STAT 151A Lecture 21

Henry Liev

15 October 2025

Remark 0.1

For (j in 1:B)
Resample $(z_1, \ldots, z_m)$
$(z_1^*, \ldots, z_m^*)$
$\hat{\theta}_j^*$
Organize $\hat{\theta}_j^*$ smallest to biggest, then form confidence interval

Method 0.2 (Studentized Interval)

For (j in 1:B)
Resample $(z_1, \ldots, z_m)$
$(z_1^*, \ldots, z_m^*)$
$\hat{\theta}_j^*$
$\frac{\hat{\theta}_j^* - \hat{\theta}}{\widehat{\text{SE}}(\hat{\theta}_j^*)}$
Use a formula to estimate $\widehat{\text{SE}}(\hat{\theta}_j^*)$ or
Use a second bootstrap
For (k in 1:B_2)
Resample $(z_1^*, \ldots, z_n^*)$
$(z_1^{**}, \ldots, z_n^{**})$
$\hat{\theta}_k^{**}$
$\widehat{\text{SE}}(\hat{\theta}_j^*) = \sqrt{\frac{1}{B_2 - 1} \sum (\hat{\theta}_k^{**} - \bar{\hat{\theta}}^{**})}$
Plug this into formula
$q_{(1)}, \ldots, q_{(B)}$, $q_{(i)}$ a quantile from smallest to largest
Extract percentiles
Reverse engineer
$\mathbb{P}(q_{(\alpha/2b)} \leq \frac{\hat{\theta}^* - \hat{\theta}}{\widehat{\text{SE}}(\hat{\theta}^*)} \leq q_{(1-\alpha/2)B}) = 1 - \alpha \approx \mathbb{P}(q_{(\alpha/2B)} \leq \frac{\hat{\theta} - \theta}{\text{SE}(\hat{\theta})} \leq q_{(1-\frac{\alpha}{2})B})$
$= \mathbb{P}(-q_{(1-\alpha/2)B} \leq \frac{\theta - \hat{\theta}}{\text{SE}(\hat{\theta})} \leq -q_{(\alpha/2)B})$
$\mathbb{P}(\hat{\theta} - q_{(1-\frac{\alpha}{2})B}\widehat{\text{SE}}(\hat{\theta}) \leq \theta \leq \hat{\theta} - q_{(\alpha/2)B}\widehat{\text{SE}}(\hat{\theta}))$

Remark 0.3 (Bootstrap for Regression Models)

$(y_1, \vec{x}_1), \ldots, (y_n, \vec{x}_n)$
How does the bootstrap help us with inference tasks here? $\hat{y} = \mathbf{X}\hat{\beta}$
Confidence intervals for $\hat{\beta}_j$
$H_0 : \beta_j = 0$ (t-stat)
$H_0 : \beta_1 = \cdots = \beta_k = 0, L\beta = c$ (F-test)
2 general bootstrapping approaches:
Bootstrapping cases: $(\vec{x}_1, y_1), \ldots, (\vec{x}_n, y_n) \to (\vec{x}_1^*, y_1), \ldots, (\vec{x}_n^*, y_n) \to \hat{y}^* = \mathbf{X}\hat{\beta}^*$
Repeat B times to get $(\hat{y}^*_{(k)}, \mathbf{X}^*_{(k)}, \hat{\beta}^*_{(j)})$
Focus on CI's for now:
Extract $(\hat{y}^*_{(k)}, \mathbf{X}^*_{(k)}, \hat{\beta}^*_{(k)})$