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TEP4156 Formula sheet

Velocity derivative

$$\frac{\partial u_i}{\partial x_j} = \dot{\varepsilon}_{ij} + \dot{\Omega}_{ij}$$

2D Shear angle rate

$$\dot{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

2D Rotation angle rate

$$\dot{\Omega}_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right)$$

3D Rotation angle rate

$$(\dot{\vec{\Omega}})_k = \frac{1}{2}\epsilon_{ijk}\dot{\Omega}_{ij}$$

$$\dot{\vec{\Omega}} = \frac{1}{2}\vec{\nabla} \times \vec{u}$$

Mass conservation

$$\frac{D\rho}{Dt} + \vec{\nabla} \cdot (\rho\vec{u}) = 0$$

Momentum conservation

$$\rho\frac{Du_i}{Dt} = \frac{\partial \sigma_{ij}}{\partial x_j} + \rho f_i$$

Energy conservation

$$\rho\frac{DE}{Dt} = \frac{\partial}{\partial x_j}(\sigma_{ij}u_i) + \rho f_i u_i - \frac{\partial q_k}{\partial x_k}$$

Total energy

$$E = e + \frac{1}{2}u_i u_i$$

Deformation

$$\sigma_{ij} = -p\delta_{ij} + \sigma'_{ij}$$

Energy equation

$$\rho\frac{De}{Dt} + p\frac{\partial u_k}{\partial x_k} = \sigma'_{ij}\frac{\partial u_i}{\partial x_j} - \frac{\partial q_k}{\partial x_k}$$

Enthalpy equation

$$\rho\frac{Dh}{Dt} - \frac{Dp}{Dt} = \sigma'_{ij}\frac{\partial u_i}{\partial x_j} - \frac{\partial q_k}{\partial x_k}$$

$$\sigma'_{ij}\frac{\partial u_i}{\partial x_j} \geq 0$$

Fourier's law

$$q_k = -k\frac{\partial T}{\partial x_k}$$

Deformation law for newtonian fluids

$$\sigma'_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial u_k}{\partial x_k}\right) + \mu_B\delta_{ij}\frac{\partial u_k}{\partial x_k}$$

Creeping flow

$$\mathrm{Re} \to 0: \quad \vec{\nabla}p = \mu\vec{\nabla}^2\vec{u}$$

Boundary layer equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \nu\frac{\partial^2 u}{\partial y^2}$$

Steady heat equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}$$

Displacement thickness

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy$$

Momentum thickness

$$\theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

Stream function

$$\psi(x,y) = \sqrt{\frac{2}{m+1}\nu x U}f(\eta)$$

Similarity variable

$$\eta = y\sqrt{\frac{m+1}{2}\frac{U}{\nu x}}$$

Falkner Skan equation

$$f''' + ff'' + \frac{2m}{m+1}(1 - f'^2) = 0$$

Kármán integral relation

$$\frac{d\theta}{dx} + \left(2 + \frac{\delta^*}{\theta}\right)\frac{\theta}{U}\frac{dU}{dx} = \frac{C_f}{2}$$

Additional formulas

Fluid enthalpy

$h = e + \frac{p}{\rho}$

Bulk viscosity

$\mu_B = \lambda + \frac{2}{3}\mu$

Dynamic viscosity

$\nu = \frac{\mu}{\rho}$

Mechanical pressure

$\bar{P} = P - \mu_B \frac{\partial u_k}{\partial x_k}$

Navier-Stokes equation

$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla})\vec{u} = -\frac{1}{\rho}\vec{\nabla}P + \nu\vec{\nabla}^2\vec{u} + \vec{g}$

Skin friction coefficient

$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2}$

Drag coefficient (flat plate)

$C_D = \frac{1}{L}\int_0^L C_f dx, \quad C_f = 2\frac{\partial \theta}{\partial x}$

Shape factor

$H = \frac{\delta *}{\theta}$

Laminar boundary layer

$\delta \ll L,$

$\mathrm{Re} \gg 1$

Stream function

$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$

Notation

Material derivative

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + (\vec{u} \cdot \vec{\nabla})$$

Levi-Civita symbol

$$\epsilon_{ijk} = \begin{cases} 1, & \text{if } ijk = 123 \text{ or cyclic permutation} \\ -1, & \text{if } ijk = 321 \text{ or cyclic permutation} \\ 0, & \text{otherwise} \end{cases}$$

Einstein summation

An index that appears twice is summed over

Example:

$$a_i b_i = \sum_{i=1}^{3} a_i b_i = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Kronecker delta

$$\delta_{ij} = \begin{cases} 0, & \text{If } i \neq j \\ 1, & \text{If } i = j \end{cases}$$

Laplace operator

$\Delta = \nabla^2$

Advanced formulas

Orr-Sommerfeld equation

$$(U-c)(v''-\alpha^2 v)-U''v+\frac{i\nu}{\alpha}(v''''-2\alpha^2 v''+\alpha^4 v)=0$$

Inviscid OS (Rayleigh) equation

$$(U-c)(v''-\alpha^2 v)-U''v=0$$

Inviscid stability theorem

Condition 1: $U''(y_p)=0$

Condition 2: $U''\cdot(U-U(y_p))<0$ somewhere

We know almost 100% that the flow is instable if both conditions are met. If not, we can not draw any conlusions.

TODO: Add Einstein notation, kronecker delta, σ', Couette flow?, Bessel functions, λ, Prandtl og gutta