Limit + Basic Derivative Speedrun

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1 Introduction

Unfortunately in mathematics we entered the start to calculus, limits. Even though limits are easy, this is just the start of a horrible and painful ride through calculus level mathematics. This document will explain linear algebra limits (Advanced Mathematics will have the trigonometric and logarithmic limits) and basic derivatives. Please take this with a grain of salt.

2 Basic Definition of Limits

The basic definition of limits is approaching the limit of the function, for example:

$$\lim_{x \to 3} f(3)$$

Where x is approaching 3 in the function of $f(x)$, that means from both sides of the function (in a graph helps to visualise) like from 2 and 4, it approaches 3 in the function of $f(x)$. It will get closer and closer to 3, so example it goes from 2.5, to 2.9, to 2.999999 and 4 gets closer from 3.5, 3.1, 3.00000001. This will give the approximate answer of the function $f(x)$, and that is the definition of limit. For example,

$$\lim_{x \to 5} x - 3$$

In this case, the x will approach the function $x - 3$ from both sides, being 4 and 6

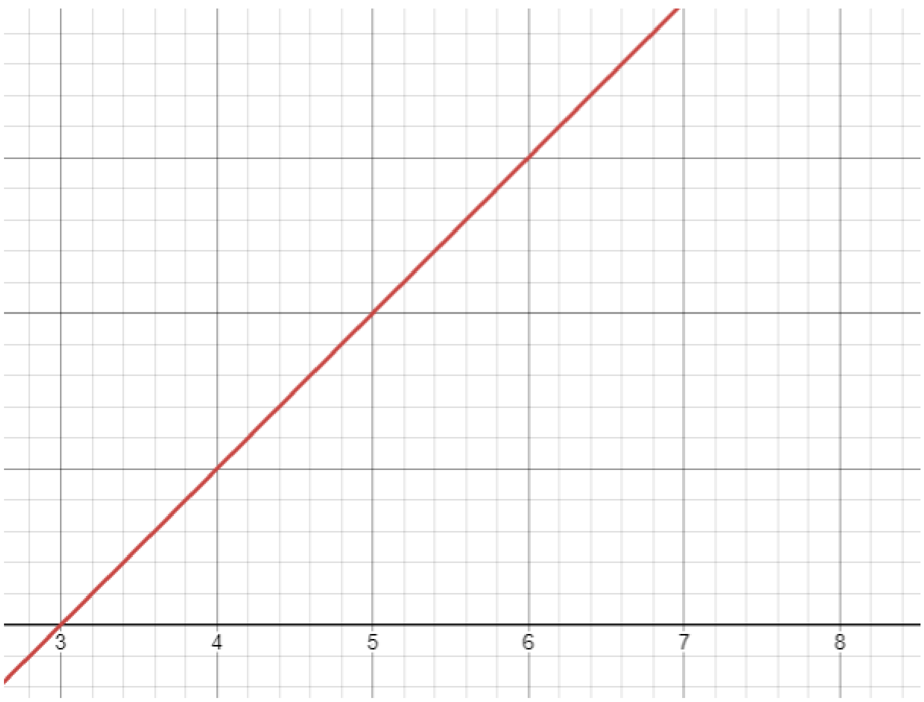


Figure 1: Graph of $x - 3$

4 will approach the limit of 5 by getting as approximately close to 5 as possible, 4.99999999 and putting it on the x, it will result in 1.999999 which is very close to the answer being 2, and 6 will approach the limit of 5 by getting as

close to 5 as possible, 5.00000001 as the x, making the answer 2.000000001, hence the answer is

x	$f(x) = x - 3$
4.9	1.9
4.99	1.99
4.999	1.999
5	2
5.001	2.001
5.01	2.01
5.1	2.1

$$\lim_{x \to 5} 5 - 3 = 2$$

3 Indeterminate Limits

Indeterminate limits are limits where just by inputting the limit directly, the answer is indeterminate, or in my words, literally impossible. examples of such are $\frac{0}{0}$ and $\frac{\infty}{\infty}$ where it is a cardinal sin to get such numbers as results in limits.

Do note that results like these $\frac{1}{0}$ are normal, and it is allowed as an answer, but indeterminate forms are cardinal sins not allowed anywhere near mathematics, also do note $\frac{1}{\infty}$ is 0, actually any number over infinity is just 0.

Now you may ask, well Darren, then how do you solve limits where the answer will be a guaranteed in a indeterminate form? Well dear reader, I have the answer for this (well mathematicians taught me), here are 4 steps on how to solve limits where you do or don't know it will be in an indeterminate form.

4 How To Solve Indeterminate Limits 101

Well as I promised, this will help you solve indeterminate limits.

4.1 Just input the x

This is just to solve just incase it is not guaranteed an indeterminate limit. The answer may just be something like $\frac{1}{0}$ or $\frac{1}{\infty}$ where it is a legitimate answer. Here is an example,

$$\lim_{x \to 2} \frac{3-x}{x-2}$$

The result will just be $\frac{1}{0}$, an actual legitimate answer.

4.2 Factoring

Oh f*ck! You might say doing your test and after inputting the x you still got an indeterminate form, sh*t you might say. Well there is still another way (and by far the most popular way in limits). Factoring.

$$\lim_{x \to 2} \frac{x^2-4}{x-2}$$

Inputting the numbers, the answer will still be in an indeterminate form of $\frac{0}{0}$. My Lord a cardinal sin! But you may realise that,

$$x^2 - 4$$

Is just in the form of $a^2 - b^2$ where it can be made into $(a + b)(a - b)$.

$$\lim_{x \to 2} \frac{(x+2)(x-2)}{x-2}$$

You see what we can do next, you are correct! Cross out the (x-2) that caused this cardinal sin.

$$\lim_{x \to 2} x + 2$$

$$\lim_{x \to 2} 2 + 2 = 4$$

Hence the answer is 4, although not all questions are in the form of $a^2 - b^2$, this by far helps, in general just factorise.

4.3 Rationalise.

My Lord you say as you pray, you found a square root! What do we do? Well of course you can still factorise, but what if it is in the form of,

$$\lim_{x \to 3} \frac{\sqrt{x+1} - 2}{x-3}$$

Well first off, calm down, just rationalise this.

$$\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}*\frac{\sqrt{x+1}+2}{\sqrt{x+1}+2}$$

$$\lim_{x\to 3}\frac{(\sqrt{x+1}-2)(\sqrt{x+1}+2)}{x-3(\sqrt{x+1}+2)}$$

$$\lim_{x\to 3}\frac{x-3}{(x-3)(\sqrt{x+1}+2)}$$

Just cross out the $x-3$

$$\lim_{x\to 3}\frac{1}{\sqrt{x+1}+2}$$

The answer is just $\frac{1}{4}$

4.4 l'Hôpital's Rule

Honestly, use this as the last resort as this maybe the hardest if you use it wrong, and easiest if you use it correctly. For now please do not use this on roots, just factorise/rationalise. This rule states that,

$$\lim_{x\to}\frac{f(x)}{g(x)}=\frac{f'(x)}{g'(x)}$$

Where $f'(x)$ is the derivative of $f(x)$ and $g'(x)$ is the derivative of $g(x)$. Where you input the limit after deriving it, and pray it doesn't comes out as an indeterminate form, and if it does, derive it again until it is a valid answer.

Now you may ask, Darren? How the f*ck do you derive. Well dear learner I will tell you, please use this as the last step if all things go wrong, don't use this on square roots (they count as $fog(x)$) because you will need to use the chain rule (don't try).

4.4.1 Derivative

Derivative is actually the next topic after limits, as it stems from limits, even it's formula uses limits, but I'll just show the shortcut, the original formula is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Long? Yeah, the short cut is called the power rule, where,

$$y = x^n$$
$$y' = nx^{n-1}$$

Where y' is the derivative of y. An example will be

$$y = x^2$$
$$y' = 2x^{2-1}$$
$$y' = 2x$$

The derivative of any constant is 0, and derivative of e^x is still e^x.

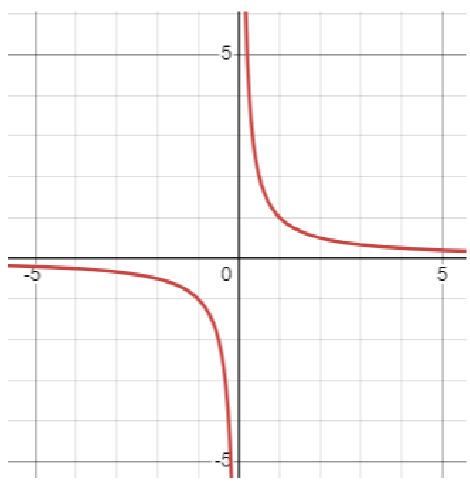
Now you know, use this in l'Hôpital's Rule, please only use when necessary and if it's not a square root encompassing the function. An example will be

$$\lim_{x\to 2} \frac{x^2 - 4}{x - 2}$$

$$\lim_{x\to 2} \frac{2x}{1}$$

$$\lim_{x\to 2} \frac{2(2)}{1} = 4$$

5 0^+ and 0^-



In this graph of $\frac{1}{x}$, let us assume x is approaching 0, if the limit is 0^+ it goes to $+\infty$ (approaching from the right) and if it's 0^- it goes to $-\infty$ (approaching from the left). In this case, ignore the other side (the +/-)