Numbers and Computers (practice)

ICS312
Machine-Level and
Systems Programming

Henri Casanova (henric@hawaii.edu)

(q1) Conversion

- What is 52_{10} in binary?

(q1) Solution

- What is 52_{10} in binary?
- Systematic method:
  - 52 = 26*2 + 0
  - 26 = 13*2 + 0
  - 13 = 6*2 + 1
  - 6 = 3*2 + 0
  - 3 = 1*2 + 1
  - 1 = 0*2 + 1
  - Answer: 110100
- Intuitive method (for “small” numbers)
  - 52 is lower than 64, so it’s 32 + something
  - 32 + 16 is 48, so 52 is 32 + 16 + something
  - 52 - 48 = 4, so: 52 = 32 + 16 + 4
  - Therefore: 110100
    - We have 32, 16, not 8, 4, not 2, not 1

(q2) Conversion

- What is 2049_{10} in binary?

(q2) solution

- What is 2049_{10} in binary?
- The systematic method is really long here
  - simple though, but tedious
- It's easier to see that 2049_{10} is $2048_{10} + 1_{10}$
  - 2048_{10} is $2^{11} = 100000000000_2$
  - 1 is $2^0 = 1_2$
- Therefore
  - $2049_{10} = 100000000001_2$

- In general, one likes to find "nearby" powers of 2

(q3) Conversion

- What is 1021_{10} in binary?

(q3) Solution

- What is 1021_{10} in binary?
- 1021_{10} is "close to" 1024_{10}
- We know that 1024_{10} is 1000000000_2
- More useful: 1023_{10} is 111111111_2
- So we can "count backwards"
- 111111110_2 comes before 111111111_2, and therefore it is 1022_{10}
- 111111101_2 comes before 111111110_2, and therefore it is 1021_{10}
- Answer: 111111101_2

(q4) Conversions

- What is $B8_{16}$ in binary?

(q4) Solution

- Just “glue” the 2 4-bit conversions together
  - $B_{16} = 1011_2$, $8_{16} = 1000_2$
  - Answer: 10111000

- How do I know that $B_{16} = 1011_2$?
  - Just go back to decimal
    - $B_{16} = 11_{10}$
    - $11_{10} = 1011_2$

(q5) Conversions

- What is 51_{10} in hexadecimal?

(q5) Solution

- What is 51_{10} in hexadecimal?
  - 51 = 3*16 + 3
  - Answer: 33

(q6) Conversion

- What is 0110_2 in hexadecimal?

(q6) Solution

- What is 0110_2 in hexadecimal?
  - $0110_2 = 6_{10} = 6_{16}$
  - Answer: 6

More Conversions

- (q7) What is 123_{10} in binary?
- (q8) What is $F3EA_{16}$ in binary?
- (q9) What is 111_{10} in hexadecimal?
- (q10) What is 100110_2 in hexadecimal?

Solutions

- Conversions:
  - (q7) What is 123_{10} in binary?

    1111011 (127 - 4)
  - (q8) What is $F3EA_{16}$ in binary?

    1111001111101010
  - (q9) What is 111_{10} in hexadecimal?

    6F (112 -1)
  - (q10) What is 100110_2 in hexadecimal?

    26

  Always try to find simple “tricks” if you can

(q11) Binary addition

- What is: 10101101 + 11001011 ?

(q11) Solution

- What is: 10101101 + 11001011 ?

```
  c    cccc
   10101101
 + 11001011
= 101111000
```

(q12) Hex addition

- What is: A5F + E32 ?

(q12) Solution

- What is: A5F + E32 ?

```
      c  c
        A5F
+       E32
=      1891
```

- Small “trick”: adding F to a digit takes that digit 1 lower and generates a carry
  - F + 7 = 6 and a carry
  - F + E = D and a carry

Another binary addition (no poll)

- What is 1010111 + 1110111?

Solution

- What is 1010111 + 1110111?

```
    ccc ccc
     1010111
+    1110111
=   11001110
```

Another hex addition (no poll)

- What is AF3F + EE8D?

Solution

- What is AF3F + EE8D?

```
    c c
    AF3F
+   EE8D
= 19DCC
```

(q13) Two's complement

- What is the 2's complement 2-byte representation of signed integer -153_{10} in hexadecimal?

(q13) Solution

- What is the 2's complement 2-byte representation of signed integer -153_{10} in hexadecimal?
  - $153_{10} = 0099_{16}$
  - complement: FF66
  - add 1 to get the answer: **FF67**

(q14) Two's complement

- What is the 2's complement 2-byte representation of signed integer 96_{10} in hexadecimal?

(q14) Solution

- What is the 2's complement 2-byte representation of signed integer 96_{10} in hexadecimal?
  - $96_{10} = 60_{16}$
  - It's a positive number, so its 2's complement representation is simply it's 2-byte binary representation
  - answer: **0060**

(q15) Two's complement

- What is the decimal value of FF4A, a 2-byte (hex) signed number stored in 2's complement fashion?

(q15) Solution

- What is the decimal value of FF4A, a 2-byte (hex) signed number stored in 2's complement fashion?
  - FF4A = $1\ldots._2$
  - Therefore it represents a negative number, let's invert it
  - Invert: 00B5
  - Add 1: 00B6 = B6
  - $B6_{16} = 11*16 + 6 = 176 + 6 = 182_{10}$
  - Therefore, in 2's complement representation, FF4A is -182_{10}

(q16) Two's complement

- What is the 2's complement 1-byte representation of signed number -81_{10} in hexadecimal?

(q16) Solution

- What is the 2's complement 1-byte representation of signed number -81_{10} in hexadecimal?
  - $81_{10} = 51_{16}$
  - complement: AE
  - add 1: AF

(q17) Two's complement

- What is the decimal value of 76 (hex), a 1-byte signed number stored in 2's complement fashion?

(q17) Solution

- What is the decimal value of 76h, a 1-byte signed number stored in 2's complement fashion?
  - It's a positive number, so 76 is simply the hex value of the integer
  - Answer: $7*16^1 + 6*16^0 = 118_{10}$

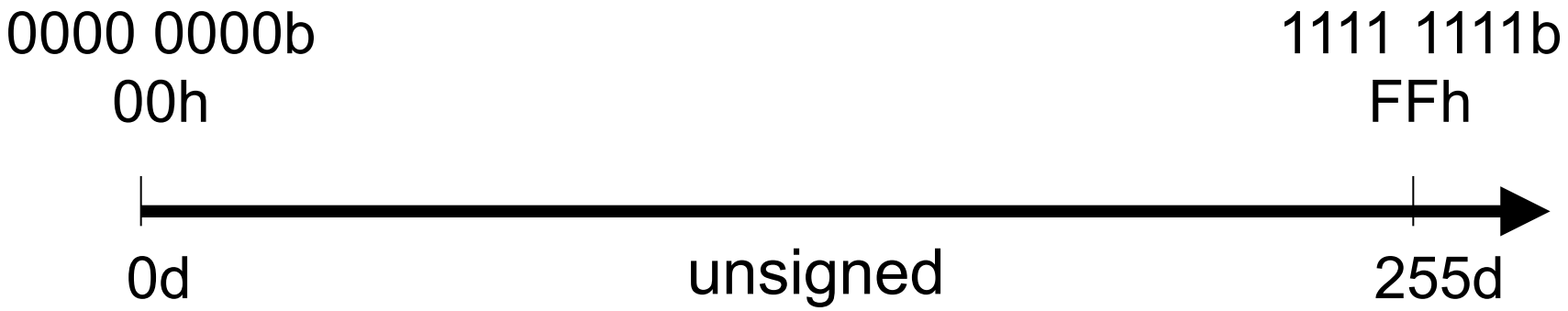
(q18) Ranges of numbers

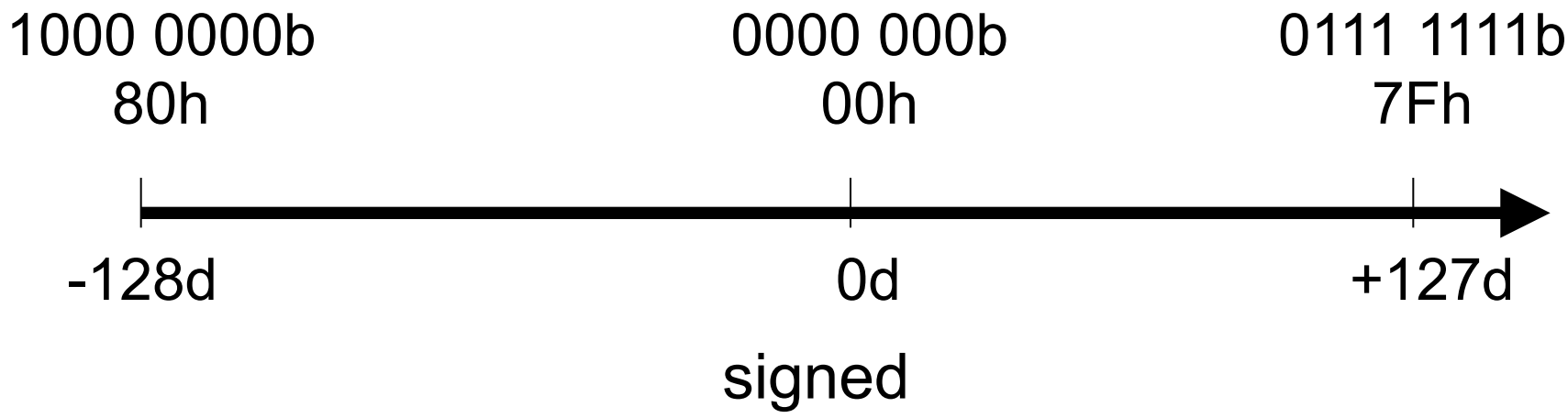
- What is the largest **unsigned** decimal number that can be encoded with 8 bits?
- What is the smallest **unsigned** decimal number that can be encoded with 8 bits?
- What is the largest **signed** decimal number that can be encoded with 8 bits?
- What is the smallest **signed** decimal number that can be encoded with 8 bits?
- What is the 2's complement representation of -1_{10} with 32 bits?

(q18) Solutions

- What is the largest **unsigned** decimal number that can be encoded with 8 bits?
  - 255 (i.e., FF in 2's complement representation)
- What is the smallest **unsigned** decimal number that can be encoded with 8 bits?
  - 0 (i.e., 00 in 2's complement representation)
- What is the largest **signed** decimal number that can be encoded with 8 bits?
  - Largest that isn't negative: 7F in 2's complement representation = 127_{10}
- What is the smallest **signed** decimal number that can be encoded with 8 bits?
  - Smallest that isn't positive: 80 in 2's complement representation = -128_{10}
- What is the 2's complement representation of -1_{10} with 32 bits?
  - 1 = 00000001; complement: FFFFFFFE; add one: FFFFFFFF

1-byte Ranges





Conclusion

- I have posted a practice quiz on Laulima (without answers). This is just some extra material you can use to practice.
- Homework #1 is all about this, let's look at it

- Yes, this is a lot of number arithmetic practice
- But as Computer Scientists you cannot afford to not be extremely comfortable with all this
  - No, you won't use it everyday at all
  - But when you need to use it, not being good at it is really, really embarrassing
    - Especially since these things pop up at job interviews!