STAT 151A Lecture 4

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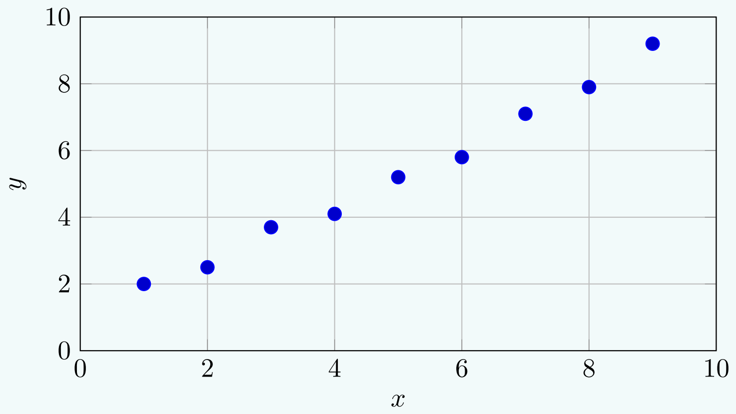
Remark 0.1 (Graphical interpretation of Simple Regression)



Line of best fit is the regression line

Remark 0.2 (Optimization approach to Simple Regression)

$$\min_c \sum_{i=1}^{n} (y_i - c)^2$$

$$\min_{a,b} \sum_{i=1}^{n} (y_i - a - bx_i)^2$$

$$\frac{\partial}{\partial a} \sum_{i=1}^{n} (y_i - a - bx_i)^2 = -2 \sum_{i=1}^{n} (y_i - a - bx_i) = 0$$

$$\frac{\partial}{\partial b} \sum_{i=1}^{n} (y_i - a - bx_i)^2 = -2 \sum_{i=1}^{n} x_i(y_i - a - bx_i) = 0$$

Above equations are known as the normal equations

$$\hat{b} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = r_{xy} \frac{\sigma_y}{\sigma_x}$$

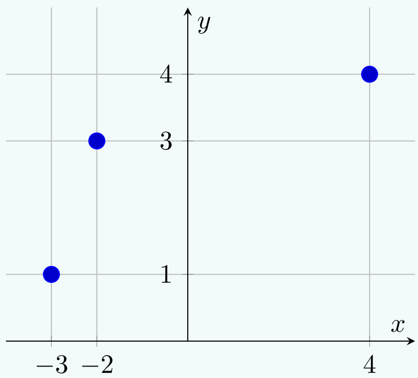
$$\hat{a} = \bar{y} - \hat{b}\bar{x}$$

Remark 0.3 (Probabilistic approach to Simple Regression)
$(x_1, y_1), \ldots$
Model: $y_i = \alpha + \beta x_i + \varepsilon_i$, $\mathbb{E}(\varepsilon_i) = 0$, $\mathrm{Var}(\varepsilon_i) = \sigma^2$
$\mathbb{E}(y_i | x_i) = \alpha + \beta x_i$
y_i, x_i, ε_i are random variables, α, β, σ are parameters

Remark 0.4 (Linear Algebra approach to Simple Regression)

y	x
3	-2
1	-3
4	4

Scatter Plot of Table Data



Remark 0.5 (Linear Algebra Review)
Vector: Length and a direction, can find angles between vectors

1. They can be scaled, $2\vec{v}$ has double length
2. They can be added, $\vec{v} + \vec{u}$

Can represent vectors in Cartesian coordinates $\vec{v} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \vec{u} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$

If you write them this way, $2\vec{v} = \begin{pmatrix} 2v_1 \\ 2v_2 \end{pmatrix}, \vec{v} + \vec{u} = \begin{pmatrix} v_1 + u_1 \\ v_2 + u_2 \end{pmatrix}$

Remark 0.6 (Geometric concepts)

$\vec{v} = \begin{pmatrix} a \\ b \end{pmatrix}$, $\text{length}(\vec{v}) = \sqrt{a^2 + b^2}$

$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_p \end{pmatrix}$, $||\vec{v}|| = \text{length}(\vec{v}) = \sqrt{v_1^2 + v_2^2 + \cdots + v_p^2} = \sqrt{\vec{v} \cdot \vec{v}}$

$\vec{u} \cdot \vec{v} = u_1 \cdot v_1 + u_2 \cdot v_2 + \cdots + u_p \cdot v_p$

Distance between two vectors? $||\vec{u} - \vec{v}||$

Alternate definition of dot product: $\vec{u} \cdot \vec{v} = ||\vec{u}|| \cdot ||\vec{v}|| \cos\theta$

Orthogonality

$\vec{u} \perp \vec{v} \Leftrightarrow \vec{u} \cdot \vec{v} = 0$

If $\vec{u} \perp \vec{v}$: $||\vec{u}+\vec{v}||^2 = (\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v}) == \vec{u}\cdot\vec{u}+\vec{v}\cdot\vec{u}+\vec{u}+\vec{v}+\vec{v}\cdot\vec{v} = ||\vec{u}||^2+||\vec{v}||^2$ by orthogonality

Definition 0.7 (Vector spaces)

$\vec{v}_1, \ldots, \vec{v}_p \in \mathbb{R}^n$

Subspace generated by $\vec{v}_1, \ldots, \vec{v}_p$ is the set of all vectors of the form: $c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_p\vec{v}_p$ where $c_i \in \mathbb{R}$

Also called $\text{span}\{\vec{v}_1, \ldots, \vec{v}_p\}$

Definition 0.8 (Vector Space Basis)

We call $\{\vec{v}_1, \ldots, \vec{v}_p\}$ if $\text{span}\{\vec{v}_1, \ldots, \vec{v}_p\} = V$