**Chapter Outline**

11.1 The Expected Return of a Portfolio  
11.2 The Volatility of a Two-Stock Portfolio  
11.3 The Volatility of a Large Portfolio  
11.4 Risk Versus Return: Choosing an Efficient Portfolio  
11.5 Risk-Free Saving and Borrowing  
11.6 The Efficient Portfolio and Required Returns  
11.7 The Capital Asset Pricing Model  
11.8 Determining the Risk Premium

**11.1 Expected return of a portfolio**

Suppose you have two stocks:

Amazon: $x\_1$: 40%, Return: 10%, Risk: 26.6%  
Southwest: $x\_2$: 60%, Return: 15%, Risk: 27.9%

If your portfolio is 40% Amazon + 60% Southwest, then your return is:

(0.4×10%)+(0.6×15%)=13%(0.4 \times 10\%) +(0.6 \times 15\%) = 13\%(0.4×10%)+(0.6×15%)=13%

There is no secret here, the return of a portfolio is the weighted average of returns.

The weights are selected by the investors and, obviously, if prices change, the weights change over time.

**11.2 Volatility 2-stock portfolio**

To compute the standard deviation of a portfolio, we cannot rely on the weighted average. We have to look to the covariances.

* $\sigma^2\_1$ = variance of asset 1.
* $\sigma^2\_2$ = variance of asset 2
* $\sigma\_{12}$ = covariance between assets 1 and 2

Covariance is the product of the assets' Sd and their correlation.

σ12=σ1×σ2×ρ12\sigma\_{12} = \sigma\_1 \times \sigma\_2 \times \rho\_{12}σ12​=σ1​×σ2​×ρ12​

Covariance is the expected product of the deviations of two returns from their means.

Cov(Ri,Rj)=E[(Ri−E[Ri])×(Rj−E[Rj])]Cov(R\_i,R\_j) = E[(R\_i-E[R\_i]) \times (R\_j-E[R\_j]) ]Cov(Ri​,Rj​)=E[(Ri​−E[Ri​])×(Rj​−E[Rj​])]

When using historical data:

Cov(Ri,Rj)=1T−1(Ri−Ri~)×(Rj−Rj~)Cov(R\_i,R\_j) = \frac{1}{T-1} (R\_i-\tilde{R\_i} ) \times (R\_j-\tilde{R\_j} )Cov(Ri​,Rj​)=T−11​(Ri​−Ri​~​)×(Rj​−Rj​~​)

Correlation:

Corr(Ri,Rj)=Cov(Ri,Rj)Sd(Ri)×Sd(Rj)Corr(R\_i,R\_j) = \frac{Cov(R\_i,R\_j)}{Sd(R\_i)\times Sd(R\_j) }Corr(Ri​,Rj​)=Sd(Ri​)×Sd(Rj​)Cov(Ri​,Rj​)​

Therefore, the portfolio variance is:

var=(x12×σ12)+(x22×σ22)+2(x1×x2×σ1×σ2×ρ12)var = (x\_1^2 \times \sigma\_1^2) + (x\_2^2 \times \sigma\_2^2) + 2(x\_1 \times x\_2 \times \sigma\_1 \times \sigma\_2 \times \rho\_{12})var=(x12​×σ12​)+(x22​×σ22​)+2(x1​×x2​×σ1​×σ2​×ρ12​)

If $\rho\_{12} = 0.26$

(0.42×26.62)+(0.62×27.92)+2(0.4×0.6×26.6×27.9×0.26)=486.1(0.4^2 \times 26.6^2) + (0.6^2 \times 27.9^2) + 2(0.4 \times 0.6 \times 26.6 \times 27.9 \times 0.26) = 486.1(0.42×26.62)+(0.62×27.92)+2(0.4×0.6×26.6×27.9×0.26)=486.1

The standard deviation of the portfolio is $\sqrt{486.1}= 22%$, which is lower than 26.6% and 27.9%.

Covariance and Correlation

Let's remember the basics about correlation.

These assets have the same historical return and volatility, **but they 'move' very differently**:

* For example, when North Air performed well, Text Oil tended to do poorly, and when North Air did poorly, Text oil tended to do well
* North Air is not positively correlated with Text Oil.
* Consider the portfolio which consists of equal investments in West Air and Tex Oil. The average return of the portfolio is equal to the average return of the two stocks...
* ...However, the volatility of 5.1% is much less than the volatility of the two individual stocks.

The amount of risk that is eliminated in a portfolio depends on the degree to which the stocks face common risks and their prices move together.

**11.3 Volatility of a large portfolio**

The variance of a three-asset portfolio can be built using a previous slide...

var=(x12×σ12)+(x22×σ22)+2(x1×x2×σ1×σ2×ρ12)var = (x\_1^2 \times \sigma\_1^2) + (x\_2^2 \times \sigma\_2^2) + 2(x\_1 \times x\_2 \times \sigma\_1 \times \sigma\_2 \times \rho\_{12})var=(x12​×σ12​)+(x22​×σ22​)+2(x1​×x2​×σ1​×σ2​×ρ12​)

... plus the following terms.

+(x32×σ32)+2(x1×x3×σ1×σ3×ρ13)+2(x2×x3×σ2×σ3×ρ23)+ (x\_3^2 \times \sigma\_3^2) + 2(x\_1 \times x\_3 \times \sigma\_1 \times \sigma\_3 \times \rho\_{13}) + 2(x\_2 \times x\_3 \times \sigma\_2 \times \sigma\_3 \times \rho\_{23})+(x32​×σ32​)+2(x1​×x3​×σ1​×σ3​×ρ13​)+2(x2​×x3​×σ2​×σ3​×ρ23​)

Note in the previous figure: as the number N of assets increases, the number of terms outside the main diagonal increases more than the main diagonal.

Therefore, the variance of a well-diversified portfolio mostly contains covariances.

For a portfolio with equal weights to all assets:

Var(Rp)=1N×average  variance+(1−1N)×average  covarianceVar(R\_p) = \frac{1}{N} \times average\;variance + (1-\frac{1}{N}) \times average \; covarianceVar(Rp​)=N1​×averagevariance+(1−N1​)×averagecovariance

As N grows to infinite, only the average covariance lasts.

So, if you increase the size of your portfolio, the risk decreases (until to a certain amount).

Usually, you can diversify about half of the initial variance.

You can also write that:

Var(Rp)=∑ixi×Cov(Ri,Rp)Var(R\_p) = \sum\_i x\_i \times Cov(R\_i,R\_p)Var(Rp​)=∑i​xi​×Cov(Ri​,Rp​)

This equation indicates that the variance of a portfolio is equal to the weighted average covariance of each stock with the portfolio.

This expression reveals that the risk of a portfolio depends on how each stock’s return moves in relation to it.

Additionally, notice that stocks can have different weights in this equation (the previous example assumed equal weights).

Final equation

The equation in the previous slide can also be written as (remember that $Cov = sd \times sd \times corr$):

Var(Rp)=∑ixi×Sd(Ri)×Sd(Rp)×Corr(Ri,Rp)Var(R\_p) = \sum\_i x\_i \times Sd(R\_i) \times Sd(R\_p) \times Corr(R\_i,R\_p)Var(Rp​)=∑i​xi​×Sd(Ri​)×Sd(Rp​)×Corr(Ri​,Rp​)

If we divide both sides of this equation by $Sd(R\_p)$, we find:

Sd(Rp)=∑ixi×Sd(Ri)×Corr(Ri,Rp)Sd(R\_p) = \sum\_i x\_i \times Sd(R\_i) \times Corr(R\_i,R\_p)Sd(Rp​)=∑i​xi​×Sd(Ri​)×Corr(Ri​,Rp​)

This equation shows the amount of risk that each security brings to portfolio.

Each asset *i* contributes to the portfolio's volatility according to its Sd scaled by its correlation with the portfolio.

Keep this equation in mind, we will use it in the future.

More about correlation

In these graphs, I am assuming that you can invest a negative amount in a stock. This is called a short position. When you buy, you have a long position.

Short sales are usually allowed if you provide enough security and collateral to the market.

The idea is that you think that a stock's price will go down so you sell it. Later, you buy it back (but if the price goes up, you lose part of your investment).

Notice that if you can short sale, you amplify the pairs return-risk available.

Correlations usually positive.

**11.4 Choosing an Efficient Portfolio**

Now, you can understand what an efficient portfolio is.

* It is the portfolio that brings the higher return for any given level of risk
* or a portfolio that brings the lower risk for any given return

As a financial manager, one crucial job you have is to find the efficient portfolios and the minimum variance portfolio.

If you add stocks you improve the frontier.

When you combine several assets, you will have what is called efficient frontier.

**11.5 Risk-Free Saving and Borrowing**

Thus far, we have considered the risk and return possibilities that result from combining risky investments into portfolios.

By including all risky investments in the construction of the efficient frontier, we achieve the maximum diversification possible with risky assets.

Now, let's see what happens when you combine a portfolio of risky assets with the risk free asset.

The return is:

E[Rpx]=x×E[Rp]+(1−x)×RfE[R\_{px}] = x \times E[R\_p] + (1-x) \times R\_f E[Rpx​]=x×E[Rp​]+(1−x)×Rf​

Which leads to:

E[Rpx]=Rf+x×(E[Rp]−Rf)E[R\_{px}] = R\_f + x \times ( E[R\_p] - R\_f ) E[Rpx​]=Rf​+x×(E[Rp​]−Rf​)

The second equation shows that: The expected return is equal to the risk-free rate plus a fraction of the portfolio’s risk premium, $E[R\_p] - R\_f$, based on the fraction x that we invest in it.

Remember that the risk free rate is assumed to have no risk, thus no variance. The standard deviation is:

Sd(Rpx)=x×Sd(Rp)Sd(R\_{px}) = x \times Sd(R\_p)Sd(Rpx​)=x×Sd(Rp​)

That is, the volatility is only a fraction of the volatility of the portfolio, based on the amount we invest in it.

Remember that we have several potential combinations between Assets 1 and 2.

In this situation, we are better off investing in the Tangent portfolio, then "moving" to adjust the level of risk.

Remember that you can have short positions:

If you borrow at the Rf to invest in a portfolio, you have a levered position.

You are investing more than 100% of your funds in the portfolio. The weight is higher than 1.

The book calls this buying stocks on margin.

To identify the tangent portfolio, we compute the Sharpe ratio.

Sharpe  ratio=E[Rp]−RfSd(Rp)Sharpe\;ratio = \frac{E[R\_p]-R\_f}{Sd(R\_p)}Sharperatio=Sd(Rp​)E[Rp​]−Rf​​

To earn the highest possible expected return for any level of volatility we must find the portfolio that generates the steepest (highest inclination) possible line when combined with the risk-free investment.

The optimal portfolio to combine with the risk-free asset will be the one with the highest Sharpe ratio, where the line with the risk-free investment just touches, and so is tangent to, the efficient frontier of risky investments.

The Sharpe ratio measures the ratio of reward-to-volatility provided by a portfolio.

Fact 1: The tangent portfolio is efficient.  
Fact 2: Once we include the risk-free investment, all efficient portfolios are combinations of the risk-free investment and the tangent portfolio.

All investors should have the tangent portfolio. All investors should combine the tangent portfolio with the risk free asset to adjust the level of risk.

If you ignore the risk free asset, you have several efficient portfolios (efficient frontier). But once you combine with the risk free rate, there is only one.

What is a good Sharpe Ratio?

**11.6 Efficient Portfolio and Required Returns**

Let's now consider how much return we will demand from a risky asset in order to make its inclusion in our portfolio worthy.

Let's say that you hold an arbitrary portfolio P (it does not matter what is inside P for the moment).

You only include an additional asset if it excess return to the level of risk, right? That is, if it increases the Sharpe ratio of the resulting portfolio (Portfolio P + New asset).

What is the excess return that this asset i brings to your portfolio P?

* $E[R\_i] - R\_f$

What is the risk that this asset i brings to your portfolio P?

* $Sd(R\_i) \times corr(R\_i,R\_p)$

So now our question is: Is the gain in return from investing in i adequate to make up for the increase in risk?

To see that, we have to test if:

E[Ri]−RfSd(Ri)×corr(Ri,Rp)>E[Rp]−RfSd(Rp)\frac{E[R\_i] - R\_f}{Sd(R\_i) \times corr(R\_i,R\_p)} > \frac{E[R\_p] - R\_f}{Sd(R\_p)}Sd(Ri​)×corr(Ri​,Rp​)E[Ri​]−Rf​​>Sd(Rp​)E[Rp​]−Rf​​

Moving the denominator to the right-hand side:

E[Ri]−Rf>Sd(Ri)×corr(Ri,Rp)×E[Rp]−RfSd(Rp)E[R\_i] - R\_f > Sd(R\_i) \times corr(R\_i,R\_p) \times \frac{E[R\_p] - R\_f}{Sd(R\_p)}E[Ri​]−Rf​>Sd(Ri​)×corr(Ri​,Rp​)×Sd(Rp​)E[Rp​]−Rf​​

Which is:

E[Ri]−Rf>Sd(Ri)×corr(Ri,Rp)Sd(Rp)×(E[Rp]−Rf)E[R\_i] - R\_f > \frac{Sd(R\_i) \times corr(R\_i,R\_p)}{Sd(R\_p)} \times (E[R\_p] - R\_f)E[Ri​]−Rf​>Sd(Rp​)Sd(Ri​)×corr(Ri​,Rp​)​×(E[Rp​]−Rf​)

Using a Beta notation:

E[Ri]−Rf>βiP×(E[Rp]−Rf)E[R\_i] - R\_f > \beta\_i^P \times (E[R\_p] - R\_f)E[Ri​]−Rf​>βiP​×(E[Rp​]−Rf​)

Then:

E[Ri]>Rf+βiP×(E[Rp]−Rf)E[R\_i] > R\_f + \beta\_i^P \times (E[R\_p] - R\_f)E[Ri​]>Rf​+βiP​×(E[Rp​]−Rf​)

That is, increasing the amount invested in i will increase the Sharpe ratio of portfolio P if its expected return $E[R\_i]$ exceeds its required return given portfolio P, defined as

Ri=Rf+βiP×(E[Rp]−Rf)R\_i = R\_f + \beta\_i^P \times (E[R\_p] - R\_f)Ri​=Rf​+βiP​×(E[Rp​]−Rf​)

The required return is the expected return that is necessary to compensate for the risk investment i will contribute to the portfolio.

The required return for an investment i is equal to the risk-free interest rate plus the risk premium of the current portfolio, P, scaled by i’s sensitivity to P, which is $\beta\_i^P$.

If i’s expected return exceeds this required return, then adding more of it will improve the performance of the portfolio.

This equation establishes the relation between an investment’s risk and its expected return.

It states that we can determine the appropriate risk premium for an investment from its beta with the efficient portfolio.

Ri=Rf+βiP×(E[Rp]−Rf)R\_i = R\_f + \beta\_i^P \times (E[R\_p] - R\_f)Ri​=Rf​+βiP​×(E[Rp​]−Rf​)

**11.7 The CAPM**

This is perhaps the most important model in Finance.

Three main assumptions:

* Investors can buy and sell all securities at competitive market prices (without incurring taxes or transactions costs) and can borrow and lend at the risk-free interest rate.
* Investors hold only efficient portfolios of traded securities.
* Investors have homogeneous expectations regarding the volatilities, correlations, and expected returns of securities. There is no information asymmetry.

If investors have homogeneous expectations, they will identify the same efficient portfolio (the highest Sharpe).

Under the CAPM assumptions, we can identify the efficient portfolio: It is equal to the market portfolio.

A Market portfolio contains all traded securities in a economy.

If investors identify the same market portfolio (the highest Sharpe), then we can identify the Capital Market Line (CML).

All investors will have a combination of the Market Portfolio and the Rf rate.

**11.8 Determining Risk Premium**

Under the CAPM assumptions, we can identify the efficient portfolio: It is equal to the market portfolio.

Thus, we can change $R\_p$ to $R\_m$

E[Ri]=Rf+βi×(E[Rm]−Rf)E[R\_i] = R\_f + \beta\_i \times (E[R\_m] - R\_f)E[Ri​]=Rf​+βi​×(E[Rm​]−Rf​)

The beta of a security measures its volatility due to market risk relative to the market as a whole, and thus captures the security’s sensitivity to market risk.

This is the CAPM!

The Security Market Line

The CAPM implies there is a linear relationship between a stock's Beta and its expected return.

This linear relationship has a name: Security Market Line (SML)

**Questions and Final comments**

Imagine that you find the SML in the economy, and you find that two assets lies below it. Would you buy these assets?

Both assets are expensive. They are offering less return than they are expected to offer according to the CAPM.

Because they are expensive, investors are expected to sell them. So prices are expected to drop.

Final comments I

The beta of a portfolio is the weighted average beta of the assets in the portfolio.

βp=∑ixi×βi\beta\_p = \sum\_i x\_i \times \beta\_iβp​=∑i​xi​×βi​

Final comments III

Rf affects the slope of the CML. Different Rf values produce different efficient frontiers.

1. **Question:** Buying shares of security *i* improves the Sharpe ratio of a portfolio if its expected return does not exceed the required return. **Answer:** F **Feedback:** The sentence is FALSE. To improve the Sharpe ratio, an asset’s expected return must exceed its required return (the risk premium); otherwise, adding it lowers the portfolio’s risk‑adjusted return.
2. **Question:** The expected variance of a portfolio is the weighted average of the expected variances of the investments within it, using the portfolio weights. **Answer:** F **Feedback:** The sentence is FALSE. Portfolio variance also includes covariance terms between assets; it is not simply the weighted average of individual variances.
3. **Question:** Diversification eliminates independent risks. The volatility of a large portfolio results from the common risk between the stocks in the portfolio. **Answer:** T **Feedback:** The sentence is TRUE. Idiosyncratic (independent) risks offset each other in a large portfolio, so only systematic risk remains in the portfolio’s volatility.
4. **Question:** Short selling extends the set of possible portfolios. **Answer:** T **Feedback:** The sentence is TRUE. Allowing short positions (negative weights) expands the range of achievable risk–return combinations.
5. **Question:** Investors mainly worry about those risks that can be eliminated through diversification. **Answer:** F **Feedback:** The sentence is FALSE. Diversifiable risk can be eliminated without compensation; investors focus on systematic risk, which cannot be diversified away.
6. **Question:** Efficient portfolios offer investors the highest possible expected return for a given level of risk. **Answer:** T **Feedback:** The sentence is TRUE. By definition, portfolios on the efficient frontier maximize expected return for a particular volatility.
7. **Question:** To find the risk of a portfolio, we need to know the degree to which stock returns move together. Covariance and correlation measure the co‑movement of returns. **Answer:** T **Feedback:** The sentence is TRUE. Covariance and correlation quantify how asset returns co‑move and are essential inputs to portfolio risk calculations.
8. **Question:** The goal of an investor who is seeking to earn the highest possible expected return for any level of volatility is to find the portfolio that generates the steepest possible line when combined with the risk‑free investment. **Answer:** T **Feedback:** The sentence is TRUE. The steepest line from the risk‑free rate represents the maximum Sharpe ratio; investors seek this optimal risky portfolio.
9. **Question:** Under the CAPM assumptions, the capital market line (CML), which is the set of portfolios obtained by combining the risk‑free security and the market portfolio, is the set of portfolios with the highest possible expected return for any level of volatility. **Answer:** T **Feedback:** The sentence is TRUE. The CML dominates all other portfolios in risk–return space under the CAPM assumptions.
10. **Question:** The variance of a portfolio depends on the covariance of the stocks within it. **Answer:** T **Feedback:** The sentence is TRUE. The portfolio variance formula includes a 2 × w₁ w₂ covariance term, so co‑movement affects total risk.
11. **Question:** Portfolios that offer the highest expected return for a given variance (or standard deviation) are known as efficient portfolios. **Answer:** T **Feedback:** The sentence is TRUE. This is the definition of efficiency in Modern Portfolio Theory.
12. **Question:** A minimum‑variance portfolio is the portfolio with the lowest possible risk for a given set of assets. **Answer:** T **Feedback:** The sentence is TRUE. The minimum‑variance portfolio minimizes volatility across all feasible weight combinations.
13. **Question:** The market portfolio is a tangency portfolio according to the CAPM. **Answer:** T **Feedback:** The sentence is TRUE. In the CAPM, the market portfolio touches the efficient frontier and has the highest Sharpe ratio.
14. **Question:** Beta measures the sensitivity of a stock's returns to the overall market movements. **Answer:** T **Feedback:** The sentence is TRUE. Beta captures how a stock’s returns respond to changes in the market.
15. **Question:** The efficient frontier represents the set of portfolios that offer the highest expected return for a given level of risk. **Answer:** T **Feedback:** The sentence is TRUE. It traces the best possible trade‑off between risk and return.
16. **Question:** The Sharpe ratio is a measure of risk‑adjusted return, calculated by dividing the excess return of an investment by its standard deviation. **Answer:** T **Feedback:** The sentence is TRUE. Sharpe ratio = (expected return – risk‑free rate) / standard deviation.
17. **Question:** The Capital Market Line (CML) represents the risk‑return tradeoff for efficient portfolios that combine the risk‑free asset with the market portfolio. **Answer:** T **Feedback:** The sentence is TRUE. The CML shows all efficient combinations of the market portfolio and the risk‑free asset.
18. **Question:** A well‑diversified portfolio always consists of assets with low or negative correlations to each other to reduce overall portfolio risk. **Answer:** F **Feedback:** The sentence is FALSE. While low correlations help, diversification can still reduce risk even with some positive correlations; the key is the overall covariance structure.
19. **Question:** The Efficient Frontier represents portfolios that offer the best possible trade-off between expected return and risk, excluding any combinations with the risk-free asset. **Answer:** T **Feedback:** The sentence is TRUE. The Efficient Frontier shows all optimal portfolios formed by risky assets only. Combinations with the risk-free asset are represented by the Capital Market Line (CML), which starts at the risk-free rate and is tangent to the Efficient Frontier.
20. **Question:** An asset's standard deviation represents its systematic risk in the context of the Capital Asset Pricing Model (CAPM). **Answer:** F **Feedback:** The sentence is FALSE. Standard deviation measures total risk (systematic plus idiosyncratic); systematic risk is captured by beta.
21. **Question:** An efficient portfolio has no risk at all. **Answer:** F **Feedback:** The sentence is FALSE. Efficient portfolios still have risk; they simply maximize return relative to that risk.
22. **Question:** The presence of a risk‑free asset enables the investor to borrow or lend at the risk‑free rate and form portfolios having greater Sharpe ratios. **Answer:** T **Feedback:** The sentence is TRUE. Borrowing or lending at the risk‑free rate and combining with the optimal risky portfolio allows movement along the CML to achieve higher Sharpe ratios.
23. **Question:** The security market line (SML) is the graph of expected rate of return on investment vs. the variance of returns. **Answer:** F **Feedback:** The sentence is FALSE. The SML plots expected return versus beta; the CML relates expected return to standard deviation.
24. **Question:** If a stock is overpriced, it would plot above the security market line. **Answer:** F **Feedback:** The sentence is FALSE. A stock above the SML offers more return than justified by its risk and is underpriced; an overpriced stock falls below the SML.
25. **Question:** A stock's alpha is the difference between the expected return and the required return according to the CAPM. **Answer:** T **Feedback:** The sentence is TRUE. Alpha measures performance relative to the return predicted by beta.
26. **Question:** Adding assets with low or negative correlations to a portfolio generally reduces overall risk. **Answer:** T **Feedback:** The sentence is TRUE. Lower correlation lowers the covariance terms in portfolio variance, reducing overall risk.
27. **Question:** In equilibrium, it is possible to earn a return that is above the efficient frontier without the existence of a risk‑free asset or some other asset that is uncorrelated with your portfolio assets. **Answer:** F **Feedback:** The sentence is FALSE. Without a risk‑free or uncorrelated asset, all feasible portfolios lie on or below the efficient frontier; one cannot surpass it.
28. **Question:** The Sharpe ratio measures the excess return per unit of risk. **Answer:** T **Feedback:** The sentence is TRUE. It expresses how much excess return is earned per unit of total volatility.
29. **Question:** A stock with a beta of 0 is expected to have no correlation with market movements.  
    **Answer:** T **Feedback:** The sentence is TRUE. Beta zero implies the stock’s returns are uncorrelated with the market.
30. **Question:** The risk‑free asset has a beta of 1. **Answer:** F **Feedback:** The sentence is FALSE. A risk‑free asset has no covariance with the market, so its beta is zero.
31. **Question:** A portfolio on the efficient frontier can always be improved by adding more assets. **Answer:** F **Feedback:** The sentence is FALSE. Portfolios on the efficient frontier already offer the best return for their risk; additional assets cannot improve that trade‑off.
32. **Question:** If two assets have perfect positive correlation, combining them in a portfolio does not reduce risk. **Answer:** T **Feedback:** The sentence is TRUE. With correlation = +1, the assets move together, and diversification provides no risk reduction.
33. **Question:** A stock with a beta greater than 1 is considered less risky than the market. **Answer:** F **Feedback:** The sentence is FALSE. Beta > 1 means the stock amplifies market movements and is more volatile than the market.
34. **Question:** An equally weighted portfolio assigns the same weight to all assets, regardless of their risk or expected return. **Answer:** T **Feedback:** The sentence is TRUE. In an equally weighted portfolio, each asset receives an identical percentage allocation.
35. **Question:** The tangency portfolio on the efficient frontier consists of only risk‑free assets. **Answer:** F **Feedback:** The sentence is FALSE. The tangency (or market) portfolio contains only risky assets and achieves the highest Sharpe ratio.
36. **Question:** The risk‑return tradeoff implies that investors must take on additional risk to achieve higher expected returns. **Answer:** T **Feedback:** The sentence is TRUE. Generally, higher expected returns are associated with greater uncertainty.
37. **Question:** The global minimum variance portfolio has the lowest possible volatility among all feasible portfolios. **Answer:** T **Feedback:** The sentence is TRUE. It is the point of minimum variance on the efficient frontier.
38. **Question:** The Sharpe ratio helps investors compare the risk‑adjusted performance of different portfolios. **Answer:** T **Feedback:** The sentence is TRUE. It standardizes excess return by risk, allowing comparison across portfolios.
39. **Question:** A portfolio with high volatility is always considered inefficient. **Answer:** F **Feedback:** The sentence is FALSE. A high‑volatility portfolio may still be efficient if its expected return is high enough.
40. **Question:** Portfolio diversification benefits decrease as the correlations between assets increase. **Answer:** T **Feedback:** The sentence is TRUE. Higher correlations mean assets move more together, reducing diversification benefits.
41. **Question:** A well‑diversified portfolio eliminates both systematic and unsystematic risk. **Answer:** F **Feedback:** The sentence is FALSE. Diversification eliminates unsystematic (idiosyncratic) risk but cannot remove systematic risk.
42. **Question:** Investors with different risk preferences will choose different points along the Capital Market Line. **Answer:** T **Feedback:** The sentence is TRUE. More risk‑averse investors select combinations with more of the risk‑free asset; less risk‑averse may leverage the market portfolio.
43. **Question:** The covariance between two stocks determines their contribution to overall portfolio risk. **Answer:** T **Feedback:** The sentence is TRUE. The covariance affects the cross‑term in the variance formula, influencing total risk.
44. **Question:** A portfolio with a beta of 1 has the same systematic risk as the market portfolio. **Answer:** T **Feedback:** The sentence is TRUE. Beta = 1 means the portfolio’s returns move in line with the market’s systematic risk.
45. **Question:** A portfolio’s expected return is the weighted average of the expected returns of the individual assets in the portfolio. **Answer:** T **Feedback:** The sentence is TRUE. Expected returns combine linearly according to the weights.
46. **Question:** An investor who only holds a single stock is still well diversified as long as that stock has a high expected return. **Answer:** F **Feedback:** The sentence is FALSE. Holding one stock exposes the investor to idiosyncratic risk; high expected return does not substitute for diversification.
47. **Question:** According to Modern Portfolio Theory, an investor should hold a combination of the risk‑free asset and the market portfolio to achieve an optimal risk‑return tradeoff. **Answer:** T **Feedback:** The sentence is TRUE. Combining the market portfolio with the risk‑free asset along the CML yields the best attainable trade‑off.
48. **Question:** Beta is calculated as the covariance between the asset’s return and the market return divided by the variance of the market return. **Answer:** T **Feedback:** The sentence is TRUE. By definition, β = Cov(Rᵢ, R\_m) / Var(R\_m).
49. **Question:** According to the CAPM, the only compensation investors receive is for unsystematic risk. **Answer:** F **Feedback:** The sentence is FALSE. Under CAPM, investors are compensated only for systematic risk; unsystematic risk is not rewarded.
50. **Question:** The intercept of the Security Market Line is equal to the risk‑free rate of return. **Answer:** T **Feedback:** The sentence is TRUE. At β = 0, the SML gives E[R] = Rf, so the intercept is the risk‑free rate.
51. **Question:** A beta of 1.5 implies that the asset is less volatile than the market portfolio. **Answer:** F **Feedback:** The sentence is FALSE. Beta greater than one means the asset is more volatile (more sensitive to market movements) than the market.
52. **Question:** Negative beta values indicate that an asset tends to move opposite to market movements. **Answer:** T **Feedback:** The sentence is TRUE. A negative beta signifies that the asset has an inverse relationship with the market.
53. **Question:** The Capital Market Line plots expected return against beta for all securities. **Answer:** F **Feedback:** The sentence is FALSE. The CML relates expected return to standard deviation for efficient portfolios; the SML relates expected return to beta.
54. **Question:** Combining a risk‑free asset with a risky portfolio can create portfolios with higher Sharpe ratios than the risky portfolio alone. **Answer:** T **Feedback:** The sentence is TRUE. Mixing with the risk‑free asset adjusts risk and can yield a higher Sharpe ratio along the CML.
55. **Question:** Beta measures both systematic and idiosyncratic risk of a security. **Answer:** F **Feedback:** The sentence is FALSE. Beta captures only systematic risk.
56. **Question:** Diversifying across many stocks can reduce systematic risk to zero. **Answer:** F **Feedback:** The sentence is FALSE. Systematic risk cannot be diversified away; only idiosyncratic risk can be eliminated.
57. **Question:** A portfolio’s beta is the weighted average of the betas of the individual assets. **Answer:** T **Feedback:** The sentence is TRUE. Portfolio beta = Σ wᵢ × βᵢ.
58. **Question:** A stock with a beta of 0 is expected to have an expected return equal to the risk‑free rate. **Answer:** T **Feedback:** The sentence is TRUE. With β = 0, CAPM gives E[R] = Rf.
59. **Question:** All investors will always choose the same combination of risky assets if they have identical expectations and there is a risk‑free asset. **Answer:** T **Feedback:** The sentence is TRUE. With homogeneous expectations and no transaction costs, everyone holds the market portfolio in equilibrium.
60. **Question:** The slope of the Capital Market Line equals the Sharpe ratio of the market portfolio. **Answer:** T **Feedback:** The sentence is TRUE. CML slope = (E[R\_m] – Rf)/σ\_m, which is the market’s Sharpe ratio.
61. **Question:** Systematic risk can be diversified away by holding many stocks. **Answer:** F **Feedback:** The sentence is FALSE. Systematic (market) risk affects all assets; diversification cannot eliminate it.
62. **Question:** A minimum‑variance portfolio has the highest possible Sharpe ratio. **Answer:** F **Feedback:** The sentence is FALSE. The minimum‑variance portfolio minimizes volatility but may not have the highest ratio of excess return to risk.
63. **Question:** The beta of the market portfolio is exactly 1. **Answer:** T **Feedback:** The sentence is TRUE. By convention, the market portfolio has β = 1.
64. **Question:** If an asset lies below the SML, it offers a higher expected return for its risk. **Answer:** F **Feedback:** The sentence is FALSE. An asset below the SML provides too low a return for its level of systematic risk and is overpriced.
65. **Question:** Correlation coefficients can only take values between −1 and +1. **Answer:** T **Feedback:** The sentence is TRUE. Correlation is bounded between −1 and +1.
66. **Question:** The risk of a portfolio is always the average of the risks of its individual assets. **Answer:** F **Feedback:** The sentence is FALSE. Portfolio risk depends on correlations; it can be lower than the weighted average of individual volatilities.
67. **Question:** A risk‑free asset has a beta of zero. **Answer:** T **Feedback:** The sentence is TRUE. A risk‑free asset has no covariance with the market, so β = 0.
68. **Question:** The CML represents portfolios formed by combining the risk‑free asset and the market portfolio. **Answer:** T **Feedback:** The sentence is TRUE. All points on the CML are linear combinations of the risk‑free asset and the market portfolio.
69. **Question:** In the CAPM, the expected return of any asset is independent of its covariance with the market. **Answer:** F **Feedback:** The sentence is FALSE. The expected return depends directly on the asset’s beta (covariance with the market).
70. **Question:** Stocks with higher betas should yield lower expected returns to be priced fairly. **Answer:** F **Feedback:** The sentence is FALSE. Higher betas imply greater systematic risk and therefore require higher expected returns.
71. **Question:** Portfolios that lie below the efficient frontier are considered inefficient. **Answer:** T **Feedback:** The sentence is TRUE. They offer lower return for the same risk or higher risk for the same return.
72. **Question:** If two assets have a correlation coefficient of +1, diversification between them reduces risk. **Answer:** F **Feedback:** The sentence is FALSE. Perfect positive correlation means no risk reduction from diversification.
73. **Question:** The Sharpe ratio increases when the risk‑free rate decreases if portfolio return and volatility remain unchanged. **Answer:** T **Feedback:** The sentence is TRUE. A lower risk‑free rate raises the excess return in the numerator, increasing the ratio.
74. **Question:** A portfolio on the efficient frontier cannot be dominated by another portfolio offering a higher expected return with the same or lower risk. **Answer:** T **Feedback:** The sentence is TRUE. By definition, no other portfolio offers a better risk–return combination.
75. **Question:** The CAPM assumes that investors can borrow and lend unlimited amounts at the risk‑free rate. **Answer:** T **Feedback:** The sentence is TRUE. Unlimited borrowing and lending at Rf is one of the CAPM’s simplifying assumptions.
76. **Question:** Idiosyncratic risk is not relevant when evaluating an asset using CAPM. **Answer:** T **Feedback:** The sentence is TRUE. CAPM prices only systematic risk; idiosyncratic risk is assumed diversifiable.
77. **Question:** The expected return of a well‑diversified portfolio depends primarily on the portfolio’s beta. **Answer:** T **Feedback:** The sentence is TRUE. In a diversified portfolio, unsystematic risk is negligible, so expected return is determined by beta.
78. **Question:** A beta greater than zero but less than one indicates that the asset is more volatile than the market. **Answer:** F **Feedback:** The sentence is FALSE. Beta between 0 and 1 means the asset is less sensitive (less volatile) than the market.
79. **Question:** Under CAPM, the reward‑to‑risk ratio of all individual assets is the same and equal to the market risk premium. **Answer:** T **Feedback:** The sentence is TRUE. (E[R] – Rf)/β is constant and equals E[R\_m] – Rf for all assets in equilibrium.
80. **Question:** If two assets have zero correlation, the covariance between them is zero. **Answer:** T **Feedback:** The sentence is TRUE. Covariance = correlation × σ₁ × σ₂; correlation zero implies covariance zero.
81. **Question:** Adding a risk‑free asset to the portfolio cannot lower the overall portfolio variance. **Answer:** F **Feedback:** The sentence is FALSE. Including a risk‑free asset can reduce portfolio variance in proportion to its weight.
82. **Question:** The Treynor ratio is useful for comparing portfolios that are already well diversified. **Answer:** T **Feedback:** The sentence is TRUE. Because it uses beta in the denominator, the Treynor ratio focuses on systematic risk and is appropriate for diversified portfolios.
83. **Question:** It is impossible for a portfolio to have a beta of zero. **Answer:** F **Feedback:** The sentence is FALSE. A portfolio can be constructed with net beta zero by combining assets with positive and negative betas.
84. **Question:** Two portfolios with the same beta must have the same expected return under CAPM. **Answer:** T **Feedback:** The sentence is TRUE. In CAPM, E[R] = Rf + β(E[R\_m] – Rf); equal betas imply equal expected returns.
85. **Question:** The variance of a portfolio is equal to the sum of the variances of each asset when correlations are zero. **Answer:** F **Feedback:** The sentence is FALSE. When correlation is zero, portfolio variance equals the sum of squared weights times the variances (Σ wᵢ²σᵢ²), not simply Σ σᵢ².
86. **Question:** CAPM implies that all securities plot exactly on the Security Market Line in equilibrium.  
    **Answer:** T **Feedback:** The sentence is TRUE. In equilibrium, securities have zero alpha and lie on the SML.
87. **Question:** If an asset has a negative beta, it is expected to have a return below the risk‑free rate. **Answer:** F **Feedback:** The sentence is FALSE. A negative beta asset may still have an expected return above or below Rf depending on the market risk premium and its magnitude.
88. **Question:** The optimal risky portfolio is the one with the highest Sharpe ratio. **Answer:** T **Feedback:** The sentence is TRUE. The tangency (market) portfolio maximizes excess return per unit of total risk.
89. **Question:** Investing more than 100 % in the market portfolio (leveraging) moves the portfolio up the CML. **Answer:** T **Feedback:** The sentence is TRUE. Borrowing at the risk‑free rate and investing more in the market portfolio increases both return and risk along the CML.
90. **Question:** The covariance between two assets is always non‑negative. **Answer:** F **Feedback:** The sentence is FALSE. Covariance can be positive, zero or negative depending on how the assets move relative to each other.
91. **Question:** The CML can be used to evaluate the performance of individual securities. **Answer:** F **Feedback:** The sentence is FALSE. The CML applies to efficient portfolios; the Security Market Line is used for individual securities.
92. **Question:** Under CAPM, investors are compensated only for bearing unsystematic risk. **Answer:** F **Feedback:** The sentence is FALSE. Compensation in CAPM is for systematic risk; unsystematic risk is not rewarded.
93. **Question**: The Security Market Line (SML) and the Capital Market Line (CML) are identical graphs in the CAPM model. **Answer**: F **Feedback**: The sentence is FALSE. The SML plots expected return against beta (systematic risk), while the CML plots expected return against total risk (standard deviation).
94. **Question**: The global minimum variance portfolio consists only of the risk-free asset. **Answer**: F **Feedback**: The sentence is FALSE. The minimum variance portfolio is the combination of risky assets with the lowest possible volatility, not necessarily involving the risk-free asset.
95. **Question**: The efficient frontier will not change if the correlation between assets changes. **Answer**: F **Feedback**: The sentence is FALSE. The shape and position of the efficient frontier depend on the correlations between asset returns; changes in correlation affect diversification benefits.
96. **Question**: If two assets have perfect negative correlation, combining them in a portfolio can theoretically eliminate all risk. **Answer**: T **Feedback**: The sentence is TRUE. Perfect negative correlation allows for a combination where the gains in one asset offset the losses in another, potentially eliminating portfolio variance.
97. **Question**: An asset with a beta of zero is assumed to have an expected return equal to the risk-free rate. **Answer**: T **Feedback**: The sentence is TRUE. According to CAPM, an asset with beta zero is not sensitive to market movements and thus earns the risk-free rate.
98. **Question**: Portfolio variance is always reduced when adding a new asset, regardless of its correlation with existing assets. **Answer**: F **Feedback**: The sentence is FALSE. Adding an asset with a high positive correlation to existing assets may not reduce and could even increase portfolio variance.
99. **Question**: The higher the standard deviation of a stock, the higher its beta. **Answer**: F **Feedback**: The sentence is FALSE. Beta measures systematic risk relative to the market, not total risk; a stock can have high volatility (standard deviation) but a low beta if it’s uncorrelated with the market.
100. **Question**: Investors who are risk-averse will prefer portfolios below the Capital Market Line (CML). **Answer**: F **Feedback**: The sentence is FALSE. Rational investors will only choose portfolios on the CML, as portfolios below it are inefficient and offer lower returns for the same risk.

**Q1.**

Parte superior do formulário

**Which of the following statements best describes an efficient portfolio?**

 A. A portfolio that maximizes return for a given level of risk  
 B. A portfolio with the lowest variance regardless of return  
 C. A portfolio that holds only a single risky asset  
 D. A portfolio that invests equal amounts in all available assets  
 E. A portfolio that minimizes beta

Parte inferior do formulário

✅ Correct: A. An efficient portfolio is one that offers the highest expected return for a given level of risk.

**Q2.**

Parte superior do formulário

**The variance of a two‑asset portfolio depends on:**

 A. Only the variances of the individual assets  
 B. Only the covariance between the assets  
 C. Both the variances of the individual assets and the covariance between them  
 D. The average return of the portfolio  
 E. None of the above

Parte inferior do formulário

✅ Correct: C. Portfolio variance combines the individual asset variances and their covariance (or correlation).

**Q3.**

Parte superior do formulário

**If the correlation between two assets is −1, combining them in a portfolio:**

 A. Eliminates all risk if the weights are properly chosen  
 B. Has no effect on the portfolio’s risk  
 C. Doubles the portfolio’s expected return  
 D. Increases the portfolio’s variance above either asset’s variance  
 E. Makes the assets perfectly positively correlated

Parte inferior do formulário

✅ Correct: A. A correlation of −1 means the assets move perfectly in opposite directions, allowing risk to be eliminated with the right weights.

**Q4.**

Parte superior do formulário

**Which measure captures only the systematic risk of a stock?**

 A. Standard deviation  
 B. Variance  
 C. Beta  
 D. Arithmetic mean  
 E. Sharpe ratio

Parte inferior do formulário

✅ Correct: C. Beta measures an asset’s sensitivity to market movements and therefore captures only systematic risk.

**Q5.**

Parte superior do formulário

**In the Capital Asset Pricing Model (CAPM), the expected return of a security is:**

 A. Equal to the risk‑free rate  
 B. Inversely proportional to its beta  
 C. Equal to the average return of the market portfolio  
 D. A linear function of the security’s beta and the market risk premium  
 E. Independent of the security’s beta

Parte inferior do formulário

✅ Correct: D. The CAPM states that E[R] = Rf + β × (E[Rm] − Rf), a linear function of beta and the market risk premium.

**Q6.**

Parte superior do formulário

**The Sharpe ratio of a portfolio is defined as:**

 A. The portfolio’s excess return divided by its variance  
 B. The portfolio’s beta divided by its return  
 C. The portfolio’s excess return divided by its standard deviation  
 D. The portfolio’s return divided by its beta  
 E. The portfolio’s variance divided by the market variance

Parte inferior do formulário

✅ Correct: C. The Sharpe ratio measures excess return per unit of total risk (standard deviation).

**Q7.**

Parte superior do formulário

**According to the CAPM assumptions, all investors:**

 A. Can only buy and hold the risk‑free asset  
 B. Have homogeneous expectations and hold the same market portfolio  
 C. Prefer portfolios with the highest idiosyncratic risk  
 D. Are risk‑neutral  
 E. Face high transaction costs and taxes

Parte inferior do formulário

✅ Correct: B. The CAPM assumes homogeneous expectations and that all investors hold the market portfolio in equilibrium.

**Q8.**

Parte superior do formulário

**The Security Market Line (SML) depicts the relationship between:**

 A. Standard deviation and expected return for individual assets  
 B. Beta and expected return for individual assets  
 C. Weight and variance for portfolios  
 D. Correlation and covariance for portfolios  
 E. Price and earnings per share

Parte inferior do formulário

✅ Correct: B. The SML shows the linear relationship between a security’s beta and its expected return under the CAPM.

**Q9.**

Parte superior do formulário

**If an asset lies above the Security Market Line, it is considered:**

 A. Overpriced and should be sold  
 B. Underpriced and offers a higher return for its level of risk  
 C. Efficient and lies on the efficient frontier  
 D. A risk‑free asset  
 E. A candidate for short selling only

Parte inferior do formulário

✅ Correct: B. Being above the SML means the asset delivers more expected return than justified by its beta and is therefore underpriced.

**Q10.**

Parte superior do formulário

**Diversification primarily reduces:**

 A. Systematic risk  
 B. Market risk  
 C. Idiosyncratic (unsystematic) risk  
 D. The risk‑free rate  
 E. Beta

Parte inferior do formulário

✅ Correct: C. Diversification eliminates firm‑specific risk but cannot eliminate systematic, market‑wide risk.

**Expected Return, Variance, and Standard Deviation of a Portfolio**

What is the **expected return** of a portfolio that invests **22.69**% in Asset 1 (expected return = **5.12**%) and the remaining **77.31**% in Asset 2 (expected return = **1.88**% )?

Parte superior do formulário

**1. What is the expected return of the portfolio (in %)?**

Parte inferior do formulário

✅ Expected return = w₁×r₁ + w₂×r₂ = **2.62**%.

What is the **variance** of the portfolio above, given correlation **0.62**, standard deviation of Asset 1 = **26.32**% and of Asset 2 = **20.2**%?

Parte superior do formulário

**2. What is the variance of the portfolio (in squared %)?**

Parte inferior do formulário

✅ Variance = w₁²σ₁² + w₂²σ₂² + 2w₁w₂ρσ₁σ₂ = **3.95**.

What is the **standard deviation** of the portfolio above?

Parte superior do formulário

**3. What is the standard deviation of the portfolio (in %)?**

Parte inferior do formulário

✅ Standard deviation = √(variance) = **19.89**%.

**Portfolio Weight Calculation for Multiple Stocks**

Suppose you purchase **155** shares of Company A at $**62.32** per share, **135** shares of Company B at $**39.38** per share, and **91** shares of Company C at $**55.37** per share.

Parte superior do formulário

**1. What is the weight on Company A in your portfolio (in %)?**

Parte inferior do formulário

✅ Weight on Company A = value\_A / total\_value × 100 = **48.26**%.

**Portfolio Variance and Standard Deviation Calculation**

You invest **84.45%** of your funds in Asset A (volatility = **9.95%**) and **15.55%** in Asset B (volatility = **23.61%**). The correlation between the two assets is **0.35**.

Parte superior do formulário

**1. What is the variance of your portfolio (in squared %)?**

Parte inferior do formulário

✅ Portfolio variance = w²×sd₁² + (1−w)²×sd₂² + 2×w×(1−w)×sd₁×sd₂×ρ = **105.51**.

Parte superior do formulário

**2. What is the standard deviation of your portfolio (in %)?**

Parte inferior do formulário

✅ Portfolio standard deviation = √(variance) = **10.27**%.

**Calculating Beta Using Covariance and Market Variance**

Given that the covariance between a stock and the market is **4.69** and the variance of the market is **4.84**,

Parte superior do formulário

**1. What is the beta of the stock?**

Parte inferior do formulário

✅ Beta = Cov(asset, market) / Var(market) = **0.97**.

**Risk Premium and Sharpe Ratio Calculation**

An investment has an expected return of **6.26%**, a risk‑free rate of **2.02%**, and a volatility (standard deviation) of **14.87%**.

Parte superior do formulário

**1. What is the risk premium (in %)?**

Parte inferior do formulário

✅ Risk premium = expected return − risk‑free rate = **4.25**%.

Parte superior do formulário

**2. What is the Sharpe ratio (return per unit of risk)?**

Parte inferior do formulário

✅ Sharpe ratio = risk premium / volatility = **0.29**.

**Combining Risky Asset and Risk‑Free Rate**

You invest **93.62%** of your funds in a risky portfolio (expected return = **10.41%**, volatility = **21.77%**) and the remainder **6.38%** in the risk‑free asset (rate = **3.55%**).

Parte superior do formulário

**1. What is the expected return of your portfolio (in %)?**

Parte inferior do formulário

✅ Expected return = w×ret\_risky + (1−w)×Rf = **9.97**%.

Parte superior do formulário

**2. What is the standard deviation of your portfolio (in %)?**

Parte inferior do formulário

✅ Since the risk‑free asset has zero volatility, the portfolio’s volatility equals w×σ\_risky = **20.38**%.

**Beta of a Three‑Asset Portfolio**

You hold a portfolio composed of three assets with betas **1.14**, **1.11**, and **0.23**, invested in proportions **66.79%**, **0.12%**, and **33.09%**, respectively.

Parte superior do formulário

**1. What is the beta of the portfolio?**

Parte inferior do formulário

✅ Portfolio beta = w₁×β₁ + w₂×β₂ + w₃×β₃ = **0.84**.

**Expected Return via CAPM**

The risk‑free rate is **3.32%**, the market risk premium is **7.61%**, and the beta of the investment is **1.27**.

Parte superior do formulário

**1. What is the expected return of the investment according to the CAPM (in %)?**

Parte inferior do formulário

✅ Expected return = Rf + β × (market risk premium) = **13**%.

**Variance and Volatility of a Three‑Asset Portfolio**

You invest **1.21%**, **26.06%**, and **72.72%** of your funds in three assets with volatilities **16.01%**, **15.88%**, and **19.11%**, respectively. The pairwise correlations are ρ₁₂ = **0.07**, ρ₁₃ = **0.09**, and ρ₂₃ = **0.08**.

Parte superior do formulário

**1. What is the variance of the portfolio?**

Parte inferior do formulário

✅ Variance = Σwᵢ²σᵢ² + 2Σ wᵢwⱼσᵢσⱼρᵢⱼ = **220.4**.

Parte superior do formulário

**2. What is the standard deviation of the portfolio (in %)?**

Parte inferior do formulário

✅ Standard deviation = √(variance) = **14.85**%.

**Risk‑Free Combination and Sharpe Ratio**

You build a portfolio by combining the risk‑free asset (rate = **4.25%**) with a tangent portfolio (expected return = **11.43%**, volatility = **20.4%**) using a weight of **88.82%** in the tangent portfolio (weights greater than 100% imply leverage).

Parte superior do formulário

**1. What is the expected return of your combined portfolio (in %)?**

Parte inferior do formulário

✅ Expected return = w×ret\_tan + (1−w)×Rf = **10.63**%.

Parte superior do formulário

**2. What is the standard deviation of your combined portfolio (in %)?**

Parte inferior do formulário

✅ Standard deviation = |w|×σ\_tan = **18.12**%.

Parte superior do formulário

**3. What is the Sharpe ratio of your combined portfolio?**

Parte inferior do formulário

✅ Sharpe ratio = risk premium / volatility = **0.35**.

**Sharpe Calculation**

An investment has an expected return of **8.31%**, a risk‑free rate of **2.09%**, and a volatility (standard deviation) of **15.9%**.

Parte superior do formulário

**1. What is the risk premium (in %)?**

Parte inferior do formulário

✅ Risk premium = expected return − risk‑free rate = **6.22**%.

Parte superior do formulário

**2. What is the Sharpe ratio?**

Parte inferior do formulário

✅ Sharpe ratio = risk premium / volatility = **0.391**.

**Q1.**

**Explain the concept of the efficient frontier and describe how adding more assets to a portfolio can change its risk and return characteristics. Use an example of assets with different correlations to illustrate your answer.**

Possible Answer:

The **efficient frontier** is the set of optimal portfolios that offer the highest expected return for a given level of risk (or the lowest risk for a given expected return), based on the available assets.  
Adding more assets to a portfolio can change its risk and return characteristics because of **diversification**. If the assets have less-than-perfect positive correlation, combining them can reduce total portfolio risk without proportionally reducing expected return.  
*Example:* Suppose Asset A has an expected return of 8% and standard deviation of 10%, and Asset B has an expected return of 6% and standard deviation of 8%. If their correlation is **+1**, diversification does not reduce risk—the portfolio’s standard deviation is just a weighted average. But if their correlation is **0.3**, combining them can produce portfolios with lower volatility than either asset individually, shifting the efficient frontier upward and to the left (higher return for less risk).

**Q2.**

**Define covariance and correlation in the context of finance. Discuss how these measures influence the variance of a portfolio and explain why diversification can reduce unsystematic risk.**

Possible answer:

In finance, **covariance** measures the degree to which two assets’ returns move together. A positive covariance means returns tend to move in the same direction; negative means they tend to move in opposite directions. **Correlation** standardizes covariance to a scale from -1 to +1, making it easier to compare across asset pairs.  
These measures influence the **portfolio variance** because the total variance depends not only on the variances of individual assets but also on their covariances. Lower correlations reduce the contribution of the covariance terms, thus lowering total risk.  
Diversification reduces **unsystematic risk** (asset-specific risk) because combining assets with imperfect correlations smooths out individual fluctuations. Systematic risk, however, cannot be eliminated through diversification.

**Q3.**

**Describe the tangent portfolio and the Capital Market Line (CML). How does combining a risky portfolio with a risk‑free asset allow investors to achieve different levels of risk and return?**

Possible answer:

The **tangent portfolio** is the portfolio on the efficient frontier that, when combined with the risk-free asset, offers the highest Sharpe ratio. It is the point of tangency between the efficient frontier and the **Capital Market Line (CML)**.  
The CML plots the risk–return trade-off for combinations of the risk-free asset and the market (tangent) portfolio. By adjusting the weights between the two, investors can achieve different levels of risk and return:

* Investing more in the risk-free asset moves the portfolio toward lower risk and lower return.
* Borrowing at the risk-free rate to invest more than 100% in the risky portfolio increases both expected return and risk.

**Q4.**

**Define the Sharpe ratio and explain how it is used to evaluate the performance of a portfolio. Discuss how changes in the risk‑free rate affect the slope of the Capital Market Line and the interpretation of the Sharpe ratio.**

Possible answer:

The **Sharpe ratio** measures risk-adjusted return:

Sharpe Ratio=Expected Return−RfStandard Deviation\text{Sharpe Ratio} = \frac{\text{Expected Return} - R\_f}{\text{Standard Deviation}}Sharpe Ratio=Standard DeviationExpected Return−Rf​​

where RfR\_fRf​ is the risk-free rate. It evaluates how much excess return an investor earns per unit of total risk.  
In the CML, the slope equals the Sharpe ratio of the market portfolio. If the risk-free rate increases, the slope of the CML decreases (assuming market return is constant), which lowers the Sharpe ratio for all portfolios. Conversely, a lower Rf​ increases the slope and makes risk-taking relatively more attractive.

**Q5.**

**List the main assumptions of the Capital Asset Pricing Model (CAPM). Then explain how the Security Market Line illustrates the linear relationship between a security’s beta and its expected return, and discuss the implications of this relationship for asset valuation.**

Possible answer:

**Main CAPM assumptions:**

1. Investors are rational, risk-averse, and seek to maximize utility over a single period.
2. Investors can borrow and lend unlimited amounts at the risk-free rate.
3. All investors have homogeneous expectations about returns, variances, and covariances.
4. Markets are frictionless (no taxes, transaction costs, or short-sale restrictions).
5. All assets are infinitely divisible and perfectly liquid.
6. All investors have access to the same set of assets.

The **Security Market Line (SML)** shows the expected return of a security as a linear function of its **beta**:

E(Ri)=Rf+βi(E(Rm)−Rf)E(R\_i) = R\_f + \beta\_i \left( E(R\_m) - R\_f \right)E(Ri​)=Rf​+βi​(E(Rm​)−Rf​)

Beta measures the asset’s sensitivity to market movements (systematic risk). The SML implies that only beta matters for expected returns; higher beta means higher required return. Securities plotting above the SML are undervalued (offering more return than required), while those below are overvalued.