

Modeling the cumulative incidence function of clustered competing risk data



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Clustered competing risk data

Idea: causes competing by the occurrence of an event such the

confiability analysis

failure of an industrial or
electronic component

survival analysis

failure or progress of a patient
or some biological process

A typical data set consists of

Group	ID	Cause 1	Cause 2	Censorship	Time	Feature
1	1	Yes	No	No	10	A
1	2	No	No	Yes	8	A
2	1	No	No	Yes	7	B
2	2	No	Yes	No	5	A

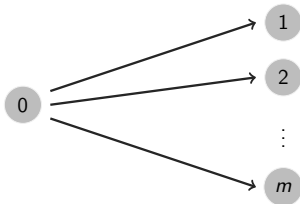


Survival data designs

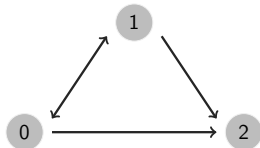
Failure time process



Competing risk process



Multistate process

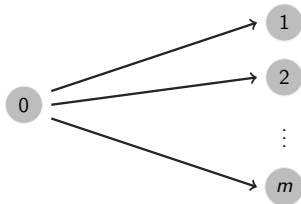


Survival data designs

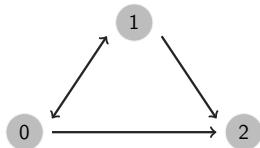
Failure time process



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Multistate process



Survival modeling framework

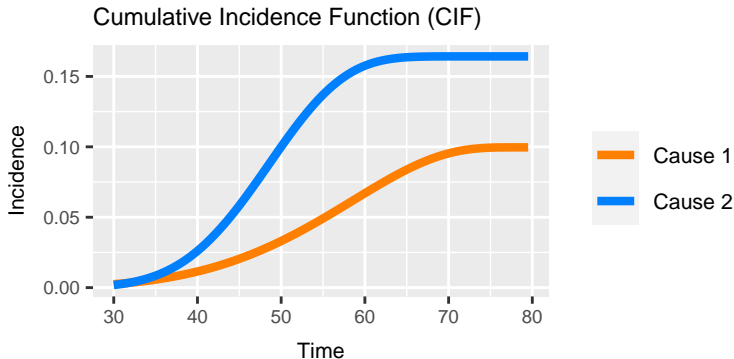
We have to choose which **scale** we model the **survival experience**.
Usually, is the

hazard (failure rate) **scale** : $\lambda(t | x) = \lambda_0(t) \times c(x, \beta)$



In the competing risk setting ...

a more attractive possibility is to work on the **probability scale**, focusing on the cause-specific



i.e.

$$\text{CIF} = \mathbb{P}[\text{failure time} \leq t, \text{ a given cause} \mid \text{features}]$$



Main focus application: cancer incidence in twins



Clustered competing risks data

↳ Clusters? Families

↳ Family studies

↳ **Twins data**

Family studies \Rightarrow **within-family dependence**

- » Taking into account the **within-family dependence** may reflect both disease **heritability** and the impact of shared **environmental effects**



And what we do? A hierarchical approach

Thinking on two competing causes

... for the outcome y_{ijt} of a subject i , family j , in the time t , we have

$$y_{ijt} \mid \underbrace{\{u_{1j}, u_{2j}, \eta_{1j}, \eta_{2j}\}}_{\text{latent effects}} \sim \text{Multinomial}(p_{1ijt}, p_{2ijt}, p_{3ijt})$$

$$\begin{bmatrix} u_{1j} \\ u_{2j} \\ \eta_{1j} \\ \eta_{2j} \end{bmatrix} \sim \text{Multivariate Normal} \left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{u_1}^2 & \rho_{u_1, u_2} & \rho_{u_1, \eta_1} & \rho_{u_1, \eta_2} \\ & \sigma_{u_2}^2 & \rho_{u_2, \eta_1} & \rho_{u_2, \eta_2} \\ & & \sigma_{\eta_1}^2 & \rho_{\eta_1, \eta_2} \\ & & & \sigma_{\eta_2}^2 \end{bmatrix} \right)$$

$$\begin{aligned} p_{kijt} &= \frac{\partial \text{CIF}}{\partial t} \\ &= \frac{\partial}{\partial t} \underbrace{\pi_k(X, u_1, u_2 \mid \beta)}_{\text{cluster-specific risk level}} \underbrace{\Phi[w_k g(t) - X^\top \gamma_k - \eta_k]}_{\text{cluster-specific failure time trajectory}}, \end{aligned}$$

$k = 1, 2$



Challenges

Thinking in the [twins data](#) application,
the small group/family size is a problem that implies in the following

cycle : small groups \Rightarrow little information \Rightarrow complex model \Rightarrow bigger number of groups

... with this,
computational challenges appear and have also to be overcome



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Thinking in the [twins data](#) application,
the small group/family size is a problem that implies in the following

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Besides, the data is very simple ...

- » we just know if the event occurred (yes or no) and the time
 - » with this, we have to be able to construct the [cumulative incidence curves](#)
- » and we have to accommodate the [within-family dependence](#)
 - » that can happen in different manners



Joint work with

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Thank you



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