Eksamensoppgave 2 - INF5620

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December 7, 2013

1 Oppgavetekst

a

Set up a wave equation problem in 2D with zero normal derivative as boundary condition. Assume a variable wave velocity.

Mention a physical problem where this mathematical model arises. Explain the physical interpretation of the unknown function.

b

Present a finite difference discretization. Explain in particular how the boundary conditions and the initial conditions are incorporated in the scheme.

c

Explain (in princple) how the 2D discretization can be extended to 3D.

d

Set up the stability condition in 3D. Also quote results on about accuracy of the method in 3D and define the accuracy measure(s) precisely.

e

Explain how you can verify the implementation of the method.

f

The scheme for the wave equation is perfect for parallel computing. Why? What are the principal ideas behind a parallel version of the scheme?

2 Bølgelikningen på grunt vann i 2D

$$\frac{\partial^2}{\partial t^2}u(t,x,y) = \frac{\partial}{\partial x}\left(q(x,y)\frac{\partial}{\partial x}u\right) + \frac{\partial}{\partial y}\left(q(x,y)\frac{\partial}{\partial y}u\right) \tag{1}$$

Denne likningen oppstår ved modellering av tsunamier. Den ukjente funksjonen u er vannivået relativt til gjennomsnittlig vannivå. $q(x,y) = v(x,y)^2$ der v er bølgehastigheten.

3 Finite difference disktetisering

Her velger jeg å bruke tilfellet at q er konstant, for å få litt mindre å skrive.