HW4

Question 1

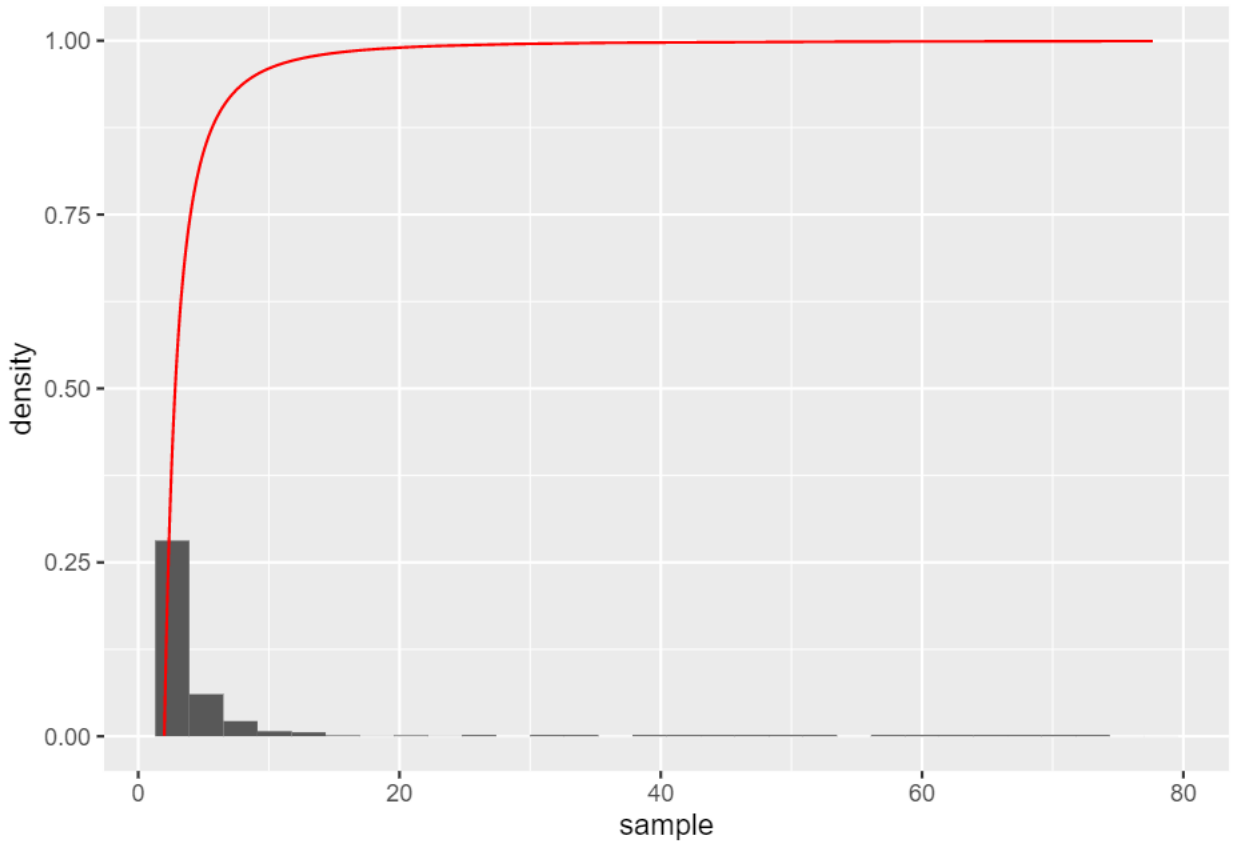
Finding the inverse

$u = F(x)$ $u = 1 - (\frac{b}{x})^a$ $x = \frac{b}{(1-u)^{\frac{1}{a}}}$ $F(U) = \frac{b}{(1-u)^{\frac{1}{a}}}$

Simulating

```
f_inv <- function(u,a,b){
  b/((1-u)^(1/a))
}

f_X <- function(x,a,b) {
  1-(b/x)^a
}

u <- runif(1000,0,1)

sample <- f_inv(u,2,2)

ggplot() +
  geom_histogram(aes(sample, y = ..density..)) +
  geom_line(aes(sample, f_X(sample,2,2)), colour = "red")
```

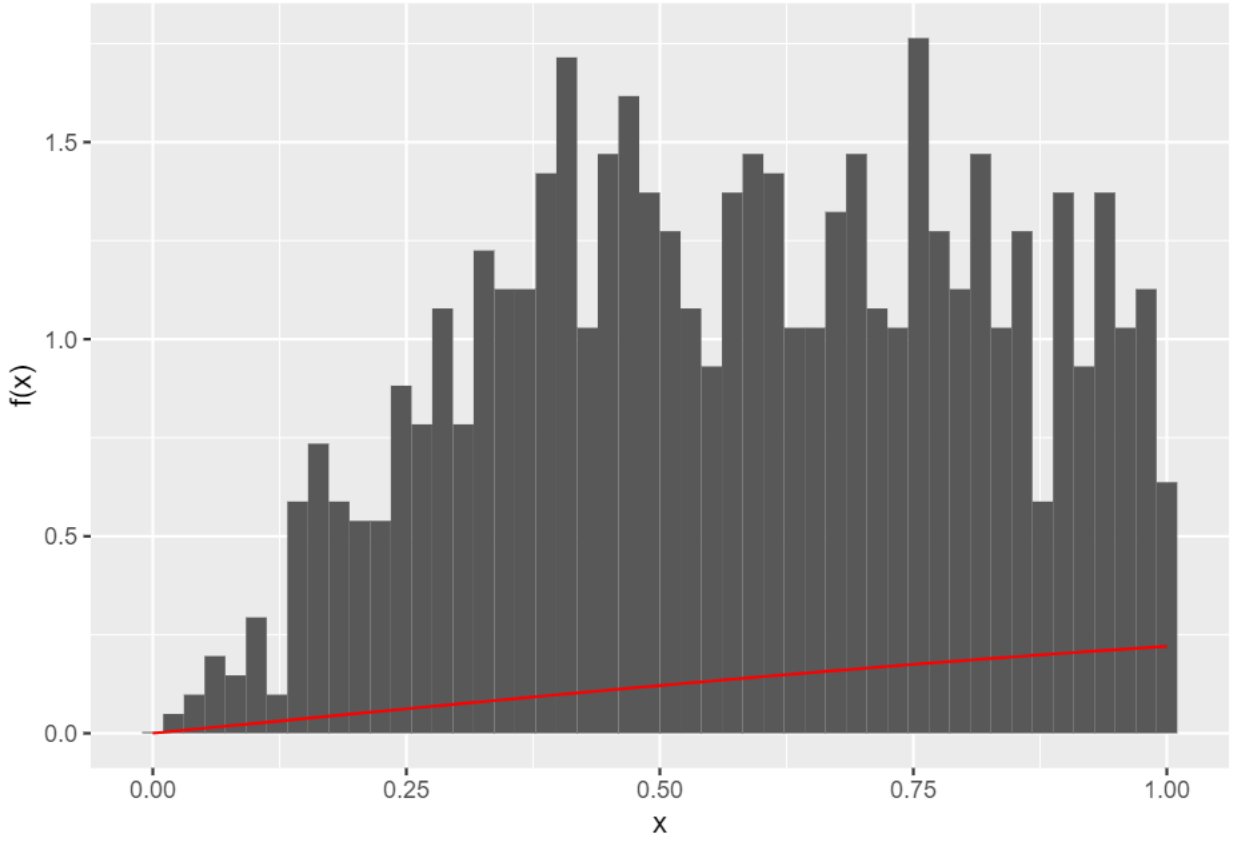
```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```



Question 2

```
f <-function(x,s) {

  exp((-x^2)/(2*s^2))*x/s^2
}


x <- seq(0, 1, length.out = 100)

envelope <- function(x) {
  f(0.4,2)
}



n <- 1000
accepted <- 0
samples <- rep(NA, n)

while(accepted < n) {

  y <- runif(1)


  u <- runif(1)

  if(u < f(y,2)/envelope(y)) {
```

```
    accepted <- accepted + 1
    samples[accepted] <- y
  }
}

ggplot() +
  geom_histogram(aes(samples, y = ..density..), bins = 50, ) +
  geom_line(aes(x, f(x,2)), colour = "red") +
  xlab("x") + ylab("f(x)")
```



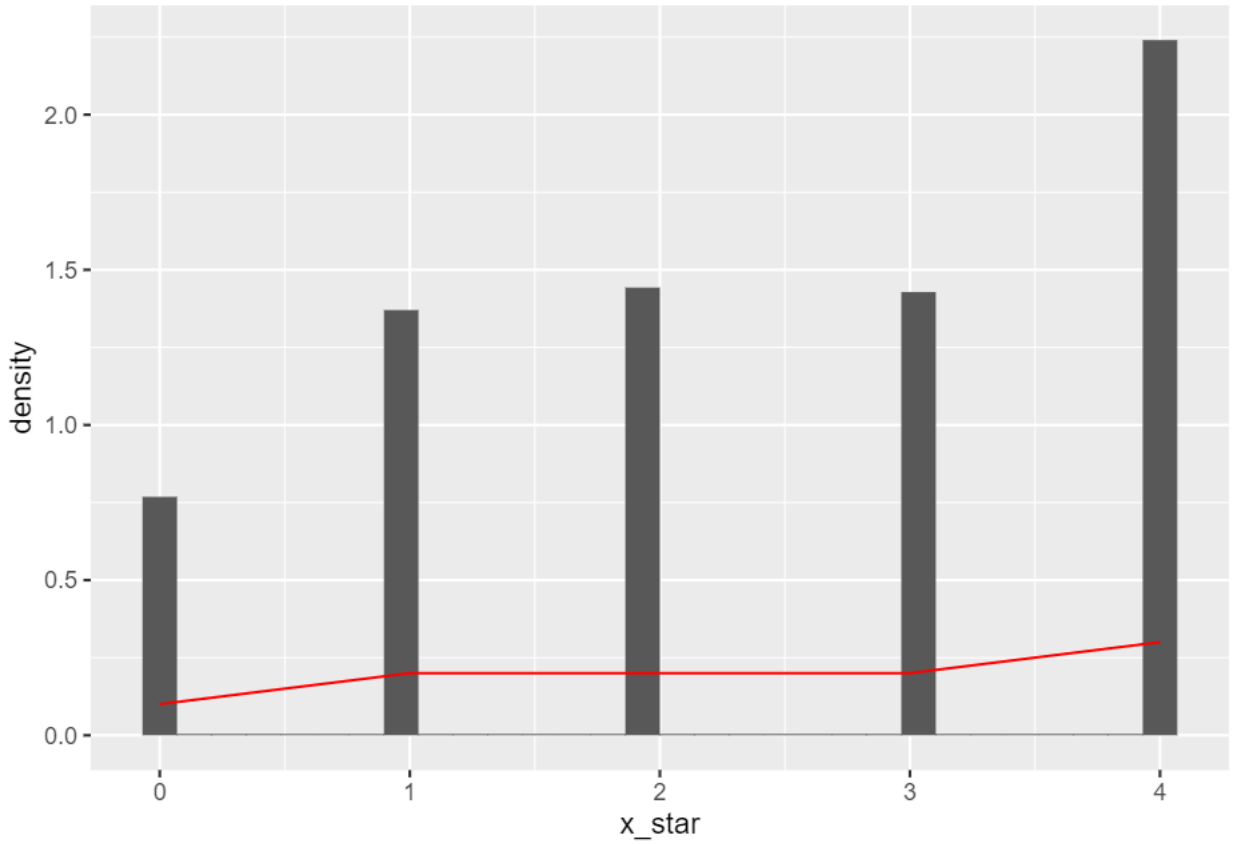
Question 3

```
x <- 0:4
f <- c(0.1,0.2,0.2,0.2,0.3)

F_x <- cumsum(f)



n <- 1000
x_star <- sample(x, n, replace = TRUE, prob = f)


table(x_star)/n
```

```
## x_star
##     0     1     2     3     4
## 0.106 0.189 0.199 0.197 0.309
```

```
ggplot()+geom_histogram(aes(x_star,y = ..density..))+geom_line(aes(x,f),colour="red")
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```



Question 4

A)

```
f <- function(x) {
    60*x^3*(1-x)^2
}

# plot pdf
x <- seq(0, 1, length.out = 100)



## ------------------------------------------------------------
envelope <- function(x) {

  f(3/5)
```

```
}


n <- 100
accepted <- 0
samples <- rep(NA, n)

while(accepted < n) {
  # sample y from g
  y <- runif(1)

  # sample u from uniform(0,1)
  u <- runif(1)

  if(u < f(y)/envelope(y)) {
    # accept
    accepted <- accepted + 1
    samples[accepted] <- y
  }
}


n100<-quantile(samples,probs = seq(0.1,0.9,0.1))



n <- 1000
accepted <- 0
samples2 <- rep(NA, n)

while(accepted < n) {

  y <- runif(1)


  u <- runif(1)

  if(u < f(y)/envelope(y)) {
    # accept
    accepted <- accepted + 1
    samples2[accepted] <- y
  }
}


n1000<-quantile(samples2,probs = seq(0.1,0.9,0.1))


n <- 10000
accepted <- 0
samples3 <- rep(NA, n)

while(accepted < n) {
```

```
  y <- runif(1)


  u <- runif(1)

  if(u < f(y)/envelope(y)) {
    # accept
    accepted <- accepted + 1
    samples3[accepted] <- y
  }
}


n10000<-quantile(samples3,probs = seq(0.1,0.9,0.1))

beta<-quantile(rbeta(1000,3,4),probs = seq(0.1,0.9,0.1))


df <- c(rbind(n100,n1000,n10000,beta))
```

B)

```
i<-1

counts<- rep(NA, 9)
while(i<=10){
i<-i+1
n <- 1000
accepted <- 0
samples2 <- rep(NA, n)
count <- 0
while(accepted < n) {
  count <- count+1

  y <- runif(1)


  u <- runif(1)

  if(u < f(y)/envelope(y)) {

    accepted <- accepted + 1
    samples2[accepted] <- y
  }
}

counts[i-1]<-count



}
```

```r
print(table(counts))
```

```
## counts
## 1999 2054 2063 2070 2086 2105 2108 2112 2131 2151
##    1    1    1    1    1    1    1    1    1    1
```

```r
print(mean(counts))
```

```
## [1] 2087.9
```

```r
pro <- 1000/counts

print(mean(pro))
```

```
## [1] 0.4791405
```