STAT 135 Lecture 18

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Remark 0.1

Fact: Mean of X's, Mean of Y's lie on the regression line
$\mathbb{1}^T = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \end{bmatrix}$
Last time: Mean residuals is 0 $\mathbb{Y} - \hat{\mathbb{Y}} = 0$

$$\mathbb{1}^T[\mathbb{Y} - \hat{\mathbb{Y}}] = 0$$
$$\mathbb{1}^T\mathbb{Y} = \mathbb{1}^T(\mathbb{X}\hat{\beta})$$
$$\mathbb{1}^T\mathbb{Y} = (\mathbb{1}^T\mathbb{X})\hat{\beta}$$
$$\mathbb{1}^T\mathbb{Y} = \begin{bmatrix} s_1 & s_2 & \cdots & s_p \end{bmatrix}\hat{\beta}$$
$$\frac{1}{n}\mathbb{1}^T\mathbb{Y} = \frac{1}{n}\begin{bmatrix} \cdots \end{bmatrix}\hat{\beta} \qquad s_j = j^{th} \text{ column sum}$$

Remark 0.2

Regression Diagnostics
Assumptions:

1. Normally distributed errors
2. Independence
3. Constant variance
4. Linearity

Look at residuals vs predicted values, if there is a cone, then the variance is not constant, or if there is a shape that does not look random the x-axis, then it is likely nonlinear, the residuals should also have mean 0

Remark 0.3

More about regression:
Linear in parameters, not the data (x's)

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2(X_{i2})^2 + \cdots + \beta_{p-1}(X_{i1}X_{i2})$$

Can transform Y's and combine X's

Method 0.4 (Can formulate ANOVA as a linear model)
$Y_{ij} = \mu + \alpha_j + \varepsilon_{ij} \qquad \sum \alpha_j = 0$
3 groups: μ, α_1, α_2

$$\mathbb{Y} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} \mu \\ \alpha_1 \\ \alpha_2 \end{bmatrix}$$

$\alpha_1 + \alpha_2 + \alpha_3 = 0 \to \alpha_3 = -\alpha_1 - \alpha_2$

Remark 0.5

- ANOVA is a special case of linear models (Regression)
- We can set up $\mathbb{X}$ matrix to encode which observation falls into which group

Remark 0.6 (Opinion)

- I use linear models descriptively to understand simple relationships in data
- Step up from looking at correlations
- Don't take the inference very seriously due to strong assumptions

Definition 0.7 (Bayesian Statistics)
Have been working in the Frequentist

- Parameters are non-random constants
- Expressions of uncertainty are through sampling distributions of estimators

A realized 95% confidence interval is not a statement of uncertainty about a parameter value
Frequentism is awkward
We want to talk about uncertainty in parameters
Why "confidence"?
"The probability of observing a result at least as extreme as what we observed (under the null hypothesis)"
Frequentists approach focuses on P(Data|hypothesis)
Really care about P(Hypothesis|Data)

Method 0.8 (Bayes' Rule)

$$P(B|A) = P(A|B)\frac{P(B)}{P(A)}$$

$$P(Y = y|X = x) = P(X = x|Y = y) \cdot \frac{P(Y = y)}{P(X = x)}$$

Continuous version:

$$f_{Y|X}(y|x) = f_{X|Y}(x|y)\frac{f_Y(y)}{f_X(x)}$$

Continuous random variable X and Y have some joint distribution
$f_X(x)$ marginal density of X
$f_Y(y)$ marginal density of Y
$f_{Y|X}(y|x)$ conditional density of Y given $X = x$

Remark 0.9

Big idea:
Model parameter is a random variable
Sample data: $X_1, \ldots, X_n \sim f(X|\Theta = \theta)$
$f_{\Theta|X}(\theta|x) = f_{X|\theta}(x|\theta)\frac{f_\Theta(\theta)}{f_X(x)}$ posterior = likelihood times prior
$f_X(x) = \int f_{X|\Theta}(x|\theta)f_\Theta(\theta)d\theta$

$$f_{\Theta|X}(\theta|x) \propto f_{X|\Theta}(x|\theta)f_\Theta(\theta)$$

Find a & b such that $P(a \leq \Theta \leq b|\text{data}) = 95\%$ Bayesian credible interval

Example 0.10 (Coin Flipping)

Flip a possible biased coin 10 times, observe 6 heads and 4 tails
Θ is the underlying probabiltiy of heads
Need a prior on Θ pick $U(0, 1)$

$$f_X(x|\theta) = \binom{10}{6}\theta^6(1-\theta)^4$$

$$f_{\Theta|X}(\theta|X = 6) \propto \theta^6(1-\theta)^4 \cdot 1 = \theta^6(1-\theta)^4 \sim Beta(7, 5)$$

Need to find K such that $\int_0^1 K\theta^6(1-\theta)^4 d\theta = 1 \to K = \frac{11!}{4!6!}$
Now we can construct a Bayesian Credible Interval

Remark 0.11 (Why is this not the default way to do things)

- Seems more natural
- Statistics is for science $\Rightarrow$ want "objective" methods, priors seem subjective

"Real" priors may not have nice closed forms
More realistic setup $\Rightarrow$ Computations get very difficult