STAT 151A Lecture 10

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Remark 0.1

Partial coefficients: $\vec{Y} = \mathbf{X}\hat{\beta} + \vec{e} = \begin{pmatrix}\vec{1} & \vec{x} & \vec{z}\end{pmatrix}\hat{\beta} + \vec{e}$
$y_i = \hat{\beta}_0 + \hat{\beta}_1 x_{1i} + \hat{\beta}_2 z_{1i} + e_i$
$y_i = b_{\vec{y}||\vec{1}_{\perp \vec{x},\vec{z}}} + b_{\vec{y}||\vec{x}_{\perp \vec{1},\vec{z}}} x_{1i} + b_{\vec{y}||\vec{z}_{\perp \vec{1},\vec{x}}} z_{1i} + e_i$
New recipe for preoducing $\hat{\beta}_1$:

1. Regress $\vec{y}$ on $(\vec{1}, \vec{z}) \to$ residuals $\vec{e}^{(y)}$
2. Regress $\vec{x}$ on $(\vec{1}, \vec{z}) \to$ residuals $\vec{e}^{(x)}$
3. Regress $\vec{y}$ on $\vec{e}^{(x)} \to$ resulting slope is $\hat{\beta}_1$

Fully general version $(pvariables \vec{x}_1, \ldots, \vec{x}_p) \to obtain \hat{\beta}_j$

1. Regerss $\vec{y}$ on $\vec{1}$, $\vec{x}_1, \ldots \vec{x}_{j-1}, \vec{x}_{j+1}, \ldots, \vec{x}_p \to$ get residuals $e^{(y)}$
2. Regerss $\vec{x}_j$ on $\vec{1}$, $\vec{x}_1, \ldots \vec{x}_{j-1}, \vec{x}_{j+1}, \ldots, \vec{x}_p \to$ get residuals $e^{(x_j)}$
3. Regerss $\vec{y}$ (or $e^{\vec{y}}$) on $e^{(x_i)} \to$ slope will be $\hat{\beta}_j$

Remark 0.2

Why do this?

- Interpretation of individual $\hat{\beta}_j \leftrightarrow$ average difference in y associated with 1 unit difference in $\vec{x}_j$ holding everything else fixed
- Alternative representation for standard error of $\hat{\beta}_j$

Remember: Under Gauss-Markov probability model $\text{Var}(\hat{\beta}) = \sigma^2(\mathbf{X}^\intercal\mathbf{X})^{-1} \leftrightarrow \hat{\sigma}^2 = \frac{1}{n-p+1}\sum(y_i - \hat{y}_i)^2$
$[\text{Var}(\hat{\beta})]_{jj} = \sigma^2[(\mathbf{X}^\intercal\mathbf{X})^{-1}]_{jj}$
Or use formulas from simple regression
$\text{SE}(\hat{\beta})^2 = \frac{\sigma^2}{\sum(x_i - \bar{x})^2} = \frac{\sigma^2}{\sum(e_i^{(\vec{x}_j)} - \bar{e}^{(x_j)})^2}$
Sidebar: Sum of squares for $\vec{x}_j \sim \vec{1}, \vec{x}_1, \ldots, \vec{x}_p$ except for $\vec{x}_j$
$\left\|\vec{x}_j - \bar{x}_j\vec{1}\right\|^2 = \text{RegSS} + \left\|e^{\vec{x}_j}\right\|^2$
We know $R^2 = \frac{RegSS}{TotSS} \Rightarrow ErrSS = TotSS(1 - R^2)$
$\text{Var}(\hat{\beta}_1) = \frac{\sigma^2}{\sum(\vec{x}_j - \bar{x}_j\vec{1})^2(1 - R_j^2)} \cdot \frac{\sigma^2}{\sum(\vec{x}_j - \bar{x}_j\vec{1})}$