

$$5.1 \quad p_x(x) = \frac{1}{2b} e^{-\frac{|x-\mu|}{b}}$$

$$cdf_{p_x}(x) = F_x(x) = \int_{-\infty}^x p_x(x') dx'$$

$$\begin{aligned} \text{for } x < \mu: \int_{-\infty}^x \frac{1}{2b} e^{\frac{x'-\mu}{b}} dx' &= \left[\frac{1}{2} e^{\frac{x'-\mu}{b}} \right]_{-\infty}^x \\ &= \frac{1}{2} e^{\frac{x-\mu}{b}} \end{aligned}$$

for $x \geq \mu$: take solution for $x < \mu$ and add:

$$\begin{aligned} \int_{\mu}^x \frac{1}{2b} e^{-\frac{x'-\mu}{b}} dx' &= \left[-\frac{1}{2} e^{-\frac{x'-\mu}{b}} \right]_{\mu}^x = -\frac{1}{2} e^{-\frac{x-\mu}{b}} + \frac{1}{2} \\ &= 1 - \frac{1}{2} e^{-\frac{x-\mu}{b}} \end{aligned} \quad \left| \begin{array}{l} \text{from the} \\ \text{solution} \\ \text{for } x < \mu \\ \text{setting } x = \mu \end{array} \right. + \frac{1}{2}$$

$$\Rightarrow F(x) = \frac{1}{2} + \frac{1}{2} \text{sign}(x-\mu) \left(1 - e^{-\frac{|x-\mu|}{b}} \right)$$

$$\text{to get } F^{-1}: F(F^{-1}(u)) = u$$

$$\text{for } u < 0.5 \quad (x < \mu): x = \mu + b \ln(2u)$$

$$\text{for } u \geq 0.5: x = \mu - b \ln(2(1-u))$$

$$\text{together: } x = \mu - \text{sign}(u-0.5) b \ln(1-2|u-0.5|)$$

$$5.2 \quad a) \quad p_x(x) = e^{-x}; \quad u(x) = e^{-x}$$

$$\Rightarrow x(u) = -\ln(u), \quad f(x(u)) = u$$

$$\left| \det \left(\frac{\partial \bar{x}}{\partial \bar{u}} \right) \right| = \frac{\partial x(u)}{\partial u} = \left| -\frac{1}{u} \right|$$

$$f(x(u)) \left| \det \left(\frac{\partial \bar{x}}{\partial \bar{u}} \right) \right| = 1$$

$$5.2) \quad u_1(x_1, x_2) = \sqrt{2 \ln(x_1)} \cos(2\pi x_2)$$

$$u_2(x_1, x_2) = \sqrt{2 \ln(x_1)} \sin(2\pi x_2)$$

Use both equations to obtain $\bar{x}(\bar{u})$

$$u_1^2 + u_2^2 = 2 \ln(x_1) (\sin^2 + \cos^2) \Rightarrow x_1 = e^{-\frac{1}{2}(u_1^2 + u_2^2)}$$

$$\frac{u_2}{u_1} = \tan(2\pi x_2) \Rightarrow x_2 = \frac{1}{2\pi} \tan^{-1}\left(\frac{u_2}{u_1}\right)$$

The Jacobst matrix of this transformation is:

$$\begin{pmatrix} \frac{\partial x_1}{\partial u_1} & \frac{\partial x_1}{\partial u_2} \\ \frac{\partial x_2}{\partial u_1} & \frac{\partial x_2}{\partial u_2} \end{pmatrix} = \begin{pmatrix} -u_1 e^{-\frac{1}{2}(u_1^2 + u_2^2)} & -u_2 e^{-\frac{1}{2}(u_1^2 + u_2^2)} \\ -\frac{u_2}{2\pi(u_1^2 + u_2^2)} & \frac{u_1}{2\pi(u_1^2 + u_2^2)} \end{pmatrix}$$

$$\text{with } \det \left(\frac{\partial \bar{x}}{\partial \bar{u}} \right) = -\frac{u_1^2 + u_2^2}{2\pi(u_1^2 + u_2^2)} e^{-\frac{1}{2}(u_1^2 + u_2^2)}$$

$$\text{with } 0 \leq x_1, x_2 \leq 1 \quad f(\bar{x}(\bar{u})) = 1$$

$$\Rightarrow p_{u(u)}(u) = \frac{1}{2\pi} e^{-\frac{1}{2}(u_1^2 + u_2^2)} = \frac{1}{2\pi} e^{-\frac{u_1^2}{2}} e^{-\frac{u_2^2}{2}} = p_{\text{gauss}}(u_1) \cdot p_{\text{gauss}}(u_2)$$

with $\mu = 0$ and $\sigma^2 = 1$