ECE 133

Homework 1

Due: Friday 10/13 at 11:59 pm

Problem 4 is not graded

Submission is through Gradescope
Please seperately upload all code files for all assignments

Problem 1 - Maxflow in CVX

Let's start by considering the maximum flow problem, the first problem we formulated in class as a linear program. We have a directional graph and our goal is to maximize the total flow leaving the source and reaching the destination, subject to the capacity constraints on the graph's edges and flow conservation conditions (flow in = flow out at all intermediate nodes). Remember that the optimization variables are the flows on the edges. If you wish to write the maxflow linear program in vector form, let us denote the optimization variable as the vector of all edge flows $\mathbf{f}$. Then the maxflow problem can be written in vector form as follows:

$$\begin{aligned} \max_{\mathbf{f}} \quad & \mathbf{s}^T\mathbf{f} \\ \text{s.t.} \quad & \mathbf{L}\mathbf{f} = 0, \\ & \mathbf{0} \le \mathbf{f} \le \mathbf{c}, \end{aligned} \tag{1}$$

where $\mathbf{L}$ is the so-called graph node-edge incidence matrix[1] except for the rows corresponding to the source and the destination, $\mathbf{s}$ is a column vector with a 1 for every edge leaving the source and a 0 for every other edge, and $\mathbf{c}$ is the column vector of edge capacities.

Your goal is to use CVX (or CVXPY) to solve the maxflow problem as a linear program on a directional graph of your choosing. For example, using the piece of code below in MATLAB, you can create and plot a directional graph using a list of the end nodes (heads and tails) of each edge. You may also specify node names and edge weights as separate inputs.

```
s = [1 1 1 2 2 3 3 4 5 5 6 7 5 9];
t = [2 4 8 3 7 4 6 5 6 8 7 8 9 3];
weights = [10 10 1 10 1 10 1 1 12 12 12 12 6 4];
names = {'A' 'B' 'C' 'D' 'E' 'F' 'G' 'H','I'};
G = digraph(s,t,weights,names)
%G =
%  digraph with properties:
%     Edges: [12x2 table]
 %    Nodes: [8x1 table]

plot(G,'Layout','force','EdgeLabel',G.Edges.Weight)
```

You can use A = incidence(G) to generate the node-edge incidence matrix for any graph G. Then you can remove the rows corresponding to your chosen source and destination to get $\mathbf{L}$ in (1).

[1] For a directional graph with m nodes and n arcs/edges, the node-edge incidence matrix A has a row for each node and column for each edge, with $A = [a_{i,j}]_{i=1,2,\ldots,m, j=1,2,\ldots,n}$ defined by $a_{i,j} = \begin{cases} +1 \text{ if arc } j \text{ leaves node } i \\ -1 \text{ if arc } j \text{ enters node } i \\ 0 \text{ otherwise} \end{cases}$

- Create your own graph by picking any structure you like. Then, using CVX, formulate and solve the maxflow problem from a chosen source to a destination on your graph. If you like, you can use the graph used in the lecture as an example. Print your code and your results.
- Investigate how large can you make your graph and still solve this problem on your laptop in a reasonable time. You can write code to make a large graph by choosing the all-connected graph (an edge between every two node), with the capacity of each edge being a random integer (in MATLAB, **randi** creates random integers from a discrete uniform distribution).

Problem 2 - Spare Auto Parts

A small firm specializes in making five types of spare automobile parts. Each part is first cast from iron in the casting shop and then sent to the finishing shop where holes are drilled, surfaces are turned, and edges are ground. The required worker-hours (per 100 units) for each of the parts of the two shops are shown below:

Part	1	2	3	4	5
Casting	2	1	3	3	1
Finishing	3	2	2	1	1

The profits from the parts are \$30, \$20, \$40, \$25, and \$10 (per 100 units), respectively. The capacities of the casting and finishing shops over the next month are 700 and 1000 worker-hours, respectively. Formulate the problem of determining the quanitities of each spare part to be made during the month so as to maximize profit. To get full credit, you do *not* need to solve the optimization problem, although it could be educational to solve it with CVX if you like.

Problem 3 - Chemical manufacturer

There are three types of machines that can be used to make a chemical. The chemical can be manufactured at four different purity levels.

Machine Type	Capacity: Production per Day	Production Costs per Day If Used to Manufacture Purity			
		1	2	3	4
1	50 tons	\$900	1000	1250	1500
2	40 tons	600	750	1000	1050
3	25 tons	600	700	800	900
Monthly demand for chemical at purity level (tons)		150	300	90	175
Penalty per ton short		40	60	75	90

The chemical is used by the company internally; if the company does not manufacture enough of it, it can be bought at a price, which is called the penalty for shortage. Assume that there are 30 production days per month at the company. Formulate the problem of minimizing the overall cost as a linear program. To get full credit, you do *not* need to solve the optimization problem, although it could be educational to solve it with CVX if you like.

Problem 4 - Liquor company (not graded)

A liquor company produces and sells two kinds of liquor: blended whiskey and bourbon. The company purchases intermediate products in bulk, purifies them by repeated distillation, mixes them, and bottles the final product under their own brand names. In the past, the firm has always been able to sell all that it produced. The firm has been limited by its machine capacity and available cash. The bourbon requires 3 machine hours per bottle while, due to additional blending requirements, the blended whiskey requires 4 hours of machine time per bottle. There are 20,000 machine hours available in the current production period. The direct operating costs, which are principally for labor and materials, are \$3.00 per bottle of bourbon and \$2.00 per bottle of blended whiskey. The working capital available to finance labor and material is \$4000; however, 45% of the bourbon sales revenues and 30% of the blended-whiskey sales revenues from production in the current period will be collected during the current period and be available to finance operations. The selling price to the distributor is \$6 per bottle of bourbon and \$5.40 per bottle of blended whiskey.

1. Formulate a linear program that maximizes contribution subject to limitations on machine capacity and working capital.
2. What is the optimal production mix to schedule? (You can use CVX)
3. Suppose that the company could spend \$400 to repair some machinery and increase its available machine hours by 2000 hours. Should the investment be made?
4. What interest rate could the company afford to pay to borrow funds to finance its operations during the current period?