

STAT 135 Lecture 1

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Definition 0.1

What is **statistics**?

Statistical Inference: Study of answering questions and making decisions with data. Uses a lot of probability.

Probability vs. Statistics:

Probability: You start with a model that you know. Have a biased coin that lands heads with probability $\frac{3}{4}$. We toss it 10 times, what is $P(H = 6)$?

Statistics: You start with data. Flipped it 10 times, got 6 heads.

Example 0.2

What is a good guess for the probability of heads? (Estimation)

Is the coin fair? (Hypothesis testing)

What is a reasonable range of values for p ? Good real questions but fuzzy. Statistics is not just math. How do we frame the problem clearly so you can answer mathematically?

Example 0.3

Real applications:

Is this vaccine effective? Safe? (Clinical trials)

Who will win the election? (Polling)

Should OpenAI launch new agent feature? (A/B testing)

Definition 0.4

Population:

True state of the world. Probability distribution.

Probability distribution:

- x_1, x_2, \dots, x_n heights of everyone in the US $\frac{1}{N}$ on each person
- Coin: Heads (p), Tails ($1-p$)

Definition 0.5**Parameters:**

Property of the population that you are interested in.

- Mean
- Spread
 - Variance
 - Standard deviation
- 75th percentile

Definition 0.6**Sample aka data:**

- Draws from populations.
 $X_1, X_2, \dots, X_n \leftarrow$ often independent
- Assume draws are with replacement
- Samples are random variables

Example 0.7

Flip a possibly biased coin 5 times, get 3 heads. What is a 'good guess' for p ? $\frac{3}{5}$? $\frac{2}{3}$? What if we had 0 heads?

Definition 0.8**Estimation:**

1. Construction: Come up with a way to estimate a parameter
2. Evaluation: How good is our estimator? Bias? Concentrated around the true value of the parameter θ ? Variance? Consistency?
3. Optimality Theory: Can we come up with a best estimator?

Example 0.9

Example of Estimators:

- Heights in the US: $x_1, x_2, \dots, x_n \stackrel{\text{iid}}{\sim}$ population of heights.

$$- \mu = \frac{1}{n} \sum_{i=1}^n x_i = \bar{x}$$

Sample X_1, X_2, \dots, X_n .

$$\text{Sample Mean} = \bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$$

- Possibly biased coin n flips, X heads.
 $\hat{p} = \frac{X}{n}$ (Sample Fraction)

Definition 0.10

Estimators are random variables. Concentrated around a truth θ , tighter distribution = better estimator. We can study the properties of estimators as random variables.

Example 0.11

Expectation of \bar{X}

Population with mean μ

$$\bar{x} := \frac{1}{n} \sum_{i=1}^n x_i$$

$$E(\bar{x}) \stackrel{?}{=} \mu \rightarrow E\left[\frac{1}{n} \sum_{i=1}^n x_i\right] = \frac{1}{n} \sum_{i=1}^n E[x_i] = \frac{1}{n} \sum_{i=1}^n \mu = \mu$$

x_1, x_2, \dots, x_n independent

Example 0.12

Variance of \bar{X}

X_1, X_2, \dots, X_n independent, mean μ , variance σ^2

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$$

$$Var(\bar{X}) = Var\left(\frac{1}{n} \sum_{i=1}^n X_i\right) = \frac{1}{n^2} Var\left(\sum_{i=1}^n X_i\right) = \frac{1}{n^2} n\sigma^2 = \frac{\sigma^2}{n}$$

Takeaways:

1. Statistics = working backwards from data to make model. (Inference)
2. Parameter = Feature of population
3. Estimator = Function of the data which is a guess at parameter
4. Estimators are random variables. Can study distributions of estimators to understand them