

TEP4156 Formula sheet

Velocity derivative

$$\frac{\partial u_i}{\partial x_j} = \dot{\epsilon}_{ij} + \dot{\Omega}_{ij}$$

2D Shear angle rate

$$\dot{\epsilon}_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

2D Rotation angle rate

$$\dot{\Omega}_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right)$$

3D Rotation angle rate

$$(\dot{\vec{\Omega}})_k = \frac{1}{2} \epsilon_{ijk} \dot{\Omega}_{ij}$$

$$\dot{\vec{\Omega}} = \frac{1}{2} \vec{\nabla} \times \vec{u}$$

Mass conservation

$$\frac{D\rho}{Dt} + \rho \vec{\nabla} \cdot \vec{u} = 0$$

Momentum conservation

$$\rho \frac{Du_i}{Dt} = \frac{\partial \sigma_{ij}}{\partial x_j} + \rho f_i$$

Energy conservtion

$$\rho \frac{DE}{Dt} = \frac{\partial}{\partial x_j} (\sigma_{ij} u_i) + \rho f_i u_i - \frac{\partial q_k}{\partial x_k}$$

Total energy

$$E = e + \frac{1}{2} u_i u_i$$

Deformation

$$\sigma_{ij} = -p \delta_{ij} + \sigma'_{ij}$$

Energy equation

$$\rho \frac{De}{Dt} + p \frac{\partial u_k}{\partial x_k} = \sigma'_{ij} \frac{\partial u_i}{\partial x_j} - \frac{\partial q_k}{\partial x_k}$$

Enthalpy equation

$$\rho \frac{Dh}{Dt} - \frac{Dp}{Dt} = \sigma'_{ij} \frac{\partial u_i}{\partial x_j} - \frac{\partial q_k}{\partial x_k}$$

$$\sigma'_{ij} \frac{\partial u_i}{\partial x_j} \geq 0$$

Fourier's law

$$q_k = -k \frac{\partial T}{\partial x_k}$$

Deformation law for newtonian fluids

$$\sigma'_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k} \right) + \mu_B \delta_{ij} \frac{\partial u_k}{\partial x_k}$$

Creeping flow

$$\text{Re} \rightarrow 0 : \quad \vec{\nabla} p = \mu \vec{\nabla}^2 \vec{u}$$

Boundary layer equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U \frac{dU}{dx} + \nu \frac{\partial^2 u}{\partial y^2}$$

Steady heat equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2}$$

Displacement thickness

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U} \right) dy$$

Momentum thickness

$$\theta = \int_0^\infty \frac{u}{U} \left(1 - \frac{u}{U} \right) dy$$

Stream function

$$\psi(x, y) = \sqrt{\frac{2}{m+1}} \nu x U f(\eta)$$

Similarity variable

$$\eta = y \sqrt{\frac{m+1}{2} \frac{U}{\nu x}}$$

Falkner Skan equation

$$f''' + f f'' + \frac{2m}{m+1} (1 - f'^2) = 0$$

Kármán integral relation

$$\frac{d\theta}{dx} + \left(2 + \frac{\delta^*}{\theta} \right) \frac{\theta}{U} \frac{dU}{dx} = \frac{C_f}{2}$$

Additional formulas

Fluid enthalpy

$$h = e + \frac{p}{\rho}$$

Bulk viscosity

$$\mu_B = \lambda + \frac{2}{3}\mu$$

Dynamic viscosity

$$\nu = \frac{\mu}{\rho}$$

Mechanical pressure

$$\bar{P} = P - \mu_B \frac{\partial u_k}{\partial x_k}$$

Navier-Stokes equation

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla}) \vec{u} = -\frac{1}{\rho} \vec{\nabla} P + \nu \vec{\nabla}^2 \vec{u} + \vec{g}$$

Skin friction coefficient

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2}$$

Drag coefficient (flat plate)

$$C_D = \frac{1}{L} \int_0^L C_f dx, \quad C_f = 2 \frac{\partial \theta}{\partial x}$$

Shape factor

$$H = \frac{\delta^*}{\theta}$$

Laminar boundary layer

$$\delta \ll L,$$

$$\text{Re} \gg 1$$

Notation

Material derivative

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + (\vec{u} \cdot \vec{\nabla})$$

Advanced formulas

Orr-Sommerfeld equation

$$(U - c)(v'' - \alpha^2 v) - U''v + \frac{i\nu}{\alpha}(v'''' - 2\alpha^2 v'' + \alpha^4 v) = 0$$

Inviscid OS (Rayleigh) equation

$$(U - c)(v'' - \alpha^2 v) - U''v = 0$$

Inviscid stability theorem

Condition 1: $U''(y_p) = 0$

Condition 2: $U'' \cdot (U - U(y_p)) < 0$ somewhere

We know almost 100% that the flow is unstable

if both conditions are met. If not, we can not draw any conclusions.

TODO: Add Einstein notation, kronecker delta, levi civita, σ' , Couette flow?, Bessel functions, λ , Prandtl og gutta