Calculus A2 (English) — (Mini-)Assignment 3.

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Submission instructions: same instructions as for assignment 1 and 2.

Question 1: (3 marks)

Compute the following integral by changing the order of integration:

$$\int_0^1 \left(\int_y^1 y \sqrt{1 - x^6} \, dx \right) \, dy.$$

Hint: you might find trigonometric functions useful for a substitution during the latter steps of the evaluation of the integral.

Question 2: (3 marks)

Use the Weierstrass comparison test to help you justify the change of the order of integration for the following integral, and then compute it.

$$\int_1^\infty \left(\int_2^3 \tfrac{y}{x^3} \exp(\tfrac{y}{x^2}) \, dy \right) \, dx.$$

Question 3: (5 marks)

The goal of this question is to compute the improper integral

$$\int_0^\infty \tfrac{\cos x}{x} \left(e^{-x} - e^{-2x} \right) \, dx.$$

You may find it useful to take the following steps:

- First show that the integrand (the function that you're integrating) extends continuously to a well-defined function on $[0, \infty)$. This will mean that you can simply integrate the function as a one-sided improper integral rather than a two-sided improper integral.

- Next, express the above integral as an interated improper integral (hint: think about what happens to e^{-xy} when you integrate it wrt y). (Note, it isn't necessary to especially check that when $x = 0$, the inner integral yields the limit the integrand of the original integral as x tends to 0 as that follows from continuity of integrals.)

- Then use either the Weierstrass, Dirichlet or the Abel test to change the order of integration, before finally evaluating the integral (hint: you may find complex numbers useful, I hope that you learned in Calculus A1 how to integrate functions which arise as the real or imaginary parts of some complex valued function. If not, there are other, more direct methods).

Question 4: (1 mark)

For the improper iterated integral in Question 2, justify the change in order of integration using either the Dirichlet test or the Abel test (use a test that you haven't yet used in this assignment).