Part A

Describe the “Umbrella domain” as an HMM:

• What is the set of unobserved variable(s) for a given time-slice t (denoted Xt in the book)?

The set of unobserved variables for a given time-slice t is $Rain_t$. Thus

$Rain_t = X_t$

• What is the set of observable variable(s) for a given time-slice t (denoted Et in the book)?

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E_t = $Umbrella_t$. $Umbrella_t$ has the domain (True, False)

Dynamic model**:**

$$P(X_t|X_{t-1}) = \begin{pmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{pmatrix}$$

Observable model:

$$P(E_t|X_t) = \begin{pmatrix} 0.9 & 0 \\ 0 & 0.2 \end{pmatrix} \; if\ U_t = True$$

$$P(E_t|X_t) = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.8 \end{pmatrix} \; if\ U_t = False$$

$$U_t = Umbrella\ on\ day\ t$$

• Which assumptions are encoded in this model? (Hint: Read page 568). Are the assumptions reasonable for this particular domain?

1. The current state in a Markov process depends only on a finite fixed number of previous states. This is called the *Markov Assumption.* E.g. in the umbrella domain, the probability of raining today depends only for a fixed number of previous days. This is a reasonable assumption.

2. There is also a sensor markov assumption. The probability of brinigng an umbrella to work is only dependent on if it raining the same day. In reality it would also depend on the weather the last days.

3. We also assume that the model is a stationary process, since the weather are controlled by the laws of nature.

4. We also assume that the model is a first order markov process. This is not necessarily true in the real world, and in reality, the process should be a higher order markov process, as the weather is dependent on more than yesterdays weather. We could also add other factors i.e., Pressure.

Part B

Day - t	Umbrella - e_t	$P(X_t \mid e_{1:t}) = f_{1:t}$
0		<0.5 , 0.5>
1	True	<0.8181, 0.1818>
2	True	<0.8833, 0.1166>
3	False	<0.1906, 0.8093>
4	True	<0.7307 ,0.2692>
5	True	<0.8673 .0.1326>

Note that all the answers are normalized.
From the table we can verify that: $P(X_2 \mid e_{1:2}) = < 0.8833, 0.1166 >$.

Part C

The results from implementing FORWARD-BACKWARD algorithm can be seen in in the Table under.

To clarify:
$P(X_t \mid e_{1:t}) = f_{1:t}$ = forward probabilities
$P(e_{k+1:t} \mid X_k) = b_{k+1:t}$ = backward probabilities.

Day - t	Umbrella- e_t	$f_{1:t}$	$b_{k+1:t}$	$Smoothed\ probabilites$ $P(X_t \mid e_{1:k})$
0		<0.5 , 0.5>	<0.6469 , 0.6469>	<0.6469 , 0.3530>

1	True	<0.8181, 0.1818>	<0.5923 , 0.5923>	<0.8673, 0.1326>
2	True	<0.8833, 0.1166>	<0.3762 , 0.3762>	<0.8204, 0.1795>
3	False	<0.1906, 0.8093>	<0.6533 , 0.6533>	<0.3074, 0.6925>
4	True	<0.7307 ,0.2692>	<0.6272 , 0.6272>	<0.8204 , 0.1796>
5	True	<0.8673 .0.1326>	<1 ,1.>	<0.8673,0.1326>

Note that all the answers are normalized.

From the table we can see that the probability for rain at day 1:
$P(X_1|e_{1:5})$ = < 0.8673, 0.1326>.

The unormalized b values:
Backward on day: 5 is: [[1.]], [[1.]]

Backward on day: 4 is: [[0.69]], [[0.69]]

Backward on day: 3 is: [[0.4593]], [[0.4593]]

Backward on day: 2 is: [[0.090639]], [[0.090639]]

Backward on day: 1 is: [[0.06611763]], [[0.06611763]]

Backward on day: 0 is: [[0.04438457]], [[0.04438457]]

Part D

Code is attached in the delivery on blackboard.