Generation of Random Variables

LAB L3 Report

Alessandro Masala
DAUIN
Politecnico di Torino
Torino, Italy
s305206@studenti.polito.it

***Abstract*—This paper briefly describe the solution for the third lab of the course, particularly for the *Generation of Random Variables* problem.**

I. Introduction

This report's objective are to provide an explanation and justification of the techniques and algorithms utilized to create random variables from different probability distributions and to present the results of the tests that were carried out. Five distinct probability distributions have been examined:

- *Rayleigh*
- *Lognormal*
- *Beta*
- *Chi-Square*
- *Rice*

II. Methods and algorithms

A. Random Variable Generation

For each distribution, the code creates random variables both using NumPy and inverse transform technique. Utilizing those methods each distribution has separate functions:

- *generate_rayleigh* and *generate_rayleigh_inverse*
- *generate_lognormal* and *generate_lognormal_inverse*
- *generate_beta* and *generate_beta_inverse*
- *generate_chi_square* and *generate_chi_square_inverse*
- *generate_rice* and *generate_rice_inverse*

B. Empirical Moments

For every set of generated data the code computes the empirical moments, that are the sample variance and sample mean.

C. Analytical Moments

Based on the known features of each distribution, the code computes its analytical moments (variance and mean). These moments are used to compare the empirical moments to the standard ones.

D. Comparison of Moments

The code contrasts the empirical moments with their analytical counterparts in order to assess the random number generation's accuracy. Moments that are near to the analytical predictions should be produced by a successful random number generator.

E. Comparing Probability Density Funcitons (PDFs)

The code compares the empirical and analytical Probability Density Functions (PDFs) for the Rayleigh distribution. The code evaluates how well the generated random variables replicate the distribution's shape with this visual comparison.

III. Results

For every distribution, the report presents the empirical moments and contrasts them with their analytical counterparts. The tests were carried out using both the functions with NumPy and the functions with the inverse transform. However, both methods returned the same empirical moments.The results for each distribution are shown in Table I.

TABLE I
Tested parameters

Distribution	N°Samples	Empirical moments	Analytical moments
Rayleigh	100	mean=2.4 var=1.3	mean=2.5 var=1.7
Rayleigh	10000	mean=2.5 var=1.7	mean=2.5 var=1.7
Rayleigh	100000	mean=2.5 var=1.7	mean=2.5 var=1.7
Lognormal	100	mean=2.9 var=1.8	mean=3.0 var=2.7
Lognormal	10000	mean=3.0 var=2.7	mean=3.0 var=2.7
Lognormal	100000	mean=3.0 var=2.7	mean=3.0 var=2.7
Beta	100	mean=0.3 var=0.03	mean=0.3 var=0.03
Beta	10000	mean=0.3 var=0.03	mean=0.3 var=0.03
Beta	100000	mean=0.3 var=0.03	mean=0.3 var=0.03
Chi Squared	100	mean=3.2 var=6.3	mean=3.0 var=6.0
Chi Squared	10000	mean=3.0 var=6.0	mean=3.0 var=6.0
Chi Squared	100000	mean=3.0 var=6.0	mean=3.0 var=6.0
Rice	100	mean=1.2 var=0.4	mean=0.08 var=4.0
Rice	10000	mean=1.2 var=0.4	mean=0.08 var=4.0
Rice	100000	mean=1.2 var=0.4	mean=0.08 var=4.0

Overall, almost all distributions' empirical and analytical moments agree well, according to the results. This is a common observation in statistical analysis: the moments get more accurate as the sample size grows.
An example of a PDFs comparison for the Rayleigh distribution is shown in Figure 1.

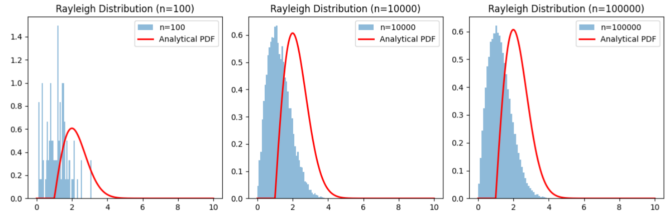

Fig. 1. Rayleigh PDFs for different sample sizes

However, compared to other distributions, the moments of the Rice distribution show greater differences, particularly for the variance. This may be because the distribution is sensitive to parameter values and is complex.