Mikroelektronische Schaltungen und Systeme

Lect.6
Frequency Response

Prof. Dr.-Ing. Ahmet Cagri Ulusoy
Institut für Hochfrequenztechnik und Elektronik (IHE)

Mikroelektronische Schaltungen und Systeme

Lect.1
Intro/CMOS Basics

Lect.2
Single Stage Amplifiers

Lect.3
Noise

Lect.4
Differential Amplifiers

Lect.5
Current Sources

Lect.6
Frequency Response

Lect.7
Feedback

Lect.8
Operational Amplifiers

Lect.9
Stability and Frequency Compensation

Lect.10
Switched-Capacitor Circuits

Lect.11
Introduction to Data Converters

Lect.12
Combinational Logic Circuits

Lect.13
Sequential Circuits

Lect.14
Timing of Sequential Circuits

Lect.15
Digital Building Blocks

Lect.16
Memory

Lect.17
Technology Mapping

Lect.18
FPGA

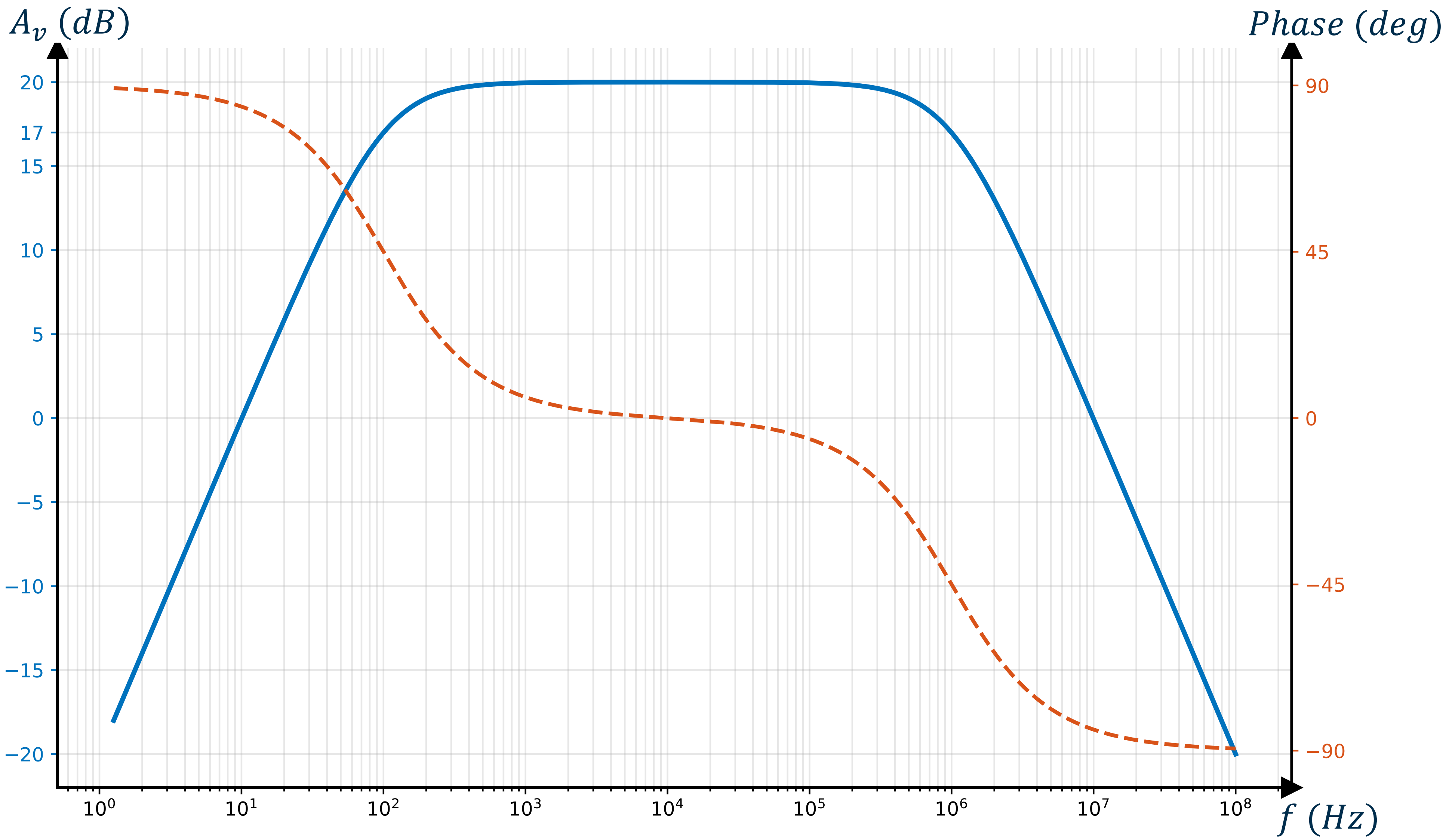
Lect.19
Physical Design I

Lect.20
Physical Design II

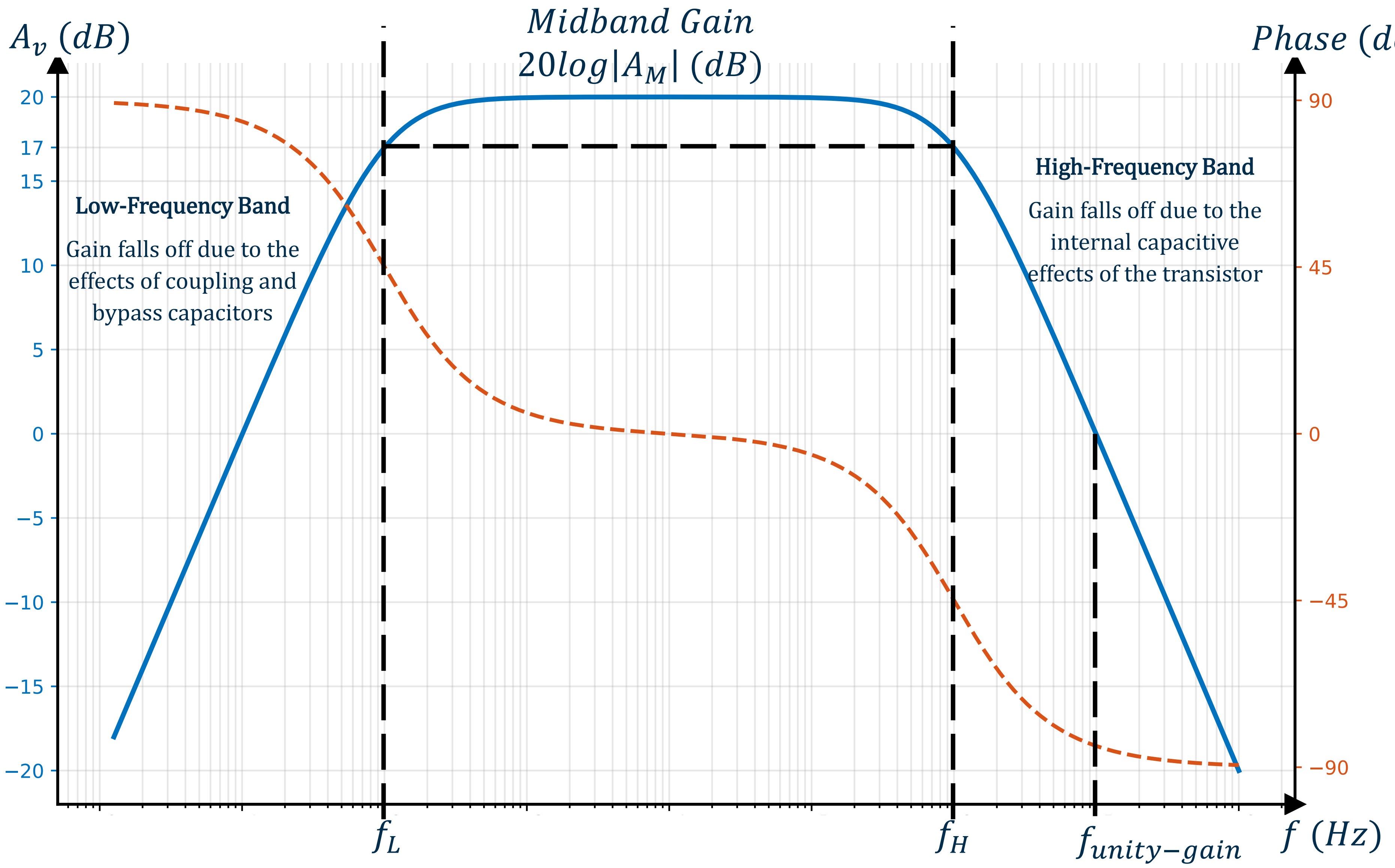
Lect.21
Q&A


to 0);
.conf.addr
v.conf.wb.wr
7 downto 0)
l(0) & wb_b

Introduction



Introduction



- **Bandwidth:**

$$\boxed{BW = f_H - f_L}$$

- **Gain-Bandwidth Product:**

$$\boxed{GBW = |A_M|BW \approx f_{unity-gain}}$$

Transfer Function

- The transfer function of a circuit at steady state can be written as:

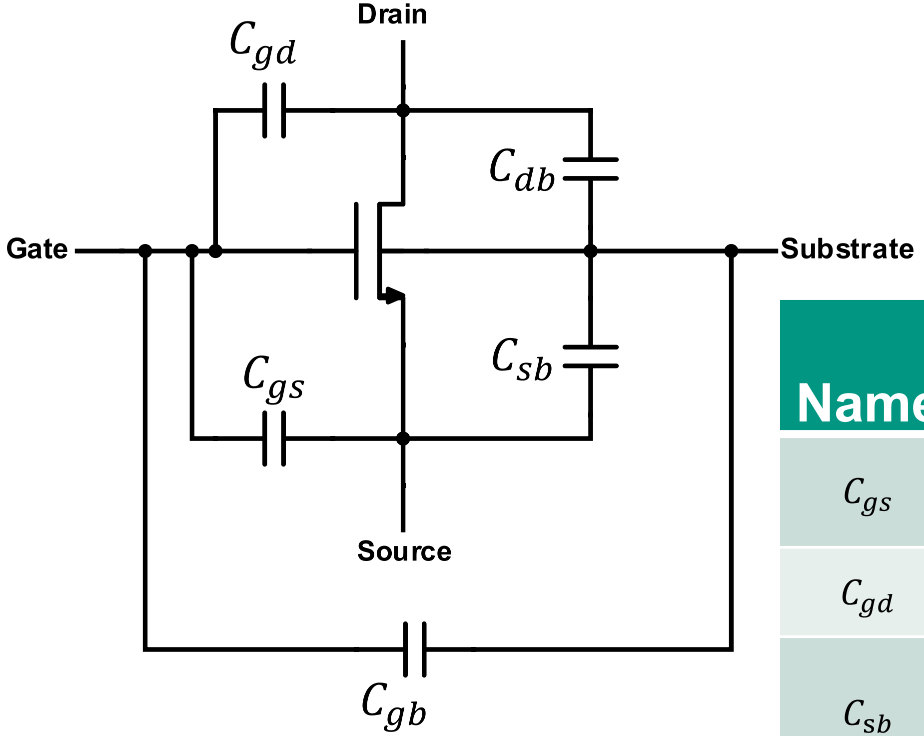
$$A(j\omega) = a_0 \frac{\left(1+\frac{j\omega}{\omega_{z_1}}\right)\left(1+\frac{j\omega}{\omega_{z_2}}\right)\dots\left(1+\frac{j\omega}{\omega_{z_n}}\right)}{\left(1+\frac{j\omega}{\omega_{p_1}}\right)\left(1+\frac{j\omega}{\omega_{p_2}}\right)\dots\left(1+\frac{j\omega}{\omega_{p_m}}\right)}$$

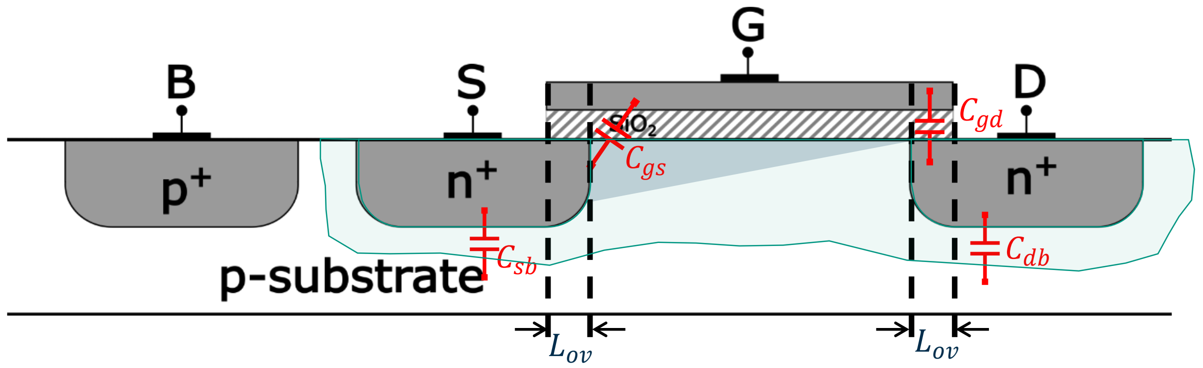
Poles: Magnitude: -3-dB, -20dB/decade, Phase: -45°asymptotically -90°

Zeros: Magnitude: +3-dB, +20dB/decade, Phase: +45° asymptotically +90°

Dominant Pole Approximation: The frequency points are far enough apart (sufficiently separated) that the poles and zeros can be considered separately.

MOSFET Parasitic Capacitances

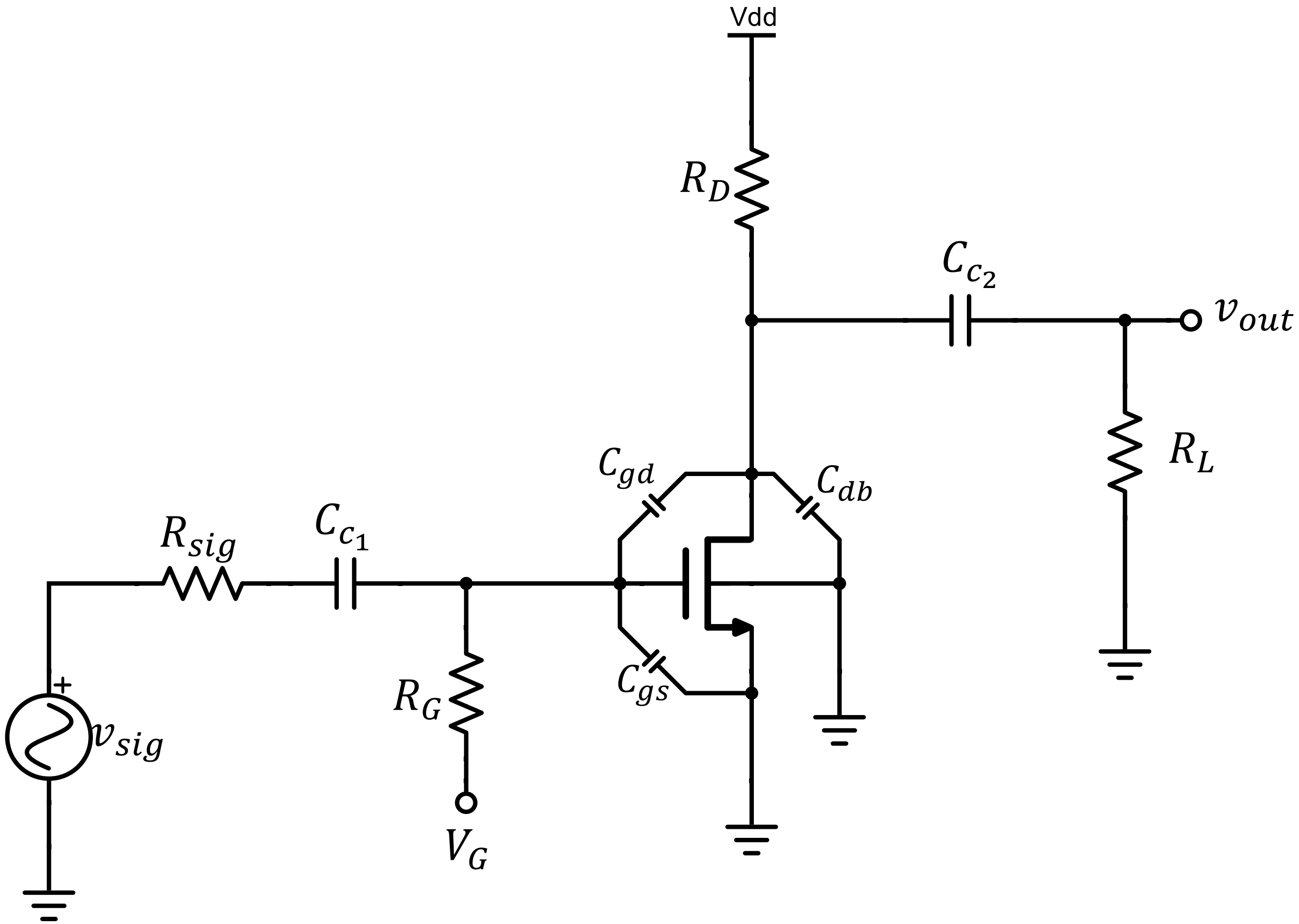




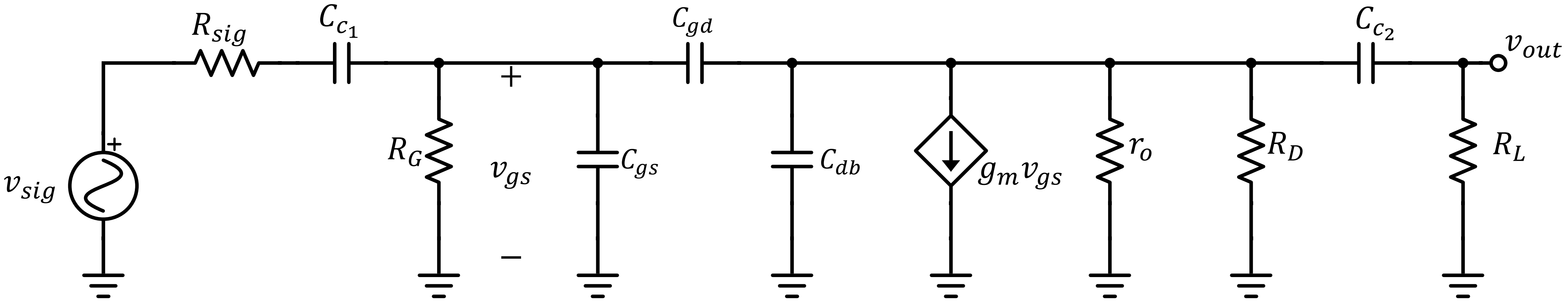
Name	Value in Saturation	Value in Triode
C_{gs}	$\frac{2}{3}WLC_{ox} + WL_{ov}C_{ox}$	$\frac{1}{2}WLC_{ox} + WL_{ov}C_{ox}$
C_{gd}	$WL_{ov}C_{ox}$	$\frac{1}{2}WLC_{ox} + WL_{ov}C_{ox}$
C_{sb}	$\frac{C_{sb0}}{\sqrt{1+\frac{V_{SB}}{\phi_0}}}$	$\frac{C_{sb0}}{\sqrt{1+\frac{V_{SB}}{\phi_0}}}$
C_{db}	$\frac{C_{db0}}{\sqrt{1+\frac{V_{DB}}{\phi_0}}}$	$\frac{C_{db0}}{\sqrt{1+\frac{V_{DB}}{\phi_0}}}$
C_{gb}	*Omitted*	*Omitted*

- L_{ov}: Overlap length
- C_{sb0}: The value of C_{sb} at zero body-source bias.
- C_{db0}: The capacitance value at zero reverse-bias voltage.
- ϕ_0: Built-in junction potential

Common-Source Amplifier with DC-Decoupling and Parasitic Capacitors

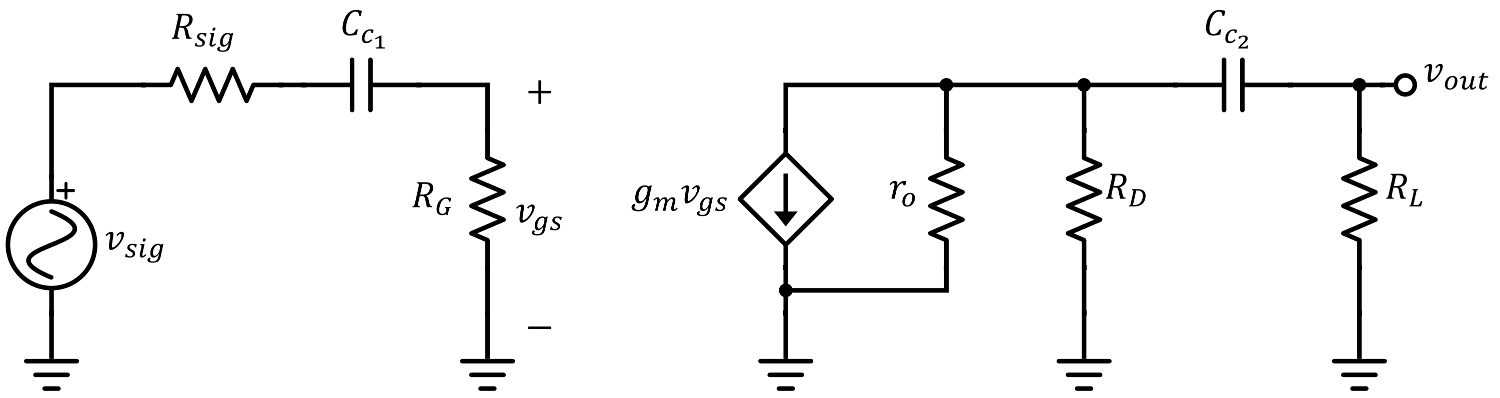


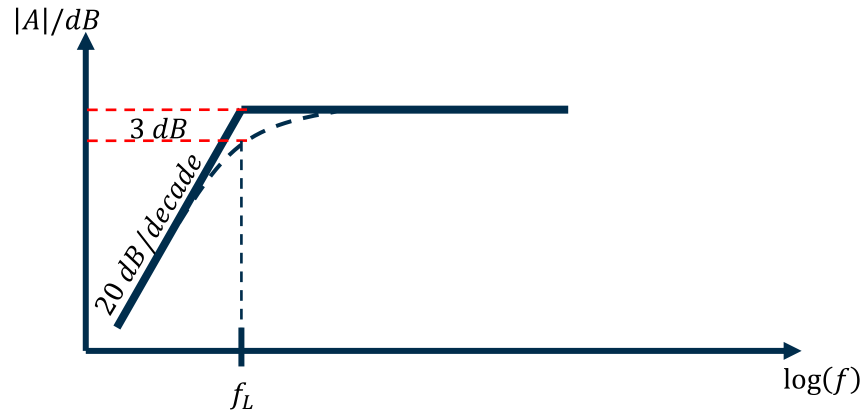
Common-Source Amplifier with DC-Decoupling and Parasitic Capacitors



Common-Source Amplifier Low-Frequency Response

- $C_{coupling} \gg C_{parasitics} \rightarrow \dfrac{1}{j\omega C_{parasitics}} \gg \dfrac{1}{j\omega C_{coupling}}$
- Parasitic capacitances are treated as open circuit in low-frequency analysis.



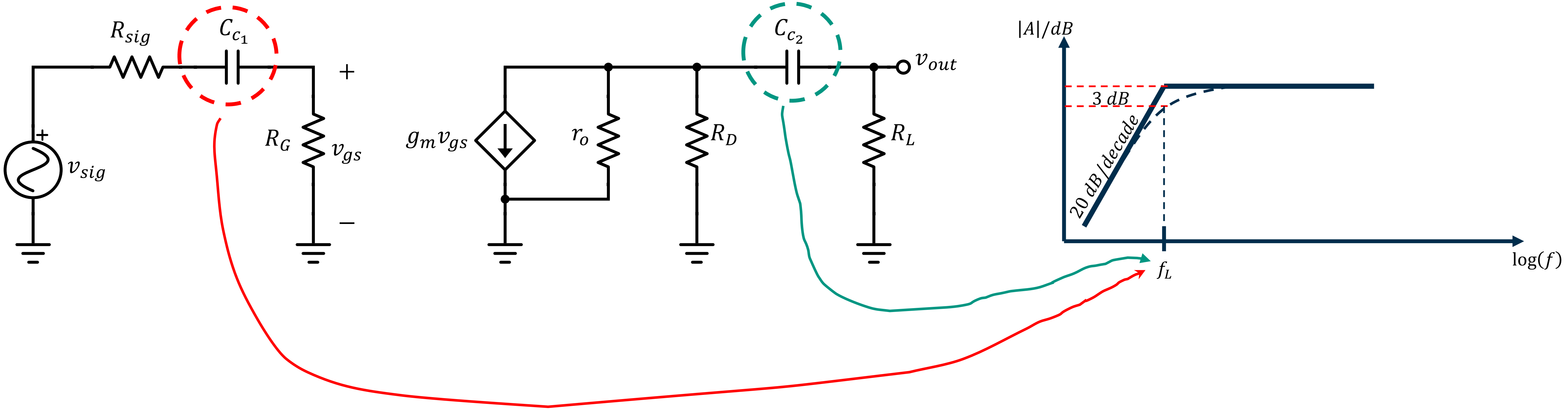


Common-Source Amplifier Low-Frequency Response

- One of the coupling capacitors dominate and determine the low-frequency corner.

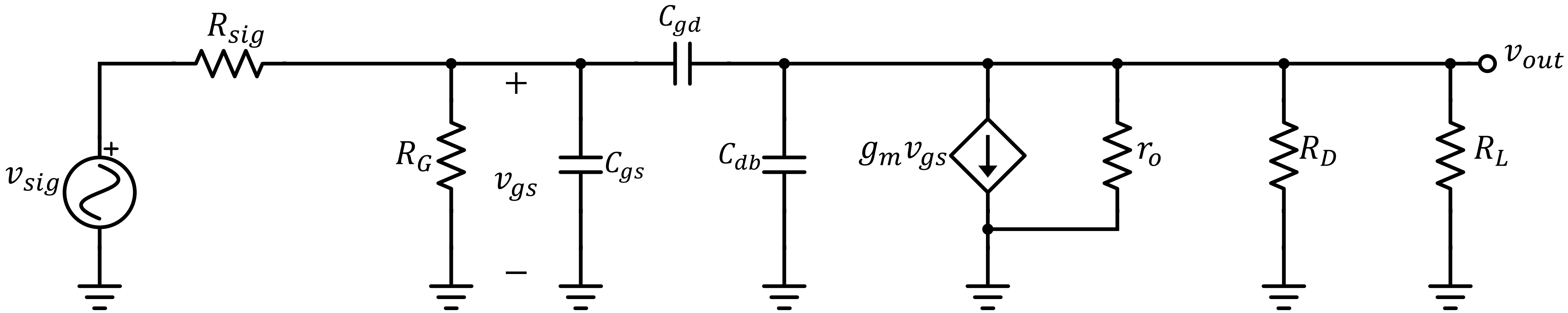
$$f_{P_1} = \frac{1}{2\pi C_{C_1}\left(R_{sig} + R_G\right)}$$

$$f_{P_2} = \frac{1}{2\pi C_{C_2}\left((r_o \parallel R_D) + R_L\right)}$$

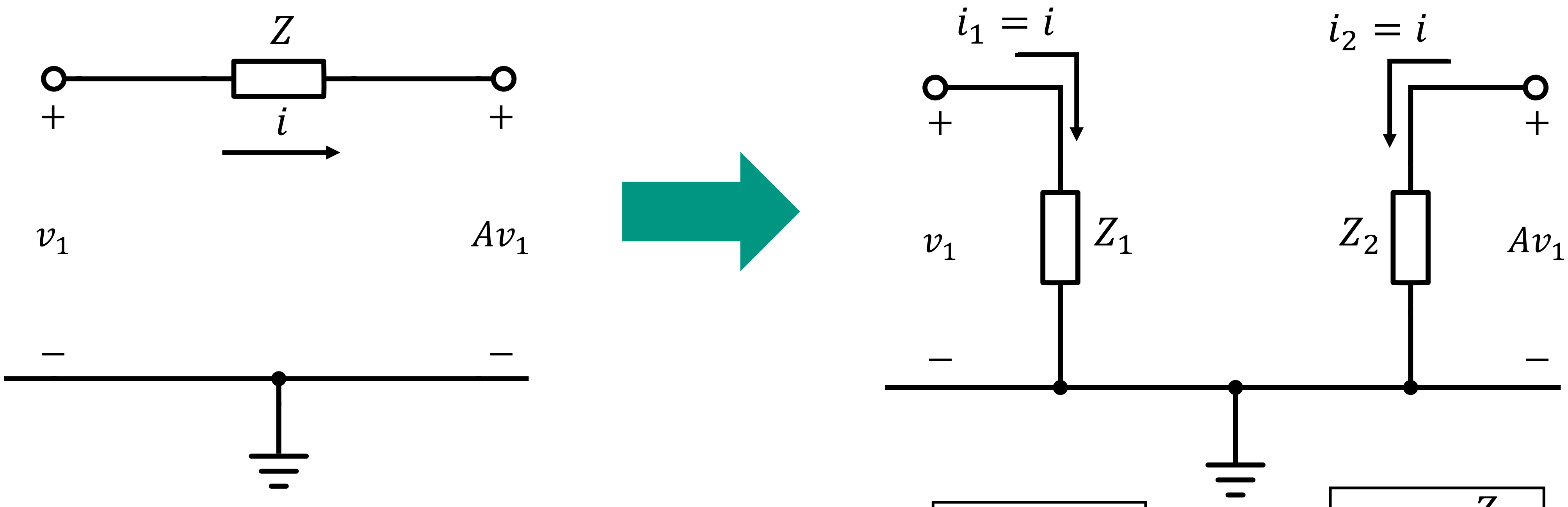


Common-Source Amplifier High-Frequency Response

- $C_{coupling} \gg C_{parasitics} \rightarrow \dfrac{1}{j\omega C_{coupling}} \rightarrow 0$ at high ω.
- Coupling capacitances are treated as short circuit in high-frequency analysis.



Miller's Theorem

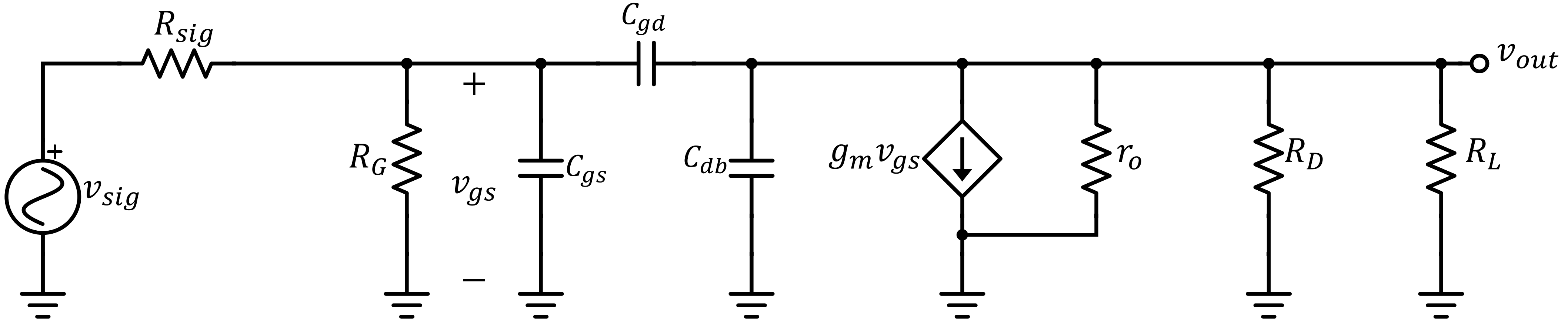


$$\boxed{Z_1 = \frac{Z}{1 - A}}$$

$$\boxed{Z_2 = \frac{Z}{1 - \frac{1}{A}}}$$

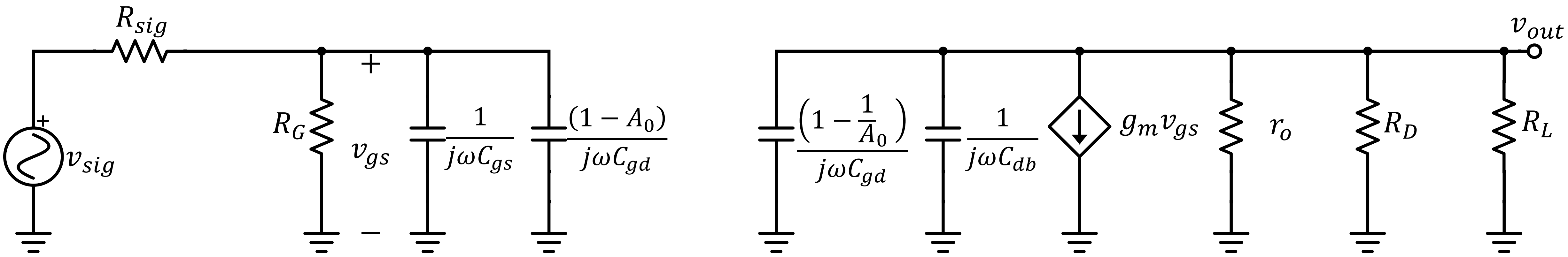
Miller's Theorem Applied in CS Amplifier

$$A_0 = -\frac{v_{out}}{v_{gs}}$$



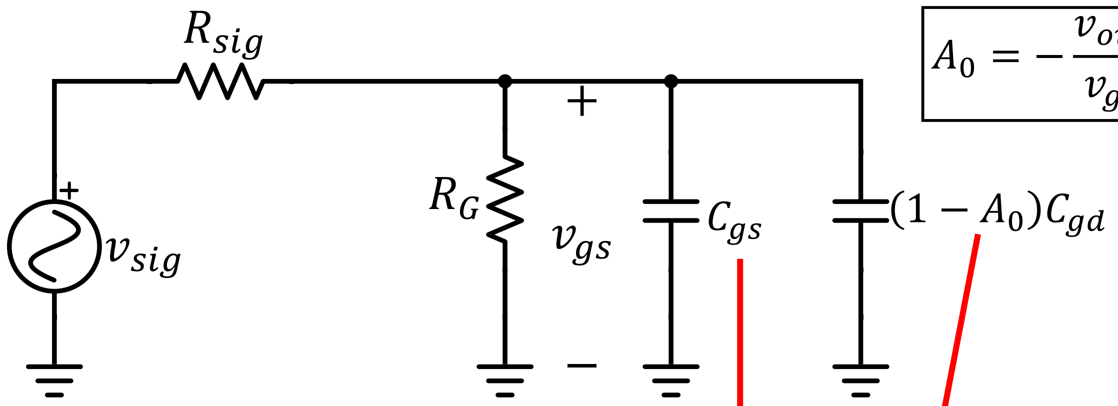
Miller's Theorem Applied in CS Amplifier

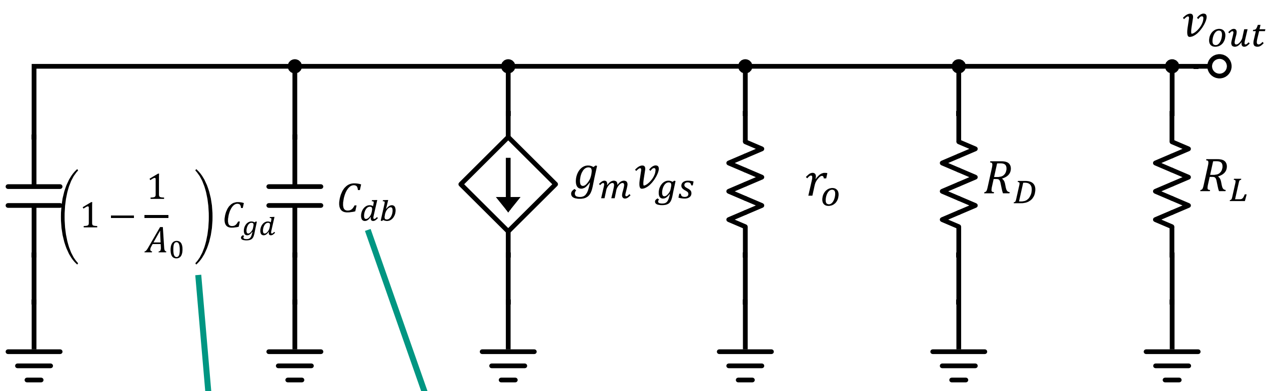
$$A_0 = -\frac{v_{out}}{v_{gs}}$$



Miller's Theorem Applied in CS Amplifier

$$A_0 = -\frac{v_{out}}{v_{gs}}$$



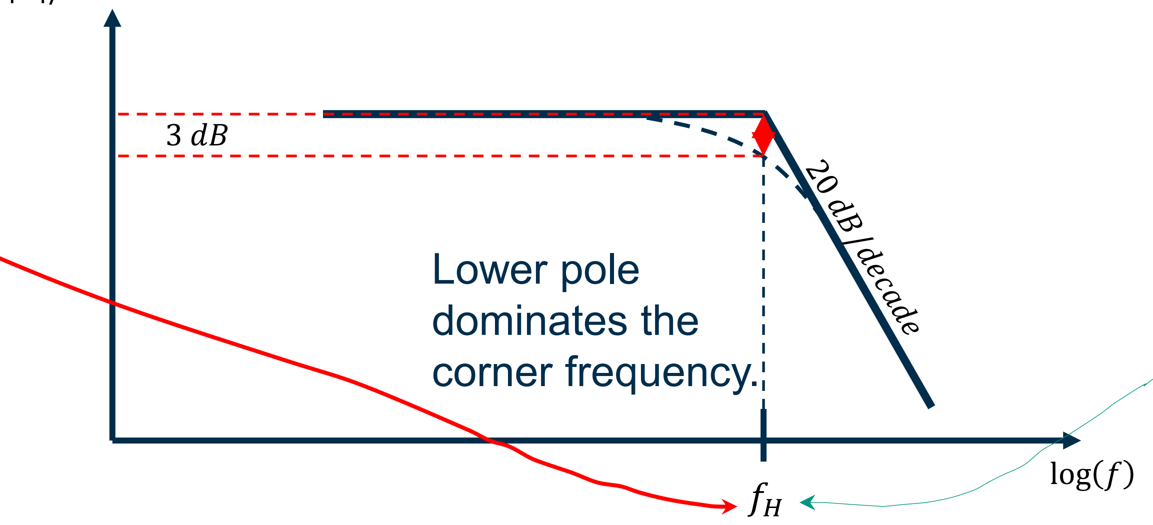


$$f_{P_1} = \frac{1}{2\pi\left(R_{sig} \parallel R_G\right)C_{in}}$$

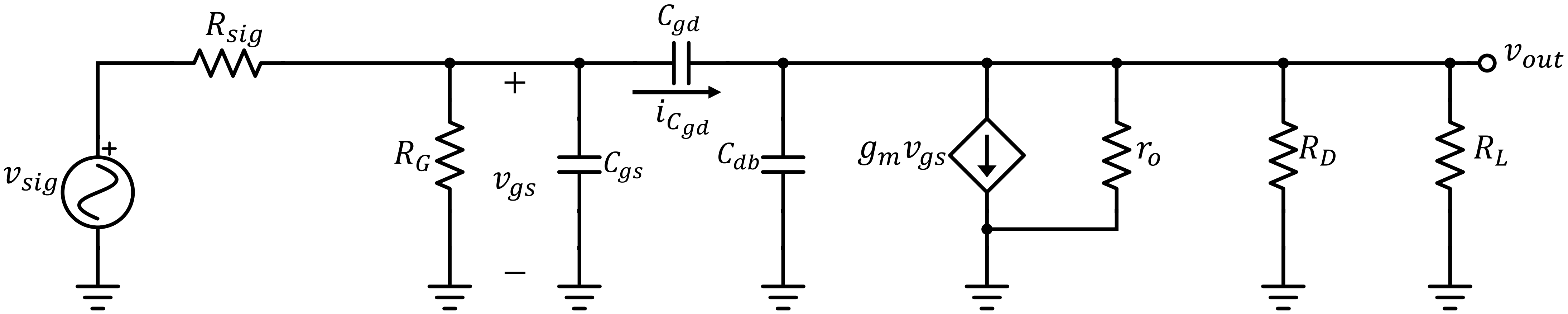
$$C_{in} = C_{gs} + (1-A_0)C_{gd}$$

$$C_{out} = \left(1-\frac{1}{A_0}\right)C_{gd} + C_{db}$$

$$f_{P_2} = \frac{1}{2\pi C_{out}(r_o \parallel R_D \parallel R_L)}$$



Common-Source Amplifier Calculation of Zero



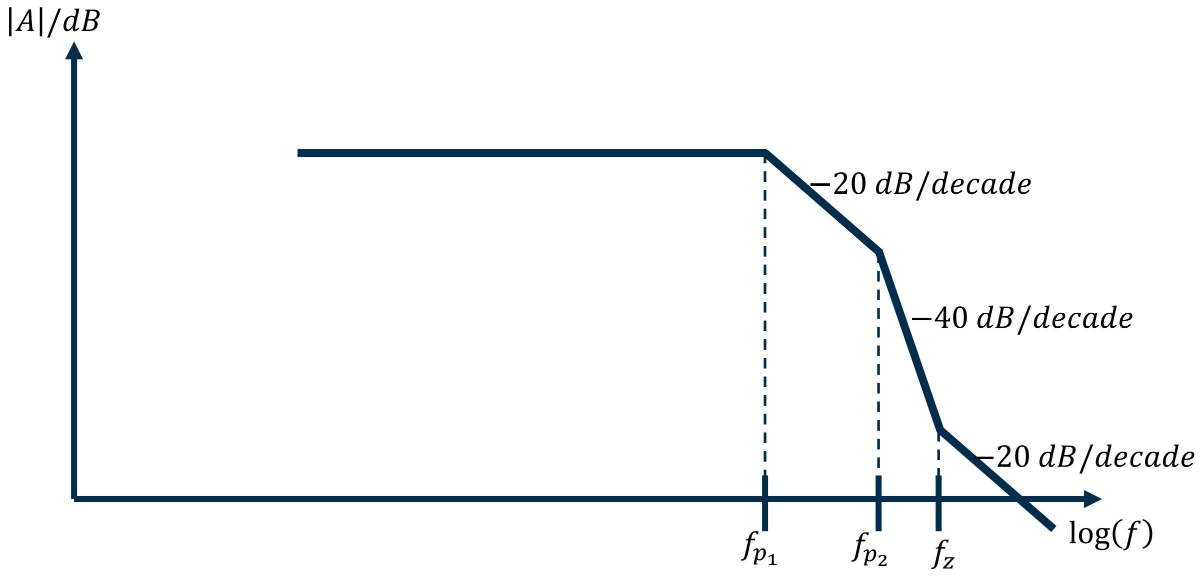
$Consider\, R_{sig} = 0 \rightarrow v_{gs} = v_{sig}$

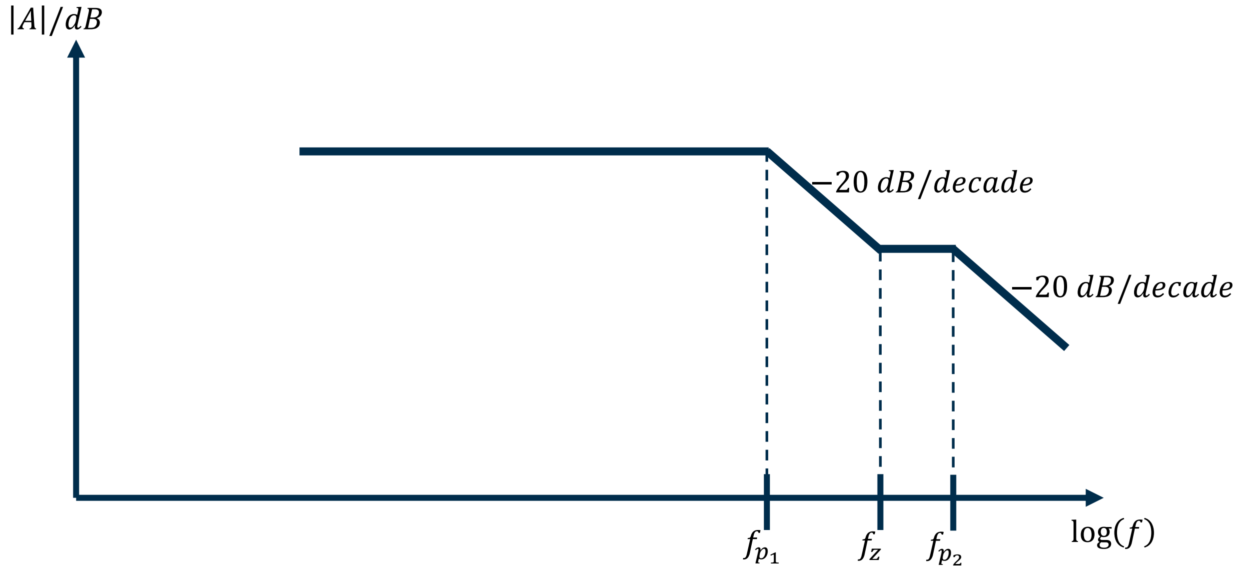
- $R_{out} = r_o \parallel R_D \parallel R_L$
- $i_{C_{gd}} = sC_{gd}(v_{sig} - v_{out}) = \frac{v_{out}}{R_{out}} + v_{out}(sC_{db}) + g_m v_{sig}$
- $v_{sig}(sC_{gd} - g_m) = v_{out}\left(\frac{1}{R_{out}} + s(C_{db} + C_{gd})\right)$
- $C_{out} = C_{db} + C_{gd}$
- $\frac{v_{out}}{v_{sig}} = \frac{(sC_{gd} - g_m)R_{out}}{1 + sR_{out}C_{out}}$

$$\boxed{\frac{v_{out}}{v_{sig}} = -\frac{g_m R_{out}\left(1 - \frac{sC_{gd}}{g_m}\right)}{1 + sR_{out}C_{out}}}$$

$$\boxed{\boldsymbol{f_z = \frac{g_m}{2\pi C_{gd}}}}$$

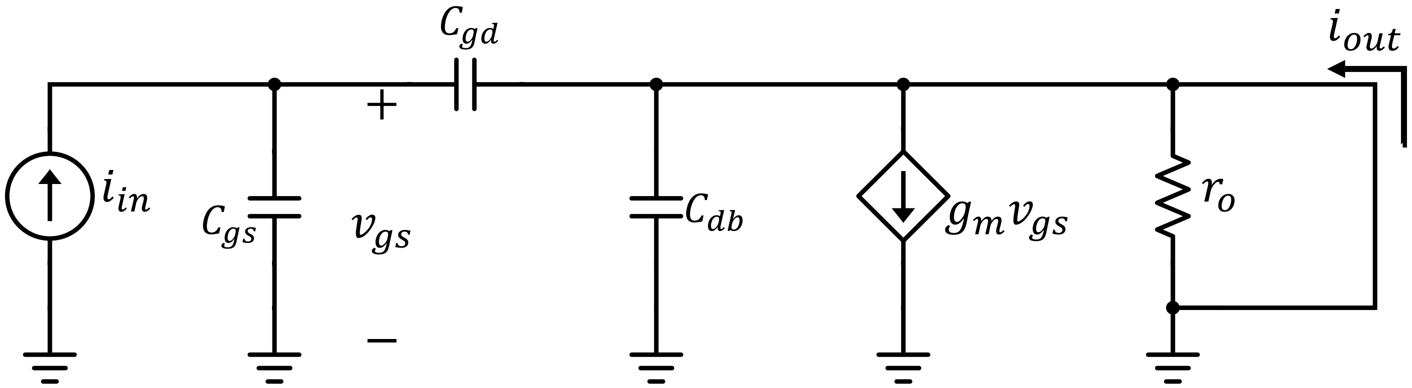
Rough Drawing of Poles and Zero on Bode Plot





Unity Gain (Transition) Frequency f_T

- The transition frequency is the frequency at which the short-circuit current gain of the common-source configuration becomes unity.
- This intrinsic device parameter is key to understanding the device's maximum speed capability and its fundamental high-frequency limitation.



$$i_{out} \approx g_m v_{gs}$$

$$v_{gs} = \frac{i_{in}}{j\omega\left(C_{gs} + C_{gd}\right)}$$

$$\frac{i_{out}}{i_{in}} = \frac{g_m}{j\omega\left(C_{gs} + C_{gd}\right)}$$

$$\left|\frac{i_{out}}{i_{in}}\right| = \frac{g_m}{\omega\left(C_{gs} + C_{gd}\right)} = 1 \rightarrow \omega_T = 2\pi f_T = \frac{g_m}{C_{gs} + C_{gd}}$$

$$\boxed{f_T = \frac{g_m}{2\pi\left(C_{gs} + C_{gd}\right)}}$$

The Method of Open-Circuit Time Constants

- It is not always a simple matter to determine the poles and zeros by quick hand analysis. In such cases an approximate value for 3-dB frequency f_h can be obtained by using the method of open-circuit time constants. This method allows you to estimate the bandwidth by analyzing the interaction of each capacitor with the circuit resistance individually.

The Method of Open-Circuit Time Constants

- Assume a system with two poles and two zeros

$$A(s) = \frac{\left(1+\frac{s}{\omega_{z_1}}\right)\left(1+\frac{s}{\omega_{z_2}}\right)}{\left(1+\frac{s}{\omega_{p_1}}\right)\left(1+\frac{s}{\omega_{p_2}}\right)} = \frac{\left(1+s\left(\frac{1}{\omega_{z_1}}+\frac{1}{\omega_{z_2}}\right)+\frac{s^2}{\omega_{z_1}\omega_{z_2}}\right)}{\left(1+s\left(\frac{1}{\omega_{p_1}}+\frac{1}{\omega_{p_2}}\right)+\frac{s^2}{\omega_1\omega_{p_2}}\right)} = \frac{(1+a_1s+a_2s^2)}{1+b_1s+b_2s^2}$$

- b_1 is the sum of the reciprocal of pole frequencies or the sum of time constants.

$$b_1 = \frac{1}{\omega_{p_1}}+\frac{1}{\omega_{p_2}} = \frac{1}{2\pi f_{p_1}}+\frac{1}{2\pi f_{p_2}} = \tau_{pole_1}+\tau_{pole_2} = \tau_{C_1}+\tau_{C_2}$$

$$\tau_{C_{1,2}} \neq \tau_{pole_{1,2}}$$

The Method of Open-Circuit Time Constants

- Assume a system with two poles and two zeros

$$A(s)=\frac{\left(1+\frac{s}{\omega_{z_1}}\right)\left(1+\frac{s}{\omega_{z_2}}\right)}{\left(1+\frac{s}{\omega_{p_1}}\right)\left(1+\frac{s}{\omega_{p_2}}\right)}=\frac{\left(1+s\left(\frac{1}{\omega_{z_1}}+\frac{1}{\omega_{z_2}}\right)+\frac{s^2}{\omega_{z_1}\omega_{z_2}}\right)}{\left(1+s\left(\frac{1}{\omega_{p_1}}+\frac{1}{\omega_{p_2}}\right)+\frac{s^2}{\omega_1\omega_{p_2}}\right)}=\frac{(1+a_1s+a_2s^2)}{1+b_1s+b_2s^2}$$

- b_1 is the sum of the reciprocal of pole frequencies or the sum of time constants.

$$b_1=\frac{1}{\omega_{p_1}}+\frac{1}{\omega_{p_2}}=\frac{1}{2\pi f_{p_1}}+\frac{1}{2\pi f_{p_2}}=\tau_{pole_1}+\tau_{pole_2}=\tau_{C_1}+\tau_{C_2}\approx\frac{1}{2\pi f_H}$$

- Dominant pole approximation

The Method of Open-Circuit Time Constants

- The result can be expanded to multiple poles:

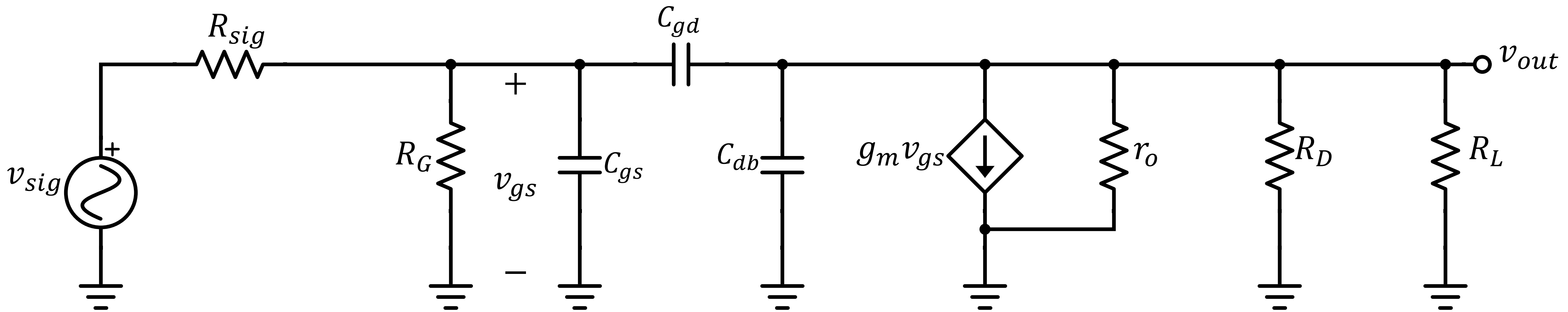
$$b_1 = \frac{1}{\omega_{p_1}} + \frac{1}{\omega_{p_2}} + \cdots + \frac{1}{\omega_{p_m}} = \sum_{k=1}^{m} \tau_{C_k} \approx \frac{1}{2\pi f_H}$$

- The lowest-frequency pole normally dominates the result; however, even when the poles are close to each other, the OCTC method still provides a good approximation of the 3-dB corner frequency.

$$\boxed{f_H = \left(\frac{1}{f_{p_1}} + \frac{1}{f_{p_2}} + \cdots + \frac{1}{f_{p_m}}\right)^{-1}}$$

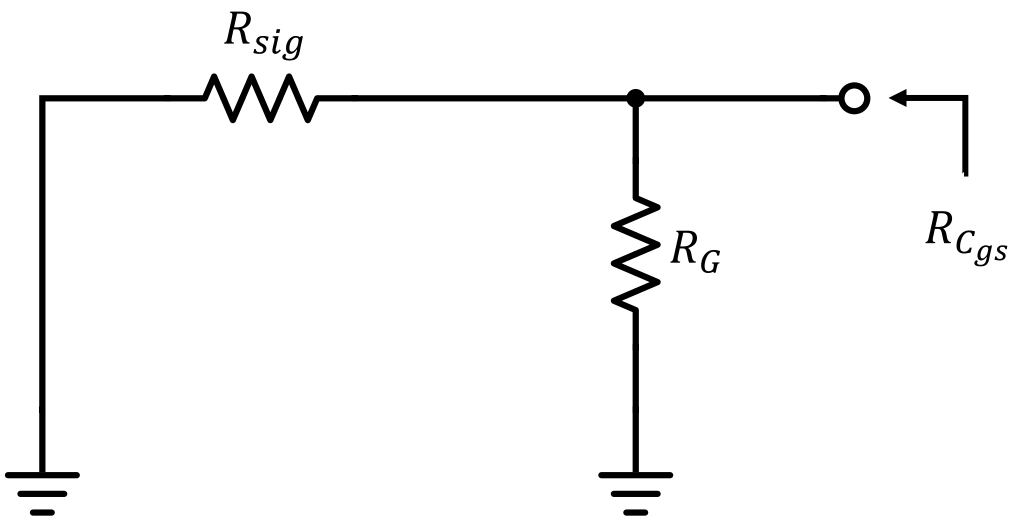
Application of the OCTC to the CS Amplifier

- We set $v_{sig} = 0$ and consider each of the three capacitances at a time, setting the other two open-circuit.



Application of the OCTC to the CS Amplifier

CGS:



$$R_{C_{gs}} = R_{sig} \parallel R_G$$

$$\boxed{\frac{1}{\tau_1} = \frac{1}{R_{C_{gs}} C_{gs}} = \frac{1}{\left(R_{sig} \parallel R_G\right) C_{gs}}}$$

Application of the OCTC to the CS Amplifier

Example: $\boldsymbol{C_{GS} = 100\ fF, C_{GD} = 10\ fF, g_m = 100mS, R_{sig} = 75\ \Omega, r_o = 10k\Omega, R_G = 1M\Omega, R_D = 500\ \Omega, R_L = 250\ \Omega}$

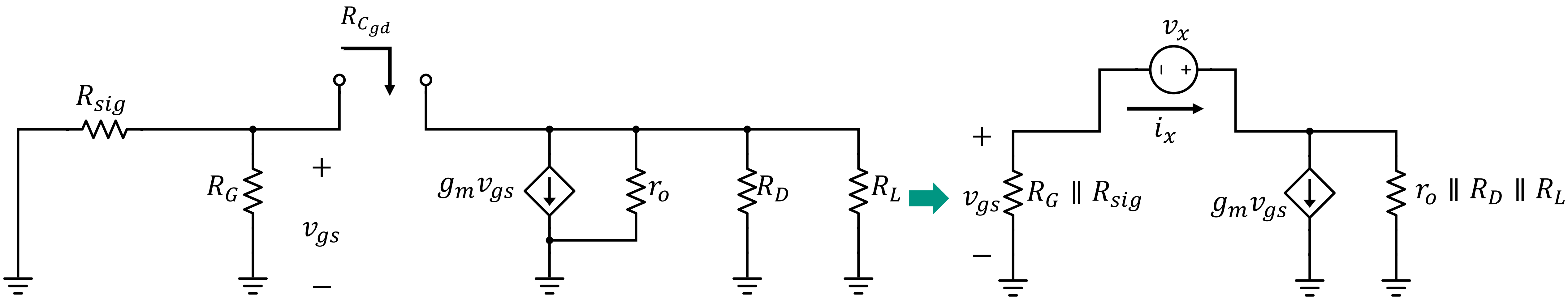
$$R_{C_{gs}} = R_{sig} \parallel R_G$$

$$\boxed{\frac{1}{\tau_1} = \frac{1}{R_{C_{gs}} C_{gs}} = \frac{1}{\left(R_{sig} \parallel R_G\right) C_{gs}}}$$

$$\tau_1 = 7.5\ \text{psec}$$

Application of the OCTC to the CS Amplifier

CGD



$$R'_{sig} = R_G \parallel R_{sig}$$

$$R'_L = r_o \parallel R_D \parallel R_L$$

$$R_{C_{gd}} = \frac{v_x}{i_x} = R'_{sig}(1 + g_m R'_L) + R'_L$$

$$\boxed{\frac{1}{\tau_2} = \frac{1}{R_{C_{gd}} C_{gd}} = \frac{1}{\left(R'_{sig}(1 + g_m R'_L) + R'_L\right) C_{gd}}}$$

Application of the OCTC to the CS Amplifier

Example: $\boldsymbol{C_{GS} = 100\ fF, C_{GD} = 10\ fF, g_m = 100mS, R_{sig} = 75\ \Omega, r_o = 10k\Omega, R_G = 1M\Omega, R_D = 500\ \Omega, R_L = 250\ \Omega}$

$$R'_{sig} = R_G \parallel R_{sig} = \boldsymbol{75\ \Omega}$$

$$R'_L = r_o \parallel R_D \parallel R_L = \boldsymbol{163\ \Omega}$$

$$R_{C_{gd}} = \frac{v_x}{i_x} = R'_{sig}(1 + g_m R'_L) + R'_L = \boldsymbol{1460\ \Omega}$$

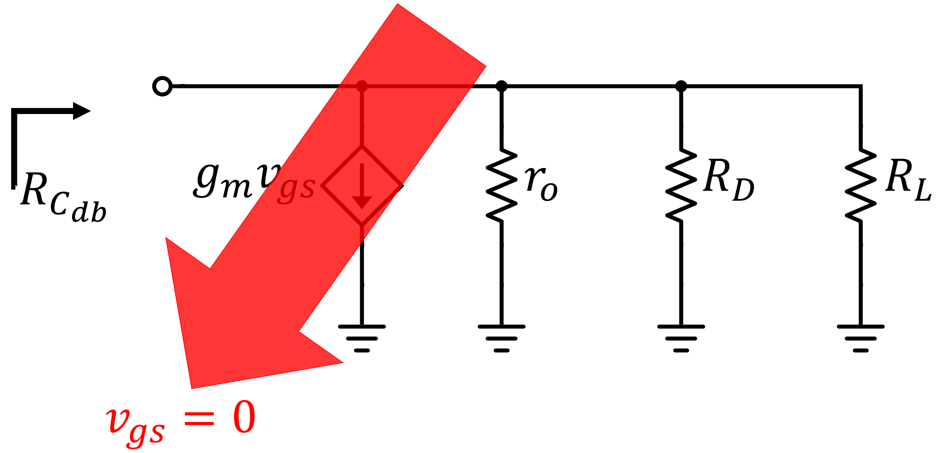
$$\boxed{\frac{1}{\tau_2} = \frac{1}{R_{C_{gd}} C_{gd}} = \frac{1}{\left(R'_{sig}(1 + g_m R'_L) + R'_L\right) C_{gd}}}$$

$$\tau_2 = 14.6\ \text{psec}$$

Application of the OCTC to the CS Amplifier

CDB

Example: $C_{GS} = 100\ fF, C_{GD} = 10\ fF, C_{DB} = 10\ fF, g_m = 100mS, R_{sig} = 75\ \Omega, r_o = 10k\Omega, R_G = 1M\Omega, R_D = 500\ \Omega, R_L = 250\ \Omega$

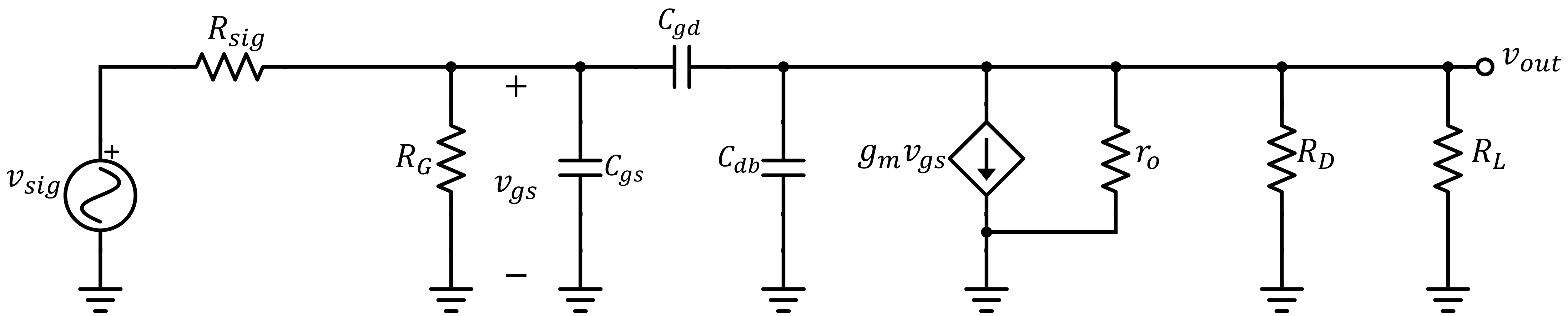


$$R_{C_{db}} = r_o \parallel R_D \parallel R_L = \mathbf{163\ \Omega}$$

$$\boxed{\frac{1}{\tau_3} = \frac{1}{R_{C_{db}} C_{db}} = \frac{1}{(r_o \parallel R_D \parallel R_L) C_{db}}}$$

$$\tau_3 = 1.63\ \text{psec}$$

Application of the OCTC to the CS Amplifier

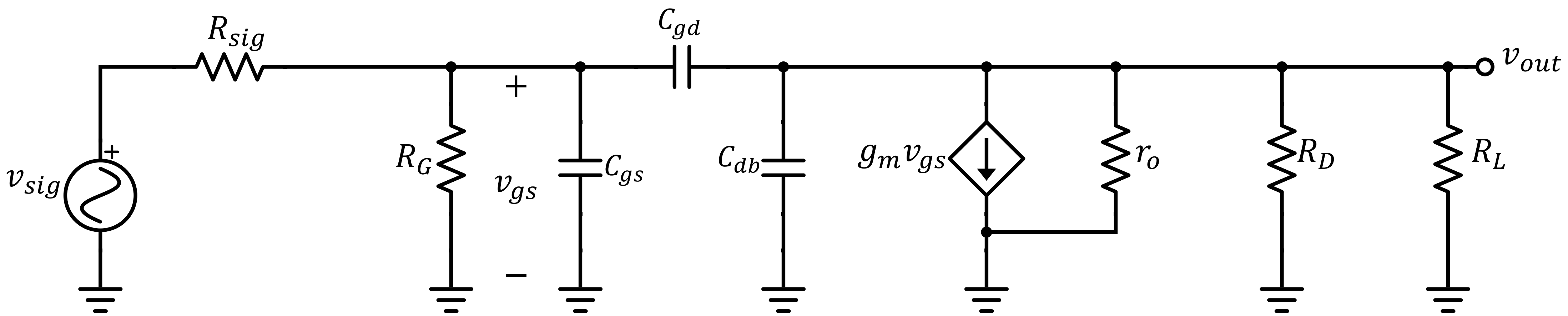


$$\boxed{f_H = \frac{1}{2\pi(\tau_1 + \tau_2 + \tau_3)}}$$

$$\boxed{f_H = \frac{1}{2\pi\left[\left(R_{sig} \parallel R_G\right)C_{gs} + R_{C_{gd}}C_{gd} + \left(r_o \parallel R_D \parallel R_L\right)C_{db}\right]}}$$

$$R_{C_{gd}} = R'_{sig}(1 + g_m R'_L) + R'_L$$

Application of the OCTC to the CS Amplifier

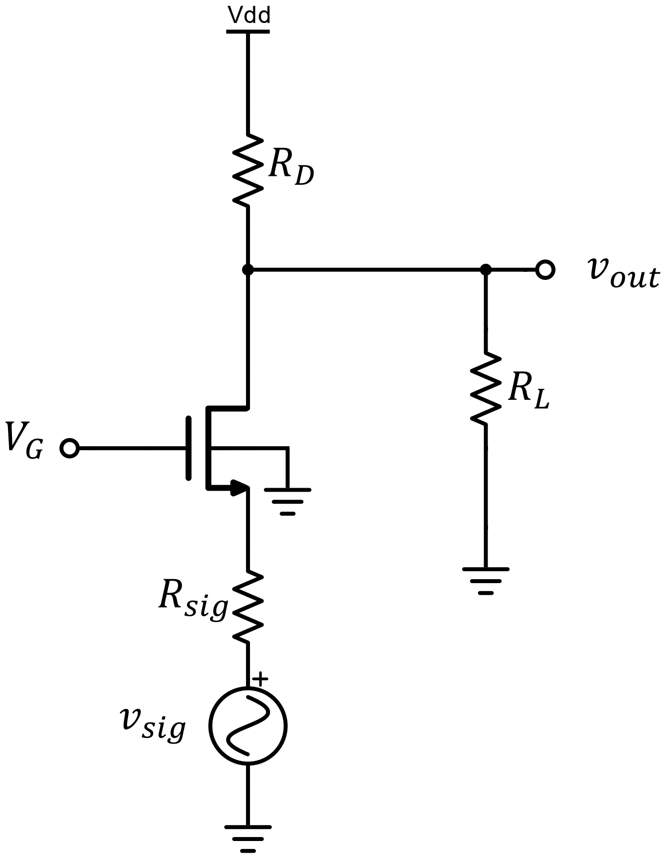


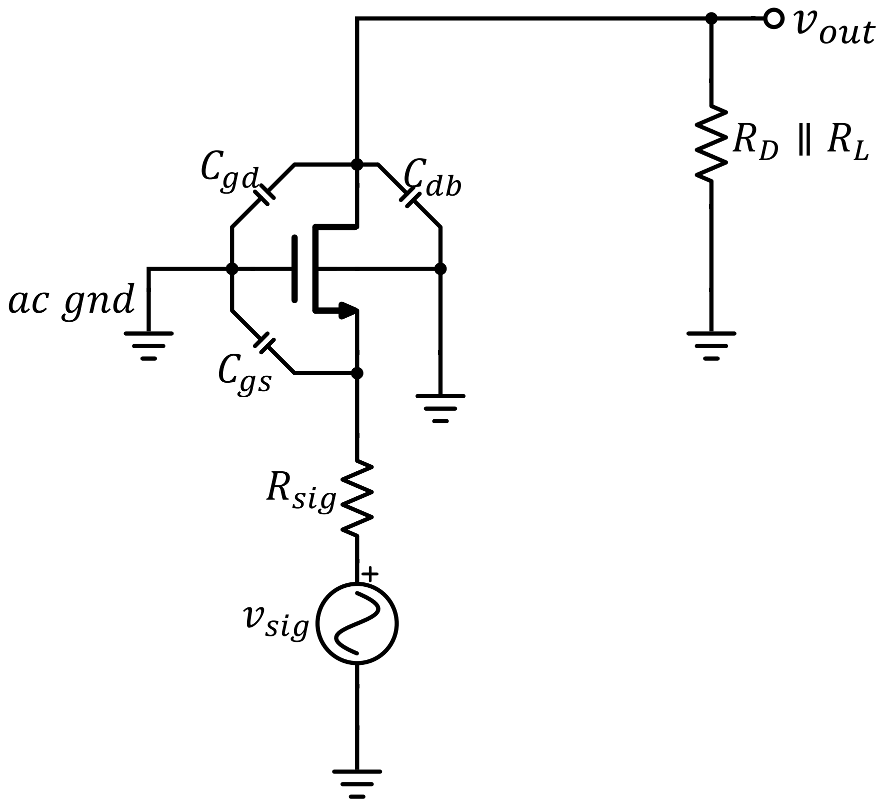
$$\boxed{f_H = \frac{1}{2\pi(\tau_1 + \tau_2 + \tau_3)}} = \frac{1}{2\pi(23.73\ \text{psec})} = \mathbf{6.7\ GHz}$$

$$\boxed{f_H = \frac{1}{2\pi\left[\left(R_{sig} \parallel R_G\right)C_{gs} + R_{C_{gd}}C_{gd} + (r_o \parallel R_D \parallel R_L)C_{db}\right]}}$$

$$R_{C_{gd}} = R'_{sig}(1 + g_m R'_L) + R'_L$$

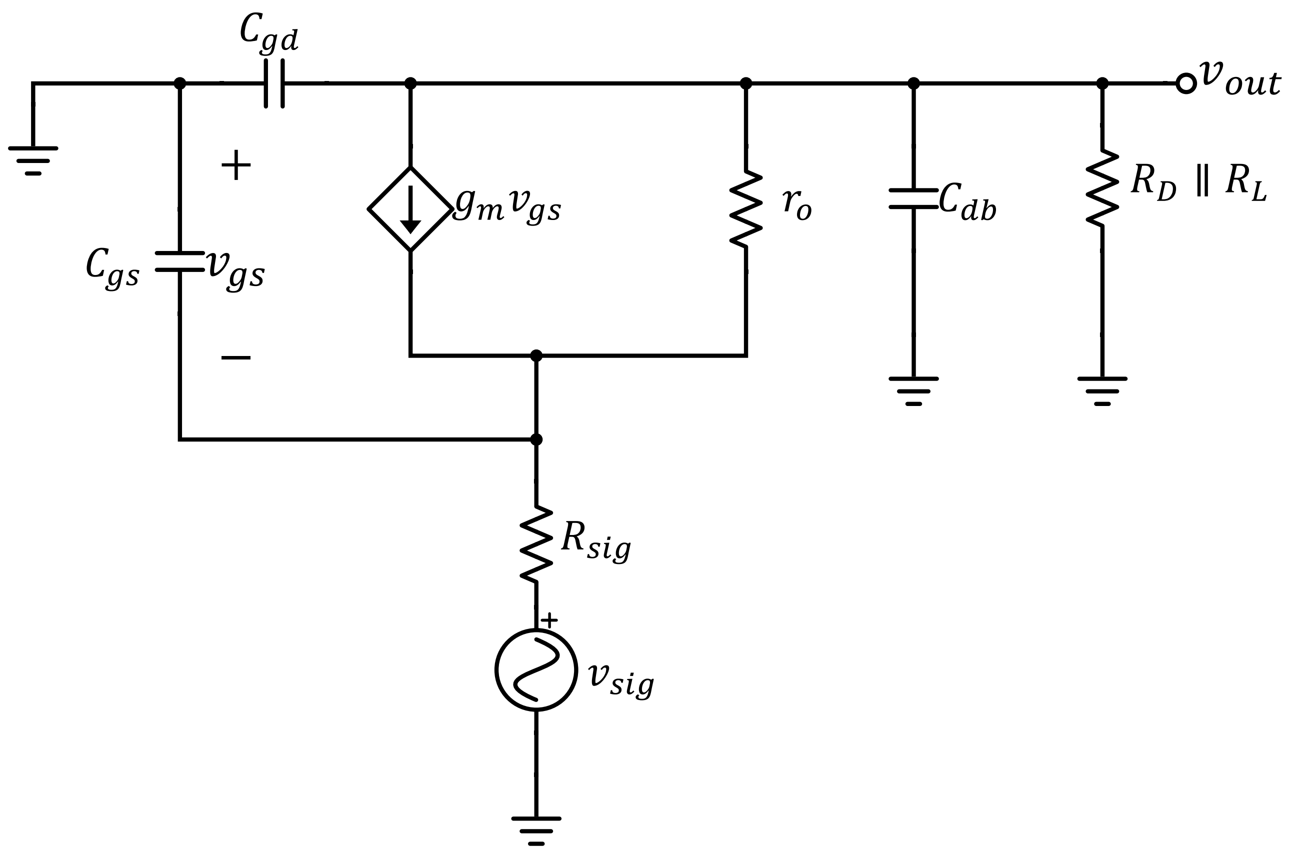
Common-Gate Amplifier High-Frequency Response





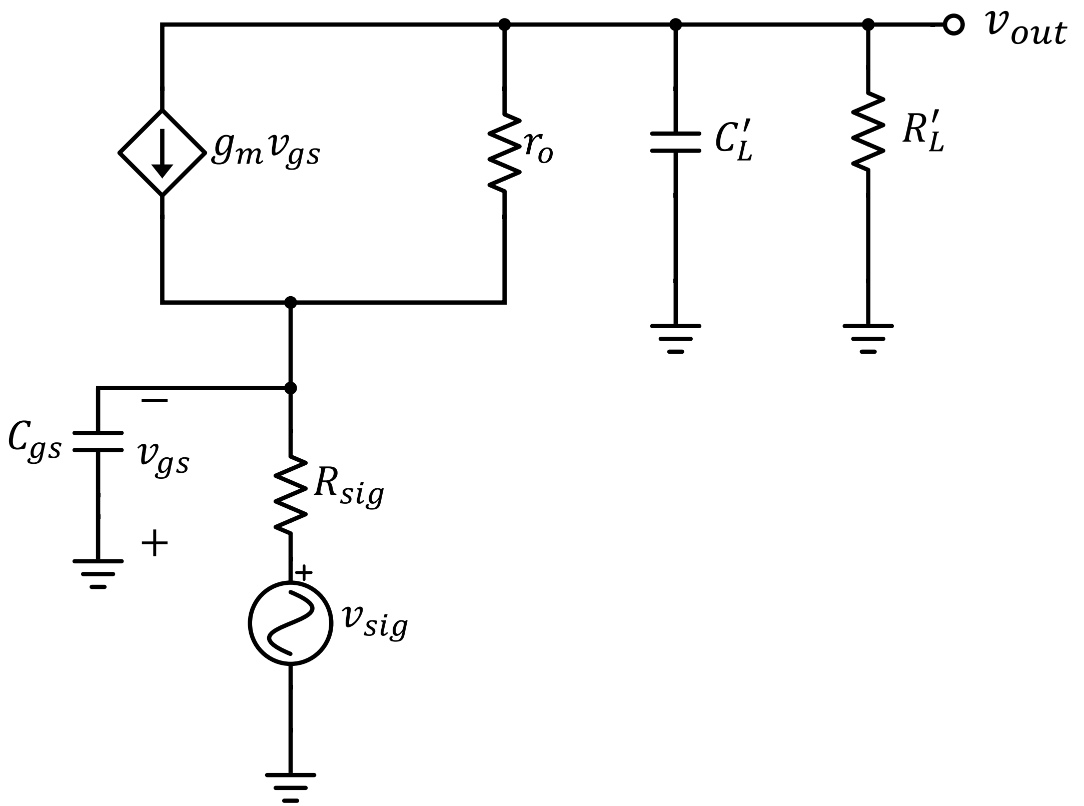
Common-Gate Amplifier High-Frequency Response

Small-Signal Equivalent



Common-Gate Amplifier High-Frequency Response

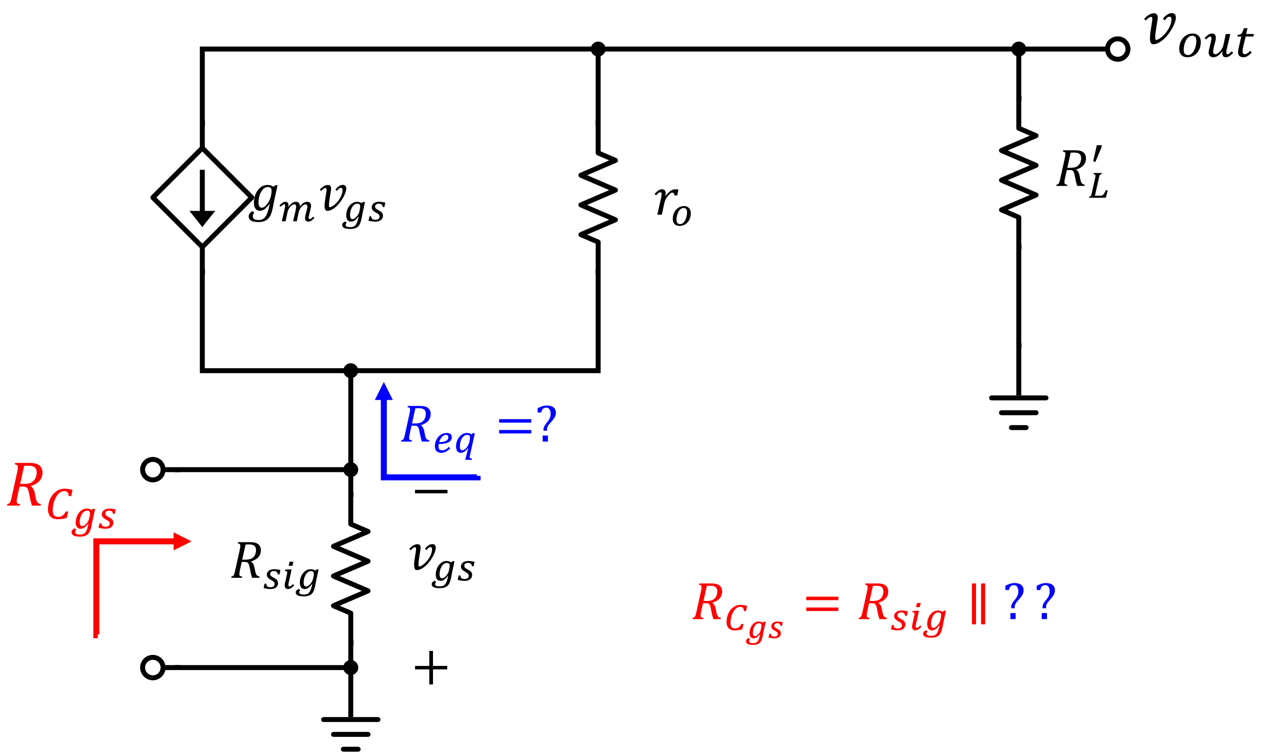
Small-Signal Equivalent Simplified



$$C'_L = C_{gd} + C_{db}$$
$$R'_L = R_D \parallel R_L$$

Common-Gate Amplifier High-Frequency Response

Open-Circuit Time Constants

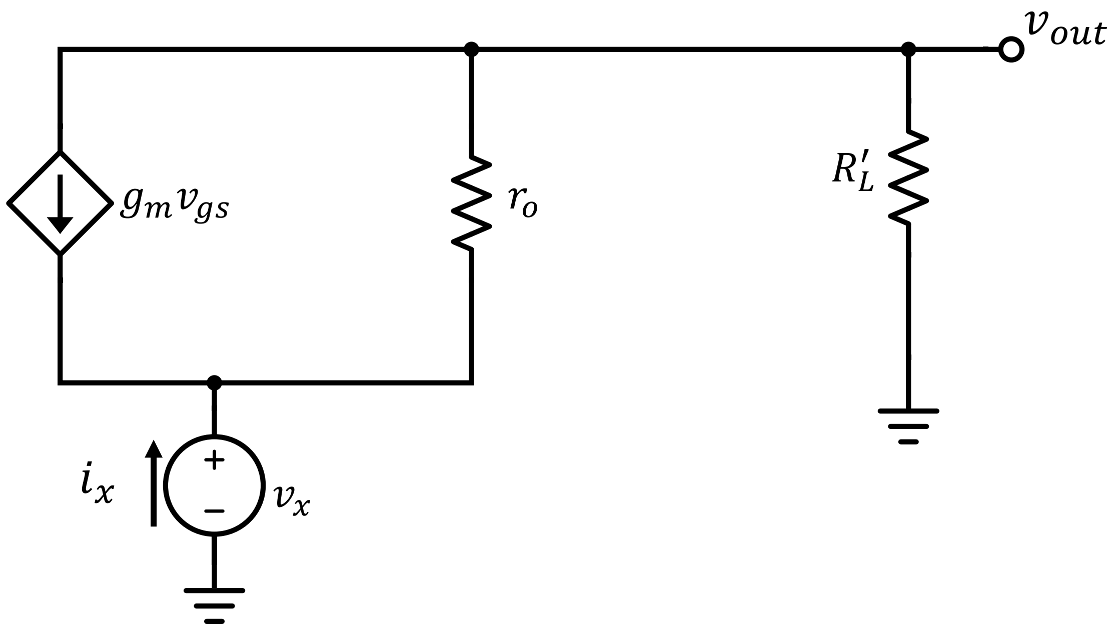


$$C'_L = C_{gd} + C_{db}$$
$$R'_L = R_D \parallel R_L$$

$$R_{C_{gs}} = R_{sig} \parallel ??$$

Common-Gate Amplifier High-Frequency Response

Open-Circuit Time Constants



$$\boxed{R_{eq} = \frac{v_x}{i_x} = \frac{R'_L\left(\frac{1}{g_m} \parallel r_o\right)}{R'_L \parallel r_o} = \frac{r_o + R'_L}{1 + g_m r_o}}$$

$$C'_L = C_{gd} + C_{db}$$
$$R'_L = R_D \parallel R_L$$

$$\boxed{R_{C_{gs}} = R_{sig} \parallel R_{eq}}$$

$$\frac{1}{\tau_1} = \frac{1}{2\pi\left[R_{sig} \parallel \left(\frac{r_o + R'_L}{1 + g_m r_o}\right)\right] C_{gs}}$$

$For\, R'_L \ll r_o, \frac{1}{g_m} \ll r_o:$

$$\boxed{\frac{1}{\tau_1} = \frac{1}{\left(R_{sig} \parallel 1/g_m\right) C_{gs}}}$$

Common-Gate Amplifier High-Frequency Response

Open-Circuit Time Constants

Example: $\boldsymbol{C_{GS} = 100\,fF, C_{GD} = 10\,fF, C_{DB} = 10\,fF, g_m = 100mS, R_{sig} = 75\,\Omega, r_o = 10k\Omega, R_G = 1M\Omega, R_D = 500\,\Omega, R_L = 250\,\Omega}$

$$\boxed{R_{eq} = \frac{v_x}{i_x} = \frac{R_L'\left(\frac{1}{g_m} \parallel r_o\right)}{R_L' \parallel r_o} = \frac{r_o + R_L'}{1 + g_m r_o}}$$

$$C_L' = C_{gd} + C_{db}$$
$$R_L' = R_D \parallel R_L$$

$$R_{sig} \parallel \frac{1}{g_m} = 8.8\,\Omega$$

$$\boxed{R_{C_{gs}} = R_{sig} \parallel R_{eq}}$$

$$R_{sig} \parallel \left(\frac{r_o + R_L'}{1 + g_m r_o}\right) = 8.945\Omega$$

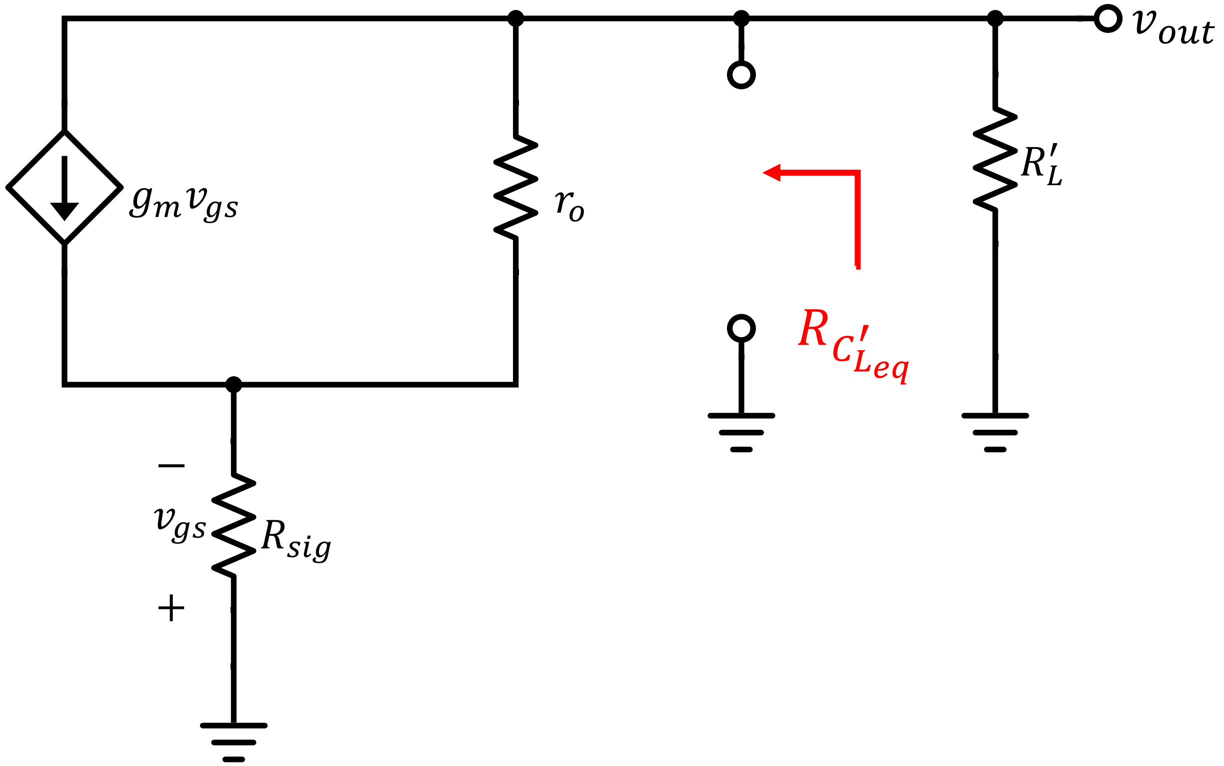
$$\frac{1}{\tau_1} = \frac{1}{2\pi\left[R_{sig} \parallel \left(\frac{r_o + R_L'}{1 + g_m r_o}\right)\right] C_{gs}}$$

$$\tau_1 = 0.88\text{ psec}$$

$For\, R_L' \ll r_o, \frac{1}{g_m} \ll r_o:$

$$\boxed{\frac{1}{\tau_1} = \frac{1}{\left(R_{sig} \parallel 1/g_m\right) C_{gs}}}$$

Common-Gate Amplifier High-Frequency Response



$$C'_L = C_{gd} + C_{db}$$
$$R'_L = R_D \parallel R_L$$

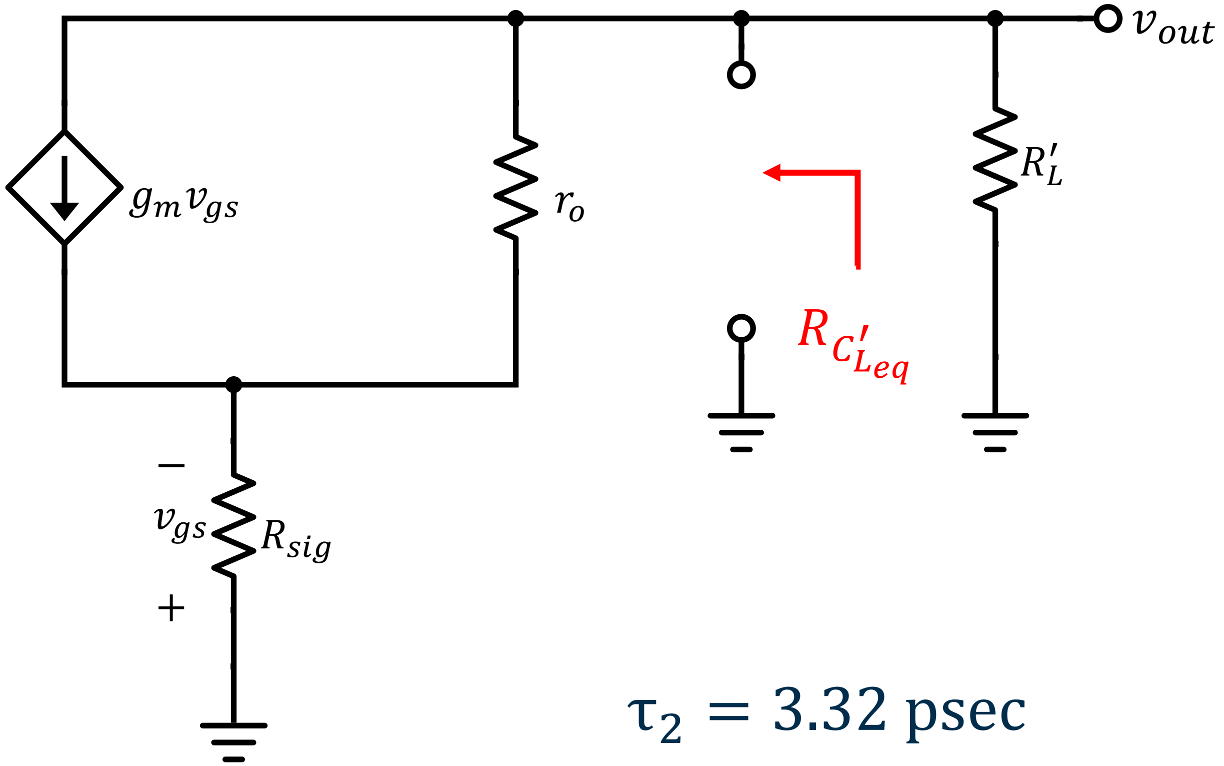
$$R_{C'_{Leq}} = g_m R_{sig} r_o + R_{sig} + r_o \approx g_m R_{sig} r_o$$

(Recall Cascode analysis)

$$\boxed{\frac{1}{\tau_2} = \frac{1}{\left(R'_L \parallel g_m R_{sig} r_o\right) C'_L}}$$

Common-Gate Amplifier High-Frequency Response

Example: $C_{GS} = 100\ fF, C_{GD} = 10\ fF, C_{DB} = 10\ fF, g_m = 100mS, R_{sig} = 75\ \Omega, r_o = 10k\Omega, R_G = 1M\Omega, R_D = 500\ \Omega, R_L = 250\ \Omega$



$$C'_L = C_{gd} + C_{db}$$
$$R'_L = R_D \parallel R_L$$

$$R_{C'_{Leq}} = g_m R_{sig} r_o + R_{sig} + r_o \approx g_m R_{sig} r_o$$

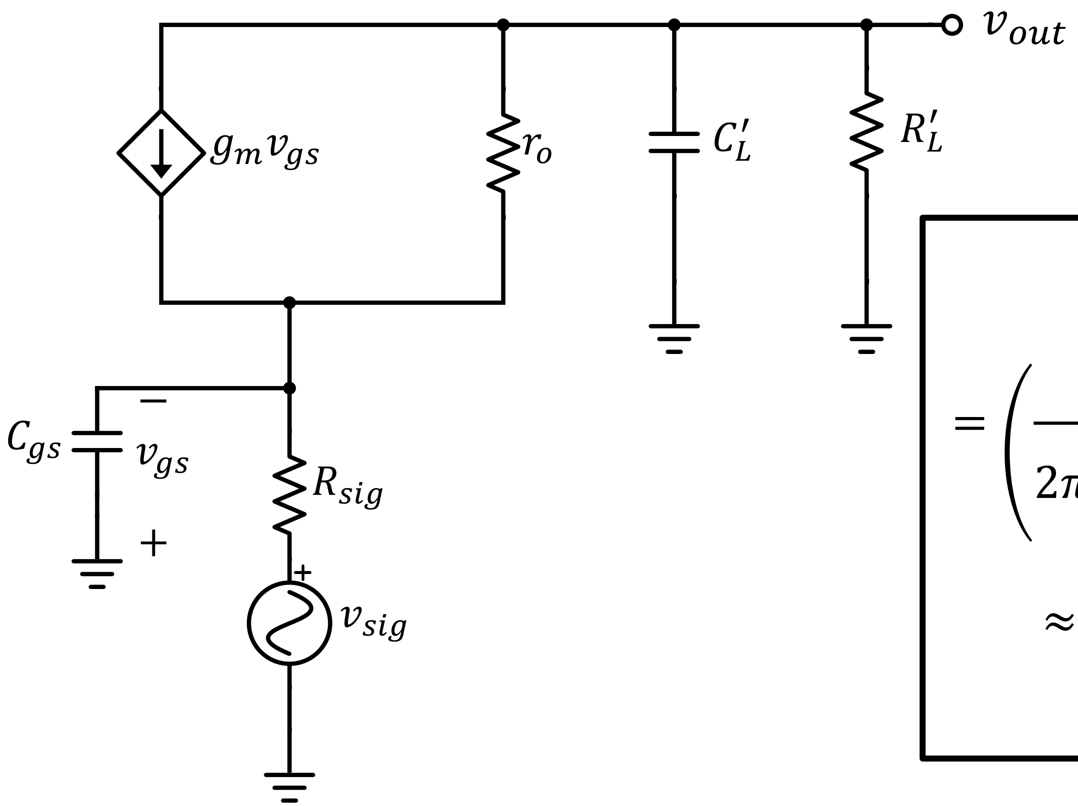
(Recall Cascode analysis)

$$\boxed{\frac{1}{\tau_2} = \frac{1}{(R'_L \parallel g_m R_{sig} r_o)C'_L}}$$

$$\underbrace{\underbrace{R'_L}_{166\ \Omega} \parallel \underbrace{g_m R_{sig} r_o}_{75k\Omega}}_{165.6\ \Omega}$$

$$\tau_2 = 3.32\ \text{psec}$$

Common-Gate Amplifier High-Frequency Response



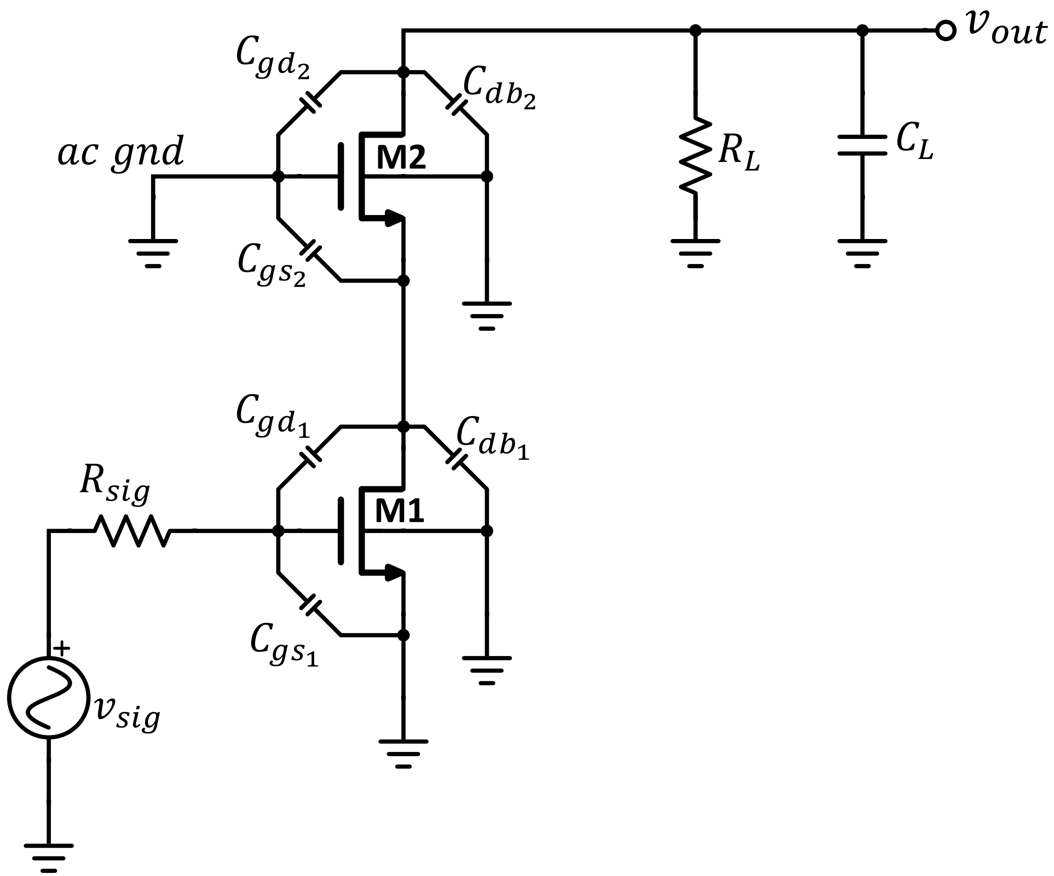
$$C_L' = C_{gd} + C_{db}$$
$$R_L' = R_D \parallel R_L$$

$$f_H = \frac{1}{2\pi(\tau_1 + \tau_2)} = \mathbf{37.8\ GHz}$$

$$= \left(\frac{1}{2\pi \left[R_{sig} \parallel \left(\frac{r_o + R_L'}{1 + g_m r_o} \right) \right] C_{gs}} + \frac{1}{2\pi \left(R_L' \parallel g_m R_{sig} r_o \right) C_L'} \right)^{-1}$$

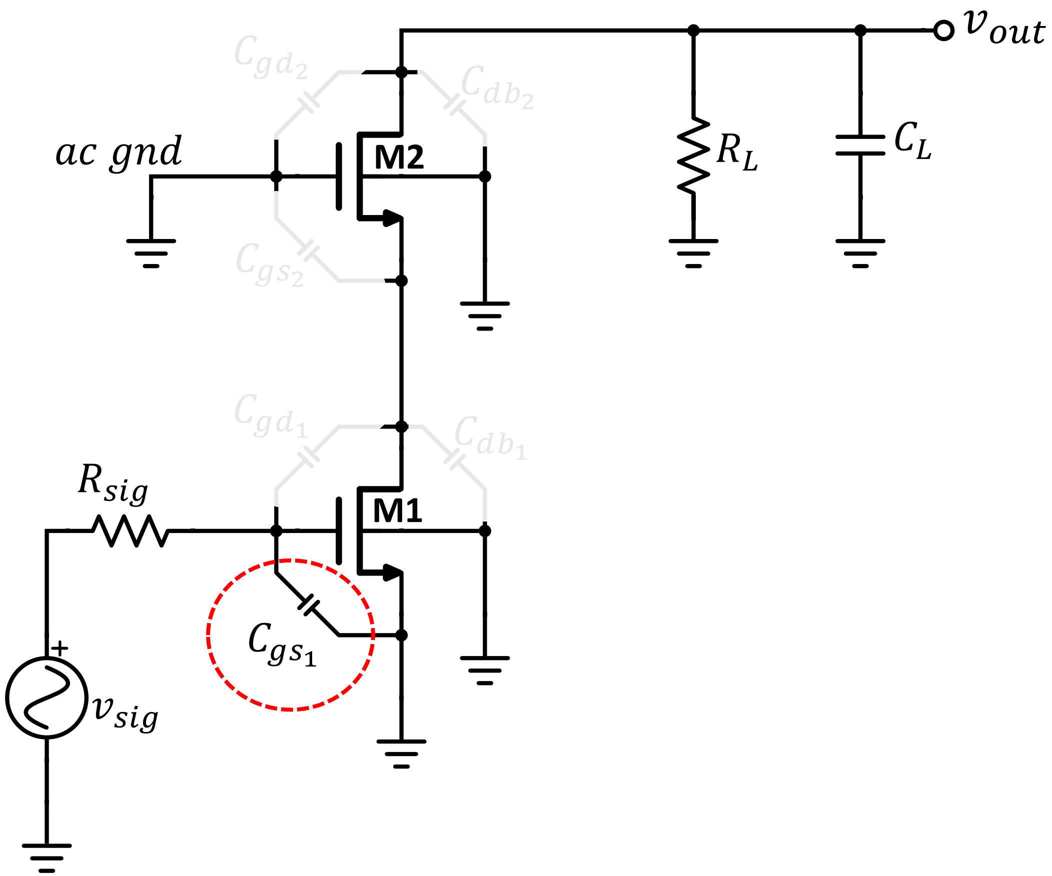
$$\approx \left(\frac{1}{2\pi \left[R_{sig} \parallel \frac{1}{g_m} \right] C_{gs}} + \frac{1}{2\pi \left(R_L' \parallel g_m R_{sig} r_o \right) C_L'} \right)^{-1}$$

Cascode Amplifier High-Frequency Response



Cascode Amplifier High-Frequency Response

Example: $C_{GS} = 100\ fF, C_{GD} = 10\ fF, C_{DB} = 10\ fF, g_m = 100mS, R_{sig} = 75\ \Omega, r_o = 10k\Omega, R_G = 1M\Omega, R_D = 500\ \Omega, R_L = 250\ \Omega$

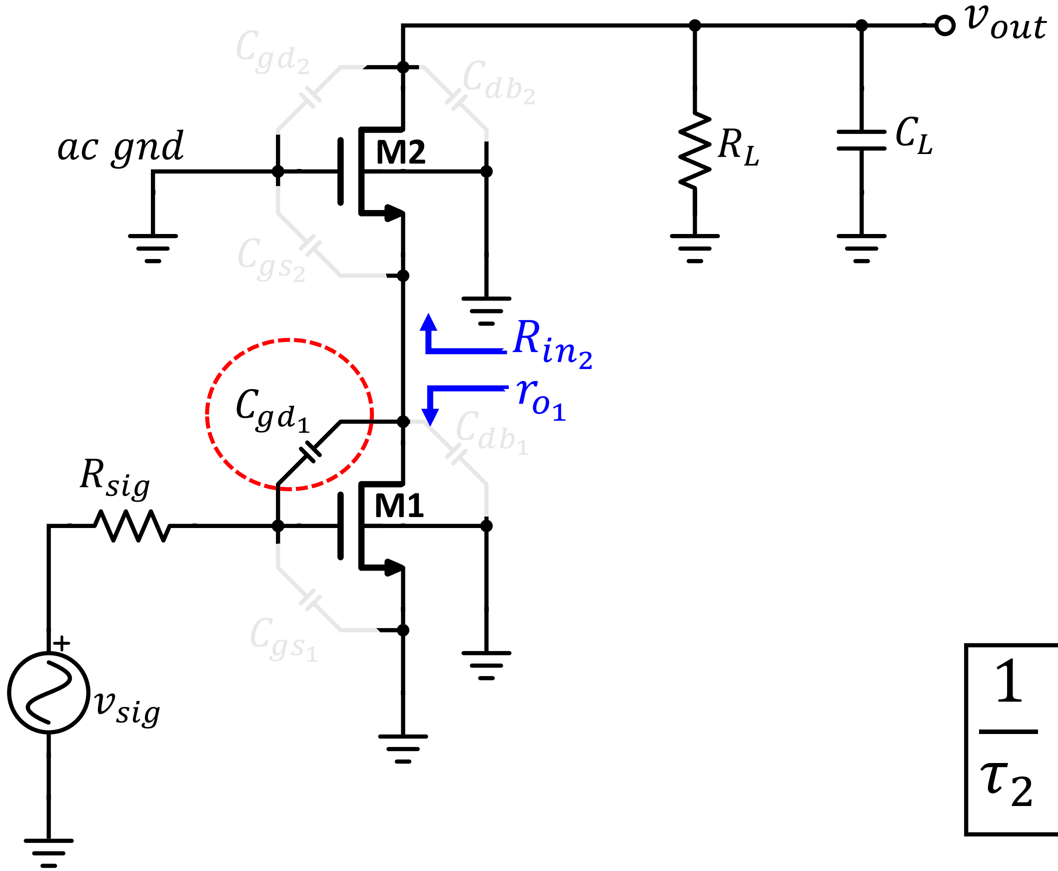


- C_{gs_1} sees a resistance R_{sig}.

$$\boxed{\frac{1}{\tau_1} = \frac{1}{R_{sig} C_{gs_1}}}$$

$$\tau_1 = 7.5 \text{ psec}$$

Cascode Amplifier High-Frequency Response



- C_{gd_1} sees a resistance R_{gd_1}, which can be obtained by adapting the formula derived in OCTC analysis of CS amplifier.

$$R_{gd_1} = g_{m_1} R_{d_1} R_{sig} + R_{sig} + R_{d_1}$$

$$R_{d_1} = r_{o_1} \parallel R_{in_2}$$

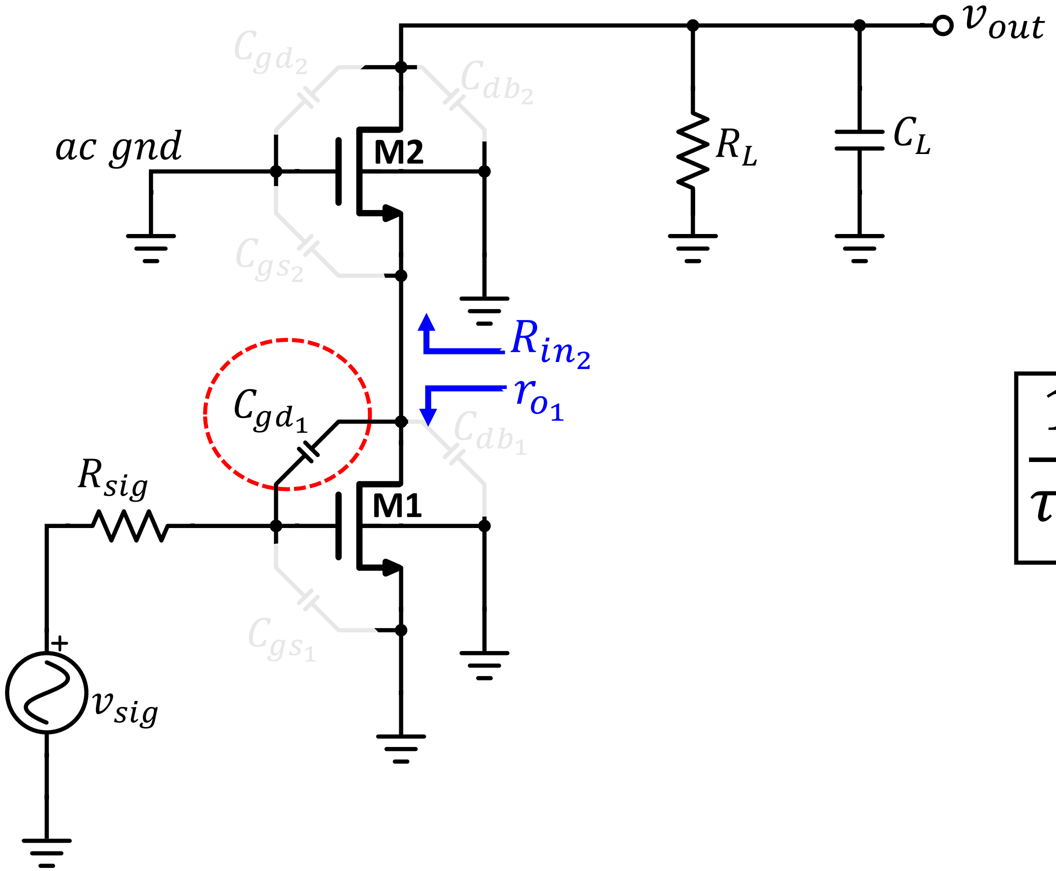
- R_{in_2} was derived in CG amplifier analysis:

$$R_{in_2} = \frac{r_{o_2} + R_L}{1 + g_{m_2} r_{o_2}}$$

$$\boxed{\frac{1}{\tau_2} = \frac{1}{R_{gd_1} C_{gd_1}} = \frac{1}{(g_{m_1} R_{d_1} R_{sig} + R_{sig} + R_{d_1}) C_{gd_1}}}$$

Cascode Amplifier High-Frequency Response

Example: $C_{GS} = 100\ fF, C_{GD} = 10\ fF, C_{DB} = 10\ fF, g_m = 100mS, R_{sig} = 75\ \Omega, r_o = 10k\Omega, R_G = 1M\Omega, R_L = 166\ \Omega$



$$R_{gd_1} = g_{m_1} R_{d_1} R_{sig} + R_{sig} + R_{d_1}$$

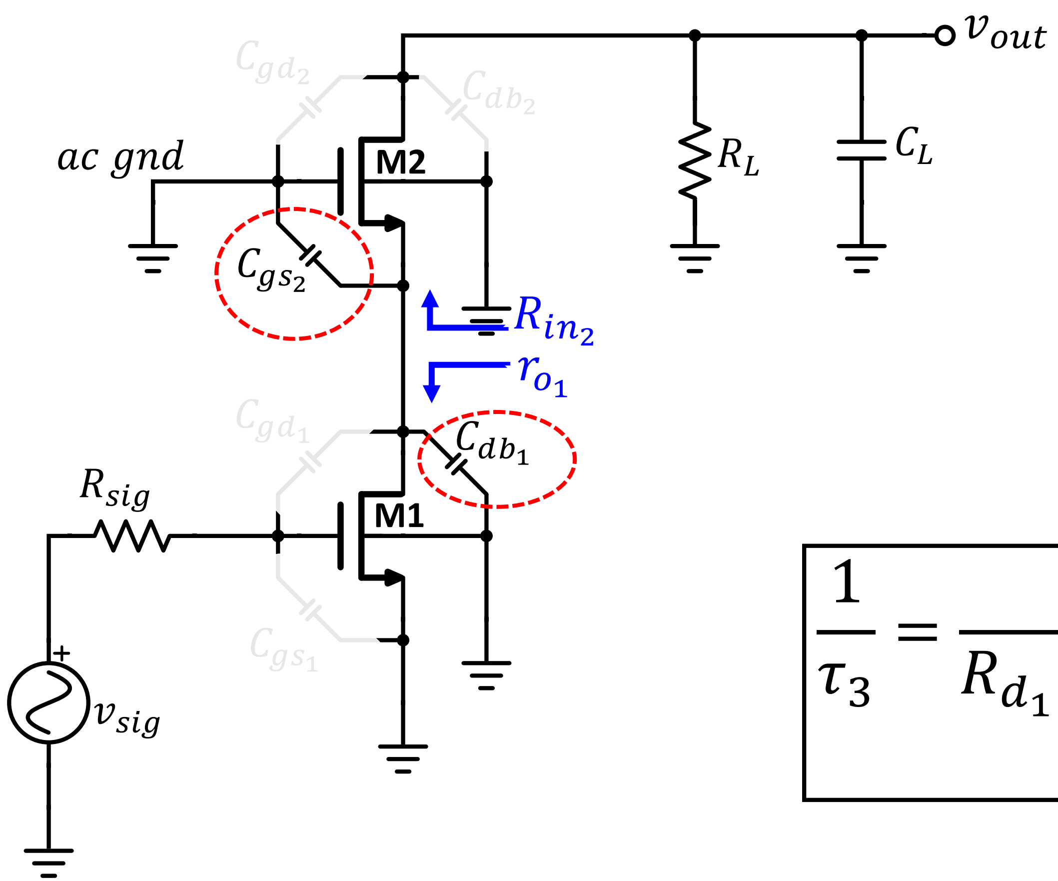
$$R_{d_1} = r_{o_1} \parallel R_{in_2} = \mathbf{10.09\ \Omega}$$

$$R_{in_2} = \frac{r_{o_2} + R_L}{1 + g_{m_2} r_{o_2}} = \mathbf{10.1\ \Omega}$$

$$\boxed{\frac{1}{\tau_2} = \frac{1}{R_{gd_1} C_{gd_1}} = \frac{1}{(g_{m_1} R_{d_1} R_{sig} + R_{sig} + R_{d_1}) C_{gd_1}}}$$

$$\tau_2 = 1.6\ \text{psec}$$

Cascode Amplifier High-Frequency Response



- Capacitance $(C_{db_1} + C_{gs_2})$ sees R_{d_1}.
- R_{d_1} was determined in previous slide as given:

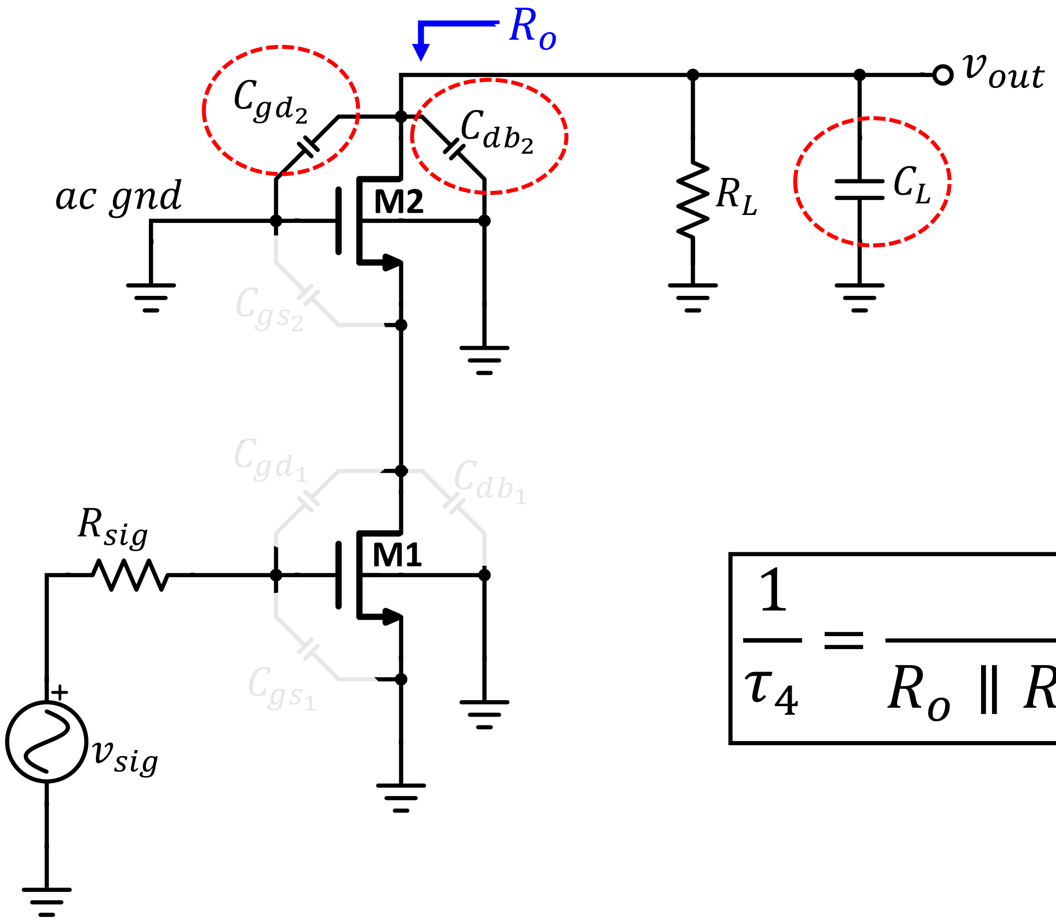
$$R_{d_1} = r_{o_1} \parallel R_{in_2} = r_{o_1} \parallel \frac{r_{o_2} + R_L}{1 + g_{m_2} r_{o_2}}$$

$$\tau_3 = 0.2 \text{ psec}$$

$$\boxed{\frac{1}{\tau_3} = \frac{1}{R_{d_1}(C_{db_1} + C_{gs_2})} = \frac{1}{\left(r_{o_1} \parallel \frac{r_{o_2} + R_L}{1 + g_{m_2} r_{o_2}}\right)(C_{db_1} + C_{gs_2})}}$$

Cascode Amplifier High-Frequency Response

Example: $C_{GS} = 100\ fF, C_{GD} = 10\ fF, C_{DB} = 10\ fF, g_m = 100mS, R_{sig} = 75\ \Omega, r_o = 10k\Omega, R_G = 1M\Omega, R_L = 166\ \Omega, C_L = 0$



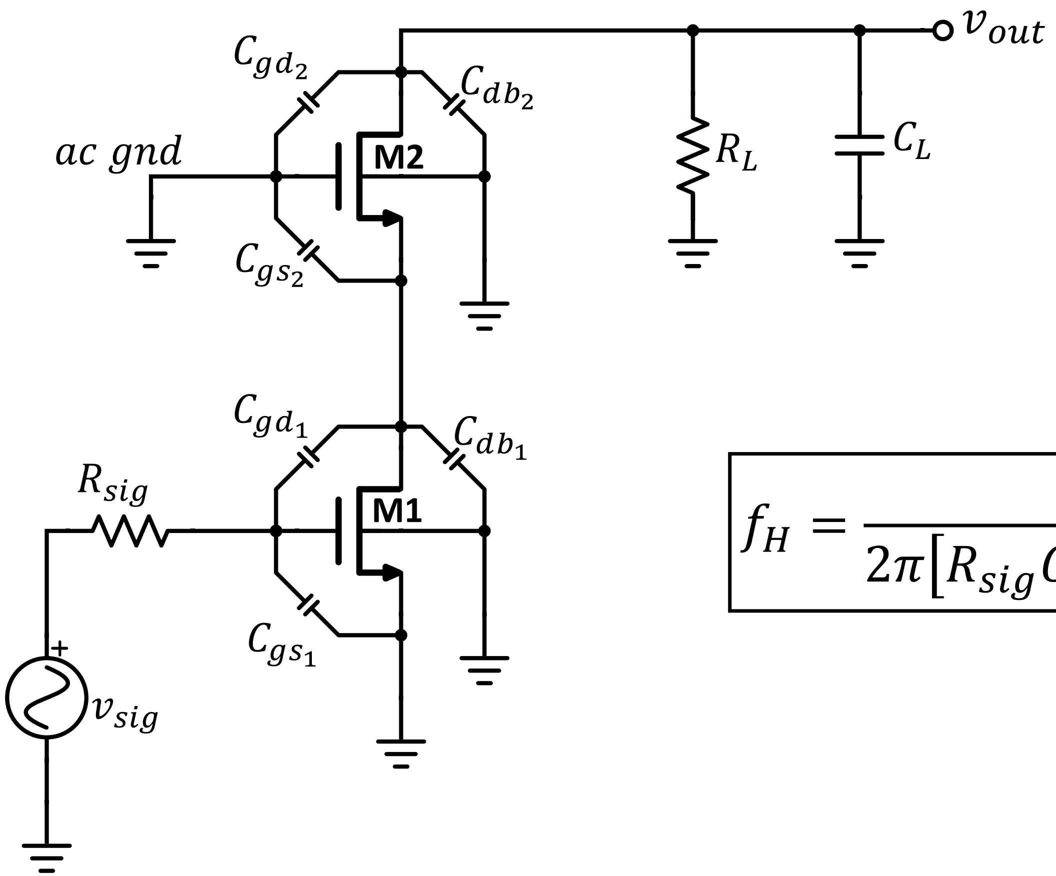
- Capacitance $\left(C_L + C_{gd_2} + C_{db_2}\right)$ see a resistance $(R_L \parallel R_o)$.
- R_o is the output resistance of the cascode amplifier, given by:

$$R_o = g_{m_2} r_{o_2} r_{o_1} + r_{o_1} + r_{o_2} \approx g_{m_2} r_{o_2} r_{o_1}$$

$$\boxed{\frac{1}{\tau_4} = \frac{1}{R_o \parallel R_L\left(C_L + C_{gd_2} + C_{db_2}\right)} \approx \frac{1}{R_L\left(C_L + C_{gd_2} + C_{db_2}\right)}}$$

$$\tau_4 = 3.32\ \text{psec}$$

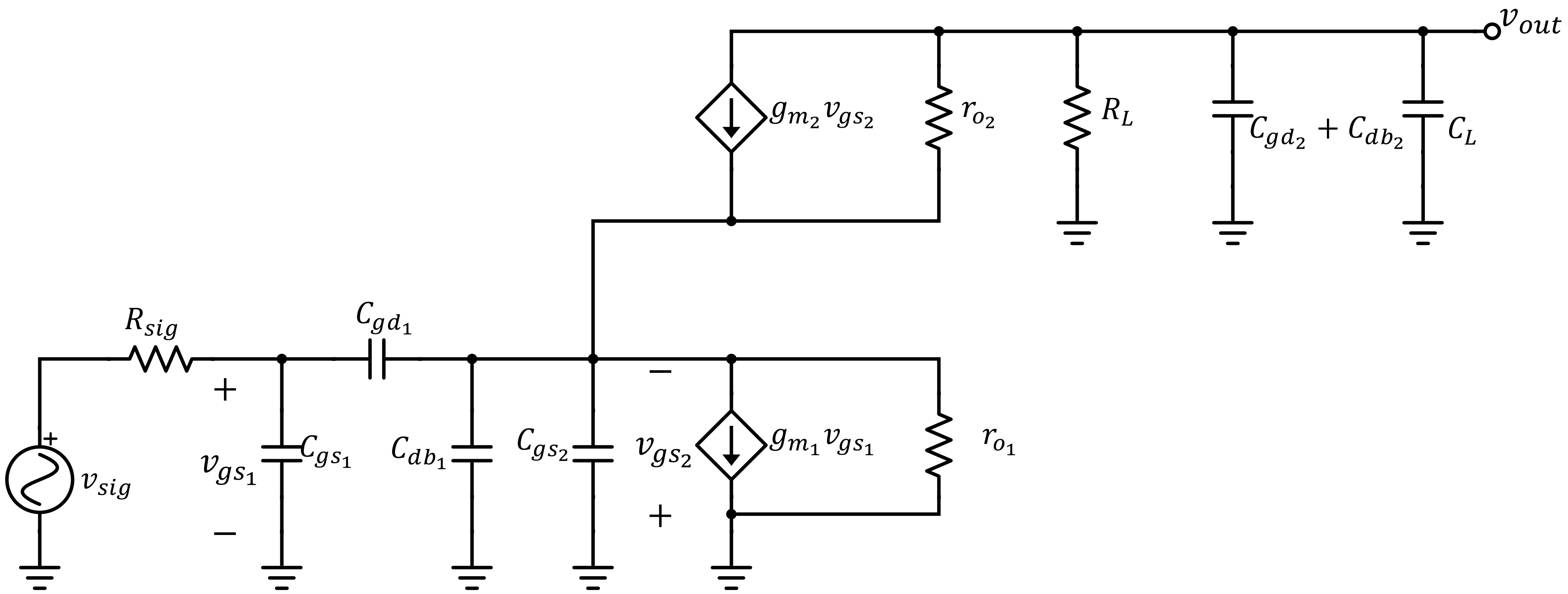
Cascode Amplifier High-Frequency Response



$$\boxed{f_H = \frac{1}{2\pi(\tau_1 + \tau_2 + \tau_3 + \tau_4)}} = \mathbf{12.6\ GHz}$$

$$\boxed{f_H = \frac{1}{2\pi\left[R_{sig}C_{gs_1} + R_{gd_1}C_{gd_1} + R_{d_1}(C_{db_1} + C_{gs_2}) + R_o\left(C_L + C_{gd_2} + C_{db_2}\right)\right]}}$$

Cascode Amplifier High-Frequency Response Small-Signal Model



2^{nd} Order Amplifiers

- For the Cascode example: we have seen that the OCTC at input ($\tau_1 = 7.5\ \mathrm{psec}$) and output ($\tau_4 = 3.3\ \mathrm{psec}$) comparable in magnitude
- Due to good isolation between input and output, these indeed can be seen as independent pole frequencies → Dominant pole approximation does not hold
- In many cases, we need to handle such circuits not as a first-order but as a second-order system with certain implications – this will be revisited in topic: Feedback and Stability

2nd Order Amplifiers

- The second-order system will be described with characteristic equation in the denominator

$$\boldsymbol{s^2 + s\frac{\omega_0}{Q} + \omega_0^2 = 0}$$

- In case of dominant pole, e.g. $\frac{1}{\omega_{p_2}} \approx 0$

$$1 + s\left(\frac{1}{\omega_{p_1}} + \frac{1}{\omega_{p_2}}\right) + \frac{s^2}{\omega_1 \omega_{p_2}}$$

2nd Order Amplifiers

- The second-order system will be described with characteristic equation in the denominator

$$\boldsymbol{s^2 + s\frac{\omega_0}{Q} + \omega_0^2 = 0}$$

- s, the Laplace parameter with $s = \sigma + j\omega$

- Q quality factor ($\frac{1}{2Q} = \zeta$, damping factor), and ω_0 the (complex conjugate) pole frequency

2nd Order Amplifiers

- The second-order amplifier behaviour