```
from machine_learning_functions import *
In [ ]: def experiment_linear(a, b, learning_rate, epochs, random_x_function):
            a, b = float(a), float(b)
            model = Model(
                FFN = FFN(
                     neurons_per_layer_list=[1, 1],
                     activation_functions_list=[None,],
                     cost_function=MSE()
                 ),
                 data_set=create_1_input_1_output_XY_data(
                     function=lambda x: a*x+b,
                     num_data_items=10000,
                     random_x_function=random_x_function
            )
            print(
                model.train_and_evaluate(
                     learning_rate=learning_rate,
                     epochs=epochs,
                     batch_size=50
            model.print_FFN_parameters()
            return model
In [ ]: experiment_linear(
            a=2, b=5,
            learning_rate=0.00025,
            epochs=1,
            random_x_function= lambda: random.uniform(-100, 100)
       (26.476959844961947, 557.0903921874293)
       Parameters of network
       {'W1': array([[1.95422062]])}
       {'B1': array([0.54006489])}
Out[]: <machine learning functions.Model at 0x257181ac510>
In [ ]:
        experiment_linear(
            a=2, b=5,
            learning_rate=0.00025,
            epochs=1,
            random_x_function= lambda: random.uniform(-100, 100)
       (30.39501657132671, 927.1757343554814)
       Parameters of network
       {'W1': array([[2.0562532]])}
       {'B1': array([0.53572177])}
Out[]: <machine_learning_functions.Model at 0x2572e5740d0>
In [ ]: experiment_linear(
            a=1/2, b=-7,
            learning_rate=0.00025,
            epochs=1,
```

```
random_x_function= lambda: random.uniform(-10, 10)
       (55.766794676885354, 38.351180903069235)
       Parameters of network
       {'W1': array([[0.57050946]])}
       {'B1': array([0.44124636])}
Out[ ]: <machine_learning_functions.Model at 0x25717fa7950>
In [ ]: experiment_linear(
            a=1/2, b=-7,
            learning_rate=0.00025,
            epochs=5,
            random_x_function= lambda: random.uniform(-10, 10)
       (44.86322336570385, 16.632833989096753)
       Parameters of network
       {'W1': array([[0.55188239]])}
       {'B1': array([-0.33177718])}
Out[ ]: <machine_learning_functions.Model at 0x25717fa6950>
In [ ]: # greater learning rate caused divergance
        experiment_linear(
            a=-3, b=-1/4,
            learning_rate=0.00025,
            epochs=20,
            random_x_function= lambda: random.uniform(-10, 10)
        )
```

```
KeyboardInterrupt
                                                 Traceback (most recent call last)
       Cell In[7], line 2
             1 # greater learning rate caused divergance
       ----> 2 experiment_linear(
             3
                   a=-3, b=-1/4,
             4
                   learning_rate=0.00025,
             5
                   epochs=20,
                   random_x_function= lambda: random.uniform(-10, 10)
             7)
       Cell In[2], line 16, in experiment_linear(a, b, learning_rate, epochs, random_x_f
             2 a, b = float(a), float(b)
             3 model = Model(
                 FFN = FFN(
             5
                       neurons_per_layer_list=[1, 1],
          (\ldots)
            13
                   )
            14 )
            15 print(
       ---> 16
                 model.train and evaluate(
                       learning_rate=learning_rate,
            17
                       epochs=epochs,
            18
            19
                       batch_size=50
            20
            21 )
            22 model.print_FFN_parameters()
            23 return model
       File c:\Users\Henry\Documents\compsci presentations\machine_learning_functions.p
       y:327, in Model.train_and_evaluate(self, learning_rate, epochs, batch_size)
           324 X, Y = self.X_data[data_item_index], self.Y_data[data_item_index]
           326 # _, cost = self.FFN.foreward_propagate(X, Y)
       --> 327 _, cost = self.FFN.foreward_propagate(
                input_vector=X,
           328
           329
                   expected_output_vector=Y
           330 )
           332 total cost += cost
           334 param gradients = self.FFN.back propogate(X, Y, False)
       File c:\Users\Henry\Documents\compsci presentations\machine learning functions.p
       y:238, in FFN.foreward_propagate(self, input_vector, expected_output_vector)
                 cost = None
           237 else:
                  cost = self.cost_function(layer_activations, expected_output_vector)
       --> 238
           240 return layer_activations, cost
       File c:\Users\Henry\Documents\compsci presentations\machine_learning_functions.p
       y:197, in MSE.__call__(self, P, Y)
           194 self.diff = (P-Y)
           196 # here I will use dot product rather than transverse vectors which are no
       t well supported in numpu :(
       --> 197 return (1/self.n) * np.dot(self.diff, self.diff)
       KeyboardInterrupt:
In [ ]: experiment linear(
            a=-3, b=-1/4,
            learning_rate=0.00025,
```

```
epochs=50,
            random_x_function= lambda: random.uniform(-10, 10)
       1.153899693613739e-09
       Parameters of network
       {'W1': array([[-3.00000003]])}
       {'B1': array([-0.24996604])}
Out[ ]: <machine_learning_functions.Model at 0x1e842aa4c10>
In [ ]: def experiment_boolean_logic(input_neurons, output_neurons, function, learning_n
            neurons_per_layer_list = [input_neurons] + [neurons_per_hidden_layer]*hidden
            activation_functions_list = [RELU() for _ in range(hidden_layers)] + [Sigmoi
            model = Model(
                FFN = FFN(
                     neurons_per_layer_list=neurons_per_layer_list,
                     activation_functions_list=activation_functions_list,
                     cost_function=MSE()
                ),
                 data_set=create_a_inputs_b_outputs_XY_data(
                     a=input_neurons,
                     b=output neurons,
                     function=function,
                     random_x_function= lambda: random.choice([0.0, 1.0]),
                     num_data_items=10000
            )
            print(
                model.train_and_evaluate(
                     learning_rate=learning_rate,
                     epochs=epochs,
                     batch_size=50
            )
            model.print_FFN_parameters()
            return model
In [ ]: def logical_and(X):
            a, b = bool(X[0]), int(X[1])
            return float(
                a and b
        experiment_boolean_logic(
            input neurons = 2,
            output_neurons = 1,
            function=logical_and,
            learning_rate=10**-4,
            epochs=10,
            hidden_layers=2,
            neurons_per_hidden_layer=4
```

```
0.12960848961263577
       Parameters of network
       {'W1': array([[-0.65294285, 0.61589101],
              [-0.42820191, 0.95443661],
              [0.78559784, -0.6332115],
              [-0.49309623, 0.0098737 ]])}
       {'B1': array([0.50288722, 0.49578408, 0.50147239, 0.50223506])}
       {'W2': array([[-0.28502613, 0.84517966, -0.02225392, -0.51014619],
              [0.41362951, 0.76604725, 0.17653435, -0.83507039],
              [-0.73251695, 0.22706445, 0.14197284, 0.32888372],
              [ 0.33971928, -0.36906122, 0.70390109, 0.38498229]])}
       {'B2': array([0.499599 , 0.49750374, 0.49630429, 0.50301195])}
       {'W3': array([[-0.08625152, -0.55111397, -0.82001057, 0.67107456]])}
       {'B3': array([0.50449989])}
Out[ ]: <machine_learning_functions.Model at 0x1e82c532ad0>
In [ ]: def logical_and(X):
            a, b = bool(X[0]), int(X[1])
            return np.array([float(a and b)])
        and_model = experiment_boolean_logic(
            input_neurons = 2,
            output_neurons = 1,
            function=logical_and,
            learning_rate=10**-2,
            epochs=10,
            hidden_layers=2,
            neurons_per_hidden_layer=4
       0.22791140785530437
       Parameters of network
       {'W1': array([[ 0.71236726, 0.92403822],
              [-0.04563622, 0.82902018],
              [-0.957013, -0.41655979],
              [ 0.366711 , -0.79908891]])}
       {'B1': array([0.58150379, 0.38601112, 0.50171204, 0.48146798])}
       {'W2': array([[ 0.61075504, -0.77864653, 0.59993966, -0.49881914],
              [0.73361503, 0.61780799, -0.86412893, -0.15360232],
              [ 0.38080145, -0.46465994, 0.07840686, 0.23716035],
              [ 0.95144912, -0.73341645, -0.65706865, 0.1223274 ]])}
       {'B2': array([0.55125803, 0.48178318, 0.47786995, 0.58917505])}
       {'W3': array([[ 0.5281708 , -0.02111933, -0.17516291, 0.94342531]])}
       {'B3': array([0.60308173])}
In [ ]: (
            and_model.FFN.foreward_propagate(np.array([.0, 0.0])),
            and_model.FFN.foreward_propagate(np.array([0.0, 1.0])),
            and_model.FFN.foreward_propagate(np.array([1.0, 0.0])),
            and_model.FFN.foreward_propagate(np.array([1.0, 1.0])),
Out[]: ((array([0.02659525]), None),
         (array([0.02199461]), None),
         (array([0.01794683]), None),
         (array([0.01604408]), None))
In [ ]: def half adder(X):
            a, b = bool(X[0]), bool(X[1])
            sum = a \wedge b
```

```
carry = a and b
            return np.array([float(sum), float(carry)])
        experiment_boolean_logic(
            input_neurons = 2,
            output_neurons = 2,
            function=half adder,
            learning_rate=10**-4,
            epochs=10,
            hidden_layers=2,
            neurons_per_hidden_layer=4
        )
       0.3135790065347722
       Parameters of network
       {'W1': array([[ 0.80417935, -0.26562906],
              [-0.87725767, -0.12092512],
              [-0.78320205, -0.27136584],
              [ 0.35091593, 0.26535257]])}
       {'B1': array([0.50024658, 0.50157549, 0.49893939, 0.49868099])}
       {'W2': array([[-0.0098028 , 0.49366171, 0.45025614, -0.34743505],
              [-0.233186, -0.22733595, 0.75709782, 0.64877799],
              [-0.22303459, -0.92618616, 0.69403422, 0.5781448],
              [-0.31351791, 0.18043455, 0.31068989, 0.77025649]])}
       {'B2': array([0.50153271, 0.49843039, 0.49935668, 0.50078602])}
       {'W3': array([[ 0.70884425, -0.92781434, -0.21895813, 0.07905441],
              [ 0.68910313, -0.41105522, -0.40206512, 0.76649372]])}
       {'B3': array([0.50129485, 0.50089337])}
Out[]: <machine_learning_functions.Model at 0x1e84292bbd0>
In [ ]: def half_adder_bool(A, B):
            return A ^ B, A and B
        def full_adder_bool(A, B, Cin):
            S1, C1 = half_adder_bool(A, B)
            S2, C2 = half_adder_bool(S1, Cin)
            sum = S2
            carry = C1 or C2
            return sum, carry
In [ ]: def chain_adder_bool(A_sequence, B_sequence):
            carry_out = 0
            sum_sequence = [None for _ in range(16)]
            for i in range(16):
                sum_sequence[i], carry_out = full_adder_bool(A_sequence[i], B_sequence[i
            return sum sequence
In [ ]: def full_adder_float(X):
            a, b, c in = [bool(e) for e in X]
            s, c_out = full_adder_bool(a, b, c_in)
            return np.array([float(s), float(c_out)])
        experiment_boolean_logic(
            input neurons = 3,
            output_neurons = 2,
            function=full_adder_float,
            learning rate=10**-4,
            epochs=10,
            hidden_layers=2,
```

```
neurons_per_hidden_layer=4
       0.42989678126870745
       Parameters of network
       {'W1': array([[-0.13046729, 0.03398441, -0.1298488],
              [-0.72423869, 0.78047567, -0.64296893],
              [0.6843604, -0.26595864, -0.22207009],
              [-0.41476843, -0.50194339, -0.78647592]])
       {'B1': array([0.500551 , 0.50042498, 0.49871512, 0.49978004])}
       {'W2': array([[-0.65096664, -0.37460568, -0.37066503, 0.75567564],
              [-0.83865972, -0.79582755, 0.49221667, -0.5953249],
              [-0.69872521, -0.02073121, -0.10209797, 0.1742665],
              [-0.587185 , -0.68434492, 0.86805485, 0.01652106]])}
       {'B2': array([0.50025891, 0.49912558, 0.50068528, 0.49920759])}
       {'W3': array([[ 0.44302166, 0.35661684, 0.8725924 , 0.55579853],
              [-0.00385711, -0.42774771, -0.12385615, -0.46034002]])}
       {'B3': array([0.50123403, 0.50315067])}
Out[]: <machine_learning_functions.Model at 0x1e82c50d3d0>
In [ ]: def nibble_chain_adder_float(X):
            X = [bool(e) \text{ for } e \text{ in } X]
            As, Bs = X[:16], X[16:]
            S = chain_adder_bool(As, Bs)
            S = np.array([float(e) for e in S])
            return S
        experiment_boolean_logic(
            input_neurons = 32,
            output_neurons = 16,
            function=nibble_chain_adder_float,
            learning_rate=10**-4,
            epochs=50,
            hidden_layers=4,
            neurons_per_hidden_layer=16
```

## 1.191507532951934

```
Parameters of network
{'W1': array([[ 6.11125611e-01, 1.14858367e-01, -4.95287944e-01,
        -6.25415027e-01, -2.53345060e-01, -7.54530891e-01,
        -2.92707485e-01, 7.76460302e-01, 5.63399112e-03,
        -7.48106081e-01, 5.69085296e-02, 5.47629344e-01,
        -6.48915951e-01, -3.63134604e-01, -9.06494622e-01,
        1.13467815e-02, -2.61377319e-01, 3.04120455e-01,
         3.08378312e-01, 6.46972992e-02, -4.59864541e-01,
         5.67630634e-01, 9.18967193e-01, -9.82585472e-01,
        -4.20437556e-01, -6.64320203e-02, -6.49667910e-01,
         4.38514490e-01, 9.86187351e-01, -3.54789901e-02,
         6.41888373e-01, 9.04014896e-01],
       [ 8.45828349e-02, 1.01889100e+00, 7.53080810e-01,
         1.55303544e-02, 3.13309410e-02, 7.60985437e-01,
        -4.85823744e-01, -2.66594230e-01, 8.18592904e-01,
         2.22874715e-01, -9.39888359e-01, -7.29218120e-02,
        -4.90382179e-01, -3.78657325e-01, -3.74402574e-01,
        6.30120738e-01, 9.87167866e-01, 5.13258405e-01,
         5.03757902e-02, -3.41695240e-01, -1.00064672e+00,
        6.35986519e-01, -1.00426109e-01, 7.86634844e-01,
        -6.60966125e-01, 5.88368983e-01, -9.68166043e-01,
         2.35563609e-01, 8.44698102e-01, 9.28508884e-01,
         9.45004844e-01, 9.70151623e-01],
       [-1.85784082e-01, -9.06666718e-01, -8.51845260e-01,
        -8.59291429e-01, -5.32047570e-01, 9.73561266e-01,
         9.31511923e-01, -8.27752574e-01, -1.71458553e-01,
         6.54541113e-01, 2.21579305e-01, -4.05657808e-01,
         4.93756126e-01, 6.31896249e-01, 7.51677054e-01,
        -2.12789193e-01, -5.74461575e-02, -7.45721909e-01,
        -9.97473594e-01, -9.15008052e-01, 4.48801437e-01,
        7.66969940e-01, -6.18043729e-01, 6.51739528e-01,
        9.69792975e-01, 2.78251040e-01, -5.08312043e-01,
        -2.44814808e-01, 8.05930067e-01, 8.17313643e-01,
        -8.15825198e-01, -2.97644866e-03],
       [-7.56217119e-01, -4.02215080e-01, -2.86944100e-01,
        -2.71043138e-01, -6.08370072e-01, 4.18348958e-01,
        -7.25725954e-01, 9.77439937e-01, -6.99873384e-01,
         1.04733842e-01, 9.60759238e-01, -4.09616599e-02,
        7.90520750e-01, -3.45184715e-01, -7.69986586e-01,
         5.83566894e-01, 6.44075035e-02, 7.36095718e-02,
        -4.97759313e-01, -1.10053275e-01, -3.04750330e-01,
        -6.59186561e-02, -2.49907906e-01, -8.50615675e-01,
         7.65347009e-01, -5.05322942e-01, 3.58509525e-01,
         8.49757925e-01, -2.82819280e-01, -6.11362068e-01,
         2.62087182e-01, -5.72196162e-01],
       [ 1.84795966e-01, -9.12741378e-01, -6.92449533e-01,
         4.00143719e-01, 6.56798737e-01, 7.38100438e-01,
        -2.66447243e-01, 2.16583200e-01, -3.14465933e-01,
        -8.33346346e-01, 4.14009147e-02, 6.07321668e-01,
         7.65581839e-01, -7.44832344e-01, -2.20606877e-01,
        -4.66204145e-01, 1.75602664e-01, -6.03383403e-01,
        -7.59678399e-02, -5.37270611e-01, 7.72403043e-01,
        9.40103593e-01, -5.89818768e-01, 4.37356990e-01,
         3.60275624e-01, -1.35680476e-02, -3.59014675e-01,
         7.29989970e-01, 2.78955784e-01, 2.66817206e-02,
        -5.47650031e-03, -6.36054812e-01],
       [ 2.54589850e-01, -4.93891516e-01, -8.93293082e-01,
         4.65429979e-01, 7.55074980e-02, 5.67330332e-01,
         2.50829632e-01, 1.25363988e-01, 2.20627255e-01,
```

```
-5.27182195e-01, -8.62259871e-01, 1.05903098e-01,
 -8.68498579e-01, 3.52009338e-01, -2.06118272e-01,
 -9.73084157e-01, 8.69450738e-01, 6.60251177e-01,
 3.08196248e-01, -4.56999556e-01, 1.42986353e-02,
 -5.61166635e-01, -8.86909866e-01, -3.28299585e-01,
 6.21882106e-01, -9.04677316e-01, -6.83608616e-01,
 5.94442242e-01, 6.25595917e-01, 9.54739989e-01,
 -6.32976426e-01, -7.42669439e-01],
[ 6.19182824e-01, 1.06586529e-01, -4.56525408e-01,
 -3.95272701e-01, 1.92262832e-01, 7.58505749e-01,
-7.98617993e-01, 6.23837327e-01, -8.39823223e-01,
-7.45487042e-01, -6.72245230e-01, 9.01233236e-01,
 -3.63990755e-02, 8.77312355e-01, -3.58361581e-01,
 3.41691840e-01, -2.34078845e-01, 5.41636835e-01,
 8.10218224e-01, -5.59245870e-01, 8.54865182e-01,
 6.71152091e-01, 2.51630024e-01, -7.86135972e-01,
 -2.72531919e-01, 9.63829595e-02, -6.76974442e-01,
 5.72282446e-01, -2.71062160e-01, 6.28848392e-01,
 5.57464238e-02, -6.38080660e-01],
[ 4.95431224e-01, -6.68696187e-01, 9.69922165e-01,
 6.55818631e-01, -4.17936009e-01, 9.79376487e-01,
 8.70919587e-01, 5.79144117e-01, -2.97420664e-01,
 3.82299986e-01, -3.60656430e-01, 2.20764302e-01,
 5.87123248e-01, -4.20441062e-01, -9.32442619e-01,
 4.05902798e-01, -6.96261138e-01, 7.95912585e-01,
 -4.10320824e-01, -1.19549996e-01, 7.83910261e-01,
 -3.34766381e-02, 8.83749219e-01, -3.12652621e-01,
 1.09917029e-01, 8.02367216e-01, -7.21541360e-01,
-1.90824884e-01, 7.20022731e-01, -9.82951676e-01,
 3.04411682e-01, 7.42385471e-01],
[ 1.82520531e-01, -7.67872325e-01, 6.59498797e-01,
 6.34979690e-01, -5.76659425e-01, 2.67982945e-01,
 -3.82649467e-01, -3.04129253e-01, -2.79411893e-01,
 2.44990322e-01, 5.94643356e-01, -8.87641420e-02,
 9.99253465e-01, 9.51483775e-03, -9.39041759e-01,
 8.28988264e-01, -1.66314529e-01, 9.46602576e-01,
 8.09703300e-01, -5.40901215e-01, 9.61724689e-02,
 -4.06366711e-01, 9.66548378e-01, -4.57393679e-01,
 -4.72011294e-01, -4.53568246e-01, 1.03985259e+00,
 9.52566426e-01, 1.01620279e-01, 8.74997847e-01,
 -3.08293198e-01, 8.62196868e-01],
[ 5.95912490e-01, 8.26015974e-01, -4.40332896e-01,
 -1.46409976e-01, -8.81900248e-01, 7.38377862e-01,
-9.13424238e-02, 7.21551601e-01, -8.47553835e-01,
-7.45589228e-02, 2.35082149e-01, 8.23937373e-01,
 -5.88036153e-01, 2.91800600e-01, 5.87820572e-01,
-3.48387166e-01, -4.06183515e-01, -1.10344805e-01,
 -8.57632983e-01, -3.46382158e-01, 2.57759451e-01,
 7.82386077e-01, -2.84309501e-01, 4.68665361e-02,
 7.81614556e-01, 7.74765246e-01, -3.93033106e-01,
 1.02599642e+00, -1.97349566e-01, -4.77376244e-01,
 9.84501098e-02, 6.51589773e-01],
[ 2.88292549e-02, -1.36484860e-01, 9.02953219e-03,
 9.35297437e-01, -1.38514020e-02, 3.40582863e-02,
 -2.20932242e-01, 9.97608269e-02, -7.83722447e-01,
-3.85872336e-01, -6.87690591e-01, -2.67407696e-01,
 -9.69969076e-01, -6.84326186e-01, -1.55366415e-01,
 -1.88460926e-01, -5.73434523e-01, -2.09156540e-02,
 8.81003317e-01, -8.04440593e-01, -9.08422713e-01,
 9.72347041e-01, -1.36704124e-01, -5.43881873e-01,
```

```
-7.36680712e-01, -9.19845593e-01, -2.60981740e-01,
        6.44810870e-01, 3.88090687e-01, 2.60865480e-01,
       -3.20995505e-01, 3.90699758e-01],
      [-3.55609000e-01, -2.94970922e-01, -9.52051343e-01,
        2.52880761e-01, -7.10176815e-01, 8.69320675e-01,
        4.39380042e-01, 6.54536542e-01, -1.91907684e-01,
        3.62475919e-01, 2.27236827e-01, -8.10052545e-01,
       -7.33974625e-01, 5.56734424e-01, 4.98404400e-01,
       -6.96181027e-01, 1.17548652e-01, 4.41861006e-01,
        8.59672490e-01, -4.11337789e-01, 9.30547187e-01,
        1.16921399e-01, 6.40018997e-01, 1.14157426e-01,
        4.48593004e-01, 9.51259893e-01, 4.23029150e-02,
       -6.26333662e-02, 7.12177887e-01, 6.21091541e-04,
       -9.63159271e-01, 8.40033530e-01],
      [ 1.85484161e-01, 1.89837437e-01, -5.75337695e-01,
        6.41982258e-01, -1.57098232e-01, -2.20290597e-01,
        1.61073208e-01, -8.44021313e-01, -8.91341322e-01,
       -8.68559766e-01, 3.38251388e-01, 4.51497108e-01,
        8.17831128e-01, 7.94919740e-01, 9.71761909e-01,
        9.84505391e-01, -2.63204298e-01, -1.05362723e+00,
        1.85377925e-01, -3.04749421e-01, 2.72479074e-01,
       -8.68044333e-01, -2.48508076e-01, 8.30137204e-01,
        5.15034765e-01, 1.90894674e-01, 5.40662369e-01,
        6.45512963e-01, 6.93392658e-01, -5.98588019e-01,
       -3.86113714e-01, 7.89279838e-01],
      [-5.25615685e-01, -4.68344345e-01, 2.37584376e-05,
       -8.49223769e-01, -5.62834222e-01, -4.64468494e-01,
       -2.60553941e-01, -4.56836397e-01, 1.54690415e-01,
        1.74395769e-02, -3.74319484e-01, -2.14169929e-01,
        1.66875229e-01, 7.52755690e-01, -3.57465892e-01,
        2.49625356e-01, -5.66322016e-02, 5.82092456e-01,
       -6.29009883e-01, -3.85910481e-01, 4.04302173e-01,
       -1.53794456e-01, 6.23616500e-01, 5.04908863e-01,
        3.27964487e-01, -3.50787808e-01, -3.88843180e-01,
       -9.52319929e-01, -8.65547493e-01, 6.90464535e-01,
        7.61271638e-01, -5.05894003e-01],
      [ 5.82466438e-01, 7.01703858e-01, -2.66588724e-01,
        3.30492136e-03, 2.04459146e-01, -6.22998688e-02,
       -1.03856974e+00, 2.98982493e-01, -9.86201668e-01,
       -8.92233807e-01, -9.14474243e-01, -4.99321517e-01,
        8.34312699e-01, -7.07867716e-01, -5.74386867e-01,
       -5.67730861e-01, 3.57988543e-01, 7.28168863e-01,
       -1.75162771e-01, 7.95825869e-01, -3.52272212e-01,
        6.52397516e-01, -2.84561036e-01, -5.41577450e-01,
        9.66899997e-01, 7.13536870e-01, -5.38683031e-01,
        2.10055455e-01, -4.15494245e-01, -1.06920697e+00,
       -5.64097057e-01, 3.93038752e-02],
      [ 6.93810860e-01, 8.66425553e-02, -9.25243532e-01,
       -4.60677966e-01, -4.84122549e-01, 5.33914557e-03,
        7.85106780e-01, 8.87126781e-01, 5.91000207e-01,
       -3.56880022e-01, -7.83294246e-01, -6.34705915e-01,
       -5.78954140e-01, -1.82010381e-01, 8.81056892e-01,
        4.03915049e-01, -8.65178472e-01, -9.05844255e-01,
        6.45363846e-01, 9.77172304e-02, -4.78320396e-01,
       -8.75685667e-02, 5.50725541e-02, -9.37629321e-01,
        8.08731386e-02, 9.84484811e-01, 1.58998752e-01,
        4.93444486e-02, -7.44101213e-01, 6.85268561e-01,
       -6.53078576e-02, 7.85905841e-01]])}
{'B1': array([0.52744278, 0.64741801, 0.52126945, 0.50745116, 0.49358085,
      0.45590788, 0.53148459, 0.49328729, 0.59640836, 0.60261716,
```

```
0.49040837, 0.51833081, 0.3988355, 0.48885021, 0.35112468,
      0.53924811])}
{'W2': array([[-0.39211907, -0.85721195, -0.05785948, -0.11894905, 0.28552203,
        0.14037353, 0.78285912, -0.19317282, -0.13035841, 0.3639726,
        0.90295421, 0.90459497, 0.92640168, 0.95084925, 0.75593934,
        0.37133172],
       [0.70640044, -0.93320966, -0.48691724, -0.69047592, -0.45888284,
        0.86786799, 0.20420154, 0.35552053, -1.04213514, -0.39336306,
        -0.35134908, -0.04540253, 0.68761144, 0.77360779, -0.1885527,
       -1.02309937],
       [0.17447297, 0.66815275, 0.56257784, -0.82854649, 0.96491516,
       -0.52220533, -0.92979356, -0.42134936, 0.50323228, -0.94589797,
       -0.06230193, -0.497384 , 0.82579439, 0.56109603, -0.68796698,
       -0.40674701],
       [ 0.78412351, -0.89423582, 0.30548243, 0.66018453, 0.701093 ,
        0.08483697, -0.07871317, -0.35073796, 0.40114346, -0.95007349,
        0.23841398, 0.80322852, 0.78187415, 0.59310363, 1.04210248,
        0.853381 ],
       [0.76310959, 0.72869497, -0.44146186, 0.9119025, -0.16915071,
        0.05564149, 0.83612259, -0.26006792, -0.38641989, -0.181703 ,
        0.76147938, 0.23569024, -0.02160998, 0.28236841, -0.3353777,
        0.43503306],
       [-0.8515748, -0.77553216, 0.02893076, -0.60446448, -0.66508294,
        0.0611998, -0.48776832, -0.70377758, 0.95887351, 0.96199714,
        0.09414913, -0.0195748, 0.62345727, -0.17568507, -0.42866115,
        0.80018724],
       [0.2300981, -0.49167989, -0.75952362, 0.41001491, 0.93924947,
        0.10657324, -0.08940458, -0.56054647, -0.75201437, -0.00839819,
        0.19375444, -0.31517569, 0.46739112, -0.75534821, -0.10165073,
        0.96034956],
       [ 0.15047732, 0.61834208, 0.7713447 , 0.58862075, -0.58781266,
        -0.32578887, 0.42587872, 1.20445116, 0.60977507, 0.97341085,
       -0.66068262, 0.05257429, 0.80182712, 0.76883645, -0.386285 ,
        0.20533193],
       [ 0.28794145, -0.18151763, 0.98663779, 0.36406816, 0.26733212,
        -0.45097831, 0.95818253, -0.48731537, 0.17900809, 0.56246732,
        0.0579746 , -0.78038287 , 0.50330403 , 0.04527049 , -0.72974603 ,
        0.34024219],
       [ 0.96800298, 0.88842858, 0.99516653, 0.85857122, 0.00360315,
        0.98309447, 0.77862351, 1.03692781, -0.26307942, -0.89970706,
       -0.9319665 , 0.70827093 ,-0.63461612 ,-0.24278307 , 0.07518342 ,
       -0.78297887],
       [-0.26694814, -0.64743783, -0.96689613, 0.7253469, 0.87831359,
        0.67351568, -0.15513389, 0.22010093, -0.47083157, -0.32801575,
        0.05565517, -0.95859212, 0.01207235, -0.63020095, -0.57971538,
        0.22138042],
       [0.74231854, -0.57106104, 0.62318283, 0.60277175, -0.89732236,
        -0.23524596, -0.02754368, -0.99104612, 0.28136241, 0.83928967,
        0.3037424 , 0.71673448 , -0.69321425 , -0.0824096 , -0.99309433 ,
        0.72781256],
       [-0.79288836, -0.64570458, -0.57762914, -0.92469518, 0.98160528,
       -0.24421691, -0.54566665, 0.24432985, -0.69068534, 0.71370142,
       -0.81581636, -0.80551447, 0.84663682, -0.02676368, 0.76557768,
        0.43140539],
       [0.9992168, 0.39140756, -0.43781149, -0.68303305, 0.19031544,
       -0.01902004, 0.7809153, -0.5344656, 0.5476201, -0.65433409,
        0.95760204, 0.09650736, 0.28826399, -0.55281956, -0.96993061,
       -0.70384836],
       [ 0.16503139, 0.60811779, -0.3194037 , -0.73558303, 0.85351362,
        -1.02015572, 0.44568708, -0.67601162, -0.89132324, -0.68556271,
```

```
-0.65939191, -0.04090106, 0.75708205, 0.19234765, 0.4658718,
        0.76133112],
       [ 0.41866869, 0.94560535, 0.69029774, -0.96615935, 0.20877882,
        0.47696605, 1.01689319, 0.79482554, -0.58718642, 0.94733853,
        0.20518229, 0.46645782, -0.81190708, -0.91586776, -0.60067482,
       -0.90109466]])}
{'B2': array([0.45344317, 0.43804704, 0.49409304, 0.52675958, 0.5737221,
      0.53227874, 0.50149061, 0.56443421, 0.50691654, 0.5184838,
      0.50319882, 0.50338705, 0.4951867, 0.51348382, 0.46422412,
      0.53803177])}
{'W3': array([[ 0.40514135, -0.06309987, 0.00447166, -0.68015257, -0.46606328,
        0.71704307, -0.79474216, -0.62339401, 0.32393417, -0.1422745,
       -0.32165744, -0.2039168, 0.01767454, 0.34289544, -0.24007413,
        0.01976158],
      [0.56012657, -0.73254112, 0.88621071, -0.02248174, 0.36674581,
        1.01938553, -0.76641891, -0.26809844, 0.55925539, -0.497184
        0.46192507, 0.88116929, -0.32276011, 0.80791552, -0.20135248,
        0.41117561],
       [0.76728671, -0.6639007, -0.58949302, -0.75536576, -0.41393699,
       -0.68885362, -0.65471634, 0.191572 , -0.3212881 , -0.1596386 ,
        0.18927589, 0.57783392, -0.32540294, 0.67038502, 0.85213956,
       -1.00818903],
      [ 1.07084977, -0.11942759, 0.56730975, -0.53320399, 0.80504062,
        -0.75575184, -0.51930179, 1.23424083, 0.34066832, 0.66460817,
       -0.82810236, 0.86542582, 0.66756347, 0.53175931, -0.42522249,
        0.48303681],
      [0.74655239, 0.24279199, -0.68803804, 0.56954098, -0.74854754,
       -0.23964836, -0.34400681, -0.80456908, -0.44825672, -0.98624097,
        0.72973024, 0.2984429, -0.41331002, -0.12934685, 0.85929151,
       -0.32937829],
      [ 0.33913512, -0.96272274, 0.73186987, -0.70154972, 0.80510817,
       -0.0775732 , 0.67096978, -0.33759906, -0.26502108, 0.01991086,
       -0.799072 , 0.90324883, 0.01816652, 0.51531655, 0.91167327,
       -0.44086908],
       [-0.47507131, 0.48148275, -0.69169342, -0.34589926, 0.608733
        0.5370588 , 0.46717855, 0.99641152, 0.5646842 , -0.26743274,
        0.54367985, 0.93684135, -0.62962548, -0.23944855, 0.49835646,
        0.09458445],
      [-0.01548792, 0.29916961, 0.20109432, 1.32399298, -0.35530339,
        0.62028903, 0.15472972, 1.00820596, -0.55597824, 1.11561165,
       -0.59662874, 0.876566 , 0.70813341, 1.12177361, -0.47260279,
        1.30645925],
       [0.60585231, -0.58161492, -0.73943155, 0.16914912, 0.12359442,
        0.69179038, -0.20640241, 0.00394084, 0.64059685, -0.37278045,
        0.71927291, 0.44392081, -0.08628115, -0.88699897, -0.47412348,
        0.17777885],
      [-0.46058535, -0.63996726, -0.17839583, 0.86240219, -0.98821227,
        0.10888624, -0.33539367, 0.61279742, 0.18891291, -0.05679742,
       -0.8610192 , -0.69551827, -0.43128095, 0.65223341, 0.25953371,
        1.0769651 ],
      [0.50391404, -0.37962954, 0.808759, -0.36162111, -0.61095574,
        0.9440685 , -0.9070952 , 0.73641805, 0.55089759, -1.22264114,
        0.81784827, 0.507832 , 0.09104002, 0.31276343, -0.33299867,
        0.53774955],
      [-1.10600986, -0.04362382, -0.32811194, -0.11548215, -0.98638883,
        0.5245861 , -0.06699116 , 0.1548176 , 0.66125118 , 0.72112408 ,
       -0.7453031 , 0.34468362, 0.0564805 , 0.42837837, 0.79635974,
        0.56629373],
       [0.18850909, 0.41039633, -0.16558396, 0.01807652, -0.39958149,
       -0.90804722, 0.03879288, -0.34509798, -0.69558074, 0.72620216,
```

```
0.28803862, 0.37001449, 0.18751458, -0.65214508, 0.82848513,
         0.68248464],
       [-0.51260467, 0.35974717, 0.54684954, -0.42320897, -0.66951586,
        -0.58816107, -0.83913038, -0.309168 , 0.02219574, 0.89759824,
        -0.8011127 , 0.6822883 , -0.63945912, -0.53658532, -0.59162569,
        -0.0532141 ],
       [-1.05772874, 0.18300736, -0.65330729, 0.09241716, -0.14120979,
        -0.63756638, 0.82392264, -0.09399829, 0.16920453, 0.07537282,
        0.49721645, -0.53653501, 0.72382943, 0.31731686, -0.49014331,
         0.70172158],
       [-0.46751439, 0.81632077, 0.05563707, 0.91693243, 0.42970813,
        -0.0638794 , 0.28198291, -0.21712884, 0.12806715, -0.68394137,
        -0.06678651, 0.23978796, 0.26914216, -0.97949784, 0.68864641,
        -0.61580391]])}
{'B3': array([0.47412365, 0.51164153, 0.48423689, 0.55921915, 0.50001228,
      0.49982586, 0.50514137, 0.56983006, 0.50899383, 0.51734298,
      0.44694731, 0.48620112, 0.5175672, 0.49493187, 0.45454214,
      0.49692697])}
{'W4': array([[-0.45942746, 0.55674512, -0.13877797, -0.37979569, 0.24473661,
       -0.58511447, 0.88850228, 0.18568081, 0.14409015, -0.72931712,
        -0.63550121, -0.02439618, 0.96324496, -0.34248203, -0.37355921,
       -0.97815383],
       [ 0.97439512, 0.14506516, -0.8888176 , -0.12448852, 0.583695 ,
         0.92738317, 0.39478346, 1.40082425, 0.01095813, 0.50211615,
        -0.58515303, 1.13291879, -0.51186864, -0.11306212, 0.23990407,
        -0.90934495],
       [ 0.32239839, -0.20061169, 0.91060922, -0.50399651, 0.5667696 ,
         0.19651178, 0.60788751, -0.32475139, -0.24487034, -0.08545436,
         0.85074101, -0.36072552, 0.87349341, 0.60337552, 0.38089077,
        0.77182135],
       [ 0.86921764, 0.44368795, -0.8574338 , -0.72589289, 0.28984464,
         0.75414826, 0.29388232, 0.36246088, 0.30000635, -0.11883045,
       -0.79459657, -0.13017828, 0.97160707, 0.18036612, -0.66343565,
        -0.23311679],
       [ 0.82035229, 0.39208505, -0.10820771, -1.00518585, 0.09152193,
        -0.3175418 , -0.35824243, -0.83378738, 0.24378321, 0.53377377,
        0.10159942, -1.05781619, -0.21467706, -0.55137339, 0.44462494,
        0.45203379],
       [0.77610342, -0.07925858, 0.09946524, -0.39125533, 0.58366998,
        -0.46178337, -0.78240214, -1.22036567, 0.50139654, -0.98196714,
        -0.13390873, 0.7138413, -0.64519454, -0.24225318, 0.20737352,
         0.59135173],
       [-0.57515071, 0.27758665, -0.6643944 , -0.60223411, -0.019941 ,
         0.60673006, 0.60715921, -0.38363124, -0.15618482, -0.82418316,
        -0.96767592, 0.47558212, -0.46384037, 0.58657959, 0.67623915,
        -0.55460389],
       [-0.34317979, -0.94092619, 0.88512056, 0.20011103, -0.529923 ,
         0.79420787, -0.587174 , 0.60877872, -0.43276421, 0.2916031 ,
        -0.50940987, 0.29912302, 0.04437085, 0.95128245, 0.5711632,
         0.24160165],
       [0.09779586, -0.07105538, -0.33683116, -0.31549847, -0.21456459,
        0.25573084, 0.70901351, -0.38105238, 0.05001975, -0.91123598,
        -0.11217879, 0.07616276, 0.46537421, 0.44196259, -0.26816274,
        -0.69659595],
       [-0.49789565, 1.00373164, -0.96090939, -0.20974857, 0.85254364,
        \hbox{-0.79898446,} \quad \hbox{0.905269} \quad \hbox{,} \quad \hbox{1.14182464,} \quad \hbox{0.66679412,} \quad \hbox{-0.40622686,}
        -0.59387778, -0.50559146, 1.08142832, 0.48122549, 0.62136786,
       -0.40751241],
       [0.01264231, -0.44954084, 0.84912342, 0.91133707, 0.91847359,
         0.43373552, 0.68856485, -0.4168292, -0.82997174, -0.40822279,
```

```
0.39595568, -0.05555057, 0.09732001, 0.10405217, 0.6783524,
        0.91953227],
      [-0.51938442, 0.07294621, -0.45315994, -0.85103824, -0.16443874,
        0.53996848, -0.10695716, 0.3043536, -0.82971285, 0.77615628,
        0.59982803, 0.9055684, -0.72655429, 0.59290891, -0.85526598,
        0.08522276],
      [-0.2015366, -0.78361943, -0.97240136, 0.34441644, -0.93749258,
        0.41240637, -0.78842757, -0.66521388, -0.30787907, -0.44190152,
        -0.57920533, 0.53065308, -0.90930268, 0.74532753, -0.53965484,
       -0.03199129],
       [-0.7655565, -0.86243564, 0.74798971, 0.7679428, 0.59654383,
       -0.34197367, 0.24094359, 0.08977391, -0.44314594, 0.15584919,
        -0.28359043, -0.17141289, -0.68644725, -0.69546265, -0.52116961,
       -0.93241082],
       [0.02327448, -0.00321274, 0.10603066, 0.94587073, -0.39600523,
        0.44146545, 0.93905005, -0.8099026, -0.07261247, -0.29241534,
        -0.91809728, -0.01726102, -0.40223318, -0.57989054, -0.40077578,
       -0.37478513],
       [ 0.30822012, 0.99506489, 0.97800812, -0.92947232, 0.08047389,
        0.02140652, -0.38987846, -0.76980172, -0.40998887, -0.87516079,
        -0.80596623, 0.40562471, -0.52990534, 0.7245955, -0.8913628,
       -0.87382439]])}
{'B4': array([0.49991911, 0.53251419, 0.49275724, 0.49984953, 0.46966592,
      0.4729307 , 0.47372738, 0.49789524, 0.5
                                                  , 0.52369411,
      0.50964112, 0.48879887, 0.5
                                    , 0.49799808, 0.50197006,
      0.5
                ])}
{'W5': array([[ 4.68437519e-01, 1.32004033e-01, 1.82258588e-01,
       -9.54015735e-01, -7.33416169e-01, -2.77290985e-01,
       -7.42451102e-01, 6.05178598e-01, -8.56431502e-01,
        1.82879662e-01, -4.93744708e-02, 9.70254680e-01,
       -8.40572461e-01, 3.74104859e-01, 6.75902352e-01,
       -9.27915705e-01],
      [-4.07983132e-01, -6.61386734e-02, -1.60581834e-01,
       -7.49607550e-01, -7.63595226e-01, -3.69831583e-01,
        -1.03223133e-01, -1.23712551e-01, -3.49045975e-01,
        4.39490941e-01, -3.30888734e-01, 2.30332163e-01,
        3.77924719e-01, -8.25226285e-01, 6.81036307e-01,
        -6.65235404e-01],
      [-1.53574727e-01, 1.00274355e+00, -6.40601372e-02,
       -9.17856433e-01, 3.19356170e-01, -4.06354433e-01,
       -4.87250460e-01, 7.23704290e-01, -7.72746162e-01,
        4.80761263e-01, 2.61123132e-01, 4.39768395e-02,
        4.54739148e-01, -2.90004090e-01, -4.92904440e-01,
       -9.62907797e-02],
      [ 8.86223880e-01, -3.18167077e-01, 8.08087753e-01,
        -2.05974462e-01, 2.22209257e-01, -8.94875301e-01,
        4.58070746e-01, 2.69704936e-01, -9.83938254e-01,
        6.62932174e-01, 8.35548147e-01, -8.20681017e-01,
       -6.32952287e-01, 9.05337998e-01, 2.61159025e-01,
        3.18724162e-01],
      [-8.47605033e-01, -1.20892453e+00, -3.54381277e-01,
        5.50999759e-01, 8.14211877e-01, 8.04128169e-01,
        6.18399948e-01, -5.81227876e-01, -9.02361221e-01,
       -8.32023496e-01, -4.50217879e-01, 6.98020419e-02,
        5.71502191e-01, 9.14183684e-01, 1.11661489e-01,
       -7.86306594e-01],
       [-8.08431195e-01, -6.41989502e-01, -8.85932131e-01,
        -1.67407501e-01, -5.71219925e-01, -5.43030244e-01,
        5.07218899e-01, -5.78888490e-01, -1.57904385e-01,
       -9.99893027e-02, 7.46329745e-01, 1.30199122e-01,
```

```
4.97322332e-01, 3.54552348e-01, 7.80175344e-01,
 5.19150651e-01],
[ 5.85604552e-01, 4.43108852e-01, 9.41155550e-01,
 -9.65056646e-01, -3.66363004e-01, 2.69819561e-02,
-8.24511637e-01, -3.05045624e-01, 3.32980870e-01,
 9.81861693e-02, 7.38671398e-01, -1.72900308e-01,
-7.81346925e-01, -4.24060241e-01, -7.48167183e-01,
 5.89221037e-03],
[-3.72562132e-02, -8.66575273e-01, -8.42399971e-01,
 7.35974683e-02, -1.63822390e-01, -9.99875026e-02,
 1.56964045e-01, -2.49789700e-01, 5.31117816e-02,
 -1.29669982e-01, -1.36536262e-01, 5.13153392e-01,
 4.30599506e-02, -6.31340172e-01, 3.95721137e-02,
 3.46418220e-01],
[ 9.98665287e-01, 6.56206876e-01, 4.96564659e-01,
 7.39952369e-01, 9.98614336e-01, 3.42668967e-01,
 7.19083859e-01, 7.00737327e-01, -1.26897790e-01,
 5.28611981e-01, -3.78147657e-01, -5.25972806e-01,
 -8.94843239e-01, -4.24751813e-01, 9.59607979e-01,
 -3.27392016e-01],
[-3.05432073e-01, -3.28154700e-01, -9.99754265e-01,
 -9.75877087e-01, 2.91796432e-02, 5.19949349e-01,
 8.69055127e-01, -1.88515902e-01, -7.89399632e-01,
 -7.79134139e-01, 4.48448106e-01, -7.57742397e-01,
-1.35316538e-01, 3.87895955e-01, 9.42915647e-01,
 3.09043698e-01],
[ 7.13038499e-03, 4.68464667e-01, -4.12069134e-01,
 4.90952588e-01, 9.15994556e-01, 4.45459385e-01,
-7.43319558e-01, -5.76472765e-01, -9.42446800e-01,
-7.09122131e-01, -8.60301480e-01, 1.63781761e-01,
-3.71239722e-01, -5.39646540e-01, -7.42423036e-01,
-8.68660290e-01],
[ 5.85460801e-01, 6.79121799e-01, -7.09404110e-02,
 1.40198181e-02, -8.79289124e-01, -4.73243454e-01,
 3.02477967e-01, -7.06151944e-01, 5.10515331e-02,
 8.41094546e-01, -8.92473327e-01, -1.70937783e-01,
 8.97514326e-01, -9.07364909e-01, -1.28966256e-01,
 8.56069429e-01],
[ 4.33987839e-01, 9.03542567e-01, -4.75296598e-01,
 -8.59104313e-01, -7.78288052e-01, 2.72610809e-01,
-2.54309930e-01, -9.36205848e-01, 2.90496606e-01,
 -6.01054820e-02, -8.08030510e-01, 6.12515151e-01,
 -8.70869529e-01, -1.99922149e-02, 3.55530298e-01,
 7.17957561e-01],
[-2.94737433e-01, 1.78938352e-01, 1.29175823e-01,
 6.42170223e-01, -8.16387384e-01, 6.52504616e-01,
 -1.74095690e-01, 9.98032328e-01, 2.10371518e-01,
 8.66445881e-01, 1.86539194e-01, -9.23217204e-01,
 1.57048165e-01, -5.31677578e-01, 2.13668351e-01,
 -4.10507403e-02],
[ 8.45923684e-01, 1.21226768e-01, 9.35312838e-01,
 -4.88074895e-01, 4.23079190e-01, 3.13808510e-01,
 -9.65347533e-01, 3.85171954e-01, -2.30065746e-01,
 8.36920248e-01, -2.17724071e-04, -8.54336584e-01,
 2.08650250e-01, 1.74500459e-01, 4.05245732e-01,
 -2.39854652e-01],
[-9.94013605e-01, -1.50116053e-01, 5.95196768e-01,
 6.96191422e-02, -6.62494801e-01, 5.75460990e-01,
 9.18472985e-01, -2.72495295e-01, 6.92286749e-01,
 2.27255431e-02, -5.71591239e-01, -3.38396444e-01,
```

```
-8.27708913e-02, 6.85262312e-02, -2.18224353e-02,
                5.58103276e-01]])}
       {'B5': array([0.50019306, 0.49986901, 0.50007161, 0.50010097, 0.46669321,
              0.50124327, 0.50026062, 0.49989151, 0.4982677, 0.49338662,
              0.49991094, 0.50027223, 0.50315545, 0.5001429 , 0.50014689,
              0.49958588])}
Out[]: <machine_learning_functions.Model at 0x1e842833d50>
In [ ]: def complex_mathematical_function_experiment(function, epochs, learning_rate, hi
            neurons_per_layer_list = [input_neurons] + [neurons_per_hidden_layer]*hidden
            activation_functions_list = [Sigmoid() for _ in range(hidden_layers)] + [Non
            model = Model(
                FFN = FFN(
                    neurons_per_layer_list=neurons_per_layer_list,
                     activation_functions_list=activation_functions_list,
                    cost_function=MSE()
                ),
                data_set=create_a_inputs_b_outputs_XY_data(
                    a=input neurons,
                    b=output_neurons,
                    function=function,
                     random_x_function=random_x_generator,
                     num_data_items=10000
            )
            print(
                model.train_and_evaluate(
                    learning_rate=learning_rate,
                    epochs=epochs,
                    batch_size=50
            model.print_FFN_parameters()
            return model
In [ ]: def pythagerous_function(X):
            a, b = X
            c = (a**2 + b**2)**1/2
            return np.array([c])
        complex mathematical function experiment(
            function=pythagerous function,
            input_neurons=2,
            output neurons=1,
            random_x_generator=lambda: random.uniform(-10, 10),
            hidden_layers=4,
            neurons per hidden layer=10,
            epochs=20,
            learning_rate=10**-4
```

## 826.1524059854028

```
Parameters of network
{'W1': array([[ 0.56304594, -0.70286613],
       [-4.45720007, 5.76080034],
       [-0.14882578, 0.78182942],
       [-0.42953618, 0.09620127],
       [ 0.06753011, -0.8559187 ],
       [ 0.44008997, 2.02998988],
       [ 0.20274 , -1.11975257],
       [ 1.1316689 , -0.21597949],
      [-0.47337577, -0.38376731],
       [ 0.26034373, -0.74415172]])}
{'B1': array([0.46792926, 1.86286723, 0.49587635, 0.45561988, 0.38114442,
       0.70457421, 0.47414505, 0.39452113, 0.52001625, 0.47902841])
{'W2': array([[-0.99656534, -0.0291583 , 0.27362312, 0.85685396, -0.288289
       -0.78383398, 0.41722696, -0.36665639, -0.94868803, 0.5635501 ],
       [ 0.65278481, -0.92997819, 0.47758893, 0.4613792 , 0.82786383,
         1.00139893, 0.5132631, 0.41194329, 0.84236233, -0.42041361],
       [0.720952, -0.16571122, -0.61105809, 0.13080033, -0.42074204,
        -0.23679621, 0.15344648, -0.98942997, 0.11656082, 0.18263149],
       [\ 0.94734904,\ -0.54561613,\ 0.70456828,\ -0.0767369\ ,\ -0.05077682,
         0.86704126, 0.0860287, 0.30713814, 0.16668957, -0.07736793],
       [0.88078037, -1.08881536, -0.32139535, -0.22129862, -0.2911662]
        -0.9506279 , -0.33667074, 0.81666622, -0.78072451, 0.53646143],
       [-0.25289453, -0.01218813, 0.27771347, -0.21624222, -0.08445848,
        -0.78130593, -0.93759185, 0.4639343, 0.58246362, -0.7311715],
       [0.54698691, -0.36398505, -0.63279683, -0.92463633, 0.98515896,
        -0.16439991, -0.10322686, 0.96923633, 0.6952061, -0.08393717],
       [0.96950517, -0.69220255, -0.1002693, -0.04357604, -0.2864734]
         0.24224327, 0.5867324, 0.71561956, -0.04078272, 0.15248874],
       [-0.06861817, 2.36522031, 0.02153851, 0.84809754, 0.45891475,
        -0.1438247 , -0.67395617, -0.44503888, 0.35250002, -0.51411047],
       [0.95202452, -0.59402296, -0.78792237, -0.44811339, 0.68962874,
        -1.03509856, -0.47956526, -0.65890561, 0.20792172, -0.83500176])}
{'B2': array([ 0.68200892, 0.49673724, 0.66838566, 0.41793783, 0.64873287,
       0.53540267, 0.56229157, 1.37361009, -1.53777074, 0.32178897))
{'W3': array([[-2.31214103e-01, -2.09877841e-01, 7.15622597e-01,
         4.47690220e-01, -1.39923828e-01, 7.52713700e-01,
        -8.38045373e-01, -7.84500704e-01, -4.68184414e-03,
         1.41693473e-01],
       [ 5.41562292e-01, 6.72181952e-01, -3.14773468e-01,
         7.56776533e-01, -2.78797383e-01, -2.42835835e-01,
         4.97739834e-01, 4.85024247e-01, 7.33692920e-01,
        -9.22909956e-01],
       [ 3.54589310e-01, -1.87239130e-01, 1.29219082e-01,
        -3.91211913e-01, 1.00536530e-01, 4.97761596e-01,
        -2.94919385e-01, -4.75037087e-01, -9.79481043e-01,
         9.27021298e-01],
       [-8.58147660e-01, -4.54787923e-01, 3.77907224e-01,
         5.74362895e-01, 6.21561235e-01, 5.46966327e-01,
         3.76762915e-01, -9.15224328e-01, 4.24053755e-01,
        -4.52386871e-01],
       [ 2.07430496e-01, -8.74862929e-01, -6.28715317e-01,
        -2.15505015e-01, 7.85029946e-01, -7.07327206e-02,
       -8.09618934e-01, 4.50426474e-01, -2.22748444e-02,
        -7.85920603e-01],
       [ 6.18837800e-01, -5.85067364e-02, -5.42871392e-01,
         6.69732880e-01, -5.93747737e-01, -1.01329981e-01,
         7.61350984e-02, -5.98589768e-02, 9.17247590e-01,
         7.67449794e-01],
```

```
[ 2.04698608e-01, -1.72432911e-02, 2.13372101e-02,
        4.21551252e-01, -2.33416582e-01, 4.98089055e-01,
       -2.49445926e-01, -3.21153145e-01, -2.01186827e-01,
       -5.26222006e-02],
      [-9.77724042e-01, -1.14241772e-01, -6.70493134e-01,
        7.12484256e-01, 6.33963721e-01, 3.17091563e-01,
        3.24749275e-01, 5.90053525e-01, 4.10184210e-01,
       -1.52003708e-01],
      [-9.70441309e-01, -1.57155112e-01, -8.25197515e-01,
        -1.77848950e-01, -8.38765226e-01, -5.08501582e-01,
       -4.75789789e-01, -2.47536193e+00, -1.02686077e+01,
        3.35061023e-011.
       [ 3.77707019e-01, 6.29077774e-01, -6.93377506e-01,
        -9.74063050e-01, -8.12422453e-01, -1.02266400e-01,
       -4.25391988e-01, -8.98064604e-01, 1.50167435e-01,
        9.08216701e-01]])}
{'B3': array([ 0.54831008, 0.52879622, -0.01269036, 0.81713357, 0.39942751,
       0.64257347, 0.26821781, 0.67760159, -7.73356512, 0.39889383])
{'W4': array([[-5.16422782e-01, -1.46960497e-01, 9.35940209e-03,
        3.27473817e-01, 5.84115558e-01, 4.71105032e-01,
        -8.70919318e-01, 9.02853765e-01, -1.31651149e-01,
       -8.83792555e-01],
      [ 7.62562715e-01, 4.99888031e-01, 3.15463444e-01,
        8.73992862e-01, 1.75297476e-01, 4.59673467e-01,
       -5.41229488e-01, -5.62673859e-01, 3.72082534e-01,
        2.56817552e-01],
      [-3.74636641e-01, 1.03111044e-01, -7.75192761e-01,
        8.47565085e-01, -4.89983542e-01, -6.53818670e-01,
       -2.21405503e-02, -1.49832729e-01, -8.02786793e-01,
        1.03012986e+00],
      [ 7.57428623e-02, 8.09695462e-01, -6.84128316e-01,
        -3.87805692e-02, -3.39088429e-01, 3.25764426e-01,
        6.60505074e-01, 8.47598686e-01, 3.86575787e-01,
       -5.81912728e-01],
       5.78005574e-01, -6.80676678e-01, 8.79234651e-01,
        -1.01053745e+00, -1.33264699e-02, 9.21959287e-01,
        1.47725604e-01, -9.67841794e-01, -1.56834694e-01,
        6.24901289e-01],
      [ 2.87310836e-01, 4.98161578e-01, 2.99886614e-01,
        6.04066989e-01, -4.94507870e-01, 4.48564657e-01,
        3.62245273e-01, -1.60662795e-01, 4.77690017e-01,
        3.76005981e-01],
      [-8.73081874e-01, 8.32546238e-01, -2.21870688e-01,
        4.75679475e-02, -2.13019748e-01, -7.26833788e-01,
       -7.20236303e-01, -4.18960322e-01, -5.43470761e-02,
        1.37287770e-01],
      [ 9.47674221e-01, -6.79126374e-01, -8.20440921e-01,
       -6.07226532e-01, 4.06647513e-01, 5.36074389e-01,
       -4.27773570e-01, -1.63736468e-02, 9.29100414e-01,
       -5.07708379e-01],
      [ 7.92879662e-01, 4.80877734e-01, 9.95582169e-02,
        9.58284160e-01, -4.15554171e-01, 8.23359019e-02,
       -8.84314615e-01, -4.29598720e-01, 9.90482540e-01,
       -3.18897121e-01],
      [-4.15443913e-01, 4.41321260e-01, -8.81867808e-03,
       -1.25404826e+00, 1.17109450e-01, -8.84881925e-01,
        5.28596273e-01, 5.52336359e-01, -1.22253184e+01,
        1.35936938e-01]])}
{'B4': array([ 0.85472172, 1.15389946, 1.08288752, 0.95161208, 0.18336971,
       0.59971387, 0.31172581, 0.86071106, 0.64176426, -5.04995126)
```

```
{'W5': array([[0.88613741, 1.52398442, 1.28366721, 1.27979239, 0.03170021,
               0.48919392, 0.31150676, 0.9005003, 0.59097771, 0.78051047]])
       {'B5': array([15.97118776])}
Out[ ]: <machine_learning_functions.Model at 0x1e842ac7250>
In [ ]: coefficients = [random.uniform(-1, 1) for _ in range(10)]
        def polynomial(X):
            x = X[0]
            result = sum(
                coefficients[i]*x**i
                for i in range(10)
            return np.array([result])
        complex_mathematical_function_experiment(
            function=polynomial,
            input_neurons=1,
            output_neurons=1,
            random_x_generator= lambda: random.uniform(-10, 10),
            hidden_layers=4,
            neurons_per_hidden_layer=10,
            epochs=20,
            learning_rate=10**-4
```

```
3.334399902288701e+16
Parameters of network
{'W1': array([[-103.94643444],
      [-928.46176283],
      [ 38.58359977],
      [ 14.93198346],
      [ 136.88349599],
      [ -15.12756174],
      [ -38.04405694],
      [ 37.27344988],
      [ -85.47113308],
      [ 55.25004604]])}
{'B1': array([ 8.71810716, -94.77411351, -5.02707505, -24.18040157,
      -14.67144544,
                    9.28587638, -36.60329994, -3.81151203,
      -12.75633944, -3.72147446])}
{'W2': array([[ -0.98802522, 0.42905777, -0.66801885, 1.57875617,
         0.74616094, -1.41459159, -5.71679553, -0.03933153,
        -0.60728857, 0.97578554],
      [-0.04280951, -0.96165479, 0.11832701, -0.59664139,
        -0.32016336, 0.3292144, 3.02099164, 0.27880595,
         0.26307068, -1.5617449],
      [ 16.52942064, -3.39167586, -6.39990724, 1.24514889,
        11.39901969, 4.24793422, 1.74686134, 4.78495963,
        -1.38773846, 1.1572887 ],
      [-0.05037074, -1.49871656, 1.82117988, -1.15952339,
        -1.59921161, 0.85903612, 2.72694364, 0.24390972,
         0.80834524, -1.25662419],
      [ 1.191204 , -0.42966708, 0.50798767, -0.20531313,
        -0.36798615, 0.59235303, 1.09480574, 0.76508775,
        -9.59727683, -1.26634825],
      [-0.25554368, 0.84675404, 2.39457224, -1.47724954,
         0.22543133, -1.56919299, -15.30803408, -1.78405598,
        -1.09589949, 1.37529248],
      [ 2.26006885, -0.34422506, 0.83240423, -1.67270418,
                                  4.52851992, 0.48221192,
        -2.09439076, 0.9108908,
         1.77593914, -0.55997113],
      [ 1.34332049, -1.3883745 , 2.28989253, -1.36223411,
                                                0.15897135,
        -2.82855135, 2.41030029,
                                   6.1652899 ,
         0.54320037, -1.49595565],
      [-0.16020459, -0.66754811, 0.20660561, -10.52305821,
         1.12425884, -0.07236424, -0.16179095, 0.74747351,
        -0.86627152, -0.47016942],
      [ 1.1070267, 9.66714712, 0.52360398, -1.5689415,
        -0.84257192, 7.60896213, 4.09096542, 0.47229777,
        -0.2856676 , -0.62924215]])}
{'B2': array([ -59.29893157, -62.66930842, 3023.36014016, 60.50843407,
        69.4328883 , -83.25979194,
                                    65.84455839, 69.57029078,
        71.27698817, -98.59564246])}
{'W3': array([[-10.19924662, -12.23138468, -19.83211509, -14.78154457,
       -15.75230311, -19.98309521, -14.89142581, -15.82921216,
       -18.00576862, -14.93105562],
      [ -9.40220465, -10.1917058 , -18.09925661, -14.64458142,
       -15.37556525, -17.55608083, -14.90898992, -14.60253552,
       -15.87585112, -13.65035958],
      [ 1.58155289, 3.61775053, 11.13642952, 5.34271382,
         6.50544148, 4.39249079, 4.22521702, 4.54861528,
         6.6263636 ,
                     4.32185522],
      [ -8.77369467, -10.40985887, -21.4929011 , -17.90774408,
        -21.23055106, -21.26396254, -19.64667854, -20.6393511 ,
       -21.3096736 , -17.4384872 ],
```

```
[ -4.86728952, -4.19014657, -6.57915394, -4.38523989,
       -11.5604303 , -9.07090026,
                                   -6.7057343 , -6.43783128,
        -7.57849745, -6.87097605],
      [ 0.60672271, 2.74862926, 11.17632932, 3.41071416,
         4.64284261, 1.88965471, 4.21520823, 4.18759635,
         4.46517626, 4.08708999],
      [ 0.49301444, 2.92056897, 9.66658771, 2.27646889,
                                   2.79988563, 2.93668302,
         3.60186517, 0.10363005,
         3.67289396, 3.63720059],
      [ 1.99986041, 5.5364781 , 12.40157686, 5.81224008,
         7.32801763, 7.02188433, 7.38575807, 7.35359668,
         8.96089013, 6.32458308],
      [-3.43293137, -1.23898285, -8.78648713, -10.42487978,
        -5.73193073, -7.64074066, -5.88023026, -6.22116232,
        -7.15275867, -4.38290511],
      [ 5.57499085, 6.79485484, 17.43536851, 8.44940893,
        11.69106305, 9.14185636,
                                   9.94424307, 9.42009171,
        10.33764424,
                     8.33667782]])}
{'B3': array([ -9.00950814, -91.40745596, 327.54111251, -956.98391997,
       110.19543539, 369.90136649, 351.9065375, 387.60120441,
       119.57923448, 507.19611729])}
{'W4': array([[-5.04634256e+00, -4.24621402e+00, 8.69043021e+00,
       -1.12899848e+01, -4.33738170e+00, 9.46546519e+00,
        8.52635611e+00, 7.90301052e+00, -4.17849133e+00,
        1.10592278e+01],
      [-2.37264019e+00, -1.76828904e+00, -9.83770859e+00,
        4.36433965e-01, -5.59499738e-02, -8.93613052e+00,
       -1.08650225e+01, -9.35795724e+00, -2.83145238e+00,
       -1.18097308e+01],
      [-2.11522330e-01, -4.48144579e-01, 8.98454136e+00,
       -6.38413669e+00, -1.15702797e+00, 9.99697313e+00,
        1.14268948e+01, 9.11605405e+00, 4.15582105e-01,
        1.26308648e+01],
      [ 6.93392859e+01, 7.86760363e+01, -2.11845795e+01,
        8.54454841e+01, 4.13155280e+01, -3.49922603e+01,
       -2.75234803e+01, -3.44955774e+01, 4.97862730e+01,
       -6.45212140e+01],
      [-1.79104056e+00, -2.02271832e+00, 8.81151560e+00,
       -8.70617597e+00, -1.44801824e+00, 9.39638794e+00,
        9.77582612e+00, 9.04099232e+00, -1.04388372e+00,
        1.03274786e+01],
      [ 1.00134255e+02, 1.13489185e+02, -2.99374501e+01,
        1.23851579e+02, 5.98034850e+01, -4.95595433e+01,
       -3.90830474e+01, -4.91981032e+01, 7.20404431e+01,
       -9.36849511e+01],
      [ 4.35963369e+01, 4.89876312e+01, -1.40340441e+01,
        5.39387780e+01, 2.60201381e+01, -2.30485750e+01,
       -1.84134930e+01, -2.30687947e+01, 3.08571319e+01,
       -4.15009886e+01],
      [-6.08628197e+00, -4.71520419e+00, 7.96517814e+00,
       -1.18032676e+01, -3.07961214e+00, 8.56564251e+00,
        1.02220795e+01, 9.65478745e+00, -3.79090751e+00,
        1.11935165e+01],
      [ 2.86784736e+01, 3.12505266e+01, -1.13855804e+01,
        3.74044655e+01, 1.89474222e+01, -1.74207446e+01,
       -1.36660773e+01, -1.66331969e+01, 1.99571574e+01,
       -2.89271121e+01],
      [ 1.61435254e+00, 1.33872728e-01, -7.50241474e+00,
        3.92192422e+00, 1.74441198e+00, -8.61356911e+00,
       -9.36719730e+00, -7.15302022e+00, -1.91534724e-01,
```

```
-1.12063221e+01]])}
       {'B4': array([12338.54935797, -8390.20764028, 12456.20719462, -8465.95343807,
              12263.90751826, -9419.26871304, -9215.35888706, 12822.43313807,
              -9576.32577688, -8358.4833314 ])}
       {'W5': array([[208.83126933, 55.70998384, 212.18839685, 48.89116254,
               206.77133928, 61.39033189, 72.52987001, 222.75088954,
                83.66332549, 54.96186237]])}
       {'B5': array([7875123.67793858])}
Out[]: <machine learning functions.Model at 0x1e842a80910>
In [ ]: def area_under_polynomial(X):
            # this find the area under some polynomial given by a set of coefficients wi
            a, b, coefficients = X[0], X[1], X[2:]
            antideriavitve = lambda x: sum(
                coefficients[i]*i*x**(i-1)
                for i in range(1, 10)
            )
            integral = antideriavitve(a) - antideriavitve(b)
            return np.array([integral])
        model_integrate = complex_mathematical_function_experiment(
            function=area_under_polynomial,
            input_neurons=12,
            output_neurons=1,
            random_x_generator= lambda: random.uniform(-10, 10),
            hidden_layers=4,
            neurons_per_hidden_layer=10,
            epochs=20,
            learning_rate=10**-4
```

## 2.1909708833412882e+18

```
Parameters of network
{'W1': array([[-3.42435081e+03, 6.20589985e+03, 5.24664565e+03,
        -3.31644773e+03, -1.01306892e+04, 3.30148138e+03,
        -1.36195433e+04, -8.17296306e+03, 9.31025815e+03,
        -9.26415117e+03, -1.59138959e+04, -2.86481845e+02],
       [ 6.90868680e+02, 4.37287117e+02, 4.01278431e+02,
         1.58047271e+02, -4.08040017e+02, 3.95926061e+02,
         5.39872022e+02, -4.86699057e+01, 3.70689019e+02,
         6.17220469e+01, -1.42281162e+02, -9.97819215e+01],
       [ 3.26573267e+02, 4.85583280e+02, -5.57248031e+01,
         4.24779321e+02, -2.54115074e+02, 2.86803779e+02,
        -5.28479806e+01, -5.97558282e+02, -1.70677935e+02,
         2.89607385e+02, -2.23927139e+02, -3.45059606e+02],
       [-1.50334357e+02, 3.13117337e+02, -2.52817286e+02,
         1.76732034e+01, -9.54030555e+01, -1.03988684e+02,
        -1.14339851e+02, -7.29703572e+01, 2.18701227e+02,
       -7.08330829e+01, -5.98659200e+01, 3.45379990e+02],
       9.48136487e+02, 1.81636488e+03, -8.35427054e+02,
         9.90313009e+02, -1.39064261e+01, -1.32771881e+03,
         1.28612433e+02, -1.99549595e+03, -8.09694826e+02,
         4.65300258e+02, -8.76238837e+02, -7.88282660e+02],
       [-8.28067995e+02, 3.55813454e+02, -1.90152796e+02,
        -4.32814560e+02, -7.64764349e+02, -7.38149018e+01,
        6.63255525e+02, 3.71920043e+02, 4.23701234e+01,
        -6.50687849e+02, -1.50426568e+02, 2.62313303e+02],
       [ 1.00291708e+03, 2.44479361e+02, -4.15239240e+02,
        -1.04678401e+03, -3.10871105e+02, -1.00100044e+02,
         1.74362498e+02, -1.01811487e+03, 8.67370967e+01,
        -7.23112504e+02, -8.04000115e+02, -1.11126778e+03],
       [ 1.81893121e+01, 1.60068388e+03, 1.05616458e+03,
        -1.19440387e+03, 2.62958989e+02, -8.73511229e+02,
         3.93043656e+02, 2.86667038e+02, 5.94614843e+02,
         3.10704525e+02, -8.38707112e+02, 4.34875938e+02],
       [ 5.58417578e+02, -1.04013598e+03, 5.50988975e+02,
         7.70522124e+02, 1.01973907e+03, 3.40647359e+01,
         4.88314625e+02, -1.55221476e+01, 1.01952349e+03,
        -4.90717358e+02, 4.94995436e+02, -1.63988843e+01],
       [ 2.80611032e+02, 5.40463745e+01, 2.19815233e+02,
         3.79218309e+02, -1.90671264e+02, -1.18224354e+02,
         2.43642576e+02, 4.91949267e+01, -2.47743458e+02,
         3.04544885e+02, -9.41606916e+01, 1.82852615e+02]])}
{'B1': array([-2316.2847465 ,
                                -7.51737667,
                                               22.20967385, -30.10761437,
        -121.86436639, -38.36966537, 24.55394701, -30.27825722,
                        13.08771805])}
         98.37576158,
{'W2': array([[ 6.47744171e+01, -1.53801897e+00, -5.93131756e+00,
        -1.54822073e+00, 1.46833593e+01, -1.03174659e-01,
        6.76062291e+00, 6.42060771e+00, 3.93864138e+00,
        -2.09557830e+00],
       [ 6.05518306e+01, -2.06335277e+00, -3.57556133e+00,
        -2.84923038e+00, 1.09263915e+01, 8.96721601e+00,
        4.84379289e+00, 1.21619704e+01, 6.37123849e+00,
        -2.50816321e+01],
       [ 1.04853789e+02, 1.06711097e+00, -5.84389709e+00,
        -3.67977827e-01, 2.86127557e+01, 8.37164223e+00,
        1.21383442e+01, 1.20717609e+01, 7.28349358e+00,
        -1.55359266e+01],
       [ 7.14375727e+01, 5.57820761e+00, -5.06552182e+00,
        -2.05061901e+00, 1.86898694e+01, -4.14883817e+00,
        8.32685518e+00, 6.44602545e+00, 5.38287680e+00,
```

```
4.95891653e+00],
       [ 1.53709364e+02, 6.44546209e+00, -9.72565914e+00,
        -3.62234328e+00, 2.83490661e+01, -4.50868614e+00,
        1.14359242e+01, 1.50179099e+01, 6.30200362e+00,
        5.49218955e-01],
       [ 1.11049703e+02, -5.00079214e-02, -6.95191515e+00,
        -7.31813062e-01, 2.27783799e+01, 8.49698513e+00,
        9.27637729e+00, 1.61128189e+01, 7.66503174e+00,
        -1.65913014e+01],
      [-1.80415534e+02, -1.17418027e+01, 9.53776007e+00,
        4.06949775e+00, -5.25349950e+01, 9.55077826e+00,
       -2.17096825e+01, -1.58182427e+01, -8.17137777e+00,
        -4.35291966e+00],
      [-2.06793341e+02, -1.07246709e+01, 1.06988095e+01,
        2.71639187e+00, -3.74756809e+01, 3.28659349e+00,
       -1.35506875e+01, -1.62523770e+01, -7.90637281e+00,
        2.06837682e+01],
      [-1.16241293e+02, -3.23093722e+00, 4.43207890e+00,
        1.50259660e+00, -3.72057506e+01, -6.68090322e+00,
        -1.64815464e+01, 2.53170492e-01, -8.64424073e+00,
        5.44080086e+00],
      [ 5.60597223e+01, -2.22364982e-01, -4.53246821e+00,
        1.49851681e-01, 1.10555979e+01, -2.28913262e-01,
        4.23055654e+00, 7.21121915e+00, 5.51974234e+00,
        -1.05514636e+01]])}
{'B2': array([ 6574.21325916, 24472.42735881, 10791.10996205, -10254.80340593,
       -9876.27040476, 13260.44613486, 15694.91245958, -10182.22871173,
        -2557.96158638, 9568.36241844])}
{'W3': array([[ 1.13091315e+03, 1.30532685e+03, 1.26033177e+03,
        9.89971978e+02, 9.93822361e+02, 1.27832775e+03,
        1.58419339e+03, 9.48943688e+02, 1.15817017e+03,
        1.00971404e+03],
      [-5.16788480e+02, -5.94256015e+02, -5.76412046e+02,
       -4.53176659e+02, -4.54352713e+02, -5.84313912e+02,
        -7.25767743e+02, -4.33230713e+02, -5.31846336e+02,
       -4.61656597e+02],
      [ 4.24293554e+02, 4.87648711e+02, 4.72751973e+02,
        3.71790679e+02, 3.71003813e+02, 4.78394672e+02,
        5.97068976e+02, 3.52642289e+02, 4.35035420e+02,
        3.78043400e+02],
      [ 4.16845810e+02, 4.77363753e+02, 4.64479729e+02,
        3.68059369e+02, 3.67996813e+02, 4.70822968e+02,
        5.84058125e+02, 3.53368012e+02, 4.30358627e+02,
        3.71506283e+02],
      [-1.37170746e+02, -1.67804550e+02, -1.64795180e+02,
        -1.21516708e+02, -1.32881636e+02, -1.68340550e+02,
       -2.15440892e+02, -1.32352928e+02, -1.61001646e+02,
       -1.21932712e+02],
      [ 1.45490573e+03, 1.66962970e+03, 1.61508421e+03,
        1.27673276e+03, 1.28258431e+03, 1.63968031e+03,
        2.03381581e+03, 1.22263942e+03, 1.49328904e+03,
        1.29543442e+03],
      [ 4.80740138e+02, 5.49762485e+02, 5.35586292e+02,
        4.26086787e+02, 4.25762824e+02, 5.40889276e+02,
        6.75225612e+02, 4.04637285e+02, 4.96685116e+02,
        4.28593546e+02],
      [ 1.55092744e+02, 1.58107891e+02, 1.53728340e+02,
        1.35617706e+02, 1.38714679e+02, 1.56214803e+02,
        1.95630968e+02, 1.12509981e+02, 1.57556931e+02,
        1.20393720e+02],
```

```
[-7.38098988e+02, -8.44891886e+02, -8.20939419e+02,
       -6.50881561e+02, -6.51500819e+02, -8.30758319e+02,
       -1.03536964e+03, -6.19826135e+02, -7.62041538e+02,
       -6.56380963e+02],
      [ 8.79491112e-01, 4.58962357e-01, 2.78271034e-02,
        4.19652697e+00, 3.64045127e+00, -2.34423134e-01,
        5.35552338e-01, 2.21806205e+00, 2.72856166e+00,
        4.66767502e-01]])}
{'B3': array([ -41706.53060903, 18737.12924154, 17885.91561922,
       -55106.28402067, -108958.85679005, 40254.5549872,
       -16306.32083792, -216307.31116644, 19591.177802 ,
       -33496.09781628])}
               556.76279698, 117.05103743,
{'W4': array([[
                                               107.85169104,
          136.33639883, 129.46579535, 540.09239317,
           89.46999647,
                         139.65457589,
                                         127.88163016,
           94.43569852],
      [ 1162.05077562,
                          244.42428739,
                                         224.48375511,
          284.0046473 , 268.7706013 , 1127.96355691,
          186.62799497,
                         293.17720426, 267.66656844,
          194.41035848],
      [ 677.29717362,
                         142.93975544,
                                         130.49508789,
          165.91307238,
                          157.06793433,
                                          657.36986408,
          108.07351771,
                         170.1721496 ,
                                         155.32927701,
          112.57128357],
      [ 1038.32496998,
                         217.55174132,
                                         199.17081613,
          252.71857264, 239.69523661, 1008.96056235,
          165.68652554,
                         261.59007292, 238.07349838,
          172.87509981],
      [ 475.24834835, 101.02248221,
                                          92.00331305,
          116.83302895,
                         111.18479548,
                                          462.12231305,
                         119.80277269,
                                         109.20572248,
          76.44741305,
           81.03937581],
      [ 1238.21665077, 260.76727863, 237.81123201,
          301.66444622, 285.6535855 , 1202.47459122,
                         312.51567492,
                                        284.91015931,
          198.57042617,
          206.653706 ],
      [-71315.02365059, -15016.02089374, -13682.09387276,
       -17326.02771559, -16244.9149399 , -69170.17424601,
       -11352.55411153, -17904.25086821, -16394.31851306,
       -11815.012566 ],
      [ 1304.50164038, 273.58597253, 250.76653164,
                         302.14640879, 1266.68612304,
          318.40458396,
                                        300.2090788,
          208.26109178,
                         329.05282608,
          216.30723232],
      [-14306.62605117, -3015.24621701, -2747.21978324,
        -3488.17830379, -3254.1344697, -13893.5917661,
        -2278.82606809, -3539.75048476, -3300.09745825,
        -2365.18572081],
      [ -550.49633907, -116.98950456, -106.45015015,
                         -122.66724028,
                                         -537.38105004,
         -139.09432834,
          -89.37662594, -139.99495211, -126.38728579,
          -90.40597461]])}
{'B4': array([59588.11264047, 89398.45415135, 67517.97336965, 70181.42496964,
      54794.07931948, 78083.69179265, 7747.12729841, 81101.33504991,
      -5472.86924777, 417.43565317])}
{'W5': array([[ 587.45156607, 1261.64613587, 739.13193502,
          792.58006824,
                         505.92545423, 969.84421766,
       -29607.09672493, 1042.86463701, -11478.18282863,
         -571.62745477]])}
{'B5': array([61929645.74348348])}
```