



**VNUHCM-UNIVERSITY OF SCIENCE**  
**FINAL EXAMINATION**  
**Semester III – Academic year 2021-2022**

**ARCHIVE  
CODE**  
(written by ET&QA Office)  
CLC CK21223\_  
CSC10004

Course name: Data Structures and Algorithms(21CLC02) Course code: CSC10004  
Time: 120 minutes Date: 08/08/2022  
Note: Students are [ ☐ allowed / ☒ not allowed ] to use electronic devices during the examination

Full name of Student: ..... Student ID: ..... No: .....

**Question 1 (20 points).**

Consider the following algorithm:

```
ALGO_A(X) {  
    d = ∞;  
    for (i = 1; i ≤ X.length - 1; i++)  
        for (j = i + 1; j ≤ X.length; j++)  
            if (|X[i] - X[j]| < d)  
                d = |X[i] - X[j]|;  
    return d;  
}
```

- Interpreting X as an array of coordinates of points on the x-axis (horizontal axis), explain concisely what ALGO\_A(X) does, and give a tight asymptotic bound for the complexity of ALGO\_A(X).
- Write an algorithm BETTER\_A(X) that is functionally equivalent to ALGO\_A(X), but with a better asymptotic complexity. Show the complexity of BETTER\_A(X).

**Question 2 (20 points).**

Given a circular sorted array of  $n$  integers, write a C/C++ function (and auxiliary functions if needed) running in  $O(\log n)$  to search an element in it. Assume that there are no duplicates in the array and the rotation is in counterclockwise direction.

The prototype of the function is as follows:

```
int searchArray(int a[], int n, int k);
```

where  $k$  is the value to be searched for and the return value of the function is the index of the element whose value is  $k$  or  $-1$  if the element is not present in the input array.

Example:

Input array: 7 9 11 1 3 5

Search value:  $k = 3$

Output: 4

Input array: 9 11 1 3 5 7

Search value:  $k = 11$

Output: 1

Input array: 3 5 7 9 11 1

Search value:  $k = 10$

Output:  $-1$

Full name of paper setter/staff code: ..... Signature: ..... [page 1/2]  
Full name of approver: ..... Signature: .....



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**Question 3 (30 points).**

Given a binary tree, you are asked to write a C/C++ function (and auxiliary functions if needed) to display the keys of all the nodes in the longest path going from the root to a leaf of the tree. If there are many such paths, display any of them. The prototype of the function is as follows:

```
void printLongestPath(Ref r);
```

**Question 4 (30 points).**

Initially, assume that the given AVL tree is empty. You are asked to expand the tree by inserting keys (one at a time) to it. Draw the tree after each node insertion. If the tree is imbalanced at a node, rebalance the (sub)tree rooted at that node and draw the resulting tree. The expanding process takes place continuously until all of four imbalanced cases: LL, RR, LR, and RL have been occurred.

*Note:*

- The keys to be inserted to the tree are positive integers selected arbitrarily.
- Rebalancing a (sub)tree with only three nodes is not counted.
- Do not use the example shown in class.