Epidemic processes

Computer-Aided Simulations Lab - Lab L7

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I. Problem overview

The goal of the present activity is simulate an epidemic (SIR) process, where we define simple strategies to control it through non pharmaceutical interventions.

We consider a homogeneous population of 50M individuals and fix the following parameters:

- Initial reproduction number: $R(0) = 4$;
- Recovering rate: $\gamma = 1/14$ (14 days);
- Hospitalization (H) rate: 10%;
- Intensive treatment (IT) rate: 6%;
- Fatality rate: 3%;

The non pharmaceutical interventions objective is to avoid H and IT overloading its maximum capacity (10k and 5k respectively) and to keep the number of deaths under a yearly threshold of 100k.

II. Proposed approach

A. Non pharmaceutical intervention

The non pharmaceutical intervention proposed consists in limiting individuals mobility when a certain level of occupancy of hospital's beds and intensive treatment units is reached. This restriction is represented by $\rho(t)$ and is kept for at least a predetermined period or until a higher level of restriction is required.

We consider 4 levels of restriction:

0) No restriction ($\rho(t) = 1.0$): H and IT occupancy is lower than 50%. There is no time limit for this level.
1) Moderate restriction ($\rho(t) = 0.5$): H and IT occupancy is between 50% and 70%. This restriction is kept for at least 21 days (3 weeks).
2) High restriction ($\rho(t) = 0.3$): H and IT occupancy is between 70% and 90%. This restriction is kept for at least 14 days (2 weeks).
3) Very high restriction ($\rho(t) = 0.1$): H and IT occupancy is greater than 90%. This restriction is kept for at least 7 days (1 week).

Now, we can develop a simulator modeling the proposed intervention.

B. Stochastic SIR

The first simulator implemented is based on stochastic events, where a Future Event Set is used to perform scheduled events in order. The events defined are:

1) `infection`:

A new individual gets infected based on a Poison process with rate:

$$\lambda_{SI}(t) = \lambda \cdot \rho(t) \cdot S(t) \cdot I(t)$$

where λ is obtained from:

$$\lambda = \frac{R(0) \cdot \gamma}{S(0)} = \frac{R(0) \cdot \gamma}{N - I(0)} \approx \frac{R(0) \cdot \gamma}{N} \tag{1}$$

2) `recovery`:

An individual recovers based on a Poison process with rate:

$$\gamma_{IR}(t) = \gamma \cdot I(t)$$

At recovery time, we randomly select an individual, check if it was hospitalized (H) or at intensive treatment (IT) based on the hospital occupancy and if it dies (D) based on the fatality rate.

C. Mean field SIR

Next, we develop the same simulator, however this time applying a mean field model, where the average dynamics of the system are described by the following ODEs:

$$\begin{aligned} dS(t) &= -\lambda \cdot \rho(t) \cdot S(t) \cdot I(t) \cdot dt \\ dI(t) &= (\lambda \cdot \rho(t) \cdot S(t) - \gamma) \cdot I(t) \cdot dt \\ dR(t) &= \gamma \cdot I(t) \cdot dt \end{aligned}$$

The other tracked variables (hospitalization H, intensive treatment IT and deaths D) are taken based on the respective rates:

$$\begin{aligned} dH(t) &= dI(t) \cdot 10\% \\ dIT(t) &= dI(t) \cdot 6\% \\ dD(t) &= R(t) \cdot 3\% \end{aligned}$$

At each instant $t + dt$, we update the variables as $S(t + dt) = S(t) + dS(t)$ (the same for the other variables).

III. Experiments and Results

A. Models comparison

With both developed simulators, it is possible to compare their results. In addition to the previously mentioned input parameters, we also consider the initial number of infected people $I(0) = 10$ and a period of one year (365 days). The outputs are displayed in Figures 1 and 2.

From the figures, we can see how the mean field strategy is a good approximation for the SIR epidemic model, since both simulations produce similar outputs. At a first moment, the growth on the number of infected people is exponential. Around the fifth week ($t = 35$), the first intervention is made, and ρ is set to 0.5. This does not cause a significant reduction on the infection rate, as the second level of restriction ($\rho = 0.3$) is required one week later ($t = 42$). The infection

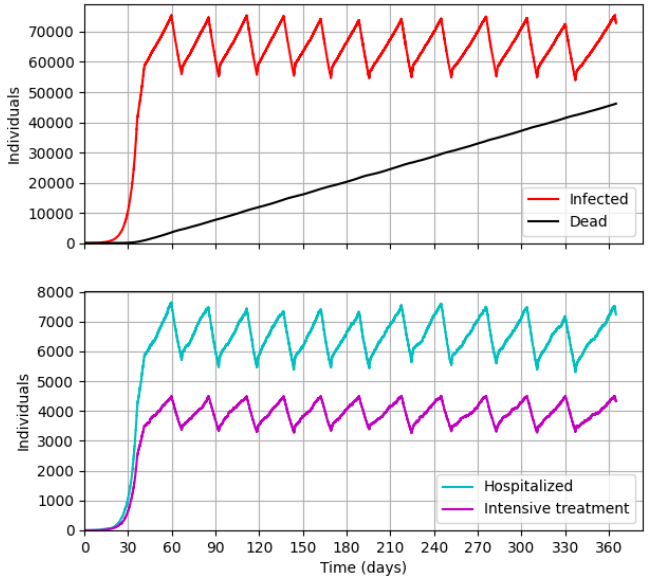

Fig. 1: Simulation A with stochastic events.

continues to spread but now in a lower rate. At the end of the second month ($t = 60$), the highest level of restriction ($\rho = 0.1$) is implemented and the number of infected people finally reduces, together with H and IT. After one week, the restriction level is set again to the first level ($\rho = 0.5$), which rapidly changes to the second ($\rho = 0.3$) and the number of infected people only reduces when the third level ($\rho = 0.1$) is re-implemented. This cycle repeats until the end of the year.

The period of the cycles increases along time. This is due to the fact that the infection rate λ_{SI} is proportional to the number of susceptible people, which decreases as people become infected and recovered. Therefore, it takes longer for the last level of restrictions to be required.

The main advantage when using the mean field strategy is the time saved during each run. While on the stochastic model, we visit every event of infection and recovery, with the mean field model, we approximate the values of the variables at instant t. For example, on this standard simulation, the stochastic model would take around a half-minute to obtain almost the same results as the mean field model obtained in a half-second. Another advantage is the easier implementation of the mean field model, since we do not need to define events and the future event set strategy.

However, with the figures, we see how both simulators were able to respect the systems constraints. The number of hospitalizations and intensive treatments required never overloaded the available places and we manage to keep the deaths after one year under 100k.

Finally, we see how the previous results agrees with the theoretical results, i.e.:

$$\text{if } \frac{\lambda N \rho}{\gamma} > 1 \implies \text{exponential growth}$$
$$\text{if } \frac{\lambda N \rho}{\gamma} < 1 \implies \text{exponential decrease}$$
$$\text{if } \frac{\lambda N \rho}{\gamma} = 1 \implies \text{constant behaviour}$$

From Eq. 1, we have that the value to be evaluated is:

$$\frac{\lambda N \rho}{\gamma} = R(0)\rho = 4\rho$$

That means that for $\rho = 0.25$, we have a constant behaviour, i.e. the infection rate and the recovery rate are equal. For values greater than 0.25, as on restriction levels 0, 1 and 2, the epidemic continues to spread exponentially. As for restriction level 3, the number of infected people decreases exponentially. In fact, as we set $\rho = 0.3$ and $\rho = 0.1$, we observe on Figures 1 and 2, that the number of infected remains almost constant between 55k and 75k, as does the number of hospitalizations and intensive treatments on their own ranges.

B. Reduced population

As a final task, we analyse our simulator performance when considering a much smaller simulation: $N = 10k$. The

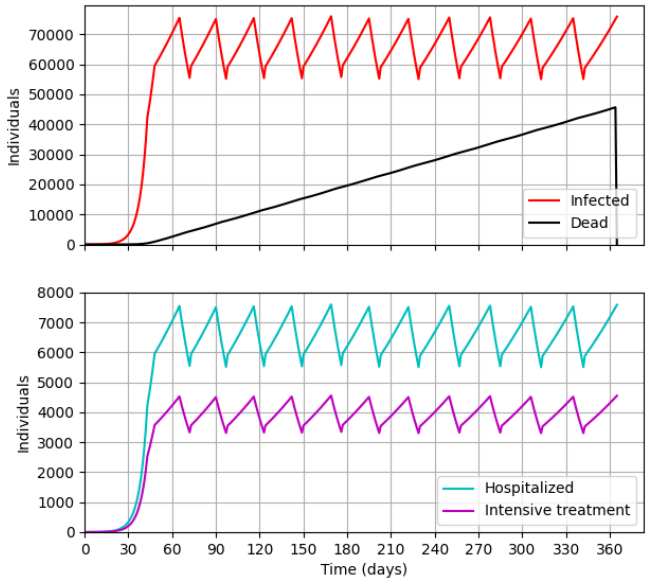

Fig. 2: Simulation A with mean field strategy.

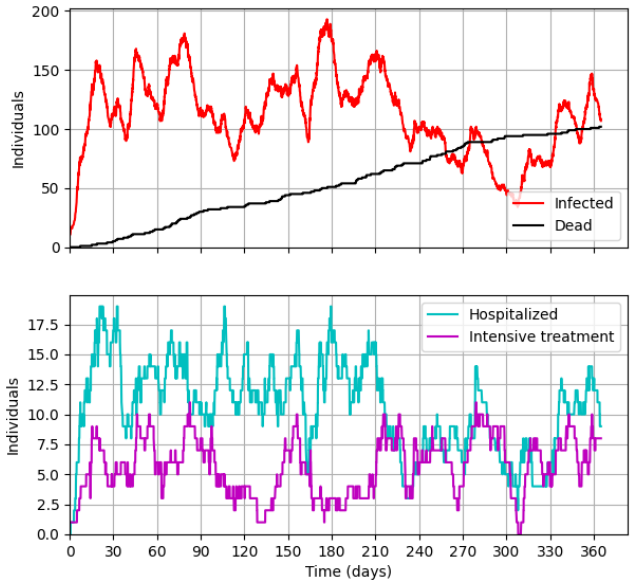

Fig. 3: Simulation B with stochastic events.

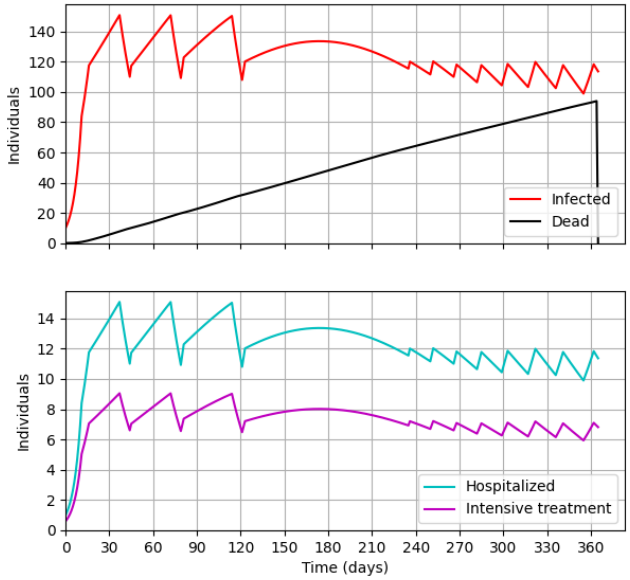

Fig. 4: Simulation B with mean field strategy.

number of H and IT places are also reduced to 20 and 10, respectively. The results are shown in Figures 3 and 4.

Some aspects are the same as on the previous case. The number of deaths grows linearly. On the mean field model, the initial growth is exponential, and then 3 cycles occur during the first 4 months.

As for the stochastic model, the cycles of growth and reduction on the number of infections is still identifiable, but in a less regular manner.

Back in the mean field model, as said before, the period of the cycles increases because with less susceptible individuals, it takes longer for the highest restriction to be implemented. In fact, after the sixth month, the number of infections starts to decrease even on the second level of restriction ($\rho = 0.3$), and after the eighth month a cycle begins, with an average number of infections lower than the previous, as it went from 130 to 110.