F3

Extremos

5

$f(x,y,z) = e^x \sin x + \cos(z-3y)$

$\frac{df}{dx} = \frac{d(e^x \sin x + \overbrace{\cos(z-3y)}^{C})}{dx} = (e^x \sin x)' = e^x \sin x + e^x \cos x$

$\frac{df}{dy} = \frac{d(\overbrace{e^x \sin x}^{C} + \cos(\overbrace{z-3y}^{C-3y}))}{dy} = -(z-3y)' \sin(z-3y) = \cancel{3y} \cancel{\cos}$
$= 3\sin(z-3y)$

$\frac{df}{dz} = \frac{d(\overbrace{e^x \sin x}^{C} + \cos(z-\overset{C}{3y}))}{dz} = -(z-3y)' \sin(z-3y) = -\sin(z-3y)$

6

a) $f(x,y) = \sqrt{xy}$

$\frac{df}{dx} = \frac{(xy)'}{2\sqrt{xy}} = \frac{y}{2} \times \frac{1}{\sqrt{xy}}$

$\frac{df}{dx}(2,2) = \frac{2}{2} \times \frac{1}{\sqrt{2\times 2}} = \frac{1}{2}$

$\frac{df}{dy} = \frac{(xy)'}{2\sqrt{xy}} = \frac{x}{2} \times \frac{1}{\sqrt{xy}}$

$\frac{df(2,2)}{dy} = \frac{2}{2} \times \frac{1}{\sqrt{2\times 2}} = 1 \times \frac{1}{\sqrt{4}} = \frac{1}{2}$

CA

$\sqrt{3x} = \frac{(3x)'}{2\sqrt{3x}} = \frac{3}{2\sqrt{3x}}$

b) $f(x,y) = \begin{cases} \frac{xy}{4-x^2-2y^2}, & \text{se } x^2+2y^2 \neq 4 \\ 0, & \text{se } x^2+2y^2 = 4 \end{cases}$

$P = (2,0)$, $2^2 + 0^2 = 4$, logo

usa-se

$\frac{df}{dx}(a,b) = \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h}$

$= \lim_{h\to 0} \frac{\frac{(2+h)\cdot 0}{4-(2+h)^2-0^2} - 0}{h} = \lim_{h\to 0} \frac{(2+h)\cdot 0}{h(4-4-4h+h^2)} = \lim_{h\to 0} \frac{0}{-4h^2+h^3} = \lim_{h\to 0} 0 = 0$

$\frac{df}{dy}(2,0) = \lim_{h\to 0} \frac{\frac{2\cdot(0+h)}{(4-2^2-2(0+h)^2)} - 0}{h} = \lim_{h\to 0} \frac{2h}{h(4-2^2-2h^2)} = \lim_{h\to 0} \frac{2}{-2h^2}$

$= \lim_{h\to 0} \frac{\cancel{2}}{-\cancel{2}h^2} = -\frac{1}{h^2} = -\infty$

não tem derivada

CA

$\frac{3x}{1-x^2} = \frac{(3x)'(1-x^2) - (3x)(1-x^2)'}{(1-x^2)^2}$

$= \frac{3(1-x^2) + 2x(3x)}{(1-x^2)^2}$

c) $f(0,0) = 0$

$$\frac{\partial f}{\partial x}(0,0) = \lim_{h \to 0} \frac{(0+h)\cdot \sin\left(\frac{1}{(0+h)^2+0^2}\right) - 0}{h}$$

$$= \lim_{h\to 0} \sin\left(\frac{1}{h^2}\right) = \sin(+\infty) \text{ não definido}$$

$$\frac{\partial f}{\partial y}(0,0) = \lim_{h\to 0} \frac{0 \cdot \sin\left(\frac{1}{0^2+(0+h)^2}\right) - 0}{h} = \lim_{h\to 0} \frac{0}{h} = 0$$

7

$$\frac{\partial f}{\partial x} = \frac{1}{x+y} - \frac{1}{x-y}$$

C.A.

$\ln(x+3) - \ln(x-3)$

$= \frac{1}{x+3} \times (x+3)' - \frac{(x-3)'}{x-3} = \frac{1}{x+3} - \frac{1}{x-3}$

$$\frac{\partial f}{\partial y} = \frac{1}{x+y} - \frac{-1}{x-y} = \frac{1}{x+y} + \frac{1}{x-y}$$

$$\frac{\partial f}{\partial x^2} = -\frac{1}{(x+y)^2} + \frac{1}{(x-y)^2}$$

$\left(\frac{1}{x+3} - \frac{1}{x-3}\right)' = \frac{-1}{(x+3)^2} + \frac{1}{(x-3)^2}$

$$\frac{\partial f}{\partial x \partial y} = \frac{\partial f}{\partial y \partial x} = -\frac{1}{(x+y)^2} - \frac{1}{(x-y)^2}$$

$\left(\frac{1}{3+y} - \frac{1}{3-y}\right)' = \frac{-1}{(3+y)^2} - \frac{1}{(3-y)^2}$

$$\frac{\partial f}{\partial y^2} = -\frac{1}{(x+y)^2} + \frac{1}{(x-y)^2}$$

Simplificar:

$$\frac{\partial f}{\partial x} = \frac{1}{x+y} - \frac{1}{x-y} = \frac{(x-y)-(x+y)}{(x+y)(x-y)} = \frac{x-x-y-y}{x^2-y^2} = -\frac{2y}{x^2-y^2}$$

$$\frac{\partial f}{\partial y} = \frac{1}{x+y} + \frac{1}{x-y} = \frac{(x-y)+(x+y)}{(x+y)(x-y)} = \frac{2x}{x^2-y^2}$$

$$\frac{\partial f}{\partial x^2} = \frac{-(x-y)^2+(x+y)^2}{(x+y)^2(x-y)^2} = \frac{-(x^2-2xy+y^2)+(x^2+2xy+y^2)}{((x+y)(x-y))^2}$$

$$= \frac{4xy}{(x^2-y^2)^2}$$

$$\frac{\partial f}{\partial x \partial y} = -\left(\frac{1}{(x+y)^2} + \frac{1}{(x-y)^2}\right) = -\left(\frac{x^2-2xy+y^2+x^2+2xy+y^2}{((x+y)(x-y))^2}\right)$$

$$= -\left(\frac{2x^2+2y^2}{(x^2-y^2)^2}\right) = -\frac{2x^2+2y^2}{(x^2-y^2)^2}$$

$$\frac{\partial f}{\partial y^2} = -\frac{\partial f}{\partial x^2} = -\frac{4xy}{(x^2-y^2)^2}$$

8

$C(x,y)$

$$\frac{dC}{dx}(x,y) = \frac{210y}{2\sqrt{xy}} + 125$$

$$\frac{dC}{d(x)}(80,20) = \frac{210 \times 10}{2\sqrt{80 \times 20}} + 125 = \frac{2100}{40} + 125 = 52{,}5 + 125$$

$$= 177.5$$

$$\frac{dC}{dy}(x,y) = \frac{210x}{2\sqrt{xy}} + 130y$$

$$= \frac{210 \times 80}{2 \times 40} + 130 = 340$$

9)

$$\frac{\partial z}{\partial x} = \cancel{x^2}\frac{2x+y}{x^2+xy+y^2}$$

$$\frac{\partial z}{\partial y} = \frac{2y+x}{x^2+xy+y^2}$$

$$\ln(x^2+2x+2^2)$$

$$= \frac{(x^2+2x+2^2)'}{x^2+2x+2^2} = \frac{2x+2}{x^2+2x+2^2}$$

$$x \times \frac{\partial z}{\partial x} + y \times \frac{\partial z}{\partial y} =$$

$$= x \times \frac{2x+y}{(x^2+xy+y^2)} + y \times \frac{2y+x}{x^2+xy+y^2}$$

$$= \frac{2x^2+yx+2y^2+yx}{x^2+xy+y^2} = \frac{2x^2+2xy+2y^2}{x^2+xy+y^2}$$

$$= \frac{2\cancel{(x^2+xy+y^2)}}{\cancel{x^2+xy+y^2}} = 2$$

10

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial^2 y} = 0$$

$$f(x,y) = \operatorname{arctg}(y/x)$$

$$\frac{\partial f}{\partial x} = -\frac{y}{x^2+y^2}$$

$$\operatorname{arctg}\left(\frac{1}{x}\right) = \frac{\left(\frac{2}{x}\right)'}{1+\left(\frac{2}{x}\right)^2} = \frac{-\frac{2}{x^2}}{1+\left(\frac{1}{x}\right)^2}$$

$$= -\frac{2}{x^2+2^2}$$

$$\frac{\partial^2 f}{\partial^2 x^2} = \frac{2xy}{(x^2+y^2)^2}$$

$$\left(-\frac{2}{x^2+2^2}\right)' = \frac{+2(x^2+2^2)'}{(x^2+2^2)^2} = \frac{2 \cdot 2x}{(x^2+2^2)}$$

$$\frac{\partial f}{\partial y} = \frac{\frac{1}{x}}{1+\frac{y^2}{x^2}} = \frac{1}{x+\frac{y^2}{x}}$$

$$\operatorname{arctg}\left(\frac{y}{2}\right) = \frac{\frac{1}{2}}{1+\left(\frac{y}{2}\right)^2}$$

$$= \frac{1}{\frac{x^2+y^2}{x}} = \frac{x}{x^2+y^2}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{-2yx}{(x^2+y^2)^2}$$ $$\left(\frac{2}{2^2+y^2}\right)' = \frac{-2(2^2+y^2)'}{(2^2+y^2)^2} = \frac{-2 \cdot 2y}{(2^2+y^2)^2}$$

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \frac{2xy}{(x^2+y^2)^2} - \frac{2xy}{(x^2+y^2)^2} = 0$$

11

a) $D_f = \{(x,y) \in \mathbb{R}^2 : x > 0\}$

b)

Eq Vetorial do Plano:

$$(x,y,z) = (a,b,f(a,b)) + \lambda\left(1,0,\frac{\partial f}{\partial x}(a,b)\right) + \mu\left(0,1,\frac{\partial f}{\partial y}(a,b)\right)$$

Eq Cartesiana

$$z - f(a,b) = \frac{\partial f}{\partial x}(a,b)\,(x-a) + \frac{\partial f}{\partial y}(a,b)\,(y-b)$$

$P = (1,2,4)$ $$z - 4 = \frac{\partial f}{\partial x}(1,2)\,(x-1) + \frac{\partial f}{\partial y}(1,2)\,(y-2) = (*)$$

$f(a,b) = f(1,2) = 4$

$a = 1$

$b = 2$

$$\frac{\partial f}{\partial x} = \frac{1}{x} + y^2 \qquad (\ln x + 4x)' = \frac{1}{x} + 4$$

$$\frac{\partial f}{\partial y} = 2xy \qquad (\ln 2 + 2y^2)' = 2 \cdot 2y$$

$$(*) \Leftrightarrow z - 4 = \left(\frac{1}{1} + 2^2\right)(x-1) + (2 \cdot 1 \cdot 2)(y-2)$$

$$\Leftrightarrow z - 4 = 5(x-1) + 4(y-2)$$

$$\Leftrightarrow z - 4 = 5x - 5 + 4y - 8$$

$$\Leftrightarrow 5x + 4y - z = 9$$

$(5,4,-1) \to$ vetor perpendicular ao plano

Reta: $(x,y,z) = (1,2,4) + k(5,4,-1),\ k \in \mathbb{R}$

12

a) $\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$

$$\frac{\partial f}{\partial x} = \sin(yz) \qquad \frac{\partial f}{\partial y} = xz\cos(yz) \qquad \frac{\partial f}{\partial z} = xy\cos(yz)$$

$$\nabla f = (\sin(yz),\ xz\cos(yz),\ xy\cos(yz))$$

b)

$D_u(1,3,0)$

$\nabla f(1,3,0) = (\sin(3\times0),\ 1\times0\times\cos(3\times0),\ 1\times3\times\cos(3\times0))$

$= (0, 0, 3)$

$\vec{u} = \frac{\vec{v}}{\|\vec{v}\|} = \left(\frac{\sqrt6}{6}, \frac{\sqrt6}{3}, -\frac{\sqrt6}{6}\right)$

$\|\vec{v}\| = \sqrt{1+4+1} = \sqrt6$

$D_u f(1,3,0) = (0,0,3)\cdot\left(\frac{\sqrt6}{6}, \frac{\sqrt6}{3}, -\frac{\sqrt6}{6}\right) = -\frac{\sqrt6}{2}$

13

a) $D_f = \{(x,y)\in\mathbb{R}^2 : x^2+y^2 > 0\} = \mathbb{R}\setminus\{(0,0)\}$

b) ~~f(x,y)~~

Considerando um $K\in\mathbb{R}$,

$C_k = \{(x,y)\in\mathbb{R}^2 : \ln(x^2+y^2) = k\}$

$= \{(x,y)\in\mathbb{R}^2 : x^2+y^2 = e^k\}$

Circunferencias de raio $\sqrt{e^k}$

c)

$D_u(1,0)$

$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$ $\qquad \frac{\partial f}{\partial x} = \frac{2x}{x^2+y^2}$

$= \left(\frac{2x}{x^2+y^2}, \frac{2y}{x^2+y^2}\right)$ $\qquad \frac{\partial f}{\partial y} = \frac{2y}{x^2+y^2}$

Considerando o ~~vetor~~ vetor unitário $\vec{u} = (a,b)$

$D_u(1,0) = \left(\frac{2}{1+0}, \frac{0}{1+0}\right)\cdot(a,b) = 2a + 0b = 2a$

14

$S = \{(x,y,z)\in\mathbb{R}^3 : 3-z = \sqrt{x^2+y^2}\}$

$= \{(x,y,z)\in\mathbb{R}^3 : 3-\sqrt{x^2+y^2} = z\}$

Considerando $f(x,y) = 3-\sqrt{x^2+y^2}$

$f(3,4) = 3-\sqrt{3^2+4^2} = 3-5 = -2$

$z - f(a,b) = \frac{\partial f}{\partial x}(a,b)(x-a) + \frac{\partial f}{\partial y}(a,b)(y-b)$

$\frac{\partial f}{\partial x} = -\frac{x}{\sqrt{x^2+y^2}}$ $\qquad \frac{\partial f}{\partial y} = -\frac{y}{\sqrt{x^2+y^2}}$ $\qquad (3-\sqrt{x^2+y})' = -\frac{2x}{2\sqrt{x^2+y}} = -\frac{x}{\sqrt{x^2+y}}$

$z-(-2) = -\frac{3}{\sqrt{3^2+4^2}}(x-3) - \frac{4}{\sqrt{3^2+4^2}}(y-4)$

$\Leftrightarrow z+2 = -\frac{3}{5}(x-3) - \frac{4}{5}(y-4)$

$\Leftrightarrow 5z+10 = -3x+9-4y+16$

$\Leftrightarrow 3x+4y+5z = 15 \quad \rightarrow$ Plano

$(x,y,z) = (3,4,-2) + k(3,4,5), \ k \in \mathbb{R}$

15

a) $D_f = (3y, 3x, 2z)$

b)

$S_4 = \{(x,y,z) \in \mathbb{R}^3 : 3xy + z^2 = 4\}$

$= \{(x,y,z) \in \mathbb{R}^3 : z^2 = 4-3xy\}$

$= \{(x,y,z) \in \mathbb{R}^3 : z = \sqrt{4-3xy}\}$

$f(x,y) = \sqrt{4-3xy}$

$f(1,1) = \sqrt{4-3\times1\times1} = 1$

$z-1 = \frac{\partial f}{\partial x}(1,1)(x-1) + \frac{\partial f}{\partial y}(1,1)(y-1)$

$\frac{\partial f}{\partial x} = \frac{-3y}{2\sqrt{4-3xy}}$ $\qquad (\sqrt{4-3\cdot6\cdot x})' = \frac{-3\cdot6}{2\sqrt{4-3\cdot6\,x}}$

$\frac{\partial f}{\partial y} = \frac{-3x}{2\sqrt{4-3xy}}$

$z-1 = \frac{-3\cdot1}{2\sqrt{4-3}}(x-1) + \frac{-3\cdot1}{2\sqrt{4-3}}(y-1)$

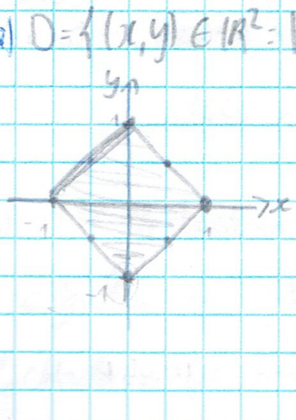
$2z-2 = -3(x-1) - 3(y-1)$

$2z-2 = -3x+3-3y+3$

$3x+3y+2z = +8$

16

a) $D = \{(x,y) \in \mathbb{R}^2 : |x|+|y| \leq 1\}$



b) D é um subconjunto fechado e limitado em $\mathbb{R}^n$, logo f tem um mínimo e máximo absoluto em D

Mínimo: $(0,0) \Rightarrow 0^2+0^2 = 0$

Máximo: $(1,0) \Rightarrow 1^2+0^2 = 1$

$(-1,0) \Rightarrow$

$(0,1) \Rightarrow$

$(0,-1) \Rightarrow$

$\cos x\, e \quad \Big|\; c_2(x) = \frac{1}{2} e \;(\cos x + \sin x)$

Folha 3

P2

17 O teorema de Weierstrass só garante extremos absolutos em intervalos fechados e limitados. Como S não é fechado, isto não se apl

18

$\forall x \in \mathbb{R}$, $\quad x^2 \geq 0 \Leftrightarrow -x^2 \leq 0$

Logo ~~f(0,y)~~ 0 é um máximo de f

Logo (0,y) é uma maximizante

Como $y \in \mathbb{R}$, f tem uma infinidade de soluções

19

a) Não é aplicável porque o domínio de f não é limitado nem fechado

b) Como $x, y, z \in \mathbb{R}$, $x^2 \geq 0 \wedge y^2 \geq 0 \wedge z^2 \geq 0$

$$\Rightarrow x^2+y^2+z^2 \geq 0$$

$$x^2+y^2+z^2=0$$

quando $x=0, y=0, z=0$

Logo $f(0,0,0)=0$ e 0 é minimizante

20

a) $\frac{\partial f}{\partial x} = -\frac{x}{\sqrt{x^2+y^2}}$

$$\left(-\sqrt{x^2+4}\right)' = -\frac{(x^2+4)'}{2\sqrt{x^2+4}} = -\frac{2x}{2\sqrt{x^2+4}} = -\frac{x}{\sqrt{x^2+4}}$$

$\frac{\partial f}{\partial x}$ Não é definido em $(0,0)$

$\frac{\partial f}{\partial y} = -\frac{y}{\sqrt{x^2+y^2}}$

$\frac{\partial f}{\partial y}$ não é definido em $(0,0)$

Logo $f(x,y)$ não é diferenciável em $(0,0)$

b) $x, y \in \mathbb{R}$

$x^2 \geq 0 \wedge y^2 \geq 0$

$\Rightarrow x^2+y^2 \geq 0$

$\sqrt{x^2+y^2} \geq 0$

$-\sqrt{x^2+y^2} \leq 0$

$f(x,y) \leq 0$

0 é máximo de f

$-\sqrt{x^2+y^2}=0 \Leftrightarrow x=0 \wedge y=0$

$(0,0)$ é maximizante de f

21

a) B é fechado e limitado logo aplica-se o teorema de Weierstrass ~~e g tem~~ Pelo teorema de Weierstrass, tendo um intervalo $B \subset \mathbb{R}^2$ fechado e limitado, g vai ter extremos globais em B

b)

$g(x,y)=y$

Logo o mínimo do g em B vai ser o mínimo de y ~~no~~ em B ; ~~ou seja~~ Logo, em $x^2+y^2\leq 1$, $y=-1$ é o mínimo de y, e, por consequência, mínimo de g, e $y=1$ é máximo de ambos

c) O teorema de Weierstrass não é aplicável em A logo g não possui extremos globais em A

22

$h(0,0)=\frac{1}{2}-\sin(0^2+0^2)=\frac{1}{2}-\sin(0)=\frac{1}{2}-0=\frac{1}{2}$

Considerando que sin é uma função limitada tal que

$-1\leq\sin(x)\leq 1$

Então considerando um $z^2=x^2+y^2$, obtém-se que

$-1\leq\sin(z^2)\leq 1$

$-1\leq-\sin(z^2)\leq 1$

$-\frac{1}{2}\leq\frac{1}{2}-\sin(z^2)\leq\frac{3}{2}$

O máximo global de $h(x,y)$ é $\frac{3}{2}$

23

a)

$f(x,y)=3xy^2+x^3-3x$

$\frac{\partial f}{\partial x}=3y^2+3x^2-3$

$\frac{\partial f}{\partial y}=6xy$

$\nabla f=(3y^2+3x^2-3,\ 6xy)$

C.A

$(3\cdot 4^2\cdot x+x^3-3x)'$
$=3\cdot 16+3x^2-3$

$(3\cdot 4\cdot y^2+4^3-3\cdot 4)'$
$=2\cdot 3\cdot 4\cdot y+0-0$

Ponto Crítico: $\nabla f=(0,0)$

$(3y^2+3x^2-3,\ 6xy)=(0,0)$

$\begin{cases}3y^2+3x^2-3=0\\6xy=0\end{cases}\Leftrightarrow\begin{cases}y^2+x^2-1=0\\xy=0\end{cases}\Leftrightarrow\begin{cases}x^2+y^2=1\\x=0\vee y=0\end{cases}$

$\Leftrightarrow\begin{cases}\overset{x=0}{0^2+y^2=1}\ \vee\ \overset{y=0}{x^2+0^2=1}\\ —\end{cases}\Leftrightarrow\begin{cases}y=\pm 1\ \vee\ x=\pm 1\\ —\end{cases}$

Pontos Críticos: $(1,0),(-1,0),(0,1),(0,-1)$

b)

$$f(x,y) = x^2y^3(6-x-y)$$
$$= 6x^2y^3 - 6x^3y^3 - 6x^2y^4$$

$$\frac{\partial f}{\partial x} = 12y^3x - 18y^3x^2 - 12y^4x$$

$$\frac{\partial f}{\partial y} = 18x^2y^2 - 18x^3y^2 - 24x^2y^3$$

CA

$$(6x^2\cdot 1^3 - 6x^3\cdot 1^3 - 6x^2\cdot 1^2)' = 2\cdot 6\cdot 1^3\cdot x - 3\cdot 6\cdot x^2 - 2\cdot 6\cdot x$$

$$(6\cdot 2^2\cdot y^3 - 6\cdot 2^3\cdot y^3 - 6\cdot 2^2 y^4)' = 3\cdot 6\cdot 4y^2 - 3\cdot 6\cdot 8\cdot y - 4\cdot 6\cdot 4\cdot y^3$$

$$\nabla f = (12y^3x - 18y^3x^2 - 12y^4x,\ 18x^2y^2 - 18x^3y^2 - 24x^2y^3)$$

$$\nabla f = (0,0)$$

$$\Leftrightarrow \begin{cases} 12y^3x - 18y^3x^2 - 12y^4x = 0 \\ 18x^2y^2 - 18x^3y^2 - 24x^2y^3 = 0 \end{cases} \Leftrightarrow \begin{cases} 2y^3x - 3y^3x^2 - 2y^4x = 0 \end{cases}$$

b)

$$f(x,y) = x^2y^3(6-x-y)$$
$$= 6x^2y^3 - x^3y^3 - x^2y^4$$

$$\frac{\partial f}{\partial x} = 12y^3x - 3x^2y^3 - 2xy^4$$

$$\frac{\partial f}{\partial y} = 18x^2y^2 - 3y^2x^3 - 4x^2y^3$$

$$\nabla f = (12y^3x - 3x^2y^3 - 2xy^4,\ 18x^2y^2 - 3y^2x^3 - 4x^2y^3) = (0,0)$$

$$\Leftrightarrow \begin{cases} 12y^3x - 3x^2y^3 - 2xy^4 = 0 \\ 18x^2y^2 - 3y^2x^3 - 4x^2y^3 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} y^3x(12 - 3x - 2y) = 0 \\ x^2y^2(18 - 3x - 4y) = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} y^3x = 0 \vee 12 - 3x - 2y = 0 \\ xy = 0 \vee 18 - 3x - 4y = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x=0 \vee y=0 \vee 2y = 12-3x \\ x=0 \vee y=0 \vee 4y = 18-3x \end{cases} \Leftrightarrow \begin{cases} x=0 \vee y=0 \vee \text{—} \\ x=0 \vee y=0 \vee 0 = -6+3x \end{cases}$$

$$\Leftrightarrow \begin{cases} x=0 \vee y=0 \vee y=3 \\ x=0 \vee y=0 \vee x=2 \end{cases}$$

Pontos Críticos: eixos ordenados e o ponto (2,3)

38

c)

$f(x,y,z) = x^4 + y^4 + z^4 - 4xyz$

$\frac{\partial f}{\partial x} = 4x^3 - 4yz = 4(x^3 - yz)$

$\frac{\partial f}{\partial y} = 4y^3 - 4xz = 4(y^3 - xz)$

$\frac{\partial f}{\partial z} = 4z^3 - 4xy = 4(z^3 - xy)$

$\nabla f = (4(x^3-yz), 4(y^3-xz), 4(z^3-xy)) = \cancel{(0,0)}\ (0,0,0)$

$\Leftrightarrow \begin{cases} 4(x^3-yz)=0 \\ 4(y^3-xz)=0 \\ 4(z^3-xy)=0 \end{cases} \Leftrightarrow \begin{cases} x^3-yz=0 \\ y^3-xz=0 \\ z^3-xy=0 \end{cases}$

Excluindo o $y=0$ e $x=0$

$\Leftrightarrow \begin{cases} z = \frac{x^3}{y} \\ z = \frac{y^3}{x} \\ - \end{cases} \Leftrightarrow \begin{cases} \frac{y^3}{x} = \frac{x^3}{y} \\ - \end{cases} \Leftrightarrow \begin{cases} y^4 = x^4 \\ - \\ - \end{cases}$

$\Leftrightarrow \begin{cases} y^4 - x^4 = 0 \\ - \\ - \end{cases} \Leftrightarrow \begin{cases} (y^2-x^2)\underbrace{(y^2+x^2)}_{\text{nunca igual a } 0} = 0 \\ - \\ - \end{cases}$

$\Leftrightarrow \begin{cases} y^2-x^2=0 \\ - \\ - \end{cases} \Leftrightarrow \begin{cases} (y-x)(y+x)=0 \\ - \\ - \end{cases} \Leftrightarrow \begin{cases} y=x \vee y=-x \\ - \\ - \end{cases}$

Se $y = x$

$\begin{cases} z = \frac{x^3}{x} \\ z^3 - x \cdot x = 0 \end{cases} \Leftrightarrow \begin{cases} z = x^2 \\ x^6 - x^2 = 0 \end{cases} \Leftrightarrow \begin{cases} - \\ x^2(x^4-1)=0 \end{cases}$

$\hookrightarrow x \neq 0$

$\Leftrightarrow \begin{cases} - \\ x^4 - 1 = 0 \end{cases} \Leftrightarrow \begin{cases} - \\ x = \pm 1 \end{cases}$

Se $x = -1$, temos o ponto crítico $(-1, -1, 1)$

Se $x = 1$, temos o ponto crítico $(1, 1, 1)$

Se $y = -x$

$\begin{cases} z = -\frac{x^3}{x} \\ z^3 - x \cdot (-x) = 0 \end{cases} \Leftrightarrow \begin{cases} z = -x^2 \\ -x^6 + x^2 = 0 \end{cases} \Leftrightarrow \begin{cases} - \\ x^2(1-x^4) = 0 \end{cases}$

$\Leftrightarrow \begin{cases} - \\ x = \pm 1 \end{cases}$

Se $x = 1$, temos o ponto crítico $(1, -1, -1)$

Se $x = -1$, " " " $(-1, 1, -1)$

As soluções atrás foram calculadas ao excluir $y=0$ e $x=0$

Se $x=0$

$$\begin{cases} 0^3-yz=0 \\ y^3-0=0 \\ z^3-0=0 \end{cases} \Leftrightarrow \begin{cases} y=0 \\ y=0 \\ z=0 \end{cases}$$

Obtem-se a solução $(0,0,0)$

24

$$\frac{\partial f}{\partial x}=2x-2$$

$$\left((x-1)^2+(y-2)^2-1\right)'$$
$$=\left((x-1)^2\right)'+0-0$$
$$=2(x-1)(x-1)'$$
$$=2x-2$$

$$\frac{\partial f}{\partial y}=2(y-2)=2y-4$$

$$\nabla f=(2x-2,2y-4)=(0,0)$$

$$\Leftrightarrow \begin{cases} 2x-2=0 \\ 2y-4=0 \end{cases} \Leftrightarrow \begin{cases} 2x=2 \\ 2y=4 \end{cases} \Leftrightarrow \begin{cases} x=1 \\ y=2 \end{cases}$$

f só tem um extremo local em $(1,2)$ $f(1,2)=\cancel{0}-1$

25

a) $f(x,y)=x^2+2xy-4(x-2y)$

$$=x^2+2xy-4x+8y$$

$$\frac{\partial f}{\partial x}=2x+2y-4$$

$$\frac{\partial f}{\partial y}=2x+8$$

$$\nabla f=(2x+2y-4,2x+8)=(0,0)$$

$$\Leftrightarrow \begin{cases} 2x+2y-4=0 \\ 2x+8=0 \end{cases} \Leftrightarrow \begin{cases} y=+6 \\ x=-4 \end{cases}$$

f não tem pontos críticos em D

b) Pelo teorema de Weierstrass,

Como D é fechado e limitado, $f(x,y)$ tem extremos absolutos em D;

Como não há pontos críticos em D, os extremos são vistos na fronteira:

$f(0,0)=0+0-0+0=0$

$f(1,0)=1+0-4+0=-3$ Minimizante: $(+1,0)$

$f(0,2)=0+0-0+16=16$

$f(1,2)=1+\cancel{2}4-4+16=17$ Maximizante: $(1,2)$

26

a) $f(x,y) = xye^{-x-y}$

$$\frac{\partial f}{\partial x} = ye^{-x-y} + yx(-1)e^{-x-y}$$
$$= ye^{-x-y} - yxe^{-x-y}$$

$$(2xe^{-x+2})' =$$
$$= 2e^{-x+2} + 2x(-x+2)'e^{-x+2}$$

$$\frac{\partial f}{\partial y} = xe^{-x-y} - xye^{-x-y}$$

$$\nabla f = (ye^{-x-y} - yxe^{-x-y}, xe^{-x-y} - xye^{-x-y}) = (0,0)$$

$$\Leftrightarrow \begin{cases} ye^{-x-y}(1-x) = 0 \\ xe^{-x-y}(1-y) = 0 \end{cases} \Leftrightarrow \begin{cases} y=0 \vee x=1 \\ x=0 \vee y=1 \end{cases}$$

Dois pontos críticos: $(0,0)$, $(1,1)$

$$\frac{\partial^2 f}{\partial^2 x} = -ye^{-x-y} - ye^{-x-y} + yxe^{-x-y}$$

$$\frac{\partial^2 f}{\partial x \partial y} = e^{-x-y} - xe^{-x-y} - ye^{-x-y} + yxe^{-x-y}$$

$$\frac{\partial^2 f}{\partial^2 y} = -xe^{-x-y} - xe^{-x-y} + xye^{-x-y}$$

$$H(0,0) = \begin{bmatrix} \cancel{0} & 1 \\ 1 & \end{bmatrix}$$

$$H(0,0) = \begin{bmatrix} \frac{\partial^2 f}{\partial^2 x}(0,0) & \frac{\partial^2 f}{\partial x \partial y}(0,0) \\ \frac{\partial^2 f}{\partial x \partial y}(0,0) & \frac{\partial^2 f}{\partial^2 y}(0,0) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\Delta_f(0,0) = -1$$

$$\Delta_1(0,0) = 0$$

Ponto de sela

$$\Delta_2(0,0) = \Delta_f(0,0) = -1$$

$$H(1,1) = \begin{bmatrix} -\frac{1}{e^2} & 0 \\ 0 & -\frac{1}{e^2} \end{bmatrix}$$

$$\Delta f = \frac{1}{e^4}$$

$$\Delta_1 = -\frac{1}{e^2}$$

$$\frac{\partial^2 f}{\partial^2 x}(1,1) = -1e^{-1-1} - 1e^{-1-1} + 1 \cdot 1e^{-1-1}$$
$$= -e^{-2} = -\frac{1}{e^2}$$

Maximizante local

$$\frac{\partial^2 f}{\partial x \partial y}(1,1) = e^{-2} - e^{-2} - e^{-2} + e^{-2} = 0$$

$$\frac{\partial^2 f}{\partial^2 y}(1,1) = -\frac{1}{e^2}$$

b)

$$\frac{\partial g}{\partial x} = 3x^2 - 4xy - 2x$$

$$\frac{\partial g}{\partial y} = -2x^2 + 8y$$

$$\nabla g = (3x^2 - 4xy - 2x, -2x^2 + 8y) = (0,0)$$

$$\Leftrightarrow \begin{cases} 3x^2 - 4xy - 2x = 0 \\ -2x^2 + 8y = 0 \end{cases} \Leftrightarrow \begin{cases} x(3x - 4y - 2) = 0 \\ x^2 = 4y \end{cases}$$

$$\Leftrightarrow \begin{cases} x = 0 \vee 4y = 3x - 2 \\ x^2 = 4y \end{cases} \Leftrightarrow \begin{cases} 4y = 3x - 2 \\ x^2 - 3x + 2 = 0 \end{cases} \vee \begin{cases} x = 0 \\ y = 0 \end{cases}$$

$PC_1 = (0,0)$

$$\Leftrightarrow \begin{cases} — \\ x = 2 \vee x = 1 \end{cases} \Leftrightarrow \begin{cases} 4y = 6 - 2 \\ x = 2 \end{cases} \vee \begin{cases} 4y = 3 - 2 \\ x = 1 \end{cases} \Leftrightarrow \begin{cases} y = 1 \\ x = 2 \end{cases} \vee \begin{cases} y = \frac{1}{4} \\ x = 1 \end{cases}$$

$$x^2 - 3x + 2 = 0 \Leftrightarrow x = \frac{3 \pm \sqrt{9-8}}{2} \Leftrightarrow x = \frac{3 \pm 1}{2} \Leftrightarrow x = 2 \vee x = 1$$

Pontos Críticos: $(0,0)$
~~$(2, \frac{1}{2})$~~ $(2,1)$
$(1, \frac{1}{4})$

$$\frac{\partial^2 g}{\partial^2 x} = 6x - 4y - 2$$

$$\frac{\partial^2 g}{\partial^2 y} = 8$$

$$\frac{\partial^2 g}{\partial x \partial y} = -4x$$

$$H(0,0) = \begin{bmatrix} -2 & 0 \\ 0 & 8 \end{bmatrix}$$

$\Delta_f(0,0) = -16$

$\Delta_1(0,0) = -2$

$\Delta_2(0,0) = \Delta_f(0,0) = -16$

Ponto de Sela

$$H(2,1) = \begin{bmatrix} 6 & -8 \\ -8 & 8 \end{bmatrix}$$

$\Delta_f = 48 + 64 = 112$

$\Delta_1 = 6$

Ponto de sela

$$H(1, \tfrac{1}{4}) = \begin{bmatrix} 3 & -4 \\ -4 & 8 \end{bmatrix}$$

$\Delta_f(1, \frac{1}{4}) = 8$

Mínimo

$\Delta_1(1, \frac{1}{4}) = 4$

c)

$h(x,y) = x^3y + 12x^2 - 8y$

$\frac{\partial h}{\partial x} = 3x^2y + 24x$

$\frac{\partial h}{\partial y} = x^3 - 8$

$\nabla h = (3x^2y + 24x, x^3 - 8) = (0,0)$

$\Rightarrow \begin{cases} 3x^2y + 24x = 0 \\ x^3 - 8 = 0 \end{cases} \Leftrightarrow \begin{cases} 3x^2y = -24x \\ x^3 = 8 \end{cases}$

$\Leftrightarrow \begin{cases} 3 \cdot 4 \cdot y = -24 \cdot 2 \\ x = 2 \end{cases} \Leftrightarrow \begin{cases} y = -\frac{48}{12} \\ x = 2 \end{cases} \Leftrightarrow \begin{cases} y = -4 \\ x = 2 \end{cases}$

$\frac{\partial h}{\partial^2 y} = 0$

$\frac{\partial^2 h}{\partial^2 x} = 6xy + 24$

$\frac{\partial h}{\partial x \partial y} = 3x^2$

$H(2,-4) = \begin{bmatrix} -24 & 12 \\ 12 & 0 \end{bmatrix}$

$\nabla H_f(2,-4) = -24$ Ponto de Sela

$\nabla H_f(2,-4) = -24$

d)

$i(x,y) = xy + \frac{1}{x} + \frac{1}{y}$ Domínio: $\{(x,y) \in \mathbb{R}^2 : x \neq 0 \vee y \neq 0\}$

$\frac{\partial i}{\partial x} = y - \frac{1}{x^2}$

$\frac{\partial i}{\partial y} = x - \frac{1}{y^2}$

$\nabla f = (y - \frac{1}{x^2}, x - \frac{1}{y^2}) = (0,0)$

$\begin{cases} y - \frac{1}{x^2} = 0 \\ x - \frac{1}{y^2} = 0 \end{cases} \Leftrightarrow \begin{cases} y = \frac{1}{x^2} \\ x - x^2 = 0 \end{cases} \Leftrightarrow \begin{cases} - \\ x(1-x) = 0 \end{cases} \Leftrightarrow \begin{cases} - \\ x = 0 \vee x = 1, \wedge x \neq 0 \end{cases}$

$\Leftrightarrow \begin{cases} y = 1 \\ x = 1 \end{cases}$

$$\frac{\partial^2 f}{\partial^2 x} = +\frac{2}{x^3}$$

$$\frac{\partial^2 f}{\partial^2 y} = \frac{2}{y^3}$$

$$\frac{\partial^2 f}{\partial x \partial y} = 1$$

$$H(1,1) = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$H_2(1,1) = 3$

Mínimo

$H_1(1,1) = 2$

27

$f(x,y) = 3x^2 - y^2 + x^3$

$$\frac{\partial f}{\partial x} = 6x + 3x^2$$

$$\frac{\partial f}{\partial y} = -2y$$

$\nabla f = (6x + 3x^2, -2y) = (0,0)$

$$\Leftrightarrow \begin{cases} 6x + 3x^2 = 0 \\ -2y = 0 \end{cases} \Leftrightarrow \begin{cases} 3x(x+2) = 0 \\ y = 0 \end{cases} \Leftrightarrow \begin{cases} x = 0 \vee x = -2 \\ y = 0 \end{cases}$$

PC $= (0,0)\ (-2,0)$ c.q.m.

$$\frac{\partial^2 f}{\partial^2 x} = 6 + 6x$$

$$\frac{\partial^2 f}{\partial x \partial y} = 0$$

$$\frac{\partial^2 f}{\partial^2 y} = -2$$

$H(0,0) = \begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix}$ $H_2 = -2$ $H_1 = 0$ Ponto cela

$H(-2,0) = \begin{bmatrix} -6 & 0 \\ 0 & -2 \end{bmatrix}$ $\Delta_2 = \cancel{24}\ 12$ $\Delta_1 = -6$ Maximizante

28

a) $f(x,y) = (x - y^3)^2 - x^3$

$$\frac{\partial f}{\partial x} = 2(x - y^3) - 3x^2$$

$$\frac{\partial f}{\partial y} = 2 \cdot (-3y^2)(x - y^3) = -6y^2(x - y^3)$$

$\nabla f = (2(x-y^3) - 3x^2, -6y^2(x - y^3)) = (0,0)$

$$\begin{cases} 2(x-y^3)-3x^2=0 \\ -6y^2(x-y^3)=0 \end{cases}$$

Considerando o ponto $(0,0)$

$$\begin{cases} 2(0-0)-3\cdot 0=0 \\ -6\cdot 0\,(0-0)=0 \end{cases} \Leftrightarrow \begin{cases} 0=0 \\ 0=0 \end{cases}$$

Confirma-se que $(0,0)$ é PC

b)

$$\frac{\partial^2 f}{\partial^2 x} = 2-6x$$

$$\frac{\partial^2 f}{\partial^2 y} = -12yx + 30y^4 \qquad -6y^2(x-y^3) = -6y^2x + 6y^5$$

$$\frac{\partial^2 f}{\partial x \partial y} = -6y^2$$

$$H(0,0) = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \qquad ??$$

29

$f(x,y) = 2x^2 - 2y^2$

$D = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \leq 1\}$

fronteira: $x^2+y^2=1 \Leftrightarrow \underbrace{x^2+y^2-1}_{g(x,y)}=0$

$g(x,y) = x^2+y^2-1$

$$\frac{\partial g}{\partial x} = 2x \qquad \frac{\partial f}{\partial x} = 4x$$

$$\frac{\partial g}{\partial y} = 2y \qquad \frac{\partial f}{\partial y} = -4y$$

$$\nabla g = (2x, 2y) \qquad \nabla f = (4x, -4y)$$

$$\begin{cases} \nabla f = \lambda \nabla g \\ g=0 \end{cases} \Leftrightarrow \begin{cases} (4x,-4y) = (2\lambda x, 2\lambda y) \\ x^2+y^2-1=0 \end{cases} \Leftrightarrow \begin{cases} 4x = 2\lambda x \\ -4y = 2\lambda y \\ x^2+y^2=1 \end{cases}$$

$$\Leftrightarrow \begin{cases} \lambda = \frac{4x}{2x} \\ \lambda = \frac{-4y}{2y} \\ \text{---} \end{cases} \Leftrightarrow \begin{cases} -4y \cdot 2x = 4x \cdot 2y \\ \text{---} \end{cases} \Leftrightarrow \begin{cases} -8yx = 8yx \\ \text{---} \end{cases}$$

$$\Leftrightarrow \begin{cases} yx=0 \\ \text{---} \end{cases} \Leftrightarrow \begin{cases} x=0 \\ y^2=1 \end{cases} \vee \begin{cases} y=0 \\ x^2=1 \end{cases} \Leftrightarrow \begin{cases} x=0 \\ y=1 \vee y=-1 \end{cases} \vee \begin{cases} y=0 \\ x=\pm 1 \end{cases}$$

~~Extremantes locais~~

Extremantes locais:

$(1,0)\ (-1,0)\ (0,-1)\ (0,1)$

$f(1,0) = 2 \leftarrow$ Maximo global

$f(-1,0) = 2$

$f(0,1) = -2 \leftarrow$ Minimo global

$f(0,-1) = -2$

30

$g = x^2+y^2-1 \qquad f = xy$

$\frac{dg}{dx} = 2x \qquad \frac{df}{dx} = y$

$\frac{dg}{dy} = 2y \qquad \frac{df}{dy} = x$

$$\begin{cases} \nabla f = \lambda \nabla g \\ g = 0 \end{cases} \Leftrightarrow \begin{cases} (y,x) = \lambda(2x,2y) \\ x^2+y^2-1=0 \end{cases} \Leftrightarrow \begin{cases} \lambda = \frac{y}{2x} \quad \lambda = \frac{x}{2y} \\ x^2+y^2=1 \end{cases}$$

$$\Leftrightarrow \begin{cases} \frac{x}{2y} = \frac{y}{2x} \\ x^2+y^2=1 \end{cases} \Leftrightarrow \begin{cases} 2x^2 = 2y^2 \\ - \end{cases} \Leftrightarrow \begin{cases} x^2 = y^2 \\ 2x^2 = 1 \end{cases} \Leftrightarrow \begin{cases} - \\ x^2 = \frac{1}{2} \end{cases} \Leftrightarrow \begin{cases} - \\ x = \pm\sqrt{\frac{1}{2}} \end{cases}$$

Extremantes locais

$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ (Notar que $y \geq 0$ em D)

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$f\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = -\frac{1}{2}$

$f\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = \frac{1}{2}$

31 $g = x^2+y^2-80 \qquad d(x,y) = \sqrt{(x-1)^2+(y-2)^2}$

$\nabla g = (2x, 2y)$

Por simplicidade, consideremos

$f(x,y) = (d(x,y))^2 = (x-1)^2+(y-2)^2$

$\nabla f = (2(x-1), 2(y-1))$

$$\begin{cases} \nabla f = \lambda \nabla g \\ g = 0 \end{cases} \Leftrightarrow \begin{cases} (2(x-1), 2(y-2)) = \lambda(2x, 2y) \\ x^2+y^2-80=0 \end{cases} \Leftrightarrow \begin{cases} 2(x-1) = 2\lambda x \\ 2(y-2) = 2\lambda y \\ x^2+y^2=80 \end{cases} \Leftrightarrow \begin{cases} \lambda = \frac{x-1}{x} \\ \lambda = \frac{y-2}{y} \\ - \end{cases}$$

$$\begin{cases} \frac{x-1}{x} = \frac{y-2}{y} \\ — \end{cases} \Leftrightarrow \begin{cases} y(x-1) = x(y-2) \\ — \end{cases} \Leftrightarrow \begin{cases} yx - y = xy - 2x \\ — \end{cases}$$

$$\Leftrightarrow \begin{cases} y = 2x \\ x^2 + (2x)^2 = 80 \end{cases} \Leftrightarrow \begin{cases} — \\ x^2 + 4x^2 = 80 \end{cases} \Leftrightarrow \begin{cases} — \\ 5x^2 = 80 \end{cases} \Leftrightarrow \begin{cases} — \\ x^2 = 16 \end{cases}$$

$$\Leftrightarrow \begin{cases} — \\ x = \pm 4 \end{cases}$$

Extremantes Globais

$(4,8)$ $(-4,-8)$

$$d(4,8) = \sqrt{(4-1)^2 + (8-2)^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$$

$$d(-4,-8) = \sqrt{(-4-1)^2 + (-8-2)^2} = \sqrt{25+100} = \sqrt{125} = 5\sqrt{5}$$

32

$$d(x,y,z) = \sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2} = \sqrt{x^2+y^2+z^2}$$

Consideremos $f(x,y,z) = (d(x,y,z))^2 = x^2+y^2+z^2$

$\nabla f = (2x, 2y, 2z)$

$g(x,y,z) = x + 2y + z - 4$

$\nabla g = (1,2,1)$

$$\begin{cases} \nabla f = \lambda \nabla g \\ g = 0 \end{cases} \Leftrightarrow \begin{cases} (2x, 2y, 2z) = \lambda(1,2,1) \\ x + 2y + z = 4 \end{cases} \Leftrightarrow \begin{cases} 2x = \lambda \\ 2y = 2\lambda \\ 2z = \lambda \\ — \end{cases}$$

$$\Leftrightarrow \begin{cases} 2x = y \\ x = z \\ x + 2(2x) + x = 4 \end{cases} \Leftrightarrow \begin{cases} — \\ — \\ x = \frac{4}{6} \end{cases} \Leftrightarrow \begin{cases} y = \frac{4}{3} \\ z = \frac{2}{3} \\ x = \frac{2}{3} \end{cases}$$

~~Extremante: $(2,1,1)$~~ Extremante: $\left(\frac{2}{3}, \frac{4}{3}, \frac{2}{3}\right)$

~~$d(2,1,1) = \sqrt{4+1+1} = \sqrt{6}$~~

33. $d(x,y,z) = \sqrt{(x-1)^2 + y^2 + (z+2)^2}$

$f(x,y,z) = (x-1)^2 + y^2 + (z+2)^2$

$\nabla f = (2(x-1), 2y, 2(z+2))$

$\nabla g = (1,2,1)$

$$\begin{cases} \nabla f = \lambda \nabla g \\ g = 0 \end{cases} \Leftrightarrow \begin{cases} (2(x-1), 2y, 2(z+2)) = \lambda(1,2,1) \\ x + 2y + z = 4 \end{cases} \Leftrightarrow \begin{cases} 2x-2=\lambda \\ 2y = 2\lambda \\ 2(z+2) = \lambda \\ — \end{cases}$$

$$\Leftrightarrow \begin{cases} 2x-2 = y \\ 2x-2 = 2z+4 \\ — \end{cases} \Leftrightarrow \begin{cases} y = 2x-2 \\ z = x - \cancel{5}3 \\ x + 4x - 4 + x - 3 = 4 \end{cases} \Leftrightarrow \begin{cases} — \\ — \\ 6x = 11 \end{cases}$$

$$\Leftrightarrow \begin{cases} y = \frac{5}{3} \\ z = -\frac{7}{6} \\ x = \frac{11}{6} \end{cases}$$

$$d\left(\frac{11}{6}, \frac{5}{3}, -\frac{7}{6}\right) = \sqrt{\left(\frac{11}{6}-1\right)^2 + \left(\frac{5}{3}\right)^2 + \left(-\frac{7}{6}+2\right)^2}$$

$$= \sqrt{\frac{25}{36} + \frac{25}{9} + \frac{25}{36}} = \sqrt{\frac{150}{36}} = \frac{\sqrt{150}}{6} = \frac{5\sqrt{6}}{6}$$

34.

$$d = \sqrt{(x-3)^2 + (y-1)^2 + (z+1)^2}$$

$$f = (x-3)^2 + (y-1)^2 + (z+1)^2 \qquad g = x^2 + y^2 + z^2 - 4$$

$$\nabla f = (2(x-3), 2(y-1), 2(z+1)) \qquad \nabla g = (2x, 2y, 2z)$$

$$\Leftrightarrow \begin{cases} \nabla f = \lambda \nabla g \\ g = 0 \end{cases} \Leftrightarrow \begin{cases} (2(x-3), 2(y-1), 2(z+1)) = \lambda(2x, 2y, 2z) \\ x^2+y^2+z^2-4 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x-3 = \lambda x \\ y-1 = \lambda y \\ z+1 = \lambda z \\ x^2+y^2+z^2 = 4 \end{cases} \Leftrightarrow \begin{cases} \lambda = \frac{x-3}{x} \\ \lambda = \frac{y-1}{y} \\ \lambda = \frac{z+1}{z} \\ — \end{cases} \Leftrightarrow \begin{cases} \frac{x-3}{x} = \frac{y-1}{y} \\ \frac{z+1}{z} = \frac{y-1}{y} \\ — \end{cases} \Leftrightarrow \begin{cases} \cancel{xy} - 3y = \cancel{xy} - x \\ \cancel{zy} + y = \cancel{zy} - z \\ — \end{cases}$$

$$\Leftrightarrow \begin{cases} x = 3y \\ -y = z \\ (3y)^2 + y^2 + (-y)^2 = 4 \end{cases} \Leftrightarrow \begin{cases} — \\ — \\ 11y^2 = 4 \end{cases} \Leftrightarrow \begin{cases} — \\ — \\ y = \pm\sqrt{\frac{4}{11}} \end{cases} \Leftrightarrow \begin{cases} x = \pm \frac{6}{\sqrt{11}} \\ z = \mp \frac{2}{\sqrt{11}} \\ y = \pm \frac{2}{\sqrt{11}} \end{cases}$$

~~Pont~~ Ext: $\left(\frac{6}{\sqrt{11}}, \frac{2}{\sqrt{11}}, -\frac{2}{\sqrt{11}}\right) \left(\frac{-6}{\sqrt{11}}, \frac{-2}{\sqrt{11}}, \frac{2}{\sqrt{11}}\right)$

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$$T(x,y,z) = 30 + 5(x+z) = 30 + 5x + 5z \qquad g(x,y,z) = x^2+y^2+z^2-1$$

$$\frac{\partial T}{\partial x} = 5 \quad \frac{\partial T}{\partial y} = 0 \quad \frac{\partial T}{\partial z} = 5$$

$$\nabla T = (5, 0, 5) \qquad \nabla g = (2x, 2y, 2z)$$