The state of the art in MILP formulations for the guillotine 2D knapsack and related problems

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Co-Advisor: Olinto Araújo

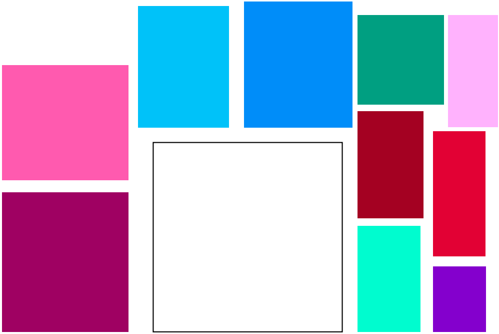
Friday, July 8, 2022

Outline

Outline for “Introduction”

Objective Function

The Guillotine 2D Knapsack Problem (G2KP) maximizes the profit obtained from cutting pieces from a single large ‘original plate’.



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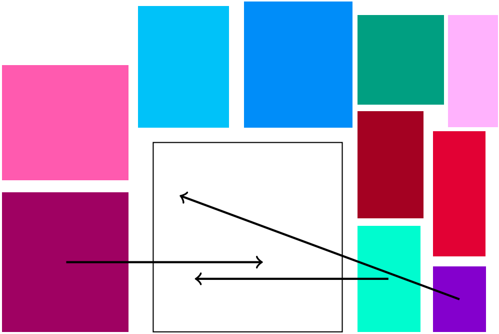

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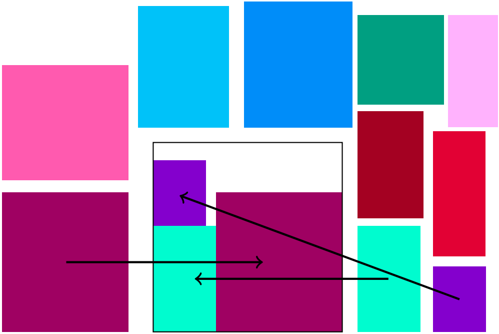

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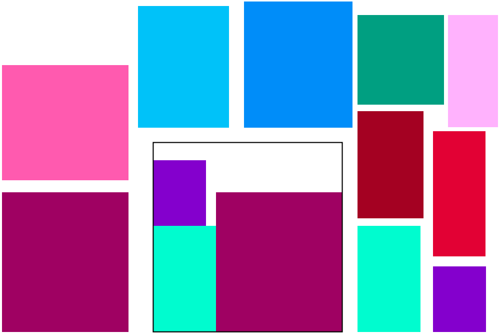
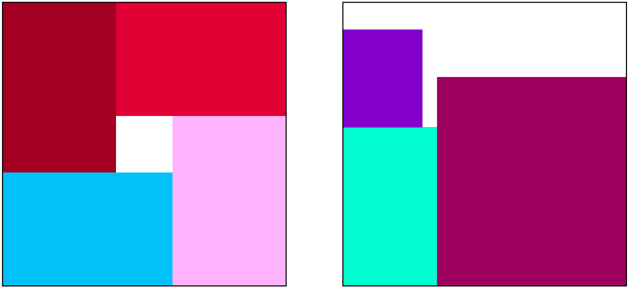


Guillotine Cuts

"every cut always go from one side of a plate to other; a cut never stops or starts from the middle of a plate"



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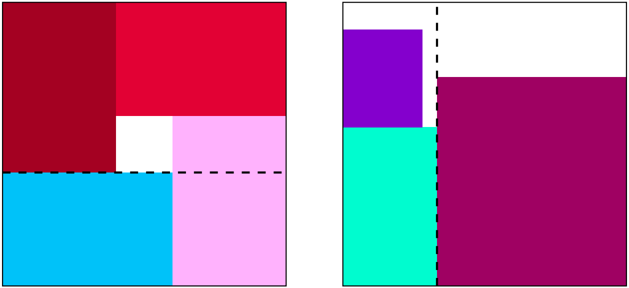

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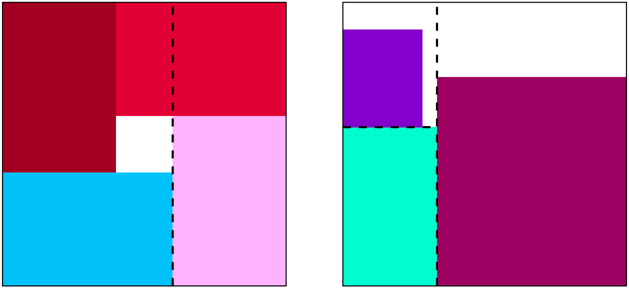

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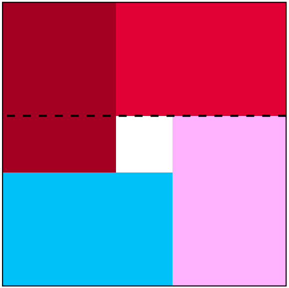

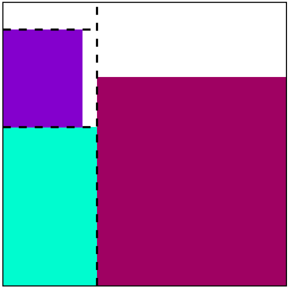

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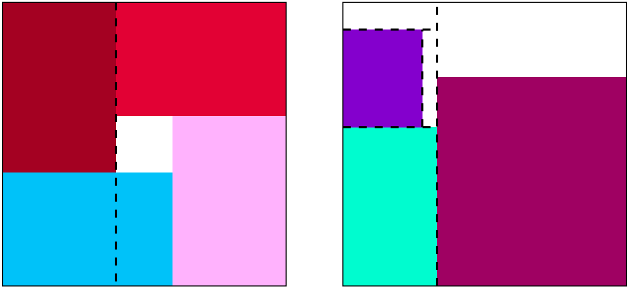

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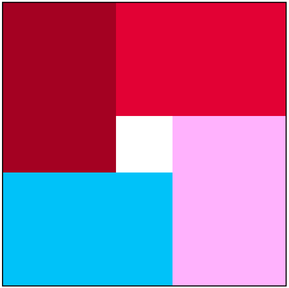

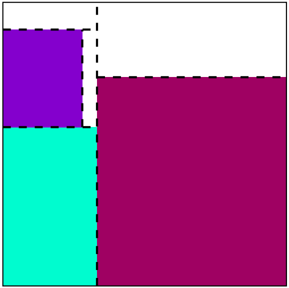

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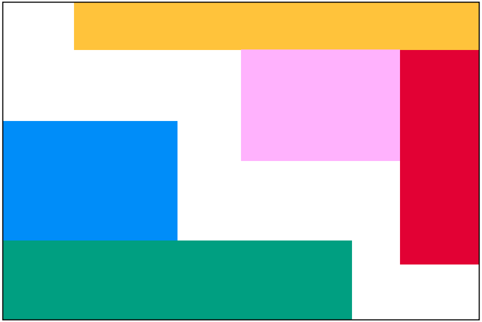

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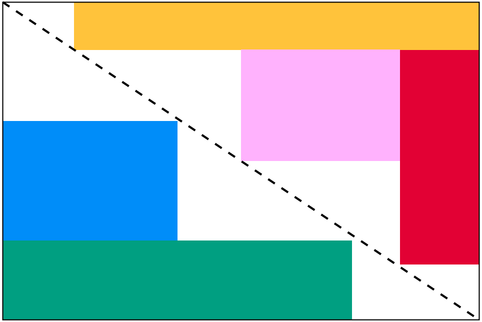

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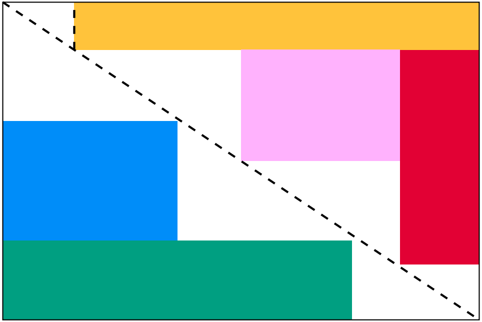

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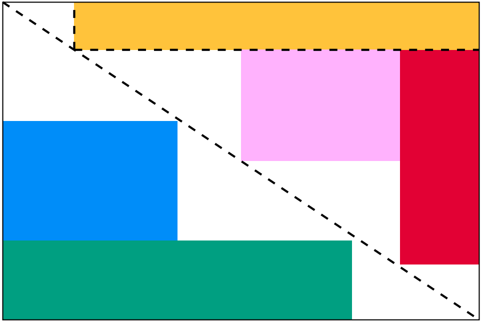

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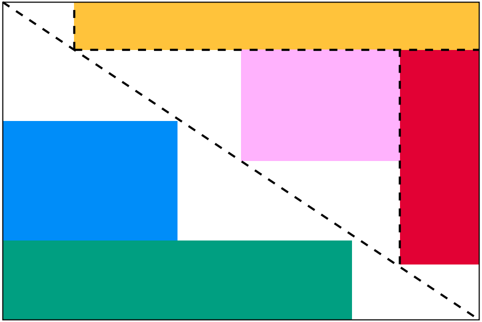

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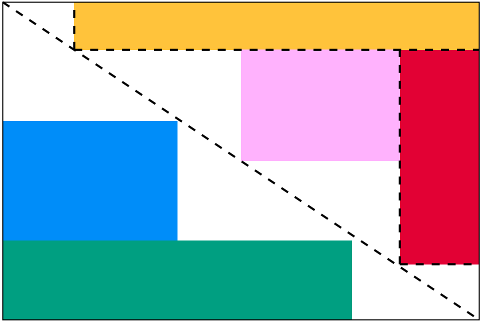

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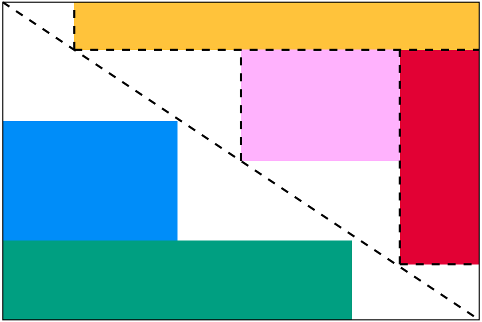

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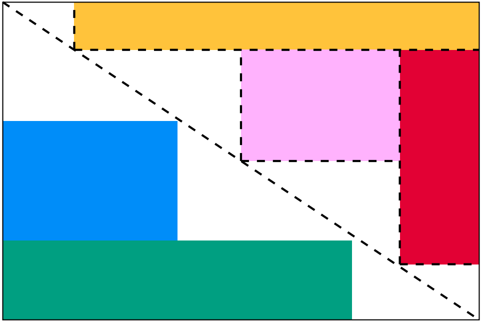

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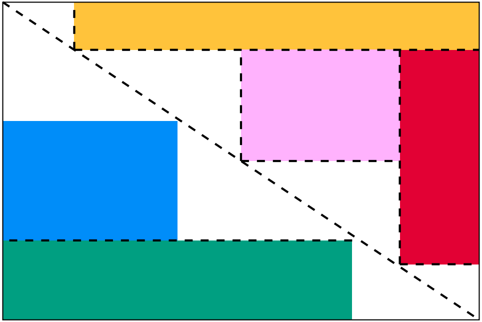

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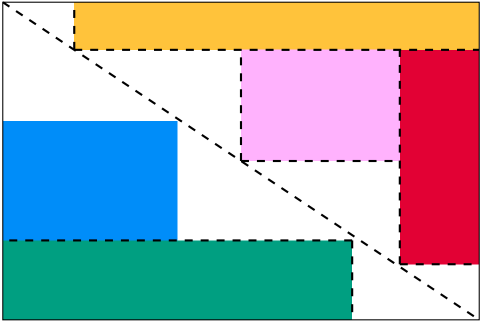

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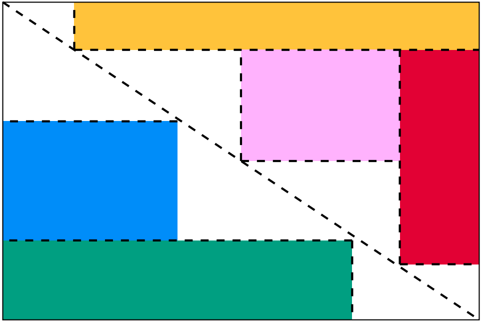

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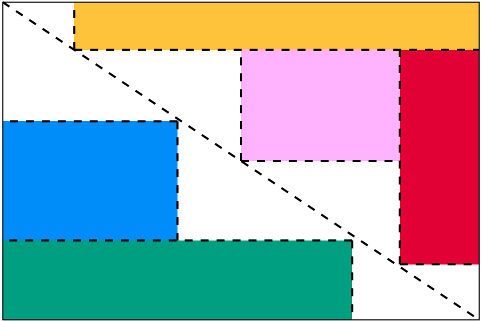

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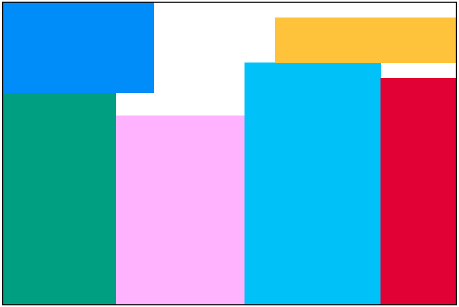

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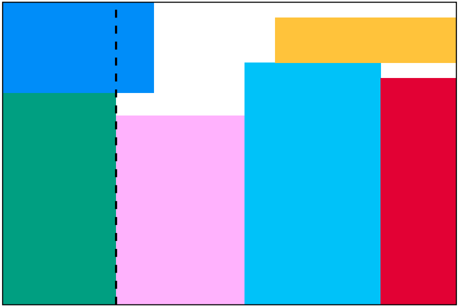

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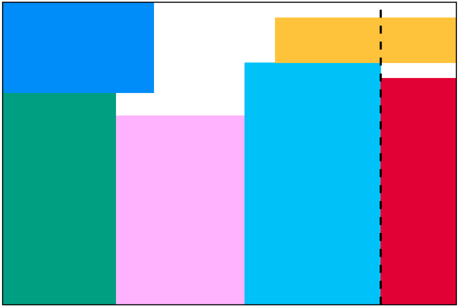

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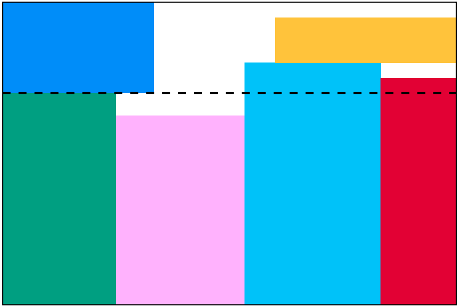

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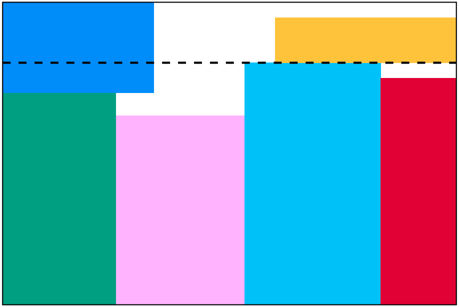

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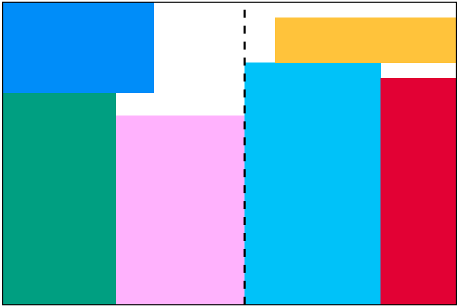

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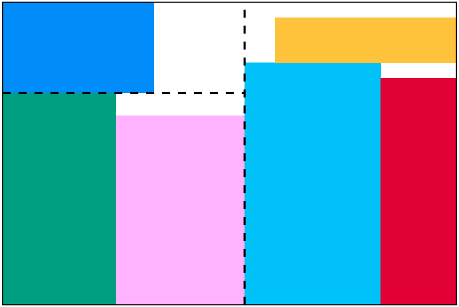

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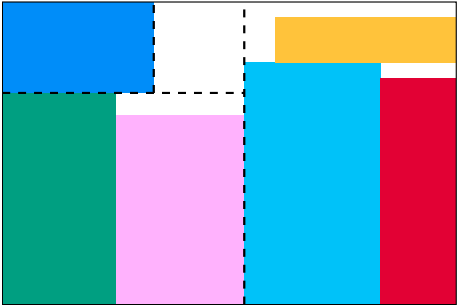

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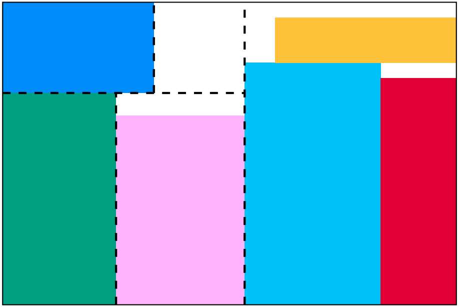

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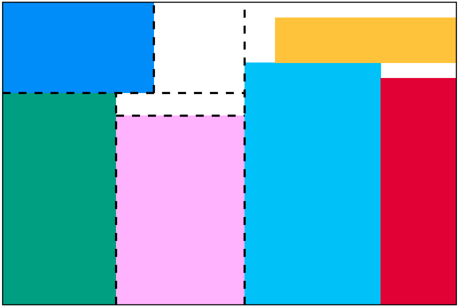

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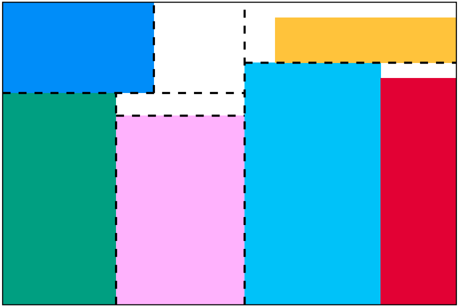

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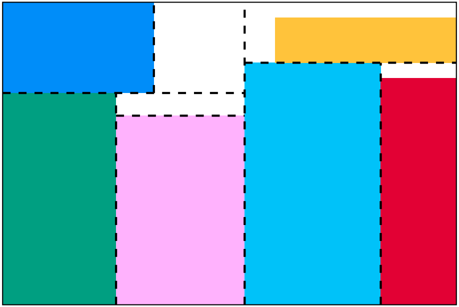

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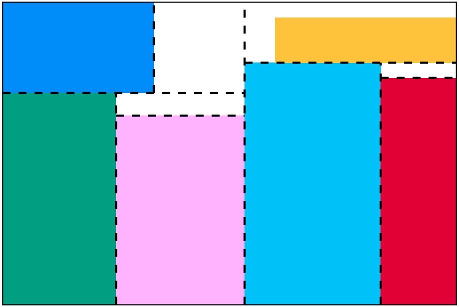

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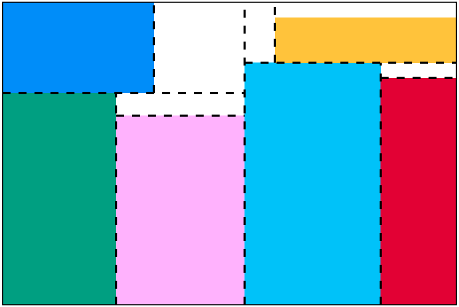

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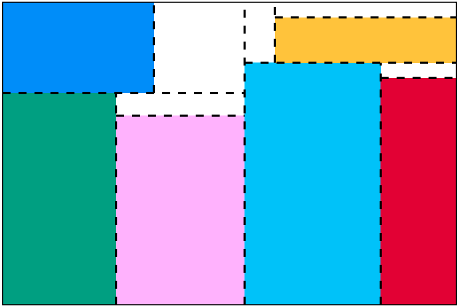
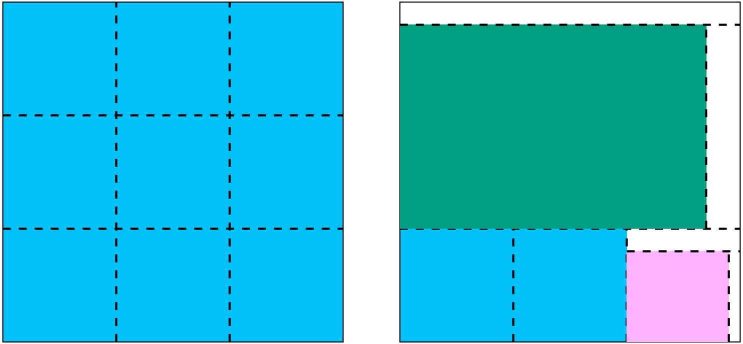


Constrained Demand

"There is a limit on the number of copies a piece may be sold."



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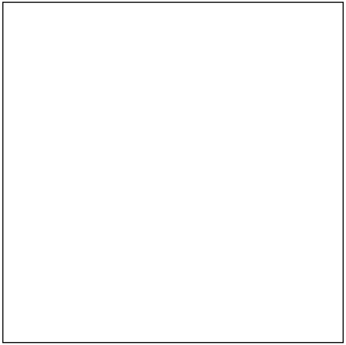

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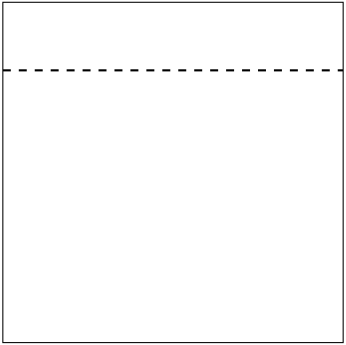

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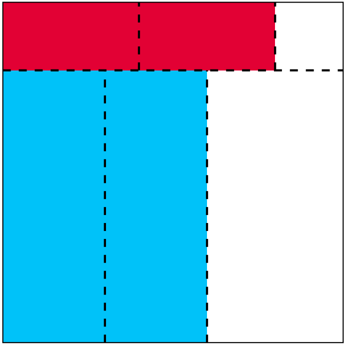

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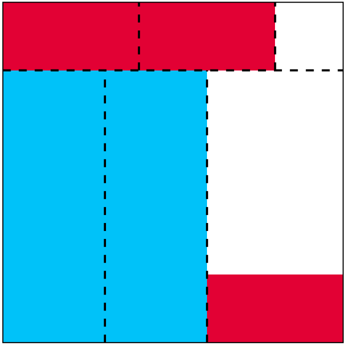

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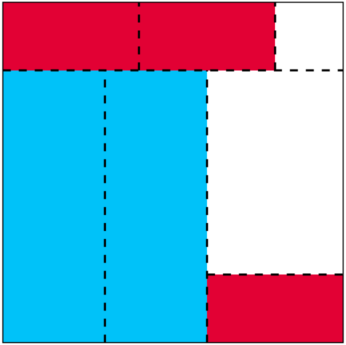

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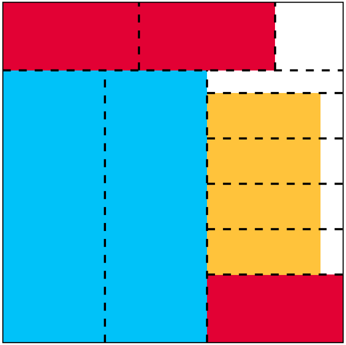

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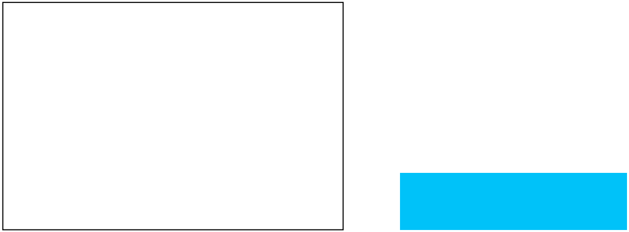

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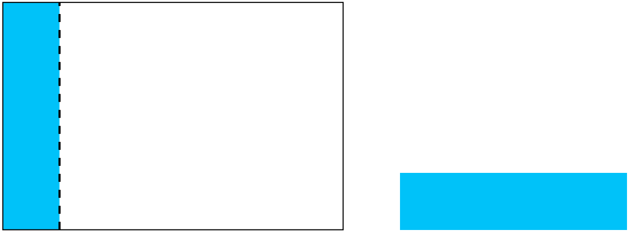

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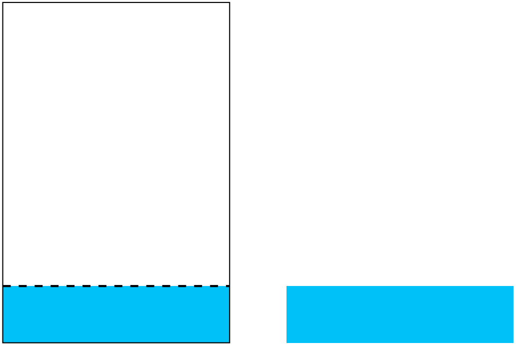

Summary of the G2KP characteristics

- Knapsack – maximises profit, single plate.
- Guillotine Cuts – from a side to another.
- Orthogonal Cuts – only cuts parallel to the sides.
- Unrestricted Cuts – may cut in any position.
- Constrained Demand – upper bound for pieces in solution.
- Unlimited Stages – no limit on cut orientation changes.
- Allow/disallow rotation – of pieces or plates.

Outline for “Prior Work”

Seminal Works and Surveys

1965 Introduces the unconstrained (weakly NP-Hard) G2KP and other variants (no experiments). [6]

1972 Seminal work on the unconstrained G2KP. Introduces 'canonical dissecations' (used pervasively).[7]

1977 Seminal work on the constrained G2KP. Treats some symmetries. Proposes classical instances.[3]

2020 Surveys exact methods and relaxations on 2D cutting and packing. [8]

2020 Surveys exact methods and relaxations the G2KP specifically; points out mistakes in the literature. [13]

[6] Gilmore, P. C.; Gomory, R. E. Multistage Cutting Stock Problems of Two and More Dimensions. 10.1287/opre.13.1.94

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[7] Herz, J. C. Recursive Computational Procedure for Two-Dimensional Stock Cutting. 10.1147/rd.165.0462

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[3] Christofides, N.; Whitlock, C. An Algorithm for Two-Dimensional Cutting Problems. 10.1287/opre.25.1.30

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[8] Iori, M.; de Lima, V. L.; Martello, S.; Miyazawa, F. K.; Monaci, M. Exact Solution Techniques for Two-Dimensional Cutting and Packing. 10.1016/j.ejor.2020.06.050

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[13] Russo, M.; Boccia, M.; Sforza, A.; Sterle, C. Constrained Two-Dimensional Guillotine Cutting Problem: Upper-Bound Review and Categorization. 10.1111/itor.12687

Formulations

- 2008 The first MILP formulation for unlimited stages. Mostly of theoretical interest. [1]
- 2010 Two- and (restricted) three-staged formulations for the G2CSP. Not the first *k*-staged formulation. [15]
- 2013 Compares three two-staged MILP formulations including the work mentioned above.[4]
- 2016 The first MILP formulation for the unlimited stages G2KP able to solve medium-sized intances.[5]
- 2020 Three new competitive formulations were proposed recently.[9]

[1] Ben Messaoud, S.; Chu, C.; Espinouse, M.-L. Characterization and Modelling of Guillotine Constraints. 10.1016/j.ejor.2007.08.029

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2020 Three new competitive formulations were proposed recently.[9]

[15] Silva, E.; Alvelos, F.; Valério de Carvalho, J. M. An Integer Programming Model for Two- and Three-Stage Two-Dimensional Cutting Stock Problems. 10.1016/j.ejor.2010.01.039

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[9] Martin, M.; Morabito, R.; Munari, P. A Top-down Cutting Approach for Modeling the Constrained Two- and Three-Dimensional Guillotine Cutting Problems. 10.1080/01605682.2020.1813640.

Current Work

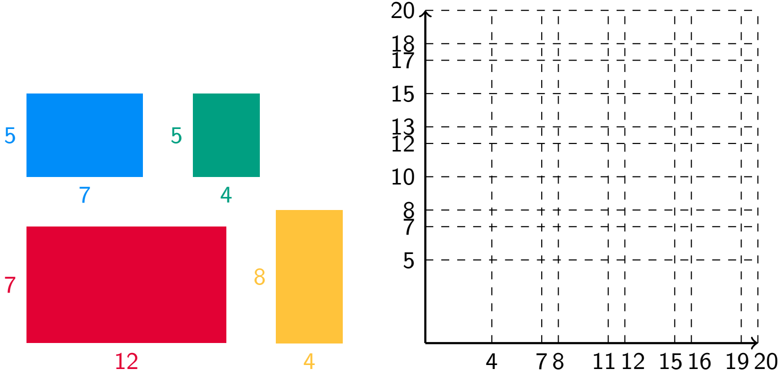
(this work)

- enhances a state-of-the-art formulation by cutting down model size and symmetries;
- adapts a previously known reduction to our context also reducing model size and symmetries;
- brings new results for recently proposed and more challenging instances;
- directly compares to the state of the art on MILP formulations for the problem;
- adapts to related but distinct problems to further test the formulation;
- proposes a hybridisation of the proposed model with a previous model for a restricted problem.

Outline for “Reductions”

Discretization

We can restrict cuts to linear combinations of piece dimensions (constrained by their demand) without losing optimality.



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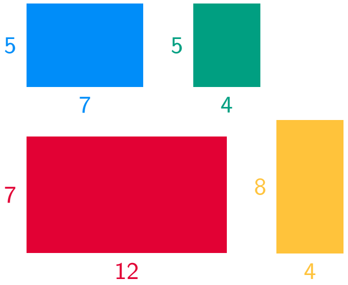

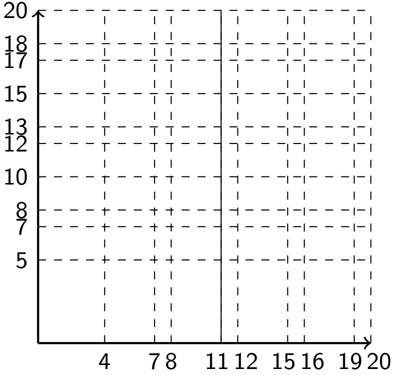

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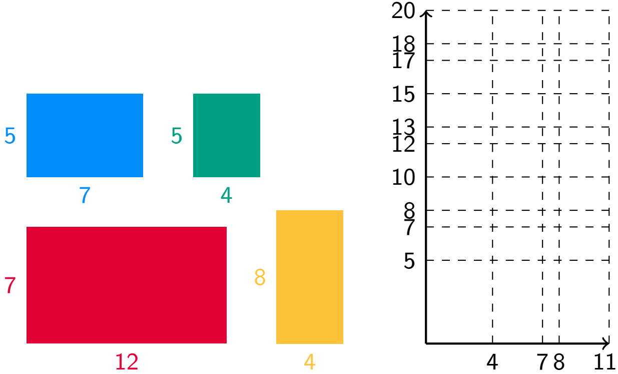

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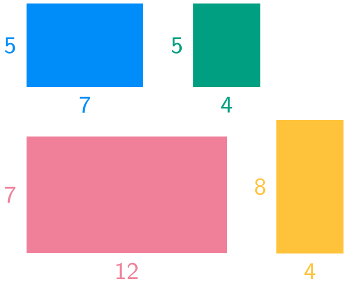

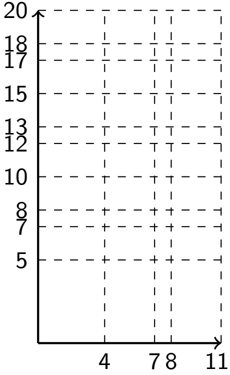

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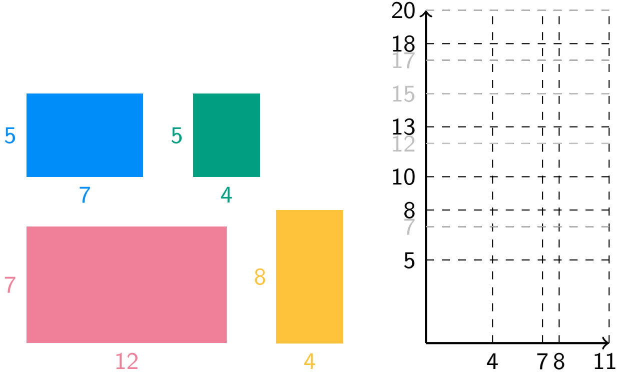


Plate-Size Normalization I

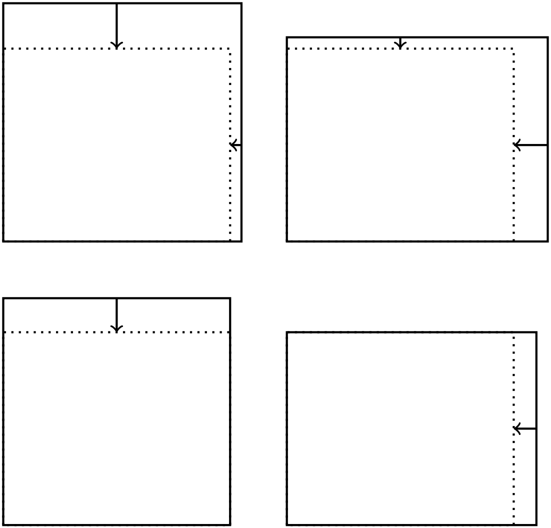
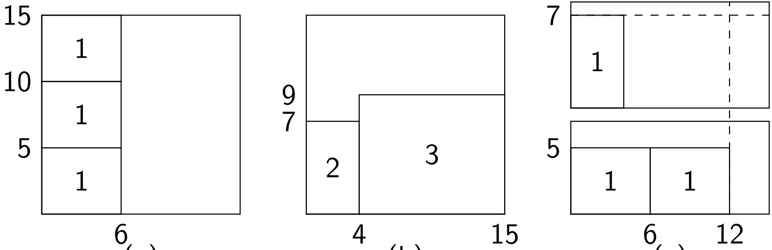


Plate-Size Normalization II



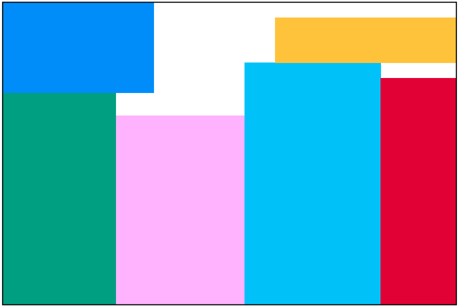
(a) (b) (c)

Previous Reductions

- Redundant-Cut
  - Remove unnecessary intermediary trim cuts.
  - Does not affect the number of plates (constraints).
  - Is superseded by our enhanced formulation.
- Cut-Position
  - Removes unrestricted cuts from small plates.
  - If 6 pieces do not fit, there is no loss.
  - Affects both number of variables and constraints.

Unrestricted Cuts (Revisited)

"The position of a cut does not need to match a single piece dimension."



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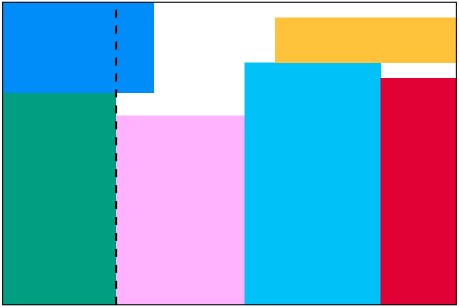

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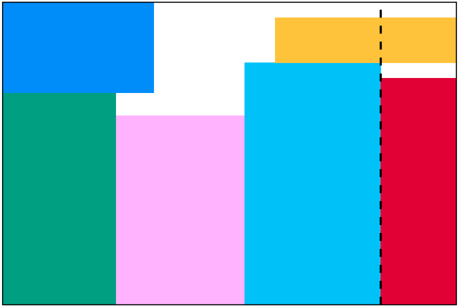

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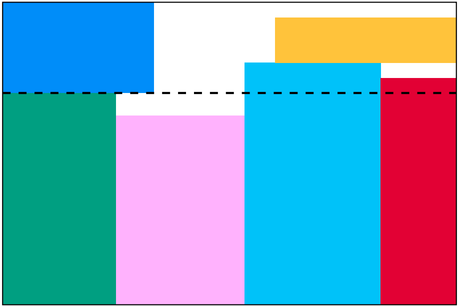

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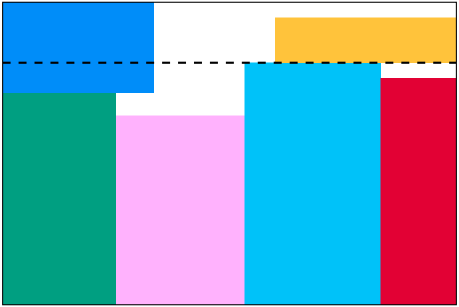

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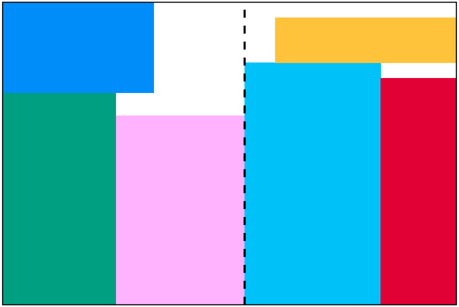

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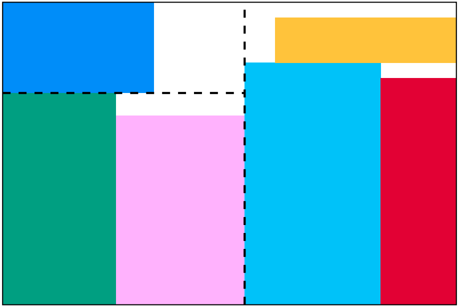

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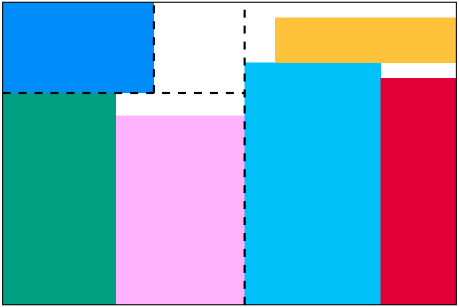

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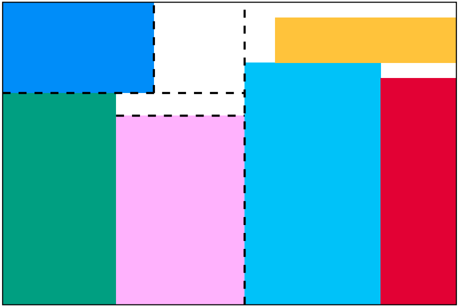

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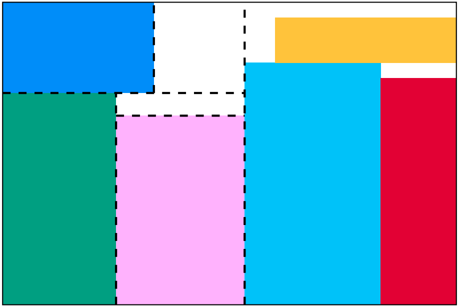

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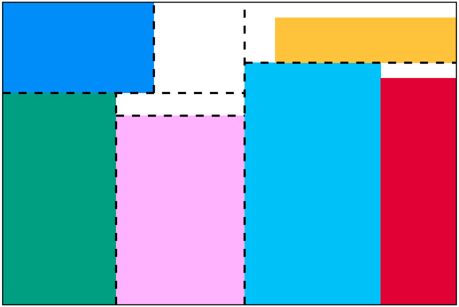

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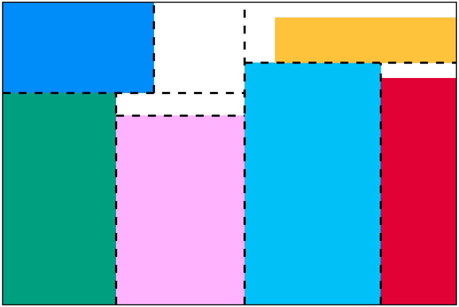

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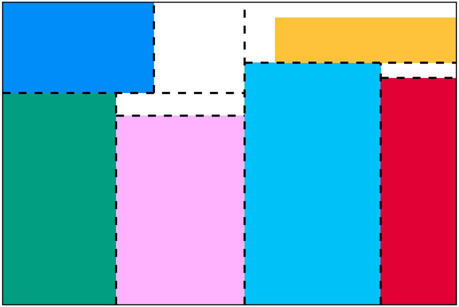

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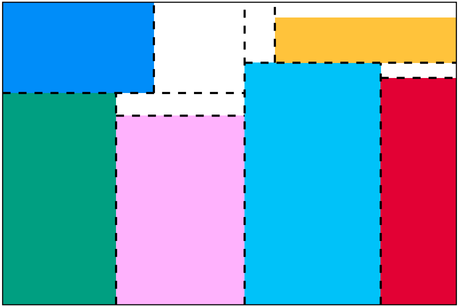

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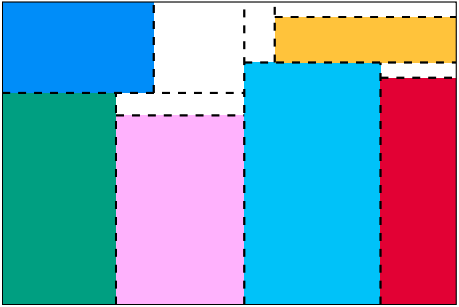

Outline for "Formulations"

Furini's Formulation

$$\textbf{max.} \sum_{j \in \bar{J}} p_j y_j$$

$$\textbf{s.t.}\, y_j \leq u_j \qquad \forall j \in \bar{J},$$

$$\sum_{o \in O} \sum_{q \in Q_{0o}} x^o_{q0} + y_0 \leq 1 \qquad ,$$

$$\sum_{o \in O} \sum_{q \in Q_{jo}} x^o_{qj} + y_j \leq \sum_{k \in J} \sum_{o \in O} \sum_{q \in Q_{ko}} a^o_{qkj} x^o_{qk} \qquad \forall j \in \bar{J}, j \neq 0,$$

$$\sum_{o \in O} \sum_{q \in Q_{jo}} x^o_{qj} \leq \sum_{k \in J} \sum_{o \in O} \sum_{q \in Q_{ko}} a^o_{qkj} x^o_{qk} \qquad \forall j \in J \setminus \bar{J},$$

Our Formulation

$$\textbf{max.} \sum_{(i,j) \in E} p_i e_{ij}$$

$$\textbf{s.t.} \sum_{j \in E_{i*}} e_{ij} \leq u_i \qquad \forall i \in \bar{J},$$

$$\sum_{o \in O} \sum_{q \in Q_{0o}} x^o_{q0} + \sum_{i \in E_{*0}} e_{i0} \leq 1 \qquad ,$$

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Outline for "Comparison to base formulation (pre-proposal)"

The three data sources

There is data from three data sources in the experiments:

Original Data from [5, 17].

- Other machine and solver (CPLEX).
- Unspecified implementation language.
- Implementation was not available.

Faithful Our reimplementation of original.

- Uses Julia, JuMP, and Gurobi.
- Matches number of variables and constraints closely.

Enhanced Our enhanced formulation based on [5, 17].

- Compatible with the Cut-Position and the pricing, supersedes Redundant-cut.

Original vs Faithful

The closer to 100.00% the better.

Variant	N. M.	O. #v	F. %v	O. #p	F. %p
Complete PP-G2KP	0	156M	100.00	1.88M	100.00
Complete +Cut-Position	0	103M	99.99	1.73M	100.01
Complete +Redundant-Cut	0	121M	109.94	1.88M	100.00
PP-G2KP (CP + RC)	0	74M	120.05	1.73M	100.01
Restricted PP-G2KP	0	≈5M	99.28	0.30M	99.99
Priced Restricted PP-G2KP	1	≈4M	102.20	0.30M	99.99
Priced PP-G2KP	10	≈15M	93.74	1.64M	100.01

Faithful vs Enhanced

Last column has the sum of running times for the instances solved.

Variant	#e	#m	#s	#b	#variables	S. T.
Faithful	–	59	53	0	88,901,964	41,257
Enhanced	–	59	58	2	3,216,774	14,738
F. +Normalizing	–	59	56	0	60,316,964	27,678
E. +Normalizing	–	59	59	52	2,733,125	14,169
F. +N. +Warming	–	59	56	0	60,316,964	28,142
E. +N. +Warming	–	59	59	4	2,733,125	9,778
Priced. F. +N. +W.	8	50	55	0	8,072,810	6,854
Priced. E. +N. +W.	8	51	59	0	1,021,526	9,209

Results over harder instances (Enhanced)

C.	V.	#m	Avg. #v	Avg. #p	T. T.	#s	Avg. S. T.
1	Not P.	20	1,787,864	22,316	172,574	5	2,114.85
	P.	5	264,315	11,978	196,733	3	4,377.77
2	Not P.	20	1,533,490	18,638	167,973	5	1,194.68
	P.	8	394,613	9,735	178,812	4	1,503.01
3	Not P.	20	2,895,300	33,249	171,155	5	1,831.11
	P.	5	372,597	13,287	179,712	4	1,728.08
4	Not P.	20	3,201,374	35,197	167,776	7	3,910.89
	P.	2	211,093	14,227	199,477	2	2,538.79

Summary of pre-proposal results

- Our reimplementation seems reasonably fair.
- Enhanced takes ≈4 hours to solve all instances...
  - ...while Faithful takes 12 hours to solve 53 of 59.
- For the new and more challenging instances:
  - 17 new optimal solutions (unrestricted).
  - Better lower bounds for 25 instances.
  - Better upper bounds for 58 instances.

Outline for “Comparison to other formulations”

Concise description of other formulations

- BCE [1] – first formulation, compact but $O(n^4)$.
- MLB [10] – grid formulation, can model defects.
- MM1 and MM2 [11] – Bottom-up formulations, pseudo-polynomial and compact.
- MM3 [12] – top-down formulation, compact, big-Ms.
- FMT [5] – faithful, no warm-start, no pricing.
- BBA (this work) – enhanced, no warm-start, no pricing.

Summary of the results

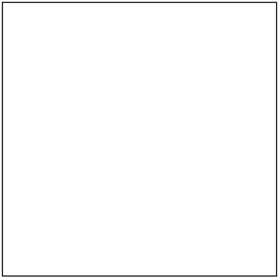
- BCE, MLB, and FMT are disregarded after preliminary experiments.
  - Solve few instances, or fail during root node, or have large gap.
- For most datasets (representative table next slide):
  - BBA solves more instances in less time.
  - BBA unsolved runs have very bad LB, ...
    - ... while MM1, MM2, and MM3 have a good LB.
    - For the APT dataset has 100% gap while MM1 and MM2 have 11% and 3%.

Representative table

	CU							
	Fixed				Rotation			
Alg.	#opt	g_{lb}	Avg. T.	g_{ub}	#opt	g_{lb}	Avg. T.	g_{ub}
BBA	10	9.09	425	0.21	9	18.18	716	0.06
MM1	3	0.54	2928	1.45	0	0.68	3600	0.57
MM2	0	0.80	3600	1.45	0	0.88	3600	0.57
MM3	3	0.78	3021	1.45	2	0.97	3400	0.57

Outline for “Hibridised Formulation”

Piece-outlining cuts



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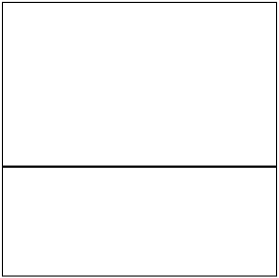

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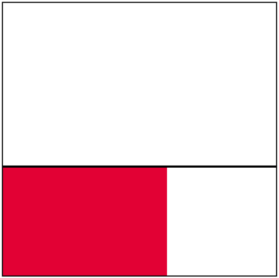

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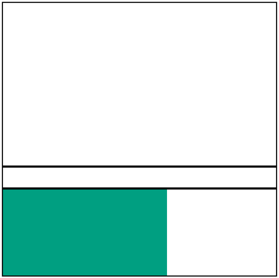

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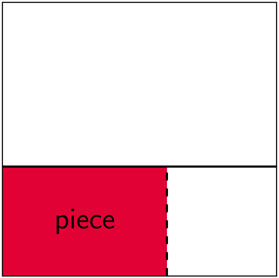

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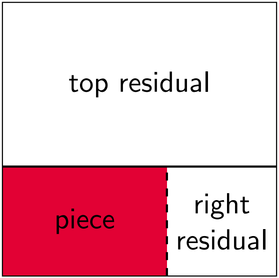
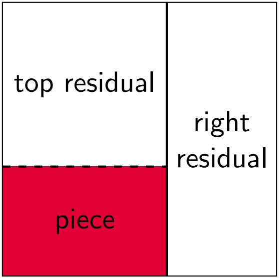


Horizontal-first POC

Piece-outlining cuts



Vertical-first POC

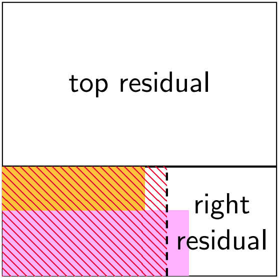
Piece-outlining cuts

Horizontal-first POC

Piece-outlining cuts

Vertical-first POC

Piece-outlining cuts



Horizontal-first POC

Changes in the formulation (hybridisation)

$$\textbf{max.} \sum_{(i,j)\in E} p_i e_{ij} + \sum_{i\in \bar{J}} p_i s_i$$

$$\textbf{s.t.}\; s_i + \sum_{j\in E_{i*}} e_{ij} \leq u_i \qquad \forall i \in \bar{J},$$

$$\sum_{o\in O} \sum_{q\in Q_{0o}} x^o_{q0} + \sum_{i\in E_{*0}} e_{i0} \leq 1 \qquad ,$$

$$\sum_{o\in O} \sum_{q\in Q_{jo}} x^o_{qj} + \sum_{i\in E_{*j}} e_{ij} \leq \sum_{k\in J} \sum_{o\in O} \sum_{q\in Q_{ko}} a^o_{qkj} x^o_{qk} \qquad \forall j \in J, j \neq 0,$$

$$s_i \leq \sum_{j\in J} \sum_{o\in O} \sum_{q\in Q_{jo}} h^o_{qji} x^o_{qj} \qquad \forall i \in \bar{J}.$$

Summary of our results (Hybridisation)

- FMT59 dataset: the run time reduction was of 20%.
  - Runs with 99%+ of the run time in the B&B phase are the most affected.
- Clautiaux42 dataset (high length/width piece repetition):
  - If positions of shared piece length/width are ignored: -1.5% run time.
  - If we create POCs for every possible piece: +236% run time.

Outline for "Related problems"

Summary of the G2MKP characteristics

- Knapsack – maximises profit, ~~single~~ multiple equal plates.
- Guillotine Cuts – from a side to another.
- Orthogonal Cuts – only cuts parallel to the sides.
- Unrestricted Cuts – may cut in any position.
- Constrained Demand – upper bound for pieces in solution.
- Unlimited Stages – no limit on cut orientation changes.
- Allow/disallow rotation – of pieces or plates.

Changes to the formulation (G2MKP)

$$
\begin{aligned}
\textbf{max.} & \sum_{(i,j) \in E} p_i e_{ij} \\
\textbf{s.t.} & \sum_{j \in E_{i*}} e_{ij} \leq u_i & \forall i \in \bar{J}, \\
& \sum_{o \in O} \sum_{q \in Q_{0o}} x^o_{q0} + \sum_{i \in E_{*0}} e_{i0} \leq \cancel{1} m & , \\
& \sum_{o \in O} \sum_{q \in Q_{jo}} x^o_{qj} + \sum_{i \in E_{*j}} e_{ij} \leq \sum_{k \in J} \sum_{o \in O} \sum_{q \in Q_{ko}} a^o_{qkj} x^o_{qk} & \forall j \in J, j \neq 0,
\end{aligned}
$$

Summary of our findings G2MKP

- Least studied of the chosen problems.
- Different datasets have different behaviours:
  - For the CW_M dataset (low u_i, high n):
    - Increasing m increased the times (many timeouts).
    - Allowing rotation increased time and model size (2.8~7.5).
  - For the A_M dataset (high u_i, low n):
    - Increasing m increased times lightly ($< 17\%$).
    - Allowing rotation increased time and model size (many OOM).

Summary of the G2CSP/G2BPP characteristics

- ~~Knapsack – maximises profit, single plate.~~
- Input minimisation – minimises number of bins.
- Guillotine Cuts – from a side to another.
- Orthogonal Cuts – only cuts parallel to the sides.
- Unrestricted Cuts – may cut in any position.
- ~~Constrained Demand – upper bound for pieces in solution.~~
- Required Demand – *lower* bound for pieces in solution.
- Unlimited Stages – no limit on cut orientation changes.
- Allow/disallow rotation – of pieces or plates.

Changes to the formulation (G2CSP/G2BPP)

$$\cancel{\textbf{max.} \sum_{(i,j) \in E} p_i e_{ij}} \textbf{min.}\ b$$

$$\textbf{s.t.} \sum_{j \in E_{i*}} e_{ij} \cancel{\leq} \geq u_i \qquad \forall i \in \bar{J},$$

$$\sum_{o \in O} \sum_{q \in Q_{0o}} x_{q0}^{o} + \sum_{i \in E_{*0}} e_{i0} \leq \cancel{1} b \qquad ,$$

$$\sum_{o \in O} \sum_{q \in Q_{jo}} x_{qj}^{o} + \sum_{i \in E_{*j}} e_{ij} \leq \sum_{k \in J} \sum_{o \in O} \sum_{q \in Q_{ko}} a_{qkj}^{o} x_{qk}^{o} \qquad \forall j \in J, j \neq 0,$$

Summary of our findings (G2CSP/G2BPP)

- Other methods have better results (but no direct comparison)
  - Either not exact, tackle a simpler problem, or not pure MILP.
- Different datasets have different behaviours:
  - For the A dataset (high u_i, low n):
    - Allowing rotation almost always lead to OOM.
  - For the CLASS dataset (low u_i, high n):
    - Allowing rotation generally reduced model size and run time.
    - Many pieces for small plate led to discretization saturation.
    - Better primals helped when model size did increase.

Summary of the G2OPP characteristics

- ~~Knapsack – maximises profit,~~ single plate.
- Decision problem – checks feasibility of packing all pieces.
- Guillotine Cuts – from a side to another.
- Orthogonal Cuts – only cuts parallel to the sides.
- Unrestricted Cuts – may cut in any position.
- ~~Constrained Demand – upper bound for pieces in solution.~~
- Required Demand – *lower* bound for pieces in solution.
- Unlimited Stages – no limit on cut orientation changes.
- Allow/disallow rotation – of pieces or plates.

Changes to the formulation (G2OPP)

$$\cancel{\boldsymbol{max.} \sum_{(i,j)\in E} p_i e_{ij}}$$

$$\boldsymbol{s.t.} \sum_{j \in E_{i*}} e_{ij} \cancel{\leq} \geq u_i \qquad \forall i \in \bar{J},$$

$$\sum_{o \in O} \sum_{q \in Q_{0o}} x^o_{q0} + \sum_{i \in E_{*0}} e_{i0} \leq 1 \qquad ,$$

$$\sum_{o \in O} \sum_{q \in Q_{jo}} x^o_{qj} + \sum_{i \in E_{*j}} e_{ij} \leq \sum_{k \in J} \sum_{o \in O} \sum_{q \in Q_{ko}} a^o_{qkj} x^o_{qk} \qquad \forall j \in J, j \neq 0,$$

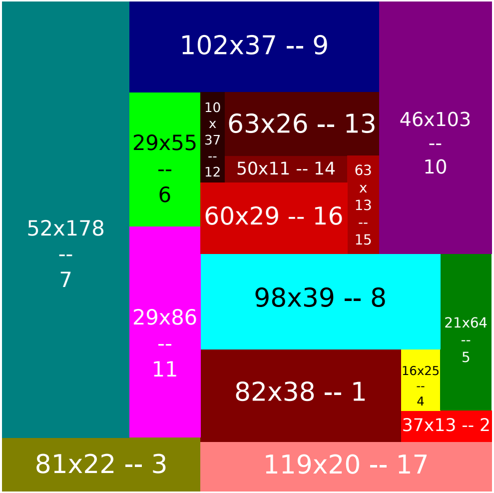
Summary of our findings (G2CSP/G2BPP)

- Competitive against CP, but not against an *ad hoc* method.
  - The *ad hoc* has a memory complexity of $2(min\{W, L\}2^n)^2$
- Allowing rotation increases model size but reduces run times.
- The run time decrease relates to instances becoming feasible.
- Mirror-plate is able to reduce model size and run time.

About the T instances from Hopper (2000)

- We found a dataset generation mistake during experiments.
- For the G2OPP, *all* T instances should return true.
  - However, it seems all return false.
- Some works may have been impacted: [2, 16, 14].
- The mistake was (probably) to use the same procedure of dataset N.

A non-guillotinable optimal solution for instance T1a



Outline for “Conclusions”

Conclusions

- The (re-)formulation and PSN show an order of magnitude improvement.
  - For instances that can be solved with MILP, solves the most.
  - For harder instances, fail to deliver good primal solutions.
- Hybridisation further reduces time spent on B&B by 20%.
- For related problems, *ad hoc* methods have the advantage.
  - However, a direct comparison is hard in most cases.
  - Our work may serve as base for future comparisons.

Conclusions II

Breaking symmetries and removing dominated variables had better results than resorting to more complicated variable pricing techniques.

These improvements are immediately compatible with related problems, and do not depend on problem-specific heuristics like some pricing techniques.

Thank you all.

References I

[1] Ben Messaoud, S., Chu, C., Espinouse, M.L.: Characterization and modelling of guillotine constraints. European Journal of Operational Research 191(1), 112–126 (Nov 2008), http://www.sciencedirect.com/science/article/pii/S0377221707009083

[2] Bortfeldt, A., Jungmann, S.: A tree search algorithm for solving the multi-dimensional strip packing problem with guillotine cutting constraint. Annals of Operations Research 196(1), 53–71 (Jul 2012), https://link.springer.com/article/10.1007/s10479-012-1084-7

References II

[3] Christofides, N., Whitlock, C.: An Algorithm for Two-Dimensional Cutting Problems. Operations Research 25(1), 30–44 (Feb 1977), https://pubsonline.informs.org/doi/abs/10.1287/opre.25.1.30

[4] Furini, F., Malaguti, E.: Models for the two-dimensional two-stage cutting stock problem with multiple stock size. Computers & Operations Research 40(8), 1953–1962 (Aug 2013), http://www.sciencedirect.com/science/article/pii/S0305054813000749

References III

[5] Furini, F., Malaguti, E., Thomopulos, D.: Modeling Two-Dimensional Guillotine Cutting Problems via Integer Programming. INFORMS Journal on Computing 28(4), 736–751 (Oct 2016), https://pubsonline.informs.org/doi/abs/10.1287/ijoc.2016.0710

[6] Gilmore, P.C., Gomory, R.E.: Multistage Cutting Stock Problems of Two and More Dimensions. Operations Research 13(1), 94–120 (Feb 1965), https://pubsonline.informs.org/doi/abs/10.1287/opre.13.1.94

[7] Herz, J.C.: Recursive computational procedure for two-dimensional stock cutting. IBM Journal of Research and Development 16(5), 462–469 (1972)

References IV

[8] Iori, M., de Lima, V.L., Martello, S., Miyazawa, F.K., Monaci, M.: Exact Solution Techniques for Two-dimensional Cutting and Packing. European Journal of Operational Research p. S0377221720306111 (Jul 2020), http://arxiv.org/abs/2004.12619, arXiv: 2004.12619

[9] Martin, M., Birgin, E.G., Lobato, R.D., Morabito, R., Munari, P.: Models for the two-dimensional rectangular single large placement problem with guillotine cuts and constrained pattern. International Transactions in Operational Research 27(2), 767–793 (2020), https://onlinelibrary.wiley.com/doi/abs/10.1111/itor.12703

References V

[10] Martin, M., Birgin, E.G., Lobato, R.D., Morabito, R., Munari, P.: Models for the two-dimensional rectangular single large placement problem with guillotine cuts and constrained pattern. International Transactions in Operational Research 27(2), 767–793 (2020), https://onlinelibrary.wiley.com/doi/abs/10.1111/itor.12703

[11] Martin, M., Morabito, R., Munari, P.: A bottom-up packing approach for modeling the constrained two-dimensional guillotine placement problem. Computers & Operations Research 115 (Mar 2020), http://www.sciencedirect.com/science/article/pii/S030505481930293X

References VI

[12] Martin, M., Morabito, R., Munari, P.: A top-down cutting approach for modeling the constrained two- and three-dimensional guillotine cutting problems. Journal of the Operational Research Society 0(0), 1–15 (Sep 2020), https://doi.org/10.1080/01605682.2020.1813640

[13] Russo, M., Boccia, M., Sforza, A., Sterle, C.: Constrained two-dimensional guillotine cutting problem: upper-bound review and categorization. International Transactions in Operational Research 27(2), 794–834 (2020), https://onlinelibrary.wiley.com/doi/abs/10.1111/itor.12687

References VII

[14] Shang, Z., Pan, M., Pan, J.: An improved priority heuristic for the fixed guillotine rectangular packing problem 1656, 012005 (Sep 2020), https://doi.org/10.1088/1742-6596/1656/1/012005, publisher: IOP Publishing

[15] Silva, E., Alvelos, F., Valério de Carvalho, J.M.: An integer programming model for two- and three-stage two-dimensional cutting stock problems. European Journal of Operational Research 205(3), 699–708 (Sep 2010), http://www.sciencedirect.com/science/article/pii/S0377221710000731

References VIII

[16] Thomas, J., Chaudhari, N.S.: Design of efficient packing system using genetic algorithm based on hyper heuristic approach. Advances in Engineering Software 73, 45–52 (Jul 2014), https://www.sciencedirect.com/science/article/pii/S0965997814000489

[17] Thomopulos, D.: Models and Solutions of Resource Allocation Problems based on Integer Linear and Nonlinear Programming. Tesi di dottorato, alma (May 2016), http://amsdottorato.unibo.it/7399/