

## Assignment 2

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### Question 1

i.

$$\begin{aligned}f(x, y) &= f(1, 1) + [(x-1) - (y-1)] + \frac{1}{2!}[(x-1)^2 - 2(x-1)(y-1) + (y-1)^2] + \\&\quad \frac{1}{3!}[(x-1)^3 - 2(x-1)^2(y-1) - 2(x-1)^2(y-1) + (y-1)^3] + o(\|(x, y) - (1, 1)\|^3) \\&= 1 + [x - y + \frac{1}{2}(x - y)^2 + \frac{1}{6}(x - y)^3] + o(\|(x, y) - (1, 1)\|^3)\end{aligned}$$

ii.

$$\begin{aligned}f(x, y) &= 1 + [x - y + \frac{1}{2}(x - y)^2] + R_k \\&= 1 + [x - y + \frac{1}{2}(x - y)^2] + \frac{1}{6}(x - y)^3 f(\vec{\xi})\end{aligned}$$

iii. Take point  $(1, \frac{1}{2})$  into Taylor expansion in i:

$$\begin{aligned}f(1, \frac{1}{2}) &= 1 + [\frac{1}{2} + \frac{1}{2}(\frac{1}{2})^2 + \frac{1}{6}(\frac{1}{2})^3] + o(\|(x, y) - (1, 1)\|^3) \\&\Rightarrow \sqrt{e} \approx \frac{79}{48}\end{aligned}$$

iv. Take point  $(1, \frac{1}{2})$  into Taylor expansion in ii:

$$\begin{aligned}f(x, y) &= 1 + [\frac{1}{2} + \frac{1}{2}(\frac{1}{2})^2] + \frac{1}{6}(\frac{1}{2})^3 f(\vec{\xi}) \\&\Leftrightarrow \sqrt{e} = \frac{13}{8} + \frac{1}{48} f(\vec{\xi})\end{aligned}$$

Since  $\vec{\xi}$  is in the open interval between point  $(1, 1)$  and  $(1, \frac{1}{2})$ , then  $1 \leq f(\vec{\xi}) \leq \sqrt{e}$ .

Therefore,

$$\begin{aligned}\sqrt{e} &\geq \frac{13}{8} + \frac{1}{48} = \frac{79}{48} \\ \sqrt{e} &\leq \frac{13}{8} + \frac{1}{48}\sqrt{e} \\ &\leq \frac{78}{47} \\ \Rightarrow \frac{79}{48} &\leq \sqrt{e} \leq \frac{78}{47}\end{aligned}$$

## Question 2