# Dynamical Processes on Graphs

Computer-Aided Simulations Lab - Lab L6

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## I. Problem overview

The goal of the present activity is to evaluate properties of dynamic process on graphs. We consider a voter model on both graphs: $G(n,p)$ and regular grids over finite portions of $\mathbb{Z}^2$ and $\mathbb{Z}^3$.

## II. Proposed approach

### A. Graph generation

The graph generation is as implemented on the previous Lab L5. For the $G(n,p)$, we use the Erdős-Rényi model, where the probability of each edge between the $n$ nodes to be present is $p$. Furthermore, if the graph is not connected, only the largest component is considered.

For the regular grids, each node is neighbor to the ones immediate next to it, i.e. if the difference between one of the coordinates of two nodes is exactly 1, and the other coordinates are the same, these nodes are neighbors. We use the grid sizes as input parameter, in a way that the product of the sizes equals the number of nodes in the graph.

### B. Voter model

On the voter model, a node at wake up assumes the state of one randomly selected neighbor. At initialization, each node assumes a state $\{-1,+1\}$, where the probability of the initial state of a node to be $+1$ is given as the input parameter $p_1$.

For handling the Future Event Set (FES) efficiently, we assume the wake up rate of the nodes to be equal $\lambda_v = 1$. By doing that, we rely on the property of poison process to schedule the next events as "a random node (uniformly chosen) wake up at rate $\lambda \cdot n$", instead of scheduling the wake up of all nodes with rate $\lambda$ simultaneously.

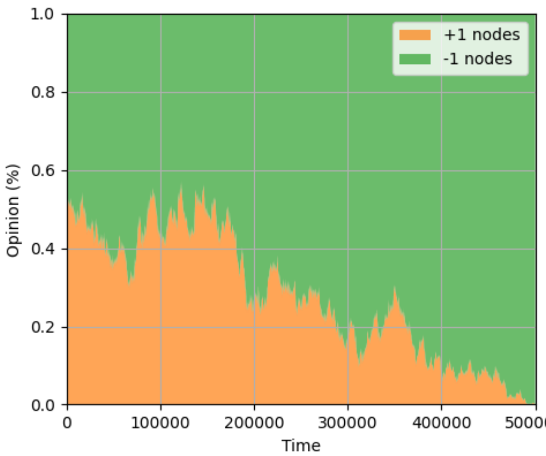


Fig. 1: Population opinion along time for $p_1 = 55\%$.

## III. Experiments and Results

Two experiments are performed, one for each type of graph.

### A. Graph $G(n,p)$

For the $G(n,p)$ graph, we fix $n = 1000$ nodes and the probability of connecting an edge is $p = 1\%$. The values considered for the initial condition $p_1$ are $[51\%, 55\%, 60\%, 70\%]$.

For this experiment, the only stop condition considered is consensus, i.e., the simulation only stops when consensus is reached. We run the voter model for these parameters and obtain the results are presented below.

- For $p_1 = 51\%$ (Figure 1), we see that -1 consensus was reached around instant 500k after almost 500k events.
- For $p_1 = 55\%$ (Figure 2), the consensus was at +1 and was reached faster, some time after instant 200k.
- For $p_1 = 60\%$ (Figure 3), once again -1 consensus was reached, and slower than $p_1 = 55\%$, around instant 250k.
- Finally, $p_1 = 70\%$ (Figure 4), +1 consensus was reached at the fastest time, at instant 110k.

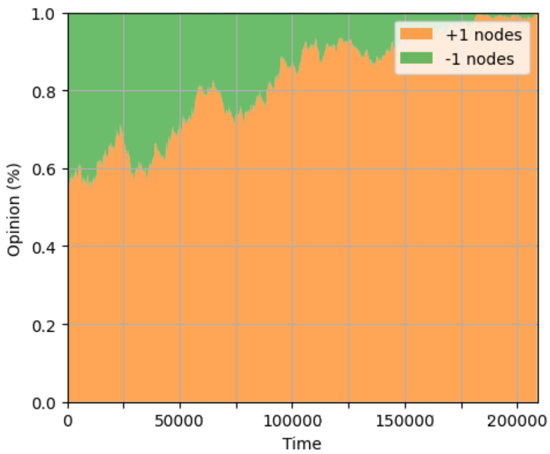


Fig. 2: Population opinion along time for $p_1 = 55\%$.

### B. Regular grids over $\mathbb{Z}^2$ and $\mathbb{Z}^3$

For the graphs over regular grids, with $p_1 = 51\%$, we consider the following sizes for the grids:

| Dimension | Sizes | Number of nodes |
|---|---|---|
| $\mathbb{Z}^2$ | $(10,10)$ | 100 |
| $\mathbb{Z}^2$ | $(32,32)$ | 1024 |
| $\mathbb{Z}^2$ | $(100,100)$ | 10000 |
| $\mathbb{Z}^3$ | $(5,5,4)$ | 100 |
| $\mathbb{Z}^3$ | $(10,10,10)$ | 1000 |
| $\mathbb{Z}^3$ | $(21,21,22)$ | 9702 |

This time, when the number of nodes is at least 1000, we consider a second stop condition since the simulation may

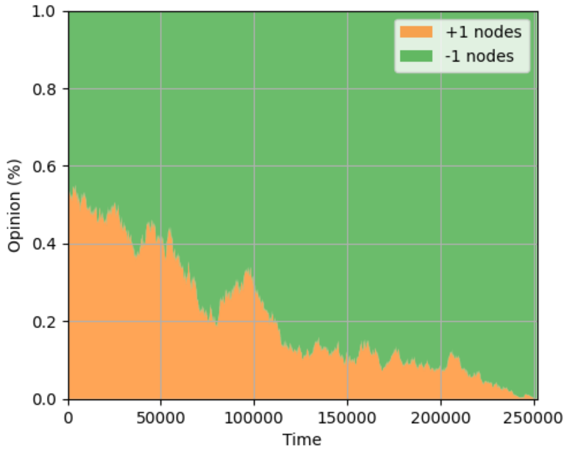


Fig. 3: Population opinion along time for $p_1 = 60\%$.

take longer to reach consensus, that is a minimum number of wake up events equals to 500k. After this many events, the simulation stops even if consensus has not been reached yet. The results are presented below.

- For the $(10, 10)$ grid (Figure 5), +1 consensus was reached around instant 450k.
- For the $(32, 32)$ grid (Figure 6), consensus was not reached after 500k events, and the final state of the graph was 64.75% of nodes at state -1 and 35.25% at state +1.
- Also for the $(100, 100)$ grid (Figure 7), consensus was not reached after 500k events, and the final state of the graph was 46.12% of nodes at state -1 and 53.88% at state +1.
- For the $(5, 5, 4)$ grid (Figure 8), -1 consensus was reached relatively fast, around instant 4k, since it was a smaller graph with more connections.
- For the $(10, 10, 10)$ grid (Figure 9), consensus was not reached after 500k events, and the final state of the graph was 49.8% of nodes at state -1 and 50.2% at state +1.
- At last, for the $(21, 21, 22)$ grid (Figure 10), consensus was not reached once again, and the final state of the graph was 50,97% of nodes at state -1 and 49.03% at state +1.

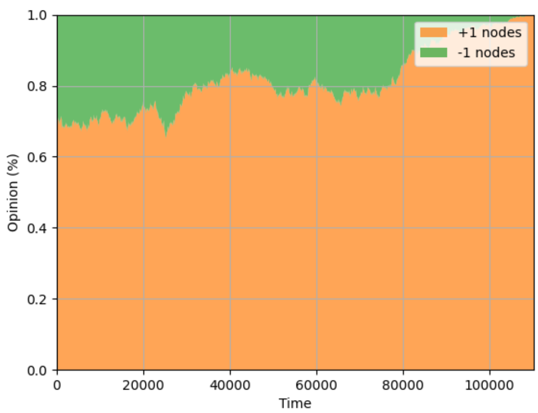


Fig. 4: Population opinion along time for $p_1 = 70\%$.

For these simulations, we can see that for greater number of nodes, it takes a long period for changes to be noticeable. For example, the grids $(100, 100)$ in $\mathbb{Z}^2$ and $(21, 21, 22)$ in $\mathbb{Z}^3$ stayed at a constant state for the entire simulation. Smaller grids were able to reach consensus or at least showed the changes in the +1/-1 ratio between the nodes along time.

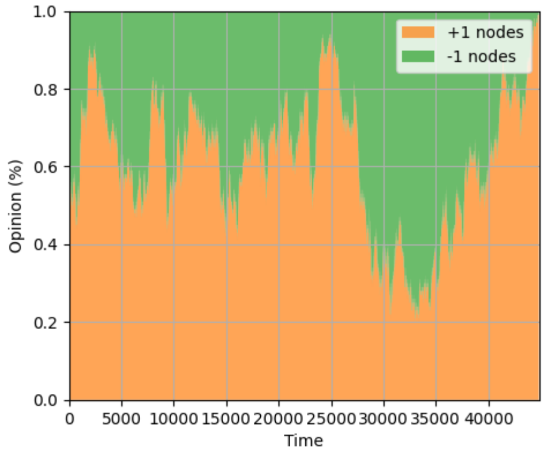


Fig. 5: Population opinion along time for $(10, 10)$ grid.

For improving the simulator, we should be able to implement it more efficiently in order to save time and run the same parameters multiple times. By doing that we could evaluate the confidence intervals for the probability of reaching consensus, as well as the time to do so.

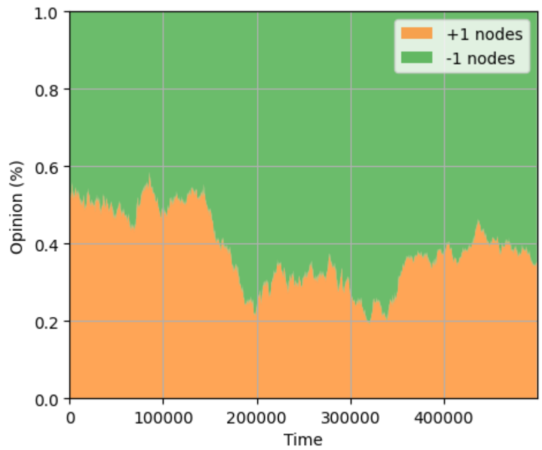


Fig. 6: Population opinion along time for $(32, 32)$ grid.

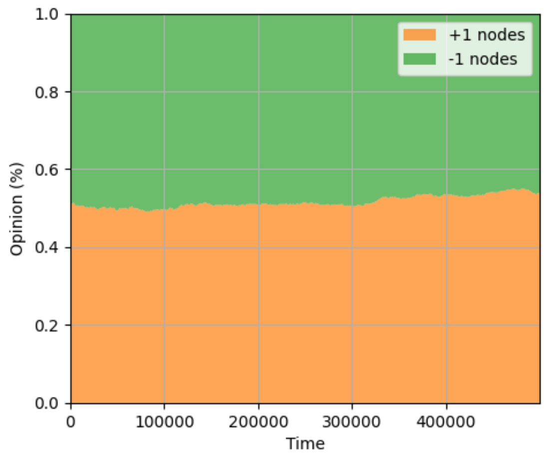


Fig. 7: Population opinion along time for $(100, 100)$ grid.

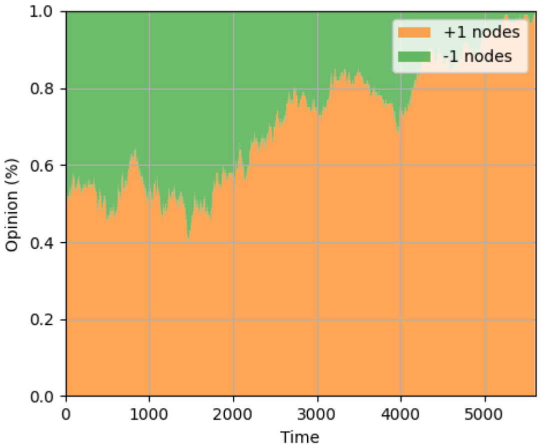

Fig. 8: Population opinion along time for $(5, 5, 4)$ grid.

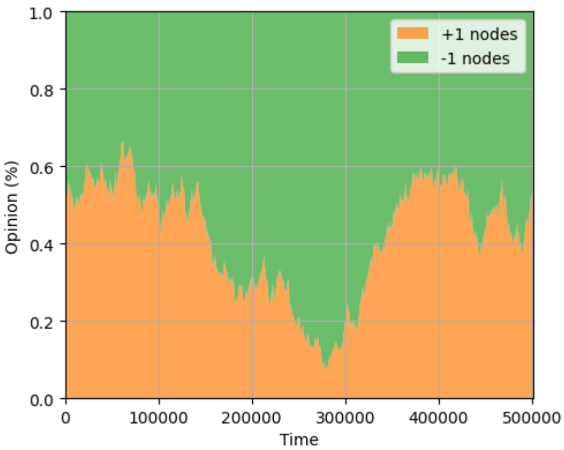

Fig. 9: Population opinion along time for $(10, 10, 10)$ grid.

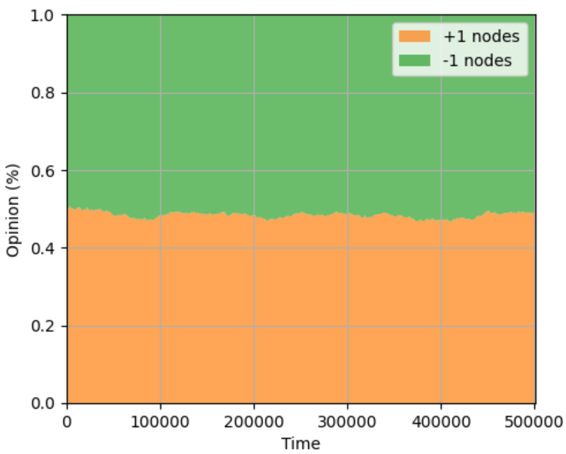

Fig. 10: Population opinion along time for $(21, 21, 22)$ grid.