5.2.b)

Interval $[0,1]$

$X := \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$ $\quad pdf := P_X(X_1, X_2) = 1$ in $[0,1]^2$
0 otherwise

$u - u(X)$ $\quad u_1(X) = \sqrt{-2\log(X_1)}\cos(2\pi X_2)$
$\quad u_2(X) = \sqrt{-2\log(X_1)}\sin(2\pi X_2)$

first ~~...~~ build the inverse $X(u)$

x_1: Phytagorian trigonometric identity

$$u_1^2 + u_2^2 = (-2\log(x_1)) \cdot (\sin^2(2\pi X_2) + (\cos^2(2\pi X_2)))$$
$$u_1^2 + u_2^2 = -2\log(x_1)$$
$$-\frac{1}{2}(u_1^2+u_2^2) = \log(x_1)$$
$$x_1(u_1, u_2) = e^{-\frac{1}{2}(u_1^2+u_2^2)}$$

then X_2:

$$\frac{u_2}{u_1} = \tan(2\pi X_2)$$
$$X_2(u_1, u_2) = \frac{1}{2\pi} \cdot \tan^{-1}\left(\frac{u_2}{u_1}\right)$$

Derive the inverse of $\frac{dx(u)}{du}$

$$\frac{dx(u)}{du} = \begin{pmatrix} \frac{dx_1}{du_1} & \frac{dx_1}{du_2} \\ \frac{dx_2}{du_1} & \frac{dx_2}{du_2} \end{pmatrix}$$

$$= \begin{pmatrix} e^{-\frac{1}{2}(u_1^2+u_2^2)} u_1 & e^{-\frac{1}{2}(u_1^2+u_2^2)} \cdot (-u_2) \\ \frac{u_2}{2\pi(u_1+u_2)^2} & \frac{u_1}{2\pi(u_1+u_2)^2} \end{pmatrix}$$