Mikroelektronische Schaltungen und Systeme

Übung 1

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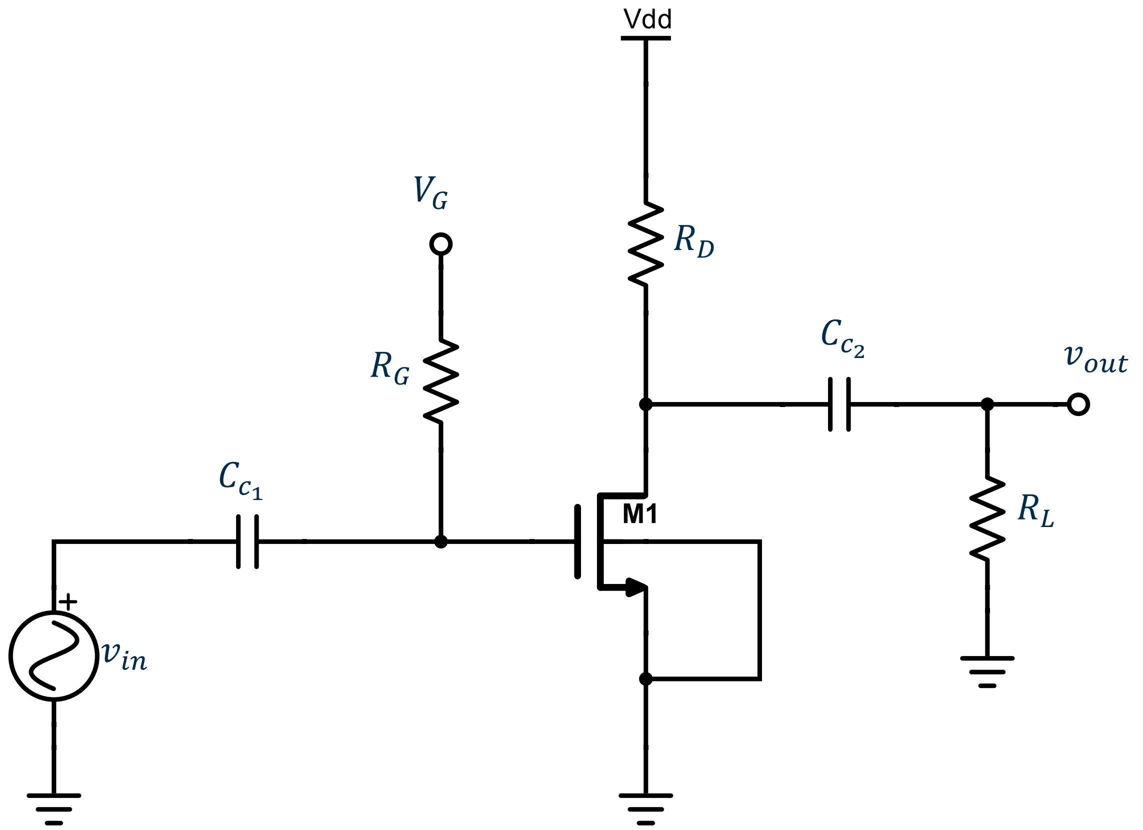
Task 1.1

In the circuit below following MOSFET and circuit parameters are provided:



- $V_{TH} = 0.7V$
- $V_{ov} = V_{GS} - V_{TH} = 0.5\ V$
- $\mu_n C_{ox} = 200 \frac{\mu A}{V^2} = k_n = \beta$
- $I_D = 1mA$
- $R_L = 6k\Omega$
- $\mathrm{R_G} \to \infty, C_{c_{1,2}} \to \infty$
- $L = 200nm$
- $A_v = \frac{v_{out}}{v_{in}} = -4$

Channel-length modulation can be ignored.

a) Calculate g_m, transistor width W, and R_D.

b) Find the minimum supply voltage $V_{DD_{min}}$ to keep the transistor in saturation and determine the DC power consumption P_{DC}.

a)

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$$\boxed{g_m = \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})}$$

$$\boxed{g_m = \sqrt{2I_D\mu_n C_{ox}\frac{W}{L}}}$$

$$\boxed{g_m = \frac{2I_D}{V_{GS}-V_{TH}}}$$

$$I_D = \frac{1}{2}\underbrace{\mu_n C_{ox}}^{\text{given}}\frac{W}{L}\Big(\overbrace{V_{GS}-V_{TH}}^{\text{given}}\Big)^2$$

given

$$1mA = \frac{1}{2}\,200\frac{\mu A}{V^2}\,\frac{W}{200n}(0.5)^2$$

Leave W alone.

$$W = \frac{(2\times10^{-3})\times(200\times10^{-9})}{(200\times10^{-6})\times(0.5)^2} = \boxed{8\mu m}$$

$$\frac{W}{L} = \frac{8\mu}{200n}$$

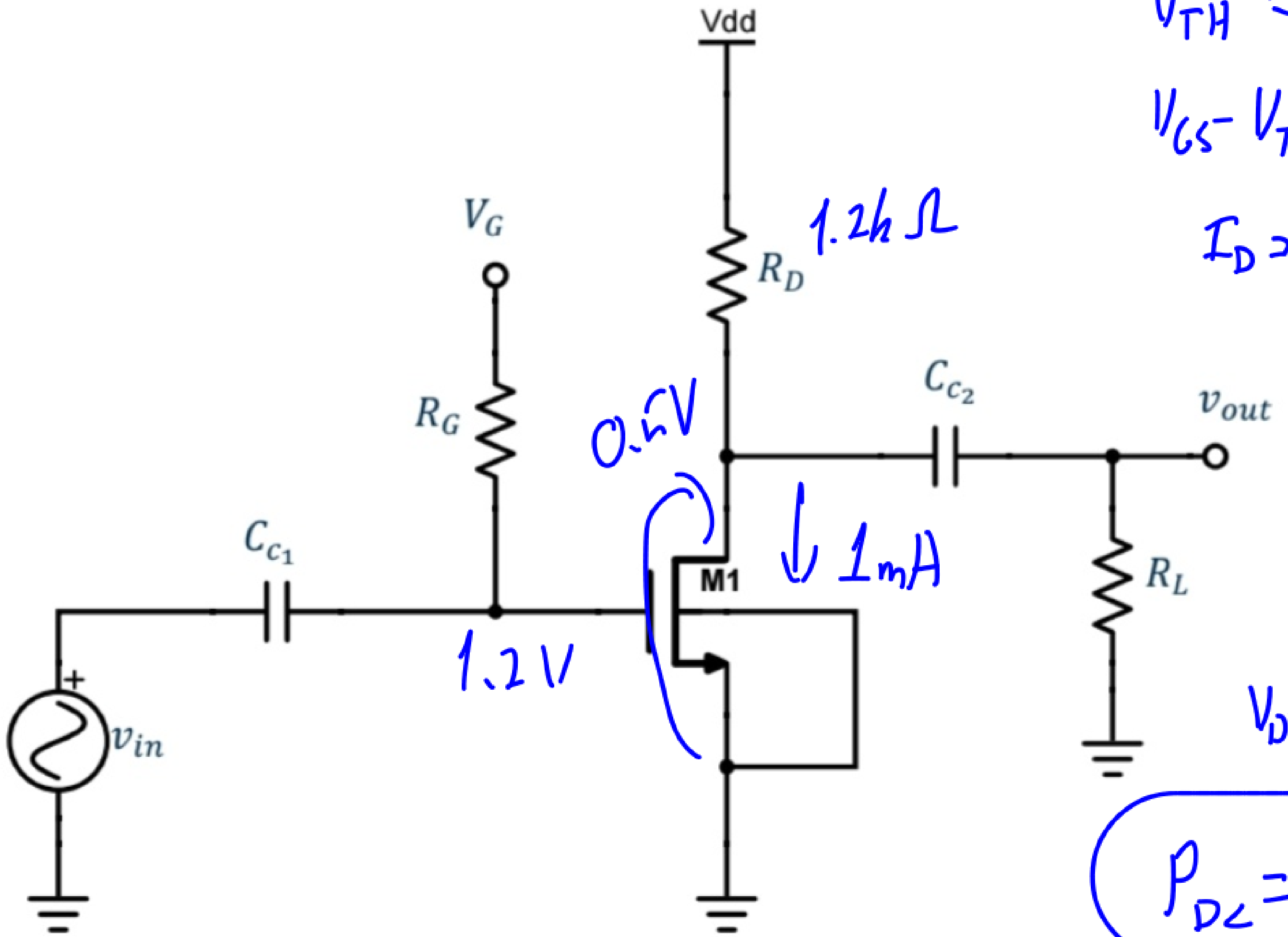
$$g_m = \frac{2I_D}{V_{ov}} = \frac{2mA}{0.5V} = \boxed{4mS}$$

$$A_v = -g_m R_{out} = -4V \qquad R_{out} = R_D \,//\, R_L$$

$$g_m = 4mS \Rightarrow R_{out} = 1k\Omega$$

$$R_D = \left(\frac{1}{1k} - \frac{1}{6k}\right)^{-1} = \frac{6k}{5} = \boxed{1.2k\Omega}$$

b)



$V_{TH} = 0.7$

$V_{GS} - V_{TH} = 0.5$

$I_D = 1mA$

$V_{GS} - V_{TH} > 0$

$V_{DS} \geq V_{OV} = 0.5$

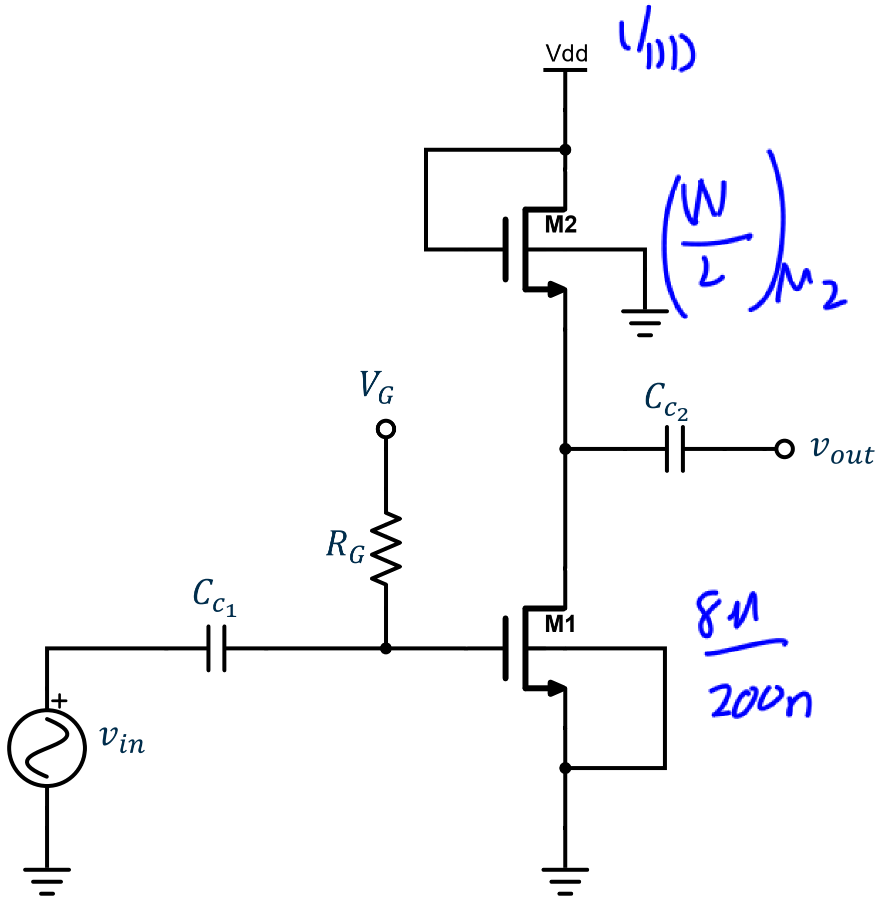
$V_D = 0.5\ V$

$V_{DD_{min}} = 0.5 + (1mA)(1.2k\Omega) = \boxed{1.7V}$

$$\boxed{P_{DC} = V_{DD} I_D = 1.7\ mW}$$

Task 1.2

The circuit from Task 1.1 will be changed as following:



- R_L is removed and R_D is replaced with a diode-connected NMOS transistor. All paramaters of M1 remains the same.

a) Determine the required $(W/L)_{M2}$ to achieve the same voltage gain magnitude $|A_v| = 4$, and calculate the new DC power consumption P_{DC}.

$I_D = 1mA$ $V_{ov} = 0.5$ $g_m = 2mS$

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- Assuming both transistors have the same $\mu_n C_{ox}$ and operate at equal drain currents, the voltage gain can be expressed as:

$$\boxed{A_v = -\frac{g_{m_1}}{g_{m_2}}\frac{1}{(1+\eta)} = -\sqrt{\frac{2I_D\mu_n C_{ox}\left(\frac{W}{L}\right)_1}{2I_D\mu_n C_{ox}\left(\frac{W}{L}\right)_2}}\frac{1}{(1+\eta)} = -\sqrt{\frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2}}\frac{1}{(1+\eta)}}$$

Bulk transconductance omitted

$$A_v = -4 = -\sqrt{\frac{(W/L)_1}{(W/L)_2}}$$

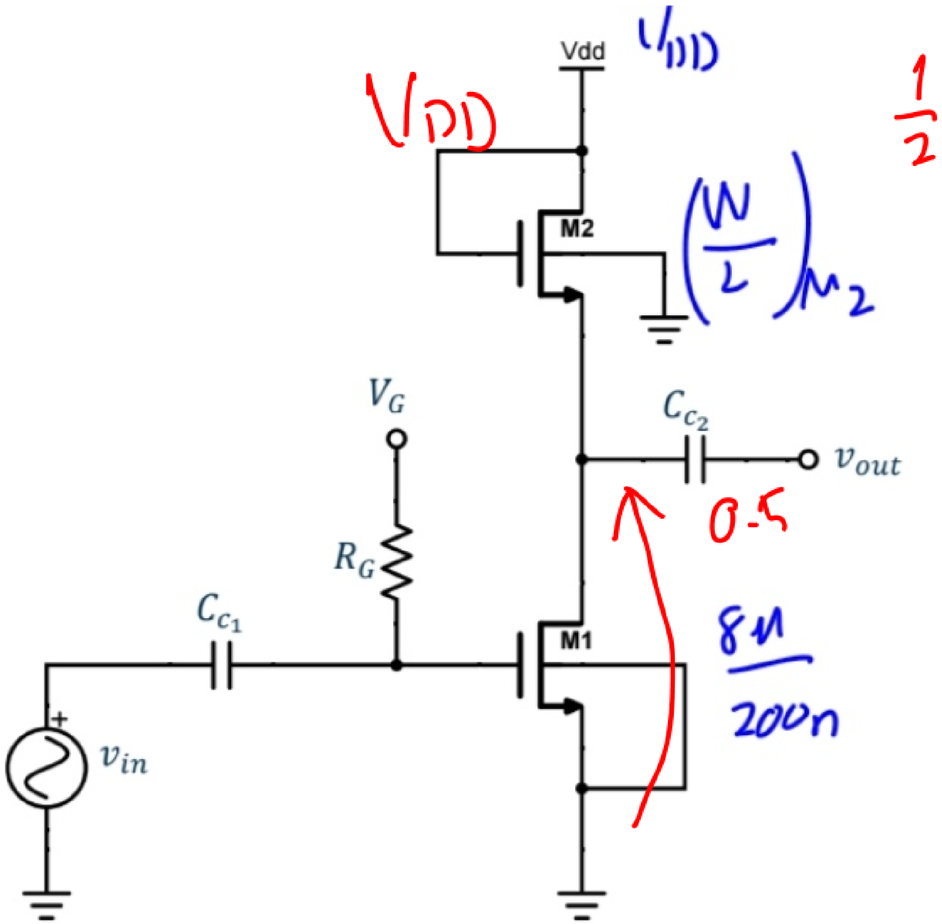
$$\left(\frac{W}{L}\right)_2 = \frac{(W/L)_1}{16}$$

$$L = 200\,nm$$

$$\boxed{W_2 = 500\,nm}$$

$$\boxed{\left(\frac{W}{L}\right)_2 = \frac{500\,nm}{200\,nm}}$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2 = 1\,mA \qquad V_{ov} = 0.5$$



$$\frac{1}{2}(200\,\mu A/V^2)(2.5)(V_{DD}-0.5-0.7)^2 = 1\,mA$$

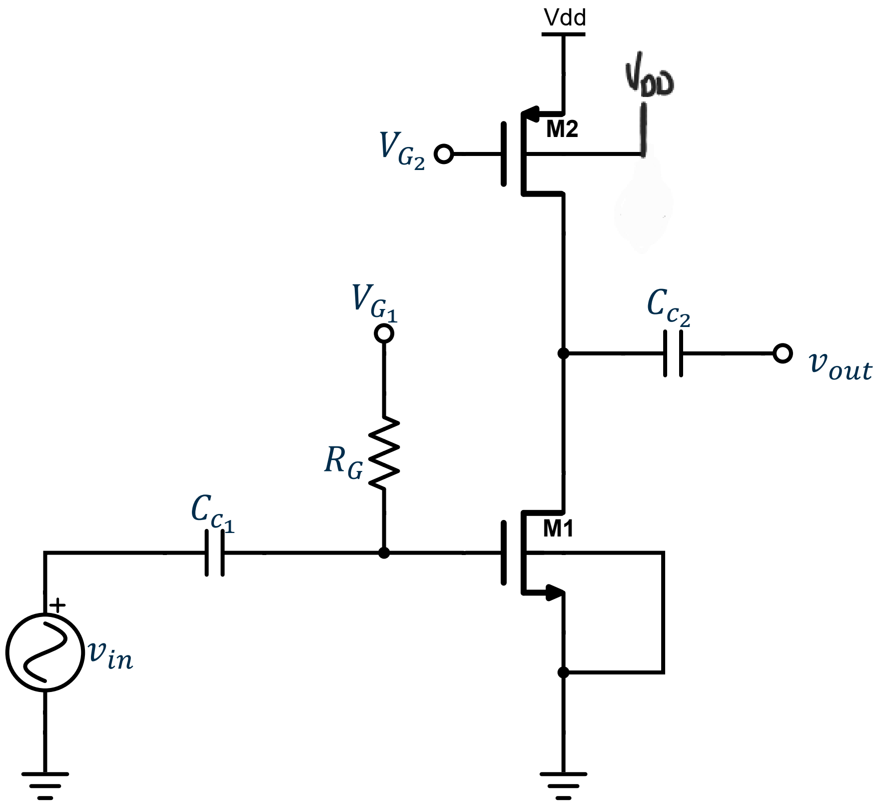
Leave V_{DD} alone

$$\boxed{V_{DD} = 3.2V}$$

$$\boxed{P_{DC} = 3.2\,mW}$$

Task 1.3

The circuit from Task 1.2 is further modified:



- Replace the diode-connected load with a **current-source PMOS load** with following parameters (M1 remains the same):
  - $|V_{TH_P}| = 0.5V$
  - $\lambda_n = \lambda_p = 0.1\ V^{-1}$
  - $\mu_p C_{ox} = 50 \frac{\mu A}{V^2}$
  - $|V_{ov}| = 0.5V$

a) Calculate the small-signal gain A_v.

b) Determine the required gate bias voltage V_{G_2}, and calculate the DC power consumption P_{DC}.

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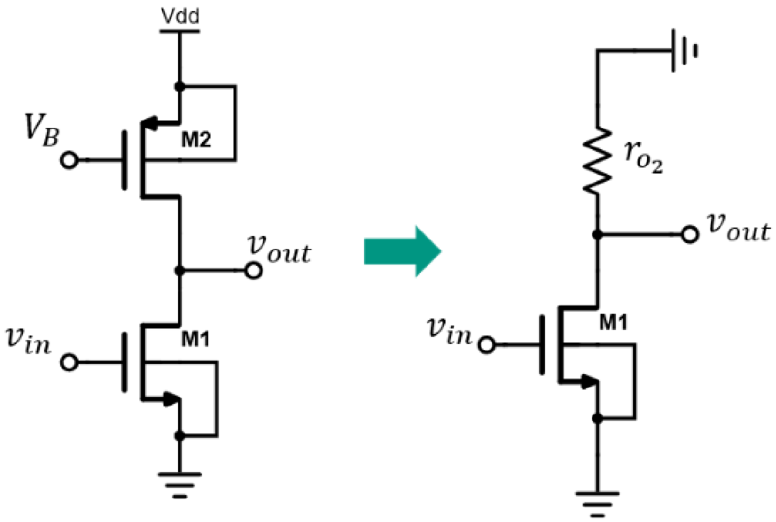


$$A_v = \frac{v_{out}}{v_{in}} = -g_{m_1}(r_{o1} \parallel r_{o_2})$$

$$I_D = \frac{1}{2}\underbrace{\mu_n C_{ox}}_{200\mu}\underbrace{\left(\frac{W}{L}\right)_1}_{\frac{8\mu}{200n}}\left(\underbrace{V_{GS}-V_{TH_N}}_{V_{ov}}\right)^2\left(1+\lambda \underbrace{V_{DS}}_{V_{ov}}\right) = \boxed{1.05\,mA}$$

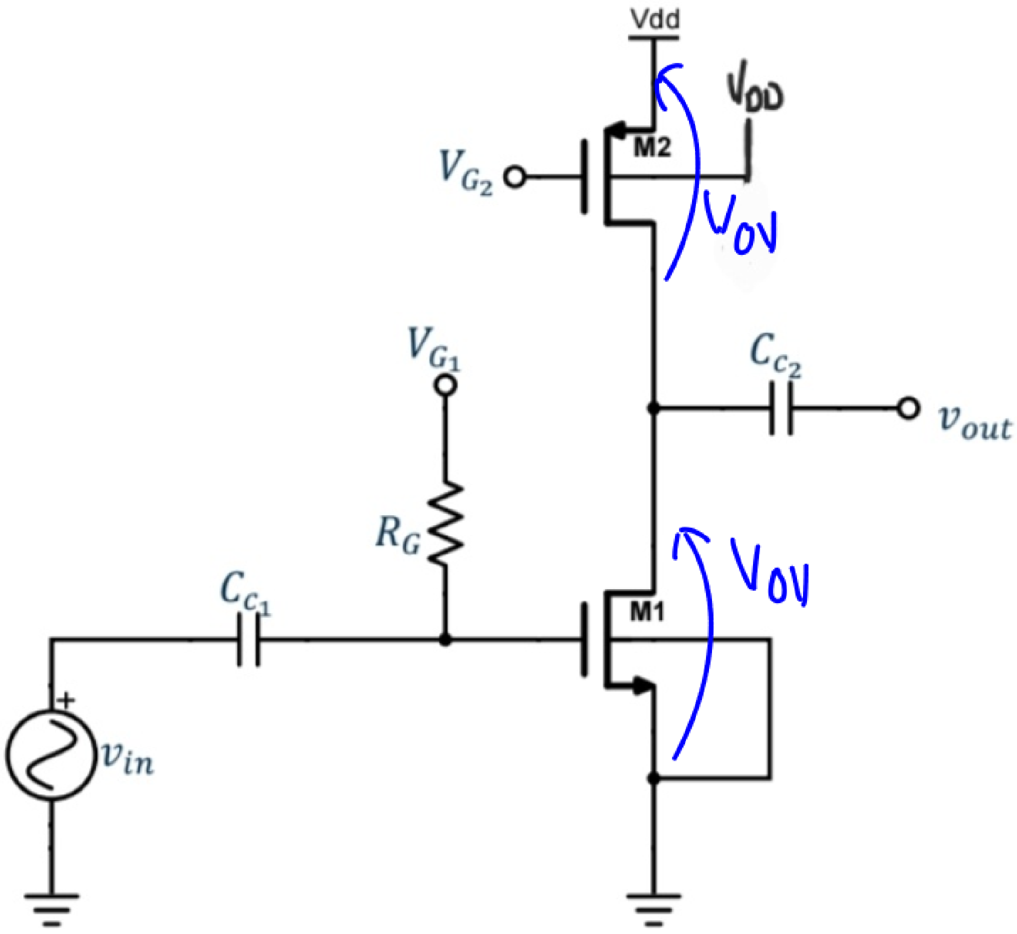
$$I_{D_{M_2}} = I_D = \frac{1}{2}\underbrace{\mu_p C_{ox}}_{50\mu}\left(\frac{W}{L}\right)_2\left(\underbrace{V_{SG}-|V_{TH}|}_{V_{ov}}\right)^2\left(1+\lambda \underbrace{V_{SD}}_{V_{ov}}\right) = 1.05\,mA$$

$$\frac{(W/L)_2}{(W/L)_1} = \frac{\mu_n C_{ox}}{\mu_p C_{ox}} = 4 \Rightarrow \boxed{\left(\frac{W}{L}\right)_2 = \frac{32\mu m}{200nm}}$$

$$g_m = g_{m_1} = g_{m_2} = \frac{2I_D}{V_{OV}} = \frac{2.1mA}{0.5V} = \boxed{4.2mS}$$

$$r_o = r_{o_1} = r_{o_2} = \frac{1}{\lambda I_D} = \frac{1}{(0.1V^{-1})(1.05mA)} \approx \boxed{9.52k\Omega}$$

$$A_V = g_{m_1}(r_{o_1} // r_{o_2}) = \frac{g_m r_o}{2} \approx 20\,V/V = 26dB$$



$$V_{TH_N} = 0.7\,V, \quad V_{TH_P} = -0.5\,V, \quad V_{ov} = 0.5\,V$$

$$V_{G_1} - V_{TH_N} = 0.5\,V \Rightarrow \boxed{V_{G_1} = 1.2\,V}$$

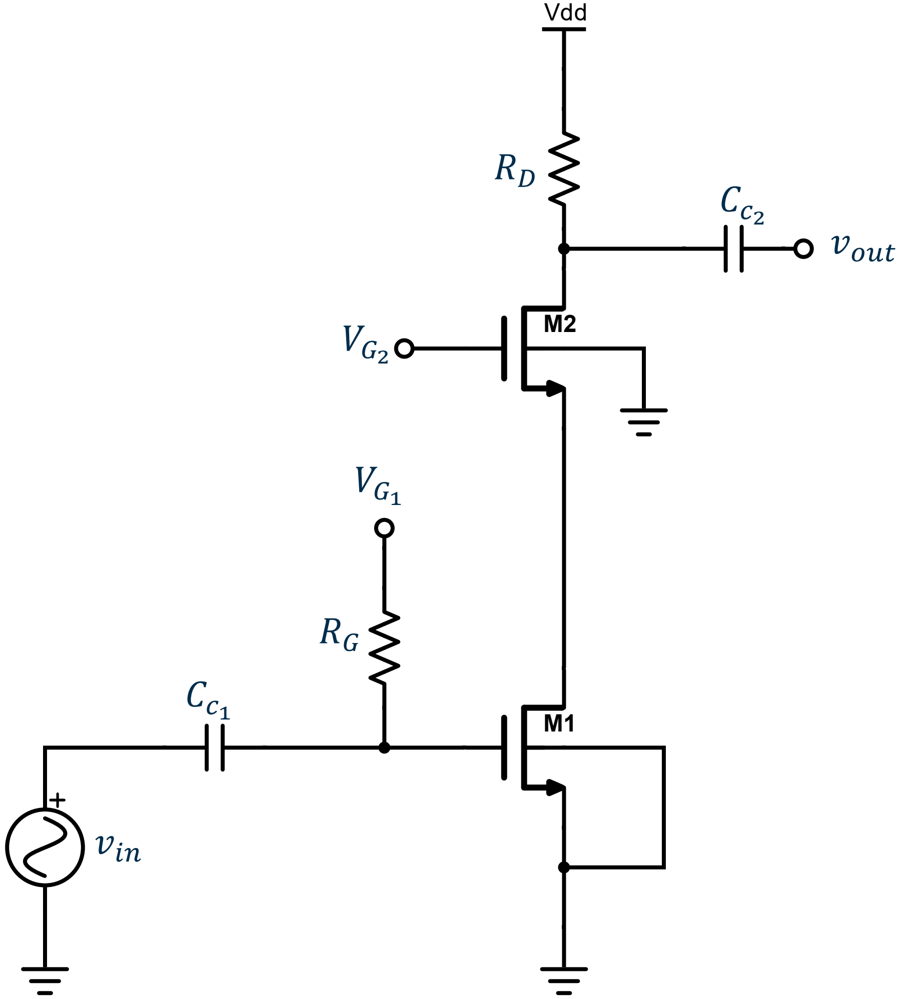
$$V_{DD} - V_{G_2} - |V_{THP}| = 0.5\,V$$

$$V_{DD} = 2\,V_{ov} = 1\,V \Rightarrow \boxed{V_{G_2} = 0\,V}$$

$$P_{DC} = \underbrace{(1\,V)}_{V_{DD}}\underbrace{(1.05\,mA)}_{I_D} = \boxed{1.05\,mW}$$

Task 2.1

For the cascode amplifier below following circuit parameters are provided:



- $V_{TH} = 0.7V$
- $V_{ov} = V_{GS} - V_{TH} = 1\ V$
- $\mu_n C_{ox} = 200 \frac{\mu A}{V^2}$
- $I_D = 1mA$
- $R_D = 1k\Omega$
- $\mathrm{R_G} \to \infty, C_{c_{1,2}} \to \infty$
- $L = 200nm$
- $\lambda_n = 0.01\ V^{-1}$

a) Calculate g_{m_1}, g_{m_2}.

b) Determine transistor width W_1, W_2.

c) Find the minimum V_{DD} to keep both transistors in saturation.

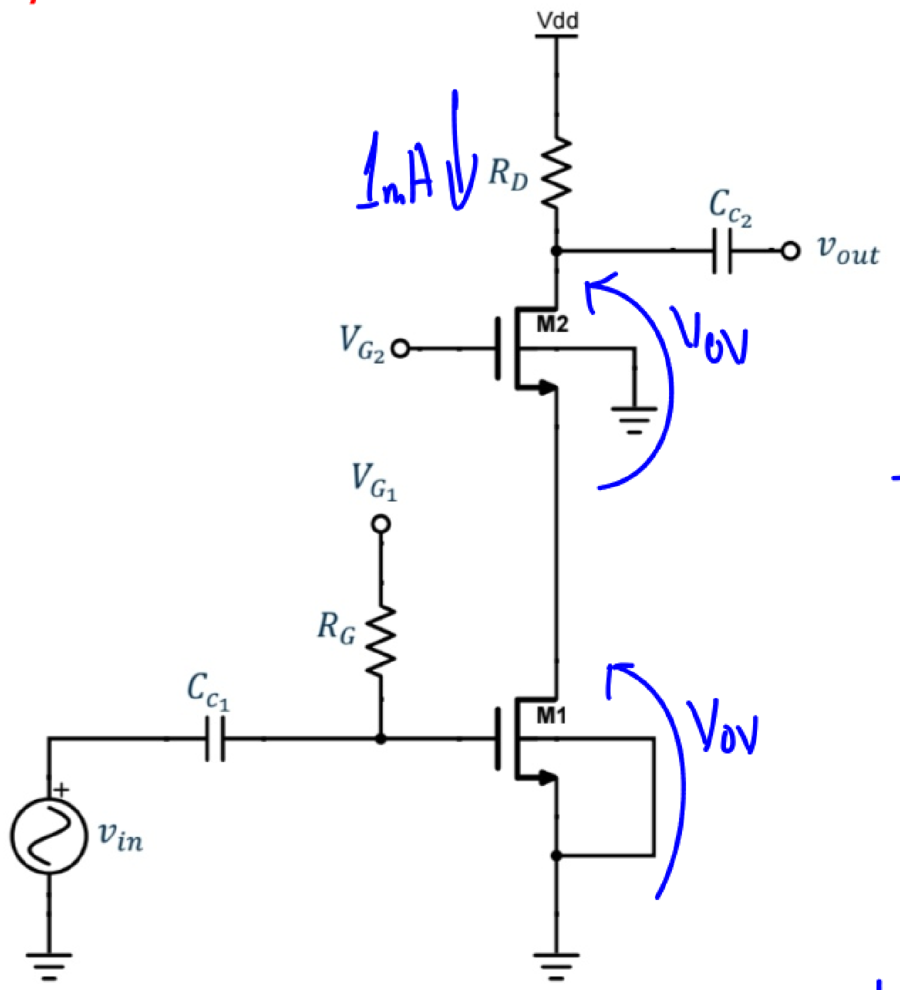
d) Calculate the small-signal gain A_v.

a) $g_m = g_{m_1} = g_{m_2} = \frac{2 I_D}{V_{OV}} = \boxed{2\,mS}$

b) $g_m = \mu_n C_{ox} \frac{W}{L}(V_{GS} - V_{TH}) \Rightarrow W = \frac{\overset{2\,mS}{g_m} \overset{200\,nm}{L}}{\underbrace{\mu_n C_{ox}}_{200\,\mu A/V^2} \underbrace{(V_{GS} - V_{TH})}_{1V}}$

$$\boxed{W = W_1 = W_2 = 2\,\mu m}$$

c)



Please note that both I_D and λ were given in the question.
Since λ is too low, it is excluded in I_D calculation but will be used in r_o calculation.

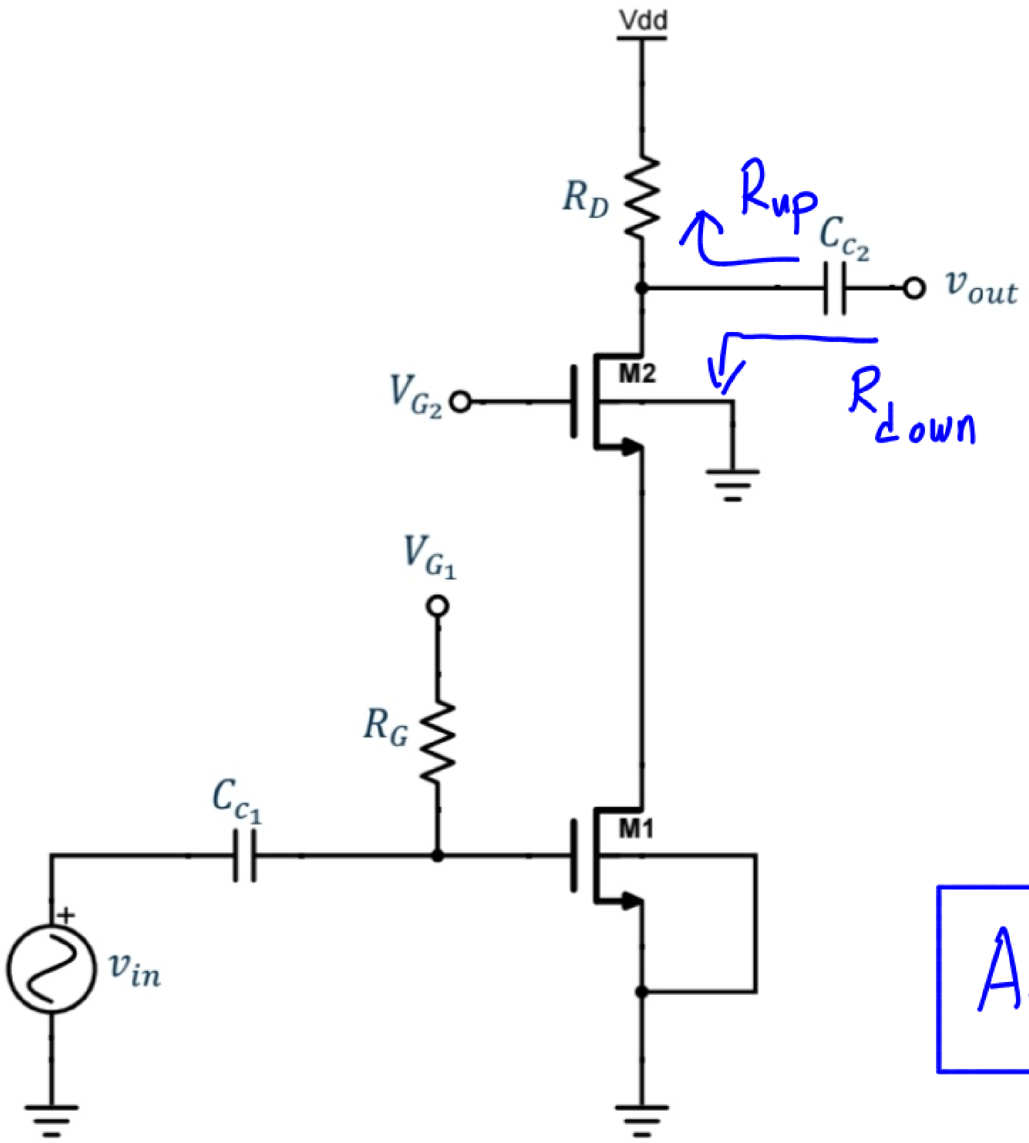
$$\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(\underbrace{V_{GS}-V_{TH}}_{V_{OV}=1V})^2 = 1mA$$

$$V_{G_1} - 0.7V = 1V \Rightarrow \boxed{V_{G_1} = 1.7V}$$

$$V_{G_2} - 1V - 0.7V = 1V \Rightarrow \boxed{V_{G_2} = 2.7V}$$

$$V_{DD} = 2\underbrace{V_{OV}}_{1V} + \underbrace{I_D}_{1mA}\underbrace{R_D}_{1k\Omega} = \boxed{3V}$$

c)



$R_{down} \approx g_{m_2} r_{o_2} r_{o_1}$ (Cascode R_{out})

$R_{up} = R_D$

$A_v = -g_{m_1} (R_{up} // R_{down})$

$r_o = r_{o_1} = r_{o_2} = \frac{1}{\lambda I_D} = 100k\Omega$

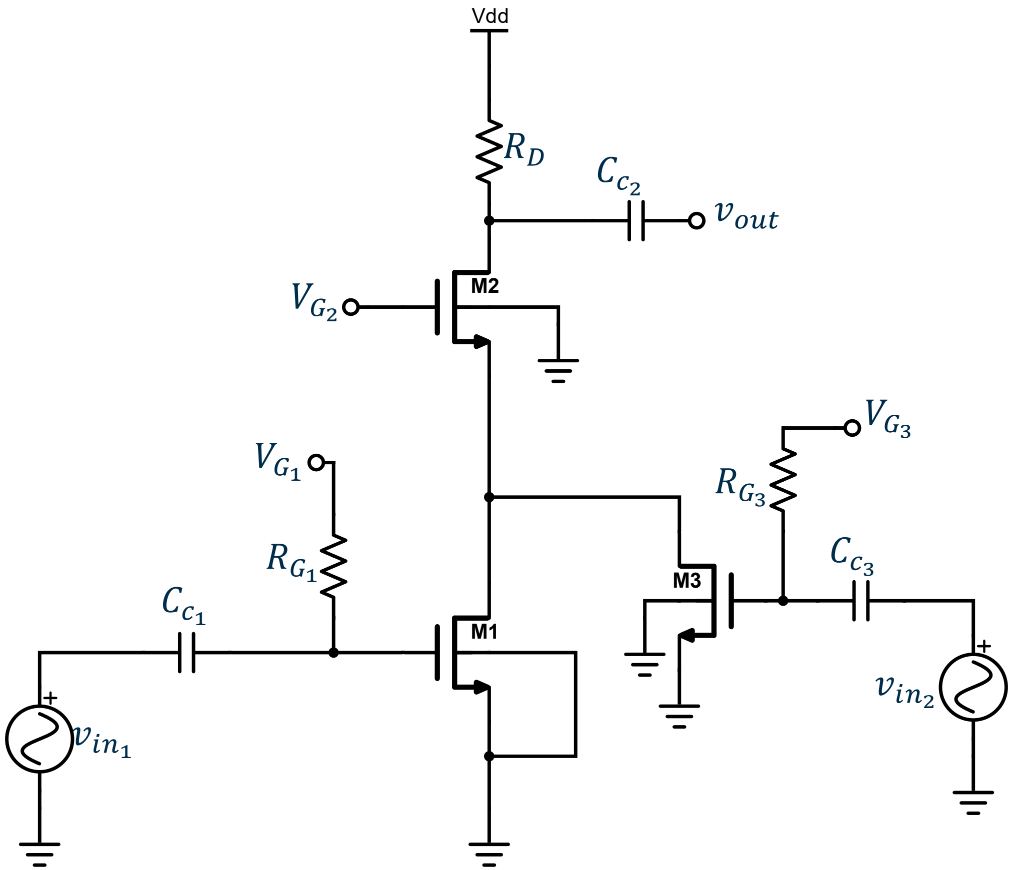
$R_{down} = (2mS)(100k\Omega)^2 = 20M\Omega$

$R_{up} = R_D = 1k\Omega \longrightarrow R_{up} // R_{down} \approx R_{up} = 1k\Omega$

$$\boxed{A_v = -g_{m_1} R_{up} = -2\ V/V \quad (6dB)}$$

Task 2.2

The amplifier from Task 2.1 is modified as following by include a second input stage. Following circuit parameters are provided with M1 and M3 being identical:



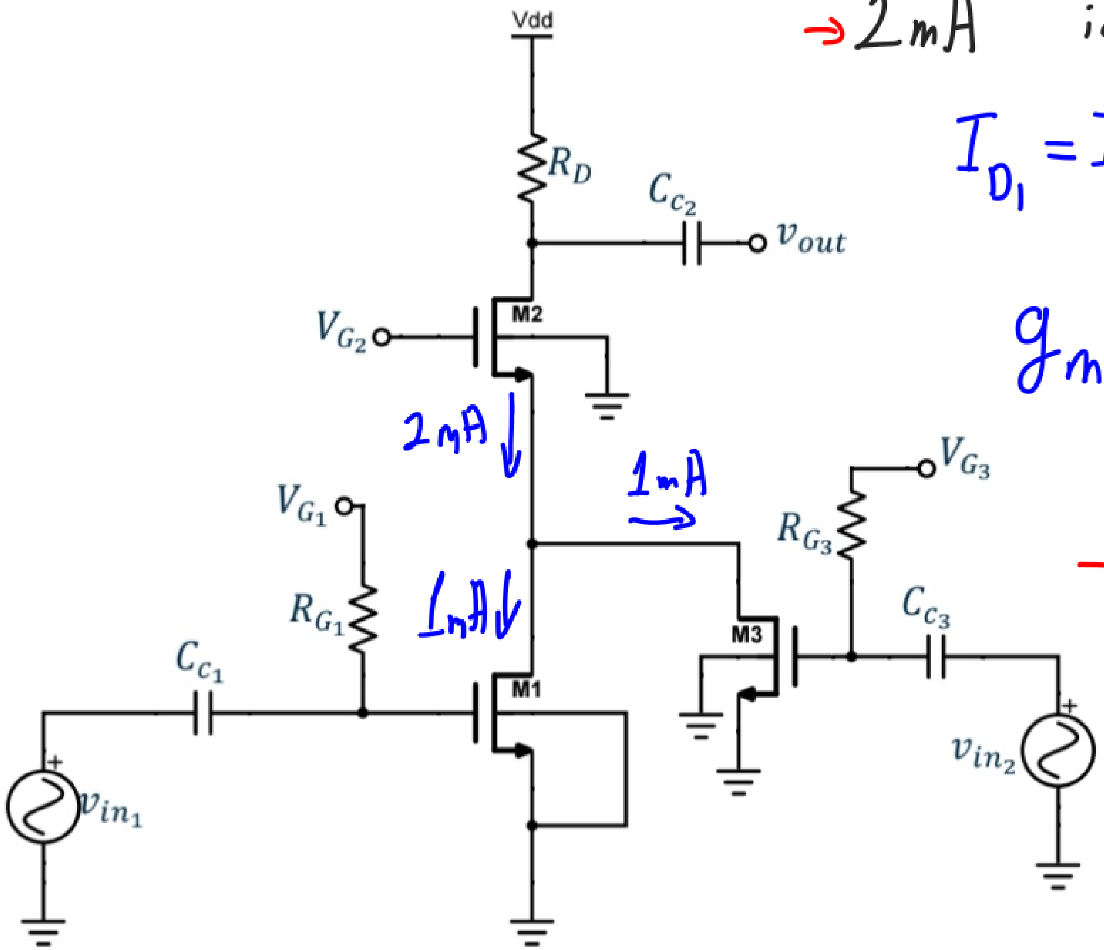
- $V_{TH} = 0.7V$
- $V_{ov} = V_{GS} - V_{TH} = 1\,V$
- $\mu_n C_{ox} = 200\,\frac{\mu A}{V^2}$
- $I_{D_1} = I_{D_3} = 1mA$
- $R_D = 1k\Omega$
- $\mathrm{R}_{\mathrm{G}_{1,3}} \to \infty, C_{c_{1,2,3}} \to \infty$
- $L = 200nm$

The two input signals are:

- $v_{in_1}(t) = 1\mu V cos(2\pi 10^6 t + 5°)$
- $v_{in_2}(t) = 5\mu V cos(2\pi 10^6 t + 35°)$.

a) Determine $v_{out}(t)$.

First check the DC Performance



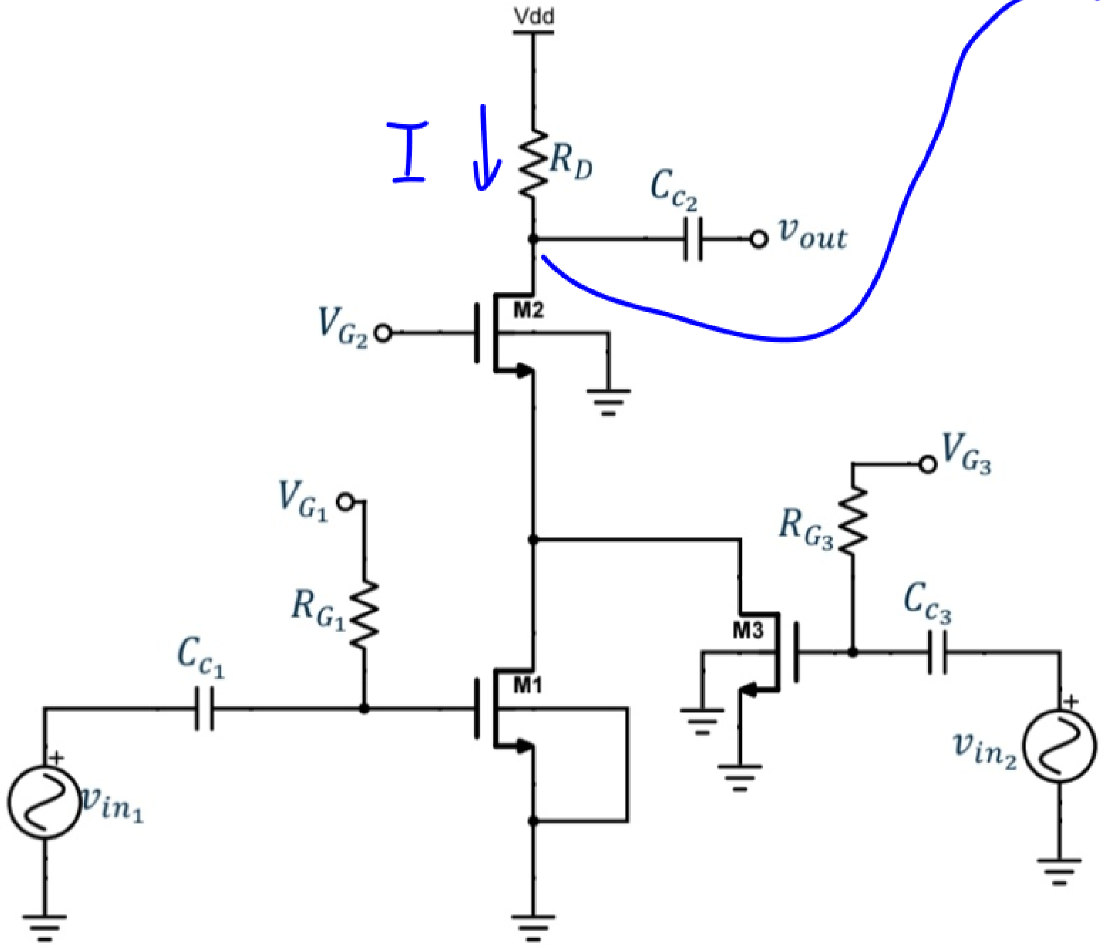
→ 2mA is distributed to M_1 and M_2 equally.

$$I_{D_1} = I_{D_3} \Rightarrow \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_3 \Rightarrow V_{G_1} = V_{G_3} = 1.7V$$

$$g_{m_1} = g_{m_3} = \frac{2 \times 1mA}{1V} = 2mS \qquad g_{m_2} = 2g_{m_{1,2}} = 4mS$$

→ The current flowing through M_2 is doubled.
It could maybe adjusted by changing V_{GS_2}.
However, it'd effect the whole circuit.
Doubling $\frac{W}{L}$ is more efficient.

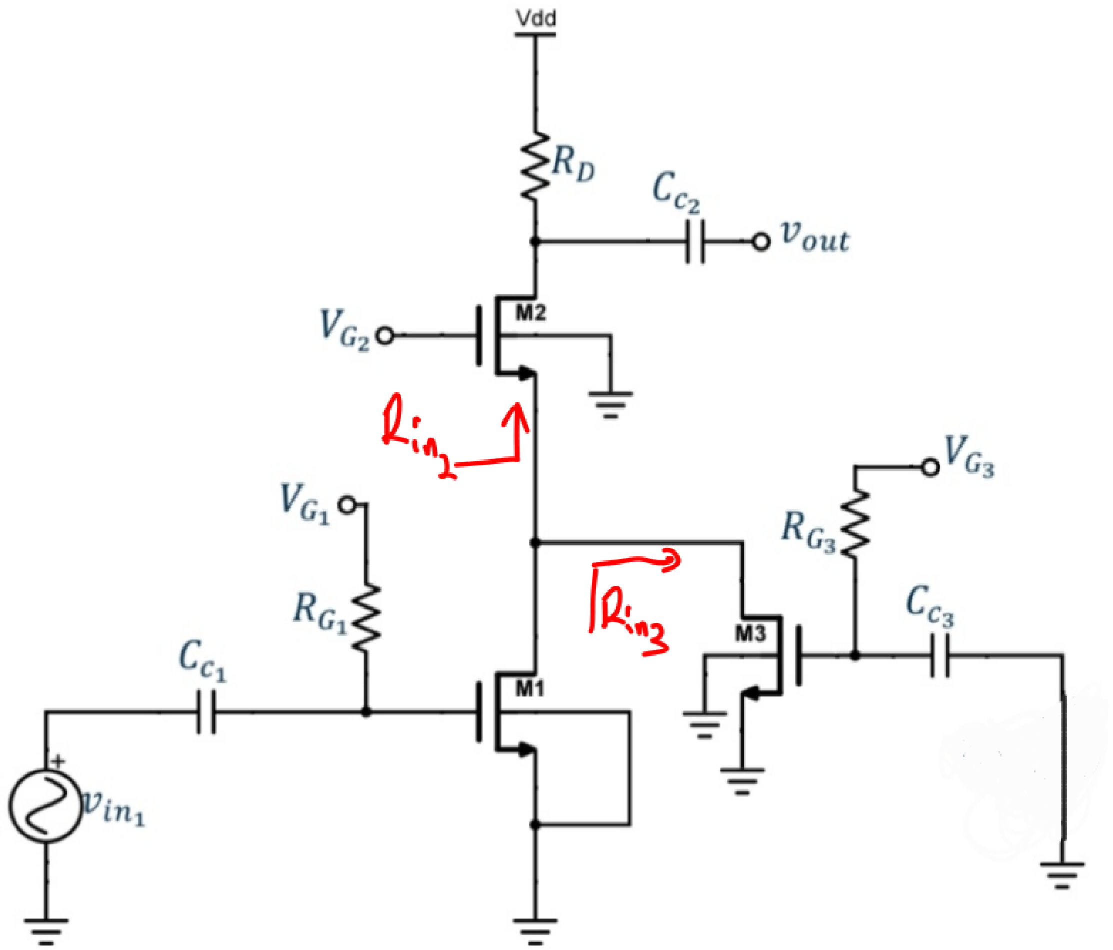
$$\left(\frac{W}{L}\right)_2 = 2\left(\frac{W}{L}\right)_1 = 2\left(\frac{W}{L}\right)_2$$



$V_{D_{M_2}} = 2V_{ov} = 2V, \quad R_D = 1k\Omega, \quad I = 2mA$

$$V_{DD} = 2V + (2mA)(1k\Omega) = \boxed{4V}$$

Using superposition. v_{in_2} Shorted.



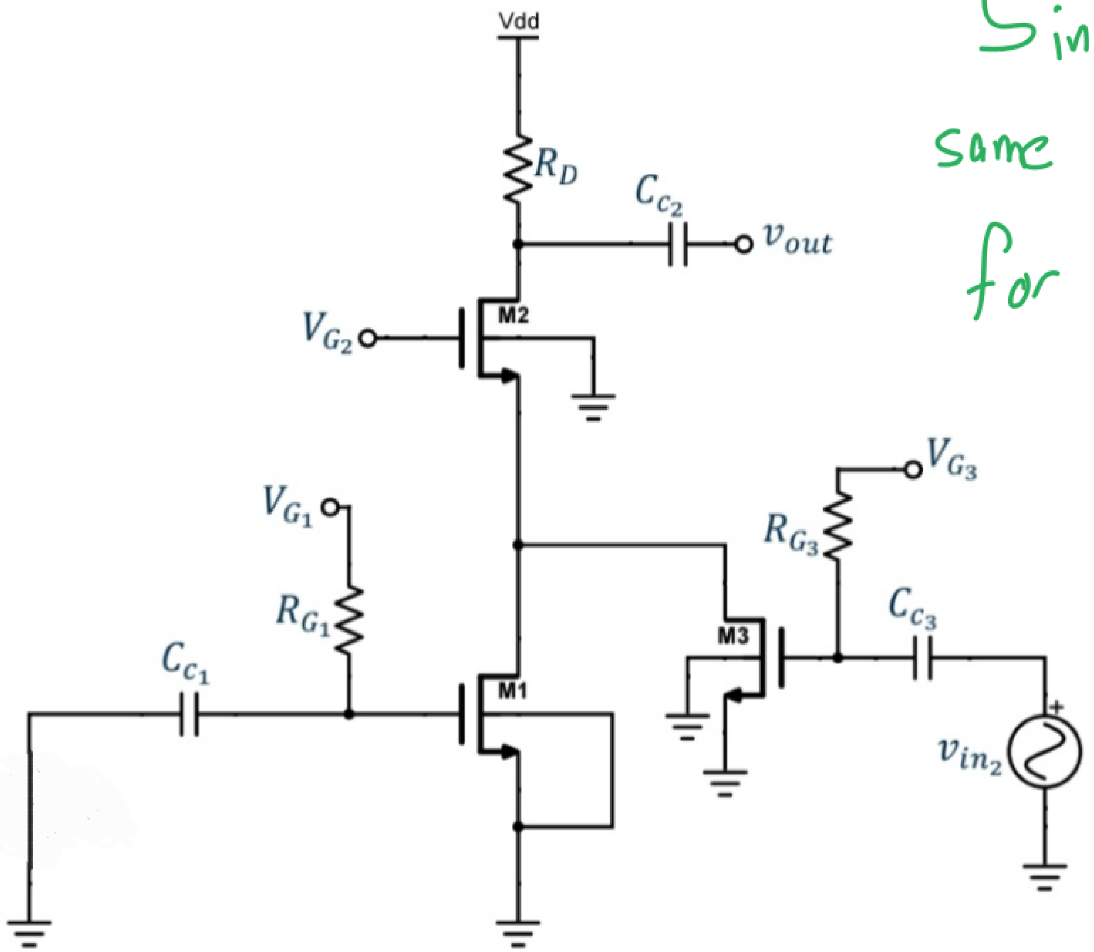
$$R_{in_2} \approx \frac{1}{g_{m_2}} << R_{in_3} = r_{o_3}$$

So, due to its higher impedance R_{in_3} can be considered as open circuit. Then standard cascode amplifier remains left.

$$A_V \approx -g_{m_1} R_D = -2V/V$$

$$\boxed{v_{out_1}\Big|_{v_{in_1}} = 2\,mV \cos(2\pi 10^6 t + 185^\circ)}$$

v_{in_1} Shorted



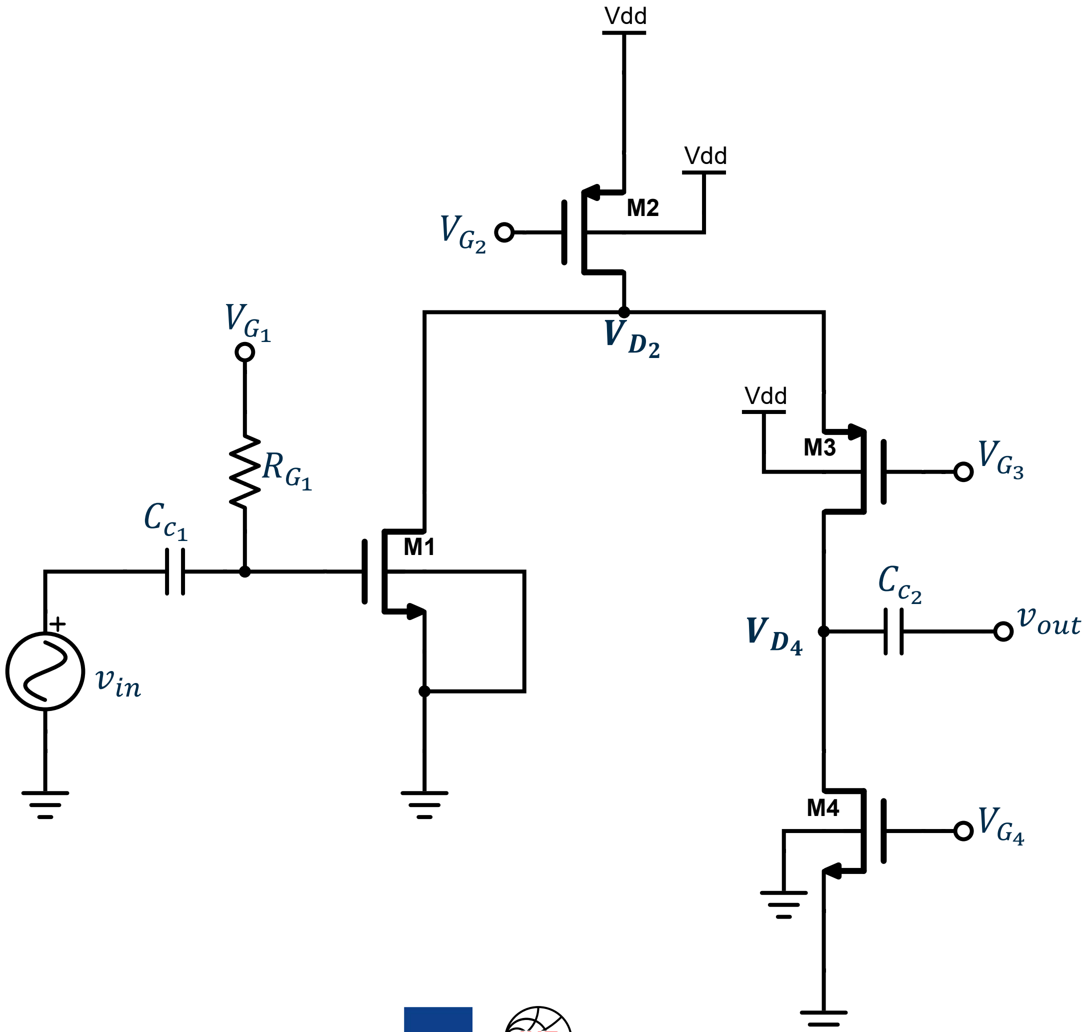
Since M1 and M3 are equivalent, the same principle will apply for v_{in_2}. No need for repeating the calculation.

$$\boxed{v_{out_2}\Big|_{v_{in_2}} = 10\,mV\cos(2\pi 10^6 t + 215^\circ)}$$

$$\boxed{v_{out} = v_{out_1} + v_{out_2} = 2\,mV\cos(2\pi 10^6 t + 185^\circ) + 10\,mV\cos(2\pi 10^6 t + 215^\circ)}$$

Task 2.3

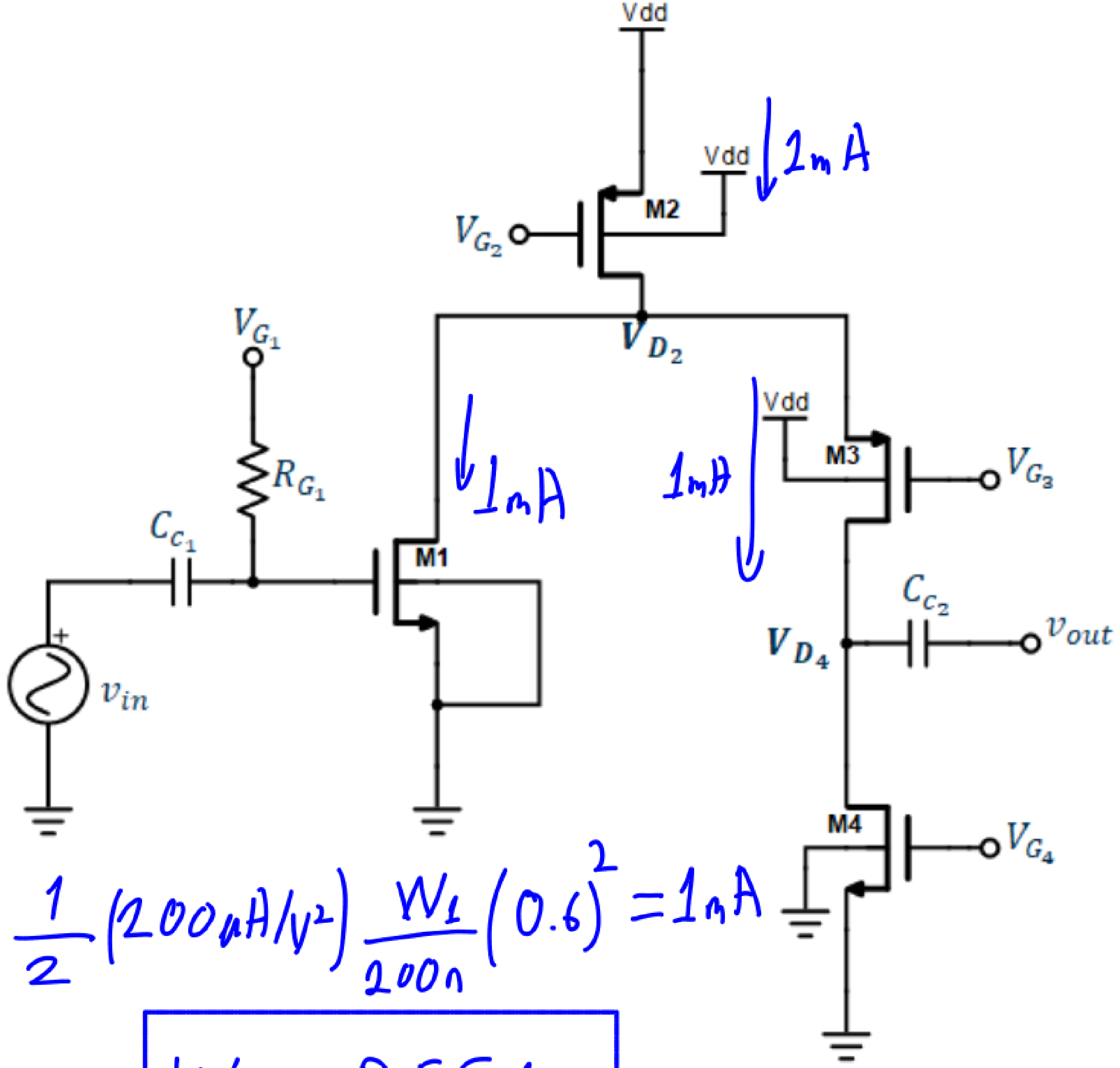
For the circuit below, following circuit parameters are provided:



- $V_{TH_N} = |V_{TH_P}| = 0.4V$
- $V_{ov} = |V_{GS}| - |V_{TH}| = 0.6\ V$
- $\mu_n C_{ox} = 200 \frac{\mu A}{V^2}$
- $\mu_p C_{ox} = 50 \frac{\mu A}{V^2}$
- $I_{D_1} = I_{D_{3,4}} = 1mA$
- $\mathrm{R}_{G_1} \to \infty, C_{c_{1,2}} \to \infty$
- $L = 200nm$
- $\lambda_N = \lambda_P = 0.05V^{-1}$

Furthermore, assume $V_{D_4} = 0.6V$, $V_{D_2} = 1.2V$

a) Calculate DC operating points of each transistor and calculate the small-signal gain A_v.



$$\frac{1}{2}(200\mu A/V^2)\frac{W_1}{200n}(0.6)^2 = 1mA$$

$$\boxed{W_1 \approx 5.55\mu m}$$

$$W_1 = W_4 = \frac{W_3}{4} = \frac{W_2}{8} \quad \left(\frac{\mu_n C_{ox}}{\mu_p C_{ox}} = 4\right)$$

$$V_{G_1} - 0.4V = 0.6V \Rightarrow \boxed{V_{G_1} = 1V}$$

$$V_{G_4} - 0.4V = 0.6V \Rightarrow \boxed{V_{G_4} = 1V}$$

$$1.2 - V_{G_3} - 0.4V = 0.6V \Rightarrow \boxed{V_{G_3} = 0.2V}$$

$$V_{DD} = V_{D_2} + V_{OV} = \boxed{1.8V}$$

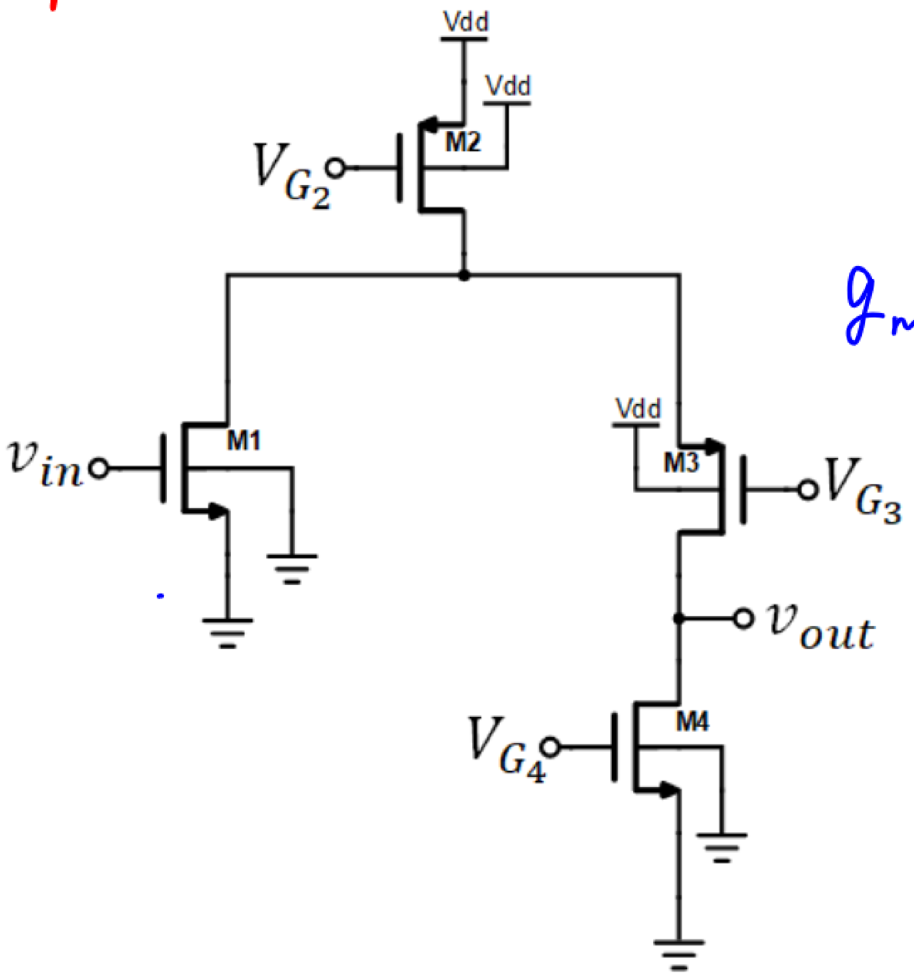
$$1.8V - V_{G_2} - 0.4V = 0.6V \Rightarrow \boxed{V_{G_2} = 0.8V}$$

All transistors operate in saturation region.

$$g_m = g_{m_1} = \frac{g_{m_2}}{2} = g_{m_3} = g_{m_4} = \frac{2 \times 1mA}{0.6V} = \boxed{3.33mS}$$

$$r_o = r_{o_1} = 2r_{o_2} = r_{o_3} = r_{o_4} = \frac{1}{\lambda(1mA)} = \boxed{20k\Omega}$$

Folded Cascode Gain Slide 2 P.44



$$\boxed{A_v = -g_{m_1}([g_{m_3} r_{o_3}(r_{o_1} \parallel r_{o_2})] \parallel r_{o_4})}$$

r_{o_4}: $20k\Omega$

$$g_{m_3} r_{o_3}(r_{o_1} // r_{o_2}) = g_m \frac{r_o^2}{3} = 444\,k\Omega$$

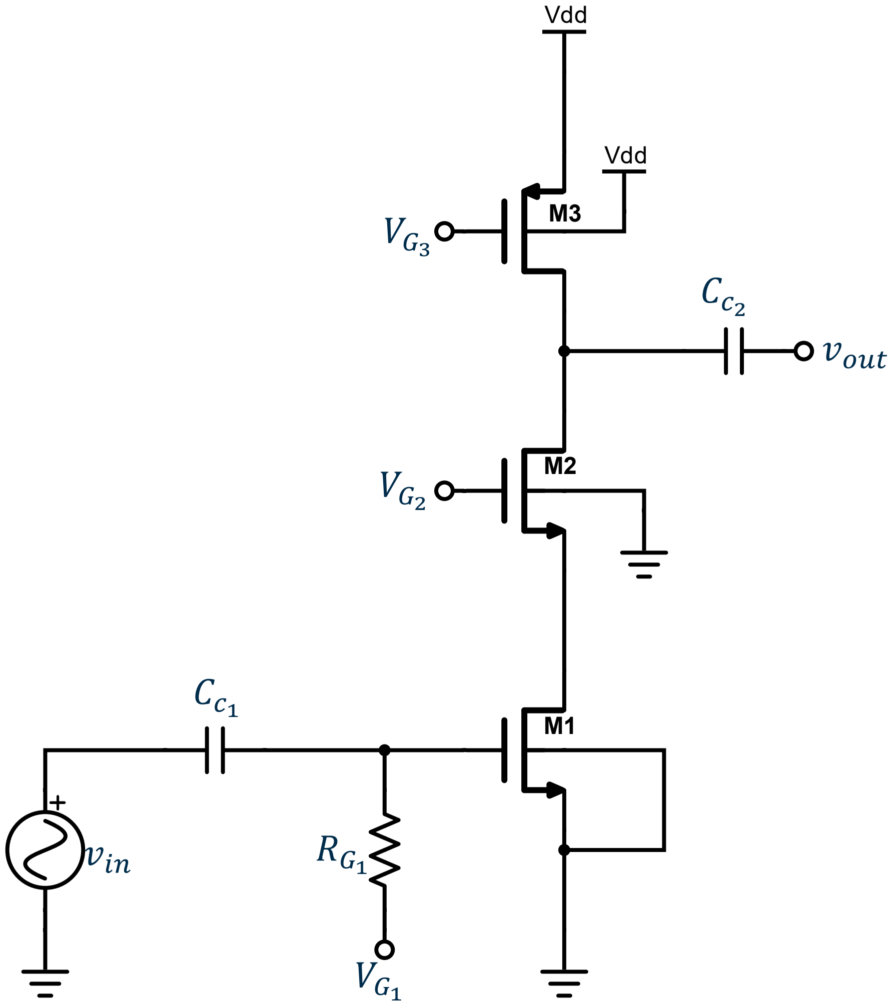
$$(444\,k\Omega) // 20\,k\Omega \approx 19\,k\Omega$$

$$A_v = -g_m(19\,k\Omega) = -(3.33\,mS)(19\,k\Omega)$$

$$\boxed{A_v \approx 63\ V/V}$$

Task 3

For the circuit below, following parameters are provided:



- $V_{TH_N} = |V_{TH_P}| = 0.2V$
- $V_{ov} = |V_{GS}| - |V_{TH}| = 0.4\ V$
- $\mu_n C_{ox} = 200 \frac{\mu A}{V^2}$
- $\mu_p C_{ox} = 50 \frac{\mu A}{V^2}$
- $I_D = 4mA$
- $\mathrm{R}_{G_1} \rightarrow \infty, C_{c_{1,2}} \rightarrow \infty$
- $L = 200nm$
- $\lambda_N = \lambda_P = 0.01V^{-1}$

It can be assumed that all transistors are operated in saturation region.

a) Calculate the output noise voltage density and input-referred noise voltage density.

b) Compare the noise performance of this amplifier to that of a single-ended common-source amplifier with a current-source load (i.e., without M2).

$$I_D = I_{D_{M_1}} = I_{D_{M_2}} = I_{D_{M_3}} = 4\,mA$$

$$V_{ov} = V_{ov_{M_1}} = V_{ov_{M_2}} = V_{ov_{M_3}} = 0.4\,V$$

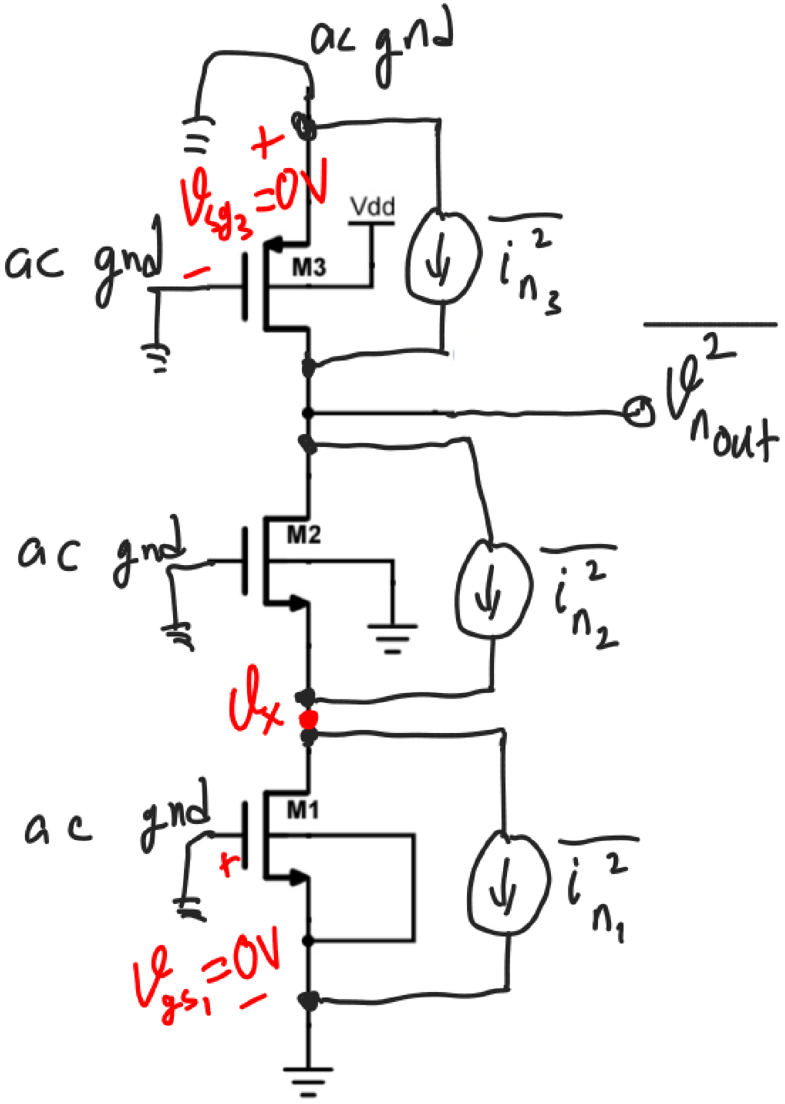
$$g_m = \frac{2 I_D}{V_{ov}} = \boxed{20\,mS}$$

$$r_o = \frac{1}{\lambda I_D} = \boxed{25\,k\Omega}$$

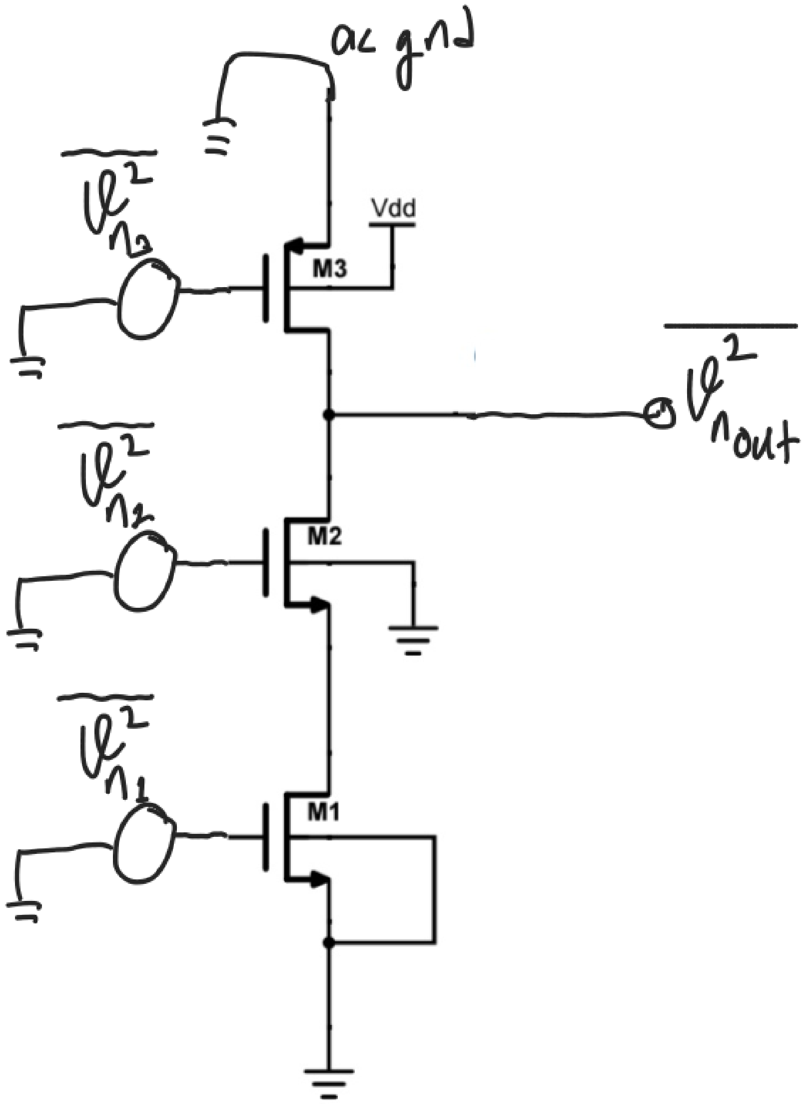
$$A_v = -g_{m_1}\left(g_{m_2} r_{o_2} r_{o_1} \,//\, r_{o_3}\right) \approx -g_m r_o = \boxed{-500\,V/V}$$

Each transistor generates thermal noise.

CURRENT NOISE EQUIVALENT

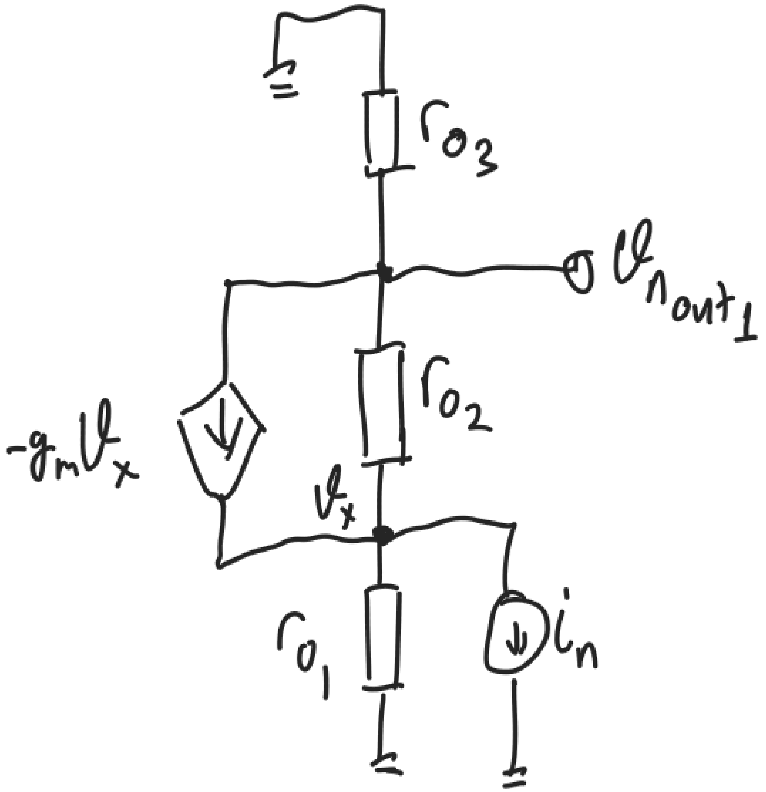


VOLTAGE NOISE EQUIVALENT



Apply superposition & draw small-signal model:

i_{n_1}



1

$$\frac{-v_{nout_1}}{r_{o_3}} = -g_m v_x + \frac{v_{nout_1} - v_x}{r_{o_2}} = \frac{v_x}{r_{o_1}} + i_n$$

2

$$v_x = v_{nout_1} \frac{1/r_{o_3} + 1/r_{o_2}}{g_m + 1/r_{o_2}}$$

3

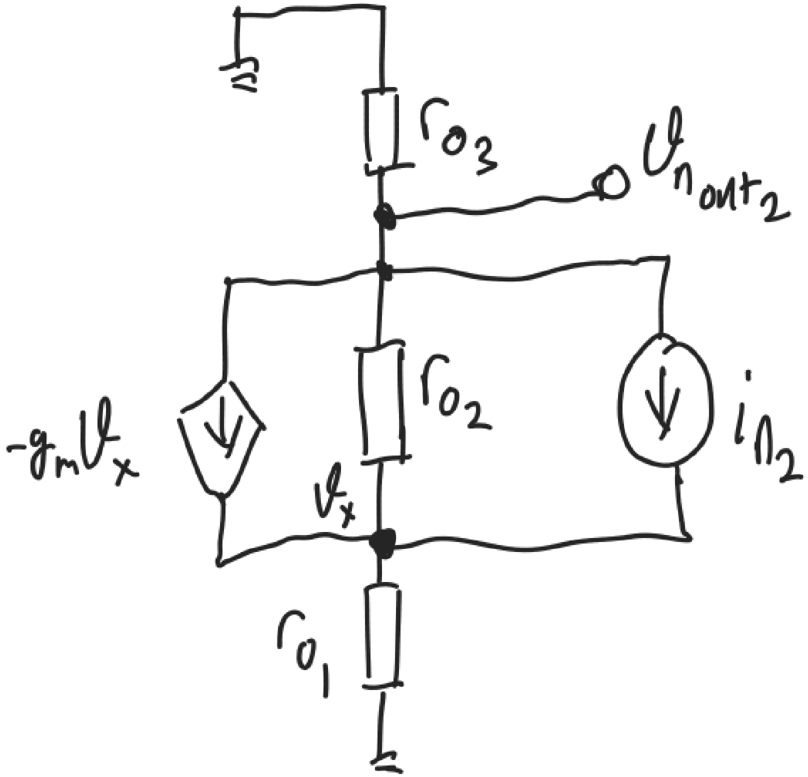
$$-v_{nout_1} \underbrace{\left(\frac{1}{r_{o_3}} + \frac{1/r_{o_3} + 1/r_{o_2}}{g_m + 1/r_{o_2}} \frac{1}{r_{o_1}} \right)}_{\frac{1}{R_{eq_1}}} = i_n$$

$$\boxed{\overline{v^2_{n_{out_1}}} = \overline{i^2_{n_1}}\, R^2_{eq_1}}$$

Please note that calculating M1's noise contribution through its voltage noise equivalent is way more easier!

$$\boxed{\overline{v^2_{n_{out_1}}} = \overline{v^2_{n_1}}\, |A_v|^2}$$

i_{n_2}:



1

$$\frac{-v_{nout_2}}{r_{o_3}} = -g_m v_x + \frac{v_{nout_2} - v_x}{r_{o_2}} + i_{n_2} = \frac{v_x}{r_{o_1}}$$

2

$$v_x = -\frac{r_{o_1}}{r_{o_3}} v_{nout_2}$$

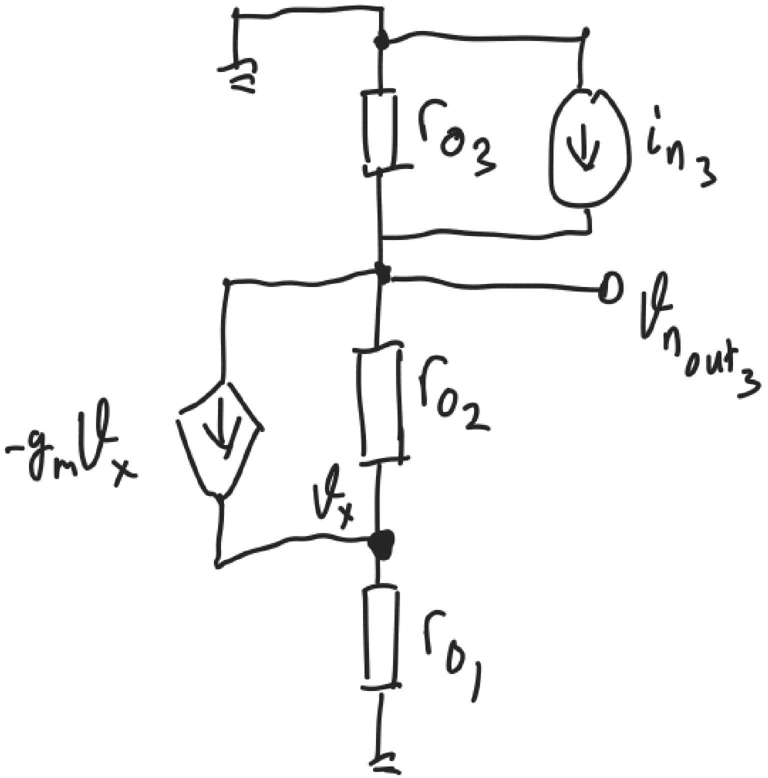
3

$$-v_{nout_2}\left(g_m \frac{r_{o_1}}{r_{o_3}} + \frac{r_{o_1}}{r_{o_2} r_{o_3}} + \frac{1}{r_{o_2}} + \frac{1}{r_{o_3}}\right) = i_{n_2}$$

$$v_{nout_2} = -\frac{r_{o_2} r_{o_3}}{g_m r_{o_1} r_{o_2} + r_{o_1} + r_{o_2} + r_{o_3}} i_{n_2}$$

$$\boxed{\overline{v^2_{nout_2}} = \left(\frac{r_{o_2} r_{o_3}}{g_m r_{o_1} r_{o_2} + r_{o_1} + r_{o_2} + r_{o_3}}\right)^2 \overline{i^2_{n_2}}}$$

i_{n_3}:



1

$$\frac{-v_{nout_3}}{r_{o3}} + i_{n_3} = -g_m v_x + \frac{v_{nout_3} - v_x}{r_{o2}} = \frac{v_x}{r_{o1}}$$

2

$$v_{nout_3}\frac{1}{r_{o2}}\left(\frac{1}{r_{o1}} + \frac{1}{r_{o2}} + g_m\right)^{-1} = v_x$$

3

$$i_{n_3} = v_{nout_3}\left[\frac{1}{r_{o3}} + \frac{1}{r_{o1} r_{o2}}\underbrace{\left(\frac{1}{g_m} \,\|\, r_{o1} \,\|\, r_{o2}\right)}_{\approx \frac{1}{g_m}}\right] \qquad \frac{1}{g_m} \ll r_o$$

$$\boxed{\overline{v^2_{nout_3}} = \overline{i^2_{n_3}}\left(r_{o3} \,\|\, g_m r_{o1} r_{o2}\right)^2}$$

$$\overline{v_{n_{out,1}}^2} = \overline{v_{n,1}^2}\,|A_v|^2 = \frac{4kT\gamma}{g_m}(g_m r_o)^2 = \boxed{4kT\gamma g_m r_o^2}$$

$$\overline{v_{n_{out,2}}^2} = \left(\frac{r_{o2} r_{o3}}{g_m r_{o1} r_{o2} + r_{o1} + r_{o2} + r_{o3}}\right)^2 \overline{i_{n_2}^2} = \boxed{4kT\gamma g_m\left(\frac{r_o^2}{g_m r_o^2 + 3 r_o}\right)^2}$$

$$\overline{v_{n_{out,3}}^2} = \overline{i_{n_3}^2}\left(r_{o3} \,//\, g_m r_{o1} r_{o2}\right)^2 \approx \boxed{4kT g_m r_o^2}$$

$r_o << g_m r_o^2$

$k = 1.38 \times 10^{-23}\ J/K$ $\quad T = 300\ °K$ $\quad \gamma = 2/3$

$$\boxed{\overline{v^2_{n_{out_1}}} = \overline{v^2_{n_{out_3}}} = 138.06\ f\frac{V^2}{Hz}}$$

$$\boxed{\overline{v^2_{n_{out_2}}} = 545.69\ z\frac{V^2}{Hz}}$$

$(z = 10^{-21})$

$$\boxed{\overline{v^2_{n_{out}}} = \overline{v^2_{n_{out_1}}} + \overline{v^2_{n_{out_2}}} + \overline{v^2_{n_{out_3}}} = 276.13\ f\frac{V^2}{Hz}}$$

$$\boxed{\text{Output Noise Voltage Density} = \sqrt{\overline{v^2_{n_{out}}}} = 524.43\ V/\sqrt{Hz}}$$

$$\text{Input-Referred Noise Voltage Density} = \sqrt{\frac{\overline{V_{n_{out}}^2}}{|A_V|^2}} = 1.05\ \mathrm{nV}/\sqrt{\mathrm{Hz}}$$

Note that the common-gate device M2 contributes negligibly to the total noise.
The dominant noise sources are the input transistor M1, and the PMOS load M3.
As a result, a cascode amplifier typically exhibits higher output noise, simply because it has a higher voltage gain than a common-source amplifier with a current-source load.
However, the input-referred noise remains almost the same for both amplifiers, since input-referred noise is independent of the amplifier gain.