Mikroelektronische Schaltungen und Systeme

Lect.7
Feedback

Prof. Dr.-Ing. Ahmet Cagri Ulusoy
Institut für Hochfrequenztechnik und Elektronik (IHE)

Mikroelektronische Schaltungen und Systeme

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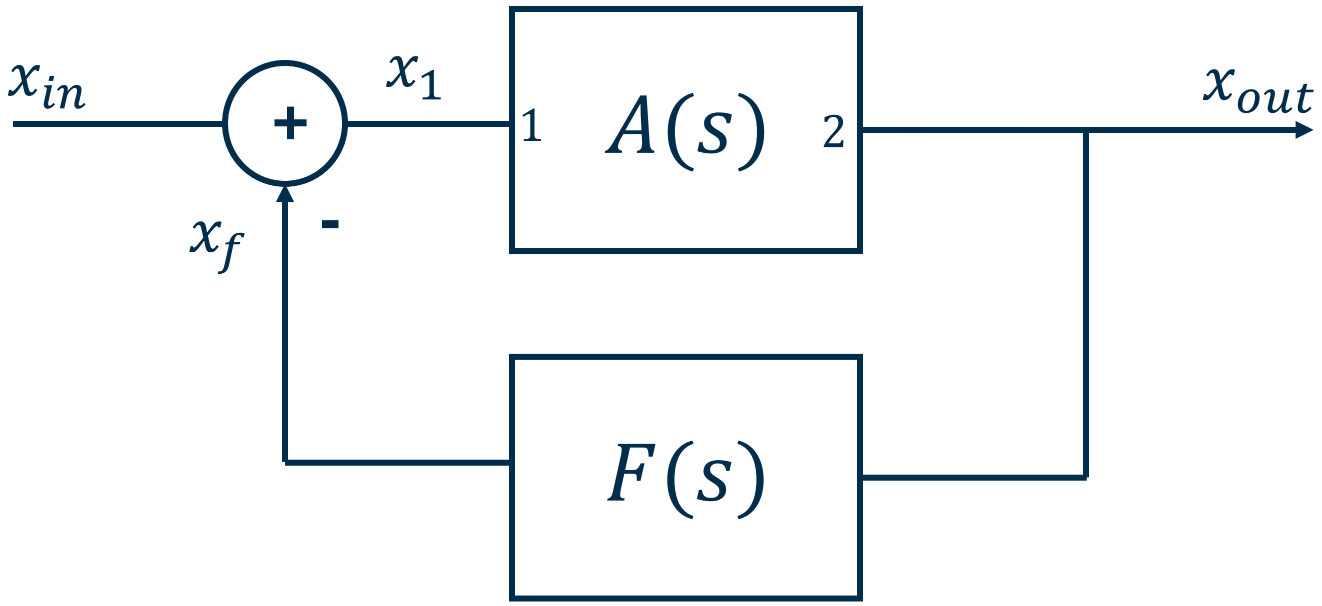
Introduction

- Microelectronic circuits are rarely used in „open loop“, they typically contain some kind of feedback mechanism to:
  - Adjust the gain
  - Adjust input/output impedances to desired values
  - Linearize/stabilize the circuit
  - Robustness against PVT
- All circuits also include unintentional feedback (e.g., Miller capacitance)
- In classic text book approach, feedback is analyzed from a system theoretical perspective
- We will start with this, but we will introduce a more intuative approach to feedback analysis

Feedback System

- A general representation of a feedback system looks as following:



- Here, x_{in}, x_{out} could be voltages, currents or a mix of voltage and current

Feedback System

- This representation leads to classic feedback expression (in steady state for voltage signals):

$$v_2 = v_{out} = v_1 A(j\omega)$$

$$v_1 = v_{in} - v_f = v_{in} - v_{out} F(j\omega)$$

$$\boxed{\frac{v_{out}}{v_{in}} = \frac{A(j\omega)}{1 + A(j\omega)F(j\omega)}}$$

Feedback System

- The challenge in circuit design with feedback is to properly draw the boundries between A and F circuits in a way that the general expressions from previous slide hold, and any loading affect between the two circuits are taken into account

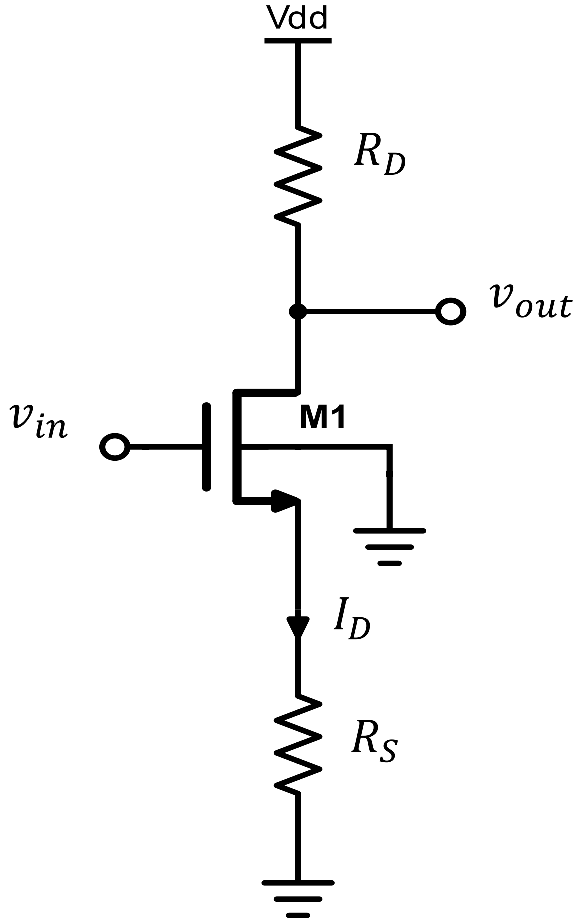
- For this purpose, we try to identify the feedback mechanism, and try to fit parts of the circuit to appropriate boxes

- These „boxes“ are two-port representations of portions of the circuits

Common-Source Amplifier with Source Degeneration

- We start with a simple example, CS stage with source degeneration:

- **First:** identify the feedback mechanism

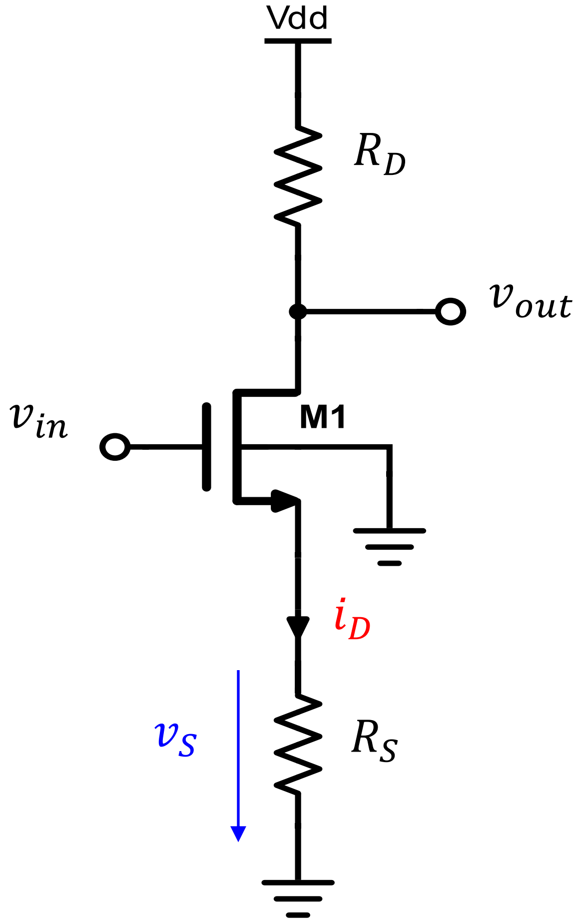


Common-Source Amplifier with Source Degeneration

- We start with a simple example, CS stage with source degeneration:

- **First:** identify the feedback mechanism
  - The current at the output i_D generates a voltage across R_S, which generates a feedback voltage that is subtracted from v_{in}
  - It is a negative feedback
  - It samples **current** at the **output** and feeds back **voltage** to the **input**

$$v_{GS} = v_{in} - v_S = v_{in} - i_D R_S$$

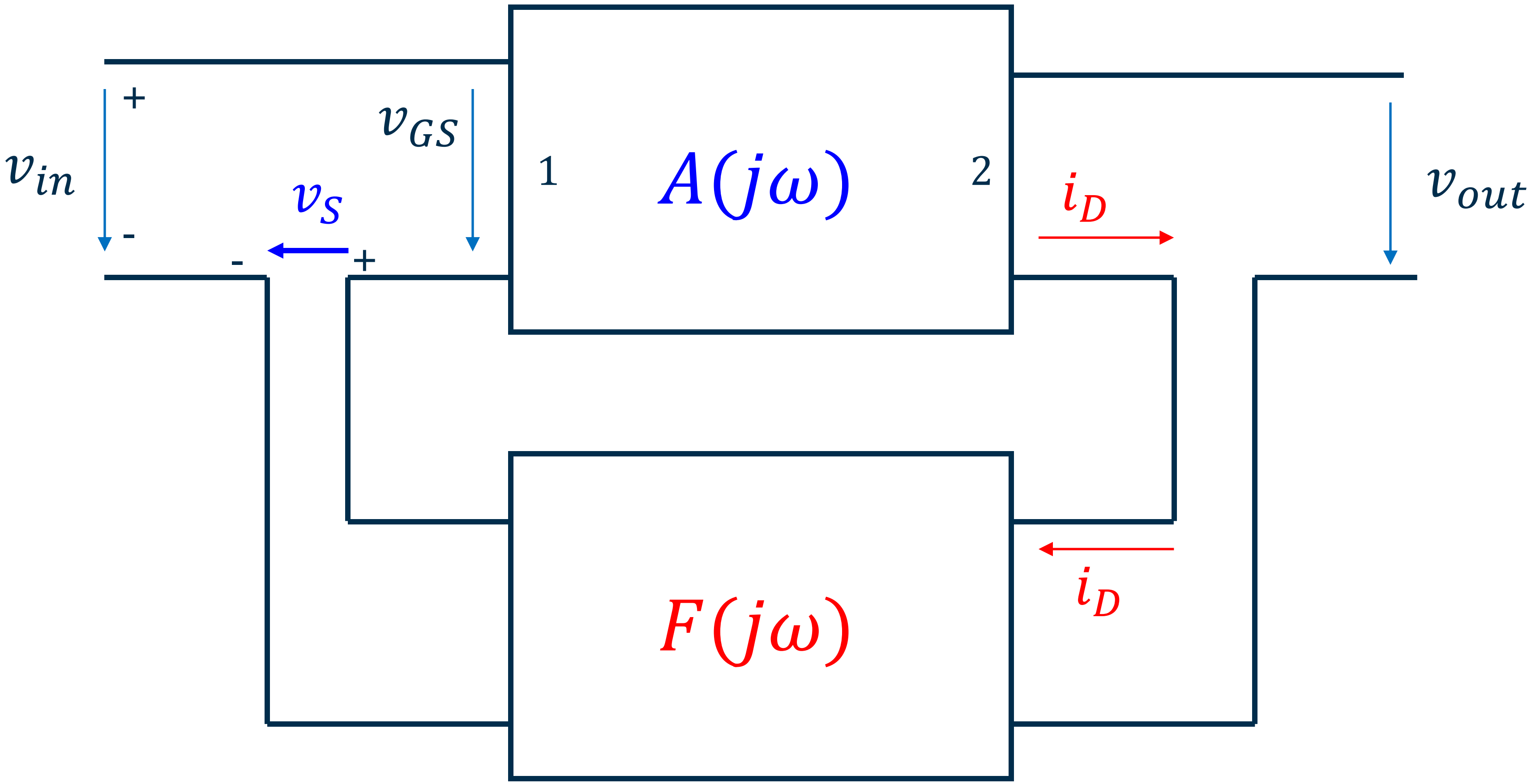


Common-Source Amplifier with Source Degeneration

- Here, the *A-circuit* is a common source amplifier, with voltage as input signal and current as output signal, ideally modelled with its y-parameters

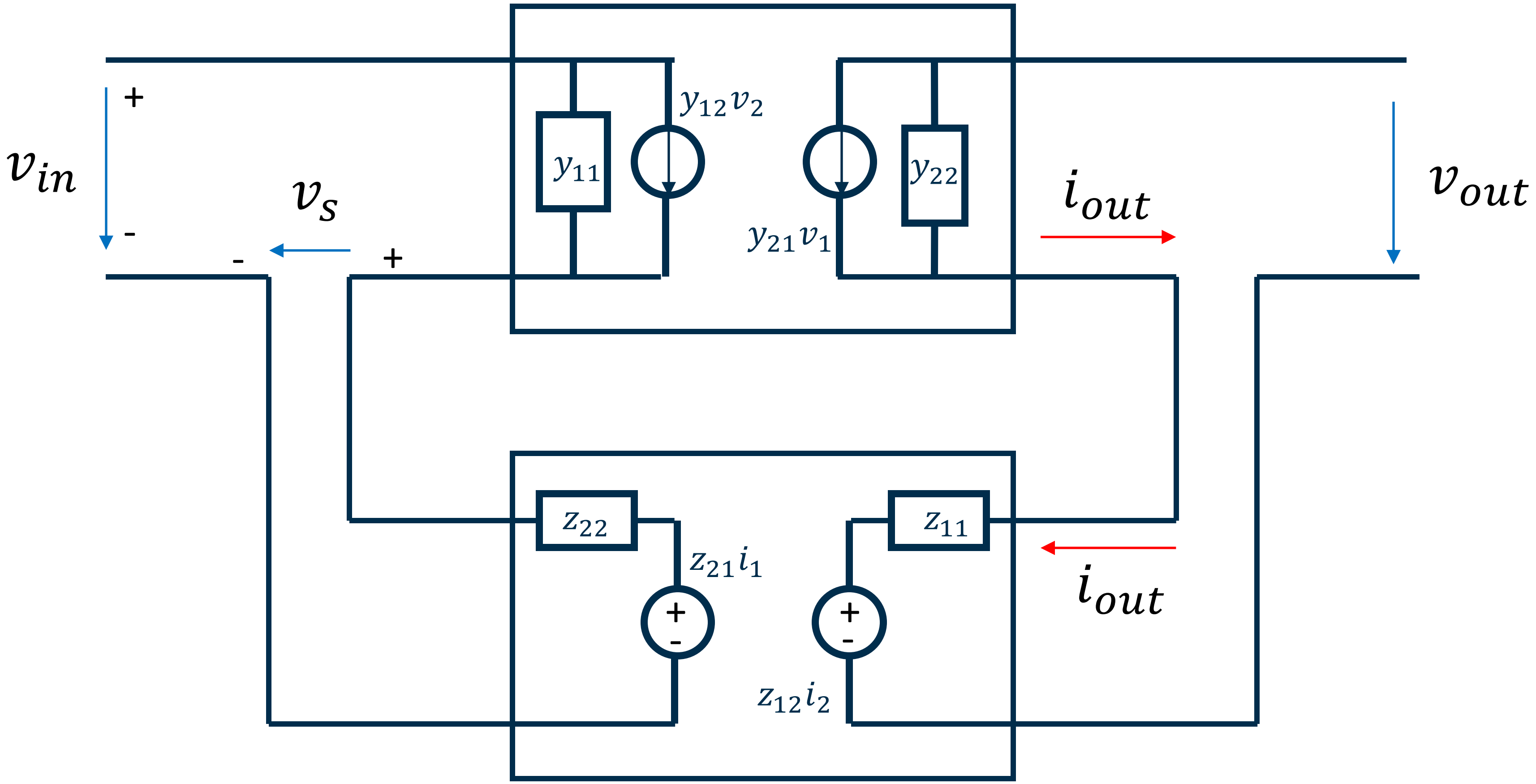
- The *F-circuit* is the feedback resistor, that takes current as input, and voltage as output, ideally modelled with its z-parameters

Common-Source Amplifier with Source Degeneration



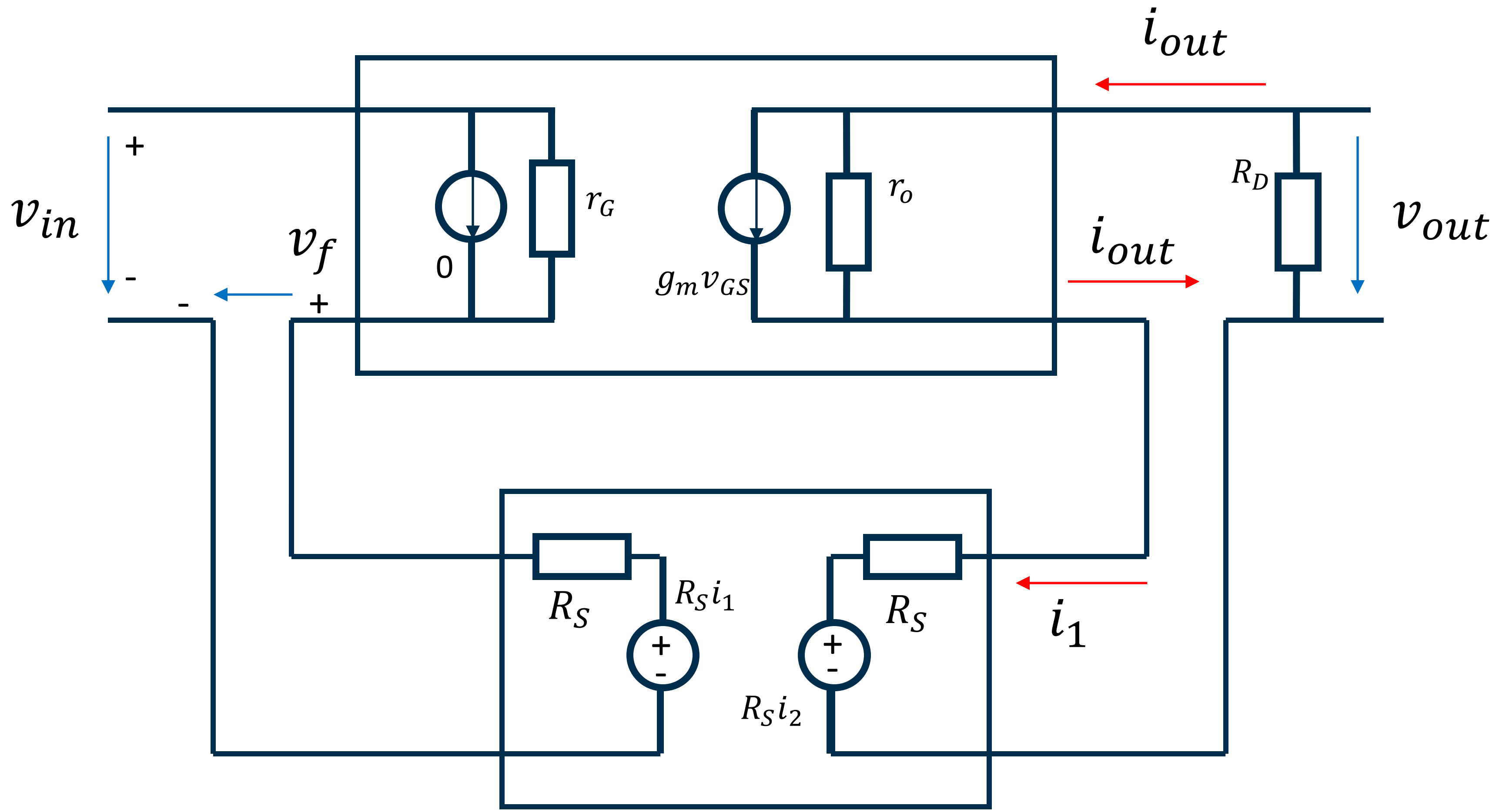
Common-Source Amplifier with Source Degeneration

$$i_1 = y_{11}v_1 + y_{12}v_2$$
$$i_2 = y_{21}v_1 + y_{22}v_2$$



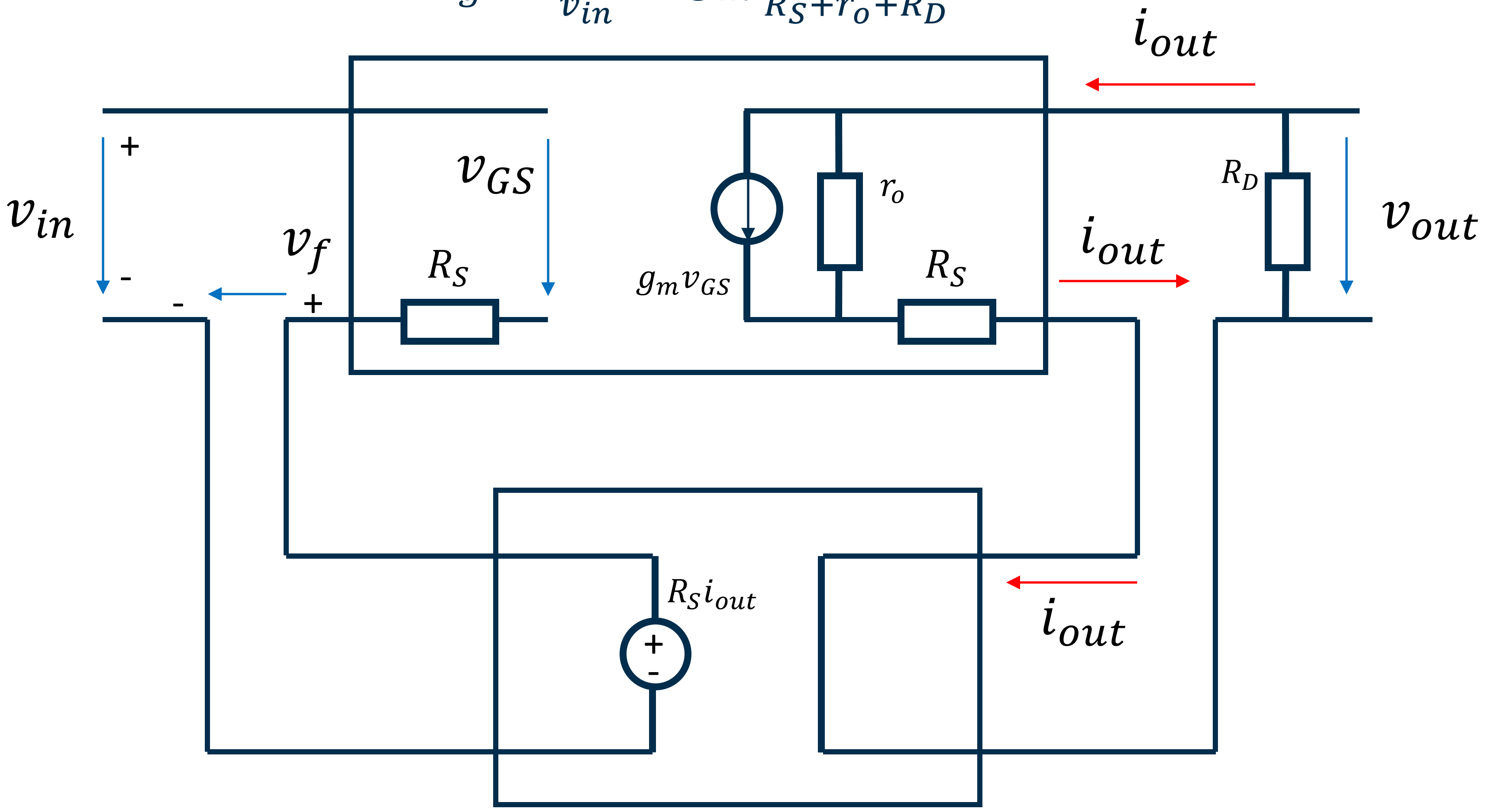
$$v_1 = z_{11}i_1 + z_{12}i_2$$
$$v_2 = z_{21}i_1 + z_{22}i_2$$

Common-Source Amplifier with Source Degeneration



Common-Source Amplifier with Source Degeneration

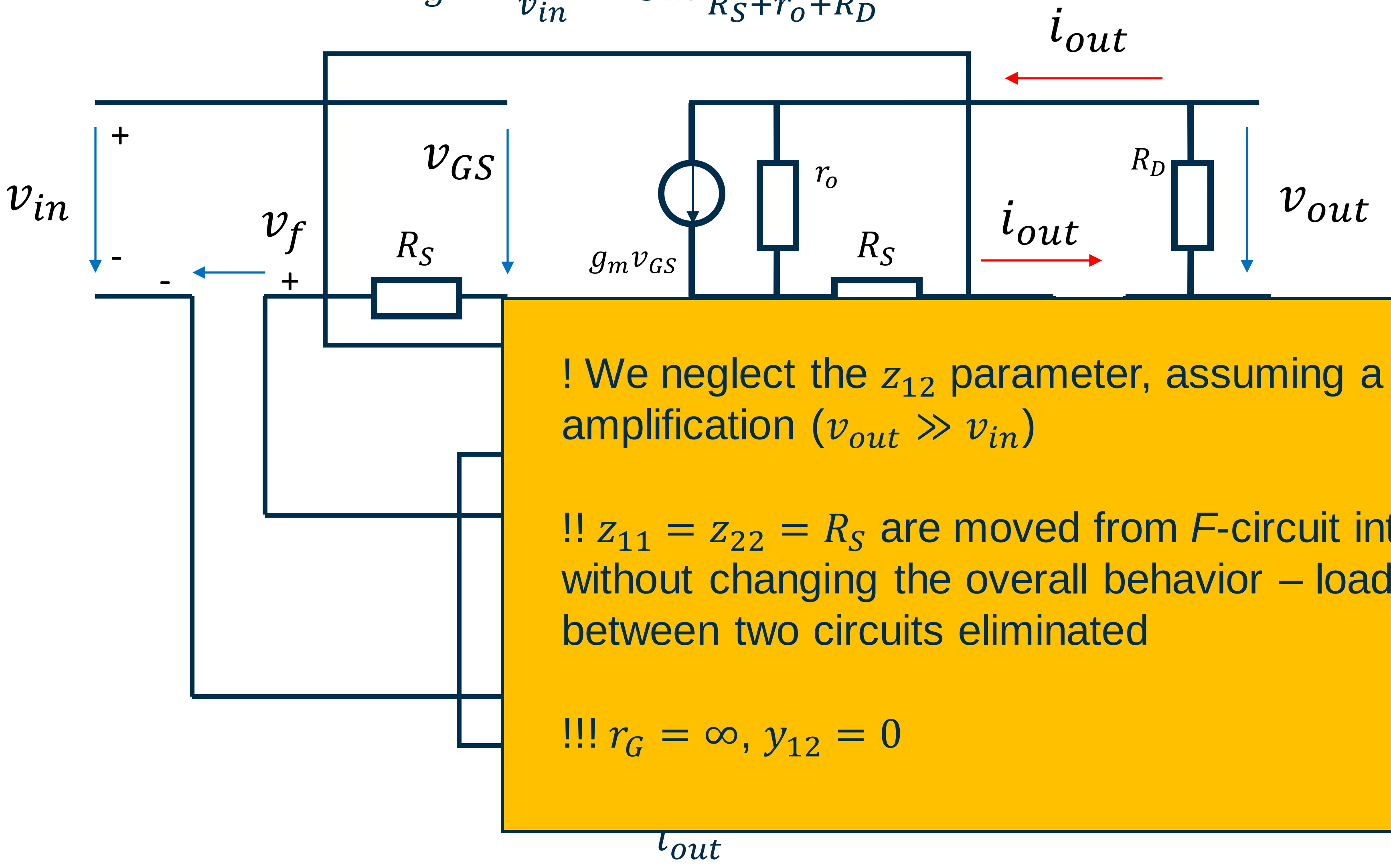
$$A_g = \frac{i_{out}}{v_{in}} = g_m \frac{r_o}{R_S + r_o + R_D}$$



$$F_Z = \frac{v_f}{i_{out}} = R_S$$

Common-Source Amplifier with Source Degeneration

$$A_g = \frac{i_{out}}{v_{in}} = g_m \frac{r_o}{R_S + r_o + R_D}$$



! We neglect the z_{12} parameter, assuming a large voltage amplification ($v_{out} \gg v_{in}$)

!! $z_{11} = z_{22} = R_S$ are moved from *F*-circuit into *A*-circuit without changing the overall behavior – loading effects between two circuits eliminated

!!! $r_G = \infty$, $y_{12} = 0$

Common-Source Amplifier with Source Degeneration

- Closed Loop Gain:

$$A_{gf} = \frac{i_{out}}{v_{in}} = \frac{\boldsymbol{A_g}}{\boldsymbol{1 + A_g F_z}} = \frac{r_o}{R_S + r_o + R_D} \frac{g_m}{1 + g_m \dfrac{r_o R_S}{R_S + r_o + R_D}}$$

- Closed Loop Voltage Gain:

$$A_{vf} = \frac{v_{out}}{v_{in}} = -\frac{A_g}{1 + A_g F_z} \times R_D = -\frac{r_o}{R_S + r_o + R_D} \frac{g_m R_D}{1 + g_m \dfrac{r_o R_S}{R_S + r_o + R_D}}$$

Common-Source Amplifier with Source Degeneration

- Closed Loop Gain:

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- Closed Loop Voltage Gain:

$$A_{vf} = \frac{v_{out}}{v_{in}} = -\frac{A_g}{1 + A_g F_z} \times R_D = -\frac{\cancel{r_o}}{\cancel{R_S + r_o + R_D}} \frac{g_m R_D}{\cancel{1} + g_m \frac{\cancel{r_o} R_S}{\cancel{R_S + r_o + R_D}}} \approx -\frac{\boldsymbol{R_D}}{\boldsymbol{R_S}}$$

for $r_o \gg R_S, R_D$

Feedback Types

	A-input	A-output	Z_{in}	Z_{out}
series-series	voltage	current	$Z_i(1 + AF)$	$Z_o(1 + AF)$
series-shunt	voltage	voltage	$Z_i(1 + AF)$	$Z_o/(1 + AF)$
shunt-series	current	current	$Z_i/(1 + AF)$	$Z_o(1 + AF)$
shunt-shunt	current	voltage	$Z_i/(1 + AF)$	$Z_o/(1 + AF)$

Feedback Analysis using Asymptotic Gain Function

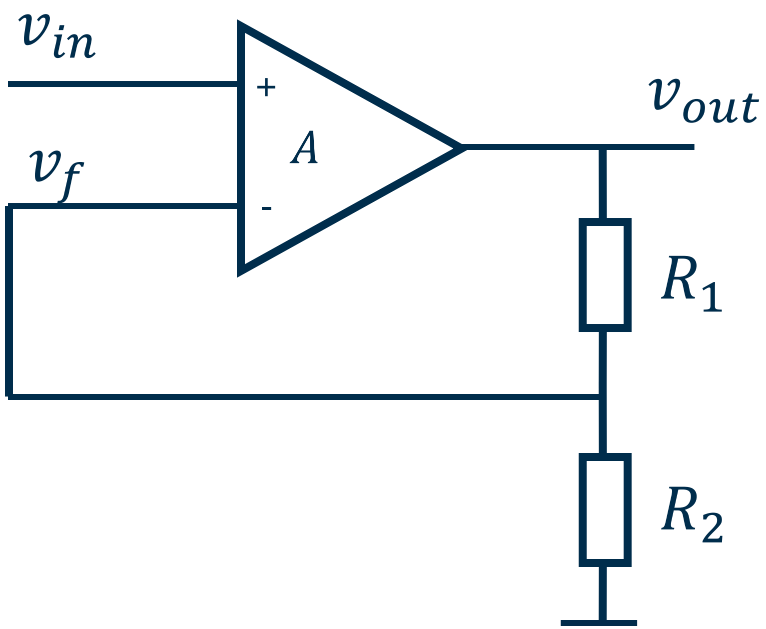
- Instead of such a formal analysis, we would like to use a more intuitive approach, namely, the asymptotic gain

- In this case we consider the extreme case for the feedback analysis, asking ourselves what happens if our feedback circuit is „ideal“, having an *A-circuit* with an infinitely high amplification

$$\frac{v_{out}}{v_{in}} = \left.\frac{A}{1+AF}\right|_{A\to\infty} = \frac{1}{F}$$

Example

- Consider following feedback circuit:



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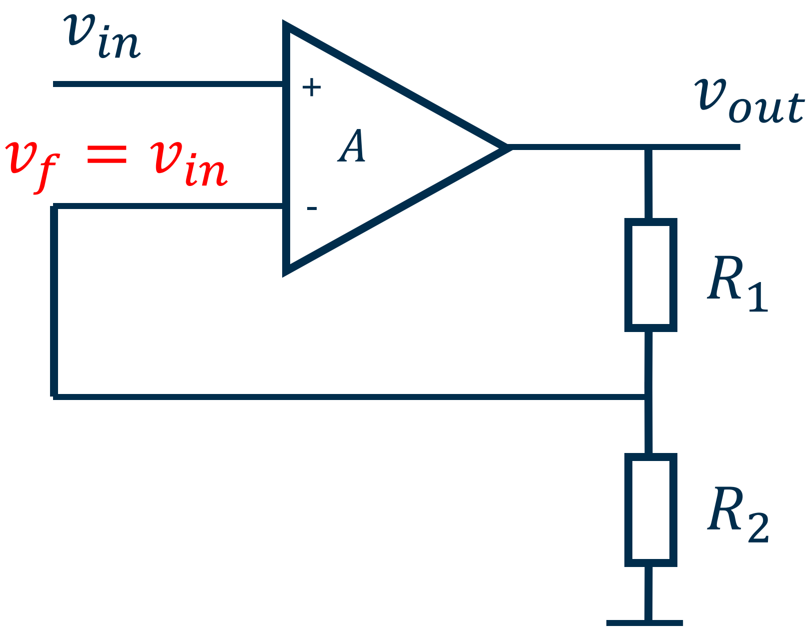

$$\frac{v_{out} R_2}{R_1 + R_2} = v_{in}$$

$$H_\infty = \left.\frac{v_{out}}{v_{in}}\right|_{A=\infty} = 1 + \frac{R_1}{R_2}$$

OPAMP is ideal: $A \to \infty$

Example

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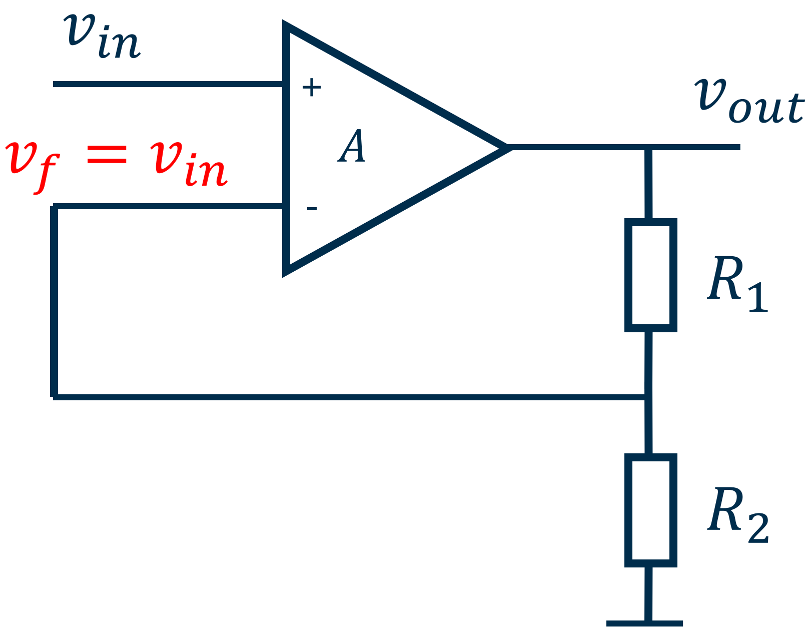

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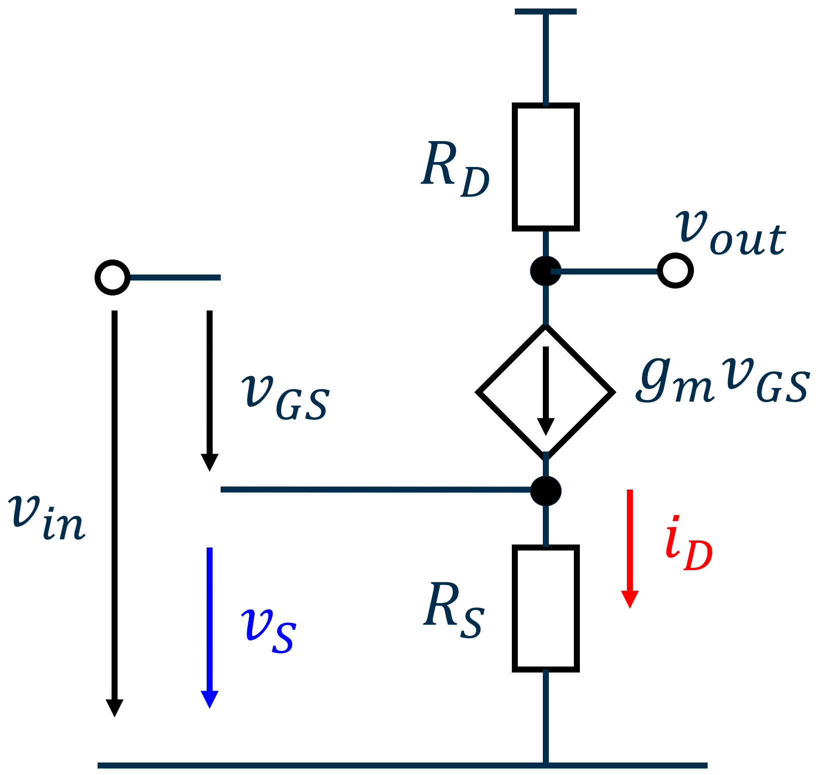
$$H_\infty = \left.\frac{v_{out}}{v_{in}}\right|_{A=\infty} = 1 + \frac{R_1}{R_2}$$

- If we want an amplifier with a gain of 10, chose $R_1 = 9\ \mathrm{k\Omega}$ and $R_2 = 1\ \mathrm{k\Omega}$

Common-Source Amplifier with Source Degeneration

- Let´s revisit previous circuit:



Common-Source Amplifier with Source Degeneration

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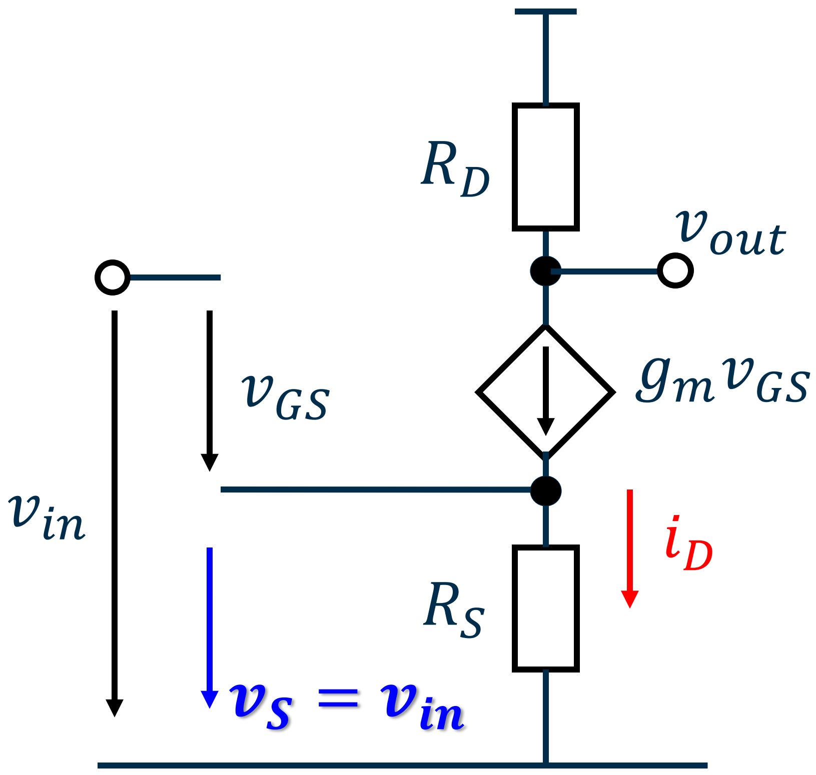


MOSFET ideal: $g_m \to \infty$

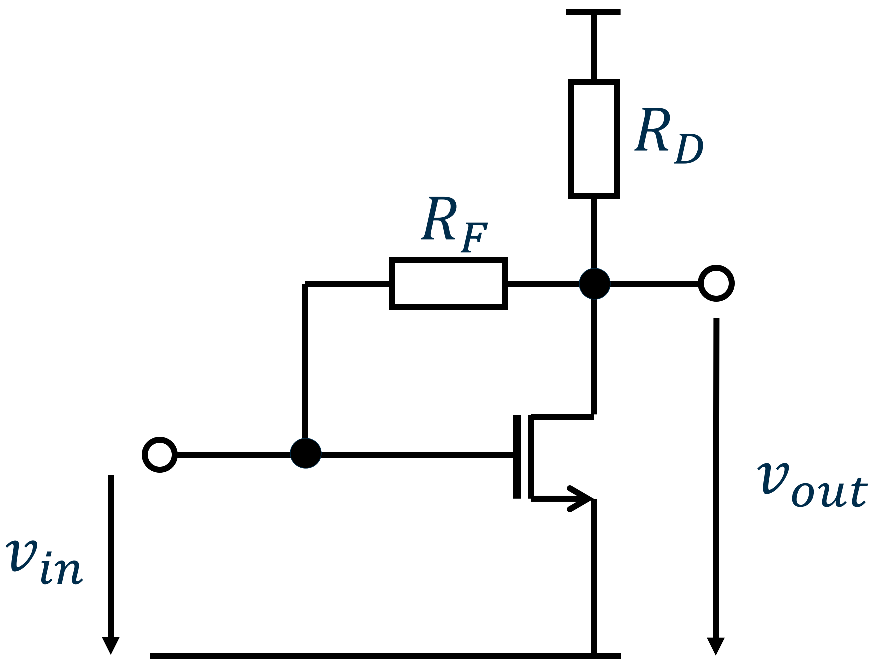
$$i_{out} = i_D = \frac{v_{in}}{R_S}$$

$$v_{out} = -i_{out} R_D = -\frac{v_{in}}{R_S} R_D$$

$$\frac{v_{out}}{v_{in}} = -\frac{R_D}{R_S}$$

Common-Source with Shunt Feedback

- Let´s consider a second common feedback circuit:



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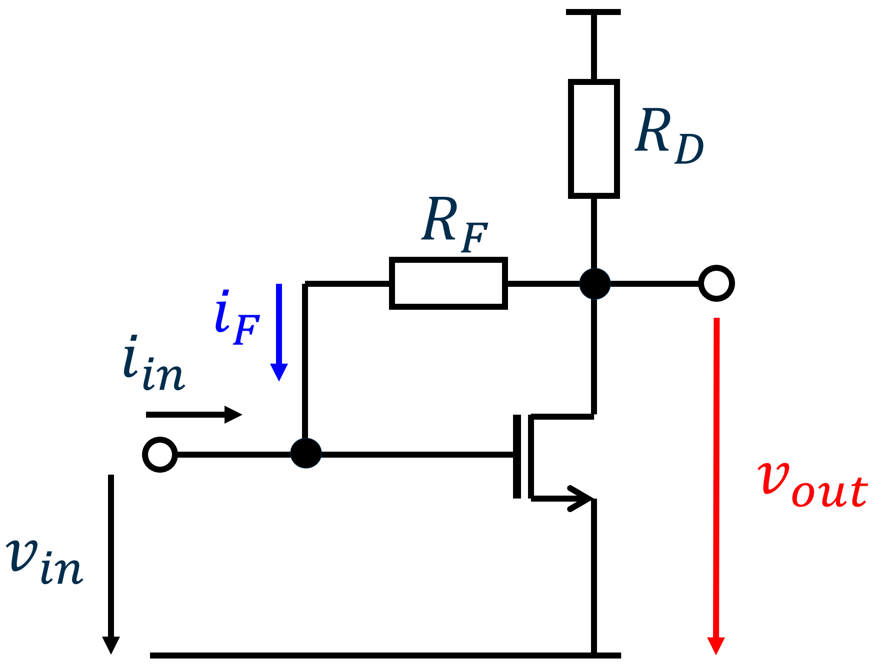

! The feedback circuit samples voltage and feeds current back to the input

Common-Source with Shunt Feedback

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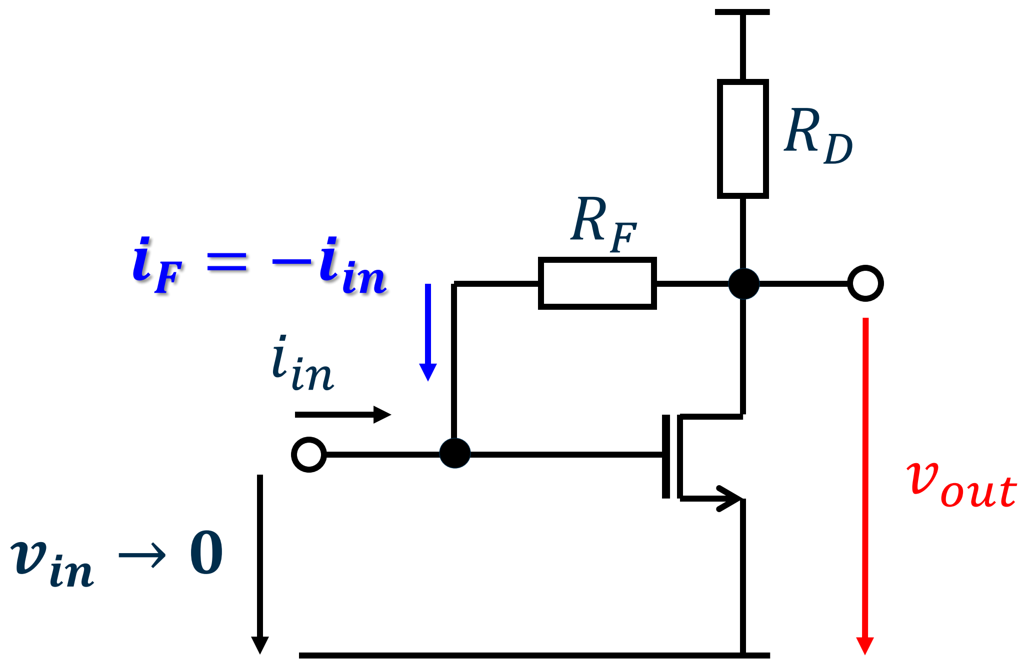


! The feedback circuit samples voltage and feeds current back to the input

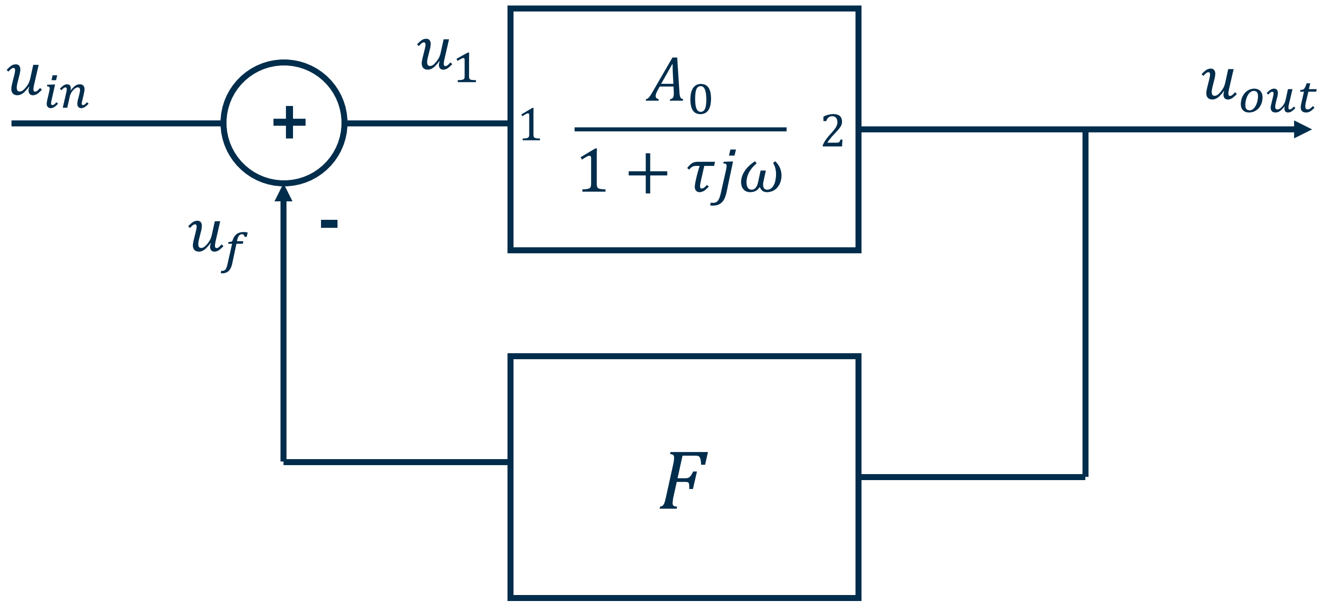
MOSFET ideal: $g_m \to \infty$

$$v_{out} - \cancelto{0}{v_{in}} = -i_{in} R_F$$

$$v_{out}/i_{in} = -R_F$$

Effects of Feedback

- Frequency response can be modified to achieve desired response:



Effects of Feedback

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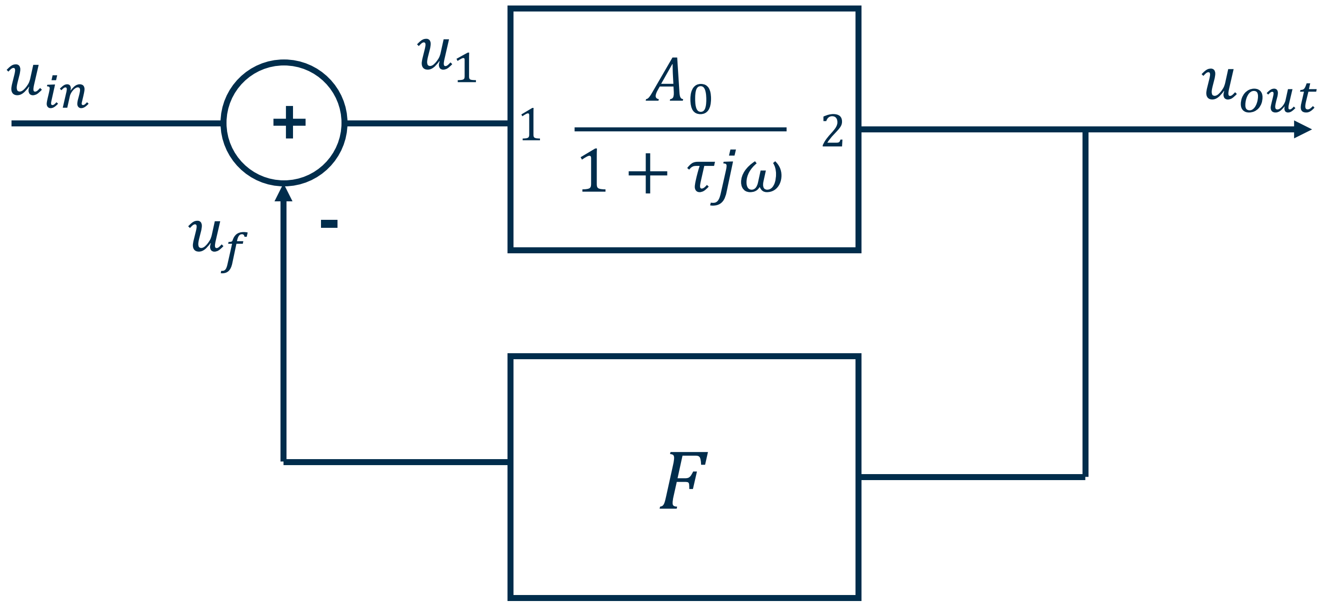
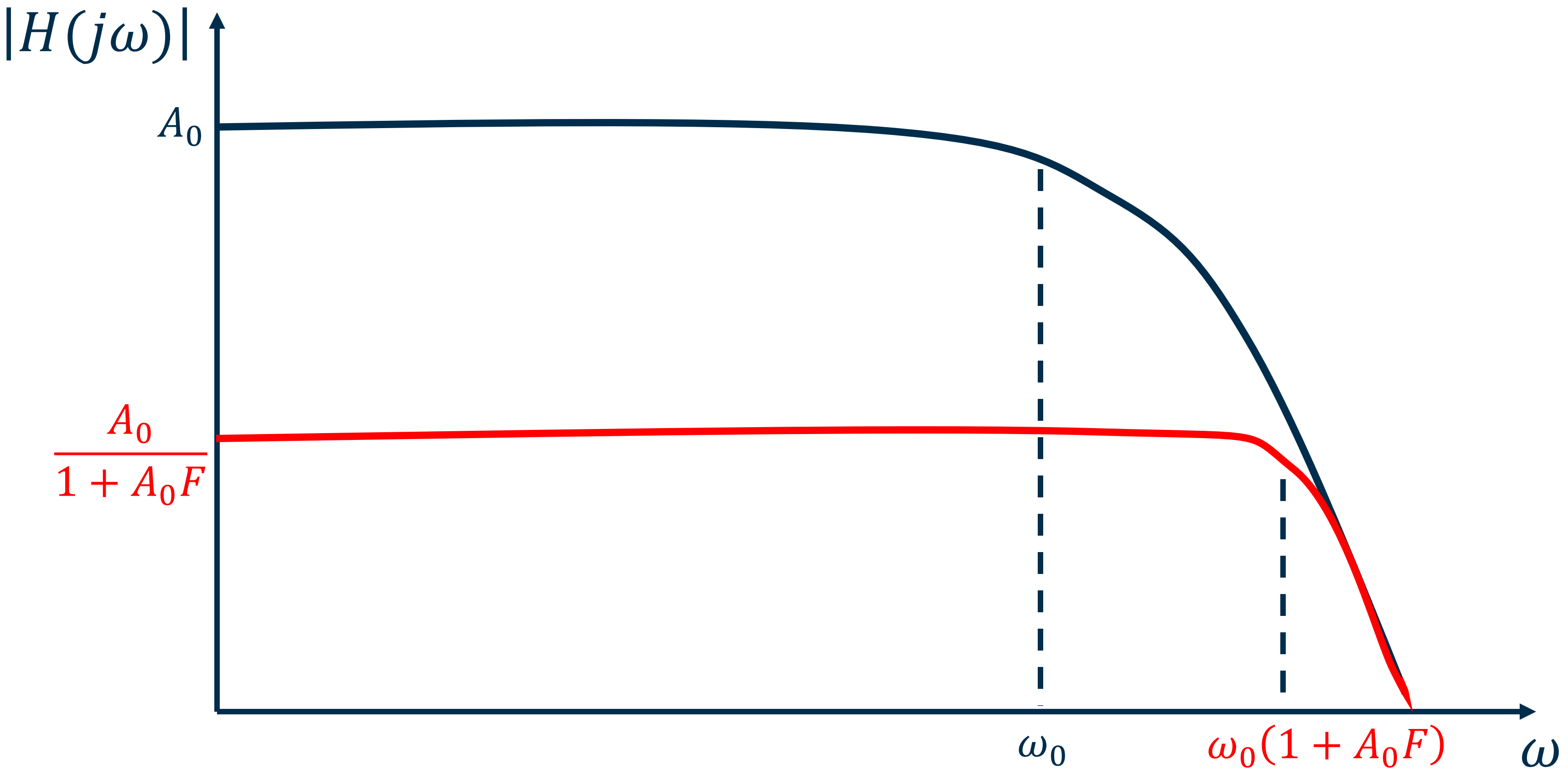


$$H(j\omega) = \frac{A(j\omega)}{1 + A(j\omega)F} = \frac{A_0}{1 + A_0F + \tau j\omega} = \frac{A_0}{1 + A_0F} \frac{1}{1 + \frac{\tau}{1 + A_0F} j\omega}$$

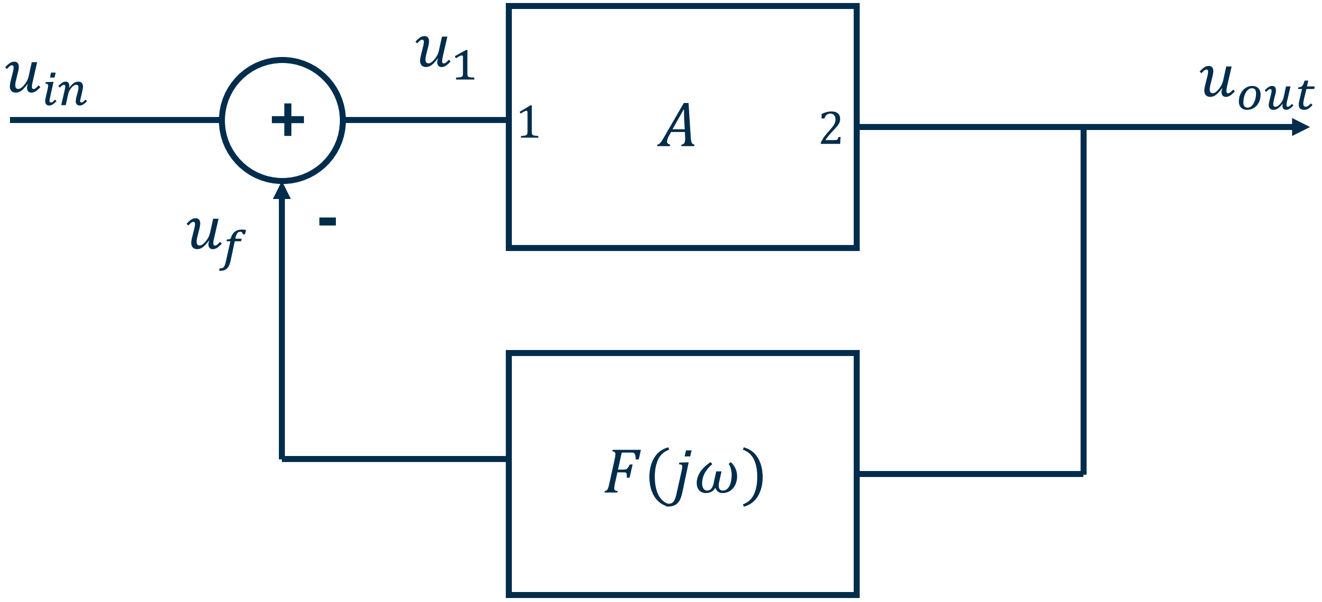
Effects of Feedback

- Frequency response with negative feedback:



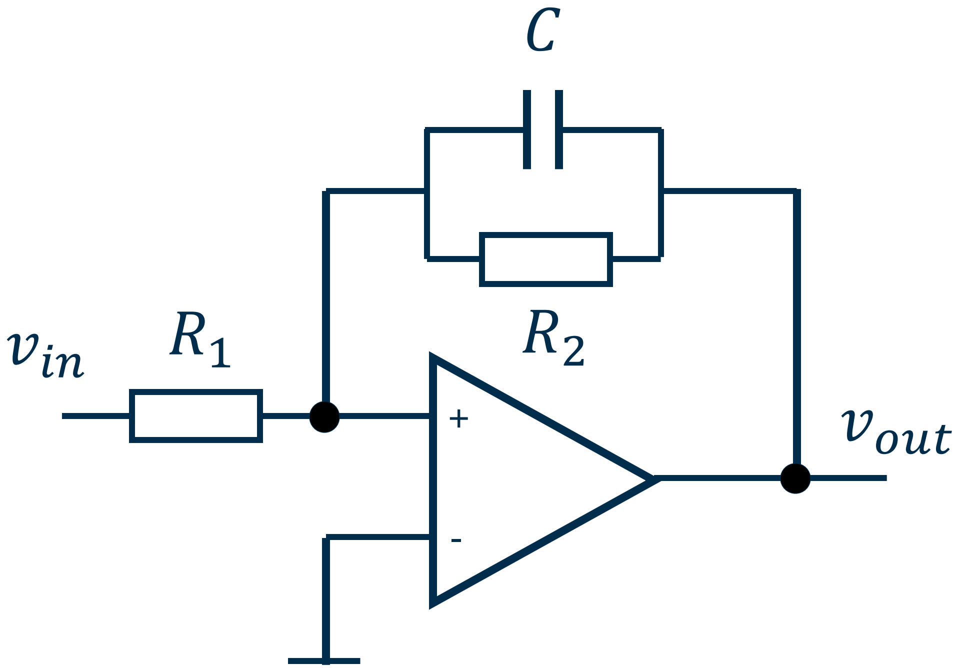
Effects of Feedback

- Frequency response can be modified, but the situation is different when the frequency dependency in the feedback circuit:



Effects of Feedback

- RC-Feedback:

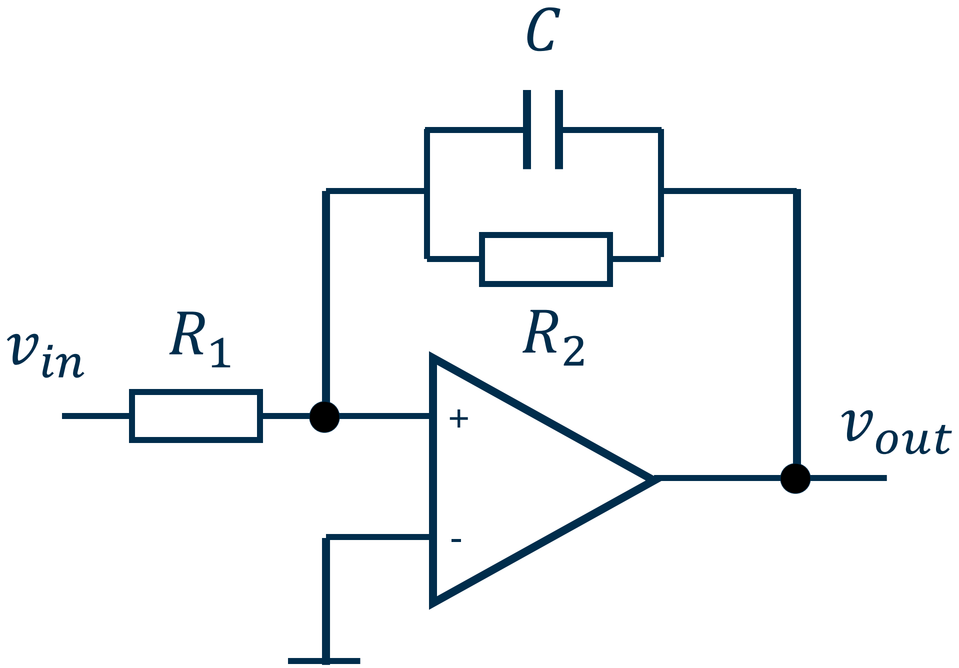


- This a high-pass feedback circuit, but:

$$\frac{v_{out}}{v_{in}} = \left.\frac{A}{1 + AF}\right|_{A \to \infty} = \frac{1}{F}$$

Effects of Feedback

- RC-Feedback:



- This a high-pass feedback circuit, but:

$$\frac{v_{out}}{v_{in}} = \left.\frac{A}{1+AF}\right|_{A\to\infty} = \frac{1}{F}$$

$$\frac{v_{in}}{R_1} = -\frac{v_{out}}{\frac{R_2}{1+j\omega R_2 C}}$$

$$H_\infty = \left.\frac{v_{out}}{v_{in}}\right|_{A=\infty} = -\frac{R_2}{R_1}\frac{1}{1+j\omega R_2 C}$$

- Which is a lowpass function!

Asymptotic Transfer Function

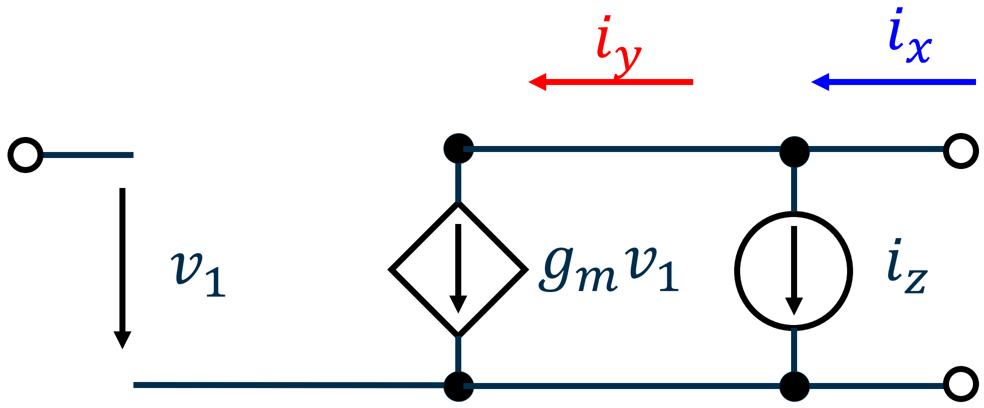
- Now we would like to move towards acquiring the complete transfer function starting from the idealized case:

$$\boxed{H = H_\infty \frac{T}{1+T} + H_0 \frac{1}{1+T}}$$

$H_\infty = H\big|_{A\to\infty}$ $\qquad$ $H_0 = H\big|_{A\to 0}$ $\qquad$ T: return ratio

Return Ratio

- The return ratio T is part of the controlled source signal that returns upon itself through the feedback, and corrects for the factor $A \neq \infty$

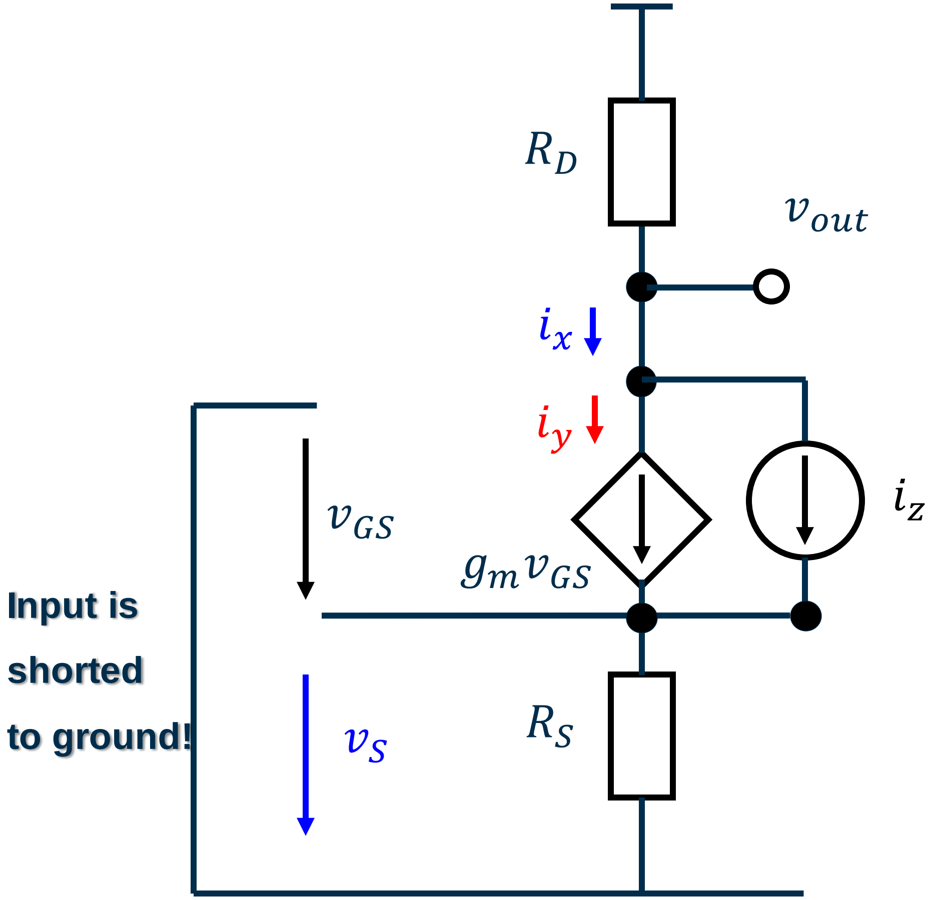


$$\boldsymbol{T = -\frac{i_y}{i_x}}$$

- Other sources are eliminated from the circuit, the quantities could be voltages or currents (they need to be of the same type)
- In this example i_z is only included so that $i_x \neq i_y$, and does not influence the calculation of T

Common-Source Amplifier with Source Degeneration

- Let´s return to our example



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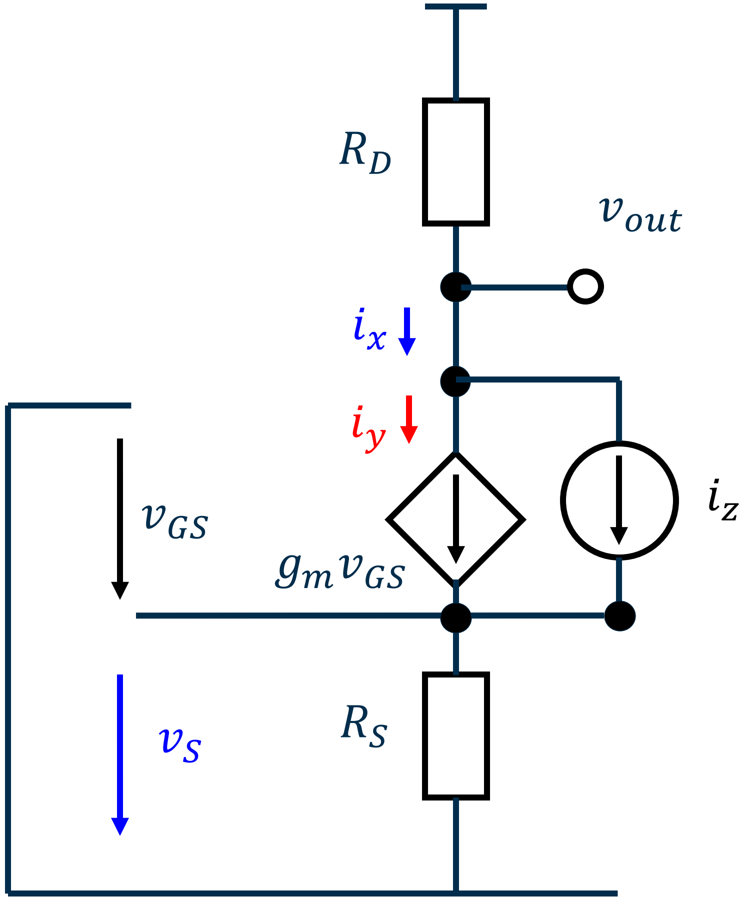


Input is shorted to ground!

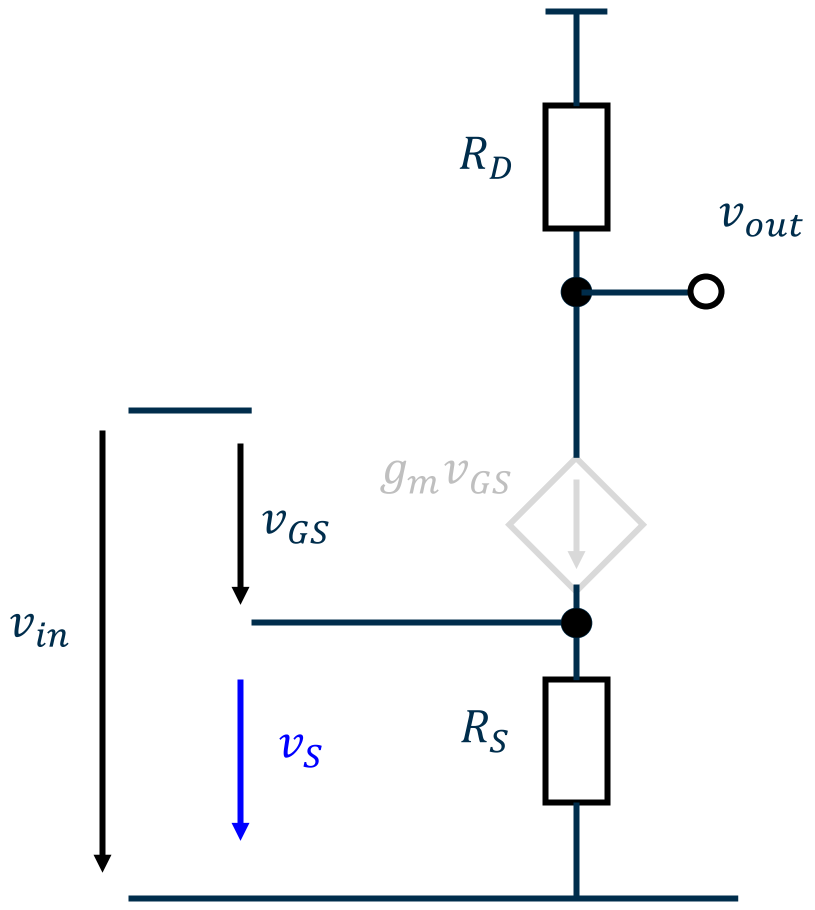
$$i_x R_S = -v_{GS}$$

$$i_y = -i_x g_m R_S$$

$$\boldsymbol{T} = -\frac{\boldsymbol{i_y}}{\boldsymbol{i_x}} = g_m R_S$$

Common-Source Amplifier with Source Degeneration

- Let´s return to our example



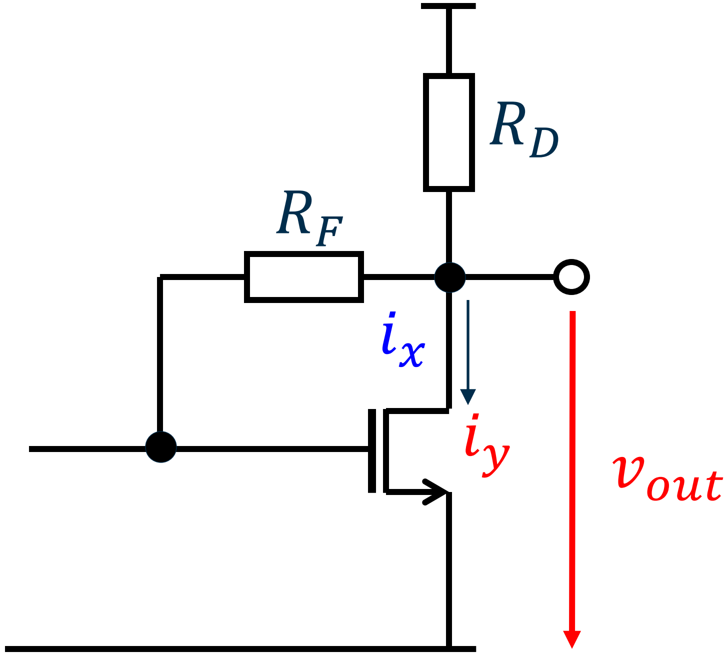
$$H_0 = H\Big|_{A\to 0} = 0$$

Common-Source Amplifier with Source Degeneration

$$H = H_\infty \frac{T}{1+T} + H_0 \frac{1}{1+T}$$

$$H = -\frac{R_D}{R_S}\frac{g_m R_S}{1+g_m R_S} = -\frac{g_m R_D}{1+g_m R_S}$$

Common-Source with Shunt Feedback



$$v_{out} = v_{in}$$

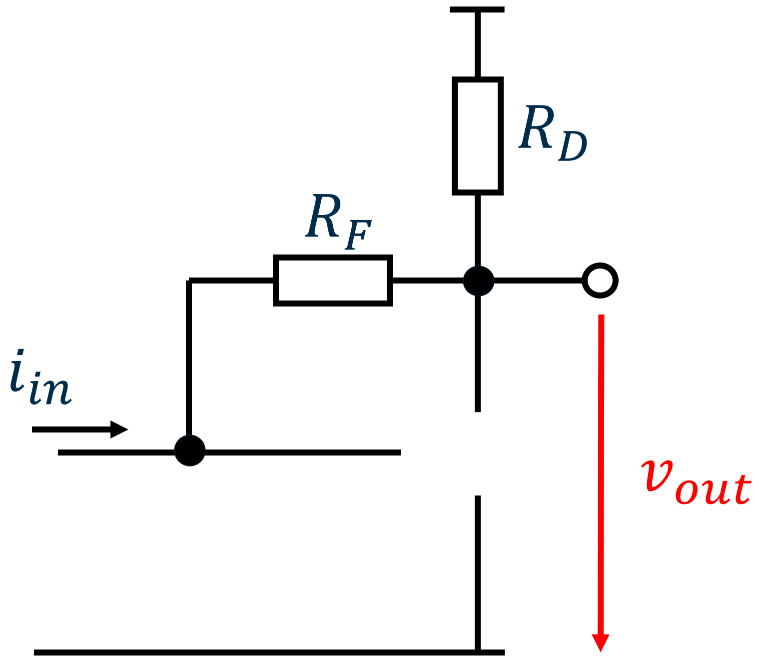
$$\frac{\cancel{v_{in} - v_{out}}}{R_F} - \frac{v_{out}}{R_D} = i_x$$

0

$$g_m v_{in} = i_y$$

$$\boldsymbol{T} = -\frac{\boldsymbol{i_y}}{\boldsymbol{i_x}} = g_m R_D$$

Common-Source with Shunt Feedback



$$H_0 = H\Big|_{A\to 0} = -R_D$$

Common-Source Amplifier with Source Degeneration

$$H = H_\infty \frac{T}{1+T} + H_0 \frac{1}{1+T}$$

$$H = -R_F \frac{g_m R_D}{1+g_m R_D} - R_D \frac{1}{1+g_m R_D}$$

$$H = -\frac{g_m R_D R_F + R_D}{1+g_m R_D}$$

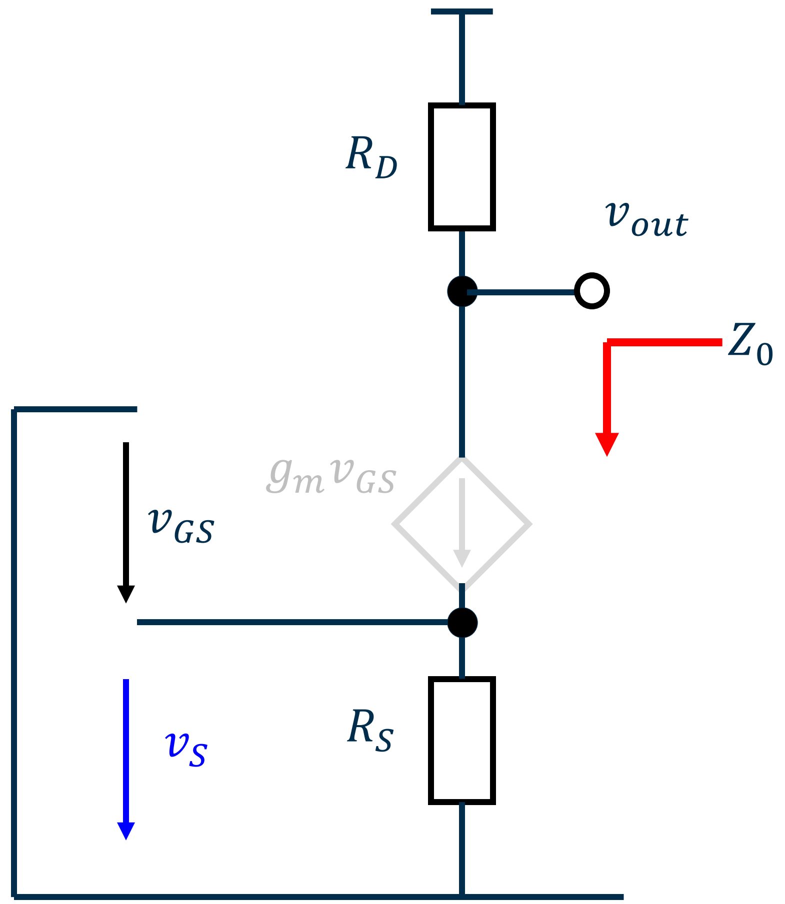
Input/Output Impedances, Blackman´s Formula

- The input and output impedances can be calculated using return ratio under open/short terminations at the port of interest

$$Z = Z_0 \frac{1 + T_{short}}{1 + T_{open}}$$

- Z_0 corresponds to the port impedance for H_0 case

Source Degenerated Amplifier



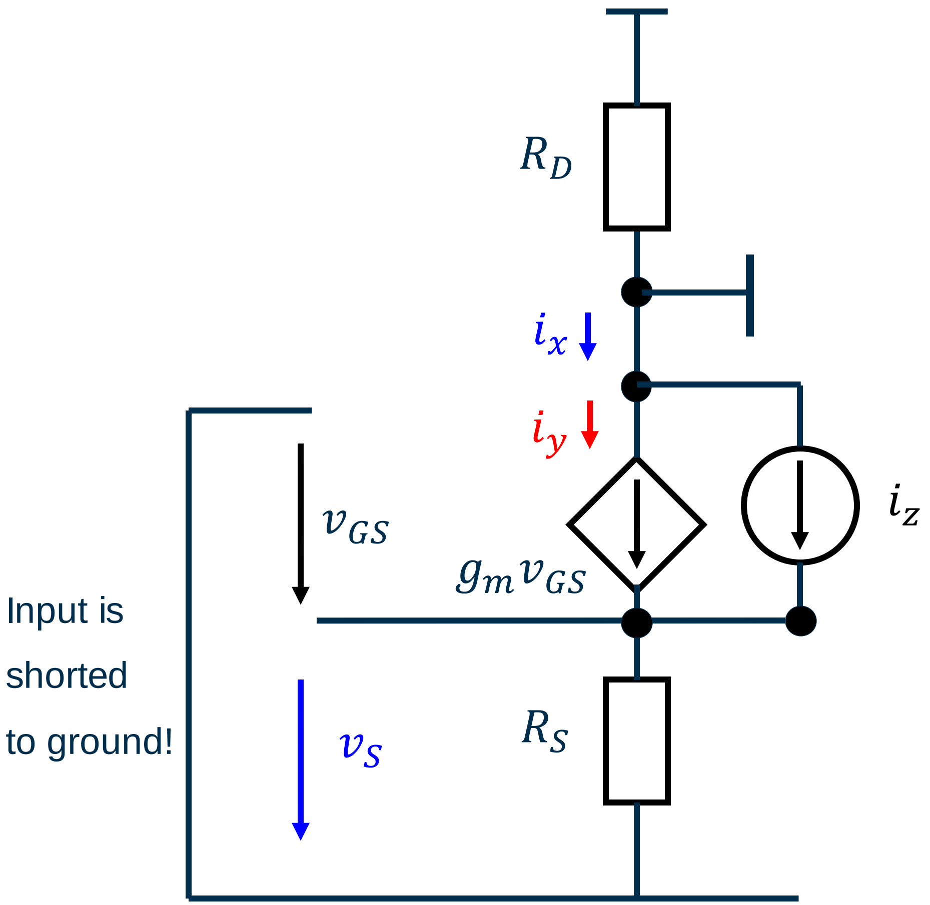
$$Z_0 = r_o$$

! Without channel length modulation $z_{out} = \infty$

!! We are also not considering R_D in the circuit, because it will be just in parallel to whatever we find looking into the amplifier

Common-Source Amplifier with Source Degeneration

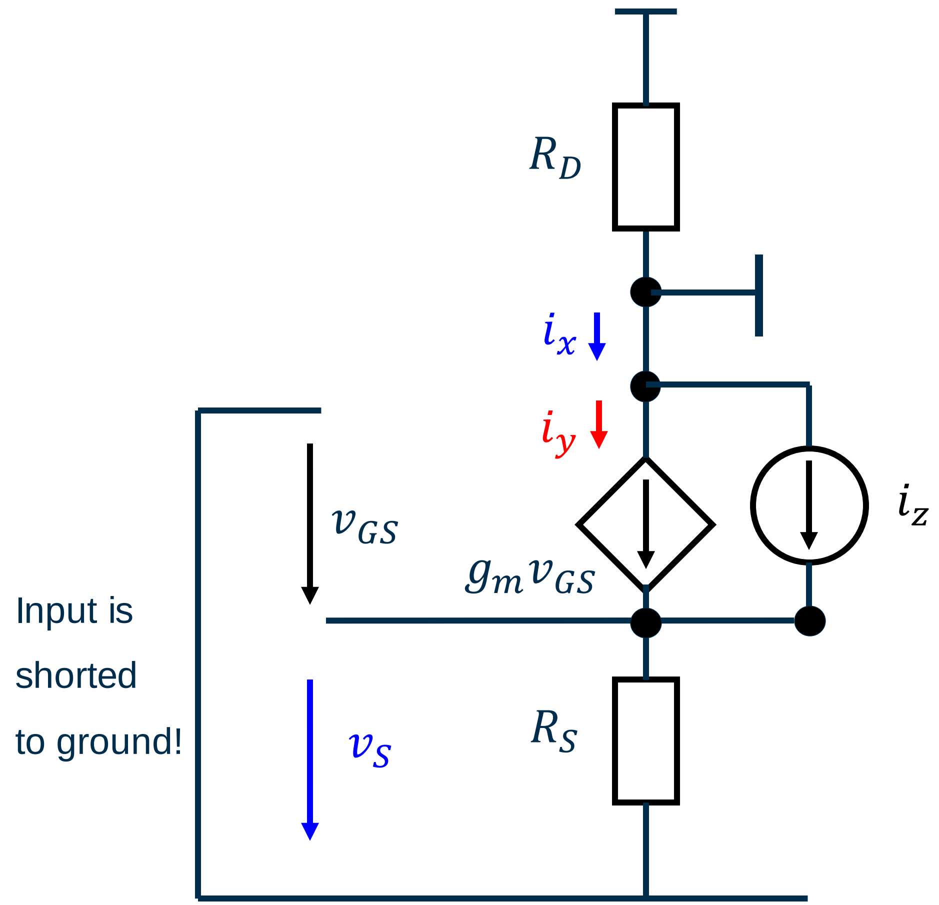
- T_{short}



$$\boldsymbol{T_{short}} = \boldsymbol{T} = -\frac{\boldsymbol{i_y}}{\boldsymbol{i_x}} = g_m R_S$$

Common-Source Amplifier with Source Degeneration

- T_{short}



$$T_{short} = T = -\frac{i_y}{i_x} = g_m R_S$$

$$T_{open} = 0 \text{ (no } \mathrm{R_D}\text{)}$$

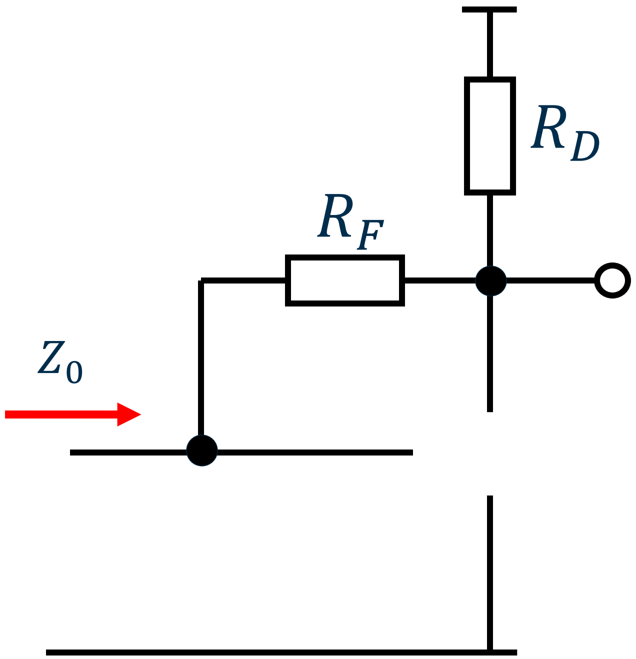
Common-Source Amplifier with Source Degeneration

$$Z = Z_0 \frac{1 + T_{short}}{1 + T_{open}}$$

$$Z = r_o \frac{1 + g_m R_S}{1 + 0} = r_o + g_m r_o R_S$$

! cascode expression that we been seeing very often

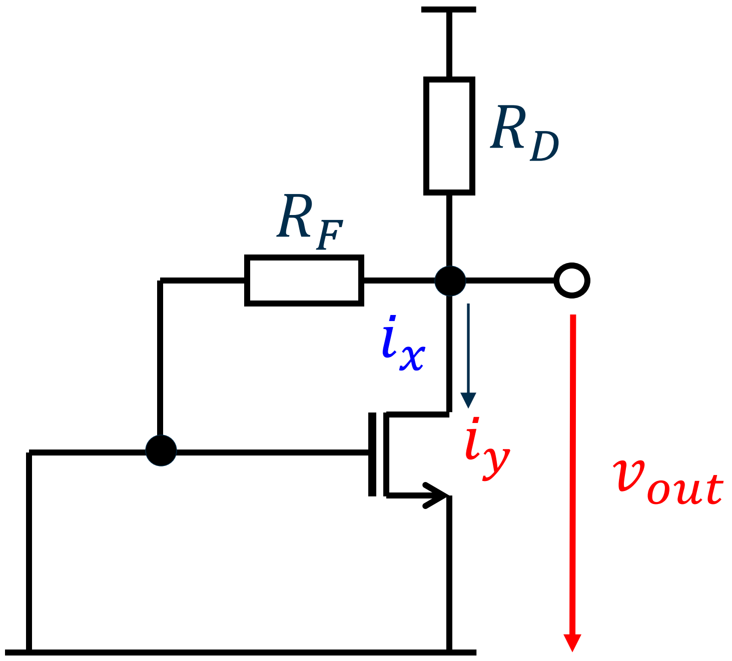
Common-Source with Shunt Feedback



$$Z_0 = R_F + R_D$$

Common-Source with Shunt Feedback

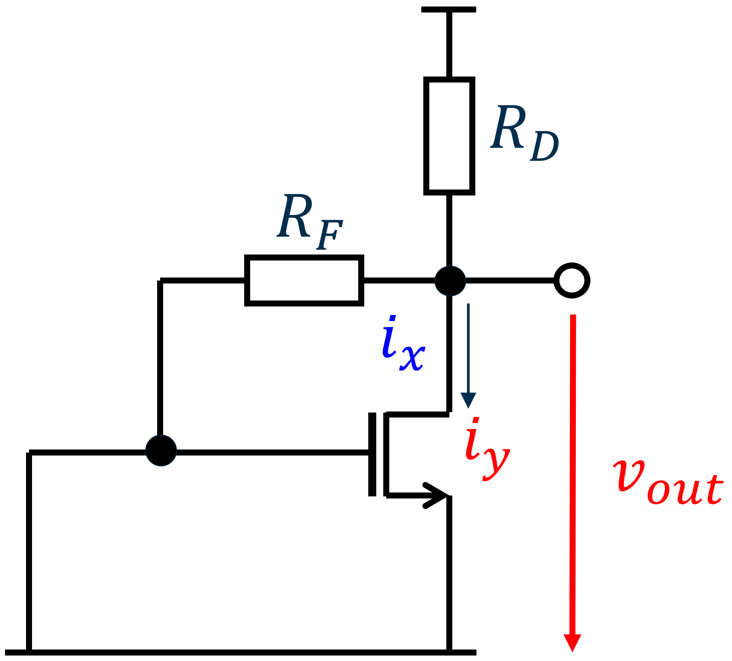
- T_{short}



$$\boldsymbol{T_{short} = 0}$$

Common-Source with Shunt Feedback

- T_{short}



$$\boldsymbol{T_{short} = 0}$$

$$\boldsymbol{T_{open} = T = g_m R_D}$$

Common-Source Amplifier with Shunt Feedback

$$Z = Z_0 \frac{1 + T_{short}}{1 + T_{open}}$$

$$Z = (R_F + R_D) \frac{1 + 0}{1 + g_m R_D} = \frac{R_F}{1 + g_m R_D} + \frac{R_D}{1 + g_m R_D}$$

Common-Source Amplifier with Shunt Feedback

$$Z = Z_0 \frac{1 + T_{short}}{1 + T_{open}}$$

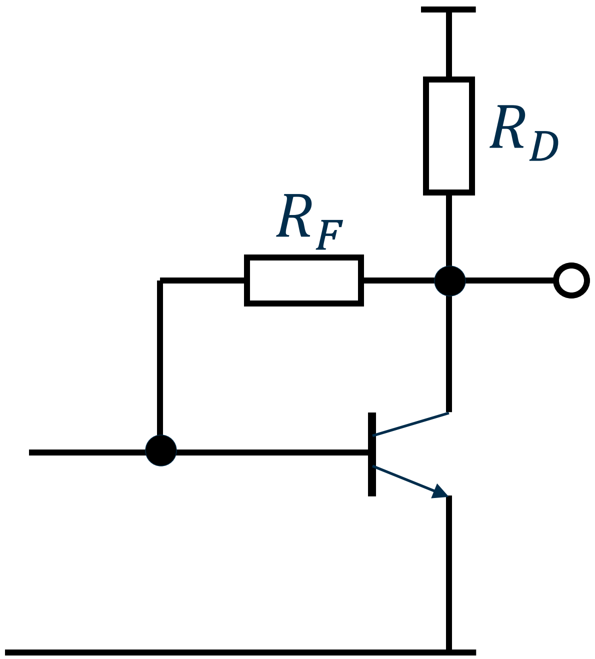
$$Z = (R_F + R_D)\frac{1 + 0}{1 + g_m R_D} = \frac{R_F}{1 + g_m R_D} + \frac{R_D}{1 + g_m R_D}$$

$$\sim \frac{1}{g_m}$$

Dominant expression

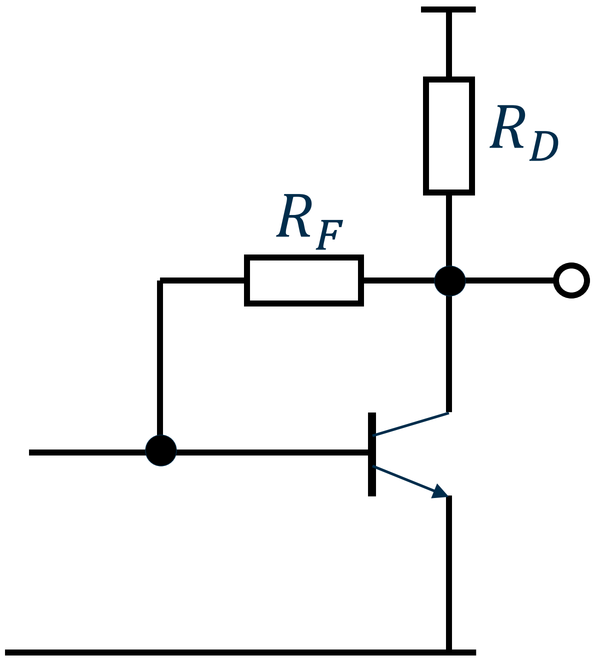
Simulations

$$R_D = 5k\Omega, I_C = 0.5\ mA, g_m = 0.015\ S$$



- No feedback: $\frac{v_{out}}{v_{in}} = 76 \quad r_{in} = 2470\ \Omega$

Simulations



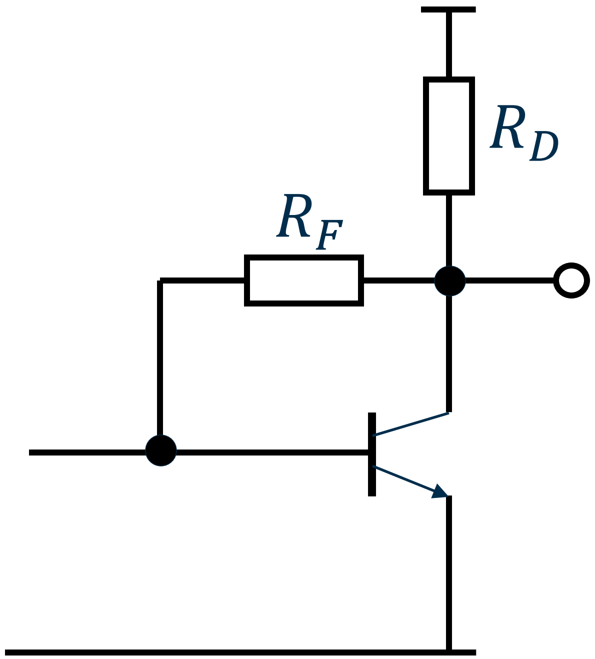
$$R_D = 5k\Omega, I_C = 0.5\ mA, g_m = 0.015\ S$$

- No feedback: $\frac{v_{out}}{v_{in}} = 76 \quad r_{in} = 2470\ \Omega$
- $R_F = 50k\ \Omega$: $\frac{v_{out}}{i_{in}} = 39200 \quad (\text{calculated } 48700)$

$$R_{in} = 563\ \Omega \quad (\text{calculated } 550\ \Omega)$$

Simulations

$$R_D = 5k\Omega, I_C = 0.5\, mA, g_m = 0.015\, S$$



- No feedback: $\frac{v_{out}}{v_{in}} = 76 \quad r_{in} = 2470\ \Omega$
- $R_F = 50k\ \Omega$: $\frac{v_{out}}{i_{in}} = 39200 \quad (\text{calculated } 48700)$

  $R_{in} = 563\ \Omega \quad (\text{calculated } 550\ \Omega)$
- $R_F = 5k\ \Omega$: $\frac{v_{out}}{i_{in}} = 4658 \quad (\text{calculated } 4935)$

  $R_{in} = 123\ \Omega \quad (\text{calculated } 123\ \Omega)$

Loop Gain

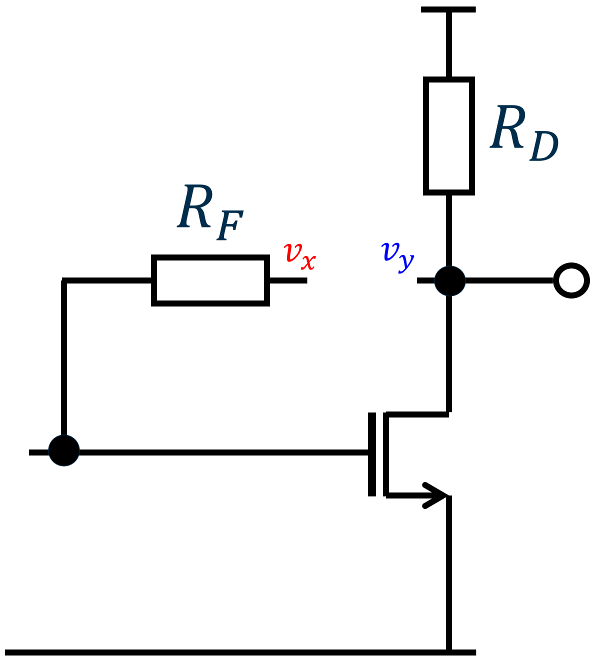
- We can see that the loop gain *AF* has significant importance in the feedback circuit – there is also a practical way the calculate this important parameter!

- To calculate the loop gain, we can break the feedback loop at any point, and calculate corresponding voltage and current gains; the loop gain will be:

$$\frac{1}{AF} = \frac{1}{A_{vop}} + \frac{1}{H_{os}}$$

Common-Source with Shunt Feedback

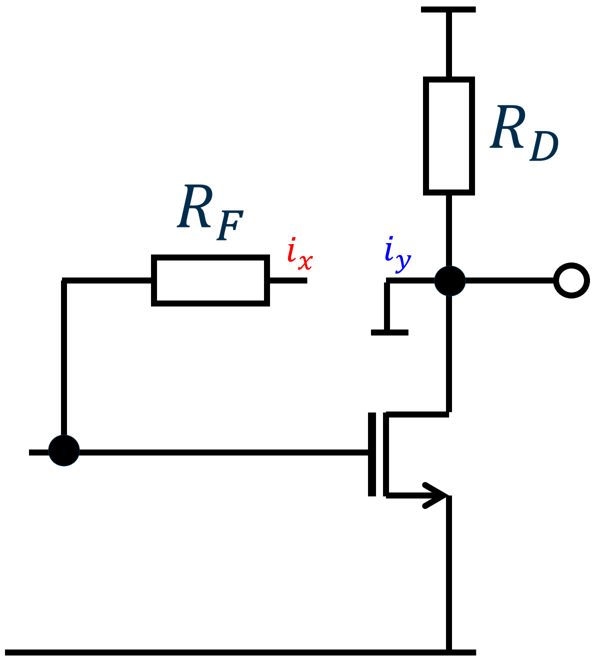
- Loop gain calculation



$$A_{vop} = -\left.\frac{v_y}{v_x}\right|_{i_y=0} = g_m R_D$$

Common-Source with Shunt Feedback

- Loop gain calculation

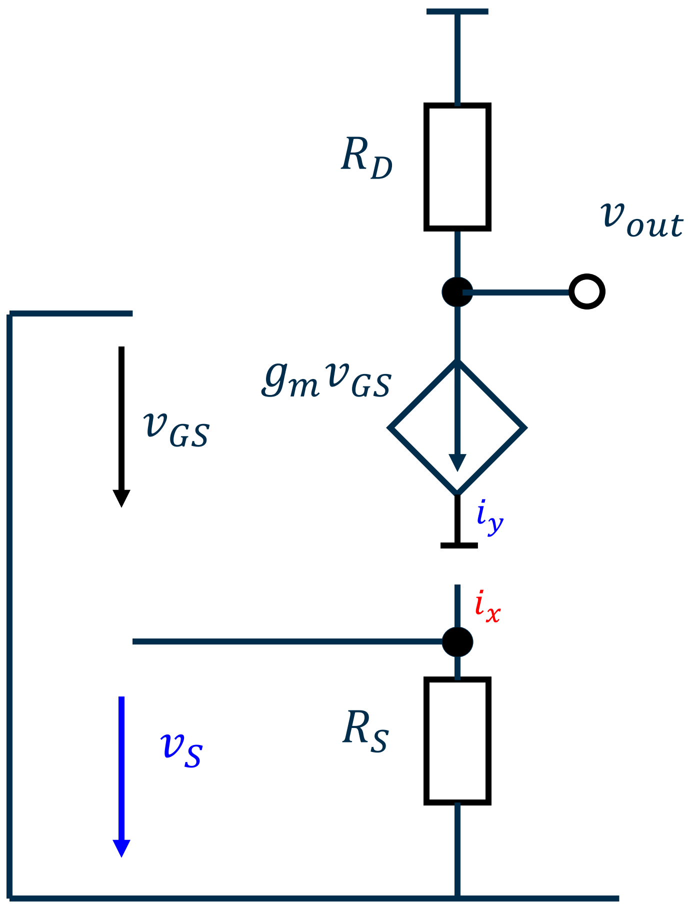


$$H_{os} = -\left.\frac{i_y}{i_x}\right|_{v_y=0} = \infty$$

$$AF = g_m R_D$$

Common-Source Amplifier with Source Degeneration

- Loop gain calculation

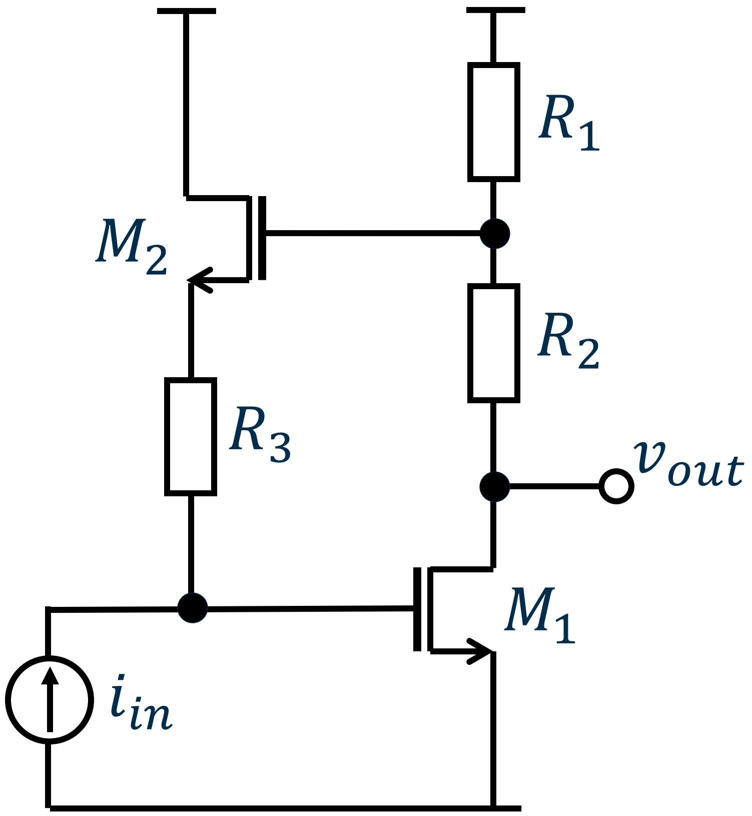


$$A_{vop} = -\left.\frac{v_y}{v_x}\right|_{i_y=0} = \infty$$

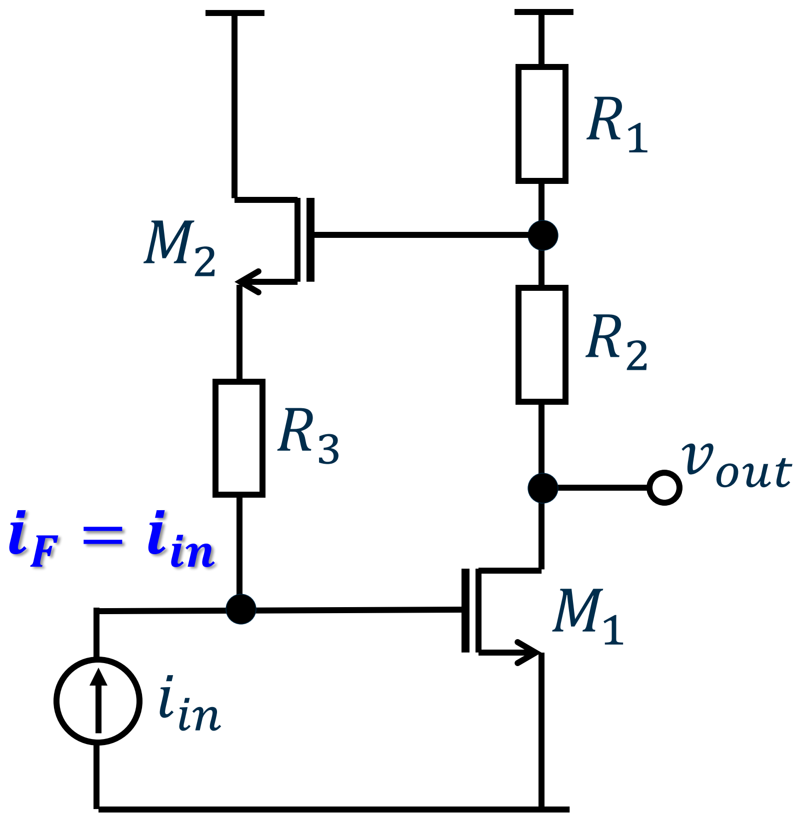
$$H_{os} = -\left.\frac{i_y}{i_x}\right|_{v_y=0} = g_m R_S$$

$$AF = g_m R_S$$

Advanced Example: Cherry-Hooper Amplifier

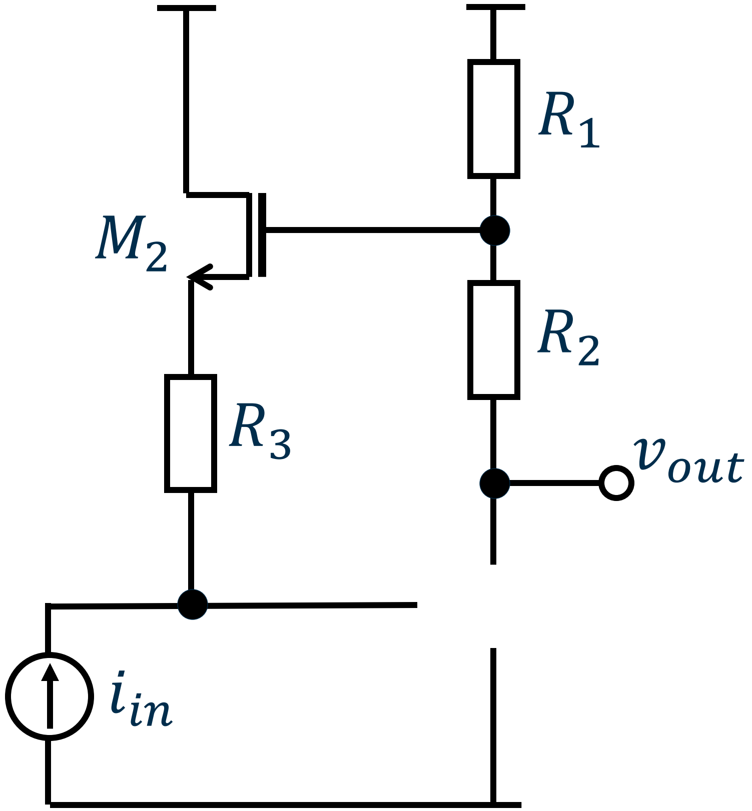


Advanced Example: Cherry-Hooper Amplifier



$$H_\infty = \left.\frac{v_{out}}{i_{in}}\right|_{g_{m1}=\infty} = \left(R_3 + \frac{1}{g_{m2}}\right)\left(1 + \frac{R_2}{R_1}\right)$$

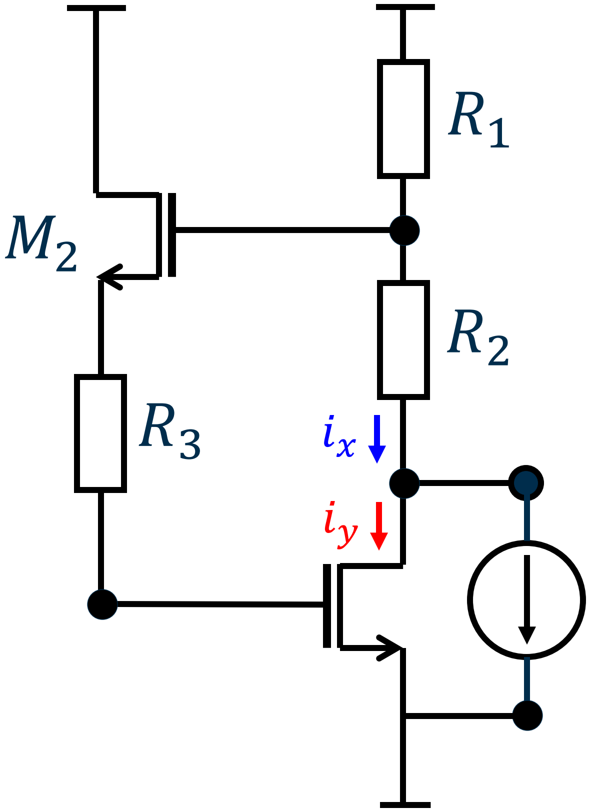
Advanced Example: Cherry-Hooper Amplifier



$$H_\infty = \left.\frac{v_{out}}{i_{in}}\right|_{g_{m1}=\infty} = -\left(R_3 + \frac{1}{g_{m2}}\right)\left(1 + \frac{R_2}{R_1}\right)$$

$$H_0 = \left.\frac{v_{out}}{i_{in}}\right|_{g_{m1}=0} = 0$$

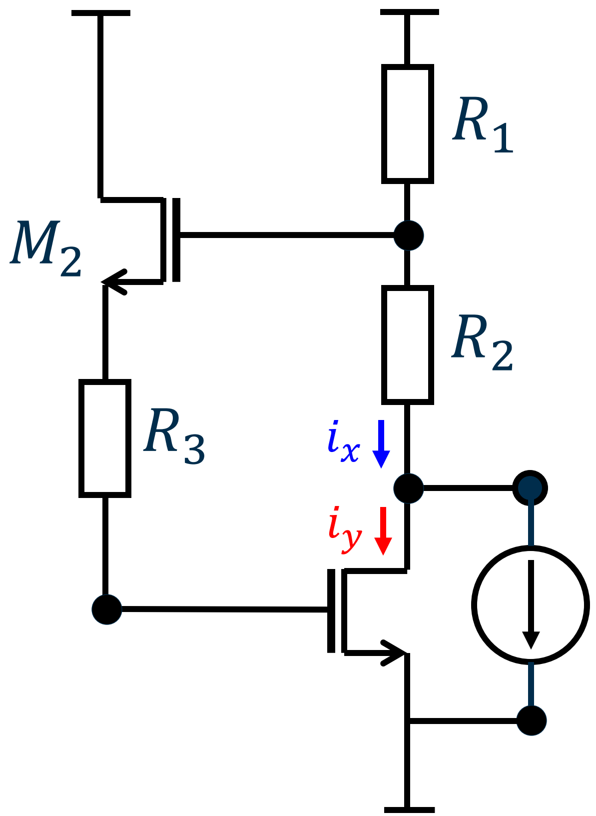
Advanced Example: Cherry-Hooper Amplifier



$$i_x R_1 = v_{G2} = v_{G1}$$ (M2 is common drain)

$$\boldsymbol{T = -\frac{i_y}{i_x} = g_{m1} R_1}$$

Advanced Example: Cherry-Hooper Amplifier



$i_x R_1 = v_{G2} = v_{G1}$ (M2 is common drain)

$$\boldsymbol{T = -\frac{i_y}{i_x} = g_{m1} R_1}$$

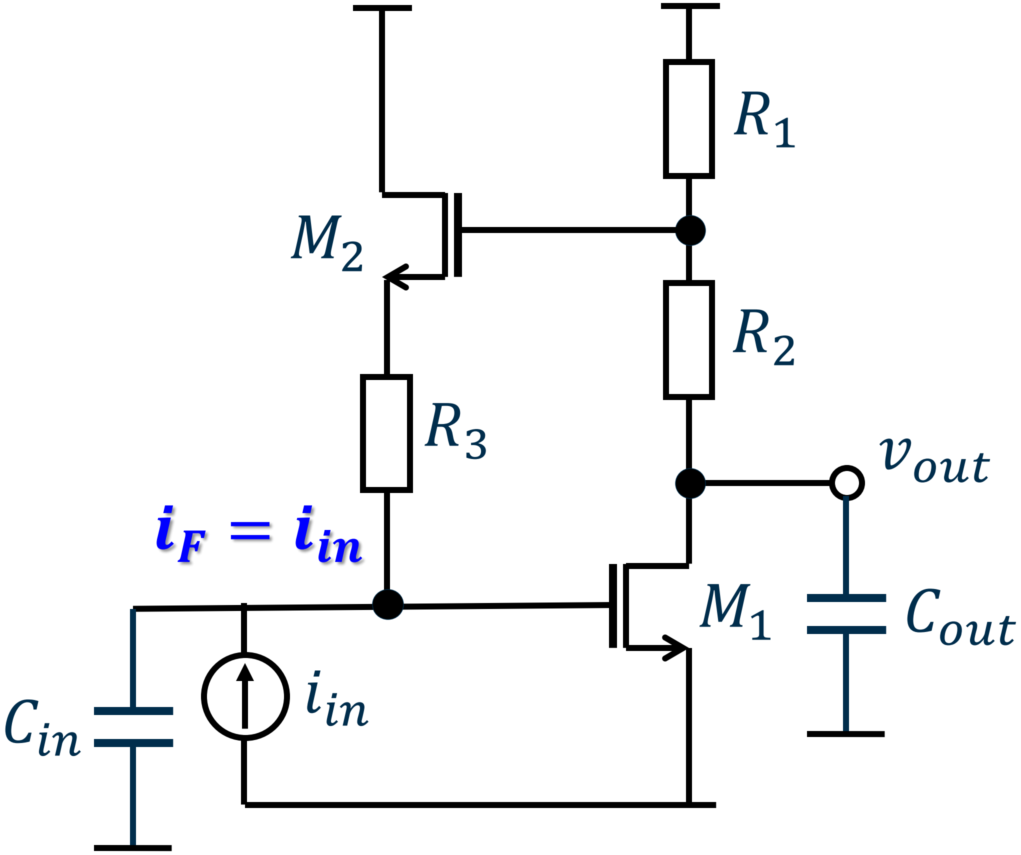
$$H = H_\infty \frac{T}{1+T} + H_0 \frac{1}{1+T}$$

$$H = -\left(R_3 + \frac{1}{g_{m2}}\right)\left(1 + \frac{R_2}{R_1}\right)\frac{g_{m1} R_1}{1 + g_{m1} R_1}$$

$$H \approx -R_3 \left(1 + \frac{R_2}{R_1}\right)$$

Advanced Example: Cherry-Hooper Amplifier

- With input and output capacitors

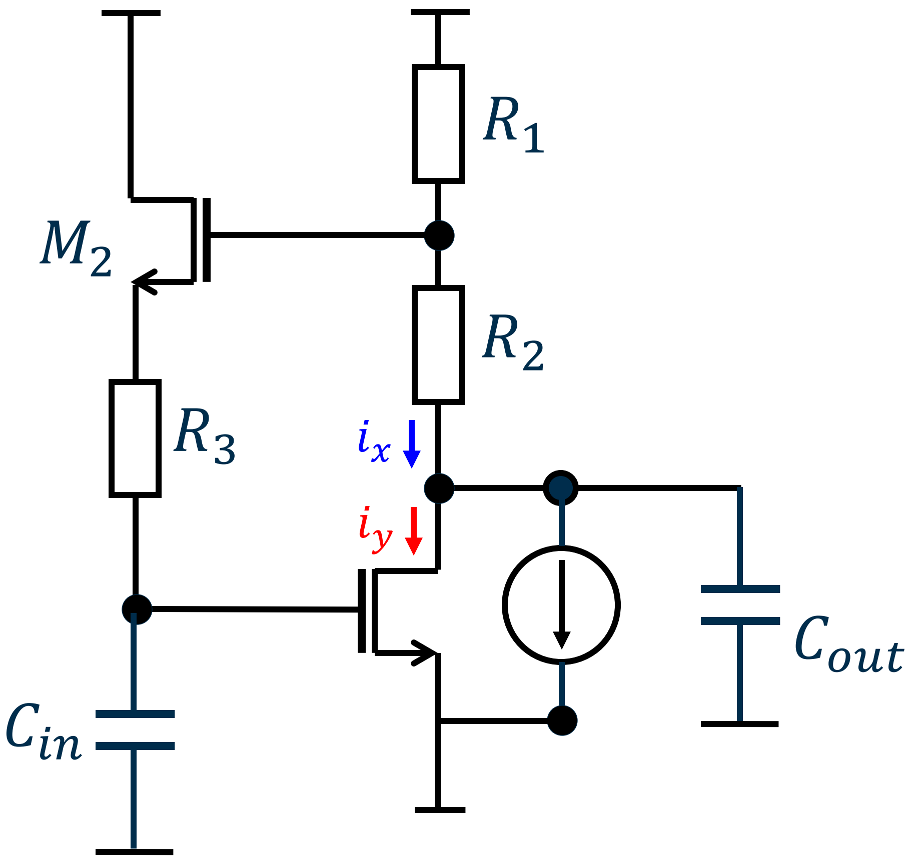


$$H_\infty = \left.\frac{v_{out}}{i_{in}}\right|_{g_{m1}=\infty} = -\left(R_3 + \frac{1}{g_{m2}}\right)\left(1 + \frac{R_2}{R_1}\right)$$

$$H_0 = \left.\frac{v_{out}}{i_{in}}\right|_{g_{m1}=0} = 0$$

- ! Asymptotic gain expressions do not change, because H_∞ forces v_{in} to 0, and C_{out} has no influence in the v_{out} expression

Advanced Example: Cherry-Hooper Amplifier



$$\boldsymbol{T} = -\frac{\boldsymbol{i_y}}{\boldsymbol{i_x}}$$

- The capacitors will impact return ratio with two independant time constants:

$$\boldsymbol{T} = -\frac{\boldsymbol{i_y}}{\boldsymbol{i_x}} = T_0 \frac{1}{(1+\tau_1 j\omega)(1+\tau_2 j\omega)}$$

$$\tau_1 = \left(R_3 + \frac{1}{g_{m2}}\right) C_1$$

$$\tau_2 = (R_1 + R_2) C_2$$

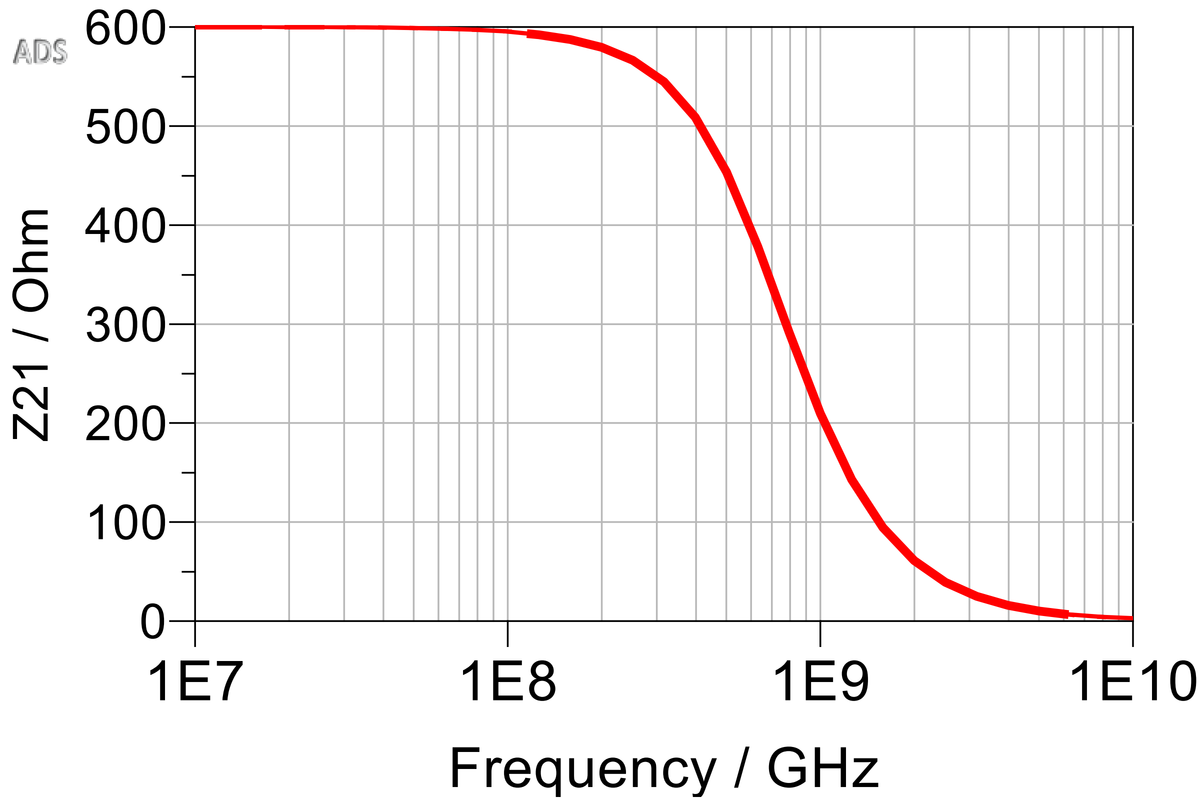
Advanced Example: Cherry-Hooper Amplifier

$$\frac{T}{1+T} = \frac{T_0}{1+T_0} \frac{1}{1 + \frac{\tau_1 + \tau_2}{1+T_0} s + \frac{\tau_1 \tau_2}{1+T_0} s^2}$$

- We have a second-order response, where we can control the damping factor as desired!

Advanced Example: Cherry-Hooper Amplifier

- Simulations with
  - $C_{1,2} = 1\ \mathrm{pF}, \mathrm{R}_{3,1} = 100\ \Omega, R_2 = 500\ \Omega, g_{m1,2} = 10\ \mathrm{mS}$



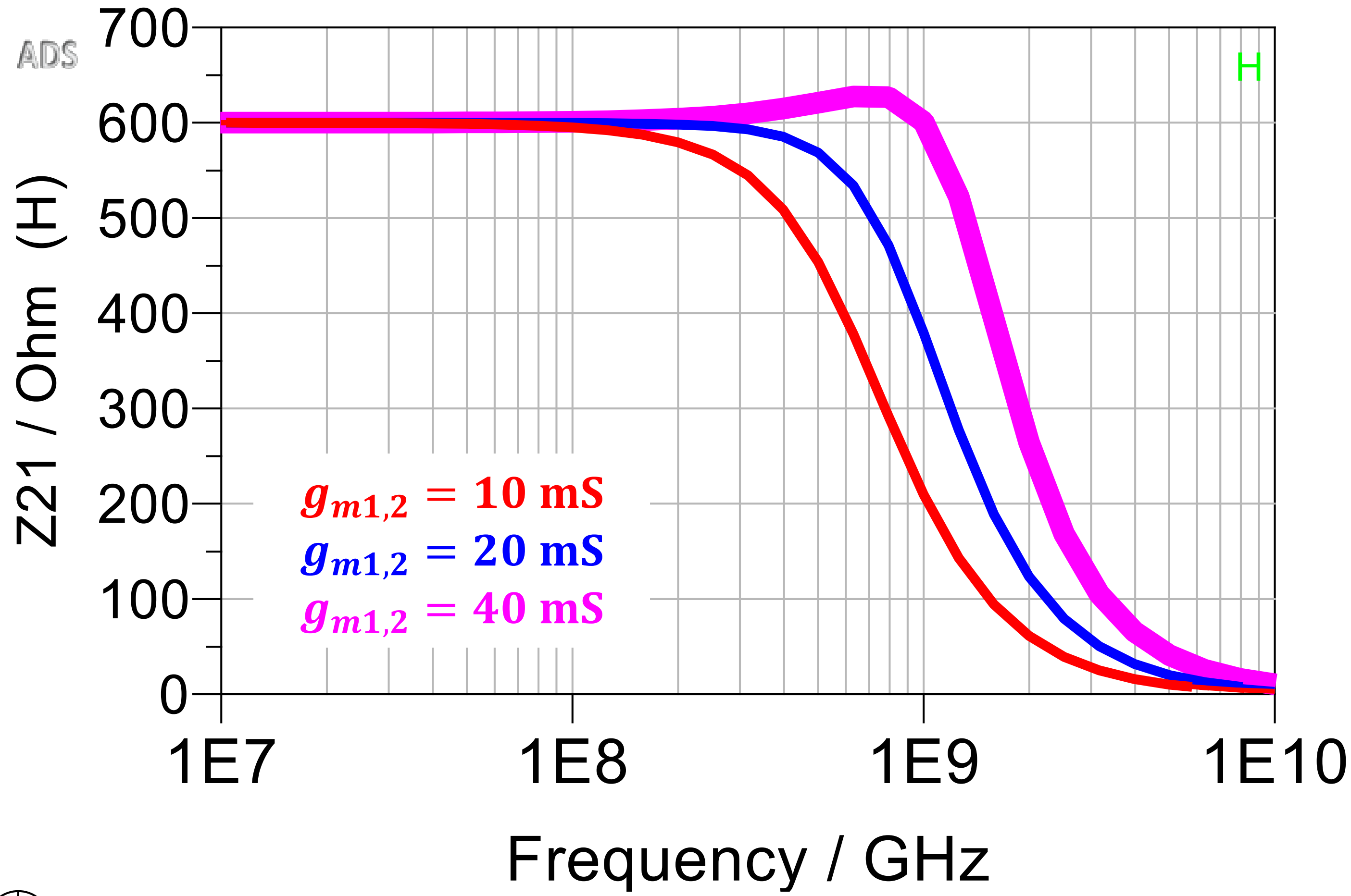
$$\tau_1 = 0.2\ \mathrm{nsec}$$

$$\tau_2 = 0.6\ \mathrm{nsec}$$

- !3-dB bandwidth should be 200 MHz
- Simulated bandwidth is 700 MHz !!

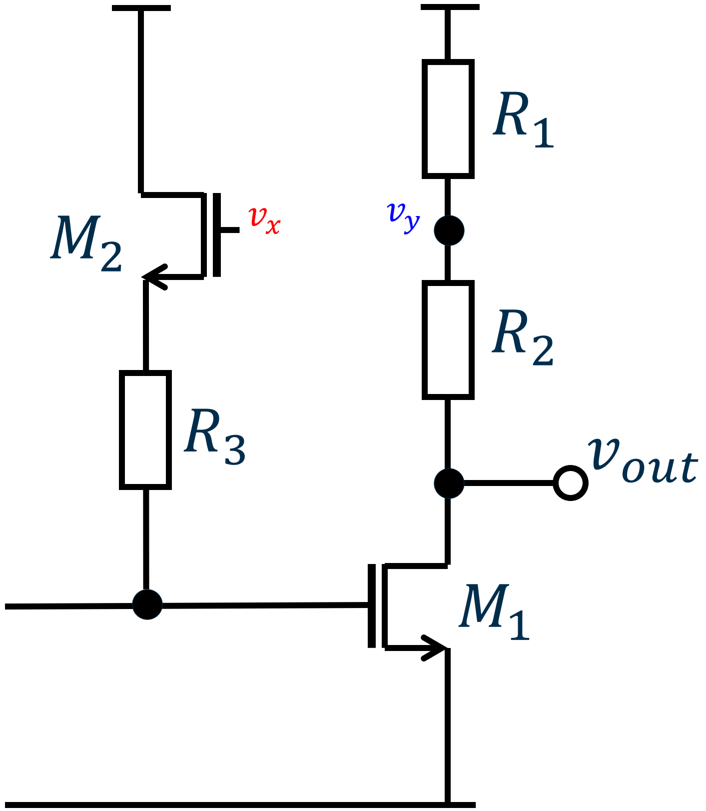
Advanced Example: Cherry-Hooper Amplifier

- Simulations with
  - $C_{1,2} = 1\ \mathrm{pF}, \mathrm{R}_{3,1} = 100\ \Omega, R_2 = 500\ \Omega, g_{m1,2} = 10\ \mathrm{mS}$



Advanced Example: Cherry-Hooper Amplifier

- Loop gain



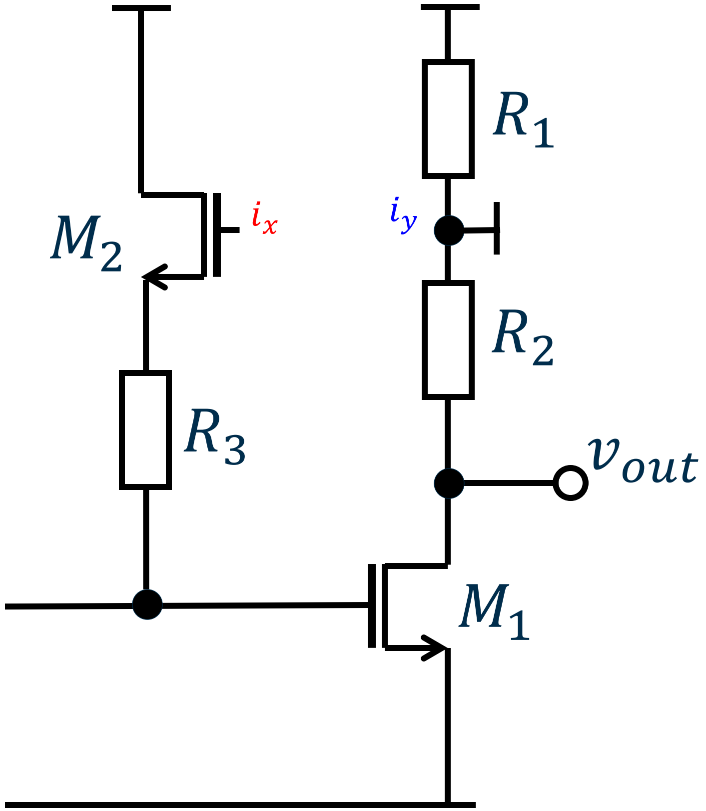
$$v_{out} = g_{m1} v_x$$

$$v_y = \frac{v_{out}}{R_1 + R_2} R_1$$

$$A_{vop} = -\left.\frac{v_y}{v_x}\right|_{i_y=0} = \frac{g_{m1} R_1}{R_1 + R_2}$$

Advanced Example: Cherry-Hooper Amplifier

- Loop gain

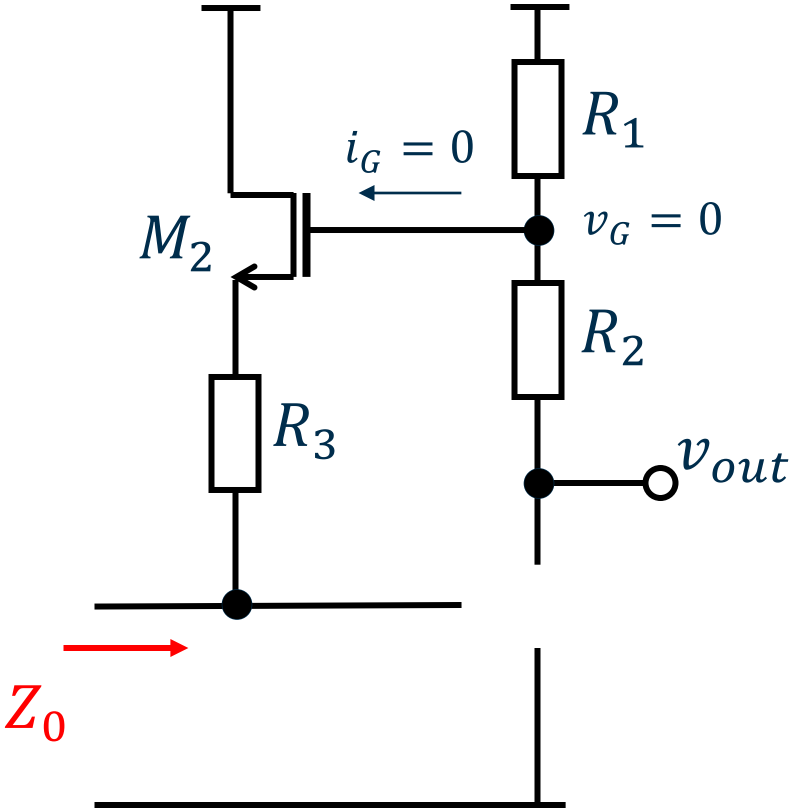


$$H_{os} = -\left.\frac{i_y}{i_x}\right|_{v_y=0} = \infty$$

$$AF = \frac{g_{m1}R_1}{R_1 + R_2}$$

Advanced Example: Cherry-Hooper Amplifier

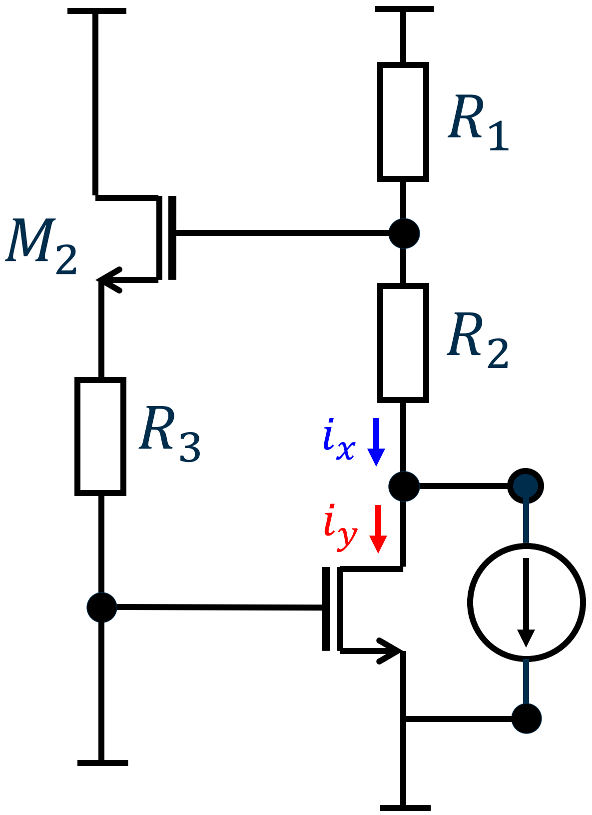
- Input Impedance



$$Z_0 = R_3 + \frac{1}{g_{m2}}$$

Advanced Example: Cherry-Hooper Amplifier

- T_{short}



$$\boldsymbol{T_{short} = -\frac{i_y}{i_x} = 0}$$

$$\boldsymbol{T_{open} = T = -\frac{i_y}{i_x} = g_{m1}R_1}$$

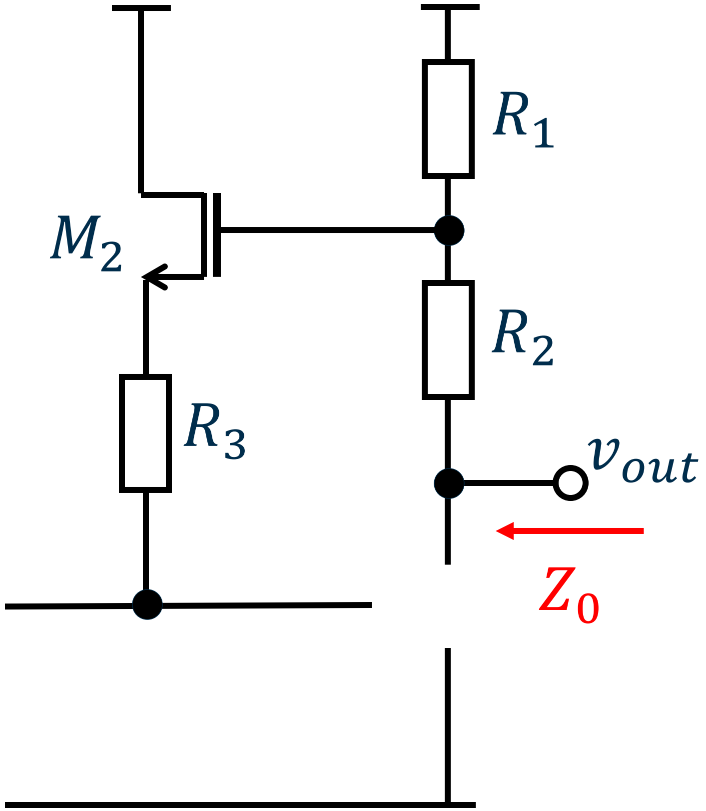
$$Z = Z_0 \frac{1 + T_{short}}{1 + T_{open}}$$

$$Z = \left(R_3 + \frac{1}{g_{m2}}\right) \frac{1}{1 + g_{m1}R_1}$$

$$Z_{in} \approx \frac{1}{g_m}\left(\frac{R_3}{R_1}\right)$$

Advanced Example: Cherry-Hooper Amplifier

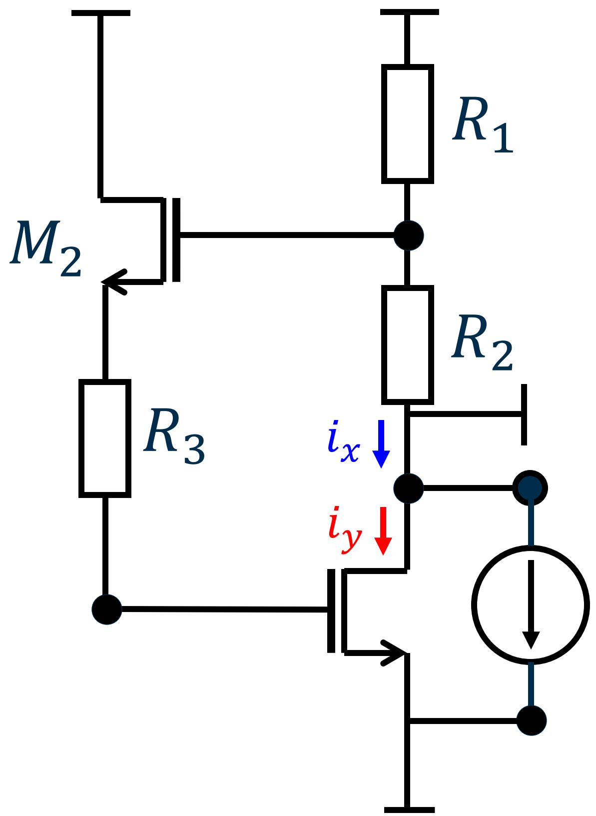
- Output Impedance



$$Z_0 = R_2 + R_1$$

Advanced Example: Cherry-Hooper Amplifier

- T_{short}



$$\boldsymbol{T_{short} = 0}$$

$$\boldsymbol{T_{open} = T = -\frac{i_y}{i_x} = g_{m1}R_1}$$

$$Z = Z_0 \frac{1 + T_{short}}{1 + T_{open}}$$

$$Z_{out} = \frac{R_1 + R_2}{1 + g_{m1}R_1}$$