Mikroelektronische Schaltungen und Systeme

Lect.4
Differential Amplifiers

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Mikroelektronische Schaltungen und Systeme

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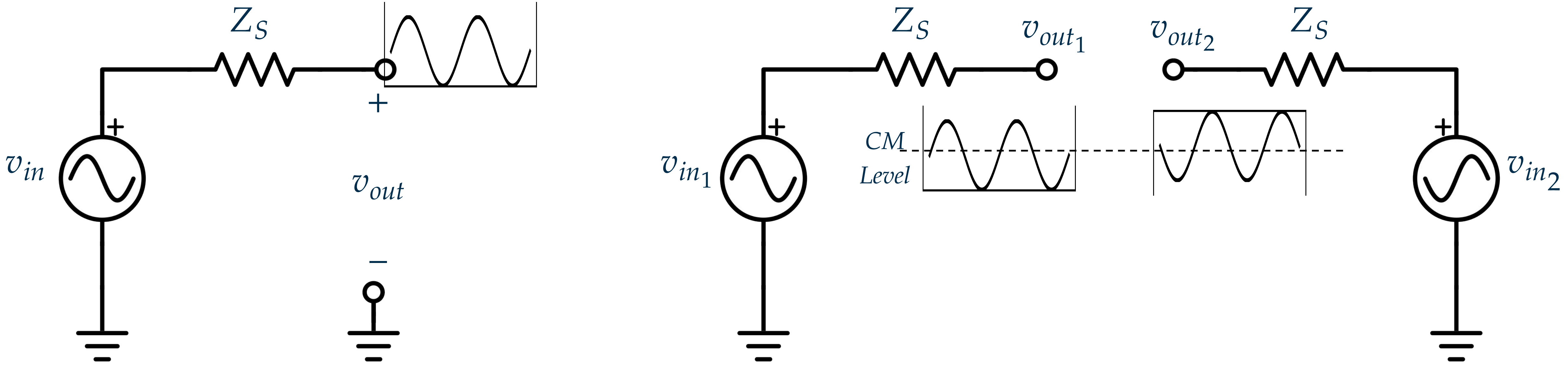
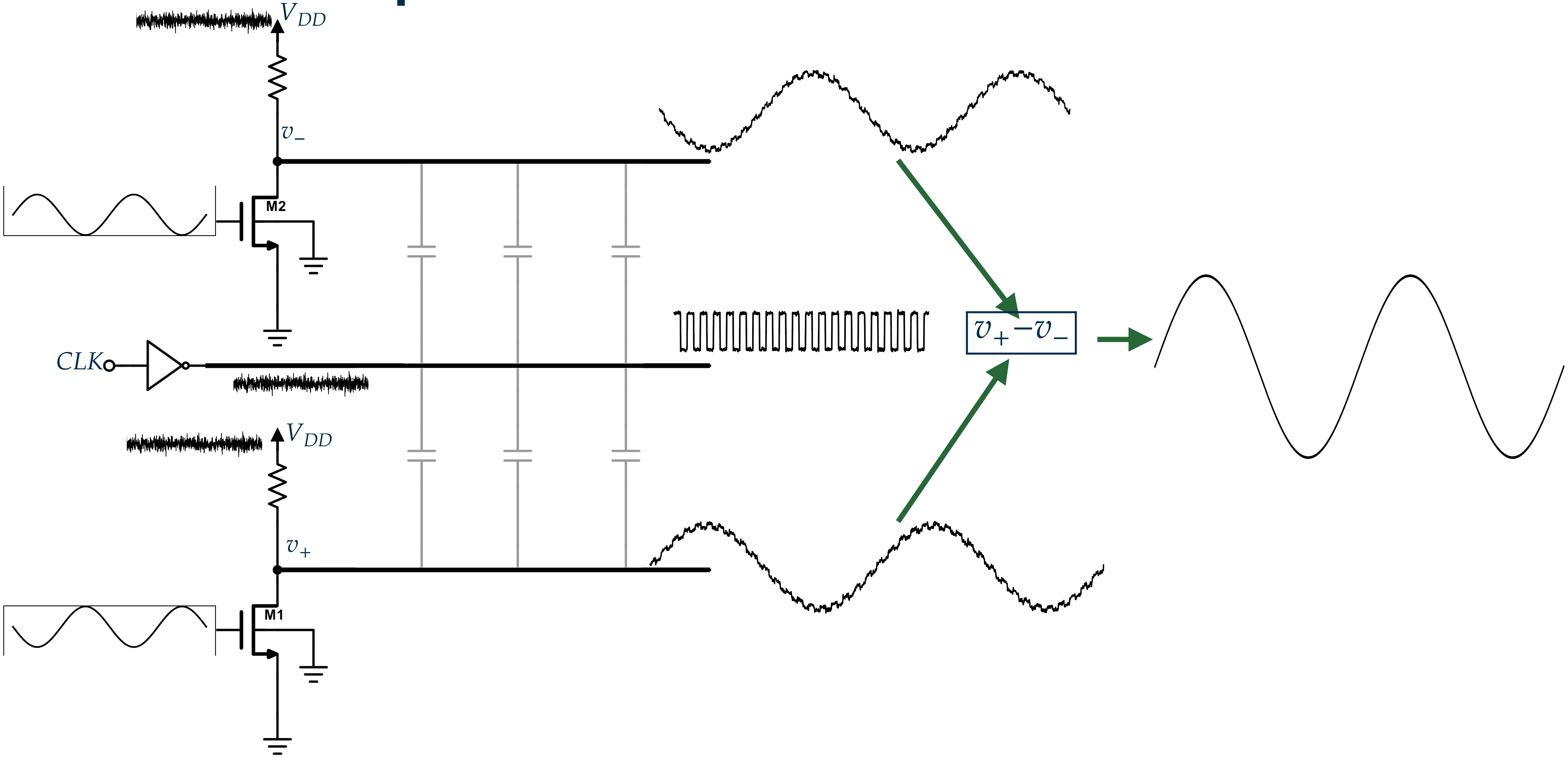
to 0);
r.conf.addr
v.conf.wb.wr
7 downto 0)
el(0) & wb_b

Differential Operation

- **Single-Ended Signal:** Measured with respect to a fixed potential, usually ground.
- **Differential Signal:** Measured between two nodes (e.g. v_{out_1}, v_{ou2}) that have equal and opposite excursions around a "common-mode" (CM) level.



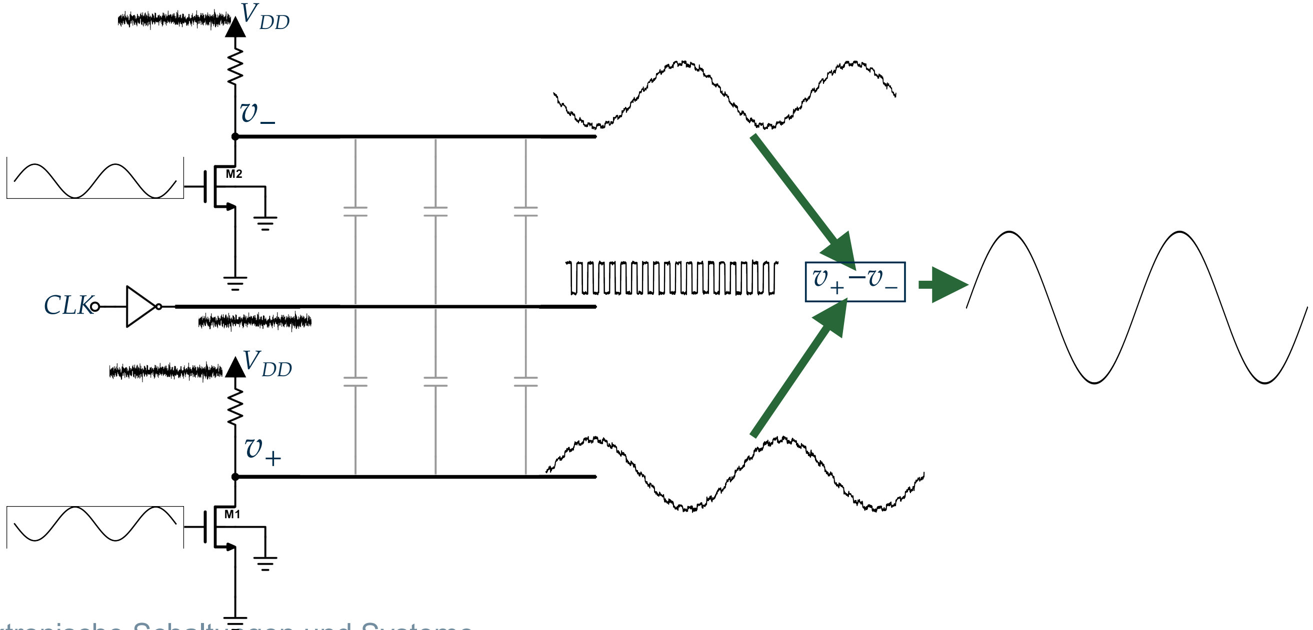
Differential Operation



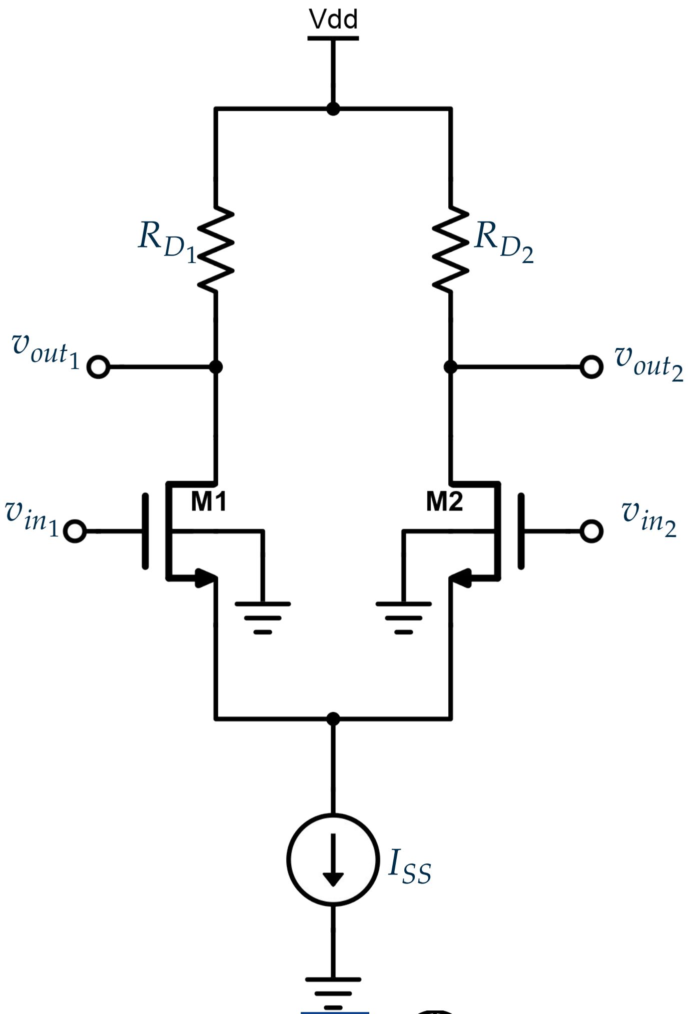
Differential Operation

Why Differential?

- A single-ended signal is highly sensitive to noise or interference (e.g., a clock line) coupling onto it. The amplifier can't distinguish the noise from the signal.
- Differential lines are physically close. The interference couples equally to both lines as a common-mode signal.
- A differential amplifier amplifies only the difference between the lines, rejecting the common-mode noise.

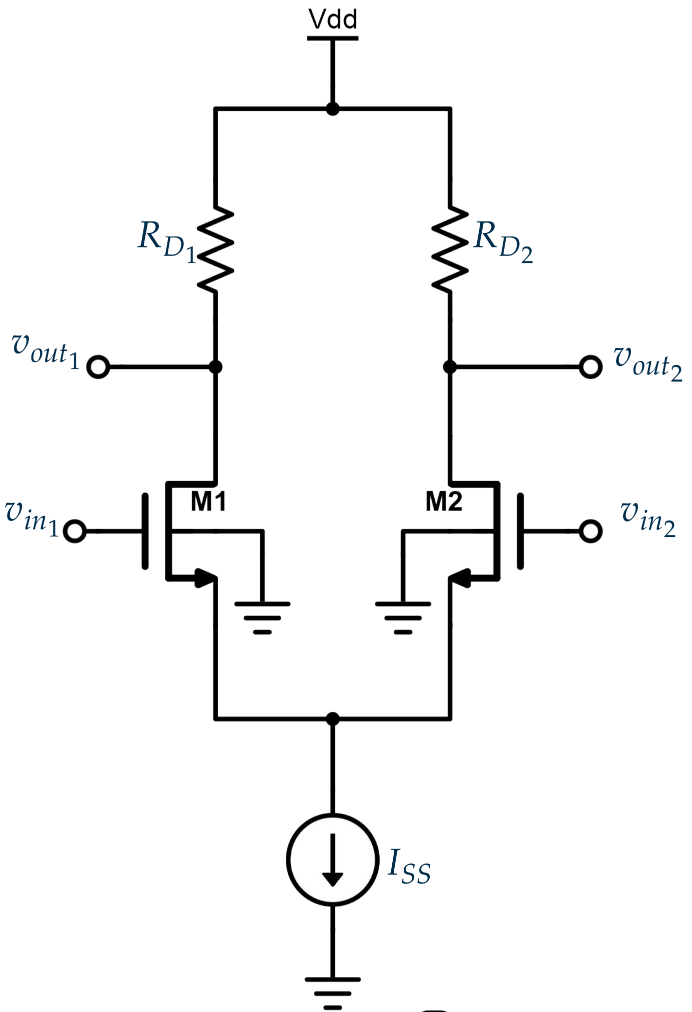


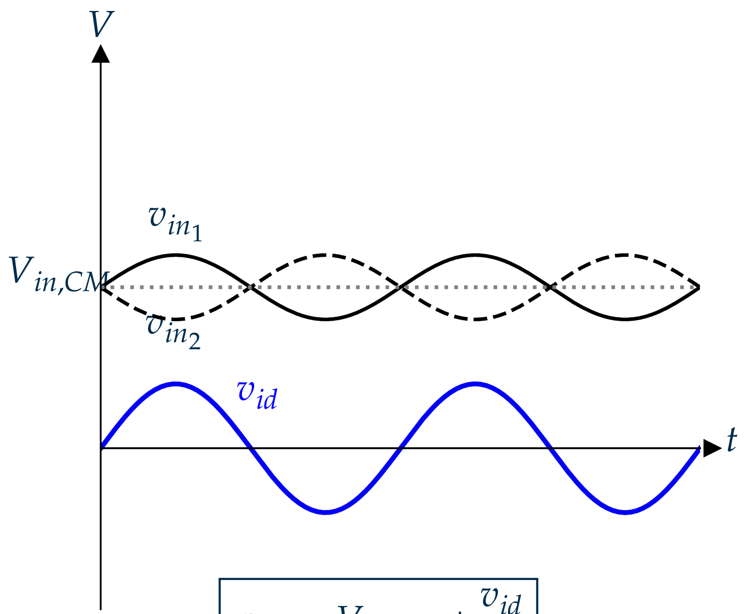
Basic Differential Pair



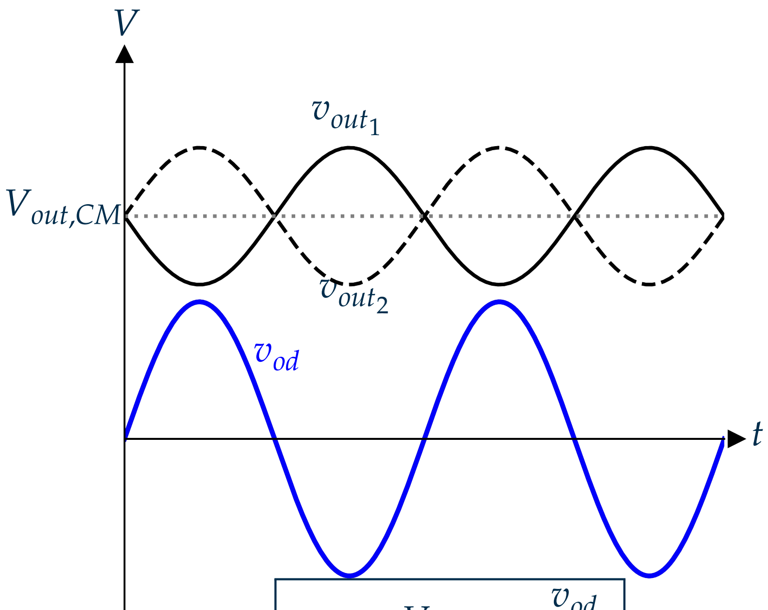
- Two identical single-ended CS amplifiers with differential inputs and a common-mode level are utilized to form a differential pair.
- The output is also differential and has its own common-mode level.
- This topology offers improved noise rejection and larger output swing. However, variations in the input common-mode level can degrade the common-mode rejection ratio (CMRR) and impact overall circuit performance.

Basic Differential Pair



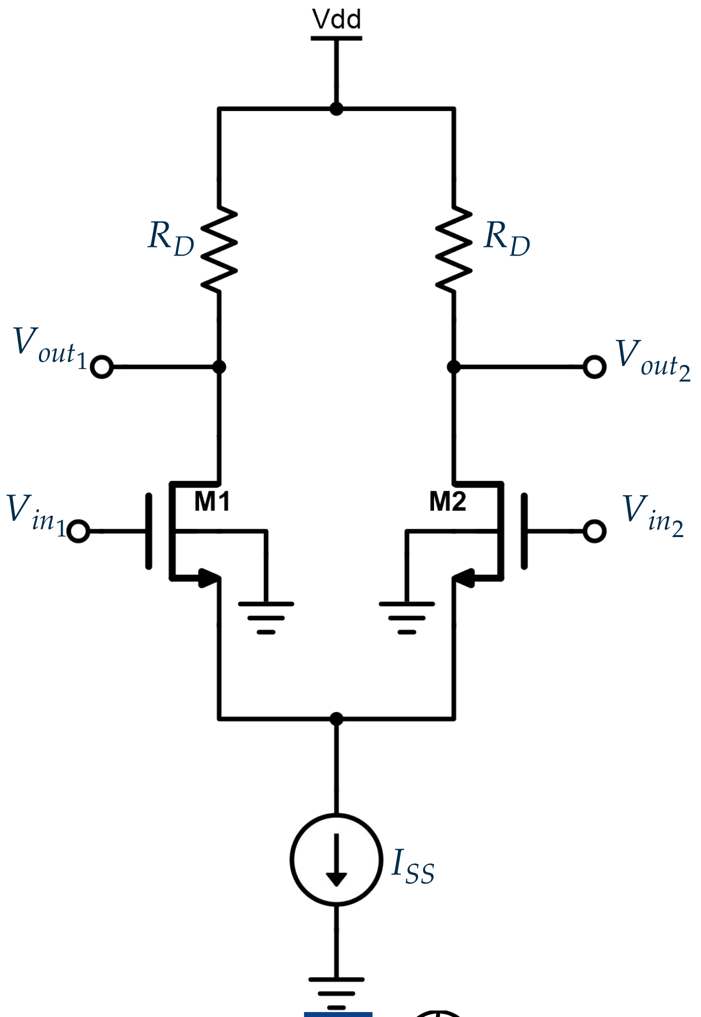


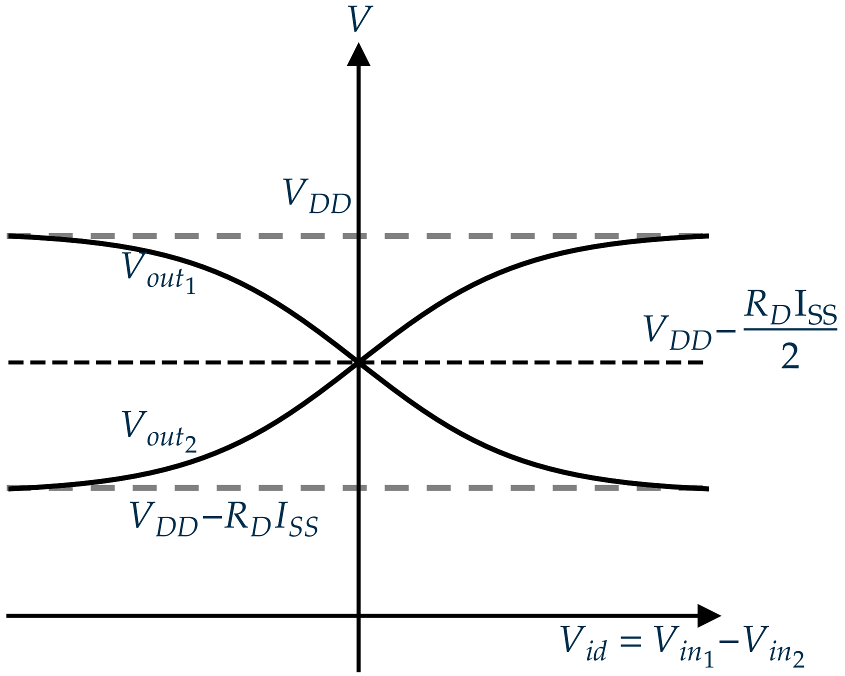
$$v_{in_1} = V_{in,CM} + \frac{v_{id}}{2}$$
$$v_{in_2} = V_{in,CM} - \frac{v_{id}}{2}$$
$$v_{id} = v_{in_1} - v_{in_2}$$

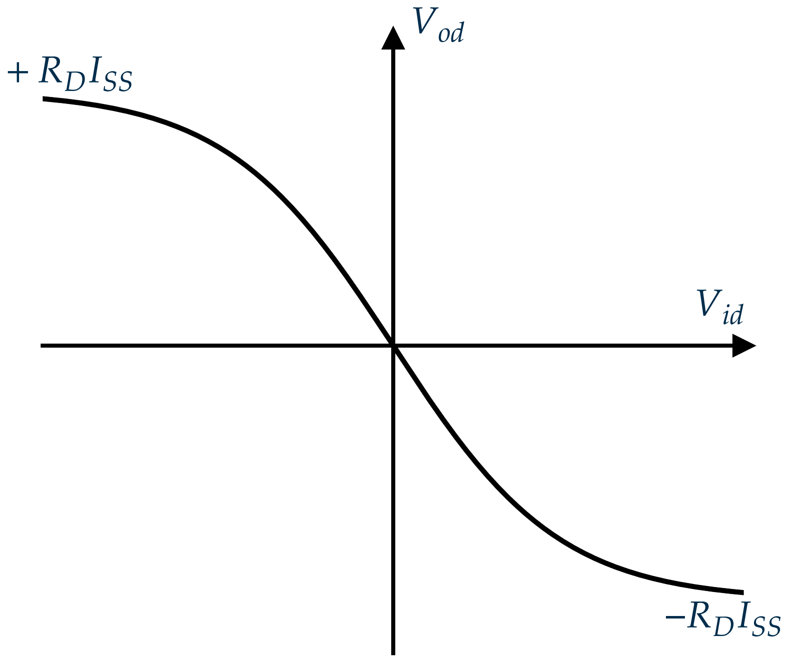


$$v_{out_1} = V_{out,CM} - \frac{v_{od}}{2}$$
$$v_{out_2} = V_{out,CM} + \frac{v_{od}}{2}$$
$$v_{od} = v_{out_2} - v_{out_1}$$

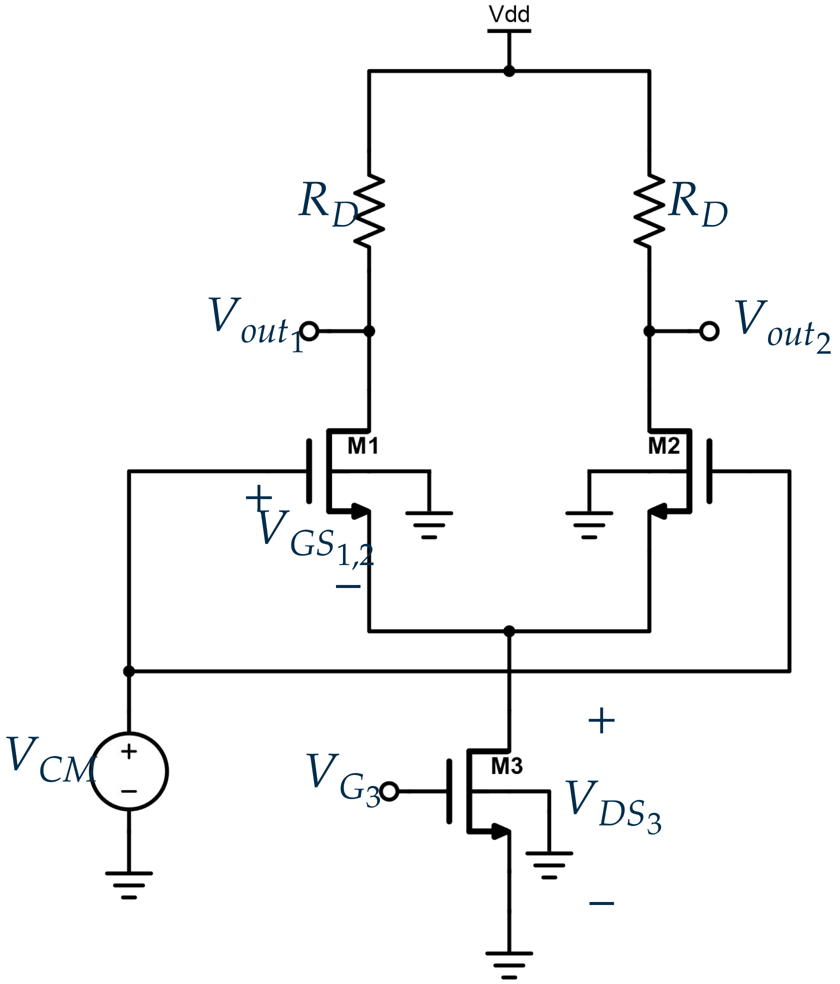
Basic Differential Pair







Voltage Swing Boundaries

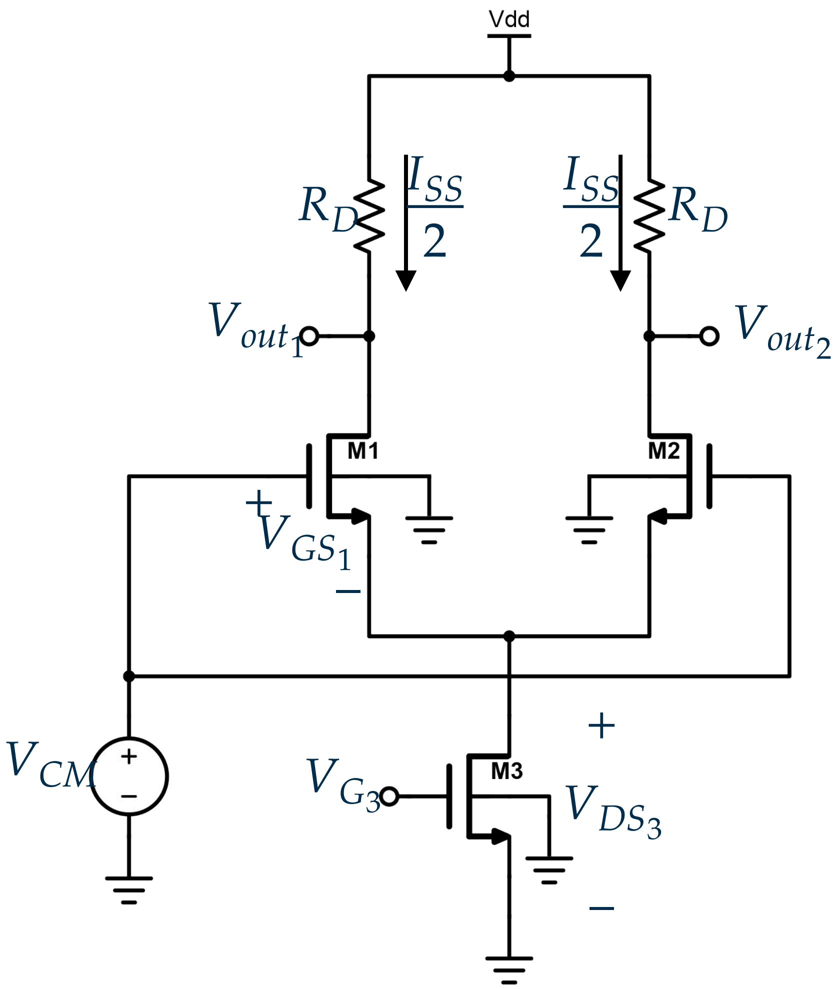


$$V_{DS_3} = V_{GS_3} - V_{TH_3}$$

$$V_{CM} \geq V_{DS_3} + V_{GS_1}$$

$$\boxed{V_{CM} \geq V_{GS_{1,2}} + \left(V_{GS_3} - V_{TH_3}\right)}$$

Voltage Swing Boundaries



$$V_{out_1} = V_{DD} - \frac{I_{SS}}{2} R_D, \quad V_{CM} = V_{DS_3} + V_{GS_1}$$

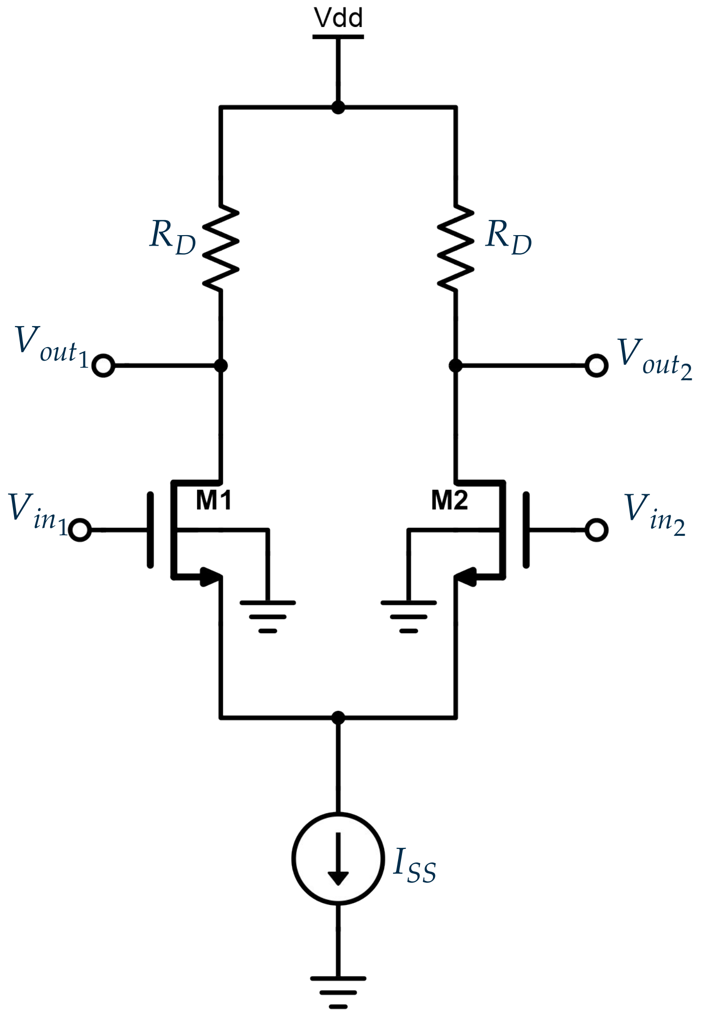
$$V_{out_1} - V_{DS_3} \geq V_{GS_1} - V_{TH_1}$$

$$V_{DD} - \frac{I_{SS}}{2} R_D \geq V_{CM} - V_{TH_1}$$

$$\boxed{V_{CM} \leq V_{TH_1} + V_{DD} - \frac{I_{SS}}{2} R_D}$$

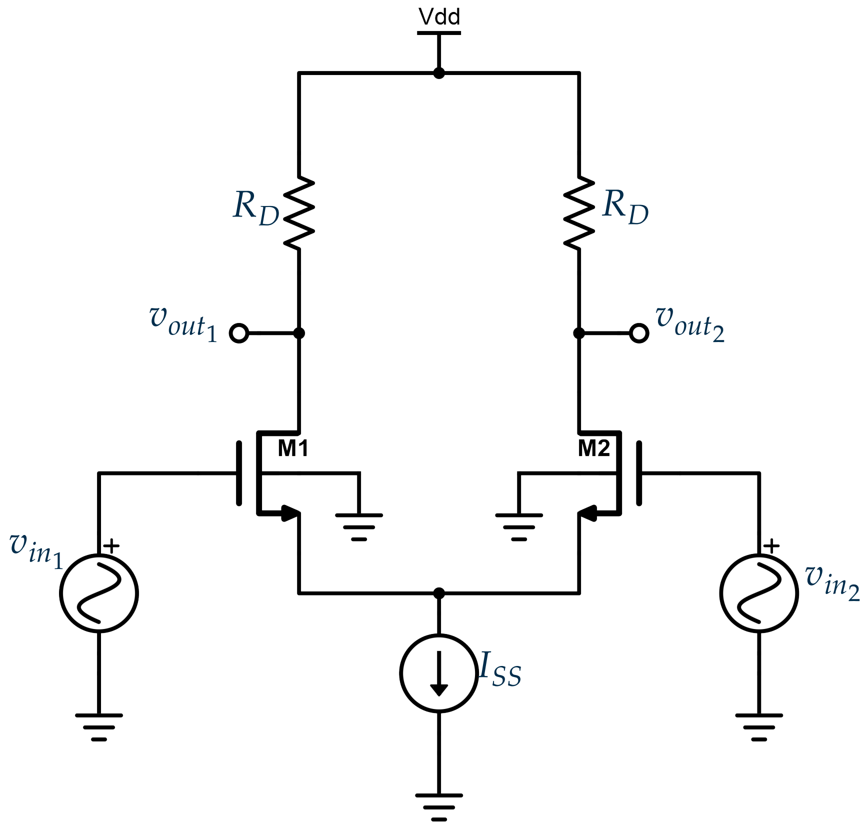
$$\boxed{V_{GS_1} + \left(V_{GS_3} - V_{TH_3}\right) \leq \boldsymbol{V_{CM}} \leq V_{TH_1} + V_{DD} - \frac{I_{SS}}{2} R_D}$$

Large Signal Analysis



- $(V_{GS} - V_{TH})^2 = \dfrac{I_D}{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}} \rightarrow V_{GS} = \sqrt{\dfrac{2I_D}{\mu_n C_{ox}\frac{W}{L}}} + V_{TH}$
- $V_{in_1} - V_{in_2} = \sqrt{\dfrac{2I_{D1}}{\mu_n C_{ox}\frac{W}{L}}} - \sqrt{\dfrac{2I_{D2}}{\mu_n C_{ox}\frac{W}{L}}}$
- $\left(V_{in_1} - V_{in_2}\right)^2 = \dfrac{2}{\mu_n C_{ox}\frac{W}{L}}\left(I_{SS} - 2\sqrt{I_{D_1} I_{D_2}}\right)$
- $\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{in_1} - V_{in_2}\right)^2 - I_{SS} = -2\sqrt{I_{D_1} I_{D_2}}$
- $\left(I_{D_1} - I_{D_2}\right)^2 = -\frac{1}{4}\left(\mu_n C_{ox}\frac{W}{L}\right)^2\left(V_{in_1} - V_{in_2}\right)^4 + I_{SS}\mu_n C_{ox}\frac{W}{L}\left(V_{in_1} - V_{in_2}\right)^2$
- $I_{D_1} - I_{D_2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{in_1} - V_{in_2}\right)\sqrt{\dfrac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - \left(V_{in_1} - V_{in_2}\right)^2}$
- $\dfrac{\partial \Delta I_D}{\partial \Delta V_{in}} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\dfrac{\frac{4I_{SS}}{\mu_n C_{ox} W/L} - 2\Delta V_{in}^2}{\sqrt{\frac{4I_{SS}}{\mu_n C_{ox} W/L} - \Delta V_{in}^2}}$
- $For\, \Delta V_{in} = 0,\, G_m = \sqrt{\mu_n C_{ox}\frac{W}{L} I_{SS}}$
- $\boxed{|A_d| = \sqrt{\mu_n C_{ox}\frac{W}{L} I_{SS}} R_D}$

Differential Amplifier Small-Signal Analysis

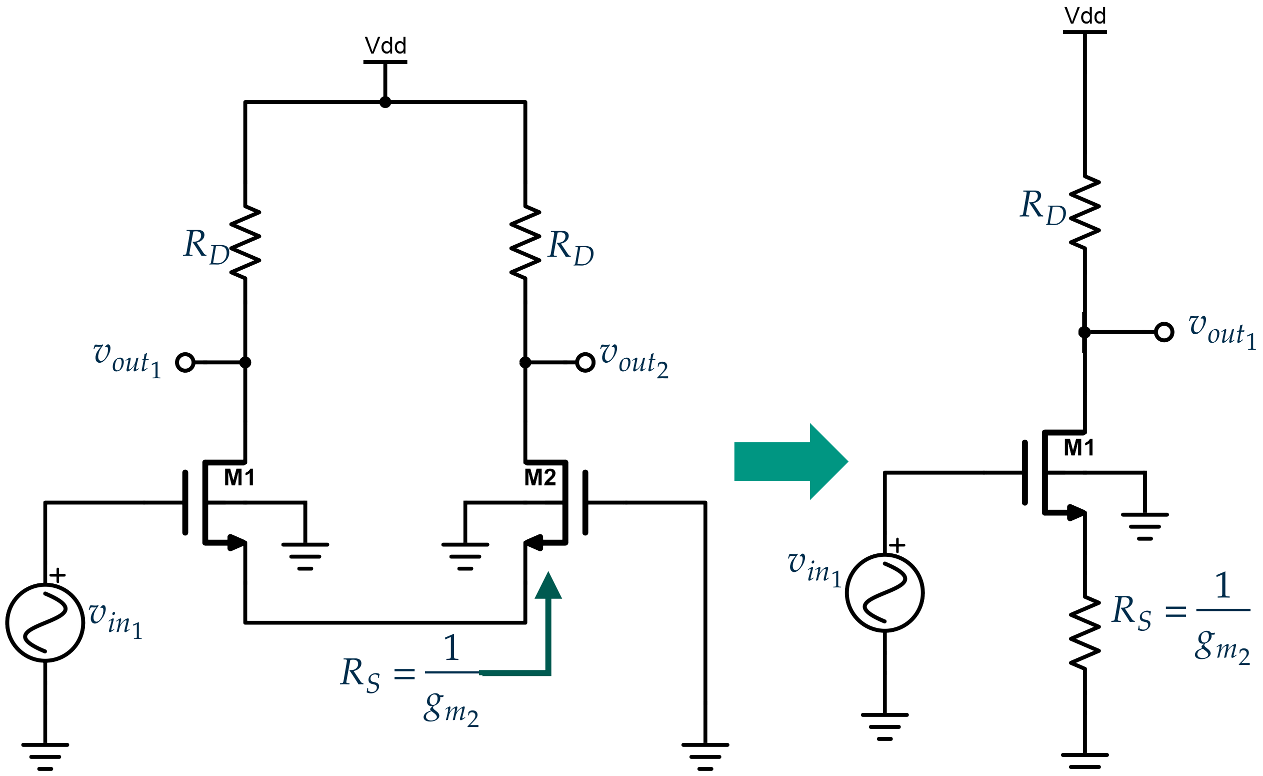


- Small signals v_{in_1} and v_{in_2} are applied as input.
- M1 and M2 are saturated.
- Two methods will be introduced for gain analysis.

Differential Amplifier Small-Signal Analysis

$$\boxed{g_{m_{eff}} = \frac{g_m}{1 + g_m R_S}}$$

Superposition (Only v_{in_1})



$$\frac{v_{out_1}}{v_{in_1}} = -\frac{R_D}{\frac{1}{g_{m_1}} + \frac{1}{g_{m_2}}}, \quad \frac{v_{out_2}}{v_{in_1}} = \frac{R_D}{\frac{1}{g_{m_2}} + \frac{1}{g_{m_1}}}$$

$$(v_{out_1} - v_{out_2}) \ \Big|_{due\ to\ v_{in_1}} = -\frac{2R_D}{\frac{1}{g_{m_1}} + \frac{1}{g_{m_2}}} v_{in_1}$$

Due to the symmetry, the effect of v_{in_2} at the output ports is identical to that of v_{in_1} except for a change in polarities.

$$(v_{out_1} - v_{out_2}) \ \Big|_{due\ to\ v_{in_2}} = \frac{2R_D}{\frac{1}{g_{m_1}} + \frac{1}{g_{m_2}}} v_{in_2}$$

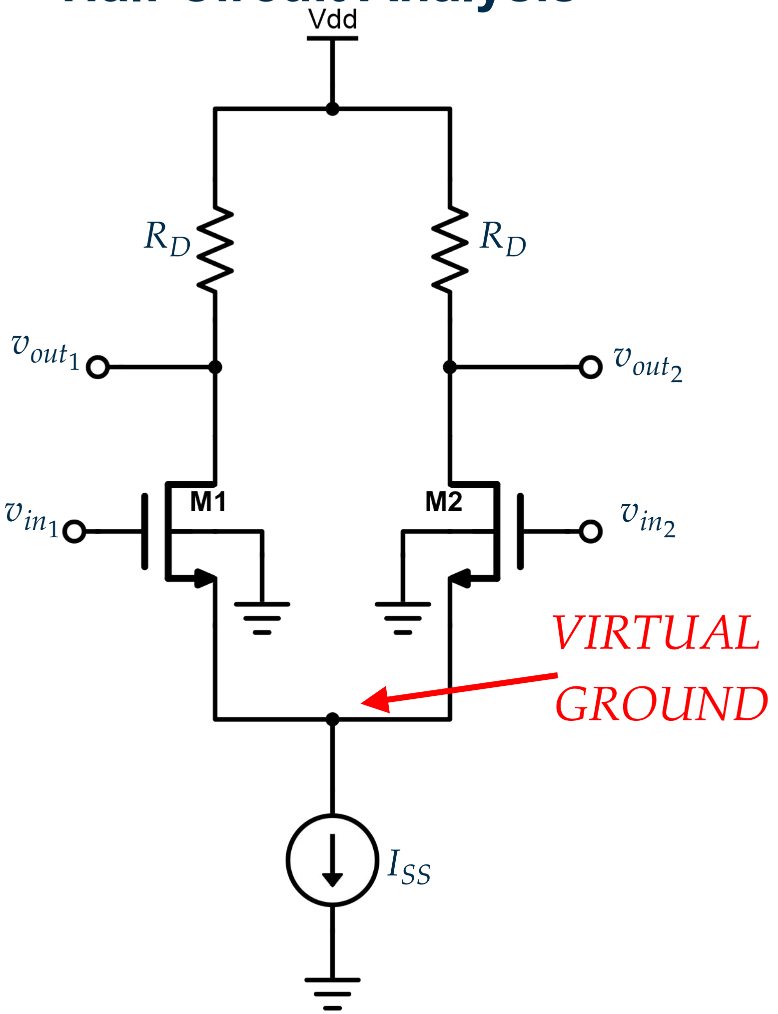
$$\frac{v_{out_1} - v_{out_2}}{v_{in_1} - v_{in_2}} = -\frac{2R_D}{\frac{1}{g_{m_1}} + \frac{1}{g_{m_2}}}$$

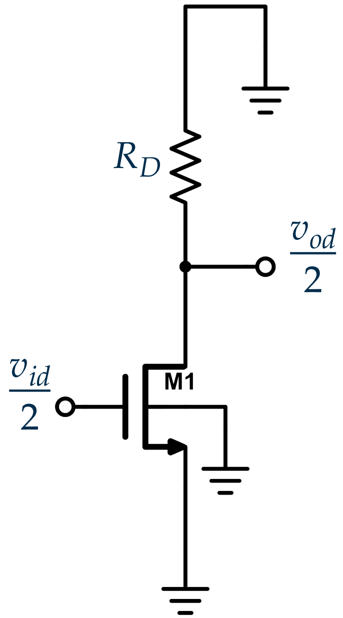
For $g_m = g_{m_1} = g_{m_2}$

$$\boxed{\frac{v_{out_1} - v_{out_2}}{v_{in_1} - v_{in_2}} = -g_m R_D}$$

Differential Amplifier Small-Signal Analysis

Half Circuit Analysis

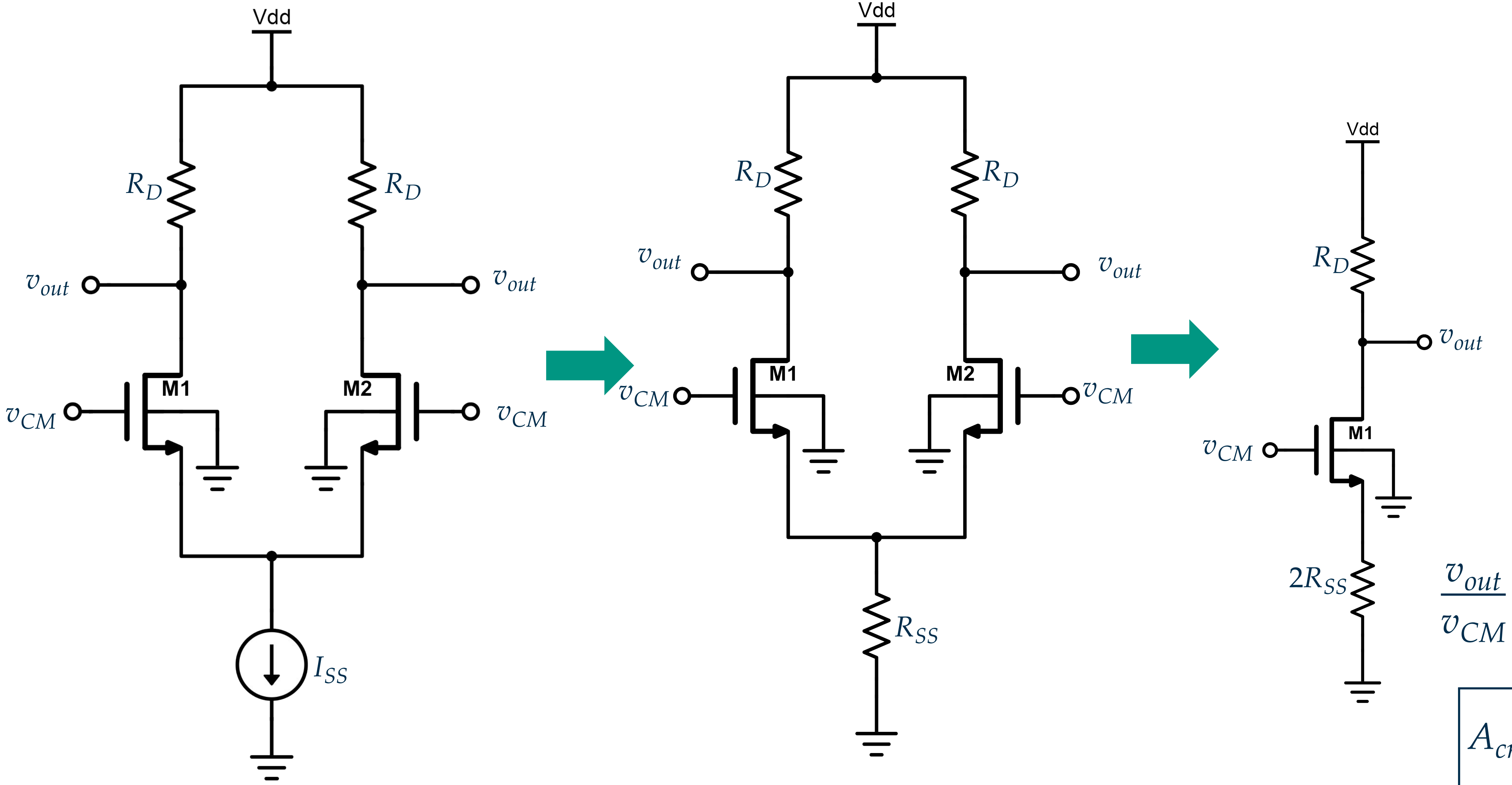




$$v_{out_1} = -v_{out_2} = \frac{v_{od}}{2}$$
$$v_{in_1} = -v_{in_2} = \frac{v_{id}}{2}$$

$$\boxed{|A_d| = \left|\frac{v_{od}}{v_{id}}\right| = g_m R_D}$$

Common-Mode Response

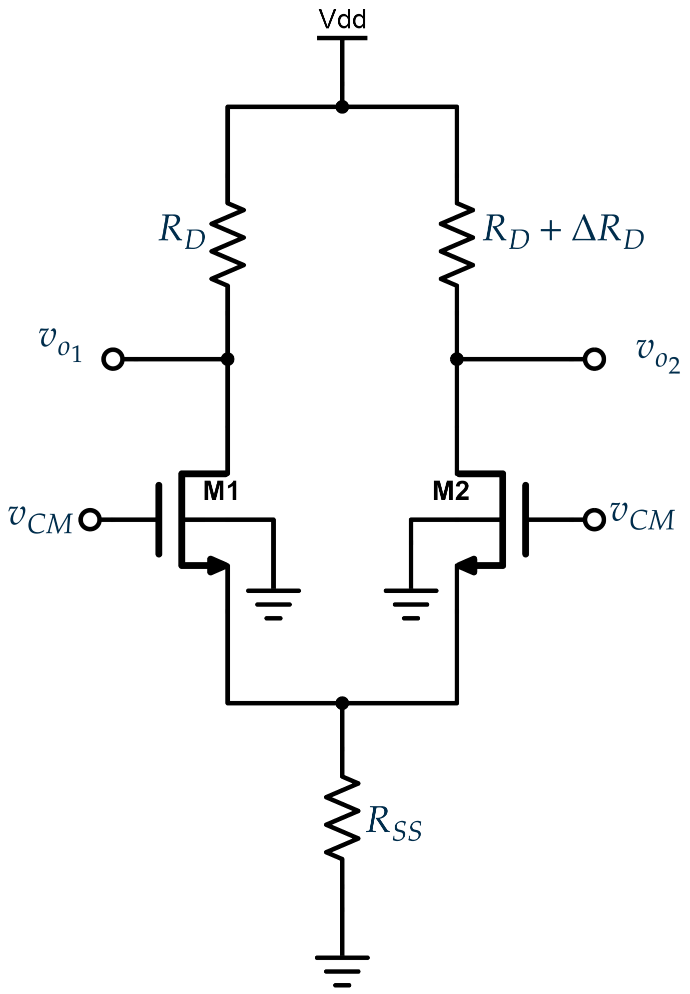


$$\frac{v_{out}}{v_{CM}} = -\frac{g_m}{1+g_m 2R_{SS}} R_D$$

$$\boxed{A_{cm} = \frac{v_{out}}{v_{CM}} \approx -\frac{R_D}{2R_{SS}}}$$

Common-Mode Response

R_D Mismatch



$$v_{o_1} = -\frac{R_D}{2R_{SS}} v_{CM}$$

$$v_{o_2} = -\frac{R_D + \Delta R_D}{2R_{SS}} v_{CM}$$

$$v_{o_d} = v_{o_2} - v_{o_1} = -\frac{\Delta R_D}{2R_{SS}} v_{CM}$$

$$\boxed{A_{cm} = \frac{v_{o_d}}{v_{CM}} = -\frac{\Delta R_D}{2R_{SS}} = -\left(\frac{R_D}{2R_{SS}}\right)\left(\frac{\Delta R_D}{R_D}\right)}$$

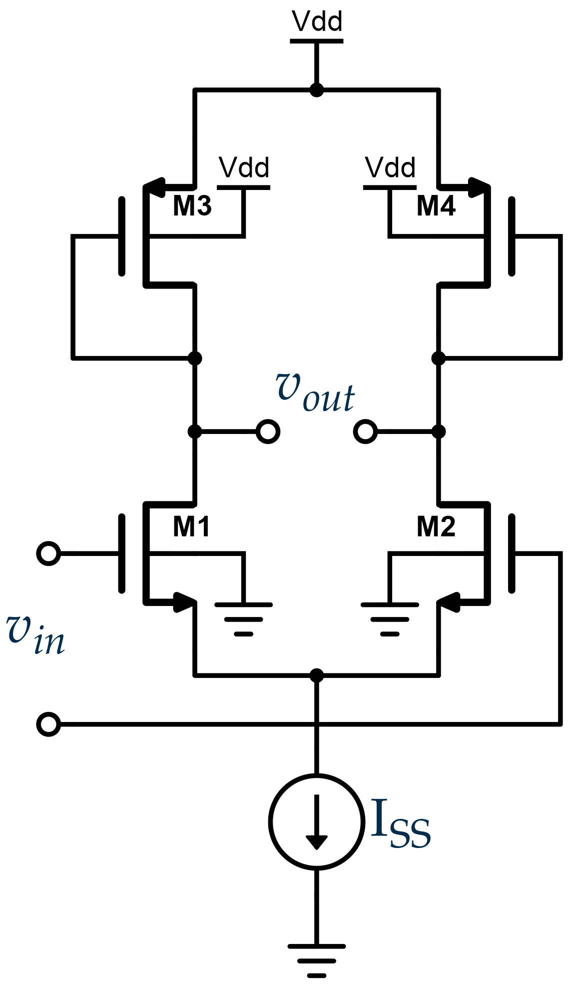
Common-Mode Rejection Ratio (CMRR)

$$CMRR = \frac{Differential\ Gain}{Common{-}Mode\ Gain} = \frac{|A_d|}{|A_{cm}|} \approx \frac{g_m R_D}{\frac{R_D}{2R_{SS}}} = 2g_m R_{SS}$$

$$CMRR(dB) = 20\log\left(\frac{|A_d|}{|A_{cm}|}\right)$$

Differential Pair with MOS Loads

Diode-Connected Load



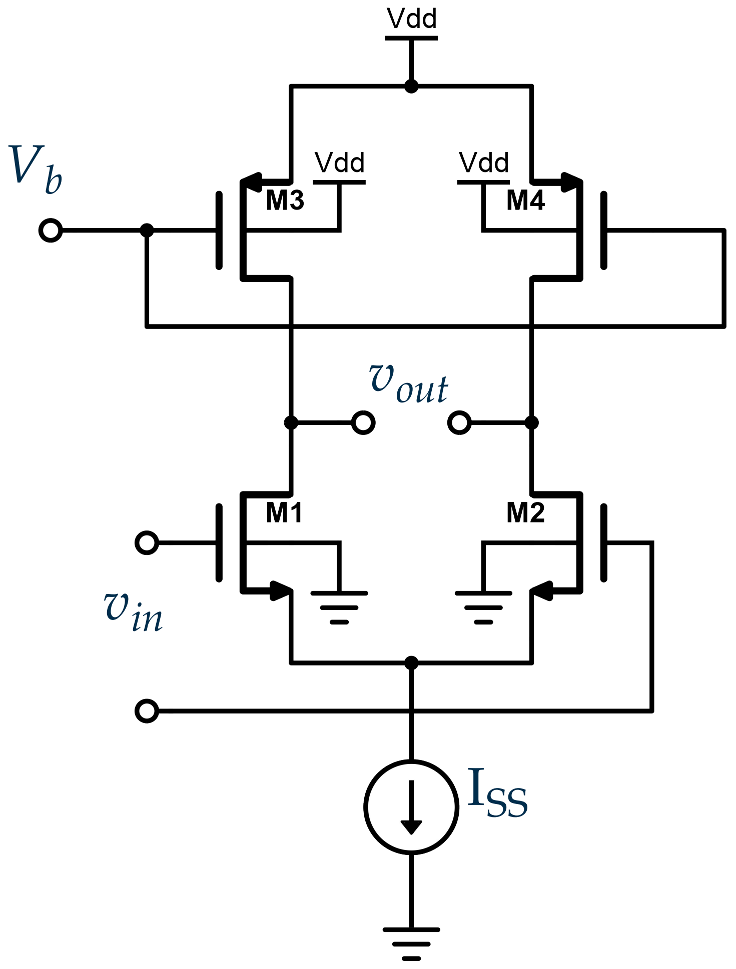
$$A_d = \frac{v_{out}}{v_{in}} = -g_{m_N}\left(\frac{1}{g_{m_P}} \parallel r_{o_N} \parallel r_{o_P}\right)$$

- N and P denote NMOS and PMOS, respectively.

$$A_d = \frac{v_{out}}{v_{in}} \approx -\frac{g_{m_N}}{g_{m_P}} = -\sqrt{\frac{\mu_n\left(\frac{W}{L}\right)_N}{\mu_p\left(\frac{W}{L}\right)_P}}$$

Differential Pair with MOS Loads

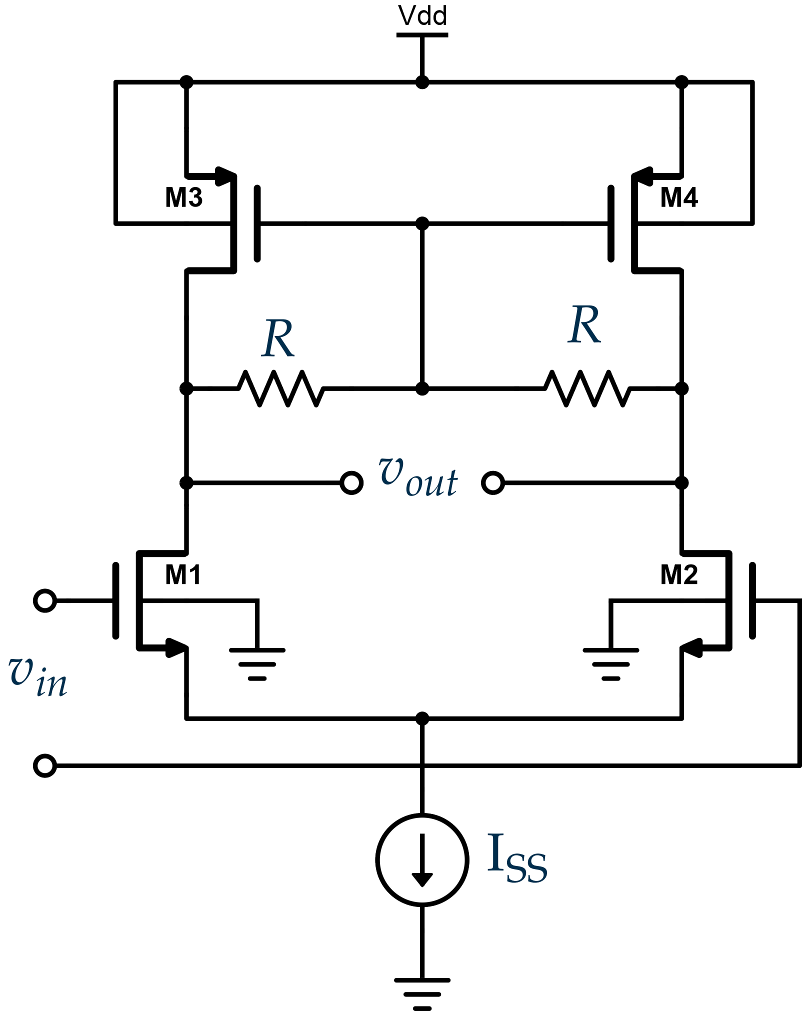
Current-Source Load



$$A_d = \frac{v_{out}}{v_{in}} = -g_{m_N}\left(r_{o_N} \parallel r_{o_P}\right)$$

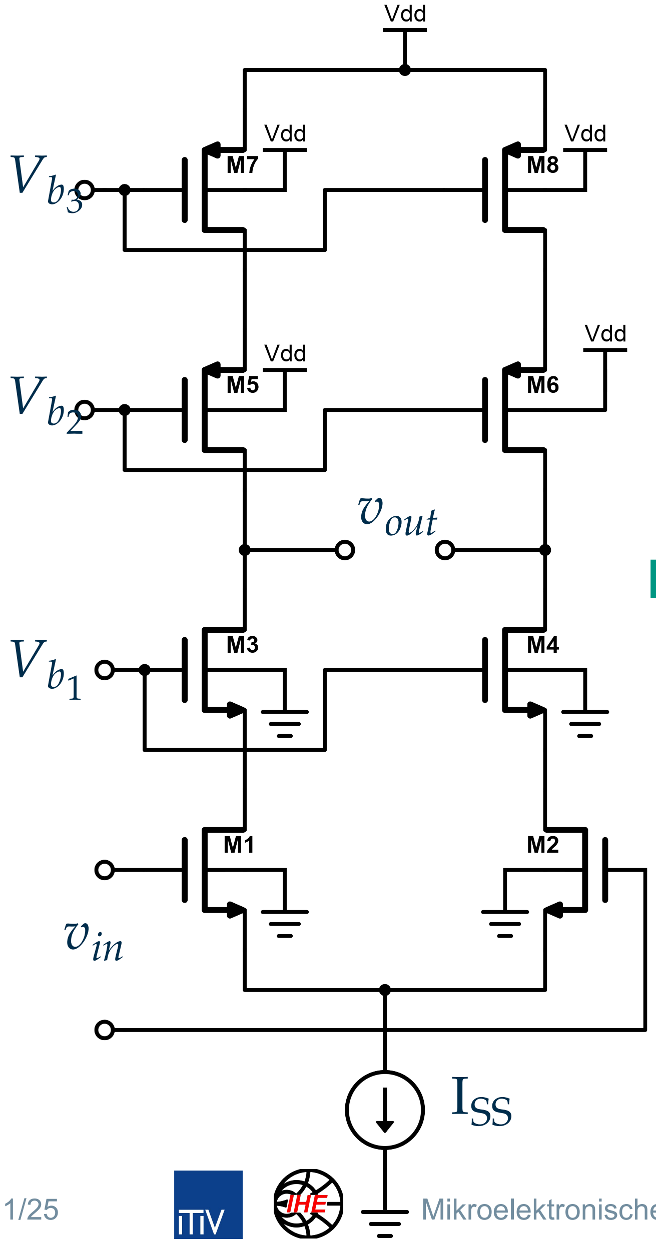
Differential Pair with MOS Loads

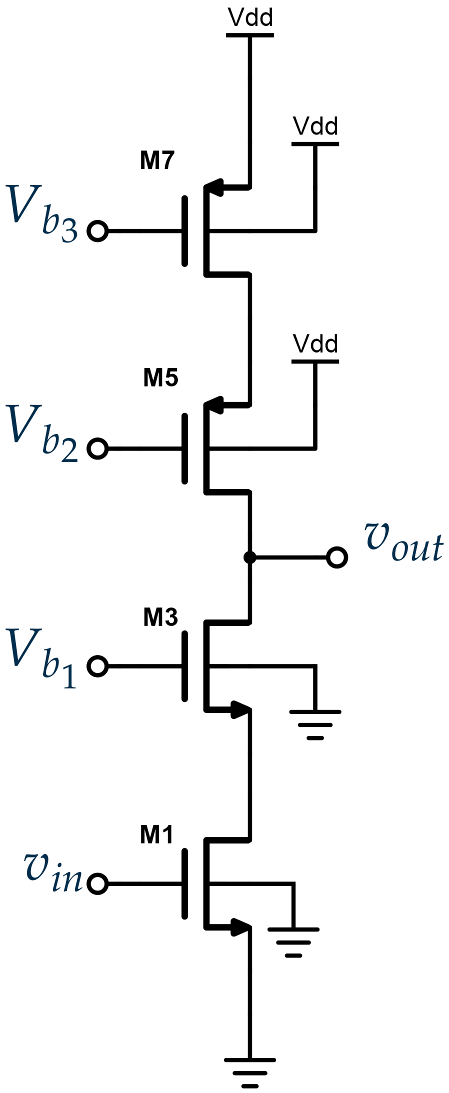
Self-Biased Load



$$\boxed{A_d = \frac{v_{out}}{v_{in}} = -g_{m_1}\left(r_{o_1} \parallel R \parallel r_{o_3}\right)}$$

Cascode Differential Pair





$$\boxed{|A_d| \approx g_{m_1}\left[\left(g_{m_3} r_{o_3} r_{o_1}\right) \parallel \left(g_{m_5} r_{o_5} r_{o_7}\right)\right]}$$