Assignment 2

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Question 1

i.

$$\boldsymbol{f}(x,y) = \boldsymbol{f}(1,1) + [(x-1)-(y-1)] + \frac{1}{2!}[(x-1)^2 - 2(x-1)(y-1) + (y-1)^2] + \\ \frac{1}{3!}[(x-1)^3 - 2(x-1)^2(y-1) - 2(x-1)^2(y-1) + (y-1)^3] + o(||(x,y)-(1,1)||^3) \\ = 1 + [x-y+\frac{1}{2}(x-y)^2 + \frac{1}{6}(x-y)^3] + o(||(x,y)-(1,1)||^3)$$

ii.

$$\boldsymbol{f}(x,y) = 1 + [x-y+\frac{1}{2}(x-y)^2] + R_k \\ = 1 + [x-y+\frac{1}{2}(x-y)^2] + \frac{1}{6}(x-y)^3\boldsymbol{f}(\vec{\xi})$$

iii. Take point$(1,\frac{1}{2})$ into Taylor expansion in **i**:

$$\boldsymbol{f}(1,\frac{1}{2}) = 1 + [\frac{1}{2} + \frac{1}{2}(\frac{1}{2})^2 + \frac{1}{6}(\frac{1}{2})^3] + o(||(x,y)-(1,1)||^3) \\ \Rightarrow \sqrt{\mathrm{e}} \approx \frac{79}{48}$$

iv. Take point $(1,\frac{1}{2})$ into Taylor expansion in **ii**:

$$\boldsymbol{f}(x,y) = 1 + [\frac{1}{2} + \frac{1}{2}(\frac{1}{2})^2] + \frac{1}{6}(\frac{1}{2})^3\boldsymbol{f}(\vec{\xi}) \\ \Leftrightarrow \sqrt{\mathrm{e}} = \frac{13}{8} + \frac{1}{48}\boldsymbol{f}(\vec{\xi})$$

Since $\vec{\xi}$ is in the open interval between point $(1,1)$ and $(1,\frac{1}{2})$, then $1 \leq \boldsymbol{f}(\vec{\xi}) \leq \sqrt{\mathrm{e}}$.

Therefore,

$$\begin{aligned}
\sqrt{e} &\geq \frac{13}{8} + \frac{1}{48} = \frac{79}{48} \\
\sqrt{e} &\leq \frac{13}{8} + \frac{1}{48}\sqrt{\mathrm{e}} \\
&\leq \frac{78}{47} \\
\Rightarrow \frac{79}{48} &\leq \sqrt{\mathrm{e}} \leq \frac{78}{47}
\end{aligned}$$