STAT 151A Lecture 12

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Remark 0.1 (More on interpreting/working with elements of $\hat{\beta}$)
$\hat{\beta} = (\mathbf{X}^\intercal \mathbf{X})^{-1} \mathbf{X}^\intercal \mathbf{Y}$
$\hat{\beta}_j \to$ partial coefficient
$\hat{\beta}_j, \hat{\beta}_k$ not directly comparable
Standardizing coefficients $\to$ makes them more comparable

Remark 0.2 (Standardizing Coefficients of Simple Regression)

$$\begin{aligned}
\vec{y} &= \hat{a}\vec{1} + \hat{b}\vec{x} + \vec{e} \\
\vec{y} &= (\bar{y} - \hat{b}\bar{x})\vec{1} + \hat{b}\vec{x} + \vec{e} \\
\vec{y} - \bar{y}\vec{1} &= \hat{b}(\vec{x} - \bar{x}\vec{1}) + \vec{e} \\
\vec{y} - \bar{y}\vec{1} &= r_{xy}\frac{s_y}{s_x}(\vec{x} - \bar{x}\vec{1}) + \vec{e} \\
\frac{\vec{y} - \bar{y}\vec{1}}{s_y} &= r_{xy}\frac{\vec{x} - \bar{x}\vec{1}}{s_x} + \frac{\vec{e}}{s_y}
\end{aligned}$$

Remark 0.3 (Standardizing Coefficients of Multiple Regression)

Starting with original model and reshuffle terms to recover what we could have gotten if we standardized $\mathbf{Y}$ and columns of $\mathbf{X}$ initially.

$$\vec{y} = \hat{\beta}_0\vec{1} + \hat{\beta}_1\vec{x}_1 + \cdots + \hat{\beta}_p\vec{x}_p + \vec{e}$$

substitute in $\hat{\beta}_0 = \bar{y} - \hat{\beta} - 1\bar{x}_1 - \cdots - \hat{\beta}_p\bar{x}_p$

$$\vec{y} - \bar{y}\vec{1} = \hat{\beta}_1(\vec{x}_1 - \bar{x}\vec{1}) + \cdots + \hat{\beta}_p(\vec{x}_p - \bar{x}_p\vec{1}) + \vec{e}$$

divide by s_y

$$\frac{\vec{y} - \bar{y}\vec{1}}{s_y} = \frac{\hat{\beta}_1(\vec{x}_1 - \bar{x}_1\vec{1})}{s_y} + \cdots + \frac{\hat{\beta}_p(\vec{x}_p - \bar{x}_p\vec{1})}{s_y} + \frac{\vec{e}}{s_y}$$
$$\frac{\vec{y} - \bar{y}\vec{1}}{s_y} = \frac{\hat{\beta}_1 s_{x_1}}{s_y}\frac{\vec{x}_1 - \bar{x}_1\vec{1}}{s_{x_1}} + \cdots + \frac{\hat{\beta}_1 s_{x_p}}{s_y}\frac{\vec{x}_p - \bar{x}_p\vec{1}}{s_{x_p}} + \frac{\vec{e}}{s_y}$$

Convert regular $\hat{\beta}_j$ to standardized $\hat{\beta}_j$:

$$\hat{\beta}_j \to \hat{\beta}_j\frac{s_{x_j}}{s_y}$$