Hash Table

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Hashing

- Binary search tree retrieval have order $O(\log_2 n)$
- Need a different strategy to locate an item
- Consider a "magic box" as an address calculator
  - Place/retrieve item from that address in an array
  - Ideally to a unique number for each key

Hashing

- Hashing is a technique to convert a range of key values into a range of indexes of an array.
- *Large* keys are converted into *small* keys by using **hash functions**.
- The values are then stored in a data structure called **hash table**.

Hashing

- Idea:
  - Distribute entries (key/value pairs) uniformly across an array.
  - Each element is assigned a key (converted key).
  - Using that key to access the element in O(1) time. (The hash function computes an index suggesting where an entry can be found or inserted.)

Hashing

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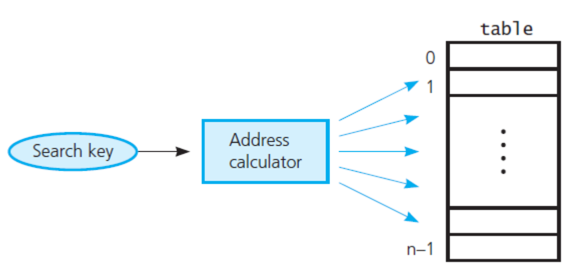

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Hash Table

- A hash table is a data structure that is used to store keys/value pairs.
- It uses a hash function to compute an index into an array in which an element will be inserted or searched.

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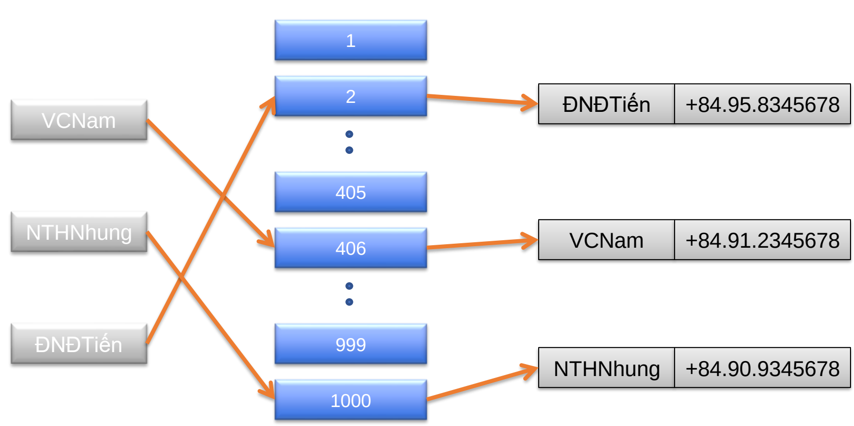
Example



Hash Function

- Hash function is a mathematical function that can be used to map/converts a key to an integer value (an array index).
- The values returned by a hash function
  - hash values
  - hash codes
  - hash sums
  - hashes.

Some Hash Functions

- Possible functions
  - Selecting digits
  - Folding
  - Modulo arithmetic
  - Converting a character string to an integer
    - Use ASCII values
    - Factor the results, Horner's rule

Some Hash Functions

- Digit-selection:
  - Select some digits in the keys to create the hash value.
  - h(001**3**6482**5**) = 35
- Folding
  - h(001364825) = 0 + 0 + 1 + 3 + 6 + 4 + 8 + 2 + 5 = 29
  - h(**001**364**825**) = 001 + 364 + 825 = 1190
- Modulo arithmetic
  - h(Key) = Key mod 101
  - h(001364825) = 12

Some Hash Functions

- A string key hash function

$$h = \sum_{i=0}^{keylength} 128^i \times char(key[i])$$

Good Hash Functions

- Properties of good hash functions

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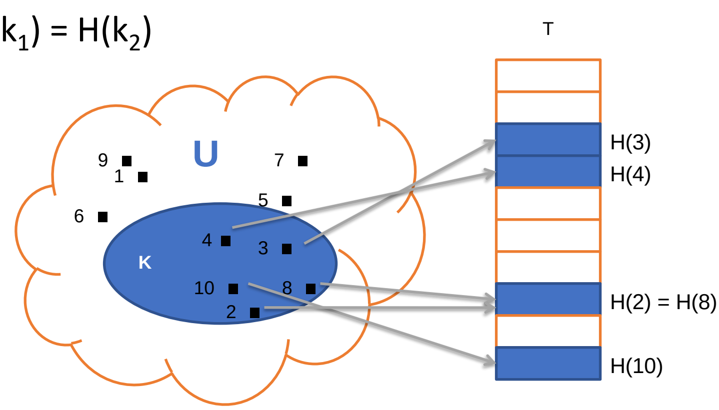
Collisions

- $\exists k_1, k_2 \in K$:

  $k_1 \neq k_2, H(k_1) = H(k_2)$



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Resolving Collisions

Resolving Collisions

- Separate Chaining (open hashing)
- Open Addressing (closed hashing)

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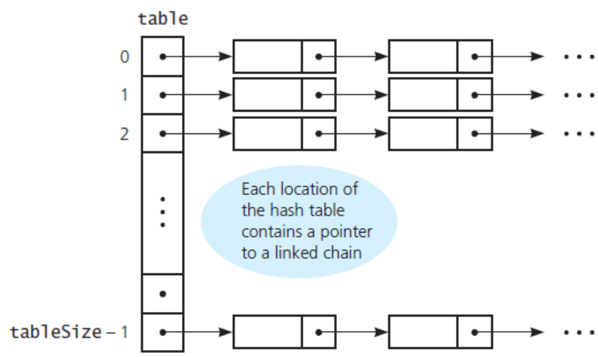
Separate Chaining



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Separate Chaining

- Each hash location can accommodate more than one item
- Each location is a “bucket” or an array itself
- Alternatively, design the hash table as an array of linked chains (“*separate chaining*”).

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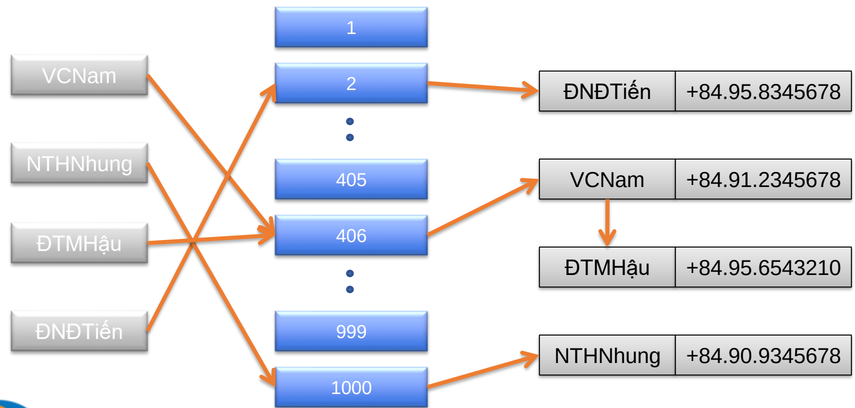
Separate Chaining



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Open Addressing

- Probe for another available location
- Some techniques:
  - Linear probing
  - Quadratic probing
  - Double hashing

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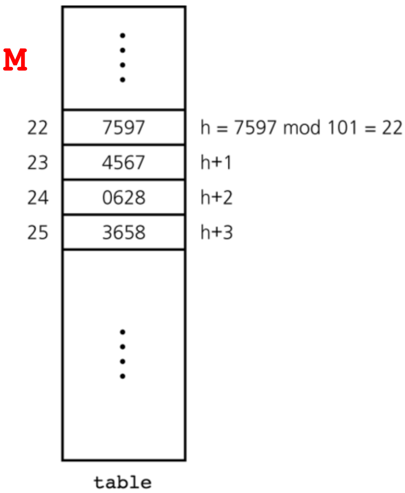
Linear Probing

$$H(k, step) = (h(k) + step) \bmod M$$

- step = 0, 1,…
- M: size of hash table



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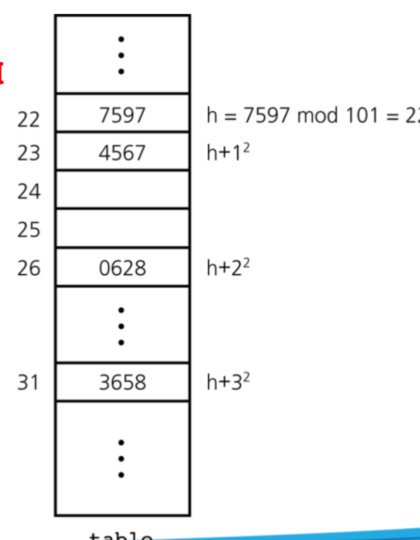
Quadratic Probing

$$H(k, step) = (h(k) + step^2) \bmod M$$

- step = 0, 1,…
- M: size of table



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Double Hashing

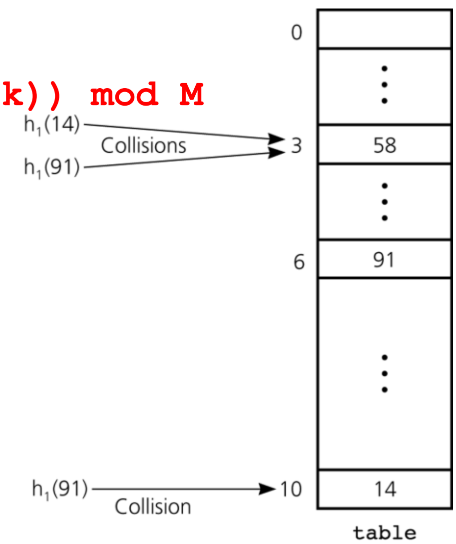
$$H(k, step) = (h(k) + step*h_2(k)) \bmod M$$

step = 0, 1,…

M: size of hash table

$h_2(k)$: second hash function

```
h(key) = key mod 11
h2(key) = 7 - (key mod 7)
```



Double Hashing

- Some functions recommended in the literature:
  - $h_2(Key) = m - 2 - Key \bmod (m - 2)$
  - $h_2(Key) = 8 – (Key \bmod 8)$
  - $h_2(Key) = Key \bmod 97 + 1$

Separate Chaining

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- Advantages:
  - Simple to implement.
  - Hash table never fills up, we can always add more elements to the chain.
  - Less sensitive to the hash function or load factors.
  - It is mostly used when it is unknown how many and how frequently keys may be inserted or deleted.

Separate Chaining

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- Disadvantages:
  - Cache performance of chaining is not good as keys are stored using a linked list. Wastage of space (Some parts of hash table are never used)
  - If the chain becomes long, then search time can become O(n) in the worst case.
  - Uses extra space for links.

Open Addressing

- Removal requires specify state of an item
  - Occupied, emptied, removed
- Clustering is a problem
- Double hashing can reduce clustering

The Efficiency of Hashing

The Efficiency of Hashing

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- Efficiency of hashing involves the load factor alpha (α)

$$\alpha = \frac{\textit{Current number of table items}}{\textit{tableSize}}$$

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The Efficiency of Hashing

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- Linear probing – average value for α

$$\frac{1}{2}\left[1 + \frac{1}{1-\alpha}\right] \quad \text{for a successful search, and}$$

$$\frac{1}{2}\left[1 + \frac{1}{(1-\alpha)^2}\right] \quad \text{for an unsuccessful search}$$

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The Efficiency of Hashing

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- Quadratic probing and double hashing – efficiency for given α

$$\frac{-\log_e(1-\alpha)}{\alpha} \quad \text{for a successful search, and}$$

$$\frac{1}{1-\alpha} \quad \text{for an unsuccessful search}$$

The Efficiency of Hashing

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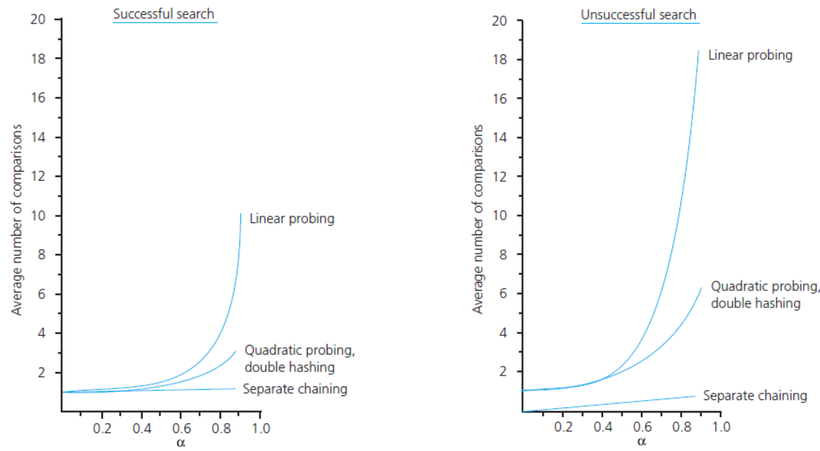
- Separate chaining – efficiency for given α

$$1 + \frac{\alpha}{2} \quad \text{for a successful search, and}$$

$$\alpha \quad \text{for an unsuccessful search}$$

The Efficiency of Hashing



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Maintaining Hashing Performance

- Collisions and their resolution typically cause the load factor α to increase
- To maintain efficiency, restrict the size of α
  - $\alpha \leq 0.5$ for open addressing
  - $\alpha \leq 1.0$ for separate chaining
- If load factor exceeds these limits
  - Increase size of hash table
  - Rehash with new hashing function

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Exercise

Given a hash table with m = 13 entries and the hash function

`h(key) = key mod m`

Insert the keys **`{10, 22, 31, 4, 15, 28, 17, 88, 59}`** in the given order (from left to right) to the hash table. If there is a collision, use each of the following open addressing resolving methods:

A. Linear probing

B. Quadratic probing

C. Double hashing with `h2(key) = (key mod 7) + 1`

Questions and Answers