Exersice 2

$$\hat{\beta} = (X^T X + \tau \mathbb{1}_p)^{-1} X^T \vec{y}$$

$$\hat{a} = (X X^T + \tau \mathbb{1}_n)^{-1} \vec{y}$$

Lemma: $$\underbrace{(X^T X + \tau \mathbb{1}_n)^{-1} X^T} = X^T (X X^T + \tau \mathbb{1}_n)^{-1}$$

$\Rightarrow$
$$= \underbrace{X^T (X^{T^{-1}})}_{\mathbb{1}} (X^T X + \tau \mathbb{1}_n)^{-1} \underbrace{(X^T)^{-1^{-1}}}_{X^T}$$

$$= X^T ((X^T X + \tau \mathbb{1}_n) X^{T^{-1}})^{-1} (X^T)^{-1^{-1}}$$

$$= X^T (X^{T^{-1}} (X^T X + \tau \mathbb{1}_n) X^T)$$

$$= X^T (\underbrace{X^{T^{-1}} X^T}_{\mathbb{1}} X X^T + \tau \underbrace{X^{T^{-1}} \mathbb{1}_n X^T}_{\mathbb{1}_n})$$

$$= X^T (X X^T + \tau \mathbb{1}_p)$$

Show: $\hat{\beta} = X^T a$

Proof:

Lemma: