

Universidade de Aveiro
Departamento de Matemática

Cálculo II - Agrupamento 4

2021/22

Folha 4: Soluções

1. (a) Sim; (b) Sim; (c) Não; (d) Sim.
2. (a) $xy' - y = 0$; (b) $y'' = 0$; (c) $xy' - y \ln(y) = 0$.
3. $y''' + y' = 0$.
4. (a) $y = C_1x - \operatorname{sen} x + C_2$, $C_1, C_2 \in \mathbb{R}$.
5. (a) $y = \ln(\operatorname{arctg} x) + C$, $C \in \mathbb{R}$;
(b) $y = \frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\arcsen x + C$, $C \in \mathbb{R}$;
(c) $y = \frac{x^3}{3} + \operatorname{arctg} x + C$, $C \in \mathbb{R}$.
6. (a) $x^2 + y^2 = C$, $C \in \mathbb{R}$;
(b) $y = Cx$, $C \in \mathbb{R}$ (compare com o ex. 2(a));
(c) $\frac{x}{t} = Ce^{-\frac{1}{x}-\frac{1}{t}}$, $C \in \mathbb{R}$;
(d) $y = \frac{1}{\ln|x^2-1|-C}$, $C \in \mathbb{R}$;
7. (a) $y = \frac{1}{x+1}$; (b) $y = -1 + 2e^{2-\sqrt{4+x^2}}$; (c) $y^3 = 4(1+x^3)$.
8. (a) $\ln|y| - \frac{x^2}{2y^2} = C$, $C \in \mathbb{R}$ ($y = 0$ é solução singular).
(b) $y = xe^{Ky}$, $x > 0$, $K \in \mathbb{R}$.
9. (b) $y = xe^{Cx}$, $x > 0$, $C \in \mathbb{R}$.
10. (a) $\operatorname{arctg}\left(\frac{y-1}{x-2}\right) - \frac{1}{2}\ln\left(1 + \left(\frac{y-1}{x-2}\right)^2\right) = \ln|x-2|$, $C \in \mathbb{R}$.
(b) $(y-x)^2 + 4y = C$, $C \in \mathbb{R}$.
11. (a) $x^2 + x \operatorname{sen} y = C$, $C \in \mathbb{R}$;
(b) $x^2 + y^2 + 2xe^y - 2yx^2 = C$, $C \in \mathbb{R}$;
(c) $y = \frac{C-3x^2}{\ln|x|-2}$, $C \in \mathbb{R}$.
12. $x + e^{-x} \operatorname{sen} y = C$, $C \in \mathbb{R}$ (um fator integrante é $\mu(x, y) = e^{-x} \cos y$).
13. (a) $x + y^2 = Cy$, $C \in \mathbb{R}$ (um fator integrante é $\mu(y) = y^{-2}$);
(b) $yx^2 - \frac{x^5}{5} = C$, $C \in \mathbb{R}$ (um fator integrante é $\mu(x) = x$, $x > 0$).
14. (a) $y = \frac{2}{5} \cos x + \frac{1}{5} \operatorname{sen} x + Ce^{-2x}$, $C \in \mathbb{R}$;

- (b) $y = -1 + C e^{-\frac{1}{2x^2}}$, $x \neq 0$, $C \in \mathbb{R}$;
(c) $y = (C + x)\sqrt{x^2 + 1}$, $C \in \mathbb{R}$.
15. Comece por verificar que a solução geral possui a forma $y = \frac{1}{x} + \frac{C}{x^2}$, $C \in \mathbb{R}$.
16. (a) $y = \frac{1}{1 + Cx + \ln x}$, $x > 0$, $C \in \mathbb{R}$ ($y = 0$ é solução singular).
(b) $y^4 = \frac{x^2}{C - 4x^5}$, $C \in \mathbb{R}$ ($y = 0$ é solução singular).
17. (a) $y = \frac{x^4}{2} + Kx^2$, $K \in \mathbb{R}$;
(b) $y = \frac{x}{2} \operatorname{cosec} x - \frac{\cos x}{2} + K \operatorname{cosec} x$, $K \in \mathbb{R}$.
18. (a) $y = Kx^2$ ($K \neq 0$);
(b) $y = Ke^x$ ($K \neq 0$);
(c) $x^2 - y^2 = K$ ($K \neq 0$).
19. (a) $y = C_1 e^{-x} + \frac{\operatorname{sen} x}{2} - \frac{\cos x}{2}$;
(b) $y = C_1 e^x + C_2 e^{-x} + \cos x$;
(c) $y = C_1 e^x + C_2 e^{-2x} + 3x$;
(d) $y = \left(C_1 + C_2 x + \frac{x^3}{6}\right) e^{2x}$;
(e) $y = C_1 + (C_2 - x) e^{-x}$;
(f) $y = C_1 \operatorname{sen}(2x) + C_2 \cos(2x) - \frac{1}{4} \cos(2x) \ln |\sec(2x) + \operatorname{tg}(2x)|$;
(g) $y = C_1 + C_2 \cos x + C_3 \operatorname{sen} x - \frac{x}{2} \operatorname{sen} x$;
(h) $y = C_1 \operatorname{sen}(3x) + C_2 \cos(3x) + \frac{\operatorname{sen} x}{8} - \frac{e^{-x}}{10}$.
(C_1, C_2, C_3 são constantes reais arbitrárias).
20. $y = \frac{3}{4}(x - \pi) e^{2(\pi - x)} + \frac{\operatorname{sen}(2x)}{8}$.
21. $y = 1 + e^{-\operatorname{sen} x}$, $x \in \mathbb{R}$.
22. (a) $y = \frac{K}{(x^2 + 1)^2}$, $K \in \mathbb{R}$;
(b) $y = C_1 \cos x + C_2 \operatorname{sen} x + x \cos x$, $C_1, C_2 \in \mathbb{R}$;
(c) $y = C e^{\operatorname{arctg} x}$, $C \in \mathbb{R}$;
(d) $y = C_1 + C_2 \cos(2x) + C_3 \operatorname{sen}(2x) + \frac{1}{3} \operatorname{sen} x$, $C_1, C_2 \in \mathbb{R}$;
(e) $y = K e^{x^3} - \frac{1}{3}$, $K \in \mathbb{R}$;
(f) $y = C_1 e^{-2x} + (C_2 + C_3 x + 2x^2) e^x$, $C_1, C_2, C_3 \in \mathbb{R}$.
23. $y = Cx^2 + x^3 + K$, $C, K \in \mathbb{R}$.
24. (a) –
(b) $y = C_1 x + C_2 e^x$, $C_1, C_2 \in \mathbb{R}$.
(c) $y = C_1 x + C_2 e^x + x^2$, $C_1, C_2 \in \mathbb{R}$.