23120262 - Trần Dương Thái Hòa

Bài 1:

$$(A|B) = \left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 2 & 2 & 1 & 1 \\ 3 & 5 & -2 & -1 \end{array}\right) \begin{matrix} d_2 = d_2 - 2d_1 \\ \longrightarrow \\ d_3 = d_3 - 3d_1 \end{matrix} \left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & -2 & 3 & 3 \\ 0 & -1 & 1 & 2 \end{array}\right)$$

$$\begin{matrix} d_3 = -d_3 \\ \longrightarrow \\ d_3 \leftrightarrow d_2 \end{matrix} \left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & 1 & -1 & -2 \\ 0 & -2 & 3 & 3 \end{array}\right) \xrightarrow{d_3 = d_3 + 2d_2} \left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$$\Rightarrow \begin{cases} x_1 = -1 + x_3 - 2x_2 \\ x_2 = -2 + x_3 \\ x_3 = -1 \end{cases} \Rightarrow \begin{cases} x_1 = 4 \\ x_2 = -3 \\ x_3 = -1 \end{cases}$$

Vậy $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ -1 \end{pmatrix}$

Bài 2:

$$(A|B) = \left(\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ 2 & -3 & 1 & 6 \\ 3 & -5 & 0 & 7 \\ 1 & 0 & 5 & 9 \end{array}\right) \begin{matrix} d_2 - 2d_1 \\ \longrightarrow \\ d_3 - 3d_1 \\ d_4 - d_1 \end{matrix} \left(\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ 0 & 1 & 3 & 4 \\ 0 & 1 & 3 & 4 \\ 0 & 2 & 6 & 8 \end{array}\right)$$

$$\begin{matrix} d_3 - d_2 \\ \longrightarrow \\ d_4 - 2d_2 \end{matrix} \left(\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} x_1 = 1 + x_3 + 2x_2 \\ x_2 = 4 - 3x_3 \\ x_3 = \alpha \in \mathbb{R} \end{cases} \Rightarrow \begin{cases} x_1 = 9 - 5\alpha \\ x_2 = 4 - 3\alpha \\ x_3 = \alpha \in \mathbb{R} \end{cases}$$

Vậy $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 9 \\ 4 \\ 0 \end{pmatrix} + \begin{pmatrix} -5 \\ -3 \\ 1 \end{pmatrix}\alpha, \quad \forall \alpha \in \mathbb{R}$

Bài 3:

$$(A|B) = \left(\begin{array}{cccc|c} 1 & 2 & 0 & 2 & 6 \\ 3 & 5 & -1 & 6 & 17 \\ 2 & 4 & 1 & 2 & 12 \\ 2 & 0 & -7 & 11 & 7 \end{array}\right) \begin{matrix} d_2 - 3d_1 \\ \longrightarrow \\ d_3 - 2d_1 \\ d_4 - 2d_1 \end{matrix} \left(\begin{array}{cccc|c} 1 & 2 & 0 & 2 & 6 \\ 0 & -1 & -1 & 0 & -1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & -4 & -7 & 7 & -5 \end{array}\right)$$

$$\begin{matrix} d_4 + 4d_2 \\ \longrightarrow \\ d_2 = -d_2 \end{matrix} \left(\begin{array}{cccc|c} 1 & 2 & 0 & 2 & 6 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & -3 & 7 & -1 \end{array}\right) \xrightarrow{d_4 + 3d_3} \left(\begin{array}{cccc|c} 1 & 2 & 0 & 2 & 6 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right)$$

$$\to \begin{cases} x_1 = 6 - 2x_2 - 2x_4 \\ x_2 = 1 - x_3 \\ x_3 = 2x_4 \\ x_4 = -1 \end{cases} \Rightarrow \begin{cases} x_1 = 2 \\ x_2 = 3 \\ x_3 = -2 \\ x_4 = -1 \end{cases} \quad \text{Vậy } \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ -2 \\ -1 \end{pmatrix}$$

Bài 4:

$$(A|B) = \left(\begin{array}{ccc|c} 2 & -4 & -1 & 1 \\ 1 & -3 & 1 & 1 \\ 3 & -5 & -3 & 2 \end{array}\right) \xrightarrow[d_2 - 2d_1,\ d_3 - 3d_1]{d_1 \leftrightarrow d_2} \left(\begin{array}{ccc|c} 1 & -3 & 1 & 1 \\ 0 & 2 & -3 & -1 \\ 0 & 4 & -6 & -1 \end{array}\right)$$

$$\xrightarrow{d_3 - 2d_2} \left(\begin{array}{ccc|c} 1 & -3 & 1 & 1 \\ 0 & 2 & -3 & -1 \\ 0 & 0 & 0 & 1 \end{array}\right) \Rightarrow \text{Hệ phương trình vô nghiệm vì } r(A) < r(A|B) \ (2 < 3)$$

Bài 5:

$$(A|B) = \left(\begin{array}{ccc|c} 1 & 2 & -2 & 3 \\ 3 & -1 & 1 & 1 \\ -1 & 5 & -5 & 5 \end{array}\right) \xrightarrow[d_3 + d_1]{d_2 - 3d_1} \left(\begin{array}{ccc|c} 1 & 2 & -2 & 3 \\ 0 & -7 & 7 & -8 \\ 0 & 7 & -7 & 8 \end{array}\right)$$

$$\xrightarrow{d_3 + d_2} \left(\begin{array}{ccc|c} 1 & 2 & -2 & 3 \\ 0 & -7 & 7 & -8 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} x_1 = 3 - 2x_2 + 2x_3 \\ x_2 = \frac{8}{7} + x_3 \\ x_3 = \alpha \in \mathbb{R} \end{cases} \Rightarrow \begin{cases} x_1 = \frac{5}{7} \\ x_2 = \frac{8}{7} + \alpha \\ x_3 = \alpha \in \mathbb{R} \end{cases}$$

$$\text{Vậy } \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5/7 \\ 8/7 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \alpha, \ \forall \alpha \in \mathbb{R}$$

Bài 6:

$$(A|B) = \left(\begin{array}{ccc|c} 2 & -4 & 6 & 8 \\ 1 & -1 & 1 & -1 \\ 1 & -3 & 4 & 0 \end{array}\right) \xrightarrow[d_3 - d_1]{d_2 \leftrightarrow d_1} \left(\begin{array}{ccc|c} 1 & -1 & 1 & -1 \\ 0 & -2 & 4 & 10 \\ 0 & -2 & 3 & 1 \end{array}\right)$$

$$\xrightarrow{-\frac{1}{2} d_2} \left(\begin{array}{ccc|c} 1 & -1 & 1 & -1 \\ 0 & 1 & -2 & -5 \\ 0 & -2 & 3 & 1 \end{array}\right) \xrightarrow{d_3 + 2d_2} \left(\begin{array}{ccc|c} 1 & -1 & 1 & -1 \\ 0 & 1 & -2 & -5 \\ 0 & 0 & -1 & -9 \end{array}\right)$$

$$\to \begin{cases} x_1 = -1 + x_2 - x_3 \\ x_2 = -5 + 2x_3 \\ x_3 = 9 \end{cases} \Rightarrow \begin{cases} x_1 = 3 \\ x_2 = 13 \\ x_3 = 9 \end{cases}$$

$$\text{Vậy } \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 13 \\ 9 \end{pmatrix}$$

Bài 7:

$$(A|B) = \left(\begin{array}{cccc|c} 4 & -2 & -4 & 2 & 1 \\ 6 & -3 & 0 & -5 & 3 \\ 8 & -4 & 28 & -44 & 19 \\ -8 & 4 & -4 & 12 & -5 \end{array}\right) \xrightarrow[\substack{d_2 - 6d_1 \\ d_3 - 8d_1 \\ d_4 + 8d_1}]{\frac{1}{4}d_1} \left(\begin{array}{cccc|c} 1 & -\frac{1}{2} & -1 & \frac{1}{2} & \frac{1}{4} \\ 0 & 0 & 6 & -8 & \frac{3}{2} \\ 0 & 0 & 36 & -48 & 9 \\ 0 & 0 & -12 & 16 & -3 \end{array}\right)$$

$$\xrightarrow[d_4 + 2d_2]{d_3 - 6d_2} \left(\begin{array}{cccc|c} 1 & -\frac{1}{2} & -1 & \frac{1}{2} & \frac{1}{4} \\ 0 & 0 & 6 & -8 & \frac{3}{2} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} x_1 = \frac{1}{4} + \frac{1}{2}x_2 + x_3 - \frac{1}{2}x_4 \\ x_2 = \alpha \\ x_3 = \frac{1}{4} + \frac{4}{3}x_4 \\ x_4 = \beta \end{cases} \quad \forall \alpha, \beta \in \mathbb{R}$$

$$\Rightarrow \begin{cases} x_1 = \frac{1}{2} + \frac{1}{2}\alpha + \frac{5}{6}\beta \\ x_2 = \alpha \\ x_3 = \frac{1}{4} + \frac{4}{3}\beta \\ x_4 = \beta \end{cases}$$

Vậy $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 0 \\ 1/4 \\ 0 \end{pmatrix} + \begin{pmatrix} 1/2 \\ 1 \\ 0 \\ 0 \end{pmatrix}\alpha + \begin{pmatrix} 5/6 \\ 0 \\ 4/3 \\ 1 \end{pmatrix}\beta$

$\forall \alpha, \beta \in \mathbb{R}$

Bài 8:

$$(A|B) = \left(\begin{array}{ccc|c} 1 & -2 & 3 & -3 \\ 2 & 2 & 0 & 0 \\ 0 & -3 & 4 & 1 \\ 1 & 0 & 1 & -1 \end{array}\right) \xrightarrow[d_4 - d_1]{d_2 - 2d_1} \left(\begin{array}{ccc|c} 1 & -2 & 3 & -3 \\ 0 & 6 & -6 & 6 \\ 0 & -3 & 4 & 1 \\ 0 & 2 & -2 & 2 \end{array}\right)$$

$$\xrightarrow[d_3 + 3d_2]{3d_4 - d_2} \left(\begin{array}{ccc|c} 1 & -2 & 3 & -3 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} x_1 = -3 + 2x_2 - 3x_3 = -5 \\ x_2 = 1 + x_3 = 5 \\ x_3 = 4 \end{cases}$$

Vậy $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -5 \\ 5 \\ 4 \end{pmatrix}$

Bài 9:

$$(A|B) = \left(\begin{array}{ccc|c} 3 & -3 & 3 & -3 \\ -1 & -5 & 2 & 4 \\ 0 & -4 & 2 & 2 \\ 3 & -1 & 2 & -4 \end{array}\right) \xrightarrow[\substack{d_2 + d_1 \\ d_4 - 3d_1 \\ -\frac{1}{2}d_3}]{\frac{1}{3}d_1} \left(\begin{array}{ccc|c} 1 & -1 & 1 & -1 \\ 0 & -6 & 3 & 3 \\ 0 & 2 & -1 & -1 \\ 0 & 2 & -1 & -1 \end{array}\right)$$

$$\xrightarrow[\substack{d_3 - d_2 \\ d_4 - d_2}]{-\frac{1}{3}d_2} \left(\begin{array}{ccc|c} 1 & -1 & 1 & -1 \\ 0 & 2 & -1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} x_1 = -1 + x_2 - x_3 = -\frac{3}{2} - \frac{1}{2}\alpha \\ x_2 = -\frac{1}{2} + \frac{1}{2}x_3 = -\frac{1}{2} + \frac{1}{2}\alpha \\ x_3 = \alpha \in \mathbb{R} \end{cases}$$

Vậy $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -3/2 \\ -1/2 \\ 0 \end{pmatrix} + \begin{pmatrix} -1/2 \\ 1/2 \\ 1 \end{pmatrix}\alpha, \ \forall \alpha \in \mathbb{R}$

Bài 10:

$$(A|B) = \left(\begin{array}{cccc|c} 1 & -1 & 1 & -3 & 0 \\ 2 & -1 & 4 & -2 & 0 \end{array}\right) \xrightarrow{d_2 - d_1} \left(\begin{array}{cccc|c} 1 & -1 & 1 & -3 & 0 \\ 0 & 1 & 2 & 4 & 0 \end{array}\right)$$

$$\to \begin{cases} x_1 = x_2 - x_3 + 3x_4 \\ x_2 = -2x_3 - 4x_4 \\ x_3 = \alpha \\ x_4 = \beta \end{cases} \in \mathbb{R} \Rightarrow \begin{cases} x_1 = -3\alpha - \beta \\ x_2 = -2\alpha - 4\beta \\ x_3 = \alpha \\ x_4 = \beta \end{cases}$$

Vậy $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \\ 1 \\ 0 \end{pmatrix}\alpha + \begin{pmatrix} -1 \\ -4 \\ 0 \\ 1 \end{pmatrix}\beta$

$\forall \alpha, \beta \in \mathbb{R}$

Bài 11:

$$(A|B) = \left(\begin{array}{cccc|c} 2 & -3 & 4 & -1 & 0 \\ 6 & 1 & -8 & 9 & 0 \\ 2 & 6 & 1 & -1 & 0 \end{array}\right) \xrightarrow[d_2 - 6d_1,\ d_3 - 2d_1]{\frac{1}{2}d_1} \left(\begin{array}{cccc|c} 1 & -\frac{3}{2} & 2 & -\frac{1}{2} & 0 \\ 0 & 10 & -20 & 12 & 0 \\ 0 & 9 & -3 & 0 & 0 \end{array}\right)$$

$$\xrightarrow{\frac{1}{10}d_2} \left(\begin{array}{cccc|c} 1 & -\frac{3}{2} & 2 & -\frac{1}{2} & 0 \\ 0 & 1 & -2 & \frac{12}{10} & 0 \\ 0 & 9 & -3 & 0 & 0 \end{array}\right) \xrightarrow[\frac{1}{15}d_3]{d_3 - 9d_2} \left(\begin{array}{cccc|c} 1 & -\frac{3}{2} & 2 & -\frac{1}{2} & 0 \\ 0 & 1 & -2 & \frac{12}{10} & 0 \\ 0 & 0 & 1 & -\frac{18}{25} & 0 \end{array}\right)$$

$$\Rightarrow \begin{cases} x_1 = -\frac{29}{50}x_4 \\ x_2 = \frac{6}{25}x_4 \\ x_3 = \frac{18}{25}x_4 \\ x_4 = \alpha \in \mathbb{R} \end{cases}$$

Vậy $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -\frac{29}{50} \\ 6/25 \\ 18/25 \\ 1 \end{pmatrix}\alpha, \forall \alpha \in \mathbb{R}$

Bài 12:

$$(A|B) = \left(\begin{array}{ccc|c} 1 & 6 & 4 & 0 \\ 2 & 4 & -1 & 0 \\ -1 & 2 & 5 & 0 \end{array}\right) \xrightarrow[d_3 + d_1]{d_2 - 2d_1} \left(\begin{array}{ccc|c} 1 & 6 & 4 & 0 \\ 0 & -8 & -9 & 0 \\ 0 & 8 & 9 & 0 \end{array}\right)$$

$$\xrightarrow[-\frac{1}{8}d_2]{d_3 + d_2} \left(\begin{array}{ccc|c} 1 & 6 & 4 & 0 \\ 0 & 1 & 9/8 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\to \begin{cases} x_1 = -6x_2 - 4x_3 \\ x_2 = -\frac{9}{8}x_3 \\ x_3 = \alpha \in \mathbb{R} \end{cases} \Rightarrow \begin{cases} x_1 = \frac{11}{4}\alpha \\ x_2 = -\frac{9}{8}\alpha \\ x_3 = \alpha \end{cases}$$

Vậy $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} \frac{11}{4} \\ -\frac{9}{8} \\ 1 \end{pmatrix}\alpha, \forall \alpha \in \mathbb{R}$