5.1 $p_x(x) = \frac{1}{2b} e^{-\frac{|x-\mu|}{b}}$

$$\mathrm{cdf}_{p_x}(x) = F_x(x) = \int_{-\infty}^{x} p_x(x')\,dx'$$

for $x < \mu$: $\int_{-\infty}^{x} \frac{1}{2b} e^{\frac{x'-\mu}{b}} dx' = \left[\frac{1}{2} e^{\frac{x'-\mu}{b}}\right]_{-\infty}^{x}$

$$= \frac{1}{2} e^{\frac{x-\mu}{b}}$$

for $x \geq \mu$: take solution for $x < \mu$ and add:

$$\int_{\mu}^{x} \frac{1}{2b} e^{-\frac{x'-\mu}{b}} dx' =$$

$\left[-\frac{1}{2} e^{-\frac{x'-\mu}{b}}\right]_{\mu}^{x} = -\frac{1}{2} e^{-\frac{x-\mu}{b}} + \frac{1}{2}$ $\Big|\ +\frac{1}{2}$ from the solution for $x<\mu$ setting $x=\mu$

$$= 1 - \frac{1}{2} e^{-\frac{x-\mu}{b}}$$

$$\Rightarrow F(x) = \frac{1}{2} + \frac{1}{2}\,\mathrm{sign}(x-\mu)\left(1 - e^{-\frac{|x-\mu|}{b}}\right)$$

to get F^{-1}: $F(F^{-1}(u)) = u$

for $u < 0.5$ ($\cancel{x} < \mu$): $x = \mu + b\ln(2u)$

for $u \geq 0.5$: $x = \mu - b\ln(2(1-u))$

together: $x = \mu - \mathrm{sign}(u-0.5)\, b \ln\left(1 - 2|u-0.5|\right)$

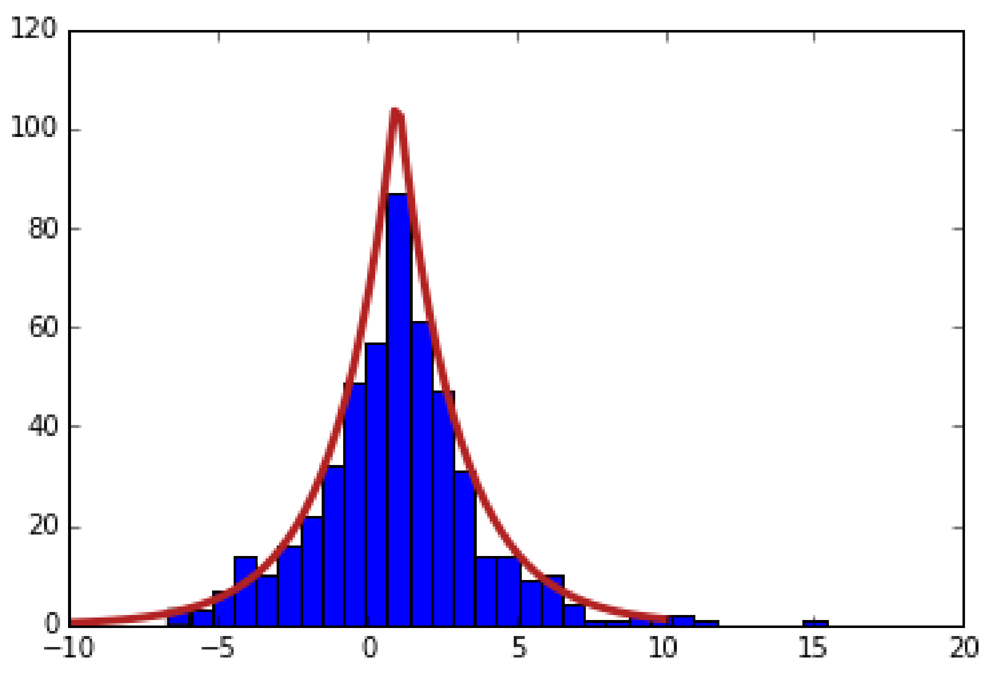
5.1

June 1, 2016

```
In [2]: import numpy as np
        import matplotlib.pyplot as plt
        % matplotlib inline

In [3]: def Finv(x, mu, b):
            return mu - b * np.sign(x - .5)*np.log(1 - 2*np.absolute(x - .5))
        def Fpdf(x, mu, b):
            return 1/(2*b)*np.exp(-np.absolute(x-mu)/b)

In [15]: samples = Finv(np.random.random(500), 1, 2)
         numbins = 30
         plt.hist(samples, numbins)
         xs = np.linspace(-10, 10, 100)
         plt.plot(xs, Fpdf(xs, 1, 2)*((26*len(samples))/numbins), color="firebrick", linewidth=3)
         plt.show()
```

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In [ ]:
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5.2 a) $p_x(x) = e^{-x}$; $u(x) = e^{-x}$

$\Rightarrow x(u) = -\ln(u)$, $f(x(u)) = u$

$$\left|\det\left(\frac{\partial \tilde{x}}{\partial \tilde{u}}\right)\right| = \frac{\partial x(u)}{\partial u} = \left|-\frac{1}{u}\right|$$

$$f(x(u))\left|\det\left(\frac{\partial \bar{x}}{\partial \bar{u}}\right)\right| = 1$$

5.2) $u_1(x_1,x_2) = \sqrt{-2\ln(x_1)}\cos(2\pi x_2)$

$u_2(x_1,x_2) = \sqrt{-2\ln(x_1)}\sin(2\pi x_2)$

Use both equations to obtain $\bar{x}(\bar{u})$

$$u_1^2 + u_2^2 = 2\ln(x_1)(\sin^2 + \cos^2) \Rightarrow x_1 = e^{-\frac{1}{2}(u_1^2+u_2^2)}$$

$$\frac{u_2}{u_1} = \tan(2\pi x_2) \Rightarrow x_2 = \frac{1}{2\pi}\tan^{-1}\left(\frac{u_2}{u_1}\right)$$

The Jacobi matrix of this transformation is:

$$\begin{pmatrix} \frac{\partial x_1}{\partial u_1} & \frac{\partial x_1}{\partial u_2} \\ \frac{\partial x_2}{\partial u_1} & \frac{\partial x_2}{\partial u_2} \end{pmatrix} = \begin{pmatrix} -u_1 e^{-\frac{1}{2}(u_1^2+u_2^2)} & -u_2 e^{-\frac{1}{2}(u_1^2+u_2^2)} \\ -\frac{u_2}{2\pi(u_1^2+u_2^2)} & \frac{u_1}{2\pi(u_1^2+u_2^2)} \end{pmatrix}$$

with $\det\left(\frac{\partial \bar{x}}{\partial \bar{u}}\right) = -\frac{u_1^2+u_2^2}{2\pi(u_1^2+u_2^2)} e^{-\frac{1}{2}(u_1^2+u_2^2)}$

with $0 \le x_1, x_2 \le 1$ $f(\bar{x}(\bar{u})) = 1$

$$\Rightarrow p_{u(x)}(u) = \frac{1}{2\pi}e^{-\frac{1}{2}(u_1^2+u_2^2)} = \frac{1}{2\pi}e^{-\frac{u_1^2}{2}}e^{-\frac{u_2^2}{2}} = p_{gauss}(u_1)\cdot p_{gauss}(u_2)$$

with $\mu = 0$ and $\sigma^2 = 1$