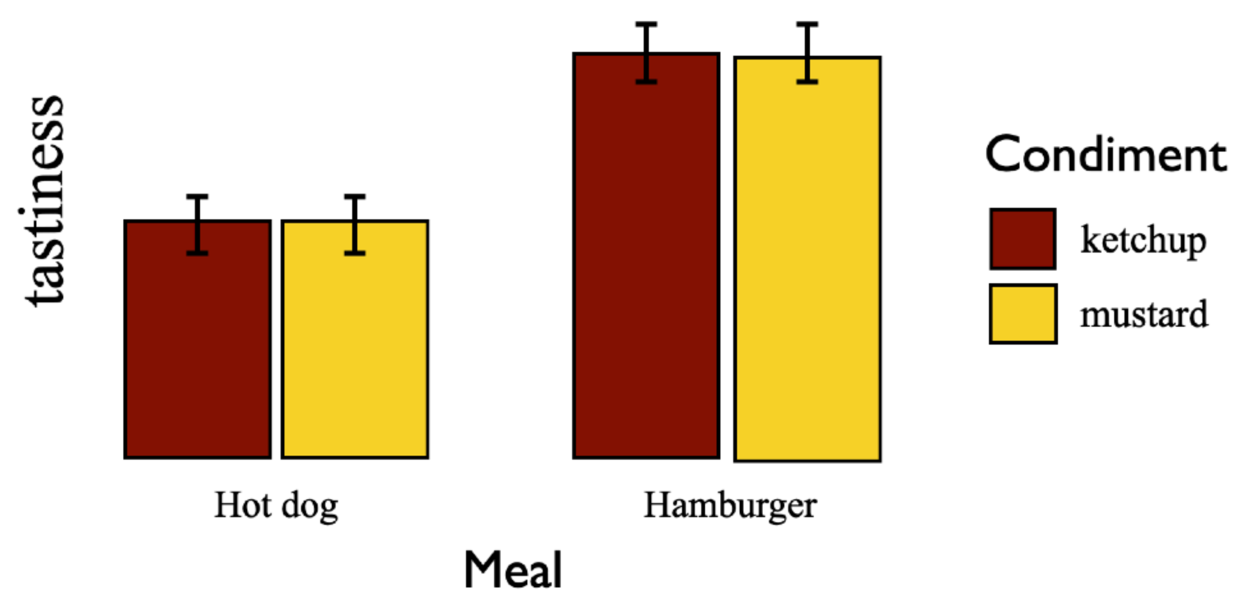
Schematic pictures of different ways of having *no* interaction in a 2x2 ANOVA

Single main effect of Factor A



Example: Gladly likes hamburgers better than hot dogs, but likes ketchup and mustard equally





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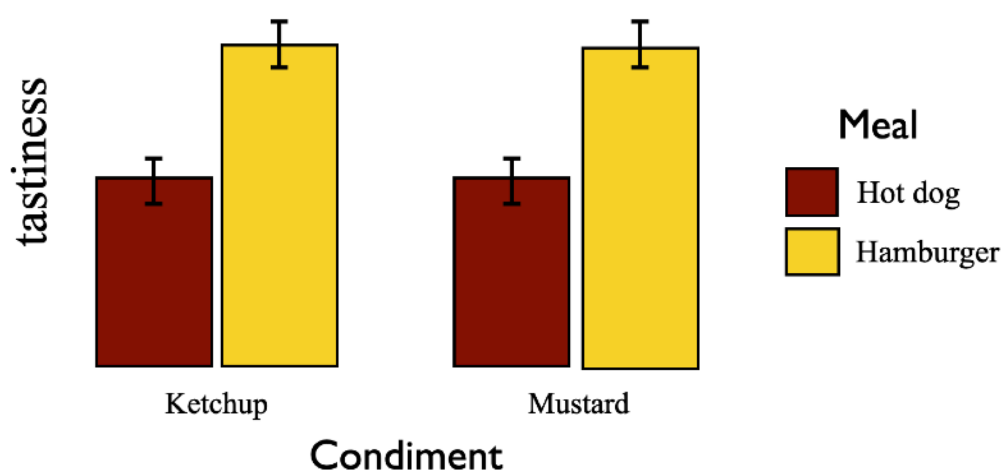
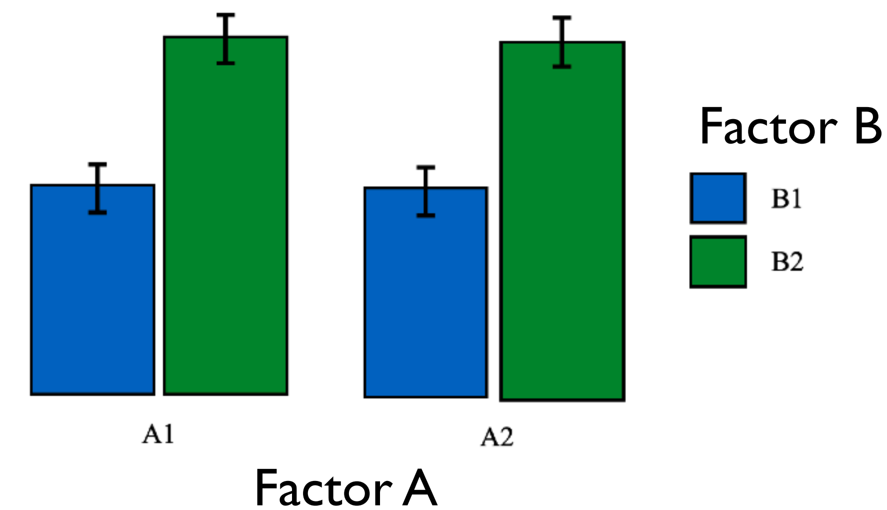
Example: Gladly likes hamburgers better than hot dogs, but likes ketchup and mustard equally



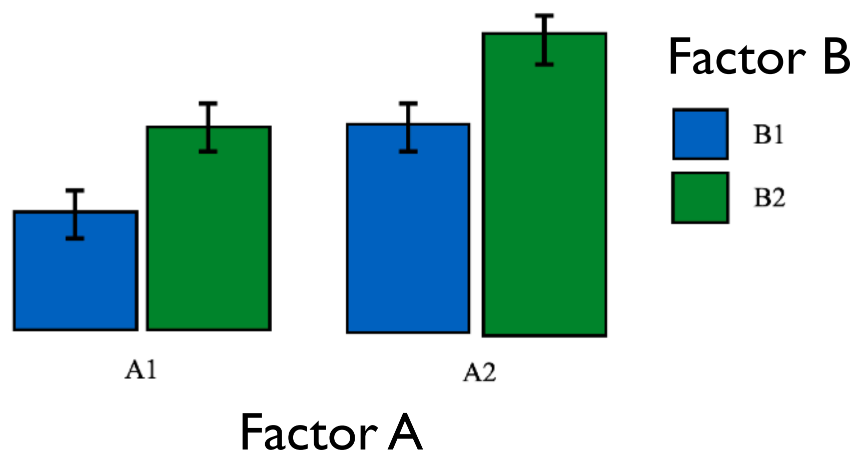
Single main effect of Factor B



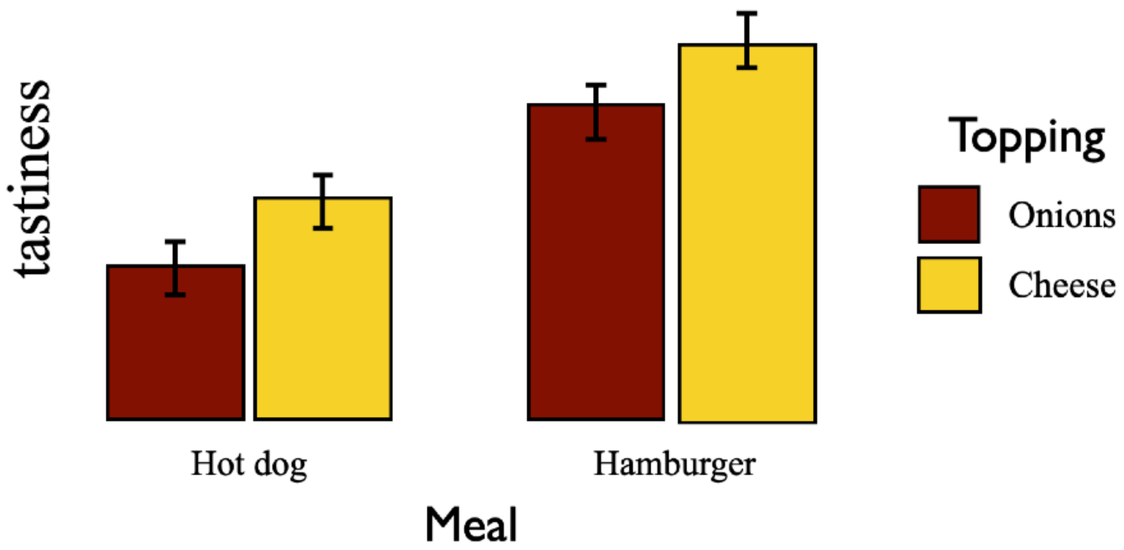
No interaction

Schematic pictures of different ways of having *no* interaction in a 2x2 ANOVA

Main effects of both A and B

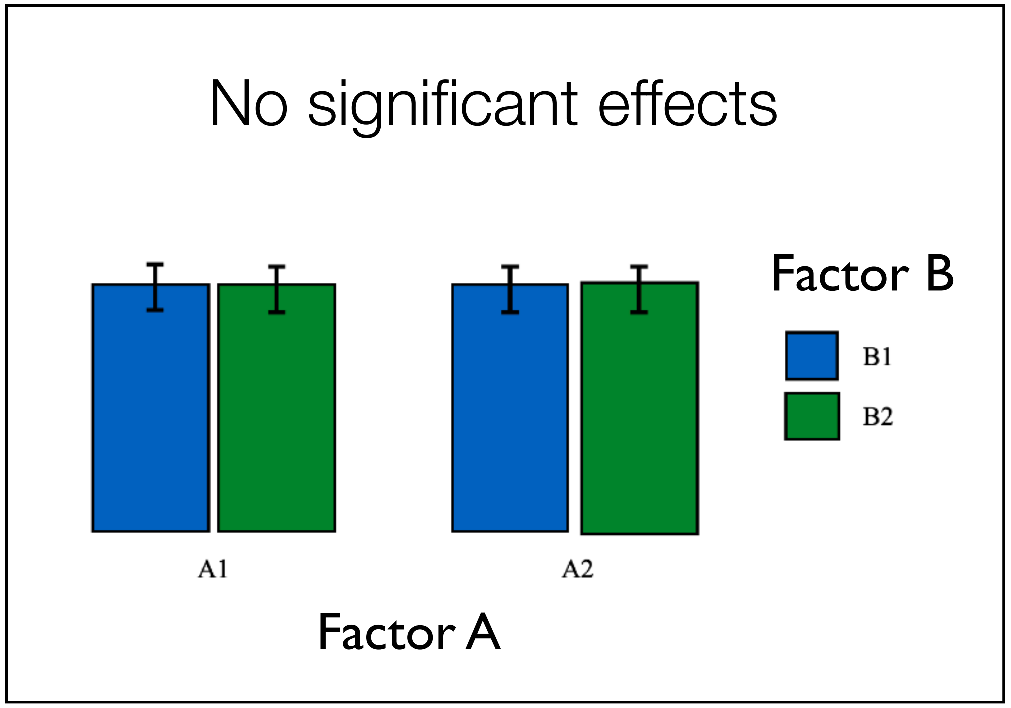


Example: Gladly likes hamburgers better than hot dogs, and cheese better than onions

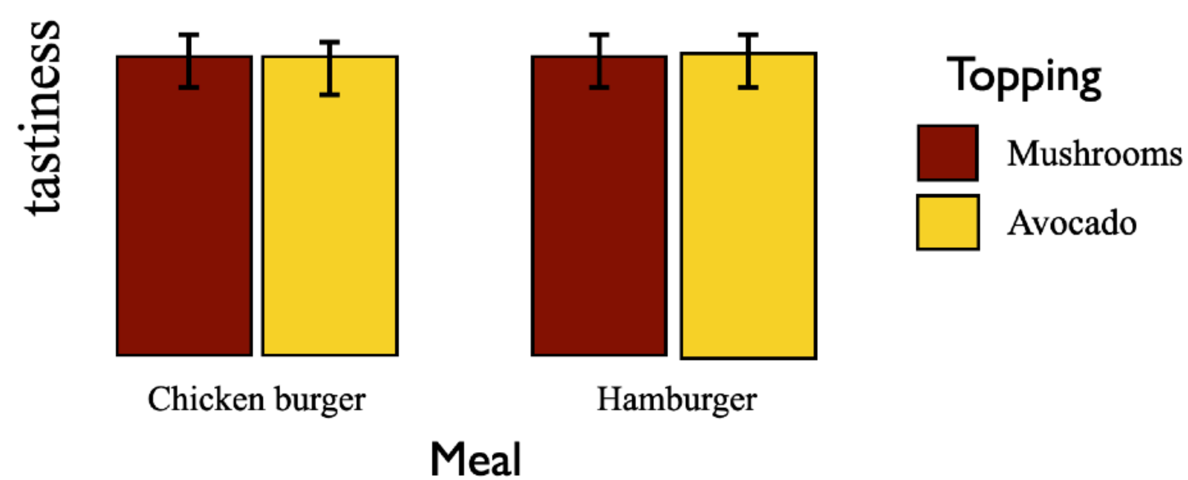


No interaction

Schematic pictures of different ways of having *no* interaction in a 2x2 ANOVA



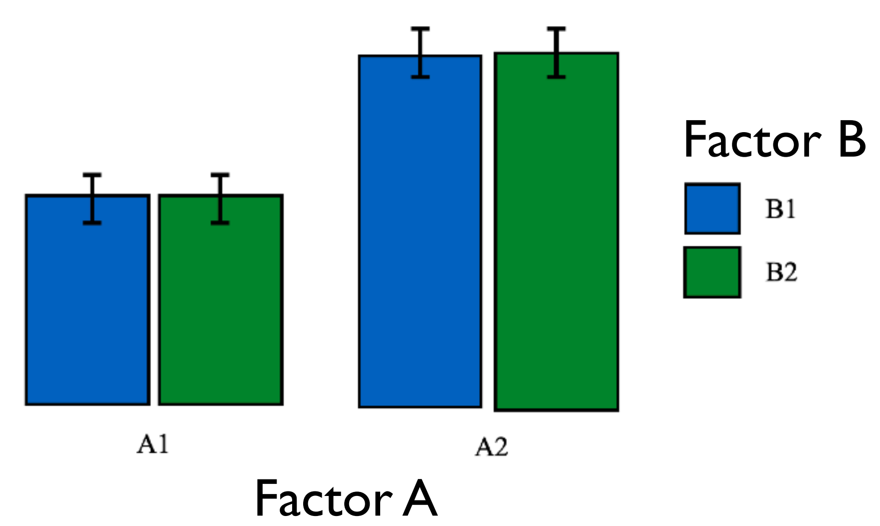
Example: Gladly likes hamburgers and chicken burgers equally, and avocado and mushrooms equally



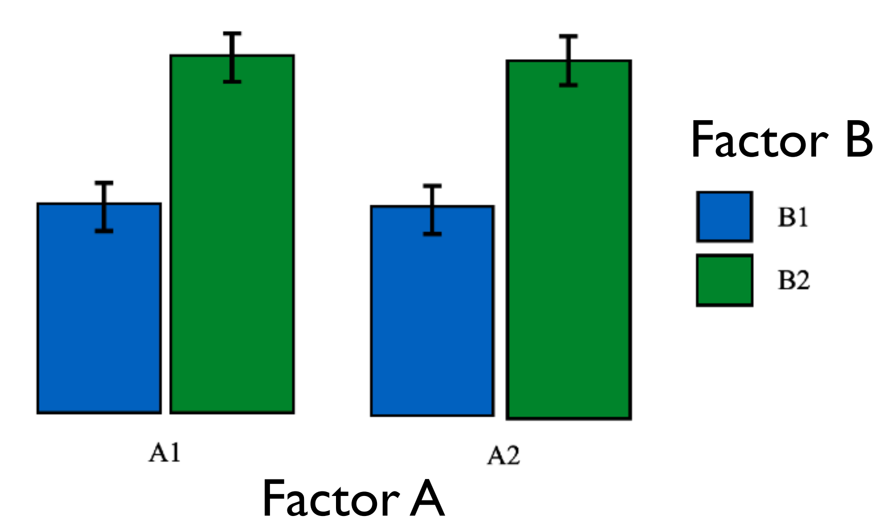
No interaction

Schematic pictures of different ways of having *no* interaction in a 2x2 ANOVA

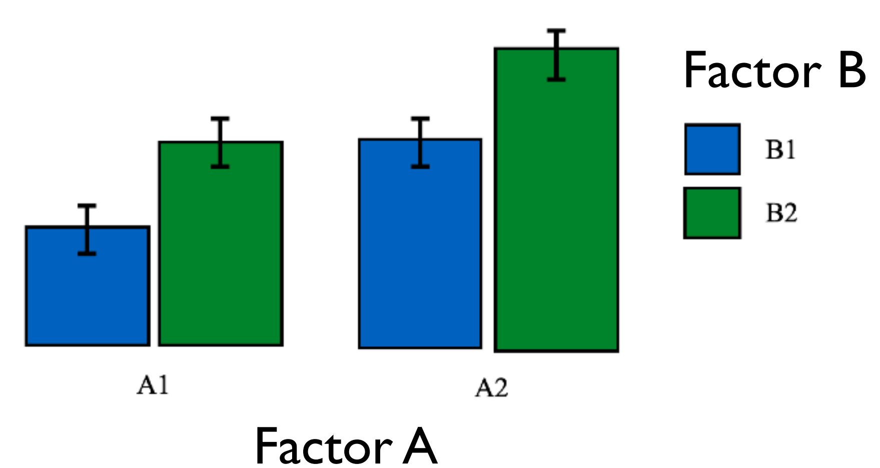
Single main effect of Factor A



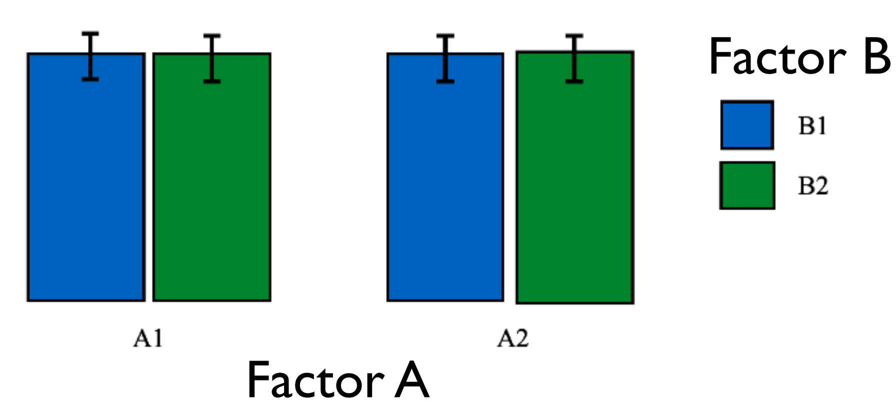
Single main effect of Factor B



Main effects of both A and B

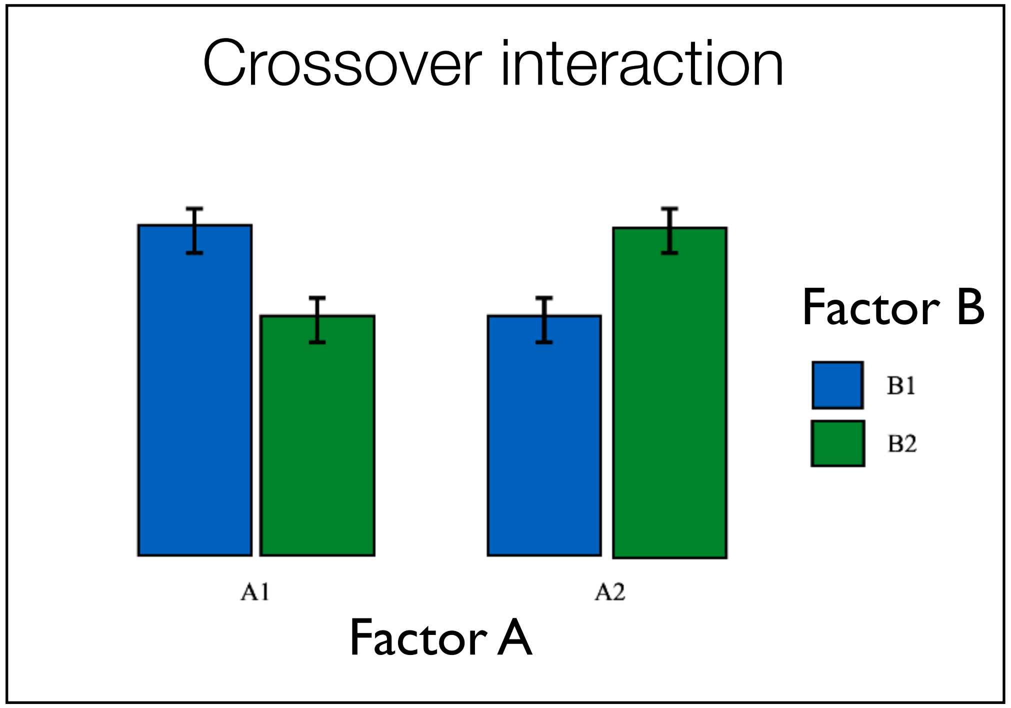


No significant effects



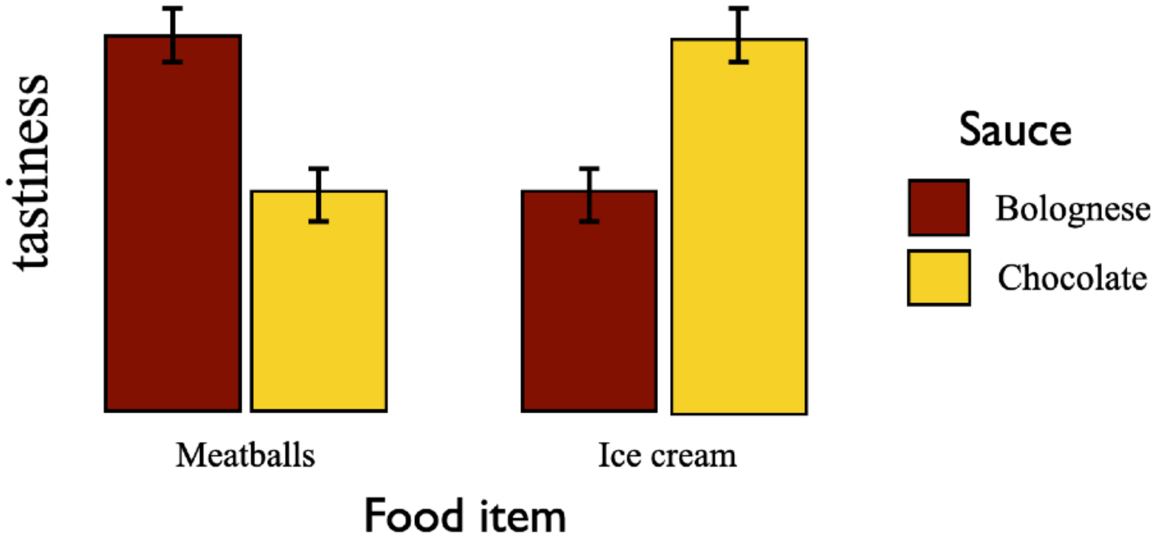
Interactions

Schematic pictures of different ways of having an interaction in a 2x2 ANOVA



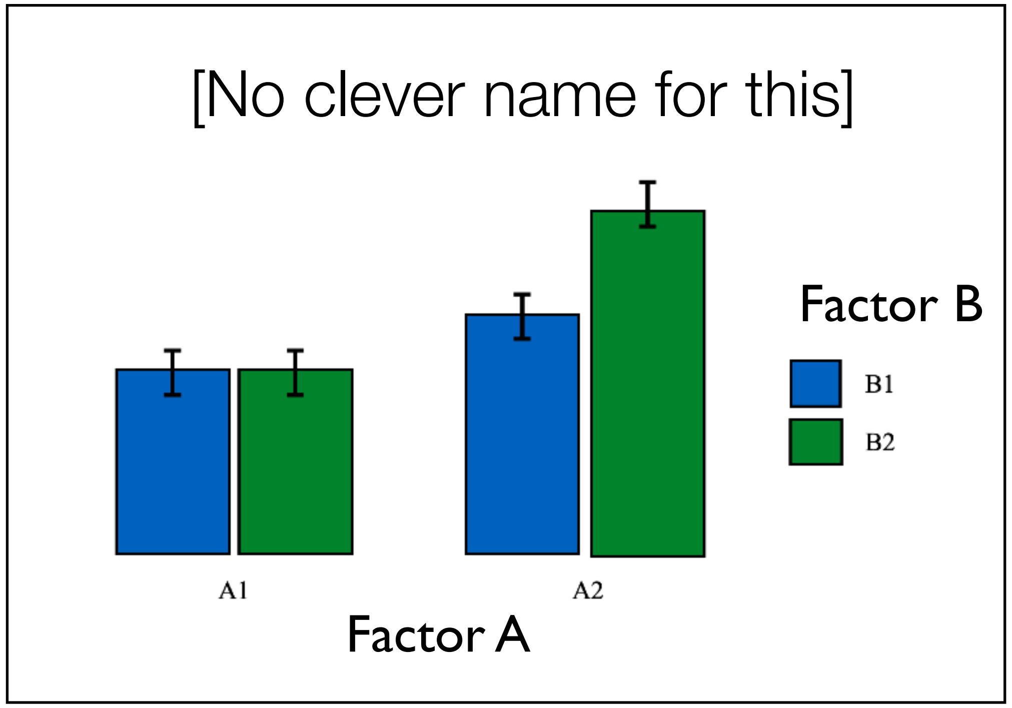
Direction of effect of a factor depends on the level of the other factor

Example: Gladly likes bolognese sauce on meatballs and chocolate sauce on ice cream, but not vice-versa



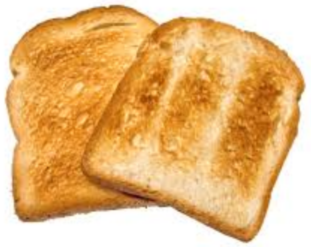
Interactions

Schematic pictures of different ways of having an interaction in a 2x2 ANOVA



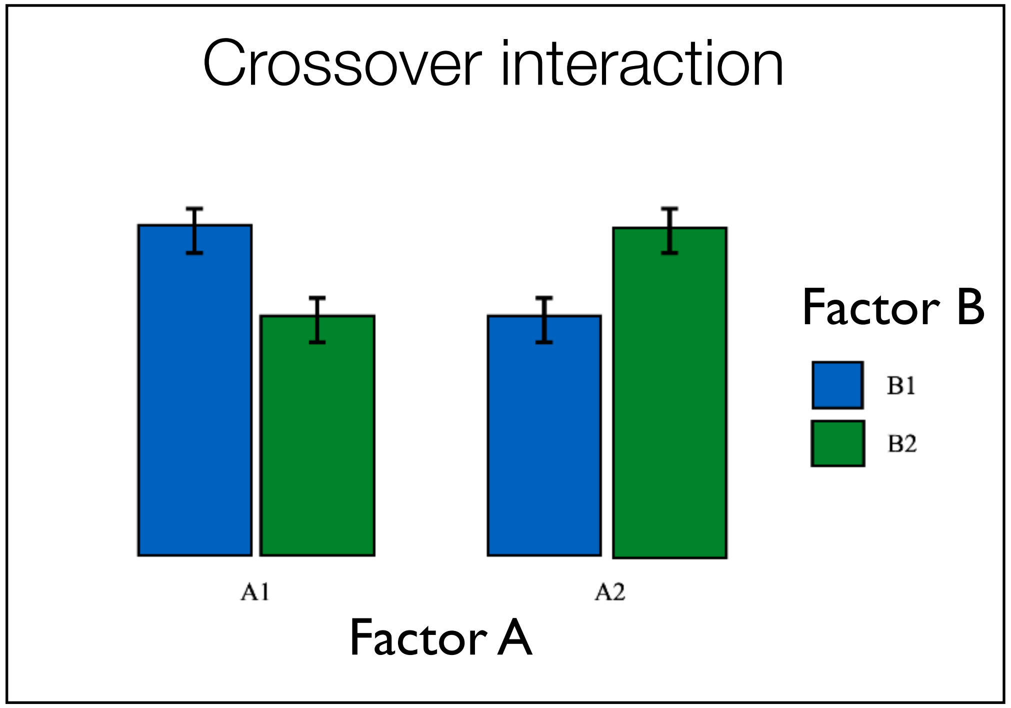
Magnitude of effect of a factor depends on the level of the other factor

Example: Gladly likes milk more than water, but only when paired with biscuits; when paired with toast, the drinks are equal

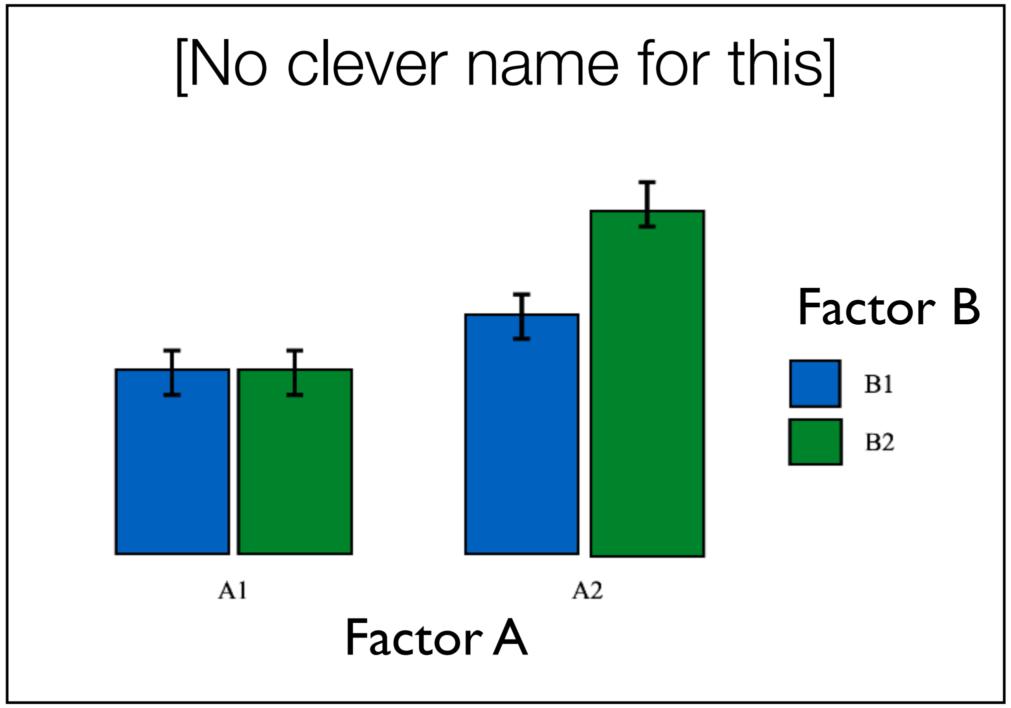


Interactions

Schematic pictures of different ways of having an interaction in a 2x2 ANOVA



Direction of effect of a factor depends on the level of the other factor



Magnitude of effect of a factor depends on the level of the other factor

Interactions

We can test for interactions by calculating a sum of squares for **each possible interaction term** (e.g., A:B indicates an interaction between Factor A and B). I'm going to gloss over the math, but it basically involves calculating how different each data point is from what you'd predict taking *both* factors into account, vs only one.

- You end up with multiple sums of squares
  - **Between groups sum of squares** (SS_A, SS_B): for each factor A and B, how different are the group means from one another?
  - **Interaction sum of squares** ($SS_{A:B}$): how much additional variation in the outcome variable is accounted for by taking into account both factors at once?
  - **Residual sum of squares** (SS_R): how much variation in the outcome variable is there after you take into account the variation associated with both factors and their interaction?

As before, the residuals change because less variation will be leftover since there are more things in there to account for it!

Doing interactions in R

Specify an interaction term by putting a :
between the two variables you care about

```
> foodtypeintoldModel <- aov(quantity ~ type + food + type:food, data = do_long)
> foodtypeintoldModel

Call:
   aov(formula = quantity ~ type + food + type:food, data = do_long)

Terms:
                    type     food type:food Residuals
Sum of Squares    200.53 63851.67   5551.19  14231.37
Deg. of Freedom        2        2         4       171
```

Doing interactions in R

R also lets you use * as a shorthand to mean "look for all main effects and interactions between these variables"

```
> foodtypeintoldModel <- aov(quantity ~ type*food, data = do_long)
> foodtypeintoldModel

Call:
   aov(formula = quantity ~ type*food, data = do_long)

Terms:
                    type     food type:food Residuals
Sum of Squares    200.53 63851.67   5551.19  14231.37
Deg. of Freedom        2        2         4       171
```

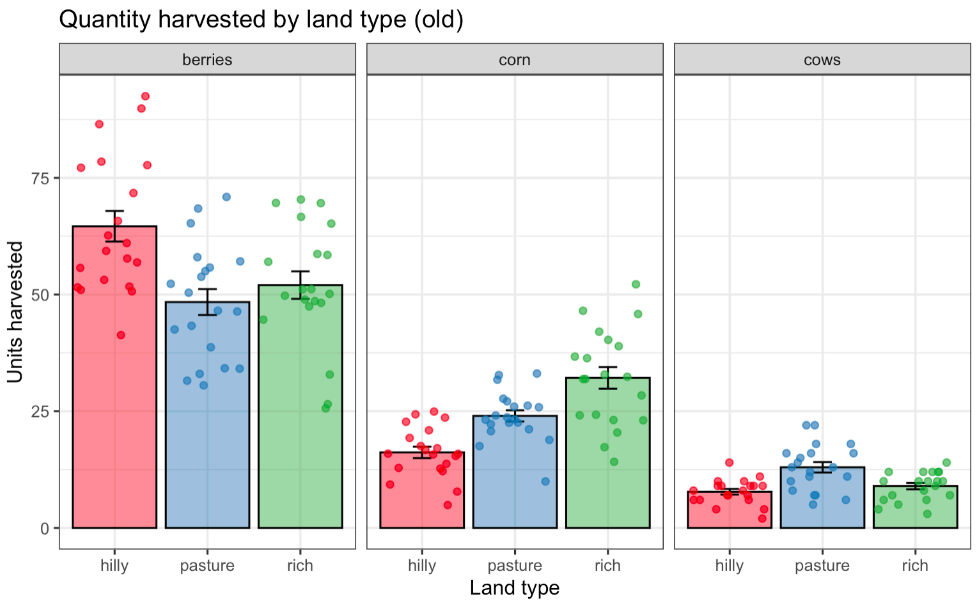
Doing interactions in R

```
> foodtypeintoldModel <- aov(quantity ~ type*food, data = do_long)
> summary(foodtypeintoldModel)

              Df Sum Sq Mean Sq F value   Pr(>F)    
type           2    201     100   1.205    0.302    
food           2  63852   31926 383.612  < 2e-16 ***
type:food      4   5551    1388  16.675 1.47e-11 ***
Residuals    171  14231      83                     
```

This shows a significant main effect of food, $F(2,171)=383.61$, $p<.0001$. There is also a significant interaction between food and land type, $F(4,171)=16.68$, $p<.0001$. There is no significant effect of land type, $F(2,171)=1.21$, $p=.302$.

All of this makes sense given the figure!
Berries are better on hilly land, corn best on rich land, and cows best on pasture



Doing interactions in R

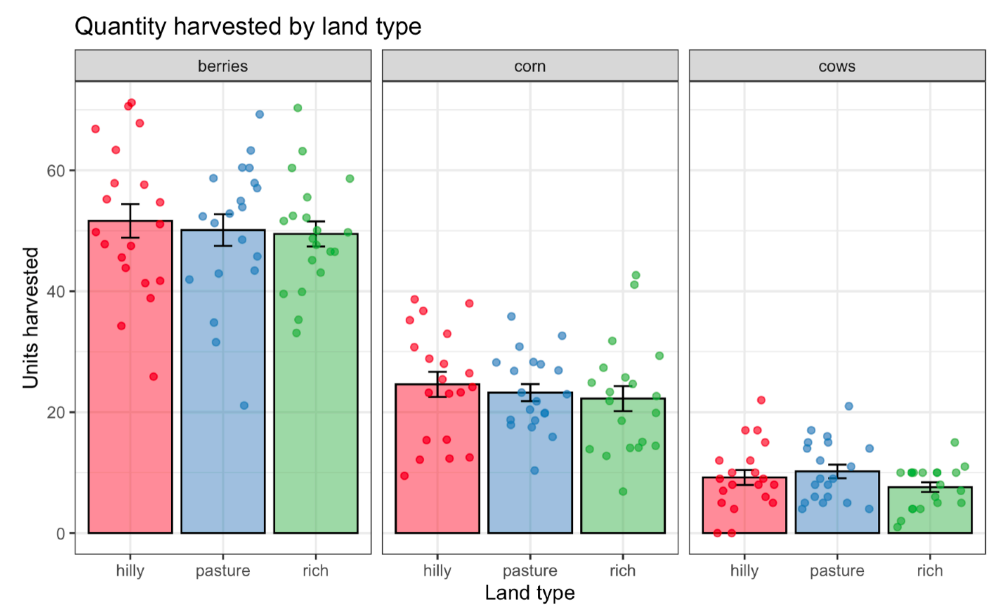
We can do exactly the same thing on the new data

```
> foodtypeintnewModel <- aov(quantity ~ type + food + type:food, data = dn_long)
> summary(foodtypeintnewModel)

              Df Sum Sq Mean Sq F value Pr(>F)
type           2    130      65   0.894  0.411
food           2  53068   26534 363.645 <2e-16 ***
type:food      4     43      11   0.148  0.964
Residuals    171  12477      73
```

This shows a significant main effect of food, F(2,171)=363.65, p<.0001. But there is no significant interaction between food and land type, F(4,171)=0.15, p=.964, and no significant effect of land type, F(2,171)=0.89, *p*=.411.

All of this *also* makes sense given the figure!
The land is no longer being used well, with different crops on different types of land



Effect sizes for two-way ANOVAs

We can calculate effect size in a similar way as before, but there are a few caveats on interpretation

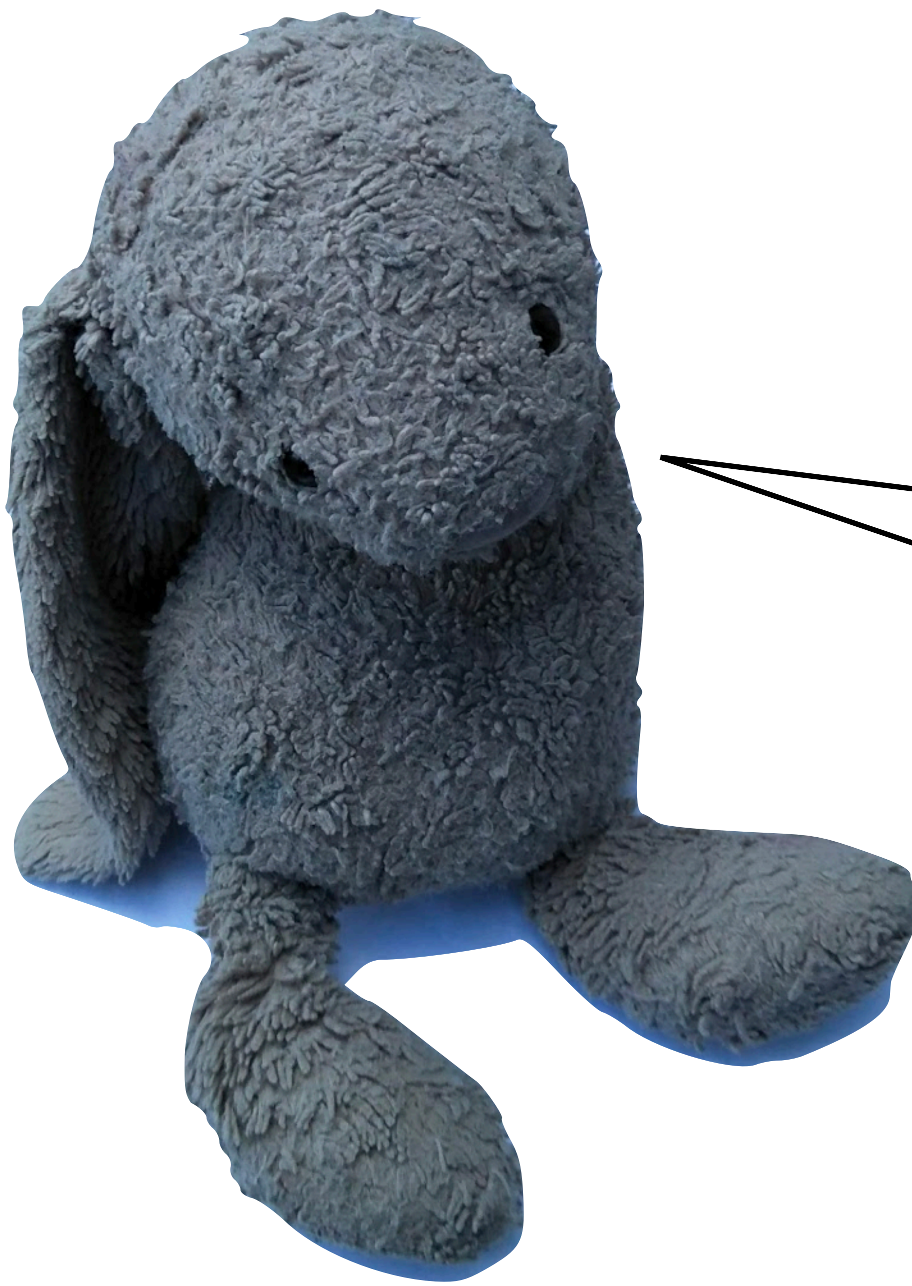
```
> etaSquared(foodtypeintoldModel)

               eta.sq eta.sq.part
type      0.002391919  0.01389464
food      0.761637247  0.81774060
type:food 0.066215842  0.28061028
```

Proportion of total variance attributable to each factor (food is 76.1%, land type is 2.4%, the interaction is 6.6%)

Partial η^2: The variance attributable to each factor *assuming you ignore all of the others* (i.e., pretend that the effect sizes of all the others is zero, and calculate what it would have been). This is sometimes useful but easy to misinterpret: it's just an effect size relative to the residual variation - you can't necessarily compare them across terms, because the residual variation in each case is different.

What does this mean?

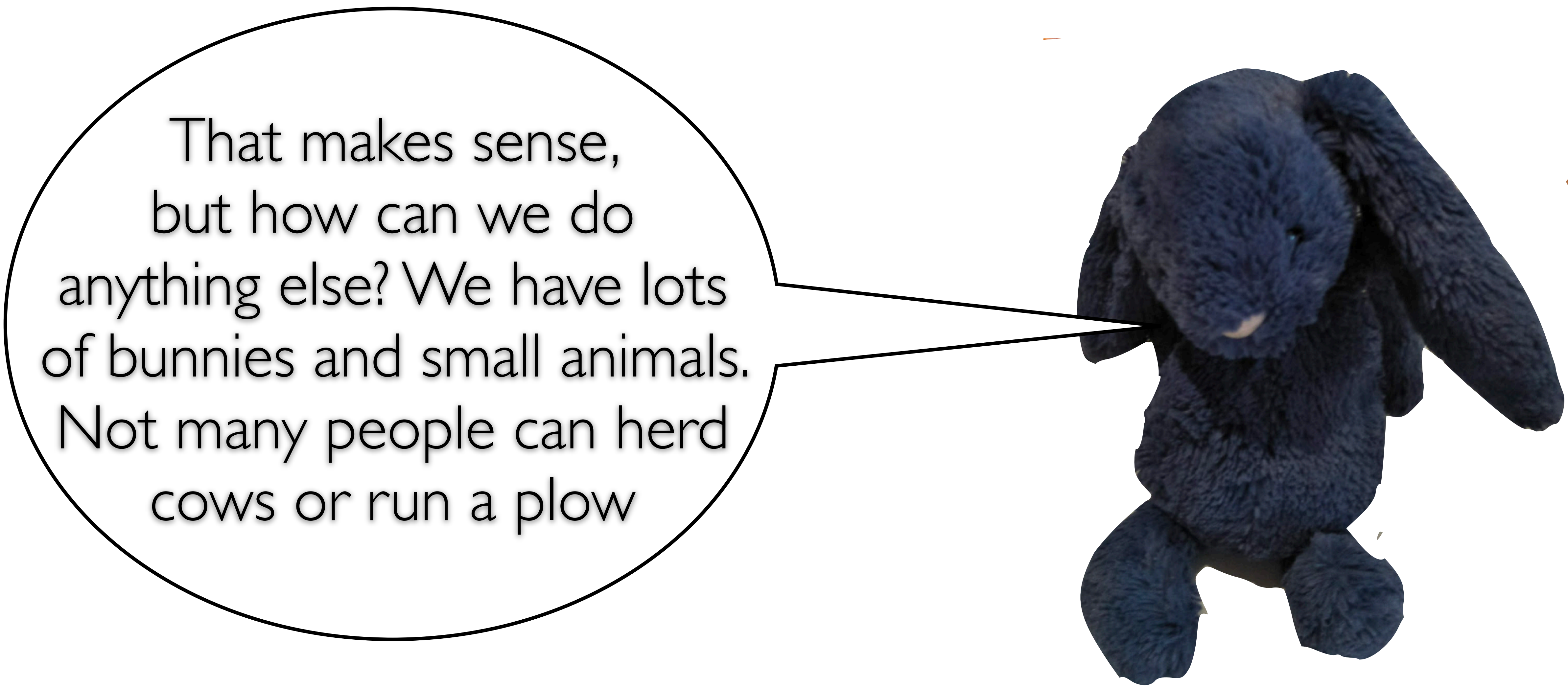


I think we *are* using our land in a very silly way, but we didn't used to

What does this mean?

Also it looks like a lot of it is because we're growing too many berries and not enough corn and cows

What does this mean?



Exercises are in w8day2exercises.Rmd