

Method	Input	Iteration	Idea Behind Method	Rate of Convergence	Pros	Cons
Bisection	continuous function $f$ on $(a, b)$ s.t. $f(a)$ has diff sign than $f(b)$	for $x_n \in (a_n, b_n)$ check if $f$ changes sign in $(a_n, x_n)$ ( $x_n, b_n$ ), repeat till convergence.	IUT	$O(\frac{1}{2^n})$	<ul style="list-style-type: none"> <li>• Easy to implement</li> <li>• Guarantees convergence if initial conditions met.</li> </ul>	<ul style="list-style-type: none"> <li>• Strict conditions on input</li> <li>• slower convergence than other methods.</li> </ul>
FPI	initial guess $x_0$	$x_n = f(x_{n-1})$ until $ x_n - f(x_{n-1})  < \text{TOL}$	Move along line $y=x$ until intersection with $f(x)$ since this implies $f(x)=x$ .	linearly for $f(x)$ if $f$ is contractive map on $[a, b]$	<ul style="list-style-type: none"> <li>• No derivative required</li> <li>• Guaranteed convergence if <math>f</math> is contractive map on <math>[a, b]</math></li> </ul>	<ul style="list-style-type: none"> <li>• <math>f' &lt; 1</math> on <math>[a, b]</math></li> <li>• slower convergence than Newton's method</li> <li>• struggles to find multiple roots.</li> </ul>
Newton	twice cont & diff function $f$ , initial guess $x_0$	$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$	Uses Taylor approximations to follow tangent lines until convergence.	quadratic if $f' \neq 0$ at root, else likely faster.	<ul style="list-style-type: none"> <li>• If <math>x_0</math> near solution we get quadratic convergence</li> </ul>	<ul style="list-style-type: none"> <li>• Not reliable if <math>x_0</math> not near solution.</li> <li>• Must have differentiable function</li> </ul>
Secant	Same as Newton's Method	$x_n = x_{n-1} + \frac{f(x_{n-1})(x_{n-2} - x_{n-1})}{f(x_{n-1}) - f(x_{n-2})}$	Similar idea to Newton's method except we approximate derivatives instead of direct computation.	$\approx 1.618$ (Golden Ratio)	<ul style="list-style-type: none"> <li>• Don't need to compute derivatives like in Newton's method.</li> </ul>	<ul style="list-style-type: none"> <li>• Slower convergence than similar Newton's method.</li> </ul>