## TEP4156 Formula sheet

### Velocity derivative

$$\frac{\partial u_i}{\partial x_j} = \dot{\varepsilon}_{ij} + \dot{\Omega}_{ij}$$

#### 2D Shear angle rate

$$\dot{\varepsilon}_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_i} + \frac{\partial u_j}{\partial x_i} \right)$$

### 2D Rotation angle rate

$$\dot{\Omega}_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right)$$

#### 3D Rotation angle rate

$$(\dot{\vec{\Omega}})_k = \frac{1}{2} \epsilon_{ijk} \dot{\Omega}_{ij}$$

$$\dot{\vec{\Omega}} = \frac{1}{2} \vec{\nabla} \times \vec{u}$$

### Mass conservation

$$\frac{D\rho}{Dt} + \vec{\nabla} \cdot (\rho \vec{u}) = 0$$

#### Momentum conservation

$$\rho \frac{Du_i}{Dt} = \frac{\partial \sigma_{ij}}{\partial x_i} + \rho f_i$$

#### Energy conservation

$$\rho \frac{DE}{Dt} = \frac{\partial}{\partial x_j} (\sigma_{ij} u_i) + \rho f_i u_i - \frac{\partial q_k}{\partial x_k}$$

## Total energy

$$E = e + \frac{1}{2}u_i u_i$$

#### Deformation

$$\sigma_{ij} = -p\delta_{ij} + \sigma'_{ij}$$

#### **Energy equation**

$$\rho \frac{De}{Dt} + p \frac{\partial u_k}{\partial x_k} = \sigma'_{ij} \frac{\partial u_i}{\partial x_j} - \frac{\partial q_k}{\partial x_k}$$

#### Enthalpy equation

$$\rho \frac{Dh}{Dt} - \frac{Dp}{Dt} = \sigma'_{ij} \frac{\partial u_i}{\partial x_j} - \frac{\partial q_k}{\partial x_k}$$

$$\sigma'_{ij}\frac{\partial u_i}{\partial x_j} \geq 0$$

#### Fourier's law

$$q_k = -k \frac{\partial T}{\partial x_k}$$

#### Deformation law for newtonian fluids

$$\sigma'_{ij} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k} \right) + \mu_B \delta_{ij} \frac{\partial u_k}{\partial x_k}$$

### Creeping flow

Re 
$$\rightarrow 0$$
:  $\vec{\nabla} p = \mu \vec{\nabla}^2 \vec{u}$ 

## Boundary layer equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + v\frac{\partial^2 u}{\partial y^2}$$

#### Steady heat equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2}$$

## Displacement thickness

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy$$

## Momentum thickness

$$\theta = \int_{0}^{\infty} \frac{u}{U} \left( 1 - \frac{u}{U} \right) dy$$

#### Stream function

$$\psi(x,y) = \sqrt{\frac{2}{m+1}\nu x U} f(\eta)$$

## Similarity variable

$$\eta = y\sqrt{\frac{m+1}{2}\frac{U}{vx}}$$

### Falkner Skan equation

$$f''' + ff'' + \frac{2m}{m+1}(1 - f'^2) = 0$$

#### Kármán integral relation

$$\frac{d\theta}{dx} + \left(2 + \frac{\delta^*}{\theta}\right) \frac{\theta}{U} \frac{dU}{dx} = \frac{C_f}{2}$$

# Additional formulas

## Fluid enthalpy

$$h = e + \frac{p}{\rho}$$

## Bulk viscosity

$$\mu_B = \lambda + \frac{2}{3}\mu$$

## Dynamic viscosity

$$\nu = \frac{\mu}{\rho}$$

## Mechanical pressure

$$\bar{P} = P - \mu_B \frac{\partial u_k}{\partial x_k}$$

## Navier-Stokes equation

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla}) \vec{u} = -\frac{1}{\rho} \vec{\nabla} P + \nu \vec{\nabla}^2 \vec{u} + \vec{g}$$

## Skin friction coefficient

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2}$$

## Drag coefficient (flat plate)

$$C_D = \frac{1}{L} \int_0^L C_f dx, \quad C_f = 2 \frac{\partial \theta}{\partial x}$$

## Shape factor

$$H = \frac{\delta *}{\theta}$$

## Laminar boundary layer

$$\delta \ll L,$$

$$\text{Re} \gg 1$$

## Stream function

$$u = \frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x}$$

# Notation

#### Material derivative

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + (\vec{u} \cdot \vec{\nabla})$$

## Levi-Civita symbol

$$\epsilon_{ijk} = \begin{cases} 1, & \text{if } ijk = 123 \text{ or cyclic permutation} \\ -1, & \text{if } ijk = 321 \text{ or cyclic permutation} \\ 0, & \text{otherwise} \end{cases}$$

#### Einstein summation

An index that appears twice is summed over Example:

$$a_i b_i = \sum_{i=1}^3 a_i b_i = a_1 b_1 + a_2 b_2 + a_3 b_3$$

### Kronecker delta

$$\delta_{ij} = \begin{cases} 0, & \text{If } i \neq j \\ 1, & \text{If } i = j \end{cases}$$

## Laplace operator

$$\Delta = \nabla^2$$

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# Advanced formulas

## Orr-Sommerfeld equation

$$(U - c)(v'' - \alpha^2 v) - U''v + \frac{i\nu}{\alpha}(v'''' - 2\alpha^2 v'' + \alpha^4 v) = 0$$

## Inviscid OS (Rayleigh) equation

$$(U - c)(v'' - \alpha^2 v) - U''v = 0$$

## Inviscid stability theorem

Condition 1:  $U''(y_p) = 0$ 

Condition 2:  $U'' \cdot (U - U(y_p)) < 0$  somewhere We know almost 100% that the flow is instable if both conditions are met. If not, we can not draw any conclusions.

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TODO: Add Einstein notation, kronecker delta,  $\sigma'$ , Couette flow?, Bessel functions,  $\lambda$ , Prandtl og gutta