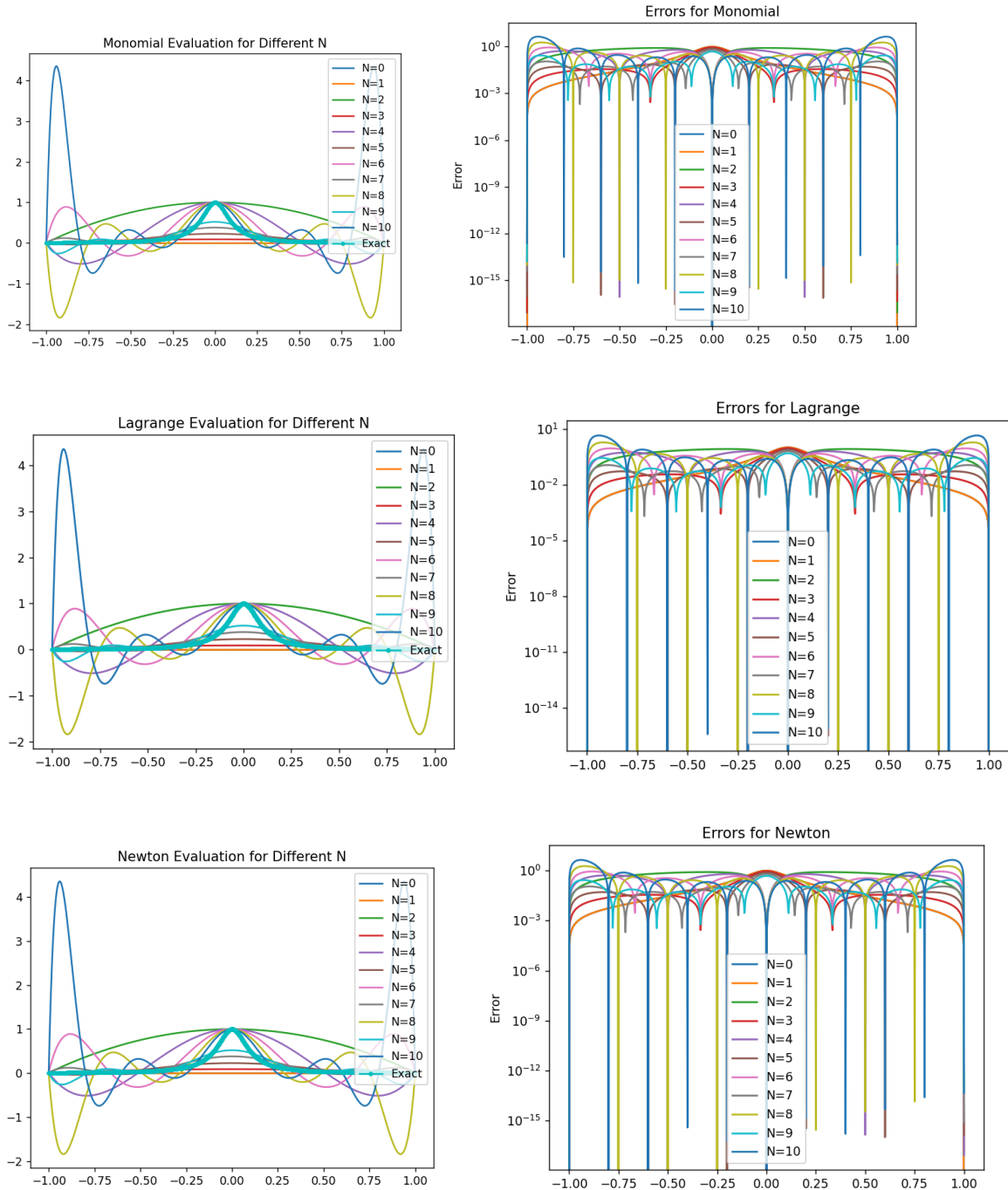


2. As N increases, the errors (particularly at the interpolation points where ideally the approximation should be at the machine epsilon) for the monomial model are the largest, likely due to the high condition number of the Vandermonde matrix. When N is still relatively small (<10) the performance of the Lagrange method is the best.

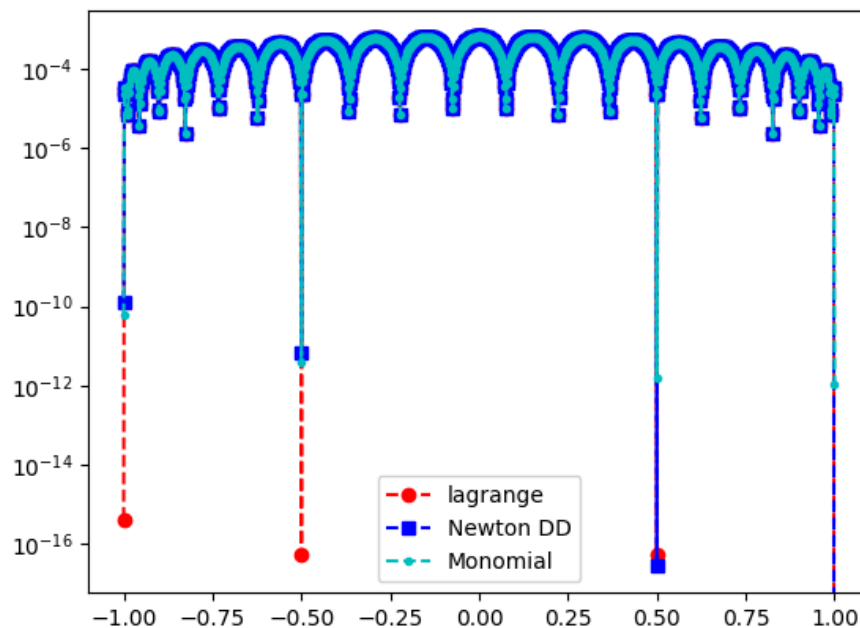


3. $p(x)$ reaches a maximum of ~ 100 for $N = 18$, here the approximations still seem roughly the same when analyzing graphically, however when analyzing the error plot the errors for all three methods are smallest at $x = -1, 0$, and 1 . Of the three methods here, Lagrange continues to perform the best (in terms of minimizing errors at these points) but at the rest of the points, the errors are roughly the same.

4. Graphically, the higher-order polynomials fit this function a lot better than the initial function we considered. However, once N reaches the high 20s or early 30s, the error plot becomes very jumbled, and as N gets even higher the error plot becomes completely unsensible.

3.2

1. Using the new interpolation method, $p(x)$ is a better fit towards the tails of the original function than it is towards the center of the plot, which is the opposite of what we observed with the uniform interpolation nodes in the previous section.
2. The errors are more uniform for the majority of the graph compared to the big swings up and down in error we observed in the first section. They also hover around a lower error level compared to the first part.



This is the plot of the error with the new interpolation method for $N = 20$, as we can see the errors are much more constant across the different methods and even within different methods hover around a similar level.