

Problem Sheet 8

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Problem 8.1

Minimize $\varphi(A, B, C, D, E) = m_0 + m_2 + m_4 + m_6 + m_9 + m_{10} + m_{13} + m_{14} + m_{15} + m_{16} + m_{17} + m_{21} + m_{26} + m_{28} + m_{30} + m_{31}$.

a) Calculate prime implicants of φ :

Since the expression defining φ is in DNF, then every minterm of DNF is an implicant of φ .
 Classify and sort minterms based on the number of positive literals they contain:

minterm	pattern	used	minterms	pattern	used	minterms	pattern	used
m_0	00000	✓	$m_{0,2}$	000-0	✓	$m_{0,2,4,6}$	00--0	
			$m_{0,4}$	00-00	✓	$m_{0,4,2,6}$	00--0	
			$m_{0,16}$	-0000				
m_2	00010	✓	$m_{2,6}$	00-10	✓	$m_{2,6,10,14}$	0--10	
			$m_{2,10}$	0-010	✓	$m_{2,10,6,14}$	0--10	
m_4	00100	✓	$m_{4,6}$	001-0	✓			
m_{16}	10000	✓	$m_{16,17}$	1000-				
m_6	00110	✓	$m_{6,14}$	0-110	✓			
m_9	01001	✓	$m_{9,13}$	01-01				
m_{10}	01010	✓	$m_{10,14}$	01-10	✓	$m_{10,14,26,30}$	-1-10	
			$m_{10,26}$	-1010	✓	$m_{10,26,14,30}$	-1-10	
m_{17}	10001	✓	$m_{17,21}$	10-01				
m_{13}	01101	✓	$m_{13,15}$	011-1				
m_{14}	01110	✓	$m_{14,15}$	0111-	✓	$m_{14,15,30,31}$	-111-	
			$m_{14,30}$	-1110	✓	$m_{14,30,15,31}$	-111-	
m_{21}	10101	✓						
m_{26}	11010	✓	$m_{26,30}$	11-10	✓			
m_{28}	11100	✓	$m_{28,30}$	111-0				
m_{15}	01111	✓	$m_{15,31}$	-1111	✓			
m_{30}	11110	✓	$m_{30,31}$	1111-	✓			
m_{31}	11111	✓						

$m_{0,2,4,6}$ and $m_{0,4,2,6}$ contain the same expressions, therefore we only use one of those in the second step of the algorithm. The same case stands for $m_{2,6,10,14}$ and $m_{2,10,6,14}$, $m_{10,14,26,30}$ and $m_{10,26,14,30}$, $m_{14,15,30,31}$ and $m_{14,30,15,31}$.

Prime Implicants of φ :

- $m_{0,2,4,6} = 00--0 = \neg A \wedge \neg B \wedge \neg E$
- $m_{2,6,10,14} = 0--10 = \neg A \wedge D \wedge \neg E$
- $m_{10,14,26,30} = -1-10 = B \wedge D \wedge \neg E$
- $m_{14,15,30,31} = -111- = B \wedge C \wedge D$
- $m_{0,16} = -0000 = \neg B \wedge \neg C \wedge \neg D \wedge \neg E$
- $m_{9,13} = 01-01 = \neg A \wedge B \wedge \neg D \wedge E$
- $m_{13,15} = 011-1 = \neg A \wedge B \wedge C \wedge E$
- $m_{16,17} = 1000- = A \wedge \neg B \wedge \neg C \wedge \neg D$
- $m_{17,21} = 10-01 = A \wedge \neg B \wedge \neg D \wedge E$
- $m_{28,30} = 111-0 = A \wedge B \wedge C \wedge \neg E$

b) Prime Implicant Chart:

	m₀	m₂	m₄	m₆	m₉	m₁₀	m₁₃	m₁₄	m₁₅	m₁₆	m₁₇	m₂₁	m₂₆	m₂₈	m₃₀	m₃₁
$m_{0,2,4,6}$	✓	✓	✓	✓												
$m_{2,6,10,14}$		✓		✓		✓		✓								
$m_{10,14,26,30}$						✓		✓					✓		✓	
$m_{14,15,30,31}$								✓	✓						✓	✓
$m_{0,16}$	✓									✓						
$m_{9,13}$					✓		✓									
$m_{13,15}$							✓		✓							
$m_{16,17}$										✓	✓					
$m_{17,21}$											✓	✓				
$m_{28,30}$														✓	✓	
E.P.I.C*	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓

Table 1: E.P.I.C stands for Essential Prime Implicant Coverage - the coverage of the minterms by the essential prime implicants found below

From the table, we can see that the essential prime implicants are:

- $m_{0,2,4,6} = 00--0 = \neg A \wedge \neg B \wedge \neg E$ (only one that covers m_4)
- $m_{10,14,26,30} = -1-10 = B \wedge D \wedge \neg E$ (only one that covers m_{26})
- $m_{14,15,30,31} = -111- = B \wedge C \wedge D$ (only one that covers m_{31})
- $m_{9,13} = 01-01 = \neg A \wedge B \wedge \neg D \wedge E$ (only one that covers m_9)
- $m_{17,21} = 10-01 = A \wedge \neg B \wedge \neg D \wedge E$ (only one that covers m_{21})
- $m_{28,30} = 111-0 = A \wedge B \wedge C \wedge \neg E$ (only one that covers m_{28})

Minterm m_{16} is the only one that isn't covered by the essential prime implicants found above. Therefore we can choose the implicant $m_{0,16}$ or $m_{16,17}$ to cover it in our resulting minimal expression.

c) The resulting minimal expressions defining $\varphi(A, B, C, D, E)$:

- $(\neg A \wedge \neg B \wedge \neg E) \vee (B \wedge D \wedge \neg E) \vee (B \wedge C \wedge D) \vee (\neg A \wedge B \wedge \neg D \wedge E) \vee (A \wedge \neg B \wedge \neg D \wedge E) \vee (A \wedge B \wedge C \wedge \neg E) \vee (\neg B \wedge \neg C \wedge \neg D \wedge \neg E)$
- $(\neg A \wedge \neg B \wedge \neg E) \vee (B \wedge D \wedge \neg E) \vee (B \wedge C \wedge D) \vee (\neg A \wedge B \wedge \neg D \wedge E) \vee (A \wedge \neg B \wedge \neg D \wedge E) \vee (A \wedge B \wedge C \wedge \neg E) \vee (A \wedge \neg B \wedge \neg C \wedge \neg D)$

The cost of our minimal expressions is 24, while the original expression's cost is 63 ($16 \cdot 3 + 15$).