

Applying Panel Regression in the Structural Equation Modeling Framework to Assess Relationships between Environmental Attitudes, Behavioural Intentions and Behaviours

Henrik K. Andersen Jochen Mayerl

Technische Universität Chemnitz

March 18, 2021



TECHNISCHE UNIVERSITÄT
CHEMNITZ

Introduction

Background

Modeling Strategy

Methodological issues

Analysis

Conclusion

References

Introduction

Introduction I

- ▶ When the Coronavirus pandemic has subsided, **climate change** will likely return as a central pressing issue
- ▶ Central to fighting climate change and avoiding climate-related crises is **changing individual behaviour**
- ▶ Some (mixed) evidence for environmental attitudes affecting pro-environmental behaviour
- ▶ Most studies correlational in nature, a whole range of theoretical and methodological challenges for investigating this issue
- ▶ Question of **whether attitudes causally affect behaviour** is not sufficiently answered

Introduction II

- ▶ We investigate this question in a **longitudinal design**
 - ▶ Allows us to control for **unobserved time-invariant confounders**
 - ▶ Introducing temporal lags lets us investigate **causal predominance**
- ▶ Structural equation modeling (SEM) has advantages in for this topic
 - ▶ Attitudes and (to a lesser extent) behaviour are difficult to observe/measure with survey data
 - ▶ SEM lets us account for **measurement error** and gets us closer to underlying constructs of interest
 - ▶ **Range of very flexible models:** fixed (FE) and random effects (RE), latent curve models (LCM), cross-lagged panel models (CLPM), dynamic panel models (DPM)
 - ▶ Lets us build models according to theory with (relative) ease

Introduction III

- ▶ We use the topic of environmental attitudes and behaviour to look at some current methodological topics/debates in SEM
- ▶ Focus on one in particular:
 - ▶ Fixed and random effects models with time-varying covariates: do we specify the effects of interest at the **observation/construct level**, or at the **residual-level** as advocated by the very popular Random-Intercept Cross-Lagged Panel Model (RI-CLPM) and Latent Curve Model with Structured Residuals (LCM-SR)
 - ▶ Also touch on measurement invariance, model fit, categorical indicators, etc.
- ▶ Show some preliminary findings on whether attitudes “cause” behaviours at the intra- or within-individual level

Background

Background I

- ▶ Fighting climate change depends on individuals modifying their behaviour (Barr 2007), e.g., consuming less meat, driving less, buying regional and seasonal products, installing and buying more energy efficient home components, paying higher taxes, etc.
- ▶ **But how do we modify behaviour?**
- ▶ Two main explanations:
 - ▶ raising awareness (cognitive, normative), i.e., **changing attitudes**
 - ▶ providing behavioural alternatives, i.e., situational or **low-cost hypothesis** (Barr 2007; Best and Kneip 2011)
- ▶ Likely both play a role and even interact with each other (Best and Kneip 2011)

Background II

- ▶ We focus on **environmental attitudes** using observational (longitudinal) data and whether positive attitudes lead to more environmentally friendly behaviour
- ▶ Environmental attitudes can be described as the **overall evaluation of the environment** or state of environment as well as **enduring emotional or cognitive processes** associated with the environment (Eilam and Trop 2012)

Background III

Why re-examine this topic?

- ▶ Unresolved questions:
 - ▶ **do changes in one's attitudes *causally* affect one's behaviour?**
 - ▶ **or are associations between attitudes and behaviour set by stable between-person differences?**
- ▶ If the former is true, then attitudes and behaviour can potentially be changed via policy aimed at attitude change
- ▶ If the latter is true, then behaviour is set by relatively stable personality-/values-/milieu-related unobservables. Changing attitudes without changing underlying stable characteristics will not change behaviour

Background IV

- ▶ So, if attitudes affect behaviour, then policy should focus on informing the public
- ▶ However, evidence for the environmental attitude-behaviour link is mixed, e.g., **heterogeneous effects** depending on the **type of behaviour** (e.g., recycling vs. transport vs. willingness to pay)
- ▶ Also, studying the relationship is fraught with **unobserved confounders** (Barr 2007), e.g., psychological factors, intrinsic motivation, income, education, etc. and most studies are **cross-sectional and observational**

Background V

- ▶ It is not even clear from the literature whether **attitudes cause behaviour or behaviour may actually cause attitudes** (Eilam and Trop 2012)
- ▶ Potentially a mix of both
- ▶ Due to **cognitive dissonance**, when one's behaviour changes (initially unrelated to an attitudinal change) attitudes may fall in line to conform with behaviour
- ▶ Cross-lagged models or, more generally, models that account for **reverse causality** allow us to investigate causal predominance, i.e., does x lead to y or does y lead to x ?

Background VI

- ▶ Besides reverse causality, it could be that previous behaviour also affects future behaviour, i.e., the **behavioural snowball** effect (Barr 2007)

Modeling Strategy

Modeling strategy I

- ▶ Based on these points, we would like to apply **panel regression models** with
 - ▶ fixed effects to account for **unobserved time-invariant confounders**
 - ▶ autoregressive effects to account for the so-called **snowball effect**
 - ▶ reverse causality to investigate **causal predominance** or rather **direction of causation**
 - ▶ multi-indicator measurement models to account for **measurement error** in the assumed independent variables

Modeling strategy II

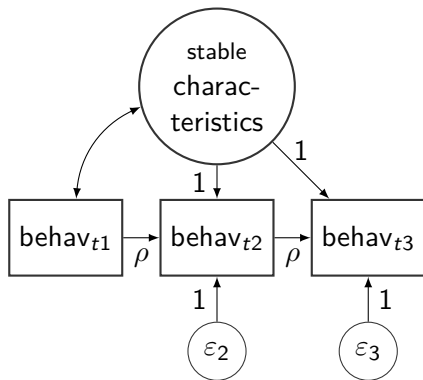


Figure 1: xxx

Methodological issues

Methodological issues I

The kitchen sink:

- ▶ Observation- or residual-level model?
- ▶ One- or two-sided models?
- ▶ Categorical measurement models and measurement invariance
- ▶ The role of fit?
- ▶ Reflective or formative indicators
- ▶ Missing values

Methodological issues II

The kitchen sink:

- ▶ **Observation- or residual-level model?**
- ▶ One- or two-sided models?
- ▶ Categorical measurement models and measurement invariance
- ▶ The role of fit?
- ▶ Reflective or formative indicators
- ▶ Missing values

Observation- vs. residual-level models I

- ▶ In the context of modeling reciprocal relations longitudinally, the RI-CLPM (Hamaker, Kuiper, and Grasman 2015) has arguably become the 'gold standard'.
- ▶ According to [Google Scholar](#), currently just under 900 citations in six years. In comparison, fundamental texts like Bollen and Curran (2004): "[ALT Models. A Synthesis of Two Traditions](#)" Curran and Bollen (2001): "[The best of both worlds](#)" each have only 400–500 after about 20 years.
- ▶ The RI-CLPM and, similarly, the LCM-SR from Curran et al. (2014), are advertised as providing benefits compared to the more conventional (bivariate) ALT models outlined in the articles above by Bollen & Curran.
- ▶ **Which one should we use?**

Observation- vs. residual-level models II

Some background:

- ▶ The DPM and RI-CLPM take **unobserved heterogeneity in terms of level** into account. The ALT and LCM-SR are more general and take stable differences in terms of **both level and trajectory** into account.
- ▶ The RI-CLPM and LCM-SR work by modeling AR and CL effects at the **residual-level**, i.e., what is left over after regressing the observed (or latent) repeated measures on the individual effects.
- ▶ When we fix the regression weights/factor loadings to 1.0 (as we usually do), then we are essentially **demeaning** or **detrending** the data.

Observation- vs. residual-level models III

- ▶ We need to be clear that the benefit of the residual-level models is that the **between-person trajectories can be interpreted more easily**.
- ▶ This is not made clear enough in the source materials (Hamaker, Kuiper, and Grasman 2015; Curran et al. 2014). E.g., “if theory posits that the over-time relation between two constructs consists of a unique between-person and a unique within-person component, an alternative parameterization to the ALT and LCM-TVC¹ is needed” (Curran et al. 2014, 884).

Observation- vs. residual-level models IV

- ▶ But the models we are talking about **always** consist of unique between- and within-person components.

$$y_{it} = \underbrace{\rho y_{it-1} + \beta x_{it-1}}_{\text{within}} + \underbrace{\eta_i}_{\text{between}} + \underbrace{\varepsilon_{it}}_{\text{within}}$$

- ▶ By including the individual effects α_i , the between-variance in the other terms is partialled out.
- ▶ So, aren't the two approaches, at the observation- and at the residual-level, equivalent?

Observation- vs. residual-level models V

For the sake of simplicity, consider an **autoregressive model with individual effects** (heterogeneity on level, not growth). This is like a univariate ALT with no slope factor or a DPM. It allows us to look at the differences and similarities between the RI-CLPM and DPM. The conclusions also apply to the LCM-SR and ALT.

Let us look at such a model with no covariates:

$$y_{it} = \rho y_{it-1} + \eta_{1i} + \nu_{it}, \quad t = 1, \dots, T \quad (1)$$

where we have observations on $t = 0, \dots, T$.

Note that $t = 0$ is the first observation, for which we currently have no equation.

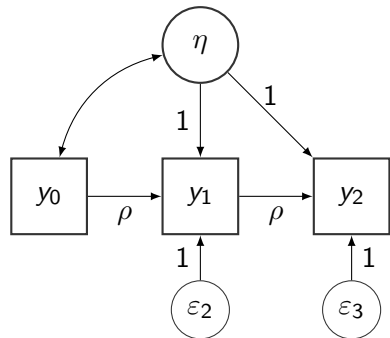
Observation- vs. residual-level models VI

There are two common ways to deal with y_{i0} :

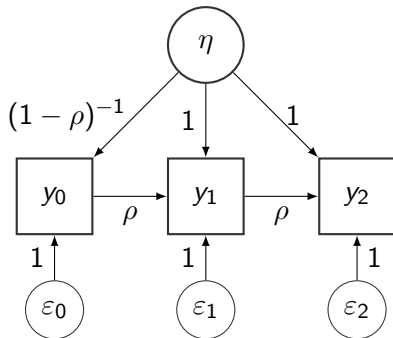
1. Treat it as **predetermined**, i.e., make no assumptions on the process before the observation period began and allow it to covary freely with the individual effects and any other covariates.
2. Assume the same process for $t = 1, \dots, T$ also applies to the previous unobserved periods: this is often referred to as the **constrained** ALT.

The constrained ALT has the benefit of **parsimony** compared to the predetermined ALT. If the assumptions about the process before the observation period began hold, then we can work out a **fixed value for the first factor loading** $\eta_{1i} \rightarrow y_{i0}$ and save a degree of freedom (or several df in specifications with slopes).

Observation- vs. residual-level models VII



(a) Predetermined



(b) Constrained

Figure 2: Simplified three-wave AR(1) models with unobserved heterogeneity

Observation- vs. residual-level models VIII

For the constrained model, we assume the process for $t = 1, \dots, T$ applies to the previous unobserved time points.

$$\begin{aligned}y_{i0} &= \rho y_{i,-1} + \eta_{1i} + \nu_{i0} \\&= \rho(\rho y_{i,-2} + \eta_{1i} + \nu_{i,-1}) + \eta_{1i} + \nu_{i0} \\&= \rho(\rho(\rho y_{i,-3} + \eta_{1i} + \nu_{i,-2}) + \eta_{1i} + \nu_{i,-1}) + \eta_{1i} + \nu_{i0} \\&= \rho^3 y_{i,-3} + \rho^2 \eta_{1i} + \rho \eta_{1i} + \eta_{1i} + \rho^2 \nu_{i,-2} + \rho \nu_{i,-1} + \nu_{i0} \\&= \dots\end{aligned}$$

and keep going with this back to $-\infty$.

Observation- vs. residual-level models IX

This results in

$$y_{i0} = \sum_{j=0}^{\infty} \rho^j \eta_{1i} + \sum_{i=0}^{\infty} \rho^j \nu_{i(t-j)}$$

(Bollen and Curran 2004; Curran and Bollen 2001; Hamaker 2005). If the autoregressive effect ρ is time-constant and less than 1 in absolute value, then the equation converges to

$$y_{i0} = (1 - \rho)^{-1} \eta_{1i} + \varepsilon_{i0} \quad (2)$$

and where ε_{i0} is the weighted sum of the previous residuals (Jongerling and Hamaker 2011).

So we set the first factor loading $\eta_{1i} \rightarrow y_{i0}$ to $(1 - \rho)^{-1}$ in the constrained model.

Observation- vs. residual-level models X

Now look at the residual-level formulation:

$$\begin{aligned}y_{it} &= \alpha_{1i} + \varepsilon_{it}, \quad t = 0, \dots, T \\ \varepsilon_{it} &= \rho \varepsilon_{it-1} + \nu_{it}, \quad t = 1, \dots, T.\end{aligned}$$

Substitute the second equation into the first:

$$y_{it} = \alpha_{1i} + \rho \varepsilon_{it-1} + \nu_{it}, \quad t = 1, \dots, T$$

and then rewrite the residuals in terms of the observed variables:

$$\begin{aligned}y_{it} &= \alpha_{1i} + \rho(y_{it-1} - \alpha_{1i}) + \nu_{it} \\ &= (1 - \rho)\alpha_{1i} + \rho y_{it-1} + \nu_{it}, \quad t = 1, \dots, T.\end{aligned}$$

Observation- vs. residual-level models XI

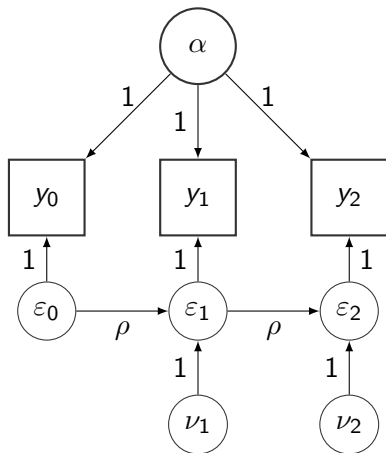


Figure 3: Simplified three-wave residual-level AR(1) model with unobserved heterogeneity

Observation- vs. residual-level models XII

Now, define $\eta_{1i} = (1 - \rho)\alpha_{1i}$ for

$$y_{it} = \rho y_{it-1} + \eta_{1i} + \nu_{it}, \quad t = 1, \dots, T$$

which is the observation-level equation for $t = 1, \dots, T$ from (1). By rearranging, we know $\alpha_{1i} = (1 - \rho)^{-1}\eta_{1i}$, so for $t = 0$, we have

$$\begin{aligned} y_{i0} &= \alpha_{1i} + \varepsilon_{i0} \\ &= (1 - \rho)^{-1}\eta_{1i} + \nu_{i0} \end{aligned}$$

(since without $\varepsilon_{i,-1}$, $\varepsilon_{i0} = \nu_{i0}$). This is the equation for y_{i0} from the constrained observation-level model shown in Equation (2).

Observation- vs. residual-level models XIII

- ▶ **This shows that we can rewrite the residual-level model as a *constrained* observation-level model.**
- ▶ The residual-level models are thus equivalent to their constrained observation-level counterparts in terms of estimated AR and CL parameters, degrees of freedom and model fit.
- ▶ The individual effects in the constrained model are scaled versions of their residual-level model counterparts and can be easily recovered.
- ▶ It can be shown easily using real data.

Observation- vs. residual-level models XIV

For example, take three waves of antisocial behaviour in children from the freely available NLSY study (Allison 2009; Center for Human Resource Research 2006).

We model autoregressive effects while accounting for unobserved heterogeneity at the observation- and residual-level.

The variables are `anti90`, `anti92` and `anti94` for antisocial behaviour measures in 1990, 1992 and 1994.

Observation- vs. residual-level models XV

The constrained observation-level model:

```
library(lavaan)

dpm_con1 <- '
# Identify the individual effects
eta1 =~ 1*anti92 + 1*anti94 + a*anti90
# Regressions, time-invariant effect
anti92 ~ rho*anti90
anti94 ~ rho*anti92
# Constraints
a == (1 - rho)^-1
'

dpm_con1.fit <- sem(model = dpm_con1, data = nlsy)
```

Observation- vs. residual-level models XVI

And the residual-level model:

```
riclpm1 <- '  
# Identify the individual effects  
alpha1 =~ 1*anti90 + 1*anti92 + 1*anti94  
# Identify the residuals as new latent variables  
e90 =~ 1*anti90  
e92 =~ 1*anti92  
e94 =~ 1*anti94  
# Set variance of the observed variables to zero  
anti90 ~~ 0*anti90  
anti92 ~~ 0*anti92  
anti94 ~~ 0*anti94  
# Regressions, time-invariant effect  
e92 ~ rho*e90  
e94 ~ rho*e92  
# Constrain covariance alpha1 and e1 to zero (override default)  
alpha1 ~~ 0*e90  
'  
riclpm1.fit <- sem(model = riclpm1, data = nlsy)
```

Observation- vs. residual-level models XVII

		RI-CLPM			Constrained DPM		
		Estimate	Std. Err.	p	Estimate	Std. Err.	p
<u>Regression Slopes</u>							
e92/anti92	←						
	e90/anti90	0.23**	0.08	0.005	0.23**	0.08	0.005
e94/anti94	←						
	e92/anti92	0.23**	0.08	0.005	0.23**	0.08	0.005
<u>(Residual) Variances</u>							
	· e90/· anti90	0.89***	0.10	0.000	0.89***	0.10	0.000
	· e92/· anti92	1.12***	0.12	0.000	1.12***	0.12	0.000
	· e94/· anti94	1.46***	0.11	0.000	1.46***	0.11	0.000
	alpha1/eta1	1.28***	0.13	0.000	0.76***	0.21	0.000
<u>Fit Indices</u>							
	$\chi^2(df)$	0.43(1)		0.512	0.43(1)		0.512
	CFI	1.00			1.00		
	RMSEA	0.00			0.00		

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. A dot (·) indicates a residual variance. **e** stands for the structured residual of **anti**. **alpha1** and **eta1** are the latent intercepts for the RI-CLPM and DPM, respectively.

Figure 4: Antisocial behaviour models: Residual- and constrained observation-level

Observation- vs. residual-level models XVIII

Notice the AR effects (size, standard error, p-value), degrees of freedom and model fit are all identical in both models. These results can be shown to generalize to models with latent slopes, see [this preprint](#) (currently under review), as well as Ou et al. (2016).

The problem is that **there are situations in which the assumptions necessary to derive the constraints for $t = 0$ are not appropriate.**

Observation- vs. residual-level models XIX

Crucially, the constrained observation-level models and thus also the residual-level models assume the covariance between the initial observation and the individual effects is

$$\begin{aligned}\text{Cov}(y_{i0}, \alpha_{1i}) &= \text{Var}(\alpha_{1i}), && \text{or, alternatively} \\ \text{Cov}(y_{i0}, \eta_{1i}) &= (1 - \rho)^{-1} \text{Var}(\eta_{1i}).\end{aligned}$$

However, **it takes time for this covariance to set in**, and it is based on the assumption that **the AR effects are less than 1 in absolute value** and are **constant over time**.

Observation- vs. residual-level models XX

For example, assume the model

$$y_{it} = \rho y_{it-1} + \eta_{1i} + \nu_{it}$$

and for the very first realization (independent of our observations), it is $y_{i0} = \eta_{1i} + \nu_{i0}$.

Observation- vs. residual-level models XXI

```
# Set some parameters
n <- 100000
rho <- 0.5
# Generate the data
eta1 <- rnorm(n, 0, 1)
y0 <- eta1 + rnorm(n, 0, 1)
y1 <- rho*y0 + eta1 + rnorm(n, 0, 1)
y2 <- rho*y1 + eta1 + rnorm(n, 0, 1)
y3 <- rho*y2 + eta1 + rnorm(n, 0, 1)
y4 <- rho*y3 + eta1 + rnorm(n, 0, 1)

df <- data.frame(eta1, y0, y1, y2, y3, y4)
```


Observation- vs. residual-level models XXII

We know the covariance between the observations and individual effects (in the population) converges to $\text{Cov}(y_{it}, \eta_{1i}) = (1 - \rho)^{-1} \text{Var}(\eta_{1i}) = (1 - 0.5)^{-1}(1) = 2$.

```
cov(df)
```

```
##           eta1          y0          y1          y2          y3          y4
## eta1 0.9959133 1.000152 1.497967 1.750683 1.870434 1.923952
## y0    1.0001519 2.007565 2.006544 2.002715 1.997991 1.991031
## y1    1.4979666 2.006544 3.500353 3.259493 3.129705 3.055673
## y2    1.7506827 2.002715 3.259493 4.379583 3.942672 3.710931
## y3    1.8704345 1.997991 3.129705 3.942672 4.840641 4.286977
## y4    1.9239521 1.991031 3.055673 3.710931 4.286977 5.048865
```

After five realizations, the covariance is 1.924. After two realizations, though, it is only 1.498, for example.

Observation- vs. residual-level models XXIII

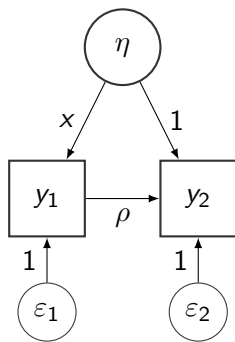


Figure 5: Autoregressive effect depends on factor loadings

Fixing the first factor loading to $x = (1 - \rho)^{-1}$ or, equivalently, running a residual-level model like the RI-CLPM or LCM-SR is only appropriate when

- ▶ the data is stationary,
- ▶ the process has been going on long enough to have reached equilibrium and
- ▶ has not been temporarily 'knocked' out of equilibrium (e.g., by an intervention)

Observation- vs. residual-level models XXIV

Obviously, $\text{Cov}(y_1, \eta) \neq (1 - \rho)^{-1} \text{Var}(\eta)$, but rather $\text{Cov}(y_1, \eta) = \rho \text{Var}(\eta) + \text{Var}(\eta)$. In the simulated example here, the actual covariance is around 1.5, but the constrained model wou

Observation- vs. residual-level models XXV

Here, for example the constrained model overestimates the AR effect substantially:

```
# Run a three-wave constrained ALT (otherwise unidentified)
ex1 <- '
alpha =~ 1*y2 + 1*y3 + a*y1
y3 ~ rho*y2
y2 ~ rho*y1
a == (1 - rho)^-1
'
ex1.fit <- sem(ex1, data = df)
summary(ex1.fit)
```

Observation- vs. residual-level models XXVI

Here, for example the constrained model overestimates the AR effect substantially:

```
...  
## Regressions:  
##           Estimate Std.Err  z-value  P(>|z|)  
##   y3 ~  
##     y2      (rho)    0.730    0.004   194.676    0.000  
##   y2 ~  
##     y1      (rho)    0.730    0.004   194.676    0.000  
...
```

Observation- vs. residual-level models XXVII

Now, the predetermined model:

```
# Run a three-wave predetermined ALT (otherwise unidentified)
ex2 <- '
alpha =~ 1*y2 + 1*y3
y3 ~ rho*y2
y2 ~ rho*y1
alpha ~~ y1
'
ex2.fit <- sem(ex2, data = df)
summary(ex2.fit)
```

Observation- vs. residual-level models XXVIII

Now, the predetermined model:

```
...  
##              Estimate  Std.Err  z-value  P(>|z|)  
##  y3 ~  
##    y2      (rho)    0.539    0.035    15.280    0.000  
##  y2 ~  
##    y1      (rho)    0.539    0.035    15.280    0.000  
##  
...
```

Analysis

Conclusion

Conclusion I

Some issues:

- ▶ Panel models with latent variables and measurement models become very cumbersome to code, especially when
 - ▶ the number of indicators per latent construct is large
 - ▶ the number of waves of observations is large
- ▶ FIML for categorical data **does not seem to be supported in lavaan at the moment**: we lose a large number of observations due to this
- ▶ Modeling sequential exogeneity seems to boost the effects of interest. Why? This was observed in a **preliminary results** of a study of Seifert & Andersen. **Why is this?**

Conclusion II

Some (more) issues:

- ▶ Nothing new, but measurement invariance tends to reduce model fit. Publishing results could be eased by **more awareness for the fact that strict invariance (scalar or threshold) is the default assumption of non-SEM methods.**

References

References I

- Allison, Paul D. 2009. *Fixed Effects Regression Models*. Thousand Oaks: Sage Publications. <https://doi.org/https://dx.doi.org/10.4135/9781412993869>.
- Barr, Stewart. 2007. "Factors Influencing Environmental Attitudes and Behaviors." *Environment and Behavior* 39 (4): 435–73. <https://doi.org/10.1177/0013916505283421>.
- Best, Henning, and Thorsten Kneip. 2011. "The Impact of Attitudes and Behavioral Costs on Environmental Behavior: A Natural Experiment on Household Waste Recycling." *Social Science Research* 40: 917–30.
- Bollen, Kenneth, and Patrick Curran. 2004. "Autoregressive Latent Trajectory (ALT) Models: A Synthesis of Two Traditions." *Sociological Methods and Research* 32 (3): 336–83. <https://doi.org/10.1177/0049124103260222>.

References II

- Center for Human Resource Research. 2006. *NLSY97 Codebook Supplement Main File Round 8*. U.S. Department of Labor. <https://www.nlsinfo.org/usersvc/NLSY97/NLSY97Rnd8CodebookSupplement/maincstoc.htm>.
- Curran, Patrick, and Kenneth Bollen. 2001. "The Best of Both Worlds: Combining Autoregressive and Latent Curve Models." In *New Methods for the Analysis of Change*, edited by L. Collins and A. Sayer, 107–35. Washington, DC: American Psychological Press. <https://doi.org/%20doi:10.1037/10409-004>.
- Curran, Patrick, Andrea Howard, Sierra Bainter, Stephanie Lane, and James McGinley. 2014. "The Separation of Between-Person and Within-Person Components of Individual Change over Time: A Latent Growth Curve Model with Structured Residuals." *Journal of Consulting and Clinical Psychology* 82 (5): 879–94. <https://doi.org/10.1037/a0035297>.
- Eilam, Efrat, and Tamar Trop. 2012. "Environmental Attitudes and Environmental Behavior — Which Is the Horse and Which Is the Cart?" *Sustainability* 4: 2210–46.

References III

- Hamaker, Ellen. 2005. "Conditions for the Equivalence of the Autoregressive Latent Trajectory Model and a Latent Growth Curve Model with Autoregressive Disturbances." *Sociological Methods and Research* 33 (3): 404–16.
<https://doi.org/10.1177/0049124104270220>.
- Hamaker, Ellen, Rebecca Kuiper, and Raoul Grasman. 2015. "A Critique of the Cross-Lagged Panel Model." *Psychological Methods* 20 (1): 102–16.
<https://doi.org/10.1037/a0038889>.
- Jongerling, Joran, and Ellen Hamaker. 2011. "On the Trajectories of the Predetermined ALT Model: What Are We Really Modeling?" *Structural Equation Modeling: A Multidisciplinary Journal* 18 (3): 370–82.
- Ou, Lu, Sy-Miin Chow, Linying Ji, and Peter C. M. Molenaar. 2016. "(Re)evaluating the Implications of the Autoregressive Latent Trajectory Model Through Likelihood Ratio Tests of Its Initial Conditions." *Multivariate Behavioral Research* 52 (2): 178–99. <https://doi.org/https://doi.org/10.1080/00273171.2016.1259980>.