# Rehabilitating the Lagged Dependent Variable with Structural Equation Modeling

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It has been argued that including the lagged dependent variable in panel models will open up unintended back-door paths and bias the estimates of the causal variable. We show that panel analysis in the structural equation modeling framework gets around this issue. Including the lagged dependent variable has the benefit of closing back-door paths due to unobserved time-varying confounders. We demonstrate this by looking at simulated data in the `lavaan` package for `R`.

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## 1 Introduction

There is a long tradition in sociology and psychology of using cross-lagged panel models to investigate dynamic processes (Rogosa 1980) in structural equation modeling (SEM). These usually look at the lagged bidirectional effects of two variables (cross-lagged paths, CL) while holding the previous values of these variables constant (autoregressive paths, AR).

At the same time, such models have been shown to be biased in the presence of time-invariant unobserved heterogeneity. Effective ways to incorporate time-invariant unobserved heterogeneity into cross-lagged panel models have been around for several decades now (e.g., Kenneth A. Bollen and Curran 2004; Curran and Bollen 2001) and this topic has experienced renewed interest in the last several years (Curran et al. 2014; Hamaker, Kuiper, and Grasman 2015).

The basic idea of cross-lagged panel models that account for unobserved time-invariant confounders can also be generalized to 'unidirectional' models that focus on the effect of one variable on another while de-emphasizing the question of reverse causality (Allison, Williams, and Moral-Benito 2017; Moral-Benito, Allison, and Williams 2018; Williams, Allison, and Moral-Benito 2018). We could call these 'dynamic fixed effects' models because they all account for both unobserved time-invariant heterogeneity as well as state dependence by including the lagged dependent variable (lagged DV or LDV).

Indeed (Kühnel and Mays 2018) argue that the inclusion of the LDV make such models more desirable than 'static fixed effects' models (which do not include the lagged dependent variable) because they potentially account for confounding by certain time-varying variables, as well.

Still, many are skeptical of the use of such models. Often, the skepticism centers directly on the use of the LDV. Indeed, many articles warn of including the LDV in a panel regression model (e.g., Brüderl and Ludwig 2014; Dafoe 2014, 2015; Foster 2010; Collischon and Eberl 2020; Keele and Kelly 2006; Leszczensky and Wolbring 2019; Mouw 2006; Walters 2019). One of the most convincing arguments is given by Morgan and Winship (2014, 111, Figure 4.3). There, they show that the inclusion of the LDV may bias the causal effect of interest in the presence of time-invariant unobserved heterogeneity.

In this brief article, we will outline the arguments for and against the inclusion of the LDV and then show that the usual SEM approach to panel modelling is not generally affected by the criticism. We hope to convince readers of the usefulness of the broad class of (cross-lagged) dynamic panel models with fixed effects in SEM.

## 2 Benefits of the LDV

Consider an empirical example by Coleman, Hoffer, and Kilgore (1982) and outlined in Morgan and Winship (2014). There, Coleman and colleagues were looking to assess the causal effect of attending a Catholic school as opposed to public school on achievement, as measured by pupils' test scores. The direct acyclic graph (DAG) shown in Figure 1a summarizes their hypothesized data generating process (DGP), where black circles represent observed variables and white ones represent unobserved variables.[1]

In this DAG, $Y_{10}$ represents the pupil's test score in grade 10, $X$ are observed determinants of test scores and $O$ are observed background factors that influence test scores, the determinants of test scores, the selection of school system, as well as the unobserved factors in $U$.

[1]Empirical examples chosen for this article were taken from the helpful video tutorial on LDVs by Mikko Rönkkö, https://www.youtube.com/watch?v=DhV5otUB3Jc.

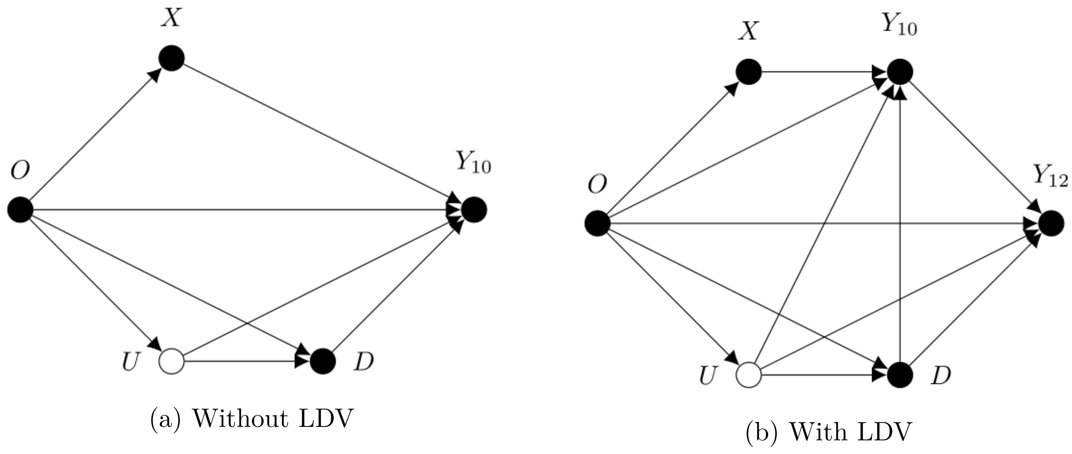



Figure 1: Catholic school example from Coleman, Hoffer, and Kilgore (1982)

For its part, $U$ could contain any number of potentially umeasured things like motivation or intelligence, that would impact the choice of school system (Catholic schools might prefer to admit intelligent pupils) and the pupil's test scores themselves.

Conditioning on $X$ and $O$ would close all back-door paths except $D \leftarrow U \rightarrow Y_{10}$. Because $U$ contains all the unobserved determinants of the causal variable and the outcome, it cannot be conditioned on and the causal effect of interest is unidentified (Morgan and Winship 2014, 270).

Coleman, Hoffer, and Kilgore (1982) came to a solution to this problem by collecting data on the pupils' test scores two years later, in grade 12. By looking at the DAG in Figure 1b, we can see that $Y_{10}$ "screen[s] off the effects of the variables in $U$ on $Y_{12}$" (Morgan and Winship 2014, 270). Indeed, by focusing on test scores in grade 12 and including the lagged measure from grade 10 in the model, the back-door paths over the unobservables in $U$ are blocked and the causal effect of $D$ on $Y_{12}$ is identified.

Note that $U$ could potentially include the unmeasured outcome even further in the past. In this way, the inclusion of the lagged dependent variable also accounts for endogeneity, where the causal variable is impacted by previous outcomes. An example given by Wooldridge (2012, 313) concerns the explanation of crime as a function of police expenditure. It is plausible that more is spent on policing in areas where crime rates have been high in the past. Simply regressing the crime rate on expenditure will likely be misleading, because where there is a high crime rate, there will be increased spending, so the effect of spending on crime may even be positive.[2] Including the lagged crime rate allows for an intuitive interpretation of the effect of expenditure: it is the difference in crime rate between two hypothetical cities with the same crime rate in the previous period given a unit change in expenditure (Wooldridge 2012).

[2]This is not to justify police expenditure, which is a difficult topic in some parts of the world. It may still be that police spending has a positive causal effect on crime, or that there is no tangible effect. But the estimated effect in a simple regression of current crime rate on current expenditure will likely be biased in one way or another.

# 3 Arguments Against the LDV

To continue with outlining the issue, let us turn to a simpler DAG as shown in Figure 2, which is adapted also from Morgan and Winship (2014, 111, Fig. 4,3).[3] In this DAG, we have dropped the observed variables captured in $X$ and $O$ from above. Since they are observed, we can drop them from the following discussion without loss of generality. We still have $D$, the causal variable of interest, and two measures of the outcome, represented by $Y_1$ and $Y_2$. Consistent with the Catholic school example, the inclusion of $Y_1$ as an observed predictor of $Y_2$ closes the back-door path across $D \leftarrow U \rightarrow Y_1 \rightarrow Y_2$.

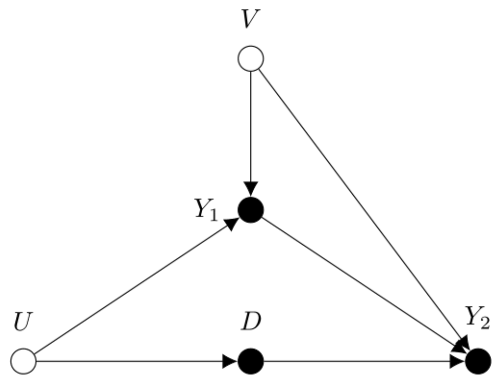



Figure 2: Lagged DV as collider from Morgan and Winship (2014)

The novelty of this DAG compared to the previous ones is the inclusion of $V$. It represents the time-invariant unobserved heterogeneity. It is an unobserved variable that affects the outcome at all points in time. It can also be thought of as the combined effect of all the time-invariant factors affecting the outcome. The issue, outlined in Morgan and Winship (2014), is that if $V$ is unobserved, then $Y_1$ is also a collider variable and controlling for it therefore opens a new back-door path over $V$, rendering the causal effect biased.

This point is often the one criticisms of the LDV focus on. Keele and Kelly (2006, 187) write, for example, that "[e]ven when a lagged dependent variable is theoretically appropriate, remaining residual autocorrelation can lead to biased coefficient results." Here, the remaining residual autocorrelation is the time-invariant unobserved heterogeneity; the unobserved stable factors that cause the outcome to be correlated with itself over time (Andersen 2022). Foster (2010, 1467) echos this, stating "[s]uch analyses [that include the LDV] are problematic. As has long been known in economics and other fields, in the presence of autocorrelation (a relationship between unobservables over time), the resulting estimates have poor statistical properties." Collischon and Eberl (2020, 297) argue against the LDV in a similar fashion.

The intuition for these arguments can be shown easily. To simplify things, let us assume variables are mean centered and scaled to have a variance of one, as Cinelli, Forney, and Pearl

[3]The same thing is shown in the variation of Model 7 in Cinelli, Forney, and Pearl (2022), where $Z$ takes the place of the LDV.

(2022) do. Then, by path tracing, labelling the structural coefficients $\lambda$, the covariances of the observed variables, $Y_2, Y_1, D$ are given by

$$\sigma_{Y_2D} = \lambda_{DY_2} + \lambda_{UD}\lambda_{UY_1}\lambda_{Y_1Y_2} \tag{1}$$
$$\sigma_{DY_1} = \lambda_{UD}\lambda_{UY_1} \tag{2}$$
$$\sigma_{Y_2Y_1} = \lambda_{Y_1Y_2} + \lambda_{VY_1}\lambda_{VY_2} + \lambda_{UY_1}\lambda_{UD}\lambda_{DY_2}. \tag{3}$$

The partial coefficient of the causal variable, $D$, on the outcome, $Y_2$, controlling for $Y_1$, is given by

$$\beta_{Y_2D\cdot Y_1} = \frac{\sigma_{Y_2D} - \sigma_{DY_1}\sigma_{Y_2Y_1}}{1-\sigma^2_{DY_1}} \tag{4}$$

(Cinelli, Forney, and Pearl 2022) which, after substitution, works out to

$$\beta_{Y_2D\cdot Y_1} = \lambda_{DY_2} - \frac{\lambda_{UD}\lambda_{UY_1}\lambda_{VY_1}\lambda_{VY_2}}{1-(\lambda_{UD}\lambda_{VY_1})^2} \tag{5}$$

which does not equal the structural coefficient $\lambda_{DY_2}$, the average causal effect.

This shows the bias resulting from the LDV and Dafoe (2015, 139) suggests that it is therefore only safe to include the LDV when it is not a collider. That is the case when either “there are no unobserved common causes of treatment and the lagged outcome” (if $U$ were missing from the DAG in Figure 2) or “no unobserved persistent causes of the outcome” (if $V$ were missing). Partly because of this (along with other theoretical reasons), Brüderl and Ludwig (2014, 342) propose flatly that “LDV models are not useful at all.”

## 4 The Structural Equation Modeling Approach

The key to the SEM approach, and the reason why cross-lagged and other panel models that include the LDV are so widespread in SEM, has to do with the fact that by using latent variables, the LDV does not open an unblocked back-door path. Panel models in SEM that account for time-invariant unobserved heterogeneity normally work by specifying a latent variable that causes the observed outcome at all points in time. This explicitly accounts for the “remaining residual autocorrelation” (Keele and Kelly 2006, 187), or “persistent causes of the outcome” (Dafoe 2015, 139).

In SEM, the time-invariant unobserved heterogeneity is not unobserved in the classical sense. It represents the conditional (on the other observed variables) covariance between the outcome and itself over time (Andersen 2022). Say we had the linear model

$$y_{it} = \mathbf{x}_{it}\boldsymbol{\beta} + \alpha_i + \epsilon_{it},\ i = 1, \ldots, N,\ t = 1, \ldots, T \tag{6}$$

where $\mathbf{x}_{it} = (d_{it}, y_{it-1})$ and $\boldsymbol{\beta} = (\beta, \rho)^\top$, $\alpha_i$ is a latent variable representing the stable factors that change between individuals but not within them, and $\epsilon_{it}$ is the time-varying error component. Then the covariance of any two columns of the wide-format outcome, conditional on the observed covariates, is just

$$\text{Cov}(y_{it}, y_{is}|\mathbf{x}_{it}) = \text{Var}(\alpha_i),\ t \neq s \tag{7}$$

since we normally assume $\text{Cov}(\epsilon_{it}, \epsilon_{is}) = 0,\ t \neq s$ and $\text{Cov}(\epsilon_{it}, \alpha_i) = 0,\ t = 1, \ldots, T$.

The SEM approach to modelling time-invariant unobserved heterogeneity as a latent variable in dynamic models relies on our ability to decompose the correlations observed between columns of the wide-format outcome into a part that is due to unobserved time-invariant factors, and the autoregressive component.

This works most reliably when the outcome is at equilibrium, i.e., where the variances and covariances between adjacent columns of the outcome (means, as well, if the mean structure is being analyzed) are not changing drastically from one point in time to the other (Andersen 2021). Ou et al. (2017) refer to such processes as "ongoing stationary." If the process is not ongoing stationary, then the covariance between adjacent columns of the outcome will be changing, and it may be difficult for the model to properly estimate the autoregressive effect, along with the variance of the latent variable representing the stable factors, see again Andersen (2021).

To ensure both sources of covariance between columns of the wide-format outcome can be properly accounted for, it is advantageous if the process is ongoing stationary, and even better when there are sufficient observations of the outcome over time (ideally more than two).[4]

While these conditions may sound restrictive at first, they simply reflect the assumptions that there are relatively stable individual levels/trajectories around which the temporal measures tend to hover, and that deviations from these tend to revert back to that underlying level/trajectory (Andersen 2021, 6). And these are basically baked-in to the DGP displayed in Figure 2, which indeed features an autoregressive effect as well as stable factors affecting the outcome over time. In fact, we can look at Figure 2 as a 'close-up' of the overall DGP,

[4] Actually, most approaches to panel modelling in SEM are robust even to data that are not yet at equilibrium. We can treat the initial observed outcome as 'predetermined', i.e., enter it as an exogenous variable into the model and allow it to covary freely with the unit effects (latent variable representing time-invariant unobserved heterogeneity), to avoid having to make strong assumptions about how long the data generating process has been going on previous to the observation period, see for example Allison, Williams, and Moral-Benito (2017); Andersen (2021).

focusing in on two measures of the outcome, as hardly any conceivable phenomena in the social sciences occur in such a vacuum.[5]

More on the method for accounting for stable unobserved characteristics in panel models in SEM has been outlined elsewhere (e.g., Allison 2011; Andersen 2022; K. A. Bollen and Brand 2010; Teachman et al. 2001), so we will not describe it in detail here. Instead, we show that by explicitly modeling the time-invariant unobserved heterogeneity, the causal effect of interest can be estimated without bias.

## 5 A Simulated Example

We construct a simulated dataset to demonstrate the use of SEM in accounting for time-varying and invariant confounders. To ensure the outcome is ongoing stationary, we simulate a 'spin-up' phase to allow the variances and covariances to reach equilibrium. Figure 3 shows the simulated DGP, where the initial outcomes, $Y_0 - Y_4$ are treated as 'unobserved.' That is, we focus on three measures of the outcome over time, $Y_5 - Y_7$, along with the causal variable, $D$, and account for the unobserved variables $U$ and $V$ indirectly.

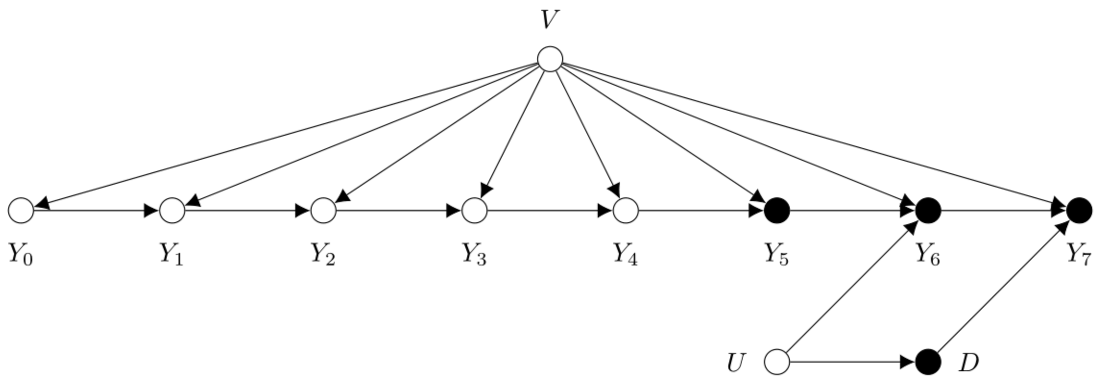



Figure 3: Simulated DGP

We look at linear additive effects. The exogenous variables, as well as the errors, are standard normal. The effect sizes were chosen arbitrarily and can be seen in the code below. The main causal effect of interest, $D \rightarrow Y7$ is set to the value 0.4.[6]

```
# Set seed
set.seed(45678)
```

[5]E.g., if we refer back to the examples mentioned above, we would assume stable factors affect the pupils' test scores not just in grades 10 and 12, but also in earlier grades, as well. And police expenditure likely hovers around a fairly stable level for long periods of time, based on stable unobserved characteristics of the city.

[6]Normally, it is assumed that the effect of the time-invariant unobserved factors is constant over time. This assumption is reflected in the usual practice of fixing the factor loadings from the latent variable to the outcome to 1.0 at each point in time (Andersen 2022; K. A. Bollen and Brand 2010). This assumption can be easily relaxed by allowing the factor loadings to be estimated freely after the first point in time. We retain it here as it is consistent with the simulated DGP.

```r

# Load packages
library(lavaan)
library(dplyr)

# Set large sample size
n <- 1000L

rho   = 0.3 # Autoregressive effect
gamma = 0.6 # Effect U -> D
delta = 0.5 # Effect U -> Y6
beta  = 0.4 # Causal effect of interest

# Time-invariant unobserved heterogeneity
V = rnorm(n, 0, 1)

# Simulate spin-up phase to allow Yt to reach equilibrium
Y0 = 1 * V + rnorm(n, 0, 1)
Y1 = rho * Y0 + 1 * V + rnorm(n, 0, 1)
Y2 = rho * Y1 + 1 * V + rnorm(n, 0, 1)
Y3 = rho * Y2 + 1 * V + rnorm(n, 0, 1)
Y4 = rho * Y3 + 1 * V + rnorm(n, 0, 1)
Y5 = rho * Y4 + 1 * V + rnorm(n, 0, 1)

# Time-varying confounder
U = rnorm(n, 0, 1)

# Causal variable
D = gamma * U + rnorm(n, 0, 1)

# Focus on effect D -> Y7, holding Y6 constant
Y6 = delta * U + rho * Y5 + 1 * V + rnorm(n, 0, 1)
Y7 = beta  * D + rho * Y6 + 1 * V + rnorm(n, 0, 1)

# Put into dataframe
df = data.frame(Y0, Y1, Y2, Y3, Y4, Y5, Y6, Y7, D, V, U)
```

Obviously, if both $U$ and $V$ were observed, then we could estimate the model without bias. We can estimate the entire model simultaneously in SEM using the `lavaan` package in `R`.

```r
m1 = "
  Y7 ~ beta*D + rho*Y6 + V
  Y6 ~ delta*U + V
  D  ~ gamma*U
"
m1.fit = sem(model = m1, data = df, estimator = "ML") %>%
```

```
  summary()
```

```
lavaan 0.6-11 ended normally after 2 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                         9

  Number of observations                          1000

Model Test User Model:

  Test statistic                                 5.641
  Degrees of freedom                                 3
  P-value (Chi-square)                           0.130

Parameter Estimates:

  Standard errors                             Standard
  Information                                 Expected
  Information saturated (h1) model          Structured

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  Y7 ~
    D       (beta)    0.435    0.028   15.659    0.000
    Y6       (rho)    0.308    0.029   10.810    0.000
    V                 1.019    0.052   19.704    0.000
  Y6 ~
    U       (delt)    0.457    0.033   13.741    0.000
    V                 1.457    0.033   43.592    0.000
  D ~
    U       (gamm)    0.597    0.032   18.704    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .Y7                1.002    0.045   22.361    0.000
   .Y6                1.082    0.048   22.361    0.000
   .D                 1.007    0.045   22.361    0.000
```

Here, the causal effect of interest (labeled `beta`) is unbiased at about 0.435 (the slight discrepancy is due to sampling error).

Now, to see the point Morgan and Winship (2014) were making, let us assume $V$ is unobserved. Including the lagged dependent variable will close the back-door path over $U$ but the bias will still be present because $V$ is unobserved.

```r
m2 = "
  Y7 ~ beta*D + rho*Y6
  Y6 ~ delta*U
  D  ~ gamma*U
"
m2.fit = sem(model = m2, data = df, estimator = "ML") %>%
  summary()
```

```
lavaan 0.6-11 ended normally after 1 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                         7

  Number of observations                          1000

Model Test User Model:

  Test statistic                                49.244
  Degrees of freedom                                 2
  P-value (Chi-square)                           0.000

Parameter Estimates:

  Standard errors                             Standard
  Information                                 Expected
  Information saturated (h1) model          Structured

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  Y7 ~
    D       (beta)    0.318    0.032    9.923    0.000
    Y6       (rho)    0.748    0.021   35.989    0.000
  Y6 ~
    U       (delt)    0.318    0.056    5.643    0.000
  D ~
    U       (gamm)    0.597    0.032   18.704    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .Y7                1.388    0.062   22.361    0.000
   .Y6                3.139    0.140   22.361    0.000
   .D                 1.007    0.045   22.361    0.000
```

The effect of $D$ on $Y_7$ is biased in this model with an estimated coefficient of 0.318.

In SEM though, $V$ is explicitly accounted for as a latent variable. It represents the correlation of the outcome with itself over time, over and above the lagged causal variable and the autoregressive effect. In the following model, both $U$ and $V$ are still treated as unobserved.

We specify a latent variable to account for time-invariant unobserved heterogeneity with `alpha =~ 1*Y6 + 1*Y7` and regress `Y7` on both `D` and `Y6`. We include `Y5` in the model as an exogenous variable, allowing it to covary freely with the stable factors represented by `alpha`. Allowing the initial observation to covary freely with the stable factors accounts for situations in which the outcome is not yet at equilibrium, see Andersen (2021) and it is the common approach in dynamic models (e.g., Allison, Williams, and Moral-Benito 2017). Including `Y5` and regressing `Y6` on it further helps the model better differentiate the autoregressive from the stable unit effects.

Note that we use the name `alpha` here instead of `V` because `lavaan` will usually return an error if one of the names of the latent variables overlaps with the name of one of the observed variables.

Finally, since `U` is now also unobserved, we allow `Y6` and `D` to covary to account for this common cause. Figure 4 shows the SEM approach graphically.

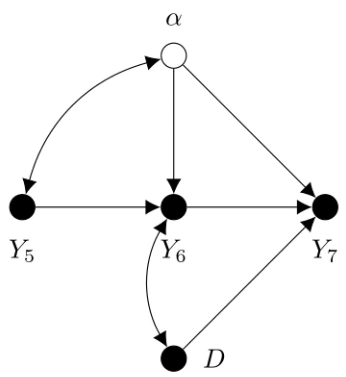



Figure 4: Modelling strategy in SEM (from m3.fit)

```
m3 = "
  # Individual effects to account for V
  alpha =~ 1*Y6 + 1*Y7
  # Regressions
  Y7 ~ beta*D + rho*Y6
  Y6 ~ rho*Y5
  # Allow initial outcome to correlate with unit effects
  alpha ~~ Y5
  # Account for U, common cause of Y6 and D
  D ~~ Y6
"
m3.fit = sem(model = m3, data = df, estimator = "ML") %>%
  summary()
```

```
lavaan 0.6-11 ended normally after 35 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        10
  Number of equality constraints                     1

  Number of observations                          1000

Model Test User Model:

  Test statistic                                 3.311
  Degrees of freedom                                 1
  P-value (Chi-square)                           0.069

Parameter Estimates:

  Standard errors                             Standard
  Information                                 Expected
  Information saturated (h1) model          Structured

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  alpha =~
    Y6                1.000
    Y7                1.000

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  Y7 ~
    D       (beta)    0.428    0.036   11.758    0.000
    Y6       (rho)    0.162    0.092    1.761    0.078
  Y6 ~
    Y5       (rho)    0.162    0.092    1.761    0.078

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  alpha ~~
    Y5                1.773    0.264    6.713    0.000
 .Y6 ~~
    D                 0.226    0.047    4.795    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .Y6                1.005    0.157    6.404    0.000
   .Y7                0.891    0.069   12.957    0.000
```

| | | | | |
|---|---|---|---|---|
| D | 1.360 | 0.061 | 22.361 | 0.000 |
| Y5 | 3.102 | 0.139 | 22.361 | 0.000 |
| alpha | 1.607 | 0.434 | 3.700 | 0.000 |

Now, the estimate of the causal effect is relatively close to the true parameter at 0.428. And this is not a fluke: we can draw many samples to show that the estimated coefficient is approximately equal to the causal effect.

```
sim_func = function(rho = 0.3, beta = 0.4, gamma = 0.6, delta = 0.5) {

  # Set large sample size
  n <- 1000L

  # Time-invariant unobserved heterogeneity
  V = rnorm(n, 0, 1)

  # Simulate spin-up phase to allow Yt to reach equilibrium
  Y0 = 1 * V + rnorm(n, 0, 1)
  Y1 = rho * Y0 + 1 * V + rnorm(n, 0, 1)
  Y2 = rho * Y1 + 1 * V + rnorm(n, 0, 1)
  Y3 = rho * Y2 + 1 * V + rnorm(n, 0, 1)
  Y4 = rho * Y3 + 1 * V + rnorm(n, 0, 1)
  Y5 = rho * Y4 + 1 * V + rnorm(n, 0, 1)

  # Time-varying confounder
  U = rnorm(n, 0, 1)

  # Causal variable
  D = gamma * U + rnorm(n, 0, 1)

  # Focus on effect D -> Y7, holding Y6 constant
  Y6 = delta * U + rho * Y5 + 1 * V + rnorm(n, 0, 1)
  Y7 = beta  * D + rho * Y6 + 1 * V + rnorm(n, 0, 1)

  # Put into dataframe
  df = data.frame(Y0, Y1, Y2, Y3, Y4, Y5, Y6, Y7, D, V, U)

  # Fit the model
  mx = "
    # Individual effects to account for V
    alpha =~ 1*Y6 + 1*Y7
    # Regressions
    Y7 ~ beta*D + rho*Y6
    Y6 ~ rho*Y5
    # Allow initial outcome to correlate with unit effects
    alpha ~~ Y5
```

```
    # Account for U, common cause of Y6 and D
    D ~~ Y6
  "
  mx.fit = sem(model = mx, data = df, estimator = "ML")

  # Get estimate of beta
  est = lavInspect(mx.fit, "list") %>%
    filter(op == "~") %>%
    filter(label == "beta") %>%
    select(est) %>%
    as.numeric()

  # Return estimate of beta
  return(est)
}

res = replicate(n = 10000L, expr = sim_func())
```

We can see that the estimated coefficient is unbiased:

```
mean(res)
```

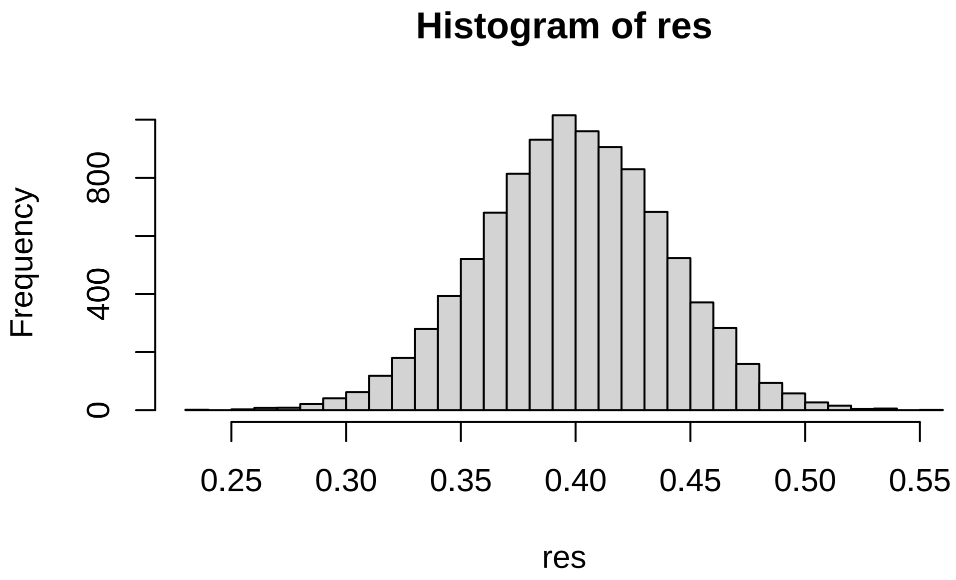
```
[1] 0.3991025
```

```
median(res)
```

```
[1] 0.39942
```

and approximately normally distributed around the true structural coefficient of 0.4:

```
hist(res, breaks = 30)
```




## 6 Conclusion

The goal of this article was to renew the reader's confidence in the use of LDVs in panel models. As we discussed, there are good reasons to consider doing so. From a theoretical standpoint, LDVs can capture inertial effects (Wooldridge 2012, 313) where there is expected to be carry-over of the outcome at one point in time to the next (Keele and Kelly 2006). The inclusion of the LDV also gives the main coefficient of interest a desirably intuitive interpretation: a comparison of outcomes between hypothetical units that displayed the same outcome in the previous period, but whose values on the causal variable differ. But perhaps most importantly, the LDV can be effective at closing back-door paths due to unobserved time-varying confounders.

The practice of including the LDV is often criticized and for good reason. In the presence of time-invariant unobserved heterogeneity, the LDV acts as a collider and opens up an unintended back-door path, thus biasing the estimate of the causal effect. But leaving the LDV out means the onus is on the researcher to measure all the potential time-varying confounders, so you're often "damned if you do, damned if you don't" (Cinelli, Forney, and Pearl 2022).

The SEM approach gets around this specific criticism of the LDV by explicitly accounting for time-invariant unobserved heterogeneity. This blocks both back-door paths, across (certain) time-varying confounders and time-invariant ones. Thus, the wide use of cross-lagged and other panel models in SEM that account for LDVs is arguably justified.

LDVs are not a silver bullet, however. The researcher's qualitative hypotheses must hold, as always. The LDV will stop confounding if the simplified DAG in Figure 2 (or something equivalent, such as Figure 3) is the true DGP. If the unobserved time-varying confounders affect the current outcome over and above the mediated path over the lagged version, then another unblocked path is opened up. And other assumptions, like the appropriateness of a linear model, must be scrutinized in SEM just as in any other methodology (Kenneth A. Bollen and Pearl 2013).

Finally, we did not look at other potential approaches to LDVs, such as the Arellano-Bond (AB) differenced model (which is perhaps the most promising approach outside of SEM), which tries to use lags even further back from the current outcome as instruments (Brüderl and Ludwig 2014). In a series of simulations, Leszczensky and Wolbring (2019) showed that AB and SEM both performed well under a wide variety of underlying DGPs and assumptions.

Instead, this paper chose to draw attention to an apparently overlooked aspect. At this very moment, a researcher at a sociological conference is likely being alerted to the fact that the LDV is a collider and that their SEM and its results are biased. We believe this should not be a blanket criticism, and that SEM provides a flexible framework for modeling various dynamic processes that suit researchers' qualitative hypotheses.

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