Mikroelektronische Schaltungen und Systeme

Lect.3
Noise

Prof. Dr.-Ing. Ahmet Cagri Ulusoy
Institut für Technik der Informationsverarbeitung (ITIV)

Mikroelektronische Schaltungen und Systeme

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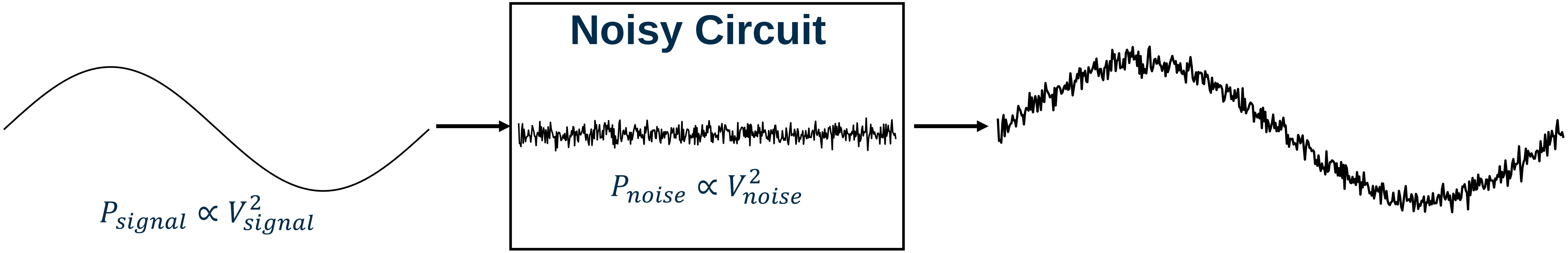
What is a Noise?

- “Noise” is used in everyday life to describe unwanted sounds.
- It is any random, unpredictable, unwanted fluctuation that adds to or interferes with the desired voltage or current signal.

This image was created using ChatGPT.

Electrical Noise

- The instantaneous value of noise in the time domain is unpredictable.
- To analyze it, we must use a "statistical model."
- The average power of noise is predictable, and it is used for circuit analysis.
- Analogy: As you get closer to a loud place, the *average* noise (power) gets predictably louder, but the sound at any given *moment* is still random.



Average Power

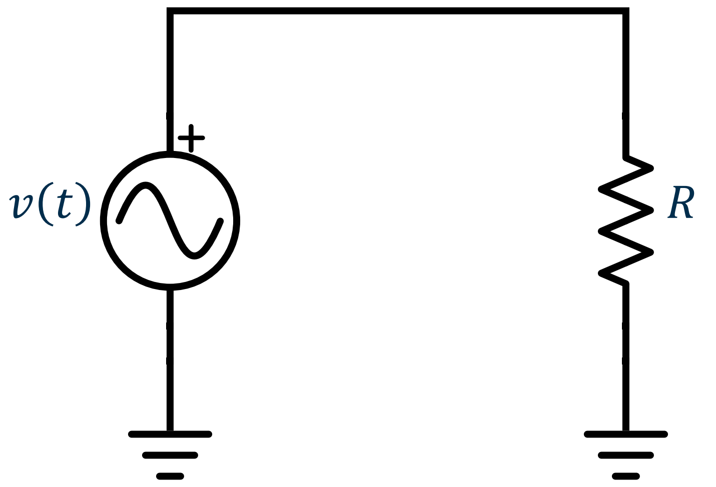
- Consider the periodic voltage

$$v(t) = V_P \sin(2\pi f t)$$

- The average power of a periodic signal delivered to resistance is given by

$$P_{av} = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{V_P^2 \sin^2\left(2\pi \frac{t}{T}\right)}{\mathrm{R}} dt$$

- T denotes the period.

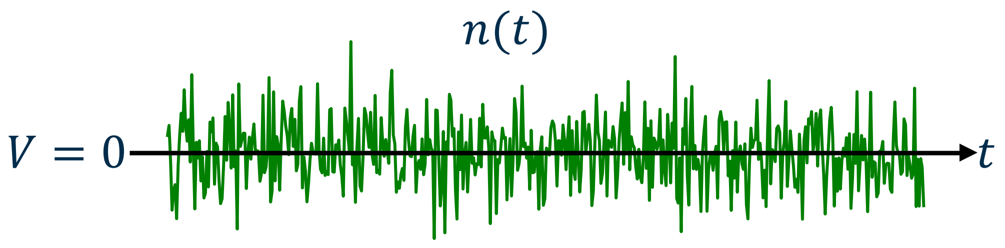


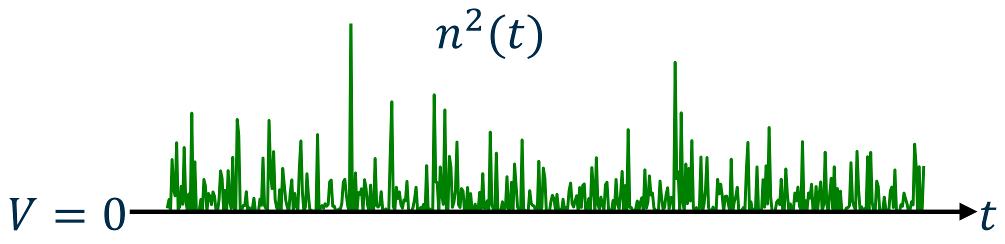
Average Power

- For a periodic signal, average power is found by squaring the voltage, integrating over one period, and dividing by the period.
- For a random signal (like noise), the process is similar, but requires a long observation time:

$$P_n = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} n^2(t)dt$$

- $n(t)$ is noise in time domain, P_n is denoted in V^2 for simplicity. It corresponds to the power delivered to a load resistance of 1Ω. Power delivered to any load can easily be calculated as $\frac{P_n}{R}$.

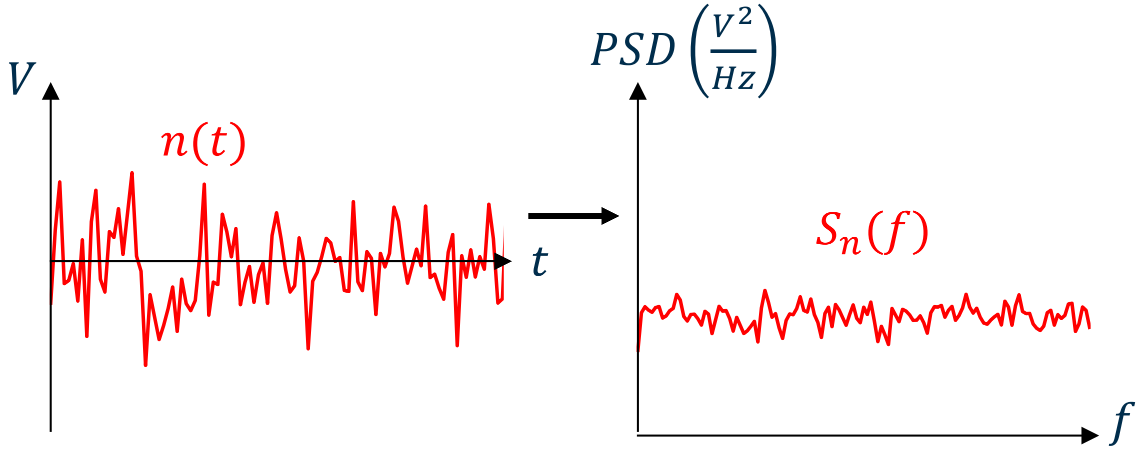




Noise Spectrum

- We analyze noise power using the **Power Spectral Density (PSD)**, the spectrum shows how much power the signal carries at each frequency.
- The PSD, $S_n(f)$, is the average noise power contained within a 1 Hz bandwidth centered at frequency f.
- The total area under the PSD curve equals the total average power of the noise.

$$\int_0^\infty S_n(f)df = \lim_{T\to\infty}\frac{1}{T}\int_0^T n^2(t)dt$$

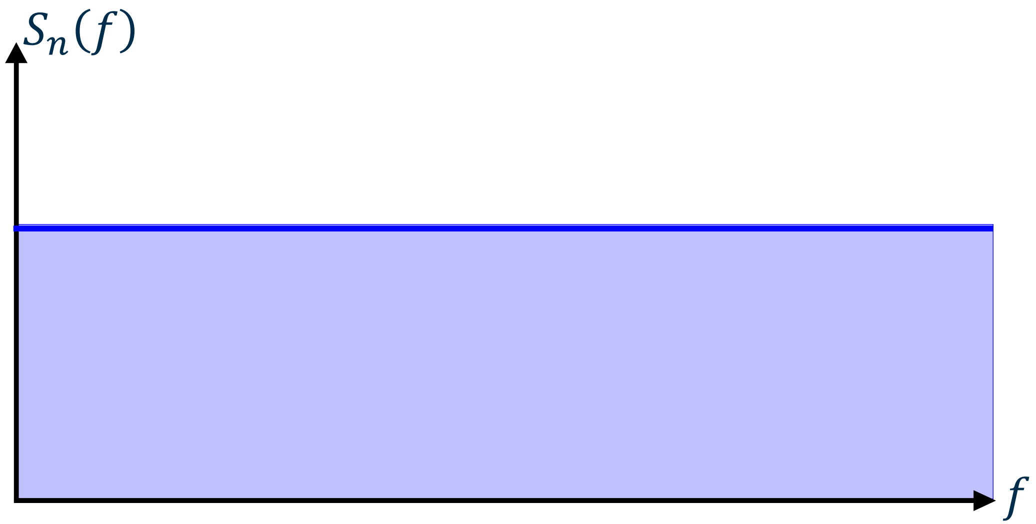


Noise Unit

- $\boldsymbol{W/Hz}$: The "true" physical unit (Watts per Hz), but rarely used in circuit analysis.
- $\boxed{\boldsymbol{V^2/Hz}}$: The standard theoretical unit. It represents the mean-square voltage per Hz (equivalent to the power in a 1Ω resistor).
- $\boxed{\boldsymbol{V/\sqrt{Hz}}}$: Another common use of the unit. Simulation tools usually provide both.
- **Example:** A noise spec of $3\,nV/\sqrt{Hz}$ means the power in a 1 Hz band is $(3\,nV)^2$, or $9 \times 10^{-18}\,V^2/Hz$.

White Noise

- An example of a common type of noise PSD is the white noise.
- A noise source with a flat, constant PSD at all frequencies.
- True white noise is physically impossible, as it would have infinite total power (infinite area under the curve).
- Noise types like white noise and pink noise are named by analogy to the spectrum of light. The one famous exception is Brown noise. It is named after the botanist Robert Brown.

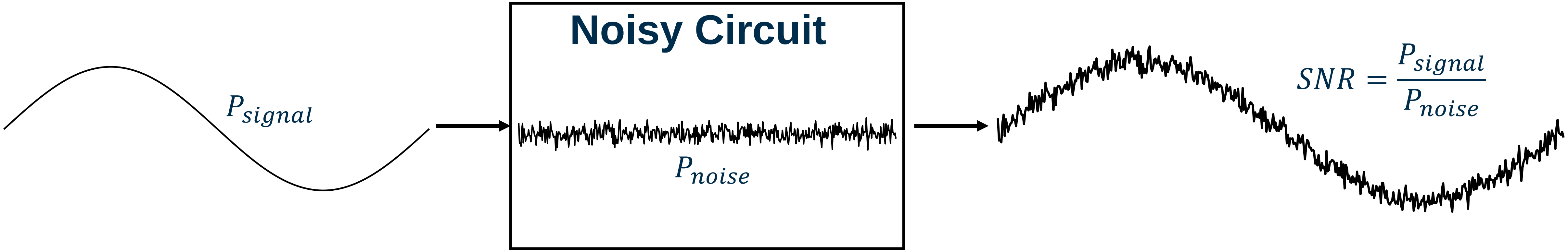


Signal-to-Noise Ratio

- For a signal to be intelligible, its power must be sufficiently higher than the circuit's noise. This relationship is measured by the Signal-to-Noise Ratio (SNR).

$$\boxed{SNR = \frac{P_{signal}}{P_{noise}}}$$

- Example: Audio signals need a minimum SNR of 20 dB, meaning $\frac{P_{signal}}{P_{noise}} = 100$.



Signal-to-Noise Ratio

- The total average noise power P_n is the total area under its Power Spectral Density (PSD) curve.

$$P_n = \int_{0}^{+\infty} S_{noise}(f)df$$

- In a bandwidth-limited system the wider the frequency range (bandwidth) of a circuit, the more noise power it will integrate.

$$P_n = \int_0^{\Delta f} S_{noise}(f)df = S_{noise} \times \Delta \mathrm{f}$$

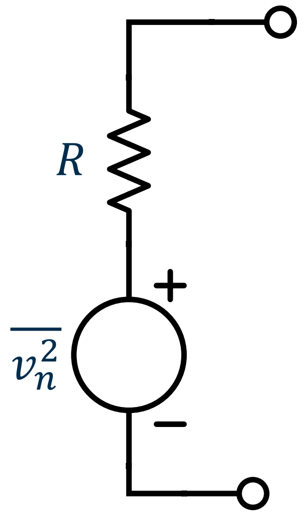
- To maximize SNR, you must limit the circuit's bandwidth to the minimum necessary for the signal.

Types of Noise

- Signals in integrated circuits are affected by two types of noise:
  - Intrinsic device noise (from within the electronic components)
  - Environmental noise (random disturbances coupled via supply, ground, or substrate)
- We will now focus on intrinsic device noise, which includes:
  - Thermal noise
  - Flicker noise
  - Shot noise (omitted)

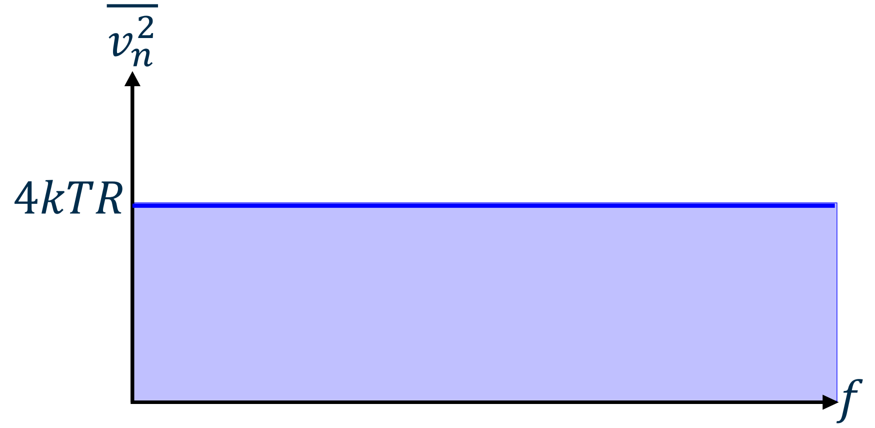
Thermal Noise

- Thermal noise is random motion of electrons in a conductor, creating voltage fluctuations. These fluctuations occur even if the average current is zero.
- It's proportional to absolute temperature (T).



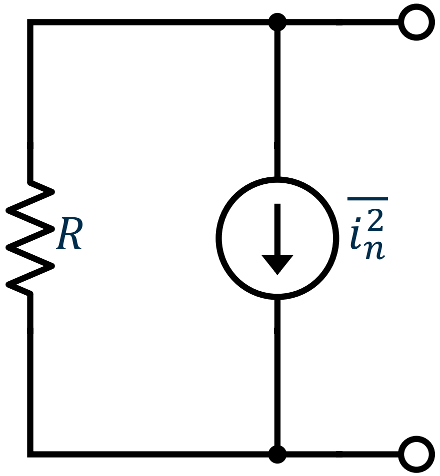
$$\boxed{\overline{v_n^2} = 4kTR\left[\frac{V^2}{Hz}\right]}$$

- $k = Boltzmann's\ constant = 13.8 \times 10^{-24}\,\frac{J}{°K}$
- $T = Temperature\ in\ °K$



Thermal Noise

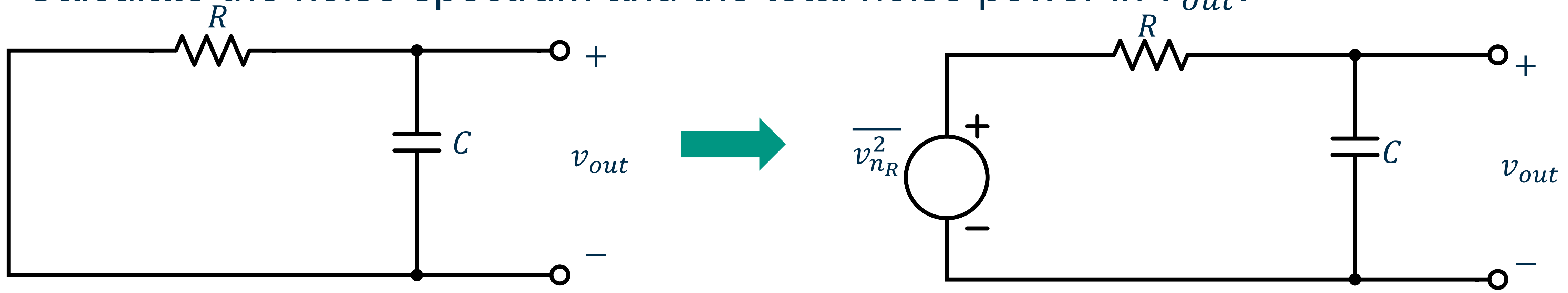
- Thermal noise can also be modeled as a current source.
- The model can be chosen based on circuit convenience.



$$\boxed{\overline{i_n^2} = \frac{\overline{v_n^2}}{R^2} = \frac{4kT}{R}\left[\frac{A^2}{Hz}\right]}$$

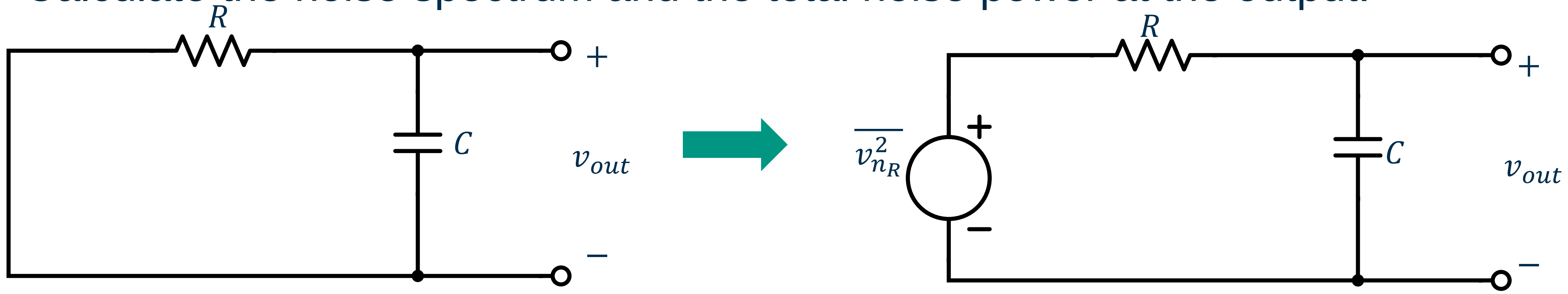
Thermal Noise of an RC circuit

- Calculate the noise spectrum and the total noise power in v_{out}.



Thermal Noise of an RC circuit

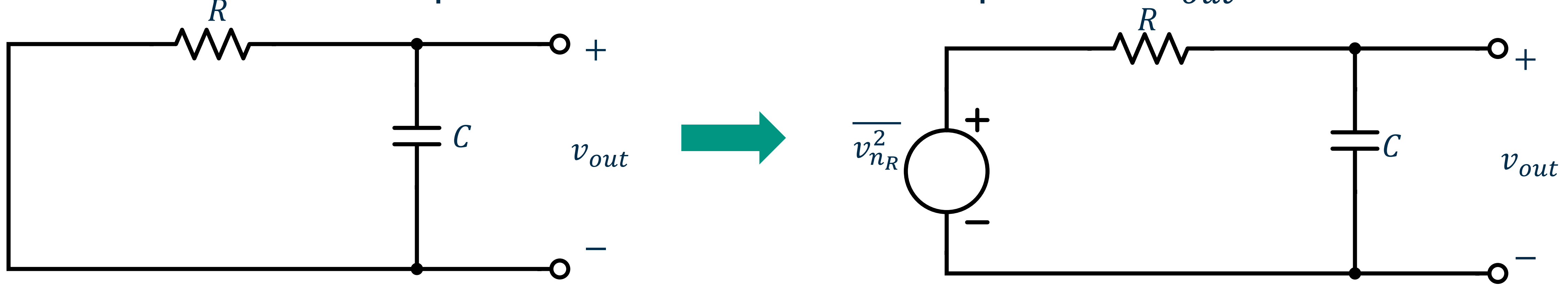
- Calculate the noise spectrum and the total noise power at the output.



1. $\overline{v_{n_R}^2} = 4kTR$
2. $v_{out} = v_{n_R}\frac{1/j\omega C}{R+1/j\omega C} = v_{n_R}\frac{1}{1+j\omega RC}$
3. $\overline{v_{out}^2} = \overline{v_{n_R}^2}\left|\frac{1}{1+j\omega RC}\right|^2 = \frac{4kTR}{1+4\pi^2R^2C^2f^2}$
4. $P_{n_{out}} = \int_0^\infty \overline{v_{out}^2}df = \int_0^\infty \frac{4kTR}{1+4\pi^2R^2C^2f^2}df$
5. $2\pi RCf = x \rightarrow df = \frac{dx}{2\pi RC}$
6. $P_{n_{out}} = \frac{4kTR}{2\pi RC}\int_0^\infty \frac{1}{1+x^2}dx = \frac{2kT}{\pi C}\left[\tan^{-1}x\right]_0^\infty = \frac{2kT}{\pi C}\frac{\pi}{2} = \frac{kT}{C}$

Thermal Noise of an RC circuit

- Calculate the noise spectrum and the total noise power in v_{out}.



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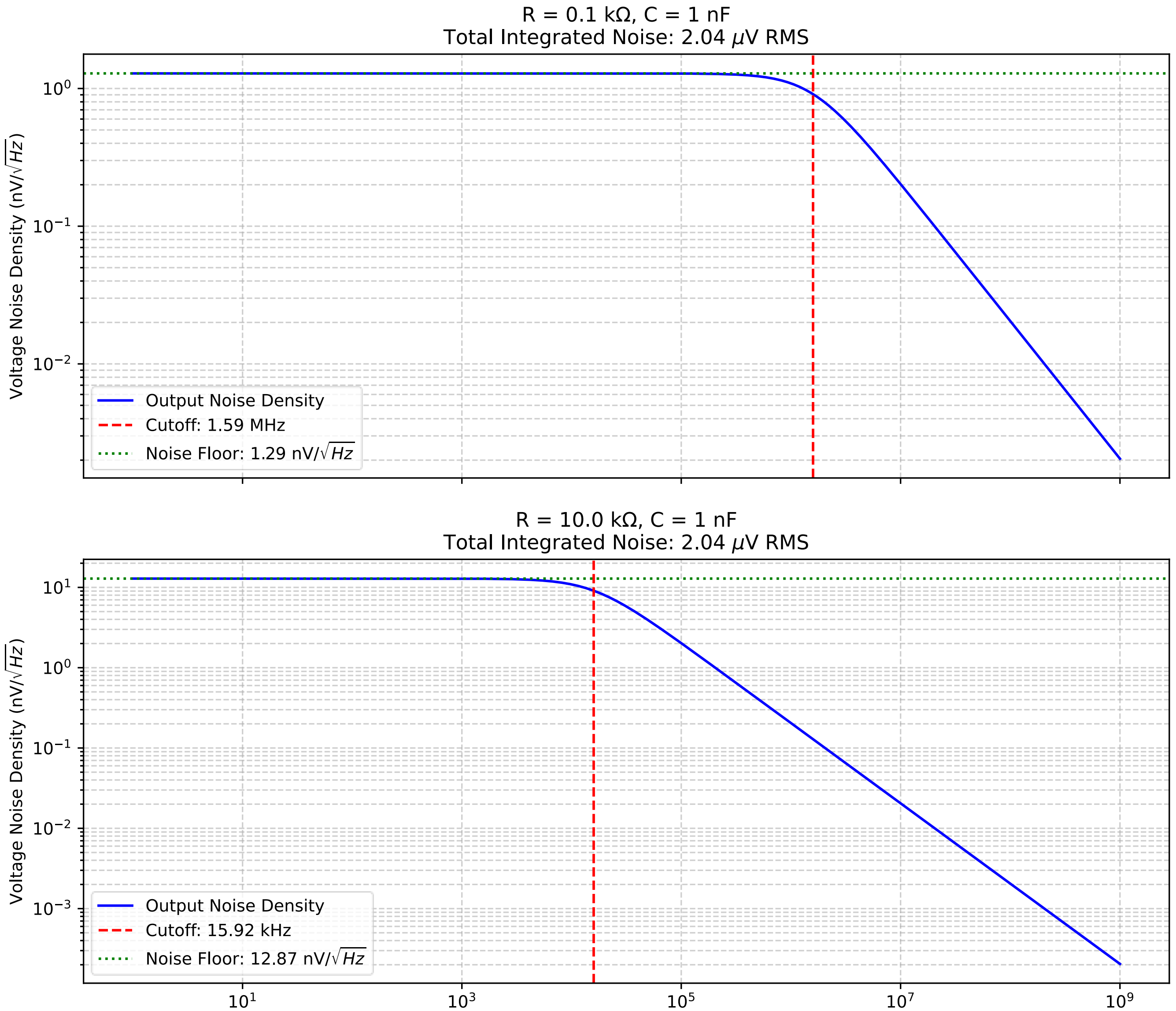
$$\boxed{P_{n_{out}} = \frac{kT}{C}}$$

- The total noise at the output of the circuit is independent of the value of R.
- Although larger values of R produces more noise per unit bandwidth, it narrows the overall bandwidth in the meantime.
- kT/C noise can be decreased only by increasing C (assuming T is fixed). This introduces many difficulties in the design of analog circuits.

Thermal Noise of an RC circuit



$$For\ C = 1nF,$$

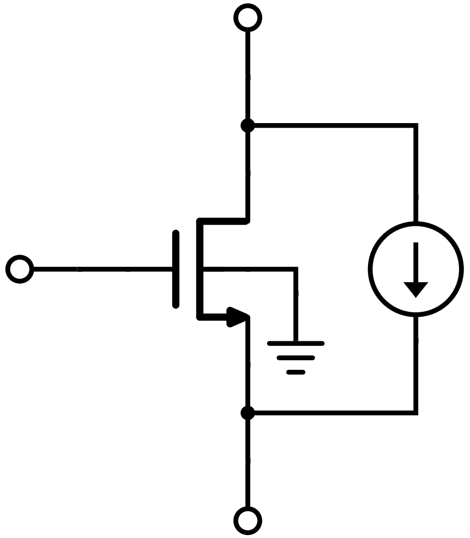
$$\frac{kT}{C} = \frac{\left(1.380649\ \times\ 10 - 23\ \frac{J}{K}\right)(300K)}{1nF}$$

$$\frac{kT}{C} = 4.142\ pV^2$$

$$\sqrt{\frac{kT}{C}} = 2.04\mu V\ RMS$$

MOSFET Thermal Noise

- MOSFETs generate thermal noise, too. Primarily due to the channel.
- For a MOSFET in saturation, it is typically modeled as a noise current source $\overline{i_n^2}$ between the drain and source.
- The output resistance r_o is a model for channel length modulation; it is not a physical resistor and does not produce thermal noise itself.

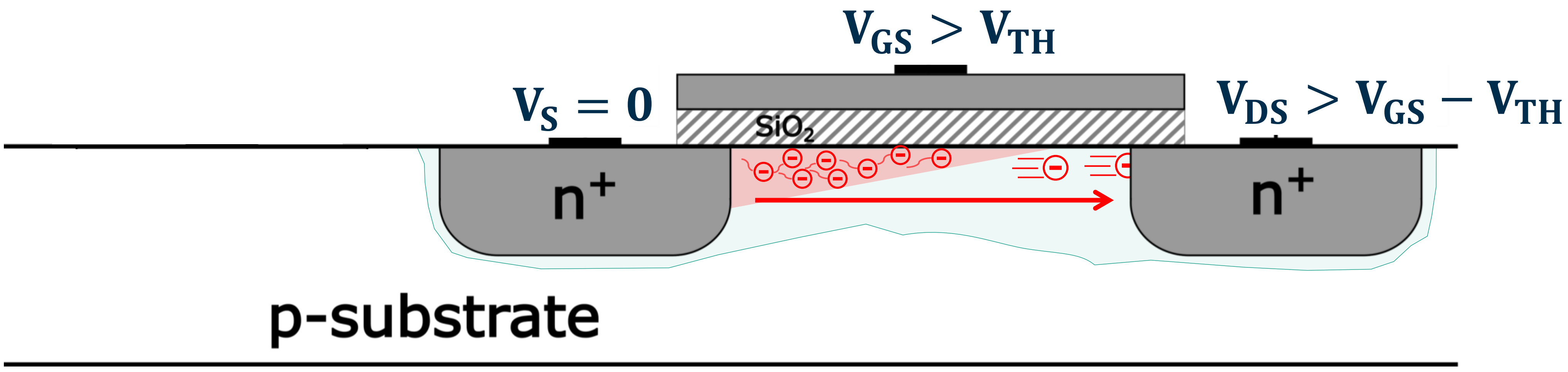


$$\overline{i_n^2} = 4kT\gamma g_m$$

- γ, Excess Noise Coefficient is typically 2/3 for long-channel devices.

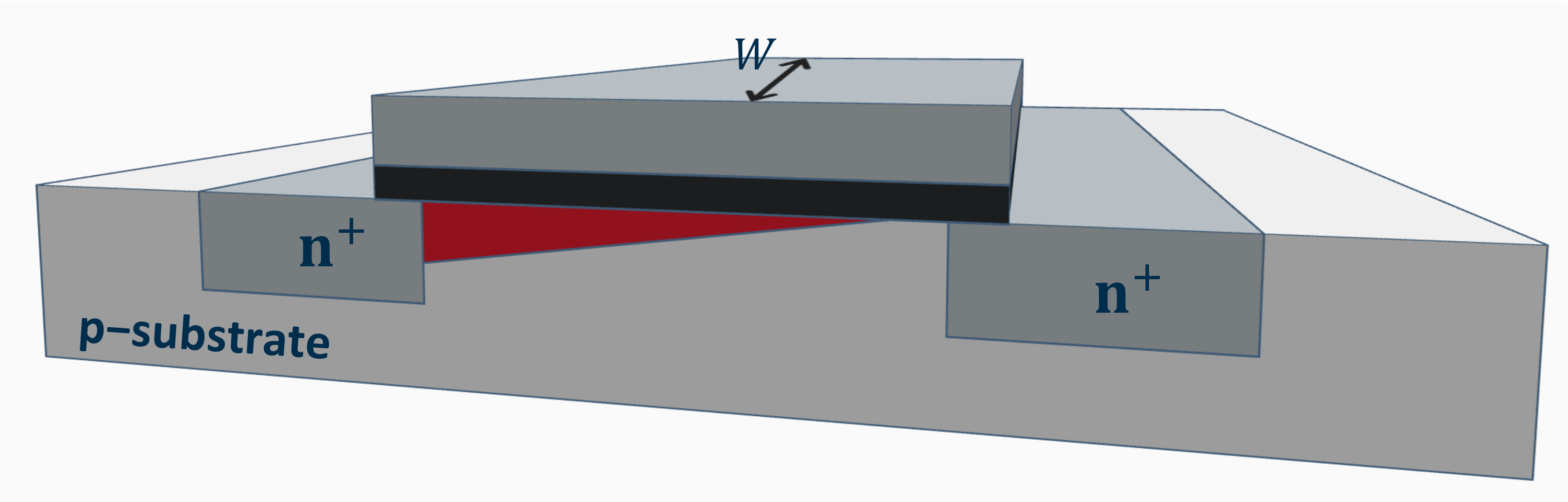
MOSFET Thermal Noise

- The channel is acting like a variable resistor and exhibiting some thermal noise.



MOSFET Thermal Noise

- Let us derive the expression analytically.



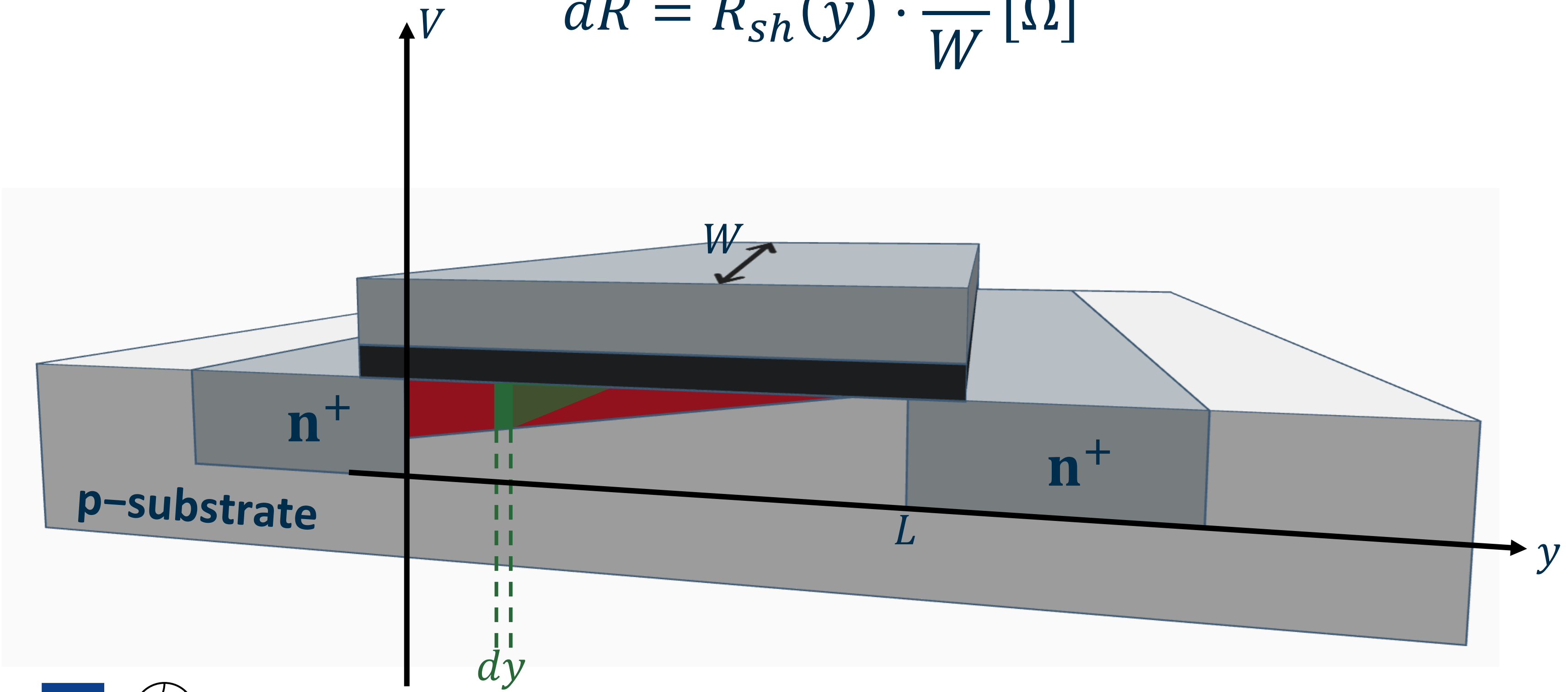
MOSFET Thermal Noise

- We need to calculate the channel resistance.
- First calculate the sheet resistance at slice dy, then integrate over whole channel.

$$R_{sh}(y) = \frac{1}{\mu_n |Q_I(y)|}\ [\Omega/\square]$$

$$dR = R_{sh}(y) \cdot \frac{dy}{W}\ [\Omega]$$

- $\mu_n: Electron\ mobility$
- $Q_I(y): Charge\ at$ slice dy
- $R_{sh} = \rho/t$ (Resistivity divided by thickness)



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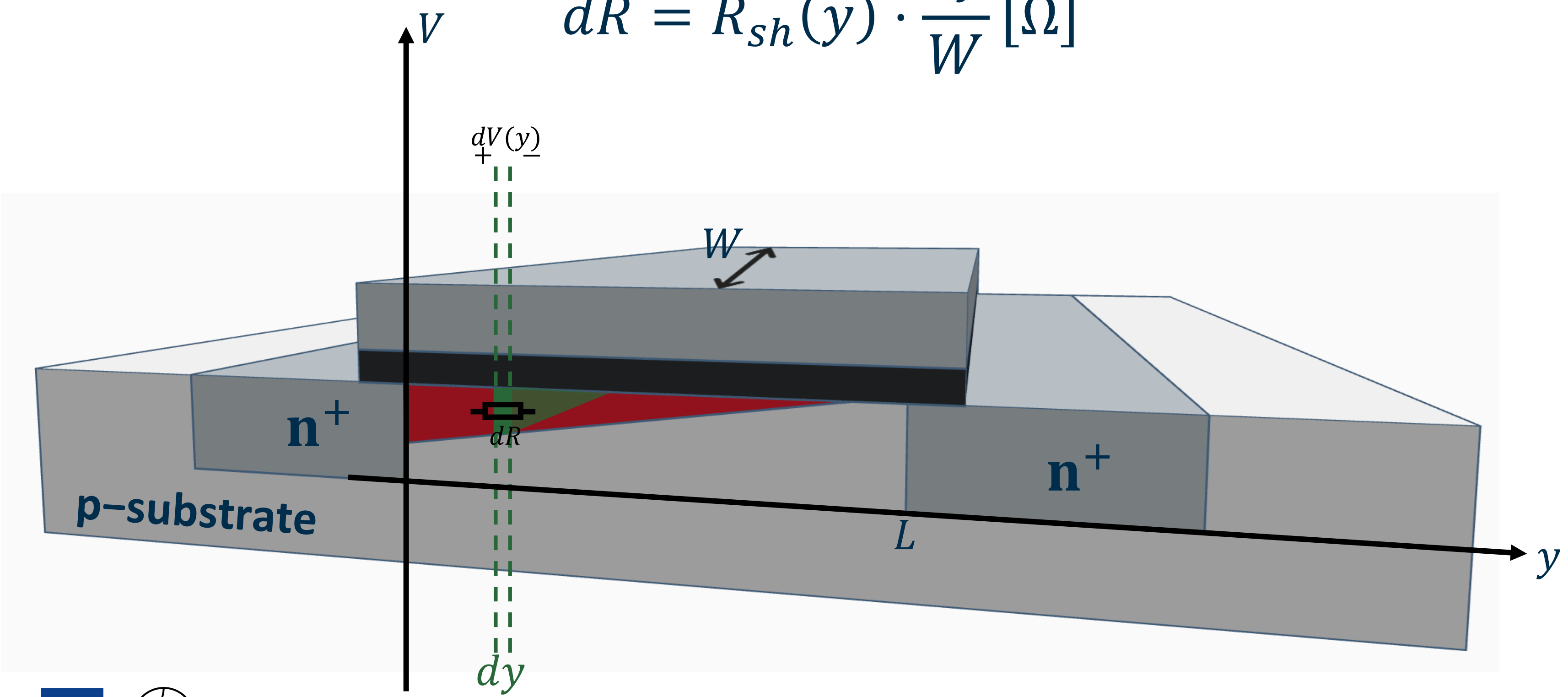
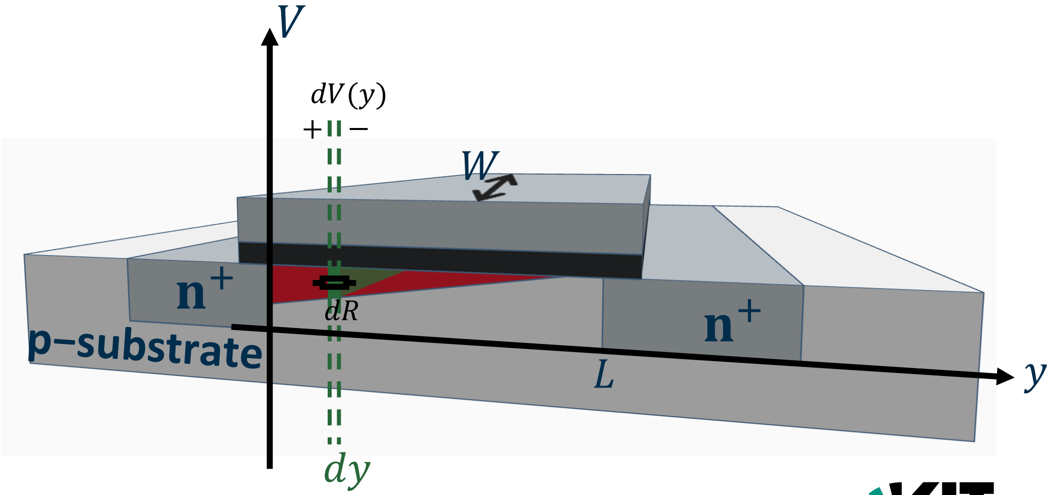


MOSFET Thermal Noise

- $|Q_I(y)| = C_{ox}(V_{GS} - V(y) - V_{TH})$
- $R_{sh}(y) = \frac{1}{\mu_n C_{ox}(V_{GS} - V(y) - V_{TH})}$
- $dR = \frac{dy}{\mu_n C_{ox} W(V_{GS} - V(y) - V_{TH})}$
- $dV(y) = I_D \cdot dR = \frac{I_D \cdot dy}{\mu_n C_{ox} W(V_{GS} - V(y) - V_{TH})}$
- $I_D \cdot dy = \mu_n C_{ox} W(V_{GS} - V(y) - V_{TH}) dV(y)$
- The total charge in the inverted channel:

$$Q_{I,T} = \int_0^L W \cdot Q_I(y) \cdot dy$$

- $dy = \frac{\mu_n C_{ox} W(V_{GS} - V(y) - V_{TH})}{I_D} dV(y)$



$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{\varepsilon_{r_{SiO_2}} \varepsilon_0}{t_{ox}} = \frac{Dielectric\ constant}{Oxide\ thickness} \left[\frac{F}{m^2}\right]$$

MOSFET Thermal Noise

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- $R_{sh}(y) = \frac{1}{\mu_n C_{ox}(V_{GS} - V(y) - V_{TH})}$
- $dR = \frac{dy}{\mu_n C_{ox} W(V_{GS} - V(y) - V_{TH})}$
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- $dy = \frac{\mu_n C_{ox} W(V_{GS} - V(y) - V_{TH})}{I_D} dV(y)$

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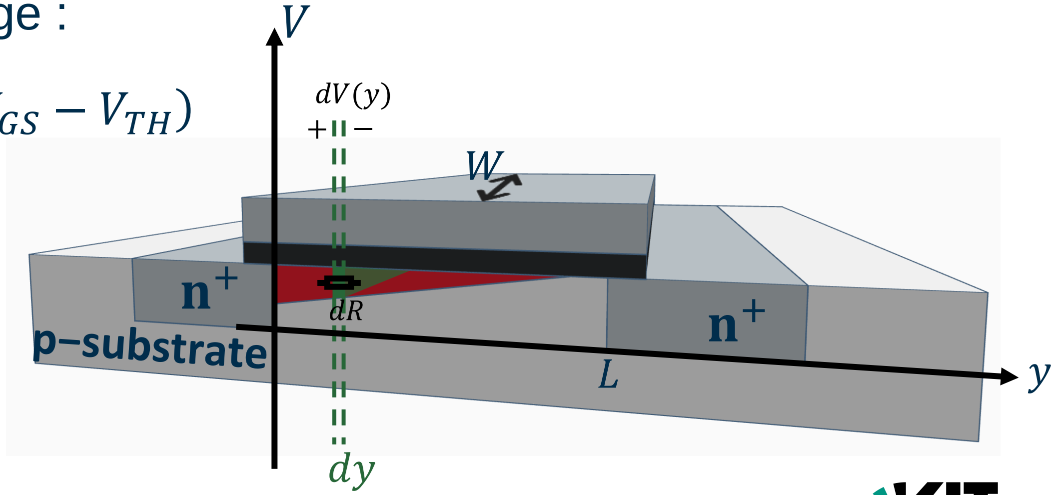
- $Q_{I,T} = \int_0^{V_{DS}} W \cdot C_{ox}(V_{GS} - V(y) - V_{TH}) \cdot \left[\frac{\mu_n C_{ox} W(V_{GS} - V(y) - V_{TH})}{I_D}\right] \cdot dV(y)$
- $Q_{I,T} = \frac{\mu_n W^2 C_{ox}^2}{I_D} \int_0^{V_{DS}} (V_{GS} - V(y) - V_{TH})^2 dV(y)$
- To evaluate the integration in the saturation region set $V_{DS} = V_{GS} - V_{TH}$. Apply for I_D the current formula in saturation region:

$$Q_{I,T} = \frac{\mu_n W^2 C_{ox}^2}{\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2} \left[- \frac{(V_{GS} - V - V_{TH})^3}{3} \right]_0^{V_{GS} - V_{TH}}$$

- $Q_{I,T} = \frac{2}{3} W L C_{ox}(V_{GS} - V_{TH})$
- The average charge :

$$\overline{Q_{I,T}} = \frac{Q_{I,T}}{WL} = \frac{2}{3} C_{ox}(V_{GS} - V_{TH})$$



MOSFET Thermal Noise

$$R_{sh} = \frac{1}{\mu_n \left|\overline{Q_{I,T}}\right|} = \frac{1}{\frac{2}{3}\mu_n C_{ox}(V_{GS} - V_{TH})}$$

$$dR = R_{sh}\frac{dy}{W} = \frac{1}{\mu_n \overline{\overline{Q_{I,T}}}}\frac{dy}{W}$$

$$R = \int_0^L dR = \int_0^L \frac{1}{\mu_n \left|\overline{Q_{I,T}}\right|}\frac{dy}{W} = \frac{L}{\frac{2}{3}\mu_n C_{ox} W (V_{GS} - V_{TH})}$$

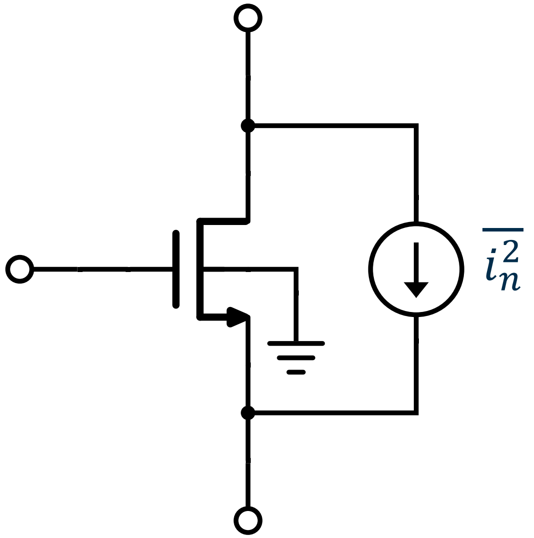
$$\boxed{R = \frac{1}{\frac{2}{3}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})} = \frac{3}{2}\frac{1}{gm}}$$

MOSFET Thermal Noise

- Consequently, for a long channel MOSFET, the equivalent thermal current noise is:

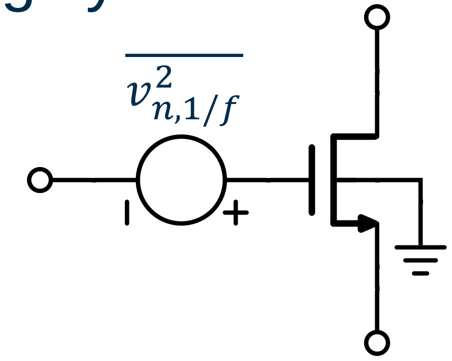
$$\boxed{\overline{i_n^2} = \frac{4kT}{R} = \frac{4kT}{\left(\frac{3}{2}\frac{1}{gm}\right)} = 4kT\frac{2}{3}g_m = 4kT\gamma g_m}$$

- γ, Excess Noise Coefficient is higher for short-channel devices.

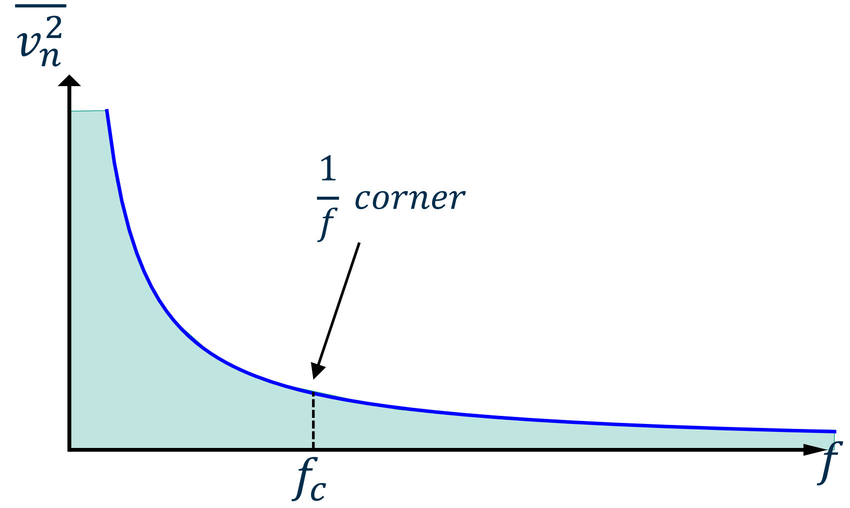


Flicker Noise (The 1/f Noise)

- Flicker noise is caused by charge trapping at oxide–silicon interface.
- Unlike thermal noise, the average power of flicker noise cannot be predicted easily.
- It's called "1/f noise" because its power is inversely proportional to frequency.
- It is roughly calculated in the saturation region by



$$\overline{v_{n,1/f}^2} = \frac{K}{C_{ox}WL} \cdot \frac{1}{f}$$

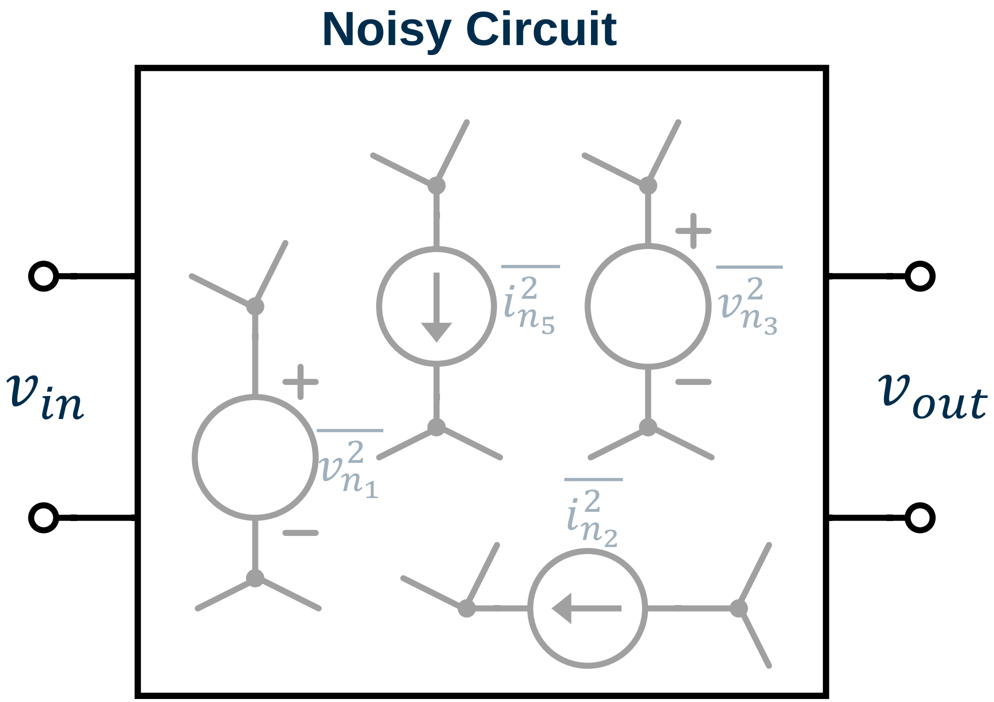


where K is a process-dependent constant.

- To reduce flicker noise, you must increase the device area WL.
- PMOS devices generally have less 1/f noise than NMOS devices.

Noise Representation in Circuits

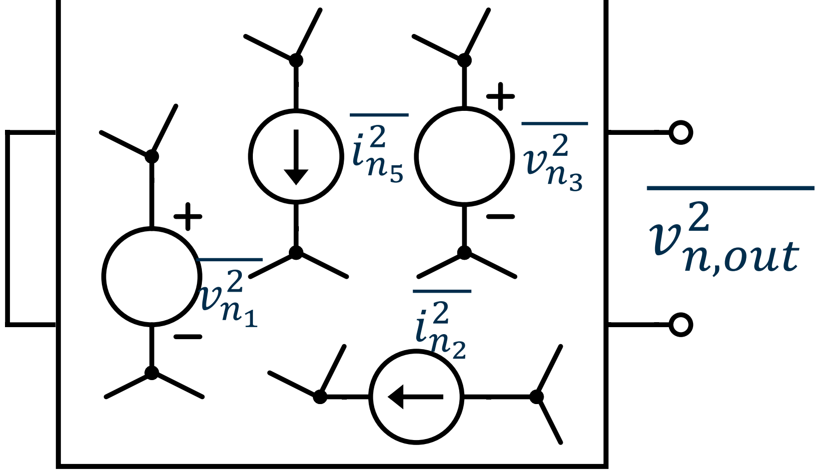
- The output noise can be calculated for each individual noise source through superposition, meaning all other independent sources in the circuit are eliminated.
- The resulting total noise at the output is called **output-referred noise**.



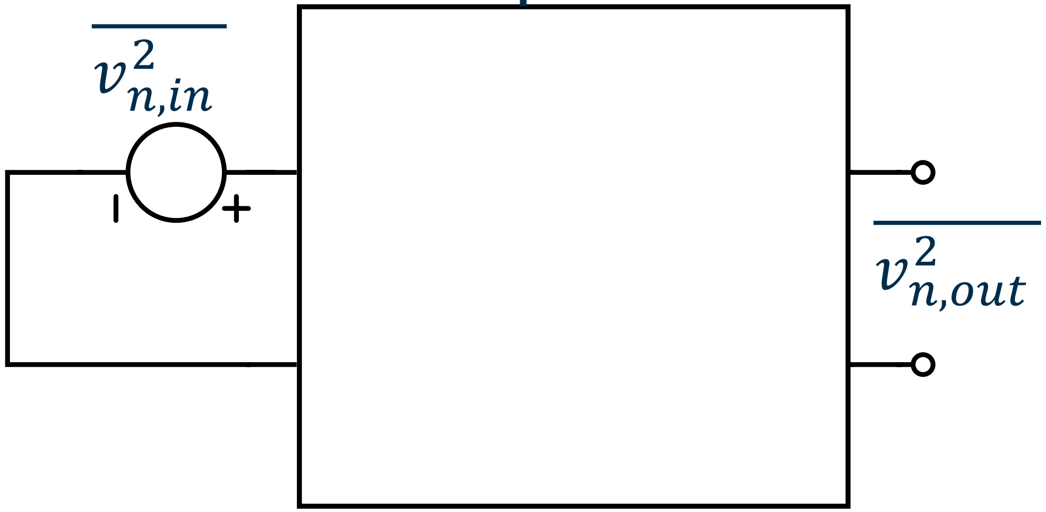
Input-Referred Noise

- The output-referred noise can be used as a comparison metric for circuits with the same gain.
- On the other hand, the **input-referred noise** is independent of gain, making it a more suitable parameter for general circuit comparisons.

Noisy Circuit

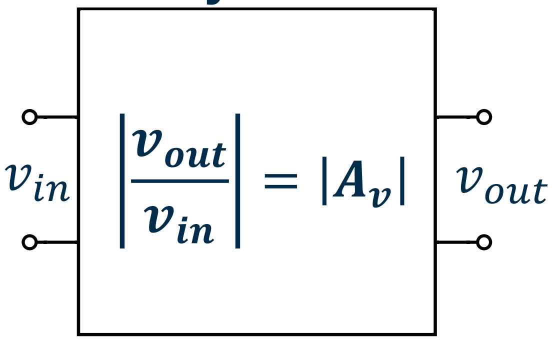


Noiseless Circuit with Input-Referred noise representation



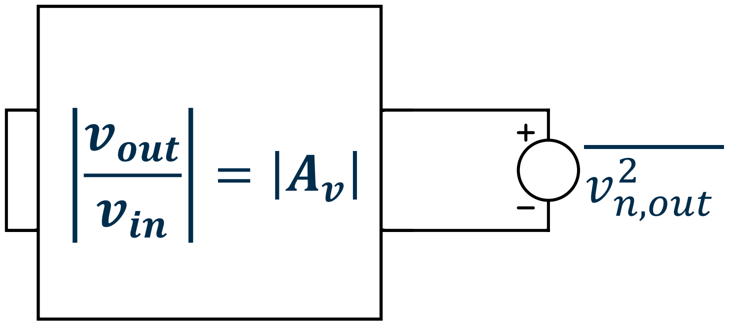
Input & Output Referred Noise

Noisy Circuit



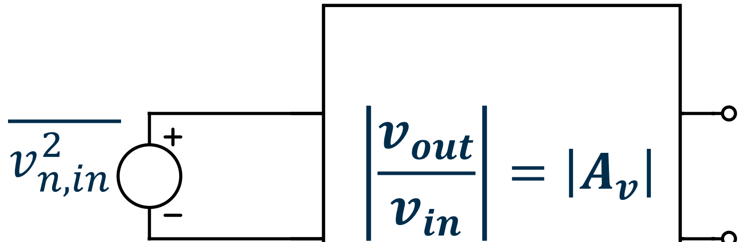
- Voltage Amplifier

Noiseless Circuit



- Output Referred Noise

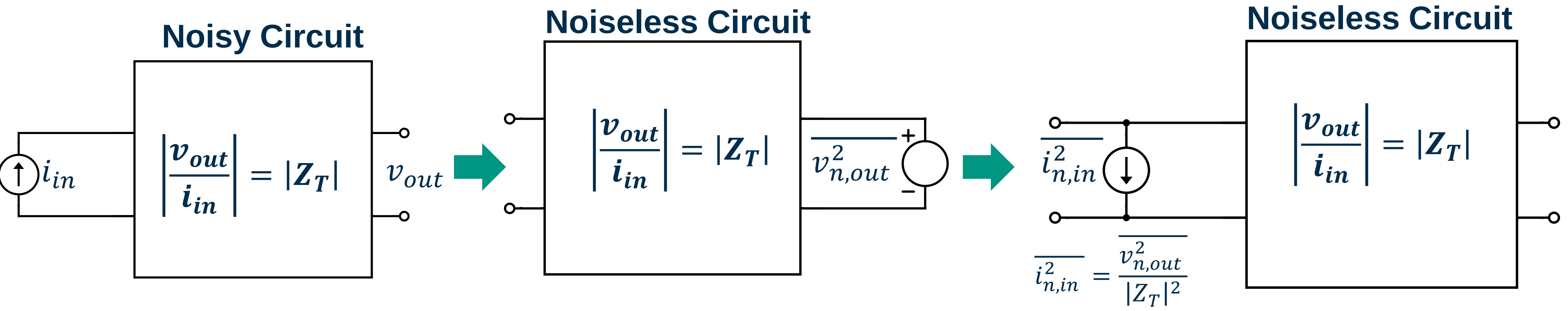
Noiseless Circuit



$$\overline{v_{n,in}^2} = \frac{\overline{v_{n,out}^2}}{|A_v|^2}$$

- Input Referred Noise

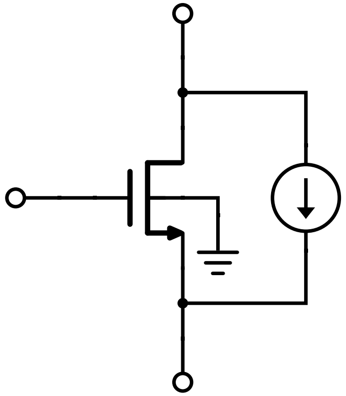
Input & Output Referred Noise



- Transimpedance Amplifier
- Output Referred Noise
- Input Referred Current Noise

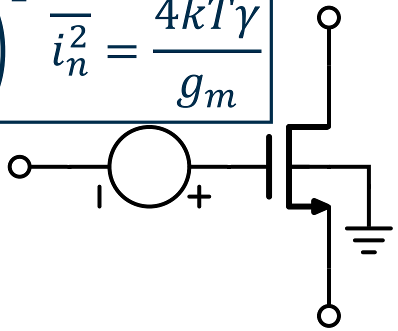
Revise MOSFET Noise

- Thermal Noise



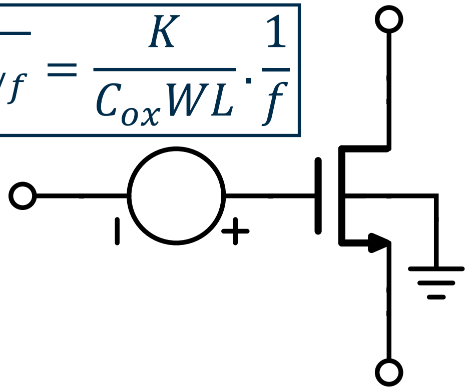
$$\overline{i_n^2} = 4kT\gamma g_m$$

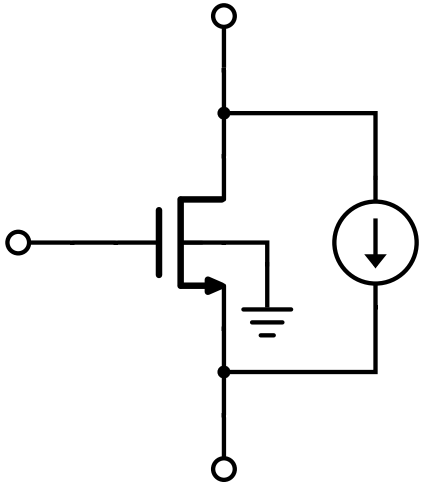
$$\overline{v_n^2} = \left(\frac{1}{g_m}\right)^2 \overline{i_n^2} = \frac{4kT\gamma}{g_m}$$



- Flicker Noise

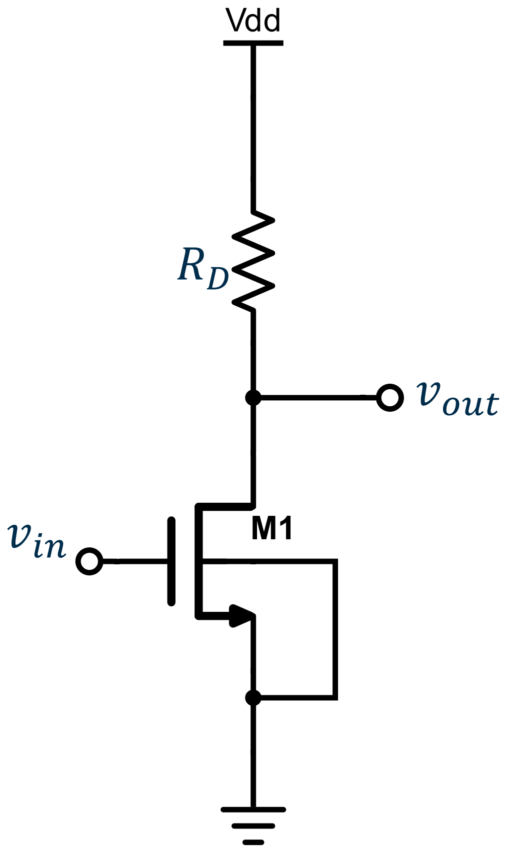
$$\overline{v_{n,1/f}^2} = \frac{K}{C_{ox}WL} \cdot \frac{1}{f}$$





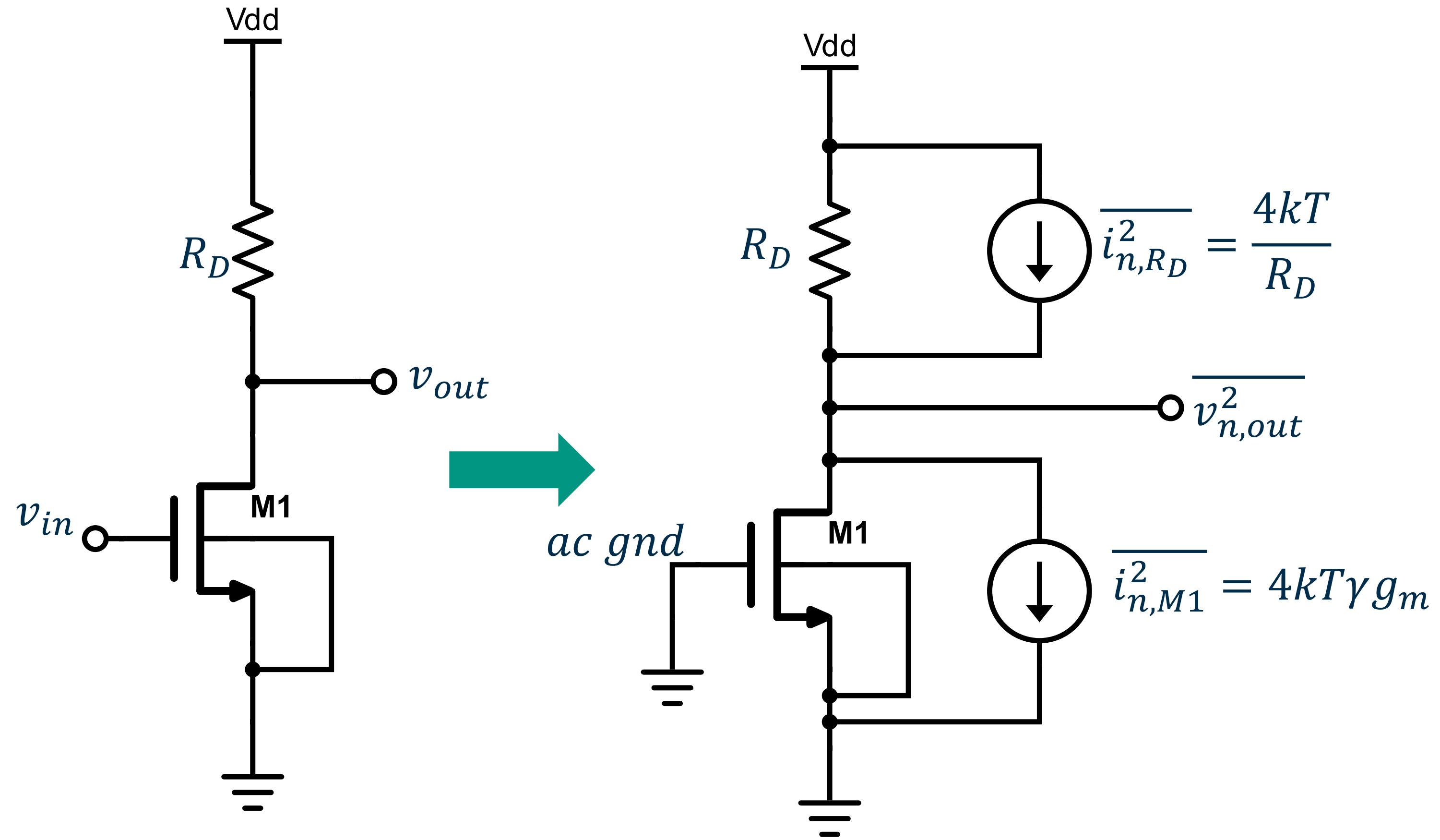
$$\overline{i_{n,1/f}^2} = \overline{v_{n,1/f}^2} \cdot g_m^2 = \frac{K}{C_{ox}WL} \cdot \frac{1}{f} \cdot g_m^2$$

Common Source Amplifier Output Noise



$$|A_v| = \frac{v_{out}}{v_{in}} = g_m R_D$$

Common Source Amplifier Output Noise



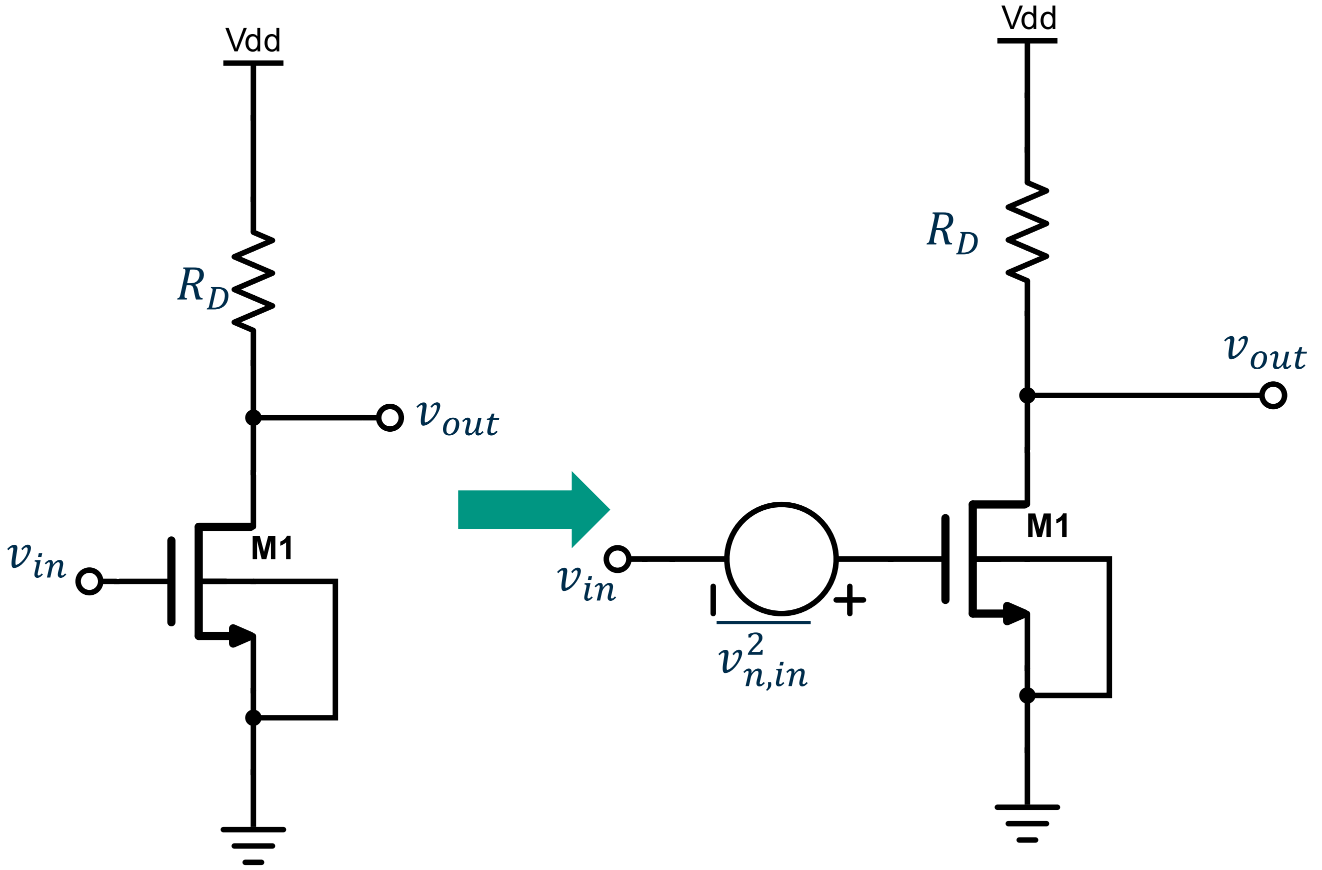
$$|A_v| = \frac{v_{out}}{v_{in}} = g_m R_D$$

$$\overline{v_{n,out}^2} = \left(\overline{i_{n,M1}^2} + \overline{i_{n,R_D}^2}\right) R_D^2$$

$$\boxed{\overline{v_{n,out}^2} = \left(4kT\gamma g_m + \frac{4kT}{R_D}\right) R_D^2}$$

Flicker noise omitted.

Common Source Amplifier Input-Referred Noise

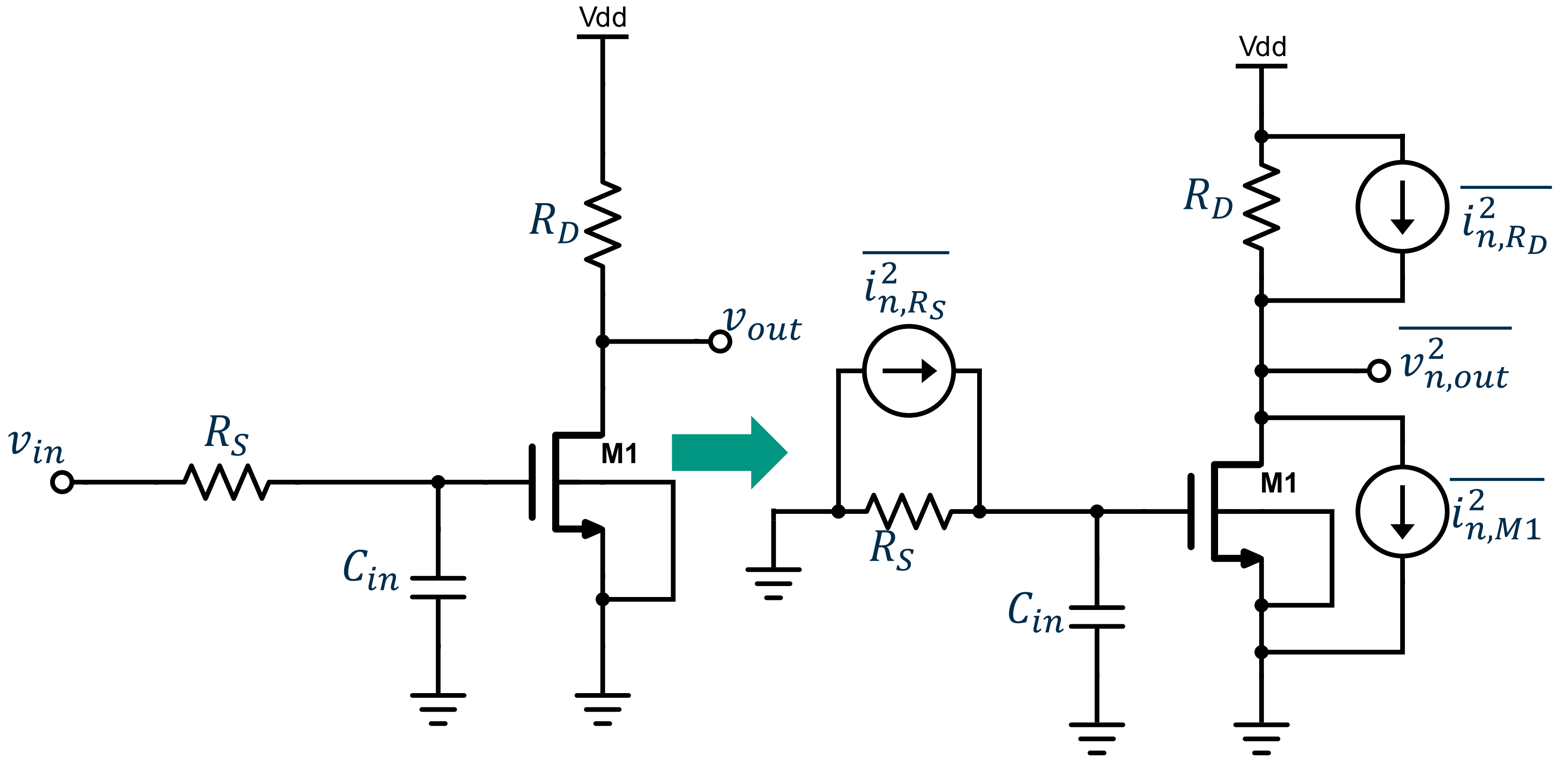


$$|A_v| = \frac{v_{out}}{v_{in}} = g_m R_D$$

$$\overline{v_{n,out}^2} = \left(4kT\gamma g_m + \frac{4kT}{R_D}\right) R_D^2$$

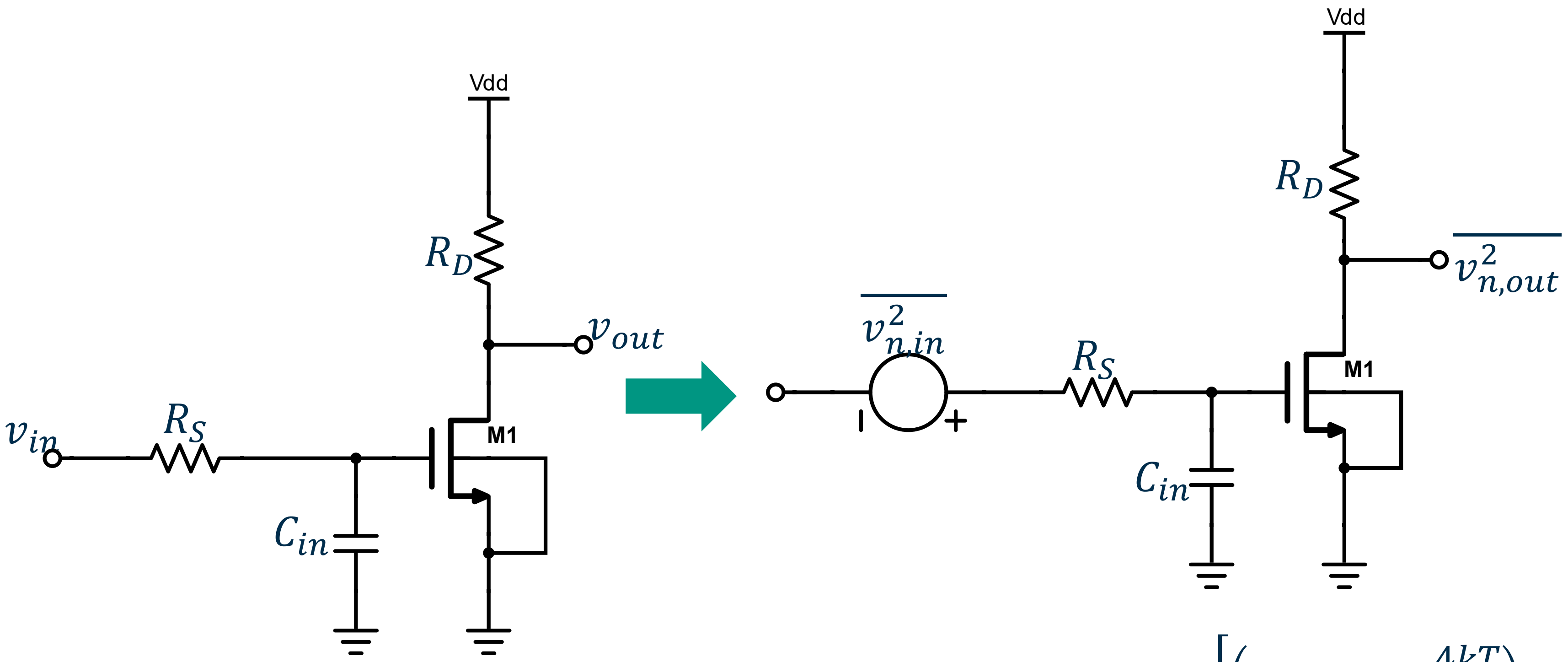
$$\boxed{\overline{v_{n,in}^2} = \frac{\overline{v_{n,out}^2}}{|A_v|^2} = 4kT\left(\frac{\gamma}{g_m} + \frac{1}{g_m^2 R_D}\right)}$$

CS Stage with Source Resistance & Input Capacitance



$$\overline{v_{n,out}^2} = \left(4kT\gamma g_m + \frac{4kT}{R_D}\right) R_D^2 + \frac{4kT}{R_s}\left(\frac{R_S^2}{1 + 4\pi f^2 R_S^2 C_{in}^2}\right)\left(g_m^2 R_D^2\right)$$

CS Stage with Source Resistance & Input Capacitance



$$\overline{v^2_{n,in}} = \frac{\overline{v^2_{n,out}}}{|A_v|^2} = \frac{\left[\left(4kT\gamma g_m + \frac{4kT}{R_D}\right)R_D^2 + \frac{4kT}{R_s}\left(\frac{R_S^2}{1+4\pi f^2 R_S^2 C_{in}^2}\right)\left(g_m^2 R_D^2\right)\right]}{\left(g_m^2 R_D^2\right)}$$

$$\boxed{\overline{v^2_{n,in}} = \left(\frac{4kT\gamma}{g_m} + \frac{4kT}{g_m^2 R_D}\right) + 4kTR_s\left(\frac{1}{1+4\pi f^2 R_S^2 C_{in}^2}\right)}$$