Transient Elimination and Batch Means

Computer-Aided Simulations Lab - Lab L4

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I. Problem overview

This exercise's main goal is to eliminate the transient from the queuing system simulator previously implemented in lab L1. A rule is proposed for removing the initial transient and then a batch means technique is used for computing the confidence intervals when considering a M/G/1 queue. The Pollaczek–Khinchine formula and the Little's Law are evaluated in order to validate the results obtained. Later, the proposed simulator is used to evaluate the average delay and dropping probability for a M/M/1 queuing system.

II. Proposed approach

A. Queuing system simulation

We start from the simulator implemented in Lab L1. As the stop condition, we use a maximum number of arrivals of 5000 clients. The simulation stops once every client has received its service.

B. Warm-up transient removal

A simple rule is proposed for removing the warm-up transient during the run of the simulation, which is removing a certain fraction (e.g. 5%) of the total values collected from the beggining. For example, if we consider 5000 clients in the system, only the delay of the 4750 last clients will be considered in the computations for the average delay.

C. Batch means

The confidence interval is computed based on the batch means technique, using 10 batches. If a given minimum accuracy for the confidence interval can be achieved with less than the 10 batches, this interval is used.

III. Experiments and Results

We consider 3 different service process:

- Deterministic, equal to 1;
- Exponential, with average 1;
- Hyper-exponential with 2 branches, with average 1.

For these service processes and a range of values for the rate of arrival λ_A, the results obtained are shown in Figures 1-3

A. Pollaczek–Khinchine formula

The Pollaczek–Khinchine formula allow us to compare our simulation results with theoretical one. With the formula

$$W = \frac{\rho + \lambda\mu Var(S)}{2(\mu + \lambda)} + \mu^{-1} \tag{1}$$

where W is the theoretical average delay, λ is the arrival rate, μ is the service rate and $Var(S)$ the variance of the service time, we compare the results with the following graphs, where a logarithmic scale is used.

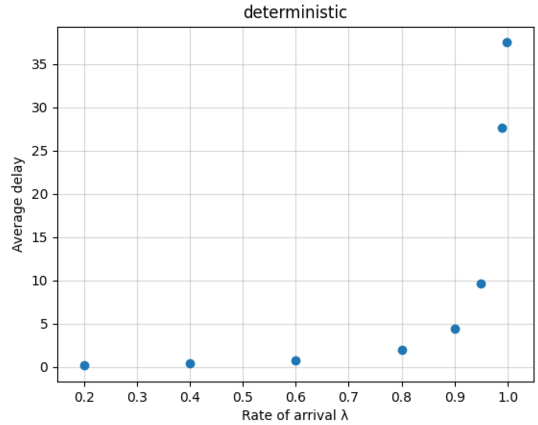

Fig. 1: Average delay for a deterministic service time.

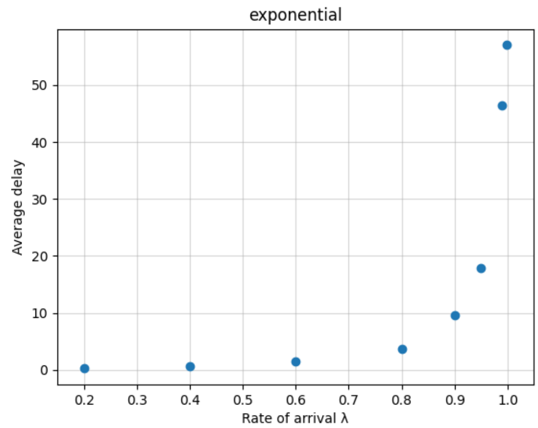

Fig. 2: Average delay for a exponential service time.

We see that the theoretical delay and the simulated one follow similar trends, but the one given by the Pollaczek–Khinchine formula is slightly higher, even for smaller arrival rates.

B. Little's Law

Next, we compare the average number of users in the simulation (given by the total number of users divided by the total time of each simulation) with the expected value given by Little's Law:

$$L = \lambda \cdot W \tag{2}$$

For each of the considered service process, we plot the difference between the average number of clients in the simulation and the one given by Little's law, showed in Figure 7, in which we see again that the values are close (difference

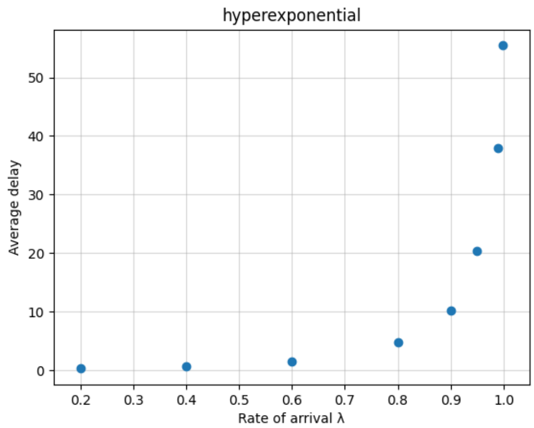

Fig. 3: Average delay for a hyper-exponential service time.

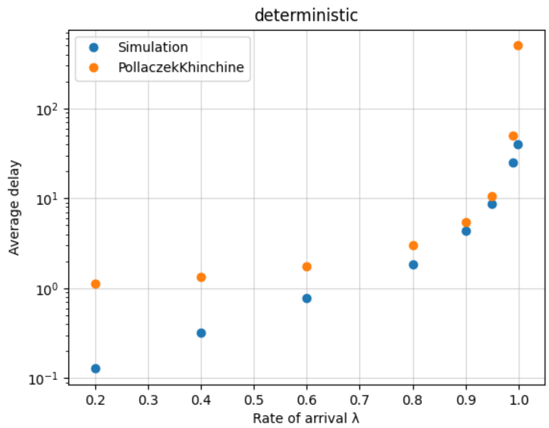

Fig. 4: Comparison between simulated and theoretical average delay for deterministic service time.

tends to zero) for smaller rates λ_A, but as λ_A increases, the difference also increases.

C. M/M/1 Queue simulation

Finally, we apply the implemented simulation for a queuing system, where the arrival of clients is a Poison process and the service time follows an exponential distribution with mean 1. We evaluate the average delay of clients' service and their dropping probability, considering a queue with finite length, respectively on Figures 8 and 9

Although both measures behave as expected, we may want to have a further look on these measures, specially the dropping probability, since there were no drops on the considered simulations. For example, we can decrease the queue length from $N = 1000$ to $N = 100$, to which we obtain the graph reproduced by Figure 11. This time, the delays keeps his previous trend, but the dropping plays a special role with this setup of parameters, since, as the arrival rate increases up to 1.2, the dropping probability reaches 32%.

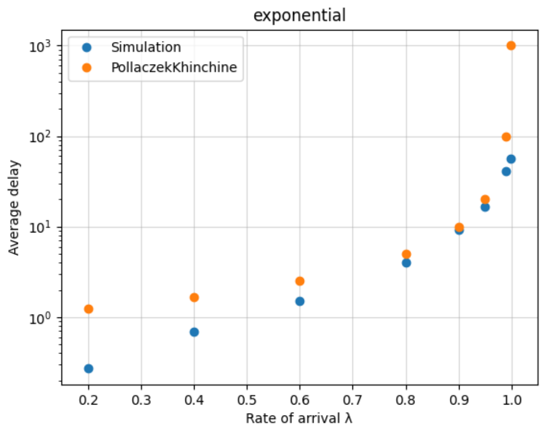

Fig. 5: Comparison between simulated and theoretical average delay for deterministic service time.

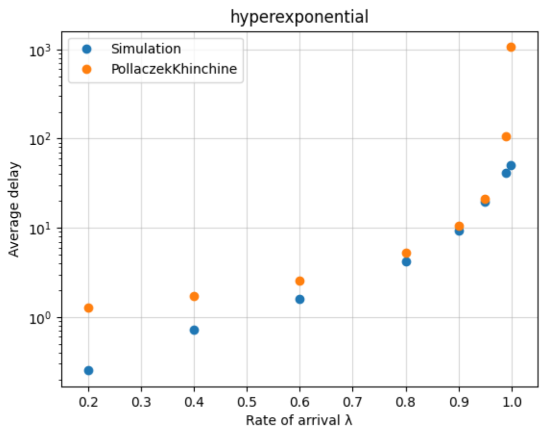

Fig. 6: Comparison between simulated and theoretical average delay for deterministic service time.

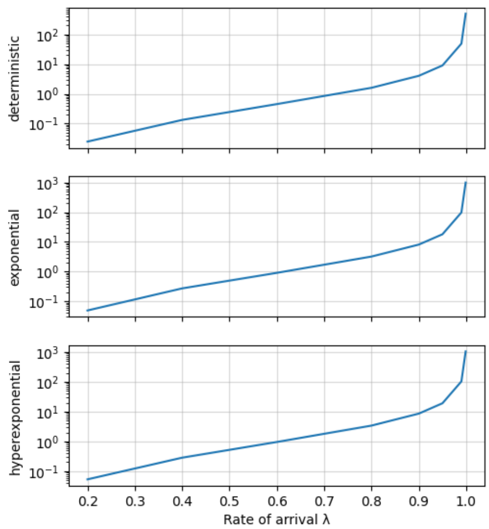

Fig. 7: Difference between theoretical and simulated average number of users on the system.

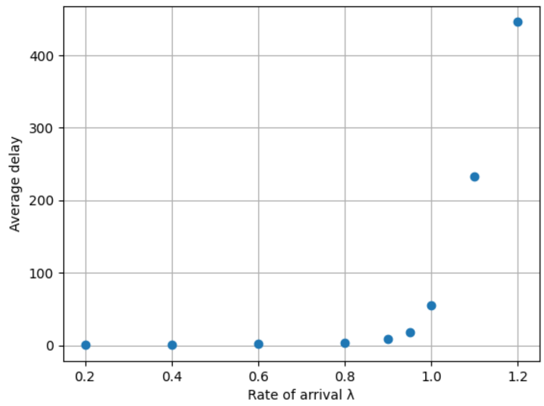

Fig. 8: Average delay of clients' service for service rate $\mu = 1$ and queue length $N = 1000$.

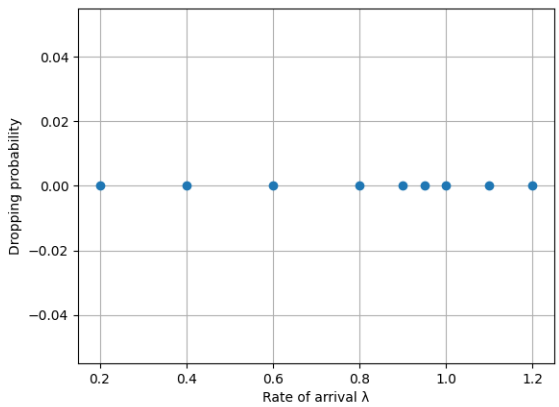

Fig. 9: Dropping probability for service rate $\mu = 1$ and queue length $N = 1000$.

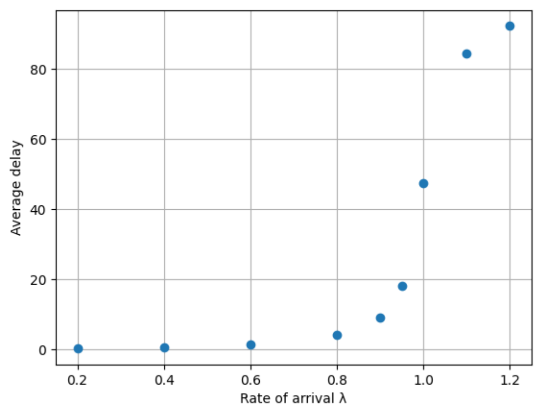

Fig. 10: Average delay of clients' service for service rate $\mu = 1$ and queue length $N = 100$.

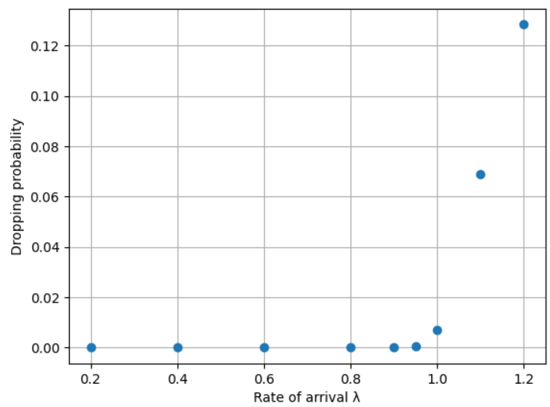

Fig. 11: Dropping probability for service rate $\mu = 1$ and queue length $N = 100$.