CHAPTER 7 HOMEWORK

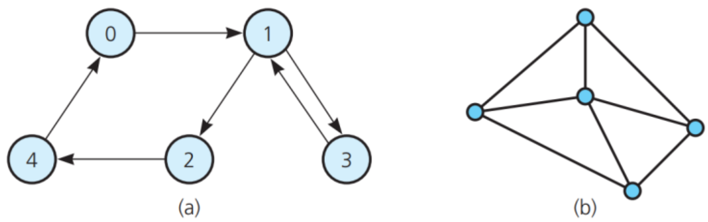

Fig 1. Graph $G1$ – used for question 1, 2, and 3

Problem 1. Describe the graph (a) and (b) in Fig 1. For example: is it undirected? Directed? Weighted? Complete? What are the orders of each vertex?

(a) G1 is a directed graph, the orders of each vertex are as follows:
deg+(0) = 1, deg-(0)=1
deg+(1) = 2, deg-(1)=2
deg+(2) = 1, deg-(2)=1
deg+(3) = 1, deg-(3)=1
deg+(4) = 1, deg-(4)=1

(b) G2 is a simple (undirected) completed graph K_5, the order of the center vertex if 4, the other vertices have order 3.

Problem 2. Use the DFS strategy and BFS strategy to traverse the graph (a) in Fig 1, begins with vertex 0. List the vertices in order in which each traversal visits them.

- DFS: 0,1,2,4,3
- BFS: 0,1,4,2,3

Problem 3. Write the adjacency matrix for the graph (a) in Fig 1.

	0	1	2	3	4
0	0	1	0	0	0
1	0	0	1	1	0
2	0	0	0	0	1
3	0	1	0	0	0
4	1	0	0	0	0

Problem 4. Add an edge to the directed graph in Fig 2 that runs from vertex d to vertex b. Write all possible topological orders for the vertices in this new graph.

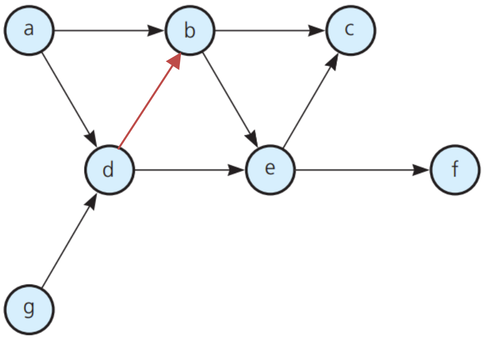

Fig 2. Graph $G2$

- *a, g, d, b, e, c, f*
- *a, g, d, b, e, f, c*
- *g, a, d, b, e, c, f*
- *g, a, d, b, e, f, c*

Problem 5. Give the adjacency matrix and adjacency list for the graph in Fig 3.

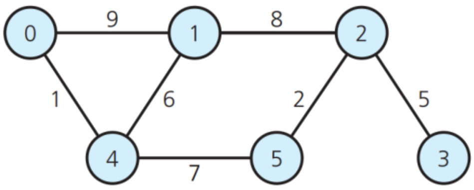

Fig 3. Graph $G3$

	0	1	2	3	4	5
0	∞	9	∞	∞	1	∞
1	9	∞	8	∞	6	∞
2	∞	8	∞	5	∞	2
3	∞	∞	5	∞	∞	∞
4	1	6	∞	∞	∞	7
5	∞	∞	2	∞	7	∞

Adjacency Matrix

Adjacency List	
0	1:9, 4:1
1	2:8, 4:6
2	1:9, 3:5, 5:2
3	2:5
4	0:1, 1:6, 5:7
5	2:2, 4:7

Problem 6. Is it possible for a connected undirected graph with five vertices and four edges to contain a simple cycle? Explain your answer.

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No, it is impossible for a connect undirected graph with 5 vertices and 4 edges to contain a cycle.

Explain: A connect undirected graph with n vertices would have at least n-1 edges (which construct its spanning tree). Therefore, a connected undirected graph with 5 vertices and 4 edges is a tree; hence, it cannot have a cycle.

Problem 7. Give the adjacency matrix and adjacency list for the following graph.

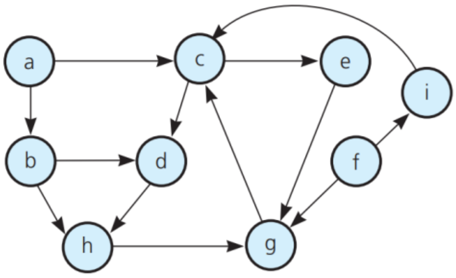

Fig 4. Graph *G4*

Problem 8. Modify the DFS traversal algorithm, write pseudocode for an algorithm that determines whether a graph conatins a cycle.

Idea: Use a visited array to keep track of the nodes that have been visited. As we traverse the graph, if we encounter a node that has already been visited, then there is a cycle in the graph.

Problem 9. Using the Topological Sorting algorithm as given in the slide (TOPOLOGICAL-SORT), write the topological order of the vertices for each graph in Fig 5.

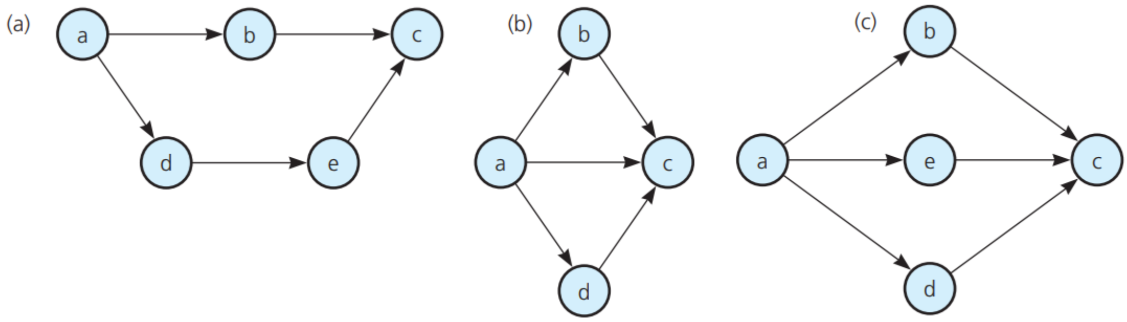

Fig 5. Graphs G*5* (a), (b), (c)

a. a, b, d, e, c
b. a, b, d, c
c. a, b, d, e, c

Problem 10. Revise the topological sorting algorithm as given in the slide (TOPOLOGICAL-SORT) by removing predecessors instead of successors. Trace the new algorithm for each graph in Fig 5.

a. a, b, d, e, c
b. a, b, d, c
c. a, b, d, e, c

Problem 11. Draw the DFS and BFS spanning trees rooted at a for the graph in Fig 6. Then, draw the minimum spanning tree rooted at a for this graph.

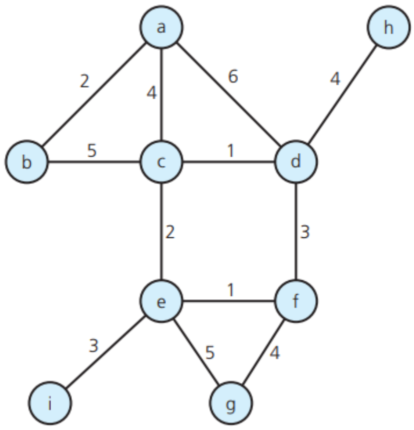

Fig 6. Graph $G6$

DFS spanning tree rooted at a: $(a,b),(b,c),(c,d),(d,f),(f,e),(e,g),(e,i),(d,h)$

BFS spanning tree rooted at a: $(a,b),(a,c),(a,d),(c,e),(d,f),(d,h),(e,g),(e,i)$

Minimum spanning tree rooted at a: $(a,b),(a,c),(c,d),(c,e),(e,f),(e,i),(d,j),(f,g)$

Problem 12. For the graph in Fig 7,

a. Draw all the possible spanning trees.
b. Draw the minimum spanning tree.

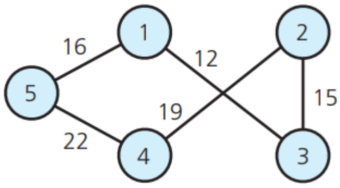

Fig 7. Graph $G7$

Problem 13. Draw the minimum spanning tree for the graph in Fig 8 when you start with

a. Vertex g
b. Vertex c

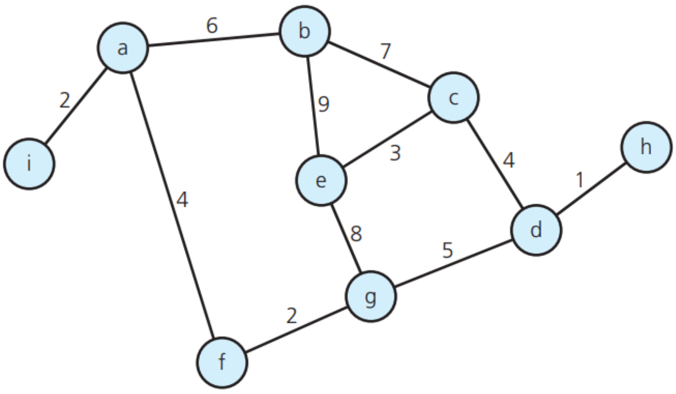

Fig 8. Graph $G8$

Problem 14. Trace the shortest-path algorithm (Dijkstra) for the graph in Fig 9, letting vertex 0 be the origin.

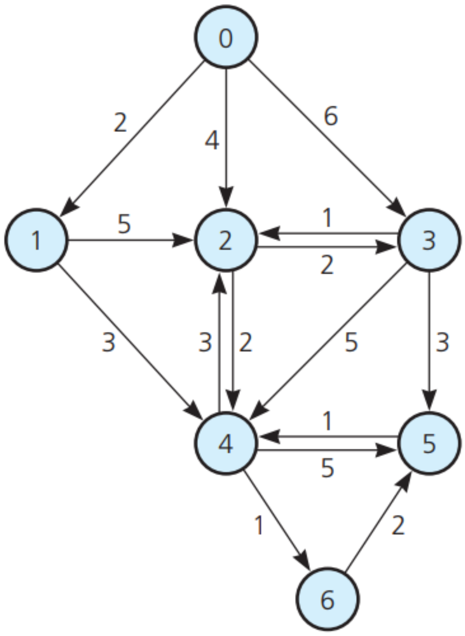

Fig 9. Graph $G9$

Problem 15. How can you modify Dijkstra algorithm so that any vertex can be the origin?

Problem 16. Show that Dijkstra's algorithm fails in graphs with negative edge weights. (Give an example and explain on your graph)