

# COS/SE

# Computersystemer

Lektion #10  
Mere algoritmer.

# Potens

Eks.:  $5^3 = 125$

Generelt:  $x^y = ?$

eller:  $\text{base}^{\text{exp}} = ?$

Vi har brug for en algoritme!

# Iterativ potens algoritme

Ide:

$x^y$  udregnes ved at gange  $x$  med sig selv  $y$  gange.

```
def power( x, y ):
    result = 1

    while( y > 0 ):
        result = result * x
        y = y - 1

    return result
```

# Rekursiv potens algoritme

Ide:

$$x^y = x * x^{y-1}. \text{ power}(x, y) = x * \text{power}(x, y-1)$$

```
def power( x, y ):
    if( y <= 0 ):
        return 1
    return x * power( x, y-1 )
```

# Iterative

# Iterative Structures

- A collection of instructions repeated in a looping manner
- Examples include:
  - Sequential Search Algorithm
  - Insertion Sort Algorithm

DET TEKNISKE FAKULTET

## Figure 5.6 The sequential search algorithm in pseudocode

```
def Search (List, TargetValue):  
    if (List is empty):  
        Declare search a failure  
    else:  
        Select the first entry in List to be TestEntry  
        while (TargetValue > TestEntry and entries remain):  
            Select the next entry in List as TestEntry  
            if (TargetValue == TestEntry):  
                Declare search a success  
            else:  
                Declare search a failure
```

# Components of repetitive control

**Initialize:** Establish an initial state that will be modified toward the termination condition

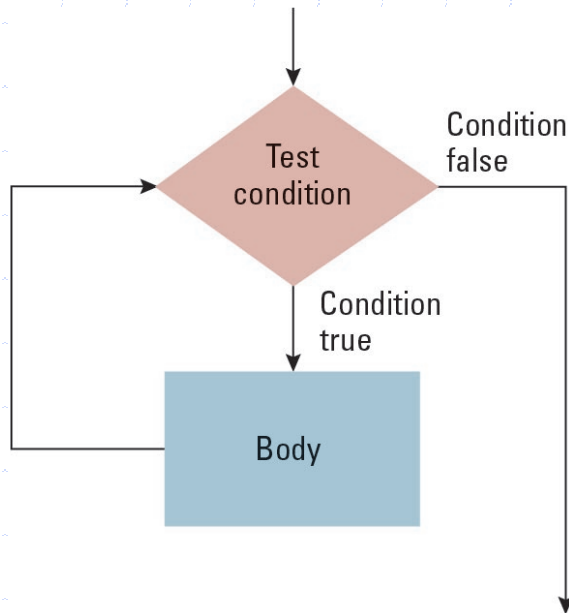
**Test:** Compare the current state to the termination condition and terminate the repetition if equal

**Modify:** Change the state in such a way that it moves toward the termination condition

DET TEKNISKE FAKULTET

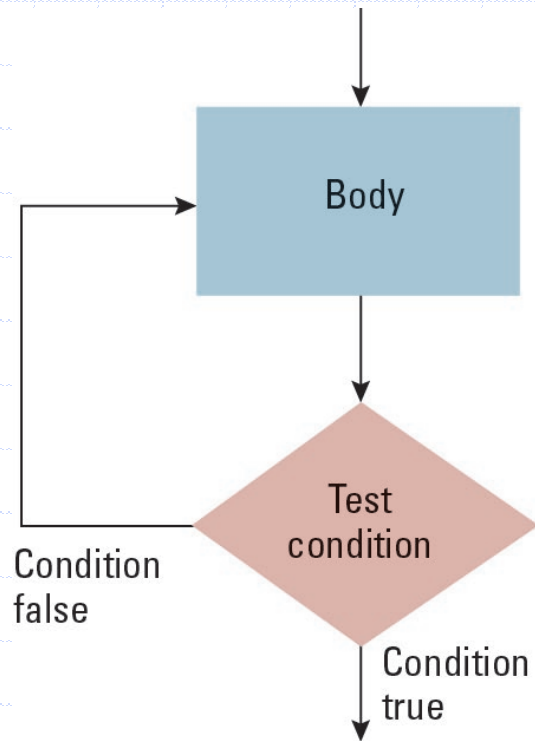


# "while" løkke



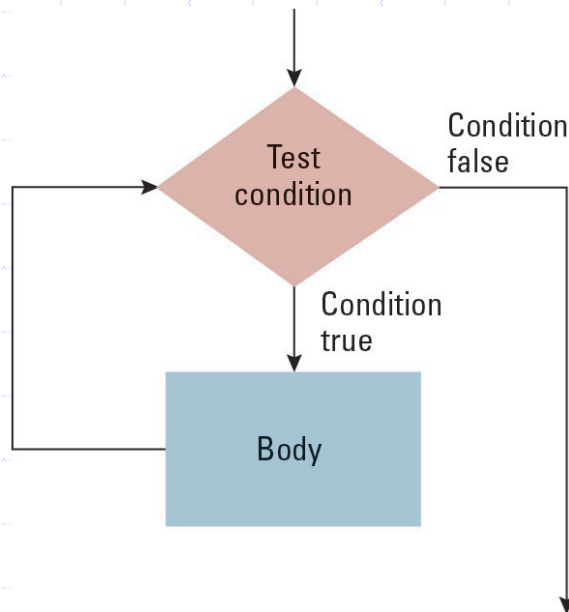
```
while ( condition ):  
    ..do your stuff
```

# “repeat until” løkke



```
clear condition
while not ( condition ) :
    ..do your stuff
```

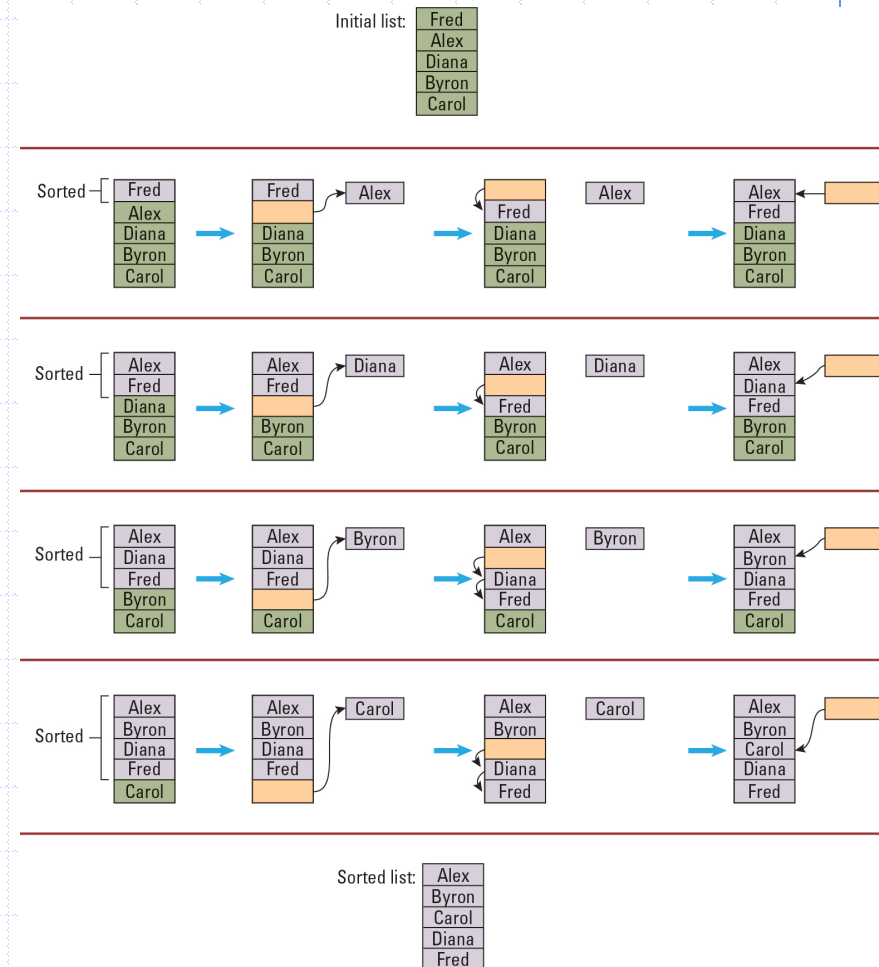
# "for" løkke looper et kendt antal gange.



```
n = 12  
i = 0  
while( i < n ):  
    ..do your stuff  
    i++
```

## Figure 5.10 Sorting the list Fred, Alex, Diana, Byron, and Carol alphabetically – insertion sort

```
def Sort(List):
    N = 2
    while (N <= length of List):
        Pivot = Nth entry in List
        Remove Nth entry leaving a hole in List
        while (there is an Entry above the
               hole and Entry > Pivot):
            Move Entry down into the hole leaving
            a hole in the list above the Entry
        Move Pivot into the hole
        N = N + 1
```

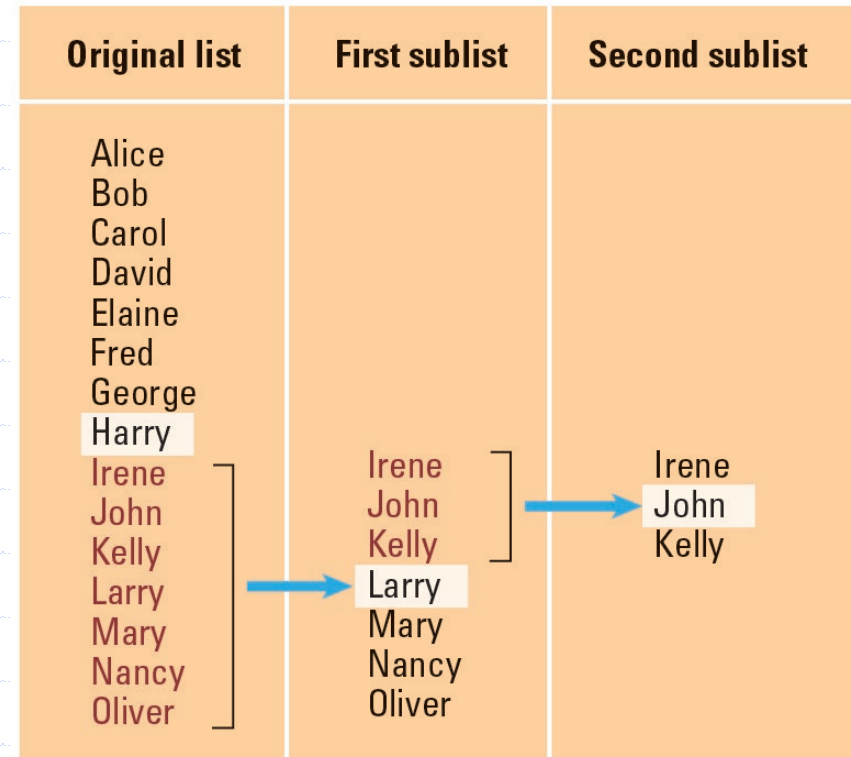


# Recursive

# Recursive Structures

- Repeating the set of instructions as a subtask of itself.
- Multiple activations of the procedure are formed, all but one of which are waiting for other activations to complete.
- Requires initialization, modification, and a test for termination (base case)
- Provides the illusion of multiple copies of the function, created dynamically in a telescoping manner
- Only one copy is actually running at a given time, the others are waiting
- Example: The Binary Search Algorithm

Figure 5.12 Applying our strategy to search a list for the entry John



```
if (List is empty):  
    Report that the search failed  
else:  
    TestEntry = middle entry in the List  
    if (TargetValue == TestEntry):  
        Report that the search succeeded  
    if (TargetValue < TestEntry):  
        Search the portion of List preceding TestEntry for  
        TargetValue, and report the result of that search  
    if (TargetValue > TestEntry):  
        Search the portion of List following TestEntry for  
        TargetValue, and report the result of that search
```

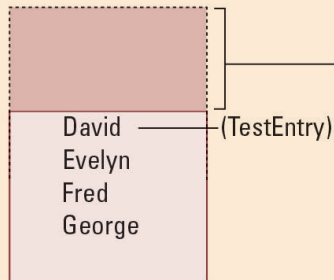
# Recursively Searching



We are here.

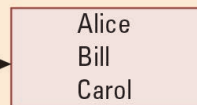
```
def Search (List, TargetValue):  
    if (List is empty):  
        Report that the search failed.  
    else:  
        TestEntry = the "middle" entry in List  
        if (TargetValue == TestEntry):  
            Report that the search succeeded.  
        if (TargetValue < TestEntry):  
            Sublist = portion of List preceding  
                TestEntry  
            Search(Sublist, TargetValue)  
        if (TargetValue > TestEntry):  
            Sublist = portion of List following  
                TestEntry  
            Search(Sublist, TargetValue)
```

List



```
def Search (List, TargetValue):  
    if (List is empty):  
        Report that the search failed.  
    else:  
        TestEntry = the "middle" entry in List  
        if (TargetValue == TestEntry):  
            Report that the search succeeded.  
        if (TargetValue < TestEntry):  
            Sublist = portion of List preceding  
                TestEntry  
            Search(Sublist, TargetValue)  
        if (TargetValue > TestEntry):  
            Sublist = portion of List following  
                TestEntry  
            Search(Sublist, TargetValue)
```

List



## Søger efter Bill



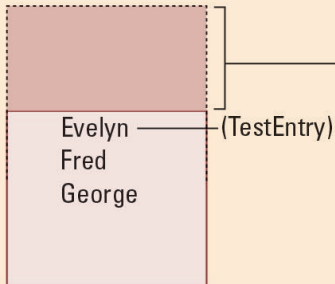
# Recursively Searching



We are here.

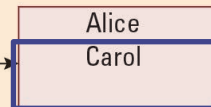
```
def Search (List, TargetValue):  
    if (List is empty):  
        Report that the search failed.  
    else:  
        TestEntry = the "middle" entry in List  
        if (TargetValue == TestEntry):  
            Report that the search succeeded.  
        if (TargetValue < TestEntry):  
            Sublist = portion of List preceding  
            TestEntry  
            Search(Sublist, TargetValue)  
        if (TargetValue > TestEntry):  
            Sublist = portion of List following  
            TestEntry  
            Search(Sublist, TargetValue)
```

List



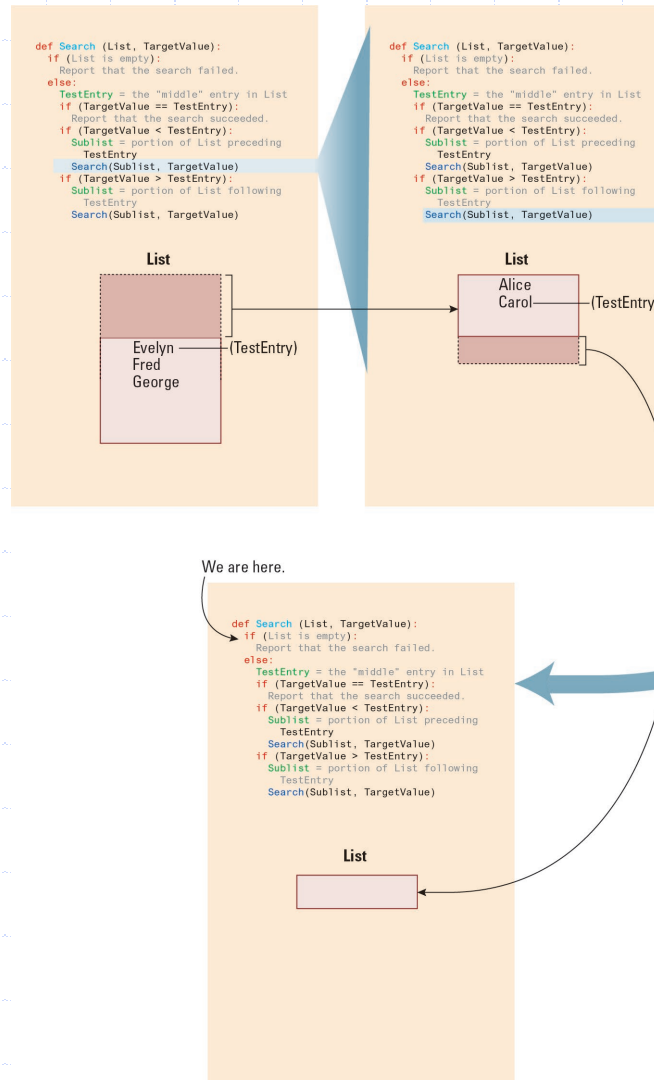
```
def Search (List, TargetValue):  
    if (List is empty):  
        Report that the search failed.  
    else:  
        TestEntry = the "middle" entry in List  
        if (TargetValue == TestEntry):  
            Report that the search succeeded.  
        if (TargetValue < TestEntry):  
            Sublist = portion of List preceding  
            TestEntry  
            Search(Sublist, TargetValue)  
        if (TargetValue > TestEntry):  
            Sublist = portion of List following  
            TestEntry  
            Search(Sublist, TargetValue)
```

List

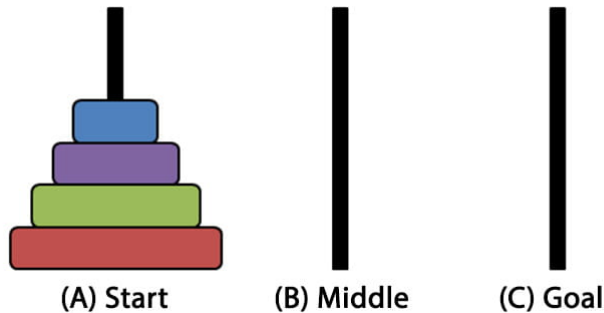


## Søger efter Charlotte

# Recursively Searching



# The Tower of Hanoi



```
def moveTower( n, start, goal ):  
  
    if( n == 1 ):  
        moveSlice( start, goal )  
    else:  
        middle = findMiddle( start, goal )  
        moveTower( n-1, start, middle )  
        moveSlice( start, goal )  
        moveTower( n-1, middle, goal )
```

# Miniprojekt

# Opgaven

- ◆ Løses som udgangspunkt i grupper af 3, fra samme klasse.
- ◆ Der skal løses 4 opgaver, som ses på de næste slides.
  - 2 Sortering algoritmer
  - 2 Søgning algoritmer
  - Test hvornår hvilke algoritmerne der er bedst på udvalgte inputs.

# Sortering algoritmer

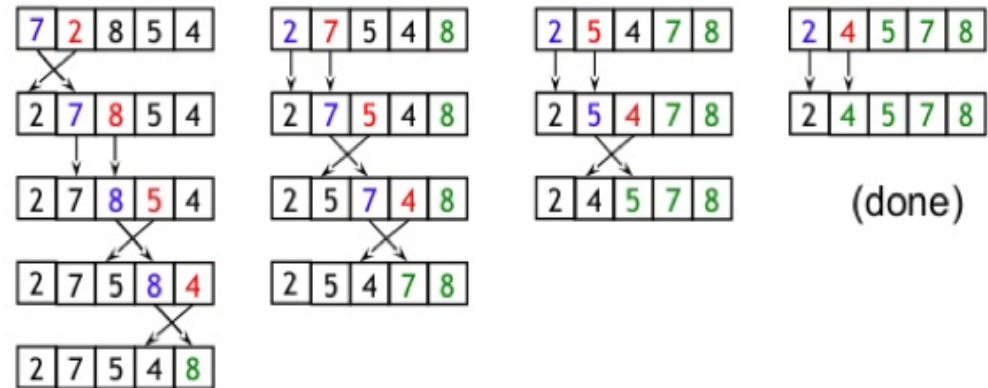
- ◆ **Bubble sort**

- ◆ **Merge sort**

# Bubble sort

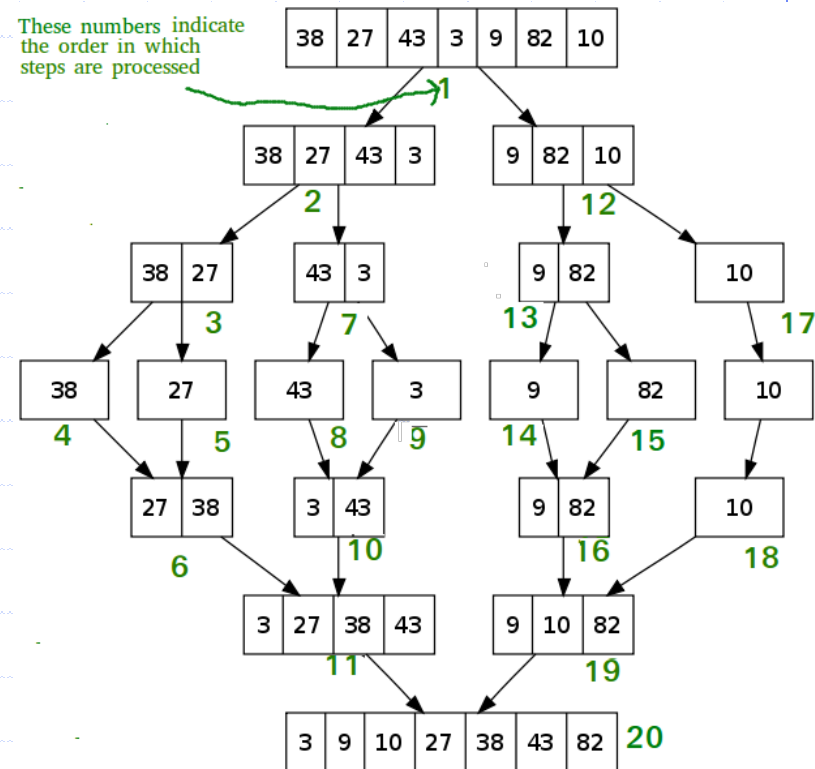
- In this example I will be outlining an example of how bubble sort works for a five-element list/array.
  - Start with the first two elements of the unsorted array.
  - Compare elements number 1 and 2. Swap the order if the second is less than the third.
  - Compare elements number 2 and 3. Swap the order if the third is less than the second.
  - Compare elements number 3 and 4. Swap the order if the fourth is less than the third.
  - Compare elements number 4 and 5. Swap the order if the fifth is less than the fourth.

## Example of bubble sort



# Merge sort

- Er en "Divide and Conquer" algoritme.
  - Deler problemet i mindre stykker
  - Sorterer rekursivt ved brug af merge
  - Ligger to dele sammen af gangen
- Består af to funktioner:
  - mergeSort
    - ◆ Deler listen op i mindre dele
  - merge
    - ◆ Ligger to dele sammen af gangen





# Test cases

- Undersøg hvordan sorteringsalgoritmerne fungerer på input:
  - Liste af 10 elementer.
  - Liste af 1000 elementer, gerne flere.
- Undersøg hvor hurtig sorteringen er i:
  - Best case – sortererede lister.
  - Worst case – omvendt sortererede lister.
  - Randomiseret input.

# Søgnings algoritmer

- **Linear search**
- **Binary search**

# Linear search

- Søger sekventielt igennem en liste for at finde en key.
- Skal ikke have sorterede input, men det bør I bruge I jeres løsninger

Key	List
3	6 4 1 9 7 3 2 8
3	6 4 1 9 7 3 2 8
3	6 4 1 9 7 3 2 8
3	6 4 1 9 7 3 2 8
3	6 4 1 9 7 3 2 8
3	6 4 1 9 7 3 2 8

<http://mathcenter.oxford.emory.edu/site/cs170/searchAndSort/>

# Binary search

- Skal have sorterede input.
- Deler problemet I mindre stykker, og kalder sig selv rekursivt:
  - Compare x with the middle element.
  - If x matches with middle element, we return the mid index.
  - Else If x is greater than the mid element, then x can only lie in right half subarray after the mid element. So we recur for right half.
  - Else (x is smaller) recur for the left half.

## Binary Search

	0	1	2	3	4	5	6	7	8	9
Search 23	2	5	8	12	16	23	38	56	72	91
	L=0	1	2	3	M=4	5	6	7	8	H=9
23 > 16 take 2 <sup>nd</sup> half	2	5	8	12	16	23	38	56	72	91
	0	1	2	3	4	L=5	6	M=7	8	H=9
23 > 56 take 1 <sup>st</sup> half	2	5	8	12	16	23	38	56	72	91
	0	1	2	3	4	L=5, M=5	H=6	7	8	9
Found 23, Return 5	2	5	8	12	16	23	38	56	72	91

# Test cases

- Undersøg hvordan sorteringsalgoritmerne fungerer på input:
  - Liste af længden 10, hvor alle elementer er forskellige.
  - Liste af længden 1000, hvor alle elementer er forskellige.
- Undersøg hvilken af søgninger er hurtigst i tilfælde af:
  - Leder efter første element i listen.
  - Søger efter sidste element i listen.
  - Søger efter det midterste element.
  - Søger efter et tilfældigt element i listen.

Spørgsmål?