Project : Sequential Uncertainty Reduction and Value of Information in Gaussian Process Models

Mina, Henrik

November 2019

Task a)

The line that maximizes the reduction in variance will be the middle line, line 13. The resulting mean variance after surveying each of the 25 lines in plotted in Figure 1. As expected, this is symmetric about the middle line. Also, surveying the line in the middle gives the most possible information about the whole forest, because the covariance is assumed symmetric about this point.

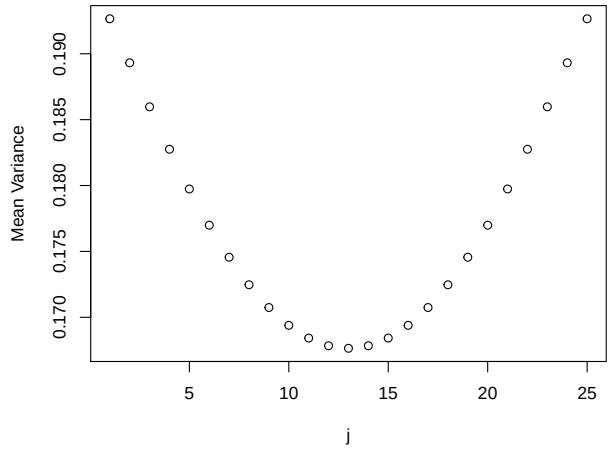


Figure 1: Mean variance for each of the possible j survey lines.

Conducting the sequential selection of the lines gives the following surveying order of lines: $\{13, 9, 18, 6, 20, 15, 5, 21, 11, 7\}$. As the covariance matrix is symmetric, line 9 and line 17 give the same reduction in mean variance at survey number 2. We then selected line 9, but could also have selected line 17. In theory, this should not influence the selection of lines, and selecting line 17 would give a selection that was mirrored around line 13 compared to selecting line 9.

Task b)

As seen in Figure 2, the North-South line to select first that gives the most profit is the one in the middle, line number 13. This is a result of the assumed symmetry in the forest, so observing the line in the middle gives the most information.

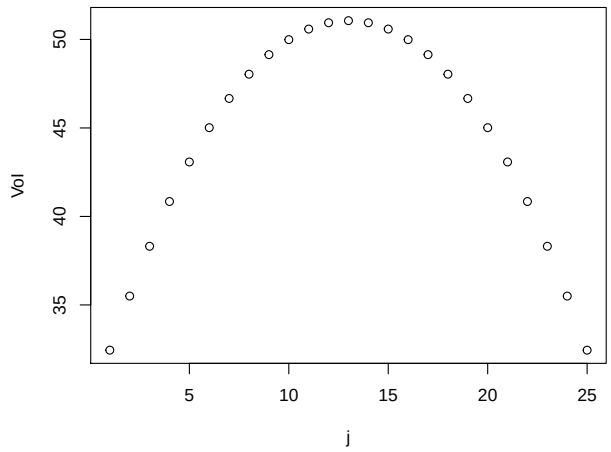


Figure 2: Value of Information for each of the possible j survey lines.

We then decide to play a game by generating data at the selected lines and gather data until the value of more sequential information is less than a price of $P = 0.5$. We repeated this for 200 replicate trials. Figure 3 shows a histogram of how many lines were inspected at each run. Note that for each run, the survey sequence is equal and the same as in task a). The only difference between runs is how many surveys of this sequence are conducted.

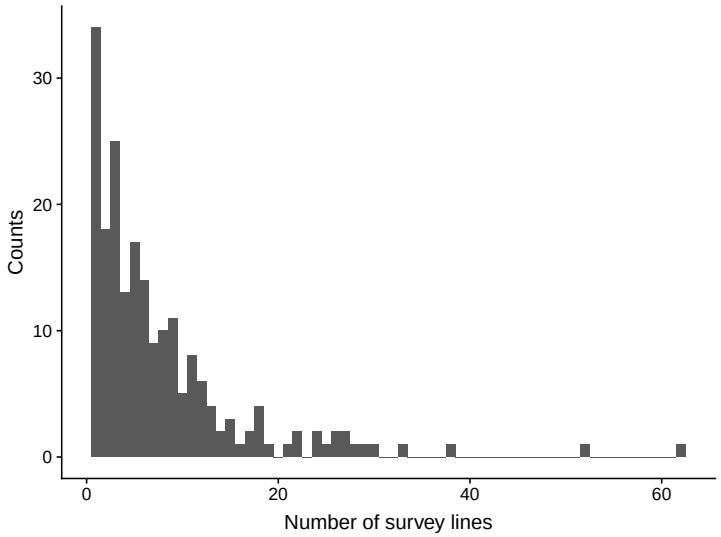
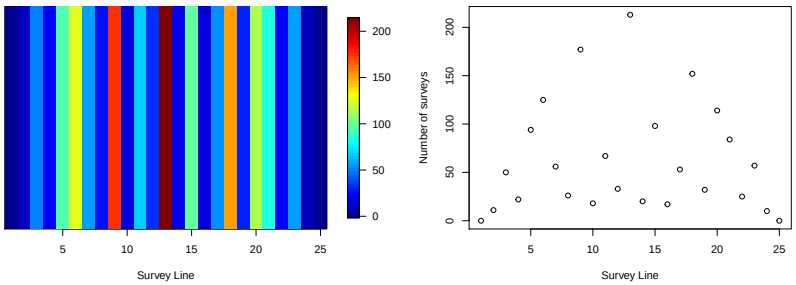


Figure 3: Histogram of number of lines selected at each of the trials.

Figure 4 shows an image plot and a scatter plot of the number of times each line is selected during the 200 trials. It is worth noting that line 13 is chosen at least once but at most 2 times in a single trial. In contrast, lines 2 and 24 are seldom picked, except for 2 single runs where they were selected 3 and 4 times each. In total, they were selected 11 times, meaning that only 4 other runs selected these lines. As in previous task, because of assumed symmetry, line 9 is chosen instead of line 17. This can be seen in Figure 4a, as line 9 has been second most surveyed after line 13.



(a) Image plot of number of times each line has been selected during the 200 trials.

(b) Scatter plot of number of times each line has been selected during the 200 trials.

Figure 4

Same conclusions as above can be drawn looking at Figure 5, that shows which survey lines have been chosen at each of the trials. One dot represents that survey line j was chosen in run i.

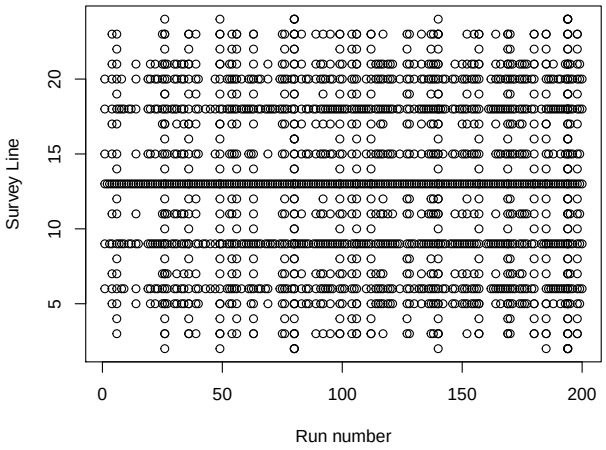


Figure 5: Lines selected during each of the 200 trials.