# STAT 151A Lecture 38

Henry Liev

24 November 2025

**Remark 0.1** (Nested Dichotomies as an Alternative to Multinomial Input Models)

They are not the same

Pros:

- No fancy tools needed
- It accounts for order among the decisions

Cons:

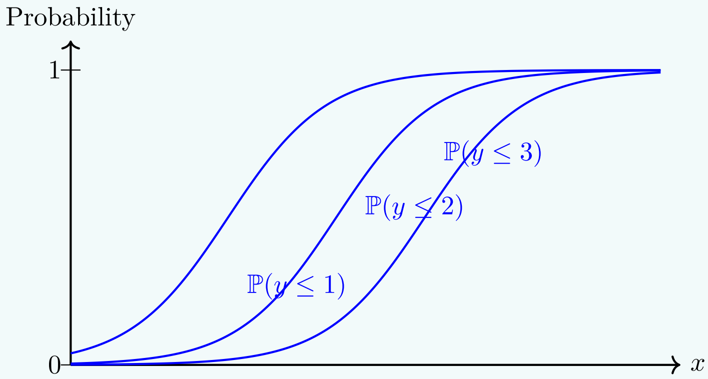
- More models are needed
- Specific type you select matters
- Less flexible about comparing arbitrary categories

$y_i \in \left\{\begin{matrix}\text{some hs} \\ \text{hs grad} \\ \text{some college} \\ \text{associates} \\ \text{bachelors} \\ \text{masters} \\ \text{prof/phd}\end{matrix}\right\}$, natural ordering, ordinal rather than nominal

Toy model: $\mathbb{P}(y_i \leq j) = \sum_{l=1}^{j} \mathbb{P}(y_i = l)$

**Remark 0.2** (Ordinal Logistic Regression/Proportional Odds Model)

$\log\left(\frac{\mathbb{P}(y\leq j)}{\mathbb{P}(y>j)}\right) = \alpha_j + \beta_1 x_{1i} + \cdots + \beta_p x_{pi}, j < m$




These grahps all have exactly the same shape: parallel → proportional → assuming we don't need ? multinomial
Benefit: $(m-1)+p$ parameters vs $(p+1)(m-1)$
Also fit this and do inference using MLE
Interpretation: $\beta_j$ is still similarly interpreted:
$e^{\beta_j}$ is factor by which odds that $y \leq$ any category are higher for a unit where $x_j$ is ? ? higher
Intercepts: pick an individual $i$
$\frac{\text{odd}(y_i|x_i)}{\text{odd}(y_k|x_i)} = \frac{\exp(\alpha_j + x_i^\intercal \beta)}{\exp(\alpha_j + x_i^\intercal \beta)} = \exp(\alpha_j - \alpha_k)$
Difference in intercepts gives the factor by which first (positive) category's odds differs from the second category's odds where all ? are the same