Lab 3 Solution

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Rayleigh Distribution Random Variable Generator

Methods:

The functions defined utilize standard Python libraries, numpy for numerical operations and scipy.stats for statistical tools, to generate random variables (R.V.s) for different distributions. For the Rayleigh distribution, the inverse transform sampling method is applied where a uniform random variable is used to produce a Rayleigh distributed R.V. through the Rayleigh inverse cumulative distribution function (CDF). Other distributions use built-in numpy functions which are optimized and reliable for generating random variables...

Testing :

For testing, the Rayleigh distribution is chosen with a sigma parameter of 2. Empirical moments (mean and variance) were evaluated for three different sample sizes (1000, 10000, 100000). These empirical moments were then compared to the analytical predictions:

Analytical Mean: $\sigma(\sqrt{\pi}/2) \approx 2.5066$

Analytical Variance: $(2-(\pi/2))\sigma^2 \approx 1.7168$

Results:

The empirical moments approached the analytical moments as the sample size increased, demonstrating the law of large numbers. Here are the empirical results:

Sample Size 1000: Mean ≈2.5198,Variance≈1.8709

Sample Size 10000: Mean≈2.4999,Variance≈1.7188

Sample Size 100000: Mean ≈2.5054,Variance≈1.7227

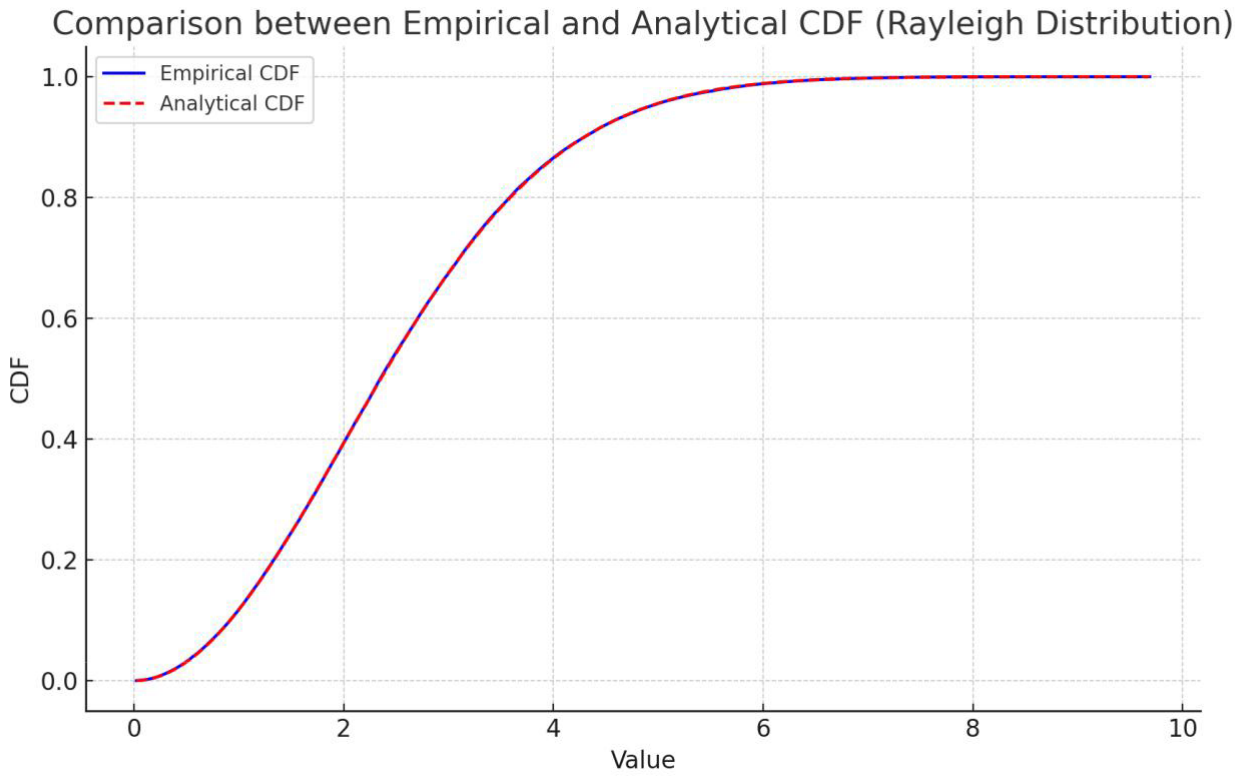
The empirical cumulative distribution function (CDF) was also plotted alongside the analytical CDF for a sample size of 100000. The plot showed a close fit between the two, indicating that the random variable generator is functioning correctly.

Conclusion:

The random variable generator functions accurately represent their respective distributions. The Rayleigh R.V. generator in particular was verified through empirical tests against analytical predictions, and it demonstrated high fidelity to the expected theoretical behavior.

Graphical Representation:

The graph presents the empirical CDF of the Rayleigh distribution and the analytical CDF for the largest sample size. The empirical CDF (blue line) closely follows the analytical CDF (red dashed line), validating the accuracy of the random variable generator created.


Comparison between Empirical and Analytical CDF (Rayleigh Distribution)
Empirical CDF
Analytical CDF
CDF
Value
0.0
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8
10