STAT 135 Lecture 11

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Lemma 0.1 (Neyman Pearson Lemma)

Fill cart with items that are the best deal $\frac{P(X|H_1)}{P(X|H_0)}$, $P(X|H_1)$ is the value and $P(X|H_0)$ is the price, until you run out of budget. Maximizes the power $(1-\beta)$ for your α.

Example 0.2 (Duality of Confidence Intervals and Hypothesis Tests)

Let $X_1, \ldots, X_n \overset{iid}{\sim} \mathcal{N}(\mu, 1)$
Testing: $H_0 : \mu = \mu_0$
$H_1 : \mu \neq \mu_0$

Solution. Get a level 0.05 test by reject when $|\bar{X} - \mu_0| > \frac{1.96}{\sqrt{n}}$
Accept when $\frac{-1.96}{\sqrt{n}} \leq \mu_0 - \bar{X} \leq \frac{1.96}{\sqrt{n}} \to \bar{X} - \frac{1.96}{\sqrt{n}} \leq \mu_0 \leq \bar{X} + \frac{1.96}{\sqrt{n}}$
$\{\mu : \mu \in 95CI\} = \{\mu : H_0 : \mu = \mu_0 \text{ not rejected}\}$
Set of μ in CI is the same as set of μ which would not be rejected testing From 95% CI $\Rightarrow$ Hypothesis test, reject outside the interval
From Hypothesis test $\Rightarrow$ CI (Those null hypothesis wouldn't be rejected) $\Rightarrow$ Can work back and forth between CI and Hypothesis Testing ■

Remark 0.3 (Comparing two samples)

$X_1, \ldots, X_n \overset{iid}{\sim} f, \qquad Y_1, \ldots, Y_m \overset{iid}{\sim} g$
Applications:

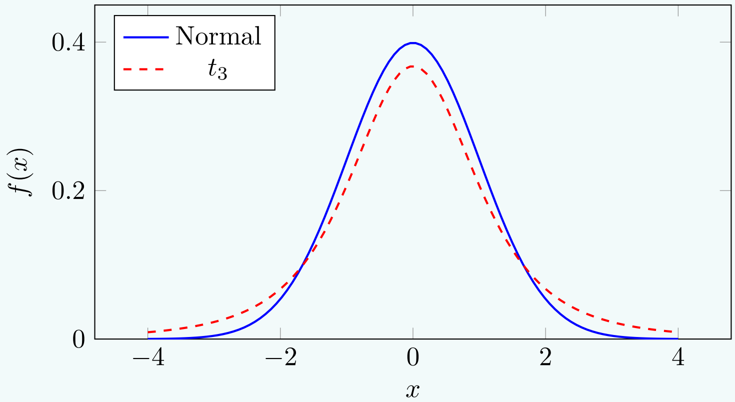
- Treatment vs control
- A/B Testing

Example 0.4

$X_1, \ldots, X_n \overset{iid}{\sim} \mathcal{N}(\mu_X, \sigma^2)$
$Y_1, \ldots, Y_m \overset{iid}{\sim} \mathcal{N}(\mu_Y, \sigma^2)$
$\bar{X} - \bar{Y} \sim \mathcal{N}(\mu_Y - \mu_X, \sigma^2(\frac{1}{n} + \frac{1}{m}))$
$\frac{\bar{X} - \bar{Y} - (\mu_x - \mu_x)}{\sigma\sqrt{\frac{1}{n} + \frac{1}{m}}} \sim N(0, 1)$
$\bar{X} - \bar{Y} \pm 1.96SE(\bar{X} - \bar{Y}) = \bar{X} - \bar{Y} \pm 1.96\sigma\sqrt{\frac{1}{n} + \frac{1}{m}}$
Reject $\mu_x = \mu_y$ when $\mu_X - \mu_Y = 0$

Method 0.5 (One sample t-test)

$X_1, \dots, X_n \overset{iid}{\sim} \mathcal{N}(\mu, \sigma^2)$ with σ^2 known
$\bar{X} \pm 1.96\frac{\sigma}{\sqrt{n}}$ Exact 95%
σ unknown
If we estimate σ^2 by $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$
Then $\frac{\sqrt{n}(\bar{X}-\mu)}{s} \sim t_{n-1}$



If we want a $(1-\alpha)\%$ Confidence interval for μ.
$\bar{X} \pm t_{n-1}(\frac{\alpha}{2})\frac{s}{\sqrt{n}}, \qquad t_{n-1}(\frac{\alpha}{2}) > 1.96$

Method 0.6 (Back to two sample problem)

$X_1, \dots, X_n \overset{iid}{\sim} \mathcal{N}(\mu_X, \sigma^2)$
$Y_1, \dots, Y_m \overset{iid}{\sim} \mathcal{N}(\mu_Y, \sigma^2)$
σ^2 unknown

$$\frac{(\bar{X}-\bar{Y}) - (\mu_X - \mu_Y)}{S_p\sqrt{\frac{1}{n}+\frac{1}{m}}} \sim t_{n+m-2}$$

$$S_p^2 = \frac{(n-1)S_X^2 + (m-1)S_Y^2}{n+m-2} \qquad \text{“Pooled Estimate”}$$

$$S_X^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$S_Y^2 = \frac{1}{m-1}\sum_{i=1}^{m}(y_m - \bar{y})^2$$

$$E(S_p^2) = \frac{(n-1)E(S_X^2) + (m-1)E(S_Y^2)}{n+m-2} = \frac{(n-1+m-1)\sigma^2}{n+m-2} = \sigma^2$$

Method 0.7 (Inference for two sample problem)

Exact $(1-\alpha)$ CI for $\mu_X - \mu_Y$

$$\bar{X} - \bar{Y} \pm t_{n+m-2}(\frac{\alpha}{2}) S_p \sqrt{\frac{1}{n} + \frac{1}{m}}$$

$H_0 : \mu_X = \mu_Y$
$H_1 : \mu_X \neq \mu_Y$
Level α test: Reject for large $\frac{|\bar{X}-\bar{Y}|}{S_p\sqrt{\frac{1}{n}+\frac{1}{m}}}$

Definition 0.8

$H_0 : \mu_X = \mu_Y$
One-Sided Hypothesis:
$H_1 : \mu_X > \mu_Y$
$H_1 : \mu_X < \mu_Y$
Two-Sided Hypothesis
$H_1 : \mu_X \neq \mu_Y$
Use two-sided test unless you know direction a priori

Takeaways:

- Newyman Pearson Lemma Intuition
- Duality of CI/Hypothesis Tests
- Two Sample Problem
- Solved it for normally distributed data with common unknown variance