STAT 135 Lecture 21: Final Review

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Remark 0.1 (Final Exam)

- Cumulative
- No notes/books/cheatsheet/devices
- Will provide
  - Density definitions
  - Moments, etc.
- No tricks
- No code
- Leave everything unsimplified (Mathematical Expression)
- Intended difficulty around the same as the midterm
- Lectures reflect what I think is important
- No nonparametric bootstrap on final (anything else can show up, permutation test is fair)

Remark 0.2 (Estimation Basics)

- Population vs sample
- Parameter, fixed constant (property of population)
- Estimator - function of data to guess parameter (Random Variable)
- Sampling Distribution

Remark 0.3 (Properties of Estimators)

- Unbiasedness - "Correct on average"
- Consistency - "Correct in the limit"
- MSE as a "performance metric" - Decomposition into Variance + Bias^2

Remark 0.4 (Techniques)

- Method of Moments
- Maximum Likelihood Estimation

Remark 0.5 (Inference (CI, SE))

- CLT (means are approximately normal)
- Delta Method (Smooth function of means)

Remark 0.6 (Maximum Likelihood)

- What it means
- How to do it
- Transformation equivariance
- Asymptotic normality
- Asymptotic efficiency (MLE may not always be unbiased but approaches CRLB)

Remark 0.7 (Optimality)

- Cramer-Rao Inequality
- Sufficient statistics
  - Motivation
  - Definition
  - Factorization Theorem
  - MLE is comprised of sufficient statistics
- Rao-Blackwell Theorem

Remark 0.8 (General Testing Setup)

P(Type I Error): α (Significance level)
P(Type II Error): β, $1-\beta$ Power

- Simple/Composite hypothesis
- Critical value
- Rejection region

Null Hypothesis
Alternative Hypothesis
Test Statistic
Neyman-Pearson Lemma (Use LRT for simple vs simple test)
Likelihood Ratio $\frac{L(\theta|H_0)}{L(\theta|H_1)}$
Duality of tests and confidence intervals

Remark 0.9 (Generalized LRT)

Composite vs Composite

$$\Lambda = \frac{\max L(\theta|H_0)}{\max L(\theta|H_1)}$$

$$\Lambda^* = \frac{\max L(\theta|H_0)}{\max L(\theta|H_0 \cup H_1)} \qquad \text{generally easier to calculate}$$

$$-2\log\Lambda \sim \chi^2_k, k = \dim\omega_1 - \dim\omega_0$$

Remark 0.10 (Two-Sample Problem)

- Two-Sample t-test
  - When to use it
  - how to carry out
  - Small samples, normal data, iid, common variance, s_p, unequal sample sizes
- Sample size calculations
- Paired t-test
- Advantages of pairing

Remark 0.11 (Nonparametric Analysis)

- Mann-Whitney
- Signed rank test

Remark 0.12 (Chi-Square)

- Goodness of Fit Test (possibly with estimated parameters)
- Independence (special case of goodness of fit)
- Homogeneity (Are the margins fixed)

Remark 0.13 (Multi-sample Problem)

One-way ANOVA

- Assumptions
- Decomposition $SS_{TOT} = SS_W + SS_B$
- How to carry out
  - Test statistic
  - Reference distribution
  - Degrees of freedom

Nonparametric version: Kruskal-Wallis

- Knowing spelling
- How it works
- Don't memorize normal approximation

Remark 0.14 (Regression)

- Matrix formulation
- OLS & MLE
- Assumptions, how to check them
- Estimates of β & σ^2
- Inference for regression coefficient
- SE(β_k), test H_0 & $\beta_k = 0$ vs H_1 & $\beta_k \neq 0$
- $\sum e_i = 0$ when there is an intercept column

Remark 0.15 (Bayesian Statistics)

- Setup & How it differs from frequentist
- Pros & cons of the formulation
- How to compute posterior given prior and likelihood
- No hard integrals

Remark 0.16 (Permutation Tests)

- How they work
- How to implement with a sample of B permutations
- Accuracy of estimate p-value, how you might choose B

No nonparametric bootstrap