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Exercises 3.2

$$
\begin{aligned}
\frac{p_{n+1}-p}{p_n-p} &\sim \frac{p_{n+2}-p}{p_{n+1}-p} \\
p_{n+1}^2 - 2(pp_{n+1}) + p^2 &\sim p_n p_{n+2} \\
-2(pp_{n+1}) + pp_n + pp_{n+2} &\sim p_n p_{n+2} - p_{n+1}^2 \\
p &\sim \frac{p_n p_{n+2} - p_{n+1}^2}{p_n - 2p_{n+1} + p_{n+2}} \\
p_n - \frac{(p_{n+1}-p_n)^2}{p_{n+2} - 2p_{n+1} + p_n} &= \frac{p_n p_{n+2} - p_{n+1}^2}{p_n - 2p_{n+1} + p_{n+2}} \\
p_n &= \frac{p_n p_{n+2} - p_{n+1}^2}{p_n - 2p_{n+1} + p_{n+2}} + \frac{(p_{n+1}-p_n)^2}{p_{n+2} - 2p_{n+1} + p_n} \\
p_n &= p_n
\end{aligned}
$$

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- The convergence if Aitkens is faster since the number of iterations needed to reach the desired tolerance is fewer than the initial fixed point method.

3.4.

Steffensons requires the function, g(x), an initial guess, an error tolerance, and a max number of iterations as inputs. It will output the fixed point

```python
def steffensons(g, p0, tol, Nmax):
   count = 0
   p1 = -1
   while count < Nmax:
       count += 1
       a = p0
       b = g(a)
       c = g(b)
       p1 = a - ((b - a) ** 2) / (c - 2 * b + a)
       if abs(p1 - p0) < tol:
           return [p1, 0]
       else:
           p0 = p1

   return [p1, 1]
```

The above Steffensen's method converged in three iterations which is faster than the the two previous iterations.