Sudut Berelasi - Trigonometri Exercise

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1 Introduction

Math advanced quiz on Monday the 20th January 2025. I truly am cooked hence I made this to make sure other people don't get cooked like I do. This discusses the trigonometric ratios of allied angles exercise given by Ms Eva on the 7th of January 2025. As my other pdf's warnings I would like to warn you again, **please take this with a grain of salt, i.e. doubtfully take this information.**

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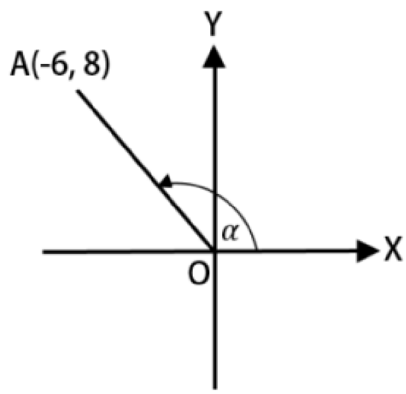
3 No. 1

1. Perhatikan gambar berikut.



Nilai $\cos \alpha$ adalah $\cdots$

A. $\frac{4}{5}$ C. $\frac{3}{4}$ E. $-\frac{4}{5}$

B. $\frac{3}{5}$ D. $-\frac{3}{5}$

We first have to make a triangle from the coordinates given of (-6,8) with the angle being adjacent to the x coordinate. From this we use tan to find the value of the initial angle.

$$\tan(\theta) = \frac{4}{3}$$
$$\tan^{-1}\frac{4}{3} = \theta$$
$$\theta = 53^\circ$$

That is something you must remember, the table goes,

มุม	0°	30°	37°	45°	53°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{1}{\sqrt{2}}$	$\frac{4}{5}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{4}{5}$	$\frac{1}{\sqrt{2}}$	$\frac{3}{5}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	$\frac{3}{4}$	1	$\frac{4}{3}$	$\sqrt{3}$	∞

This is something you must memorize, but this is about it that you need. Since α is $180 - 53$, we can safely say $\alpha = 127^\circ$. But now how do you find the value? We can use the same rule that helps with thingss like $\sin(45+30)$ but for $\cos\alpha$

$$\cos(180-53) \rightarrow \cos 180.\cos 53 + \sin 180.\sin 53$$

$$-1.\frac{3}{5}$$

$$-\frac{3}{5}$$

Hence, the answer is **D**

4 No. 2

2. Perbandingan trigonometri yang senilai dengan $\cos(180^\circ+\alpha)$ adalah $\cdots$

A. $\cos\alpha$

B. $\tan\alpha$

C. $\sin\alpha$

D. $-\cos\alpha$

E. $-\sin\alpha$

If you haven't memorized the relations, it is best to proof it and solve it. Here is the table for reference.

TABEL SUDUT BERELASI FUNGSI TRIGONOMETRI

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Sudut	$\sin\theta^\circ$	$\cos\theta^\circ$	$\tan\theta^\circ$	$\csc\theta^\circ$	$\sec\theta^\circ$	$\cot\theta^\circ$
θ°	y/r	x/r	y/x	r/y	r/x	x/y
$-\theta^\circ$	$-\sin\theta^\circ$	$\cos\theta^\circ$	$-\tan\theta^\circ$	$-\csc\theta^\circ$	$\sec\theta^\circ$	$-\cot\theta^\circ$
$(90-\theta)^\circ$	$\cos\theta^\circ$	$\sin\theta^\circ$	$\cot\theta^\circ$	$\sec\theta^\circ$	$\csc\theta^\circ$	$\tan\theta^\circ$
$(90+\theta)^\circ$	$\cos\theta^\circ$	$-\sin\theta^\circ$	$-\cot\theta^\circ$	$\sec\theta^\circ$	$-\csc\theta^\circ$	$-\tan\theta^\circ$
$(180-\theta)^\circ$	$\sin\theta^\circ$	$-\cos\theta^\circ$	$-\tan\theta^\circ$	$\csc\theta^\circ$	$-\sec\theta^\circ$	$-\cot\theta^\circ$
$(180+\theta)^\circ$	$-\sin\theta^\circ$	$-\cos\theta^\circ$	$\tan\theta^\circ$	$-\csc\theta^\circ$	$-\sec\theta^\circ$	$\cot\theta^\circ$
$(270-\theta)^\circ$	$-\cos\theta^\circ$	$-\sin\theta^\circ$	$\cot\theta^\circ$	$-\sec\theta^\circ$	$-\csc\theta^\circ$	$\tan\theta^\circ$
$(270+\theta)^\circ$	$-\cos\theta^\circ$	$\sin\theta^\circ$	$-\cot\theta^\circ$	$-\sec\theta^\circ$	$\csc\theta^\circ$	$-\tan\theta^\circ$
$(360-\theta)^\circ$	$-\sin\theta^\circ$	$\cos\theta^\circ$	$-\tan\theta^\circ$	$-\csc\theta^\circ$	$\sec\theta^\circ$	$-\cot\theta^\circ$
$(k.360+\theta)^\circ$	$\sin\theta^\circ$	$\cos\theta^\circ$	$\tan\theta^\circ$	$\csc\theta^\circ$	$\sec\theta^\circ$	$\cot\theta^\circ$

Too lazy to memorize all of this? Yeah, just prove it, it is not that hard, just use the $\sin(45+30) \to \sin(75)$ formulas. But if you are keen on memorizing this table, just ignore $\csc, \sec$, and $\cot$. But if you want to prove it, heres the way,

$$\cos(180+\alpha)$$

$$\cos 180. \cos\alpha - \sin 180. \sin\alpha$$

But you have to memorize this

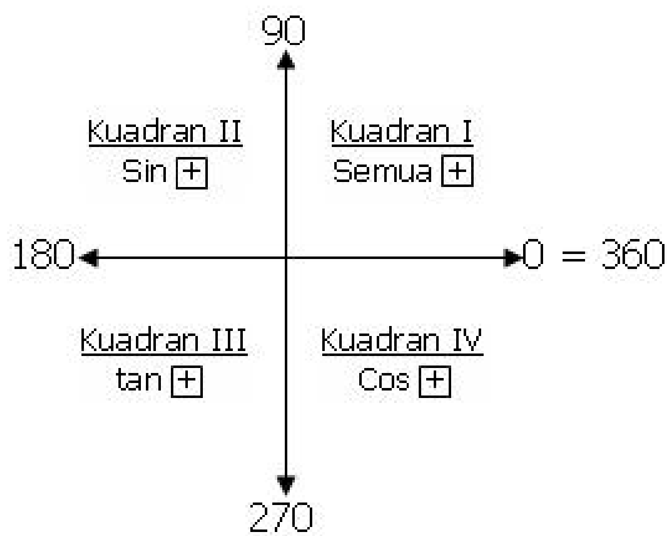

Figure 1: Enter Caption

And the $\sin, \cos$ of 90, 180, 270, 360. Hence the answer is,

$$-\cos\alpha$$

Hence, the answer is **D**

5 No. 3

3. Nilai dari $\dfrac{\sin 60^\circ}{1 + \cos 60^\circ} = \cdots$

A. $\tan 60^\circ$

B. $\tan 30^\circ$

C. $\sec 60^\circ$

D. $\csc 60^\circ$

E. $\sin 60^\circ$

This one is simple if you remember the special angles table from $0^\circ - 90^\circ$.

$$\frac{\sin 60}{1 + \cos 60}$$

$$\frac{\frac{\sqrt{3}}{2}}{1 + \frac{1}{2}}$$

$$\frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}}$$

$$\frac{\sqrt{3}}{3} \rightarrow \tan 30$$

Hence, the answer is **B**.

6 No. 4

4. Nilai dari $\sin \frac{2}{3}\pi + \sin \frac{7}{3}\pi$ adalah $\cdots$

A. $-\sqrt{3}$
B. -1
C. 0
D. 1
E. $\sqrt{3}$

It is better to use these set of formulas to solve these questions, although not particularly needed, honestly I find it better in my opinion.

$$\boxed{\begin{aligned} sin(2A) &= 2\,sin(A)\,cos(A) \\ cos(2A) &= cos^2(A) - sin^2(A) \\ &= 1 - 2sin^2(A) \\ &= 2cos^2(A) - 1 \\ \tan(2A) &= \frac{2\tan(A)}{1 - tan^2(A)} \end{aligned}}$$

Let us remind ourselves that in this case, π is radians, hence it is 180°

$$\sin\frac{2(180)}{3}+\sin\frac{7(180)}{3}$$
$$\sin 120+\sin 420$$
$$\sin(60+60)+\sin(360+60)$$
$$\sin 2(60)+\sin(360+60)$$

Again, very handy if you memorized the values of trigonometric functions of 90, 180, 270, 360. As you can see here, $\sin 2(60)$ looks like the $\sin 2(A)$ given above, it becomes $2\sin A\cos A$.

$$2\sin 60\cos 60+\sin 360.\sin 60+\cos 360.\cos 60$$
$$\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\rightarrow\sqrt{3}$$

Hence the answer is **E**.

7 No. 6

6. Jika $\tan\alpha=\frac{3}{4}$ dengan $180^\circ\le\alpha\le 270^\circ$, nilai $\sin\alpha=\cdots$

A. $-\frac{3}{4}$ C. $-\frac{3}{5}$ E. $\frac{3}{4}$

B. $-\frac{4}{5}$ D. $\frac{3}{5}$

Yeah number 5 is basic trigonometry, I'll skip it. The special angles table will come useful again for this question, if you know it already this will be very easy. $\tan 37$

is roughly equal to $\frac{3}{4}$ and since its in the 3rd quadrant, hence $\sin\theta$ will be negative. Again $\sin 37$ is rougly equal to $\frac{3}{5}$, and since its negative, hence the answer is **C**.

8 No. 7

7. Nilai $\dfrac{\sin 150^\circ + \sin 120^\circ}{\cos 210^\circ - \cos 300^\circ} = \cdots$

A. 2 C. 0 E. -1

B. 1 D $-\dfrac{1}{2}$

You can either just prove it, or directly use the angle relations, whatever is your cup of tea. I haven't memorised it so I'll prove it.

$$\frac{\sin 150 + \sin 120}{\cos 210 - \cos 300}$$
$$\frac{\sin(90+60) + \sin 2(60)}{\cos(180+30) - \cos(270+30)}$$
$$\frac{[\sin 90. \cos 60 + \cos 90. \sin 60] + [2 \sin 60 \cos 60]}{[\cos 180. \cos 30 - \sin 180. \sin 30] - [\cos 270. \cos 30 - \sin 270. \sin 30]}$$
$$\frac{\frac{1}{2} + \frac{\sqrt{3}}{2}}{-\frac{\sqrt{3}}{2} - \frac{1}{2}}$$
$$\frac{1+\sqrt{3}}{-\sqrt{3}-1}$$

Rationalizing this

$$\frac{1+\sqrt{3}}{-\sqrt{3}-1} * \frac{1-\sqrt{3}}{1-\sqrt{3}} \to -1$$

Hence the answer is **E**

9 No. 8

8. Bentuk sederhana $\sin\left(\frac{\pi}{2}+2x\right)+\sin\left(\frac{\pi}{2}-2x\right)$ adalah $\cdots$

A. $2\sin 2x$ D. $2\cos x$

B. $2\cos 2x$ E. $\cos 2x$

C. $2\sin x$

Again, I'm just gonna prove and solve this, do remember that π is $180°$.

$$\sin(90+2x)+\sin(90-2x)$$

$$[\sin 90.\cos 2x+\cos 90.\sin 2x]+[\sin 90.\cos 2x-\cos 90.\sin 2x]$$

$$2\cos 2x$$

Hence, the answer is **B**.

10 No. 9

9. Jika $\sin 70^\circ = p$ dan $\cos 70^\circ = q$, nilai dari $\cos 110^\circ \cdot \cot 160^\circ + \sin 200^\circ = \cdots$

A. $p - q$ D. $2q$

B. $p + q$ E. $2p - 2q$

C. $2p$

Just use the $\sin(A \pm B)$ but suited for 70°. Then substitute the values of $\sin 70$ and $\cos 70$.

$$\cos(180 - 70) * \cot(90 + 70) + \sin(270 - 70)$$

$$[\cos 180.\cos 70 + \sin 180.\sin 70] * [\frac{\cos(90 + 70)}{\sin(90 + 70)}] + \sin 270.\cos 70 - \cos 270.\sin 70$$

$$-q * \frac{-p}{q} - q \rightarrow p - q$$

Hence, the answer is **A**

11 No. 10

10. Jika $\tan 23^\circ = a$ dan $\sin 23^\circ = b$. Hasil dari $\frac{\tan 293^\circ + \sin 157^\circ}{\cot 113^\circ}$ adalah ....

A. $-1 + \frac{b}{a}$ D. $\frac{-1-ab}{a^2}$

B. $-1 - \frac{b}{a}$ E. $\frac{1-ab}{-a^2}$

C. $\frac{-1+ab}{-a^2}$

I have a serious feeling that it is not best to directly prove the answer, but prove the angle relations. But let us see, by first using the same formula in the likes of $\sin(A+B)$.

$$\frac{\tan(270+23)+\sin(180-23)}{\cot(90+23)}$$

My suspicion is true, the $\tan(A+B)$ formula is,

$$\frac{\tan A+\tan B}{1-\tan A.\tan B}$$

But as you know, $\tan B=\infty$, making this unsolvable, so let us solve by solving the relation.

$$\frac{\tan(270+\theta)+\sin(180-\theta)}{\cot(90+\theta)}$$

$$\frac{\frac{\sin(270+\theta)}{\cos(270+\theta)}+\sin 180.\cos\theta-\cos 180.\sin\theta}{\frac{\cos(90+23)}{\sin(90+23)}}$$

$$\frac{\frac{\sin 270.\cos\theta+\cos 270.\sin\theta}{\cos 270.\cos\theta-\sin 270+\theta}+b}{\frac{\cos 90.\cos 23-\sin 90.\sin 23}{\sin 90.\cos 23+\cos 90.\sin 23}}$$

$$\frac{-\frac{1}{a}+b}{-a}\rightarrow\frac{-1+ab}{-a^2}$$

Hence the answer is **C**.

Feel free to post this around, I already gave the labels, feel free to also ask, wallahi i'm so cooked.