

# Rehabilitating the Lagged Dependent Variable with Structural Equation Modeling

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It has been argued that including the lagged dependent variable in panel models will open up unintended back-door paths and bias the estimates of the causal variable. We show that panel analysis in the structural equation modeling framework can get around this issue. Including the lagged dependent variable has the benefit of closing back-door paths due to unobserved time-varying confounders. We demonstrate this by looking at simulated data in the `lavaan` package for R.

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## 1 Introduction

There is a long tradition in sociology and psychology of using cross-lagged panel models to investigate dynamic processes ([Rogosa 1980](#)) in structural equation modeling (SEM). These

usually look at the lagged bidirectional effects of two variables (cross-lagged paths) while holding the previous values of these variables constant (autoregressive paths).

At the same time, such models have been shown to be biased in the presence of time-invariant unobserved heterogeneity. Effective ways to incorporate time-invariant unobserved heterogeneity into cross-lagged panel models have been around for several decades now (e.g., [Kenneth A. Bollen and Curran 2004](#); [Curran and Bollen 2001](#)) and this topic has experienced renewed interest in the last several years ([Curran et al. 2014](#); [Hamaker, Kuiper, and Grasman 2015](#); [Zyphur, Allison, et al. 2020](#); [Zyphur, Voelkle, et al. 2020](#)).

The basic idea of cross-lagged panel models that account for unobserved time-invariant confounders can also be generalized to ‘unidirectional’ models that focus on the effect of one variable on another while de-emphasizing the question of reverse causality ([Allison, Williams, and Moral-Benito 2017](#); [Moral-Benito, Allison, and Williams 2018](#); [Williams, Allison, and Moral-Benito 2018](#)). We could call these ‘dynamic fixed effects’ models because they all account for both unobserved time-invariant heterogeneity as well as state dependence by including the lagged dependent variable (lagged DV or LDV).

Indeed ([Kühnel and Mays 2018](#)) argue that the inclusion of the LDV make such models more desirable than ‘static fixed effects’ models (which do not include the lagged dependent variable) because they potentially account for confounding by certain time-varying variables, as well.

Still, many are skeptical of the use of such models. Often, the skepticism centers directly on the use of the LDV. Indeed, many articles warn of including the LDV in a panel regression model (e.g., [Achen 2000](#); [Brüderl and Ludwig 2014](#); [Collischon and Eberl 2020](#); [Dafoe 2014, 2015](#); [Foster 2010](#); [Keele and Kelly 2006](#); [Leszczensky and Wolbring 2019](#); [Mouw 2006](#); [Thompson and Pickett 2021](#); [Vaisey and Miles 2017](#); [Walters 2019](#)). Often, these criticisms reference Morgan and Winship ([2014, 111](#), Figure 4.3). There, they show that the inclusion of the LDV may indeed bias the causal effect of interest in the presence of time-invariant unobserved heterogeneity. We are aware of just one methodological article that argues for the use of the LDV, and it does not make reference to the SEM approach ([Wilkins 2017](#)).

In this brief article, we will outline the arguments for and against the inclusion of the LDV and then show that the usual SEM approach to panel modelling is not generally affected by the criticism. We hope to convince readers of the usefulness of the broad class of (cross-lagged) dynamic panel models with fixed effects in SEM.

## 2 Benefits of the LDV

Consider an empirical example by Coleman, Hoffer, and Kilgore ([1982](#)) and outlined in Morgan and Winship ([2014](#)). There, Coleman and colleagues were looking to assess the causal effect of attending a Catholic school as opposed to public school on achievement, as measured by pupils’ test scores. The direct acyclic graph (DAG) shown in Figure [1a](#)

summarizes their hypothesized data generating process (DGP), where black circles represent observed variables and white ones represent unobserved variables.<sup>1</sup>

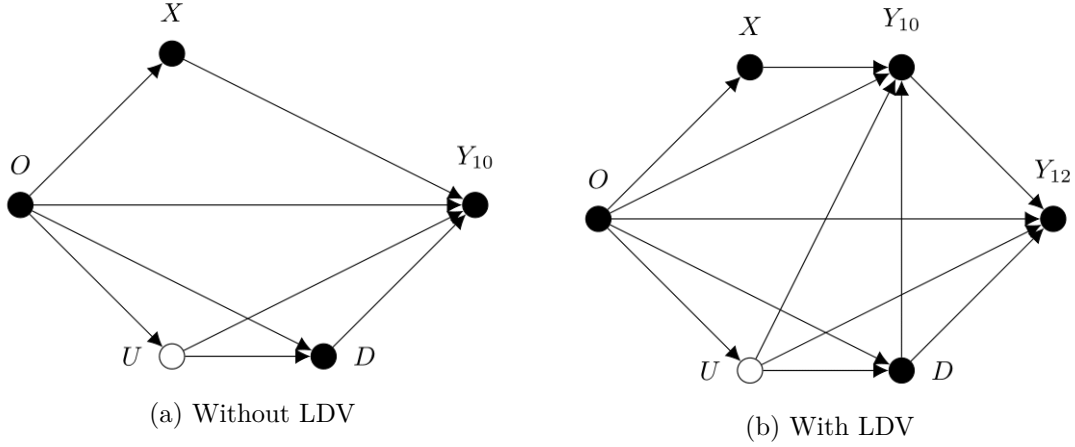


Figure 1: Catholic school example from Coleman, Hoffer, and Kilgore (1982)

In this DAG,  $Y_{10}$  represents the pupil's test score in grade 10,  $X$  are observed determinants of test scores and  $O$  are observed background factors that influence test scores, the determinants of test scores, the selection of school system, as well as the unobserved factors in  $U$ . For its part,  $U$  could contain any number of potentially unmeasured things like motivation or intelligence, that would impact the choice of school system (Catholic schools might prefer to admit intelligent pupils) and the pupil's test scores themselves.

Conditioning on  $X$  and  $O$  would close all back-door paths except  $D \leftarrow U \rightarrow Y_{10}$ . Because  $U$  contains all the unobserved determinants of the causal variable and the outcome, it cannot be conditioned on and the causal effect of interest is unidentified (Morgan and Winship 2014, 270).

Coleman, Hoffer, and Kilgore (1982) came to a solution to this problem by collecting data on the pupils' test scores two years later, in grade 12. By looking at the DAG in Figure 1b, we can see that  $Y_{10}$  "screen[s] off the effects of the variables in  $U$  on  $Y_{12}$ " (Morgan and Winship 2014, 270). Indeed, by focusing on test scores in grade 12 and including the lagged measure from grade 10 in the model, the back-door paths over the unobservables in  $U$  are blocked and the causal effect of  $D$  on  $Y_{12}$  is identified.

Note that  $U$  could potentially include the unmeasured outcome even further in the past. In this way, the inclusion of the lagged dependent variable also accounts for endogeneity, where the causal variable is impacted by previous outcomes. An example given by Wooldridge (2012, 313) concerns the explanation of crime as a function of police expenditure. It is plausible that more is spent on policing in areas where crime rates have been high in the past. Simply regressing the crime rate on expenditure will likely be misleading, because where there is a high crime rate, there will be increased spending, so the effect of spending

<sup>1</sup>Empirical examples chosen for this article were taken from the helpful video tutorial on LDVs by Mikko Rönkkö, <https://www.youtube.com/watch?v=DhV5otUB3Jc>.

on crime may even be positive.<sup>2</sup> Including the lagged crime rate allows for an intuitive interpretation of the effect of expenditure: it is the difference in crime rate between two hypothetical cities with the same crime rate in the previous period given a unit change in expenditure (Wooldridge 2012).

### 3 Arguments Against the LDV

To continue with outlining the issue, let us turn to a simpler DAG as shown in Figure 2, which is adapted also from Morgan and Winship (2014, 111, Fig. 4,3).<sup>3</sup> In this DAG, we have dropped the observed variables captured in  $X$  and  $O$  from above. Since they are observed, we can ignore them from the following discussion without loss of generality. We still have  $D$ , the causal variable of interest, and two measures of the outcome, represented by  $Y_1$  and  $Y_2$ . Consistent with the Catholic school example, the inclusion of  $Y_1$  as an observed predictor of  $Y_2$  closes the back-door path across  $D \leftarrow U \rightarrow Y_1 \rightarrow Y_2$ .

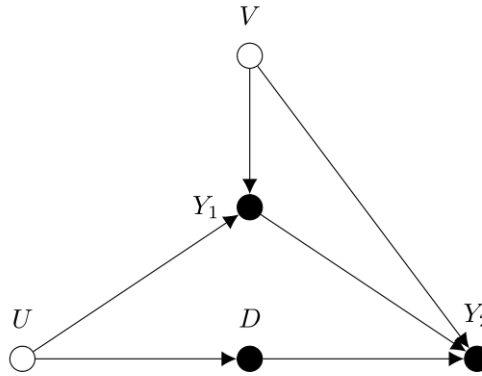


Figure 2: Lagged DV as collider from Morgan and Winship (2014)

The novelty of this DAG compared to the previous ones is the inclusion of  $V$ . It represents the time-invariant unobserved heterogeneity. It is an unobserved variable that affects the outcome at all points in time. It can also be thought of as the combined effect of all the time-invariant factors affecting the outcome. The issue, outlined in Morgan and Winship (2014), is that if  $V$  is unobserved, then  $Y_1$  is also a collider variable and controlling for it therefore opens a new back-door path over  $V$ , rendering the causal effect biased.

This point is often the one criticisms of the LDV focus on. Keele and Kelly (2006, 187) write, for example, that “[e]ven when a lagged dependent variable is theoretically appropriate, remaining residual autocorrelation can lead to biased coefficient results.” Here, the

<sup>2</sup>This is not to justify police expenditure, which is a difficult topic in some parts of the world. It may still be that police spending has a positive causal effect on crime, or that there is no tangible effect. But the estimated effect in a simple regression of current crime rate on current expenditure will likely be biased in one way or another.

<sup>3</sup>The same thing is shown in the variation of Model 7 in Cinelli, Forney, and Pearl (2022), where  $Z$  takes the place of the LDV.

remaining residual autocorrelation is the time-invariant unobserved heterogeneity; the unobserved stable factors that cause the outcome to be correlated with itself over time (Andersen 2022). Foster (2010, 1467) echoes this, stating “[s]uch analyses [that include the LDV] are problematic. As has long been known in economics and other fields, in the presence of autocorrelation (a relationship between unobservables over time), the resulting estimates have poor statistical properties.” Collischon and Eberl (2020, 297) argue against the LDV in a similar fashion.

And criticisms of the LDV continue basically to this day. Thompson and Pickett (2021) write “including lagged dependent variables does not rule out time-stable confounding and may bias estimates” [592]. Even models that attempt to combine fixed effects (accounting for bias by time-invariant unobserved heterogeneity) with the LDV are advised against: “research has shown that [LDV] estimators are in danger of being biased, this applies to linear regression as well as fixed effects environments” (Gattig and Minkus 2021, 129).

The intuition for these arguments can be shown easily. To simplify things, let us assume variables are mean centered and scaled to have a variance of one, as Cinelli, Forney, and Pearl (2022) do. Then, by path tracing, labelling the structural coefficients  $\lambda$ , the covariances of the observed variables,  $Y_2, Y_1, D$  are given by

$$\sigma_{Y_2 D} = \lambda_{DY_2} + \lambda_{UD} \lambda_{UY_1} \lambda_{Y_1 Y_2} \quad (1)$$

$$\sigma_{DY_1} = \lambda_{UD} \lambda_{UY_1} \quad (2)$$

$$\sigma_{Y_2 Y_1} = \lambda_{Y_1 Y_2} + \lambda_{VY_1} \lambda_{VY_2} + \lambda_{UY_1} \lambda_{UD} \lambda_{DY_2}. \quad (3)$$

The partial coefficient of the causal variable,  $D$ , on the outcome,  $Y_2$ , controlling for  $Y_1$ , is given by

$$\beta_{Y_2 D \cdot Y_1} = \frac{\sigma_{Y_2 D} - \sigma_{DY_1} \sigma_{Y_2 Y_1}}{1 - \sigma_{DY_1}^2} \quad (4)$$

(Cinelli, Forney, and Pearl 2022) which, after substitution, works out to

$$\beta_{Y_2 D \cdot Y_1} = \lambda_{DY_2} - \frac{\lambda_{UD} \lambda_{UY_1} \lambda_{VY_1} \lambda_{VY_2}}{1 - (\lambda_{UD} \lambda_{VY_1})^2} \quad (5)$$

which does not equal the structural coefficient  $\lambda_{DY_2}$ , the average causal effect.

This shows the bias resulting from the LDV and Dafoe (2015, 139) suggests that it is therefore only safe to include the LDV when it is not a collider. That is the case when either “there are no unobserved common causes of treatment and the lagged outcome” (if  $U$  were missing from the DAG in Figure 2) or “no unobserved persistent causes of the outcome” (if  $V$  were missing). Partly because of this (along with other theoretical reasons), Brüderl and Ludwig (2014, 342) propose flatly that “LDV models are not useful at all.”

## 4 The Structural Equation Modeling Approach

The key to the SEM approach, and the reason why cross-lagged and other panel models that include the LDV are so widespread in SEM, has to do with the fact that by using latent variables, the LDV does not open an unblocked back-door path. Panel models in SEM that account for time-invariant unobserved heterogeneity normally work by specifying a latent variable that causes the observed outcome at all points in time. This explicitly accounts for the “remaining residual autocorrelation” (Keele and Kelly 2006, 187), or “persistent causes of the outcome” (Dafoe 2015, 139).

In SEM, the time-invariant unobserved heterogeneity is not unobserved in the classical sense. It represents the conditional (on the other observed variables) covariance between the outcome and itself over time (Andersen 2022). Say we had the linear model

$$y_{it} = \mathbf{x}_{it}\boldsymbol{\beta} + \alpha_i + \epsilon_{it}, \quad i = 1, \dots, N, \quad t = 1, \dots, T \quad (6)$$

where  $\mathbf{x}_{it} = (d_{it}, y_{it-1})$  and  $\boldsymbol{\beta} = (\beta, \rho)^\top$ ,  $\alpha_i$  is a latent variable representing the stable factors that change between individuals but not within them, and  $\epsilon_{it}$  is the time-varying error component. Then the covariance of any two columns of the wide-format outcome, conditional on the observed covariates, is just

$$\text{Cov}(y_{it}, y_{is} | \mathbf{x}_{it}) = \text{Var}(\alpha_i), \quad t \neq s \quad (7)$$

since we normally assume  $\text{Cov}(\epsilon_{it}, \epsilon_{is}) = 0$ ,  $t \neq s$  and  $\text{Cov}(\epsilon_{it}, \alpha_i) = 0$ ,  $t = 1, \dots, T$ .

The SEM approach to modelling time-invariant unobserved heterogeneity as a latent variable in dynamic models relies on our ability to decompose the correlations observed between columns of the wide-format outcome into a part that is due to unobserved time-invariant factors, and the autoregressive component. Notice that this means we need at least three observed timepoints of the outcome to do so properly (Kenneth A. Bollen and Curran 2004; Ou et al. 2017). To show this, consider a simplified model with just two observed timepoints where we are trying to estimate separately the autoregressive effect and the stable unobserved factors:

$$y_{i1} = \alpha_i + \epsilon_{i1} \quad (8)$$

$$y_{i2} = \rho y_{i1} + \alpha_i + \epsilon_{i2}. \quad (9)$$

For this, we would need to estimate four parameters,  $\theta = (\phi_\alpha, \psi_1, \psi_2, \rho)$ , where  $\phi_\alpha = \text{Var}(\alpha)$  and  $\psi_t = \text{Var}(\epsilon_t)$ . But we have just three pieces of observed information,  $\text{Var}(y_1), \text{Var}(y_2), \text{Cov}(y_1, y_2)$ . This would mean our model is underidentified with  $-1$  degrees of freedom.

Furthermore, (implicitly) fixing the factor loading of  $\alpha \rightarrow y_1$  to 1.0 assumes the process under investigation is occurring within a vacuum, and that the cumulative effects of the previous realizations of the outcome have no bearing on the initial observation (Ou et al. 2017 call this the ‘history independent endogenous’ model); an unrealistic assumption in most cases. Another option is to assume the process under investigation is ‘ongoing endogenous’ (Ou et al. 2017) and fix the initial factor loading to a specific value (see, for example Andersen 2021), but this does not fix the problem of an underidentified model, either. In fact, in order to avoid making assumptions about the process before the observation period began, it is generally advised to treat the initial observations as ‘predetermined’, i.e., enter it into the model as an exogenous variable, allowing it to covary freely with the individual effects (Allison, Williams, and Moral-Benito 2017; Andersen 2021; Ou et al. 2017). This is the most robust choice but it uses up another degree of freedom in the process.

If we have three measures of the outcome, however, we can treat the initial observation as predetermined and generally have enough degrees of freedom to properly estimate the autoregressive effect as well as the variance of the time-invariant factors affecting the outcome. Doing so requires that we assume the effect of the unobserved stable factors is constant over time (this assumption is reflected in the usual practice of fixing the factor loadings from the latent variable to the outcome to 1.0 at each point in time (Andersen 2022; K. A. Bollen and Brand 2010)), and that the autoregressive effect is constant over time, as well (in other words, there is no moderation by period effects; a plausible assumption in many sociological applications based on observational data). For example, a simple model with three measures of the outcome and the above-mentioned assumptions is just-identified with zero degrees of freedom. The longer the observed timeframe, and the more observed covariates we include in the model, the more degrees of freedom we have to be able to relax these and other assumptions, if necessary.

More on the method for accounting for stable unobserved characteristics in panel models in SEM has been outlined elsewhere (e.g., Allison 2011; Andersen 2022; K. A. Bollen and Brand 2010; Teachman et al. 2001), so we will not describe it in detail here. Instead, we show that by explicitly modeling the time-invariant unobserved heterogeneity, the causal effect of interest can be estimated without bias.

## 5 Some Simulated Examples

We construct a simulated dataset to demonstrate the use of SEM in accounting for time-varying and invariant confounders. We generate three measures of the outcome to be able to treat the initial observation as predetermined.<sup>4</sup> Figure 3 shows the simulated DGP, where we focus on three measures of the outcome over time,  $Y_0 - Y_2$ , along with the causal variable,  $D$ , and account for the unobserved variables  $U$  and  $V$  indirectly.

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<sup>4</sup>We need to treat the initial observation as predetermined in this simulated example because the process will not yet have reached equilibrium (unless the autoregressive effect were to be diminishly small). This means that the ongoing endogenous and history independent endogenous specifications will be inappropriate, see Ou et al. (2017).

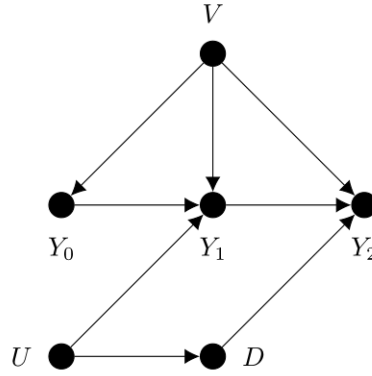


Figure 3: Simulated DGP

We look at linear additive effects. The exogenous variables, as well as the errors, are standard normal. The effect sizes were chosen arbitrarily and can be seen in the code below. The main causal effect of interest,  $D \rightarrow Y_2$  is set to the value 0.4.

```

1  # Set seed
2  set.seed(45678)
3
4  # Load packages
5  library(lavaan)
6  library(dplyr)
7
8  # Set large sample size
9  n <- 1000L
10
11 rho  = 0.3 # Autoregressive effect,  $Y_t \rightarrow Y_{t+1}$ 
12 gamma = 0.6 # Effect  $U \rightarrow D$ 
13 delta = 0.5 # Effect  $U \rightarrow Y_1$ 
14 beta  = 0.4 # Causal effect,  $D \rightarrow Y_2$ 
15
16 # Time-invariant unobserved heterogeneity
17 V = rnorm(n, 0, 1)
18
19 # Simulate initial realization of outcome
20 Y0 = 1 * V + rnorm(n, 0, 1)
21
22 # Time-varying confounder
23 U = rnorm(n, 0, 1)
24
25 # Causal variable
26 D = gamma * U + rnorm(n, 0, 1)
27
28 # Remaining realizations of outcome

```



```

29 Y1 = delta * U + rho * Y0 + 1 * V + rnorm(n, 0, 1)
30 Y2 = beta * D + rho * Y1 + 1 * V + rnorm(n, 0, 1)
31
32 # Put into dataframe
33 df = data.frame(Y0, Y1, Y2, D, V, U)

```

In a narrative fashion, we run a progression of models on this simulated data. First, to establish a baseline, we will act as though all the variables were observed (`m1`). Then, we demonstrate the point Morgan and Winship (2014) and others have made that a model with a LDV is biased in the presence of time-invariant unobserved heterogeneity (`m2`). Finally, we discuss and show the SEM approach to explicitly modelling the time-invariant unobserved heterogeneity as a latent variable and that the causal effect of interest can be estimated without bias (`m3`).

## 5.1 All Observed Variables

Obviously, if both  $U$  and  $V$  were observed, then we could estimate the model without bias. We can estimate the entire model simultaneously in SEM using the `lavaan` package in R. We call this first model `m1`, and the `lavaan` code and summary output is shown directly below.

Note that the first realization of the outcome,  $Y_0$  does not act as a confounder (any backdoor paths across it are blocked), so in this and the next model, we can safely ignore it without biasing the causal effect of interest. In the third model, where we account for the constant factors affecting the outcome as a latent variable, we will need  $Y_0$  to be able to estimate the effect of the stable factors and the autoregressive effect separately.

```

1 m1 = "
2   Y2 ~ beta*D + rho*Y1 + V
3   Y1 ~ delta*U + V
4   D  ~ gamma*U
5   "
6 m1.fit = sem(model = m1, data = df, estimator = "ML") %>%
7   summary()

```

lavaan 0.6-11 ended normally after 1 iterations

Estimator	ML
Optimization method	NLMINB
Number of model parameters	9
Number of observations	1000

Model Test User Model:

Test statistic	0.742
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Degrees of freedom	3
P-value (Chi-square)	0.863

Parameter Estimates:

Standard errors	Standard
Information	Expected
Information saturated (h1) model	Structured

Regressions:

		Estimate	Std.Err	z-value	P(> z )
Y2 ~					
D	(beta)	0.396	0.027	14.608	0.000
Y1	(rho)	0.304	0.028	10.929	0.000
V		1.007	0.048	20.934	0.000
Y1 ~					
U	(delt)	0.431	0.034	12.747	0.000
V		1.321	0.033	39.458	0.000
D ~					
U	(gamm)	0.572	0.033	17.453	0.000

Variances:

	Estimate	Std.Err	z-value	P(> z )
.Y2	0.952	0.043	22.361	0.000
.Y1	1.096	0.049	22.361	0.000
.D	1.026	0.046	22.361	0.000

Here, the causal effect of interest (labeled `beta`) is unbiased at about 0.396 (the slight discrepancy is due to sampling error).

## 5.2 LDV with $V$ Unobserved

Now, to see the point Morgan and Winship (2014) were making, let us assume  $V$  is unobserved. Including the lagged dependent variable will close the back-door path over  $U$  but the bias will still be present because  $V$  is unobserved. We call this second model `m2`. The code and summary output follow.

```

1 m2 = "
2   Y2 ~ beta*D + rho*Y1
3   Y1 ~ delta*U
4   D ~ gamma*U
5   "
6 m2.fit = sem(model = m2, data = df, estimator = "ML") %>%
7   summary()

```

lavaan 0.6-11 ended normally after 2 iterations

Estimator	ML
Optimization method	NLMINB
Number of model parameters	7
Number of observations	1000

Model Test User Model:

Test statistic	22.891
Degrees of freedom	2
P-value (Chi-square)	0.000

Parameter Estimates:

Standard errors	Standard
Information	Expected
Information saturated (h1) model	Structured

Regressions:

		Estimate	Std.Err	z-value	P(> z )
Y2 ~					
D	(beta)	0.317	0.032	9.850	0.000
Y1	(rho)	0.745	0.022	34.444	0.000
Y1 ~					
U	(delt)	0.412	0.054	7.611	0.000
D ~					
U	(gamm)	0.572	0.033	17.453	0.000

Variances:

	Estimate	Std.Err	z-value	P(> z )
.Y2	1.370	0.061	22.361	0.000
.Y1	2.802	0.125	22.361	0.000
.D	1.026	0.046	22.361	0.000

The effect of  $D$  on  $Y_2$  (again labelled **beta**) is biased in this model with an estimated coefficient of 0.317. In repeated samples with this particular combination of coefficients, the average estimate of the causal effect is around 0.3, about 25% smaller than the true effect.

### 5.3 LDV with $V$ as Latent Variable

In SEM though,  $V$  is explicitly accounted for as a latent variable. It represents the correlation of the outcome with itself over time, over and above the lagged causal variable and

the autoregressive effect. In the following model, called `m3`, both  $U$  and  $V$  are still treated as unobserved, but we will treat  $V$  as a latent variable and account for confounding by  $U$  by including the LDV.

We specify a latent variable to account for time-invariant unobserved heterogeneity with `alpha =~ 1*Y1 + 1*Y2` (line 3 of `m3`)<sup>5</sup> and regress  $Y_2$  on both  $D$  and  $Y_1$  (line 4). We include  $Y_0$  in the model as an exogenous variable, allowing it to covary freely with the stable factors represented by `alpha` (line 8). Allowing the initial observation to covary freely with the stable factors accounts for situations in which the outcome is not yet at equilibrium, see Andersen (2021) and it is the common approach in dynamic models (e.g., Allison, Williams, and Moral-Benito 2017). Finally, since  $U$  is now also unobserved, we allow  $Y_1$  and  $D$  to covary to account for this common cause (line 10). Figure 4 shows the SEM approach graphically, both as a DAG (Figure 4a) and in the style of a traditional SEM path model (Figure 4b).

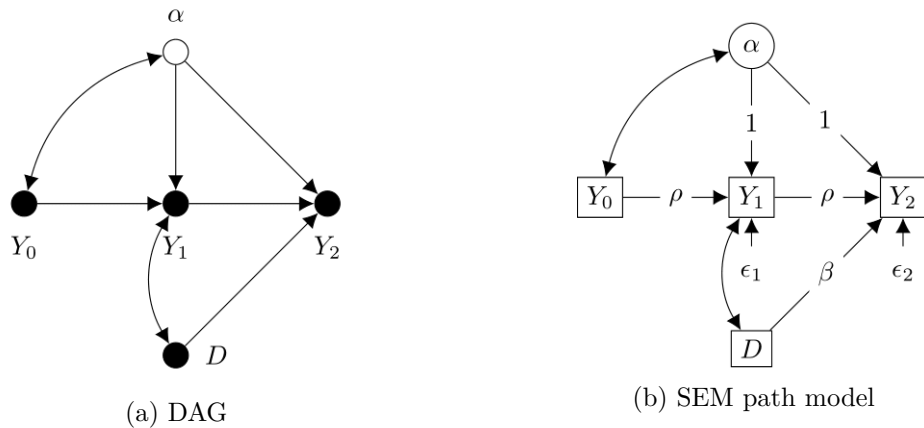


Figure 4: Modelling strategy in SEM (from `m3.fit`)

```

1 m3 = "
2   # Individual effects to account for V
3   alpha =~ 1*Y1 + 1*Y2
4   # Regressions
5   Y2 ~ beta*D + rho*Y1
6   Y1 ~ rho*Y0
7   # Allow initial outcome to correlate with unit effects
8   alpha ~~ Y0
9   # Account for U, common cause of Y1 and D
10  D ~~ Y1
11  "
12 m3.fit = sem(model = m3, data = df, estimator = "ML") %>%
13   summary()

```

lavaan 0.6-11 ended normally after 34 iterations

<sup>5</sup>Note that we use the name `alpha` here instead of `V` because `lavaan` will usually return an error if one of the names of the latent variables overlaps with the name of one of the observed variables.

Estimator	ML
Optimization method	NLMINB
Number of model parameters	10
Number of equality constraints	1

Number of observations	1000
------------------------	------

Model Test User Model:

Test statistic	0.151
Degrees of freedom	1
P-value (Chi-square)	0.697

Parameter Estimates:

Standard errors	Standard
Information	Expected
Information saturated (h1) model	Structured

Latent Variables:

	Estimate	Std.Err	z-value	P(> z )
alpha =~				
Y1	1.000			
Y2	1.000			

Regressions:

		Estimate	Std.Err	z-value	P(> z )
Y2 ~					
D	(beta)	0.378	0.042	9.006	0.000
Y1	(rho)	0.343	0.183	1.879	0.060
Y1 ~					
Y0	(rho)	0.343	0.183	1.879	0.060

Covariances:

	Estimate	Std.Err	z-value	P(> z )
alpha ~~				
Y0	0.931	0.328	2.839	0.005
.Y1 ~~				
D	0.196	0.048	4.126	0.000

Variances:

	Estimate	Std.Err	z-value	P(> z )
.Y1	1.236	0.306	4.040	0.000
.Y2	0.982	0.157	6.260	0.000
D	1.338	0.060	22.361	0.000

Y0	1.965	0.088	22.361	0.000
alpha	0.863	0.607	1.422	0.155

Now, the estimate of the causal effect is close to the true parameter at 0.378. And this is not a fluke, the discrepancy here is simply due to sampling error. We can draw many samples to show that the estimated coefficient is approximately equal to the causal effect.

```

1  set.seed(9876)
2
3  sim_func = function(rho = 0.3, beta = 0.4, gamma = 0.6, delta = 0.5) {
4
5    # Set large sample size
6    n = 1000L
7
8    # Time-invariant unobserved heterogeneity
9    V = rnorm(n, 0, 1)
10
11   # Simulate initial realization of outcome
12   Y0 = 1 * V + rnorm(n, 0, 1)
13
14   # Time-varying confounder
15   U = rnorm(n, 0, 1)
16
17   # Causal variable
18   D = gamma * U + rnorm(n, 0, 1)
19
20   # Remaining realizations of outcome
21   Y1 = delta * U + rho * Y0 + 1 * V + rnorm(n, 0, 1)
22   Y2 = beta * D + rho * Y1 + 1 * V + rnorm(n, 0, 1)
23
24   # Put into dataframe
25   df = data.frame(Y0, Y1, Y2, D, V, U)
26
27   # Fit the model
28   mx = "
29     # Individual effects to account for V
30     alpha =~ 1*Y1 + 1*Y2
31     # Regressions
32     Y2 ~ beta*D + rho*Y1
33     Y1 ~ rho*Y0
34     # Allow initial outcome to correlate with unit effects
35     alpha ~~ Y0
36     # Account for U, common cause of Y1 and D
37     D ~~ Y1
38   "
39   mx.fit = sem(model = mx, data = df, estimator = "ML")

```

```

40
41   # Get estimate of beta
42   est = lavInspect(mx.fit, "list") %>%
43     filter(label == "beta") %>%
44     select(est) %>%
45     as.numeric()
46
47   # Return estimate of beta
48   return(est)
49 }
50
51 res = replicate(n = 10000L, expr = sim_func())

```

The estimated coefficient is unbiased at 0.398 (with a standard deviation for the sampling distribution of 0.051), and approximately normally distributed, see Figure 5.

```

1 hist(res, main = NULL, xlab = NULL, breaks = 30)

```

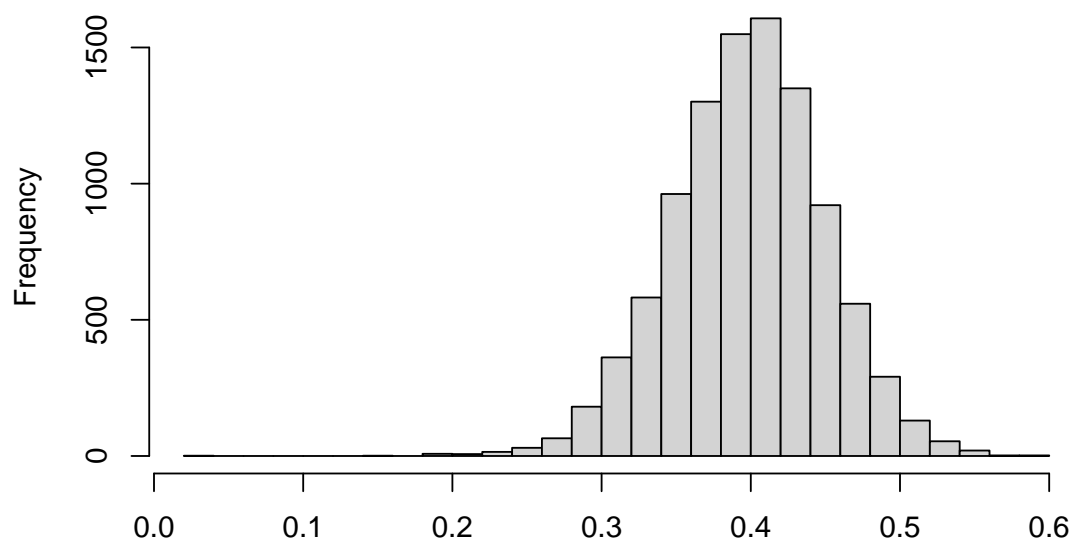


Figure 5: Histogram, sampling distribution of res

## 6 Conclusion

The goal of this article was to renew the reader’s confidence in the use of LDVs in panel models. As we discussed, there are good reasons to consider doing so. From a theoretical standpoint, LDVs can capture inertial effects ([Wooldridge 2012, 313](#)) where there is expected to be carry-over of the outcome at one point in time to the next ([Keele and Kelly 2006](#)). The inclusion of the LDV also gives the main coefficient of interest a desirably intuitive interpretation: a comparison of outcomes between hypothetical units that displayed the same outcome in the previous period, but whose values on the causal variable differ. But perhaps most importantly, the LDV can be effective at closing back-door paths due to unobserved time-varying confounders.

The practice of including the LDV is often criticized. As shown in the example in `m2`, this criticism is often warranted. In the presence of time-invariant unobserved heterogeneity, the LDV acts as a collider and opens up an unintended back-door path, thus biasing the estimate of the causal effect. But leaving the LDV out means the onus is on the researcher to measure all the potential time-varying confounders, so you’re often “damned if you do, damned if you don’t” ([Cinelli, Forney, and Pearl 2022](#)).

The SEM approach gets around this specific criticism of the LDV by explicitly accounting for time-invariant unobserved heterogeneity. This blocks both back-door paths, across (certain) time-varying confounders and time-invariant ones. Thus, the wide use of cross-lagged and other panel models in SEM that account for LDVs is arguably justified.

LDVs are not a silver bullet, however. The researcher’s qualitative hypotheses must hold, as always. The LDV will stop confounding if the simplified DAG in [Figure 2](#) (or something equivalent, such as [Figure 3](#)) is the true DGP. If the unobserved time-varying confounders affect the current outcome over and above the mediated path over the lagged version, then another unblocked path is opened up. And other assumptions, like the appropriateness of a linear model, must be scrutinized in SEM just as in any other methodology ([Kenneth A. Bollen and Pearl 2013](#)).

Finally, we did not look at other potential approaches to LDVs, such as the Arellano-Bond (AB) differenced model (which is perhaps the most promising approach outside of SEM), which tries to use lags even further back from the current outcome as instruments ([Brüderl and Ludwig 2014](#)). In a series of simulations, Leszczensky and Wolbring ([2019](#)) showed that AB and SEM both performed well under a wide variety of underlying DGPs and assumptions.

Instead, this paper chose to draw attention to an apparently overlooked aspect. At this very moment, a researcher at a conference is likely being alerted to the fact that the LDV is a collider and that their SEM and its results are biased. We believe this should not be a blanket criticism, and that SEM provides a flexible framework for modeling various dynamic processes that suit researchers’ qualitative hypotheses.



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