Calculus A2 (English) — Assignment 5.

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Submission instructions: same instructions as for assignments 1, 2, 3 and 4.

Question 1: (14 marks)

Determine if the following real number series converge/diverge, and justify your answer.

i.
$$\sum_{n=1}^{\infty} 3^{-2n+\frac{1}{n}},$$

ii.
$$\sum_{n=2020}^{\infty} \sin\left(\pi(n^2 - n + \tfrac{2}{n})\right),$$

iii.
$$\sum_{n=2020}^{\infty} \frac{\tanh(n)}{n - \cos(n)},$$

iv.
$$\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^{n^2},$$

v.
$$\sum_{n=0}^{\infty} \frac{n!}{n^n},$$

vi.
$$\sum_{n=1}^{\infty} \frac{n\cos(n)}{n^2+1},$$

vii.
$$\sum_{n=2020}^{\infty} \frac{\log(\log(n))}{n(\log(n))^2}.$$

Question 2: (6 marks)

Determine the domain of convergence (wrt x) of the following series of real functions of x (N.B.: these are not power series, so the domain of convergence might not be intervals):

i.

$$\sum_{n=2}^{\infty} \frac{n^x}{(\log n)^n}.$$

ii.

$$\sum_{n=1}^{\infty} x^{-1-\frac{1}{2}-\frac{1}{3}-\ldots-\frac{1}{n}}.$$

Bonus: (2 marks)

You're given (countably) infinitely many squares respectively with side lengths $1, \frac{1}{2}, \frac{1}{3}, \ldots$ (one of each). Obviously, the total area occupied by these squares is finite, whilst their total perimeter is infinite. Show that it is possible to simultaneously place all of these squares within a bounded region in $\mathbb{R}^2$ without them touching each other.