Modeling the cumulative incidence function of clustered competing risk data: a multinomial GLMM approach

master thesis defense

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Clustered competing risk data

Key terms:

- Clustered: groups with a dependence structure (e.g. families);
- Causes competing by *something*.

Something?

- **Failure** of an industrial or electronic component;
- **Occurence** or **cure** of a disease or some biological process;
- **Progress** of a patient clinic state.

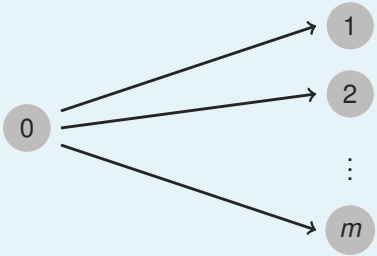
Independent of the application, always the same framework

Cluster	ID	Cause 1	Cause 2	Censorship	Time	Feature
1	1	Yes	No	No	10	A
1	2	No	No	Yes	8	A
2	1	No	No	Yes	7	B
2	2	No	Yes	No	5	A

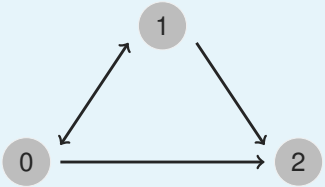
Big picture: Failure time data

Failure time process

Competing risk process



Multistate process

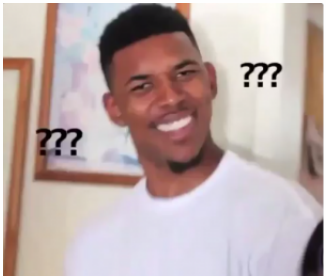


Same methodologies, different names.

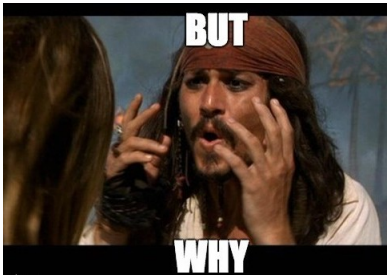
Survival analysis Industrial life testing;
Reliability analysis Biomedical studies.

A comprehensive reference is Kalbfleisch and Prentice (2002)'s book.

Modeling clustered competing risks data



What?



Why?

How?

Failure time data $\rightarrow$ Survival models

First of all, we have to choose which scale we model the **survival experience**.
Usually, is in the

$$\text{hazard (failure rate) scale}: \quad \lambda(t \mid \text{features}) = \lambda_0(t) \times c(\text{features}). \tag{1}$$

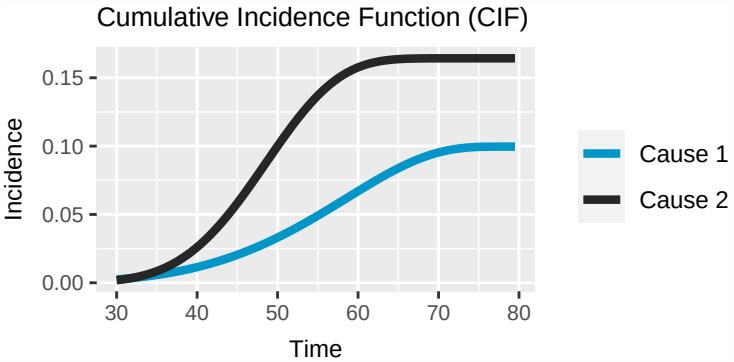
We have a Equation 1 for each competing cause.

The cluster dependence is something actually not measured...

$$\text{Not measured dependence} \rightarrow \text{random/latent effects} \rightarrow \text{Frailty models.}$$

In the competing risk setting . . .

a more attractive possibility is to work on the probability scale, focusing on the cause-specific



i.e.

CIF $= \mathbb{P}[$ failure time $\leqslant t,$ a given cause $|$ features $]$

Cancer incidence in twins

Clustered competing risks data
↳ Clusters? Families
↳Family studies
↳ Twins data

Family studies $\Rightarrow$ within-family dependence

That may reflect

- Disease **heritability**;
- The impact of shared **environmental effects**;
  - **Parental effects**: continuity of the phenotype across generations.

Our contribution: a hierarchical approach

Thinking on two competing causes

... for the outcome y_{ijt} of a subject i, family j, in the time t, we have

$$y_{ijt} \mid \underbrace{\{u_{1j}, u_{2j}, \eta_{1j}, \eta_{2j}\}}_{\text{latent effects}} \sim \text{Multinomial}(p_{1ijt}, p_{2ijt}, p_{3ijt})$$

$$\begin{bmatrix} u_{1j} \\ u_{2j} \\ \eta_{1j} \\ \eta_{2j} \end{bmatrix} \sim \text{Multivariate Normal} \left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2_{u_1} & \sigma_{u_1,u_2} & \sigma_{u_1,\eta_1} & \sigma_{u_1,\eta_2} \\ & \sigma^2_{u_2} & \sigma_{u_2,\eta_1} & \sigma_{u_2,\eta_2} \\ & & \sigma^2_{\eta_1} & \sigma_{\eta_1,\eta_2} \\ & & & \sigma^2_{\eta_2} \end{bmatrix} \right)$$

$$p_{kijt} = \frac{\partial \text{CIF}}{\partial t} = \frac{\partial}{\partial t} \underbrace{\pi_k(X, u_1, u_2 \mid \beta)}_{\substack{\text{cluster-specific} \\ \text{risk level}}} \underbrace{\Phi[w_k g(t) - X^\top \gamma_k - \eta_k]}_{\substack{\text{cluster-specific} \\ \text{failure time trajectory}}},$$

$k = 1, 2.$

Contributions & challenges

- A clear and simpler modeling structure;
- **There is no free lunch**
  Computational challenges overcame via an
  efficient implementation and estimation routines, the TMB;
- The data is very simple,
  we just know the outcome (**yes** or **no**);
- We have to be able to build the CIF curves;
- And accommodate the within-family dependence properly,
  that can happen in different manners;
- . . .

TMB: Template Model Builder

Quickly implement complex random effect models through simple C++ templates.
The R package combines

- CppAD: C++ automatic differentiation;
- Eigen: templated matrix-vector library;
- CHOLMOD: sparse matrix routines available from R;

to obtain an efficient implementation of the applied Laplace approximation with exact derivatives.

Also, key features are

- automatic sparseness detection;
- parallelism through BLAS;
- parallel user templates.

Thanks for watching and have a great day

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1. Data
2. Modeling
3. Application
4. Model
5. Contributions & challenges
6. Template Model Builder
7. Simulation study
8. Results

References

Kalbfleisch, J. D., and R. L. Prentice. 2002. *The Statistical Analysis of Failure Time Data*. Second Edition. Hoboken, New Jersey: John Wiley & Sons, Inc.