學生資訊

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a. 使用 pandas 來去讀取 College.csv 的這一份文件, 透過 index_col 一併解決 b.

- 資料來源: R 的 library 中 dump 出來的

```
import pandas as pd

df = pd.read_csv("College.csv", index_col = 0)
df.head()
```

	Private	Apps	Accept	Enroll	Top10perc	Top25perc	F.Undergrad	P.Undergrad	Outstate	Room.Board	Books	Personal	PhD	Terminal	S.
Abilene Christian University	Yes	1660	1232	721	23	52	2885	537	7440	3300	450	2200	70	78	
Adelphi University	Yes	2186	1924	512	16	29	2683	1227	12280	6450	750	1500	29	30	
Adrian College	Yes	1428	1097	336	22	50	1036	99	11250	3750	400	1165	53	66	
Agnes Scott College	Yes	417	349	137	60	89	510	63	12960	5450	450	875	92	97	
Alaska Pacific University	Yes	193	146	55	16	44	249	869	7560	4120	800	1500	76	72	

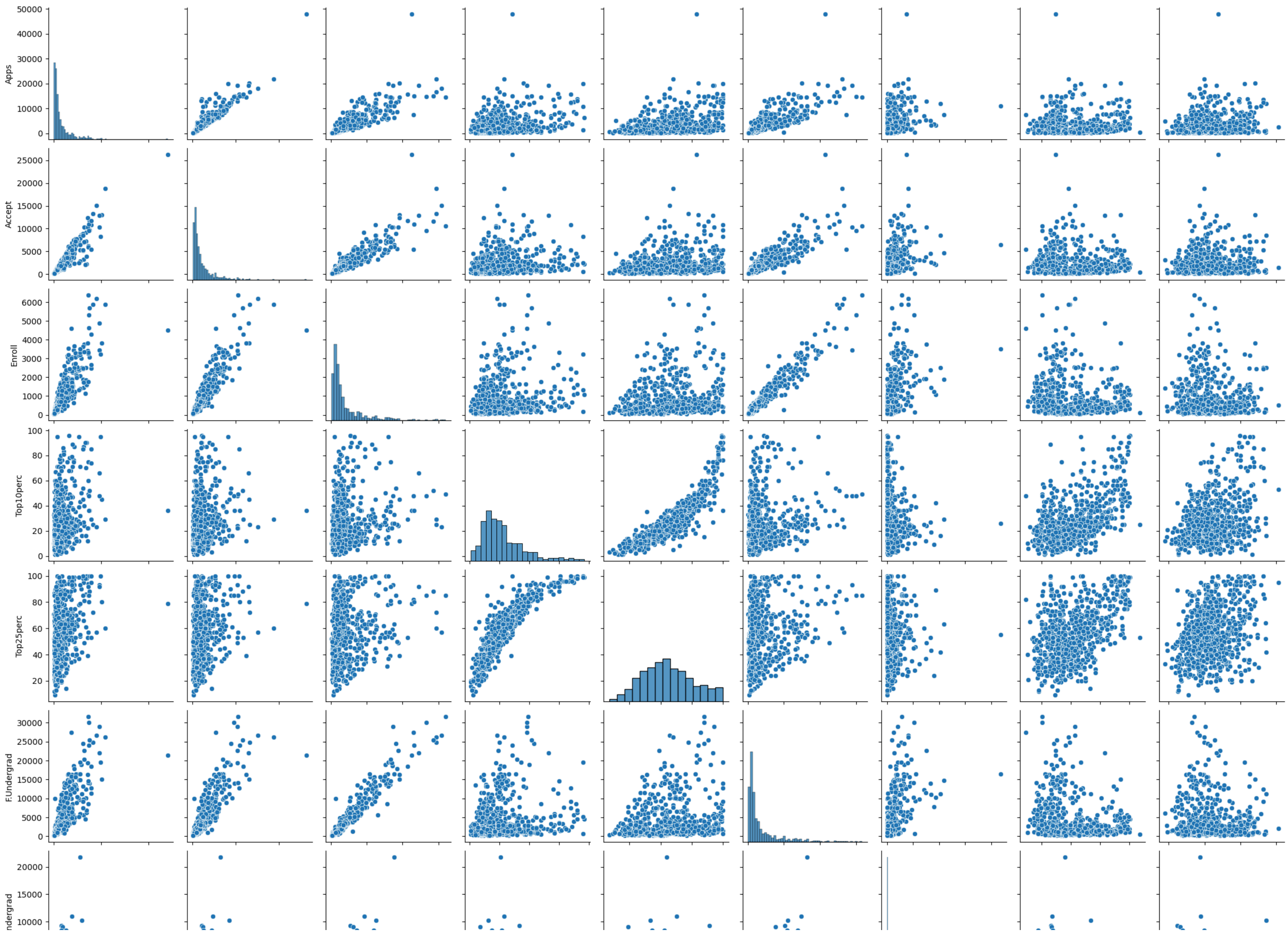
c.1. R 中的 summary 與 python 中 pandas 的 describe 類似

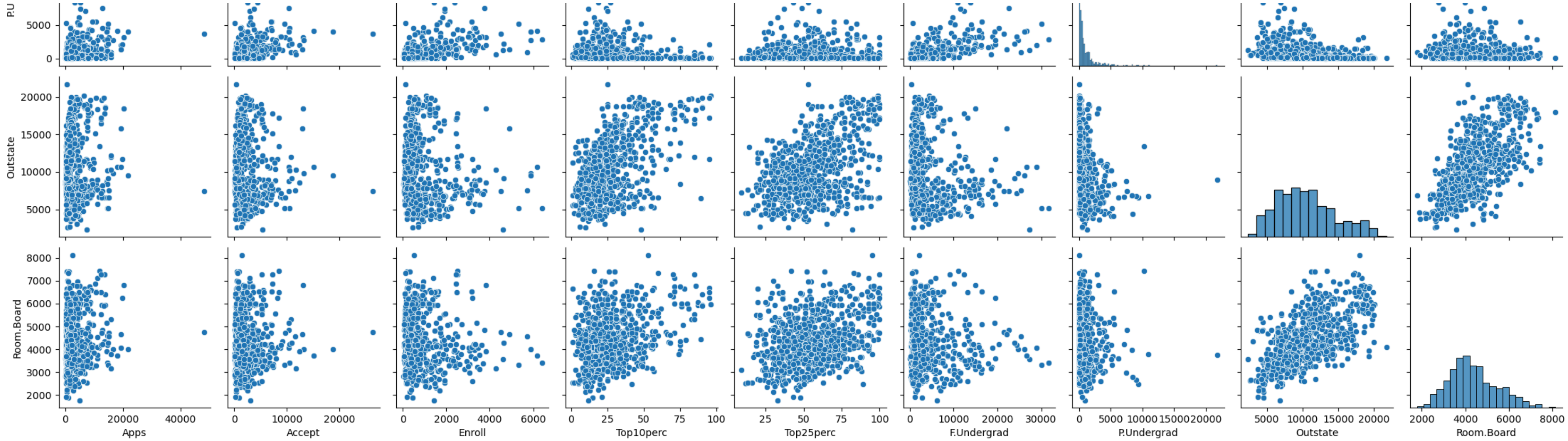
```
df.describe()
```

	Apps	Accept	Enroll	Top10perc	Top25perc	F.Undergrad	P.Undergrad	Outstate	Room.Board	Books	Personal
count	777.000000	777.000000	777.000000	777.000000	777.000000	777.000000	777.000000	777.000000	777.000000	777.000000	777.000000
mean	3001.638353	2018.804376	779.972973	27.558559	55.796654	3699.907336	855.298584	10440.669241	4357.526384	549.380952	1340.642214
std	3870.201484	2451.113971	929.176190	17.640364	19.804778	4850.420531	1522.431887	4023.016484	1096.696416	165.105360	677.071454
min	81.000000	72.000000	35.000000	1.000000	9.000000	139.000000	1.000000	2340.000000	1780.000000	96.000000	250.000000
25%	776.000000	604.000000	242.000000	15.000000	41.000000	992.000000	95.000000	7320.000000	3597.000000	470.000000	850.000000
50%	1558.000000	1110.000000	434.000000	23.000000	54.000000	1707.000000	353.000000	9990.000000	4200.000000	500.000000	1200.000000
75%	3624.000000	2424.000000	902.000000	35.000000	69.000000	4005.000000	967.000000	12925.000000	5050.000000	600.000000	1700.000000
max	48094.000000	26330.000000	6392.000000	96.000000	100.000000	31643.000000	21836.000000	21700.000000	8124.000000	2340.000000	6800.000000

c.2. 取出前 10 個 row 來去做成對的 scatter plot

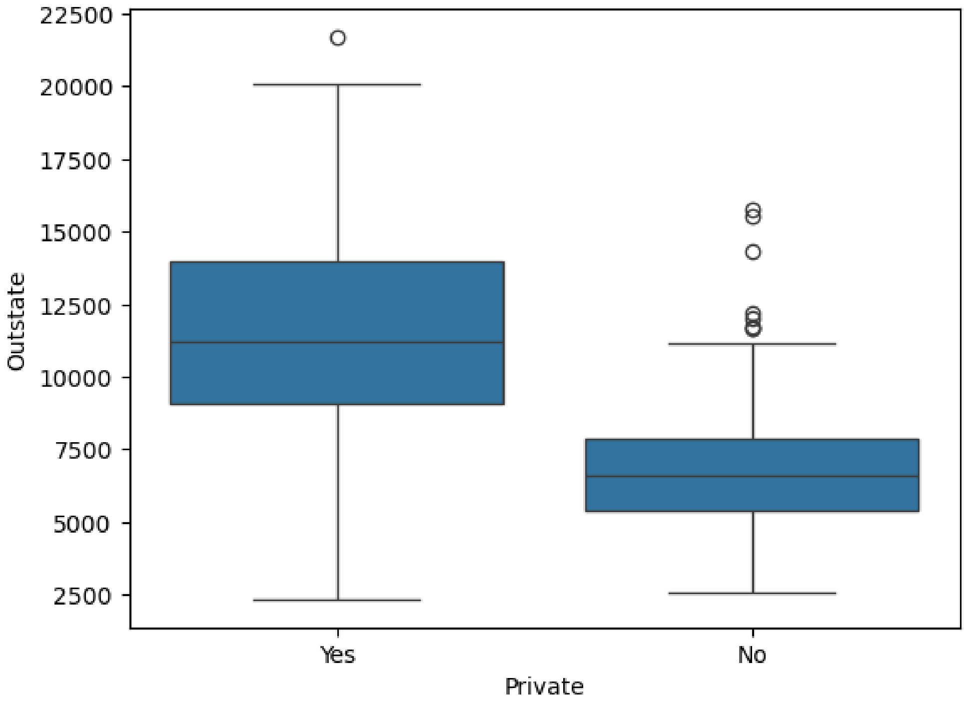
```
import seaborn as sns
import matplotlib.pyplot as plt

df_subset = df.iloc[:, :10]

sns.pairplot(df_subset)

plt.show()
```


Apps
Accept
Enroll
Top10perc
Top25perc
F.Undergrad



c.3. 目標就是希望繪製 Outstate 和 Private 的並排箱形圖

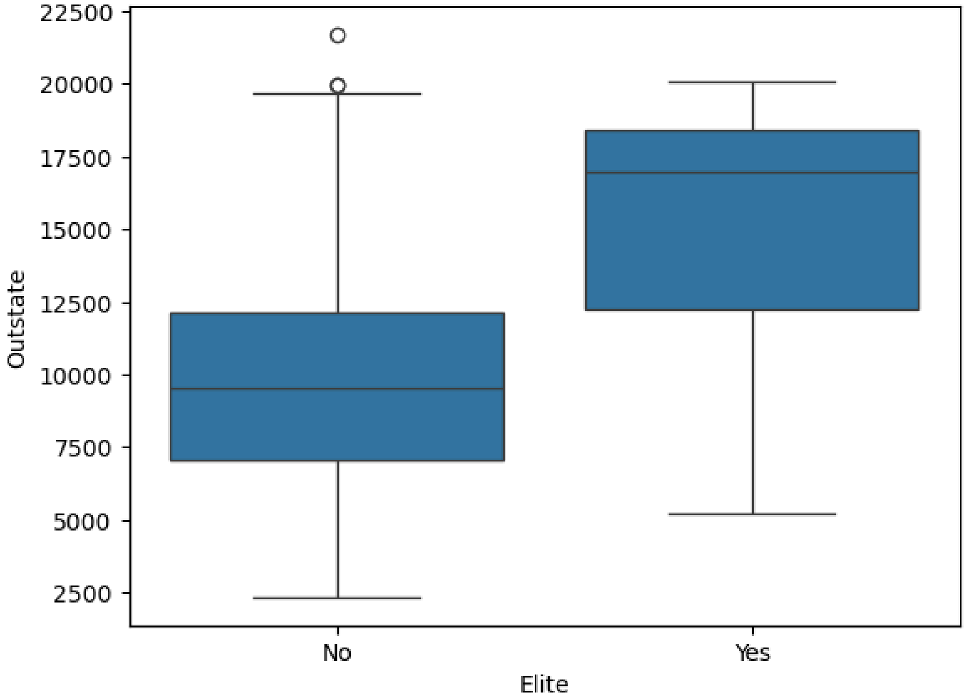
```
import seaborn as sns
import matplotlib.pyplot as plt


sns.boxplot(x='Private', y='Outstate', data=df)

# 顯示圖表
plt.show()
```



4. Use the summary() function to see how many elite universities there are. Now use the plot () function to produce side-by-side boxplots ofOutstate versus Elite.

```
import pandas as pd
import seaborn as sns
import matplotlib.pyplot as plt

# 創建新的質變量 Elite，將 Top10perc 超過 50 的標記為 "Yes"，其餘為 "No"
df['Elite'] = pd.cut(df['Top10perc'], bins=[-float('inf'), 50, float('inf')], labels=['No', 'Yes'])

print(df['Elite'].value_counts())
print(df['Elite'].describe())

# 生成 Outstate 與 Elite 的並排箱形圖
sns.boxplot(x='Elite', y='Outstate', data=df)

# 顯示圖表
plt.show()
```

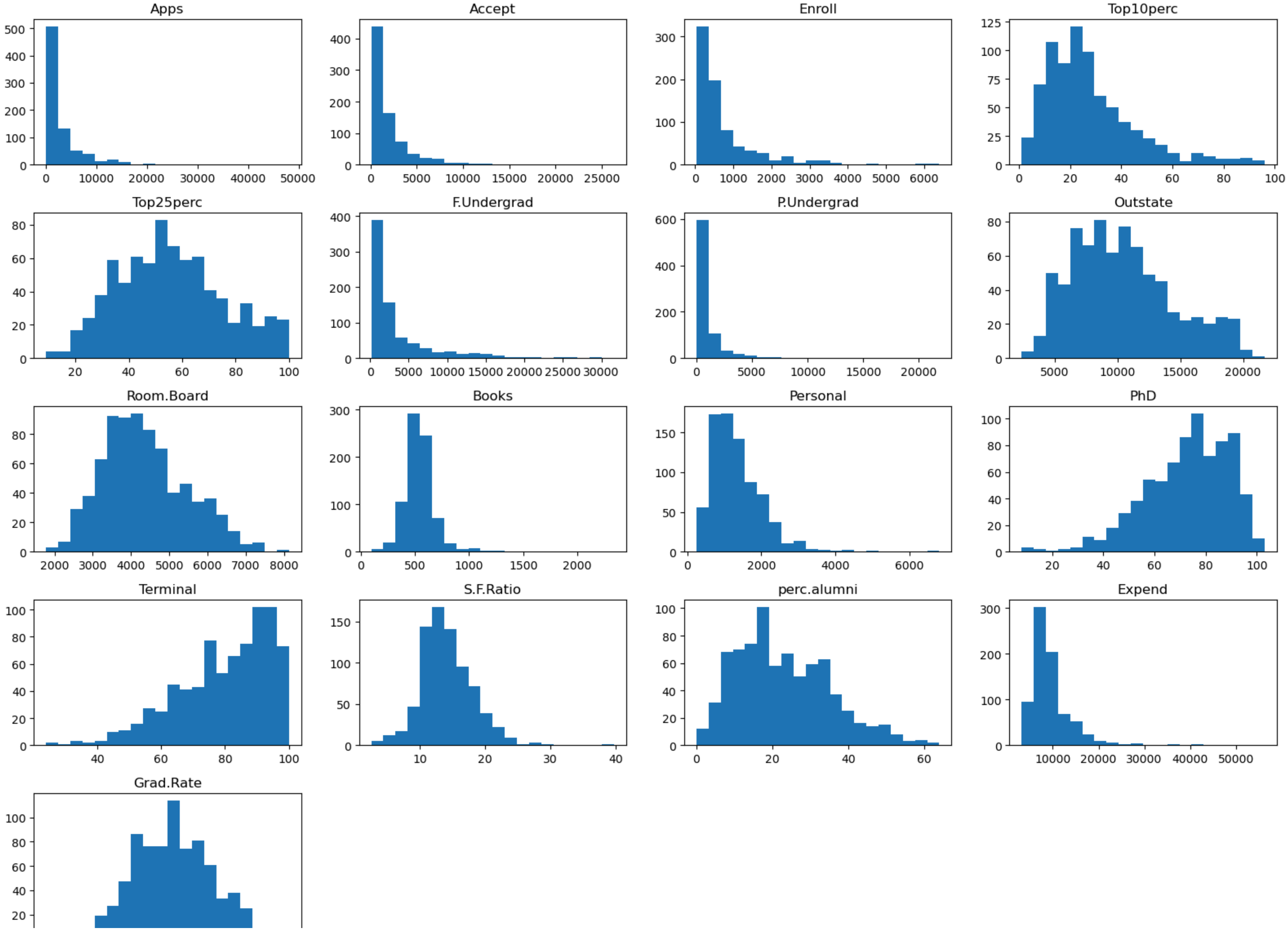
```
Elite
No     699
Yes     78
Name: count, dtype: int64
count     777
unique      2
top        No
freq      699
Name: Elite, dtype: object
```

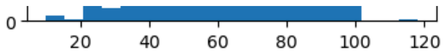


5. Use the hist() function to produce some histograms with differing numbers of bins for a few of the quantitative variables.

```
import matplotlib.pyplot as plt

# 欄位名稱列表
columns = ['Apps', 'Accept', 'Enroll', 'Top10perc', 'Top25perc', 'F.Undergrad',
           'P.Undergrad', 'Outstate', 'Room.Board', 'Books', 'Personal', 'PhD',
           'Terminal', 'S.F.Ratio', 'perc.alumni', 'Expend', 'Grad.Rate']

# 計算行與列數量，方便繪製子圖
```

```
n_cols = 4  # 每行4個圖表
n_rows = (len(columns) + n_cols - 1) // n_cols  # 計算總行數

# 設置子圖
fig, axes = plt.subplots(n_rows, n_cols, figsize=(16, 12))

# 繪製每個欄位的直方圖
for i, col in enumerate(columns):
    row = i // n_cols
    col_idx = i % n_cols
    axes[row, col_idx].hist(df[col], bins=20)  # 設置 bins 為 20
    axes[row, col_idx].set_title(col)

# 移除空白的子圖（如果有）
for j in range(i + 1, n_rows * n_cols):
    fig.delaxes(axes.flatten()[j])

# 調整子圖之間的間距
plt.tight_layout()

# 顯示圖表
plt.show()
```


Apps
Accept
Enroll
Top10perc
Top25perc
F.Undergrad
P.Undergrad
Outstate
Room.Board
Books
Personal
PhD
Terminal
S.F.Ratio
perc.alumni
Expend
Grad.Rate



6. Continue exploring the data, and provide a brief summary of what you discover.

觀察

- 美國大學平均收到約 3001 份申請，其中約 2018 人被錄取，約 779 人註冊入學。
- App（申請）、Accept（錄取）、F.Undergrad（全職本科生人數）、P.Undergrad（兼職本科生人數） 和 Enroll（註冊）的標準差較大，這表明數據分布廣泛。
- 約有 565 所大學是私立的。
- 平均房租、書籍費用和個人開支分別為 4350 美元、550 美元和 1350 美元。
- 書籍費用整體呈現常態分佈，但有離群值，最大值來到了 2340 美元，也代表著有部分學校的書籍費特別的貴

資料問題

- PhD 是百分比，但最大值達到 103%，需要檢查這一點。
- Grad.Rate 也是百分比，但最大值達到 118%，也需要檢查這一點。

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- [資料來源]()

  a. How many rows are inthis data set? How many columns? What do the rows and columns represent?

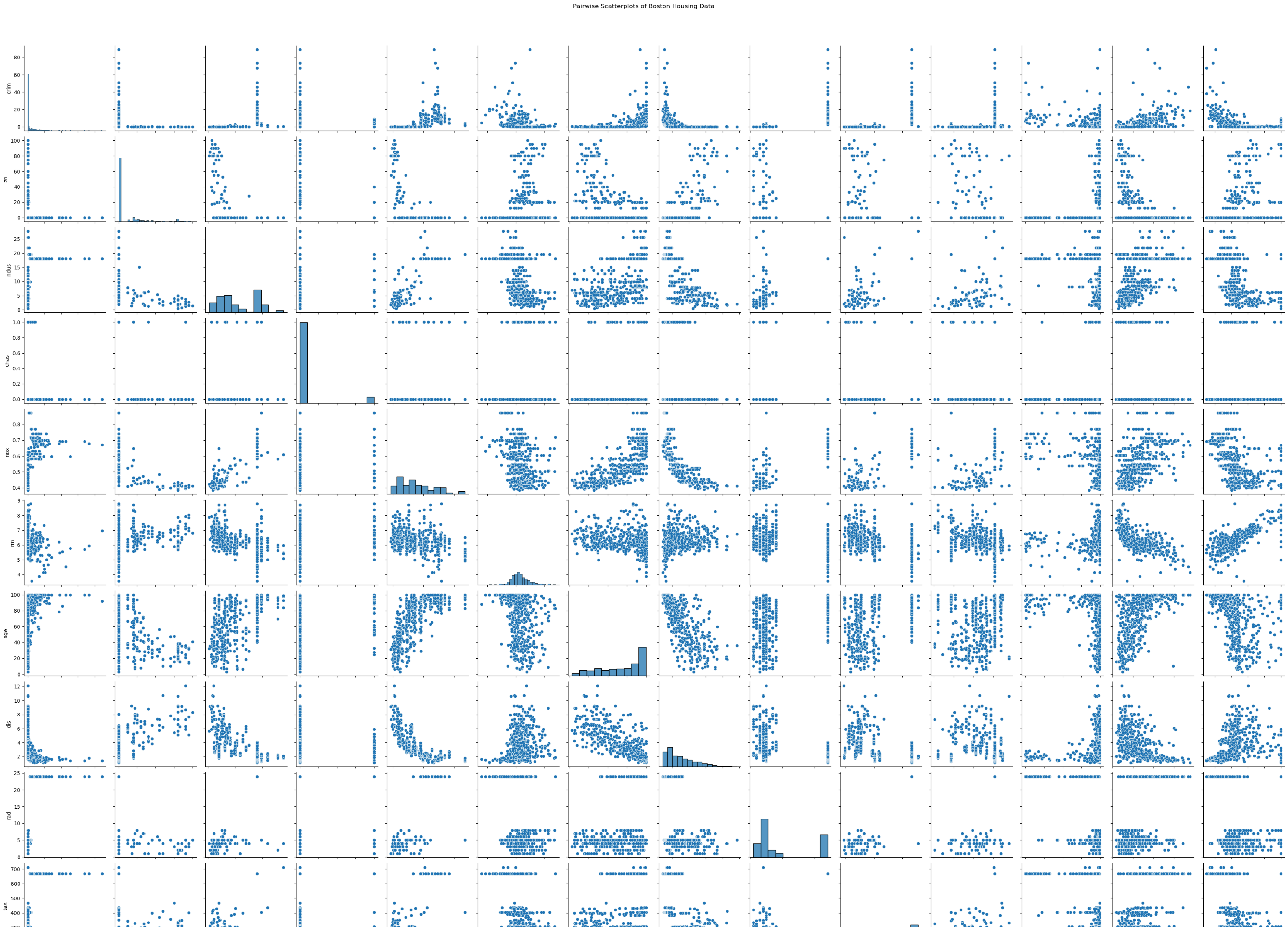
```
import pandas as pd

boston = pd.read_csv("boston_data.csv")

print(boston.head())

rows, cols = boston.shape
print(f'rows: {rows}, columns: {cols}')
```

```
      crim    zn  indus  chas    nox     rm   age     dis  rad  tax  ptratio  \
0  0.00632  18.0   2.31     0  0.538  6.575  65.2  4.0900    1  296     15.3
1  0.02731   0.0   7.07     0  0.469  6.421  78.9  4.9671    2  242     17.8
2  0.02729   0.0   7.07     0  0.469  7.185  61.1  4.9671    2  242     17.8
3  0.03237   0.0   2.18     0  0.458  6.998  45.8  6.0622    3  222     18.7
4  0.06905   0.0   2.18     0  0.458  7.147  54.2  6.0622    3  222     18.7

    black  lstat  medv
0  396.90   4.98  24.0
1  396.90   9.14  21.6
2  392.83   4.03  34.7
3  394.63   2.94  33.4
4  396.90   5.33  36.2
rows: 506, columns: 14
```

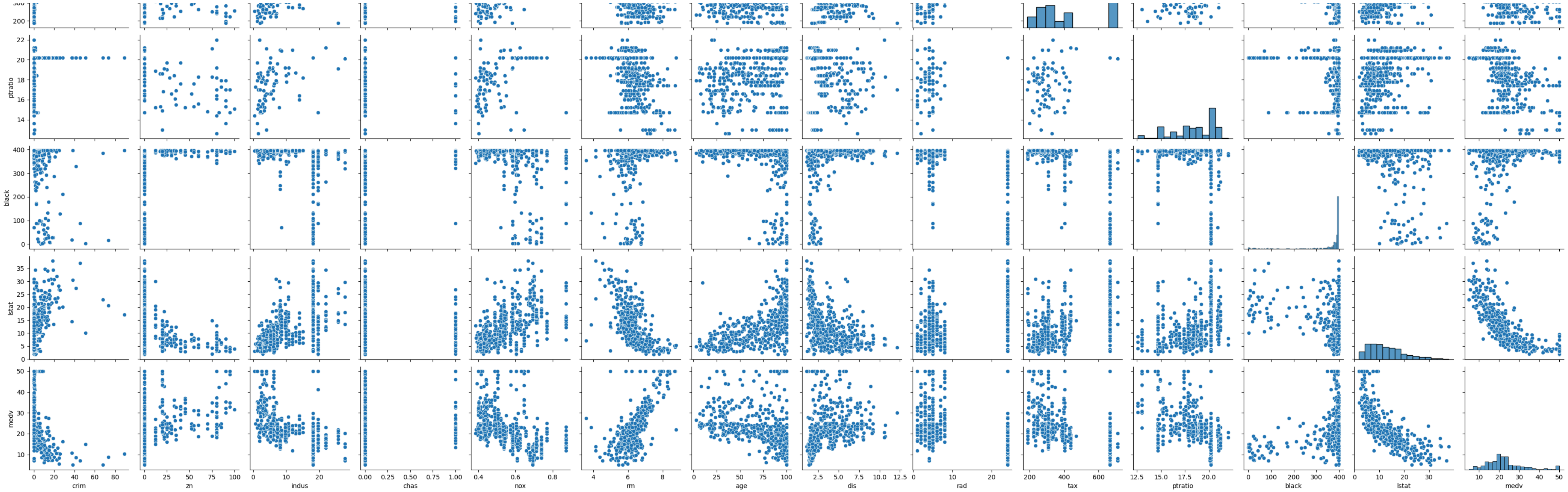
b. Make some pairwise scatterplots of the predictors (columns) in this data set.

- RM（每個住宅的平均房間數）與房價（通常用 MEDV 表示）之間的正相關，顯示房間數越多，房價通常也越高。
- LSTAT（低收入人口的百分比）與房價之間的負相關，顯示低收入人口比例越高，房價越低。

```
# 繪製成對散點圖
sns.pairplot(boston)
plt.suptitle('Pairwise Scatterplots of Boston Housing Data', y=1.02)  # 調整標題位置
plt.show()
```



Pairwise Scatterplots of Boston Housing Data
crim
zn
indus
chas
nox
rm
age
dis
rad
tax

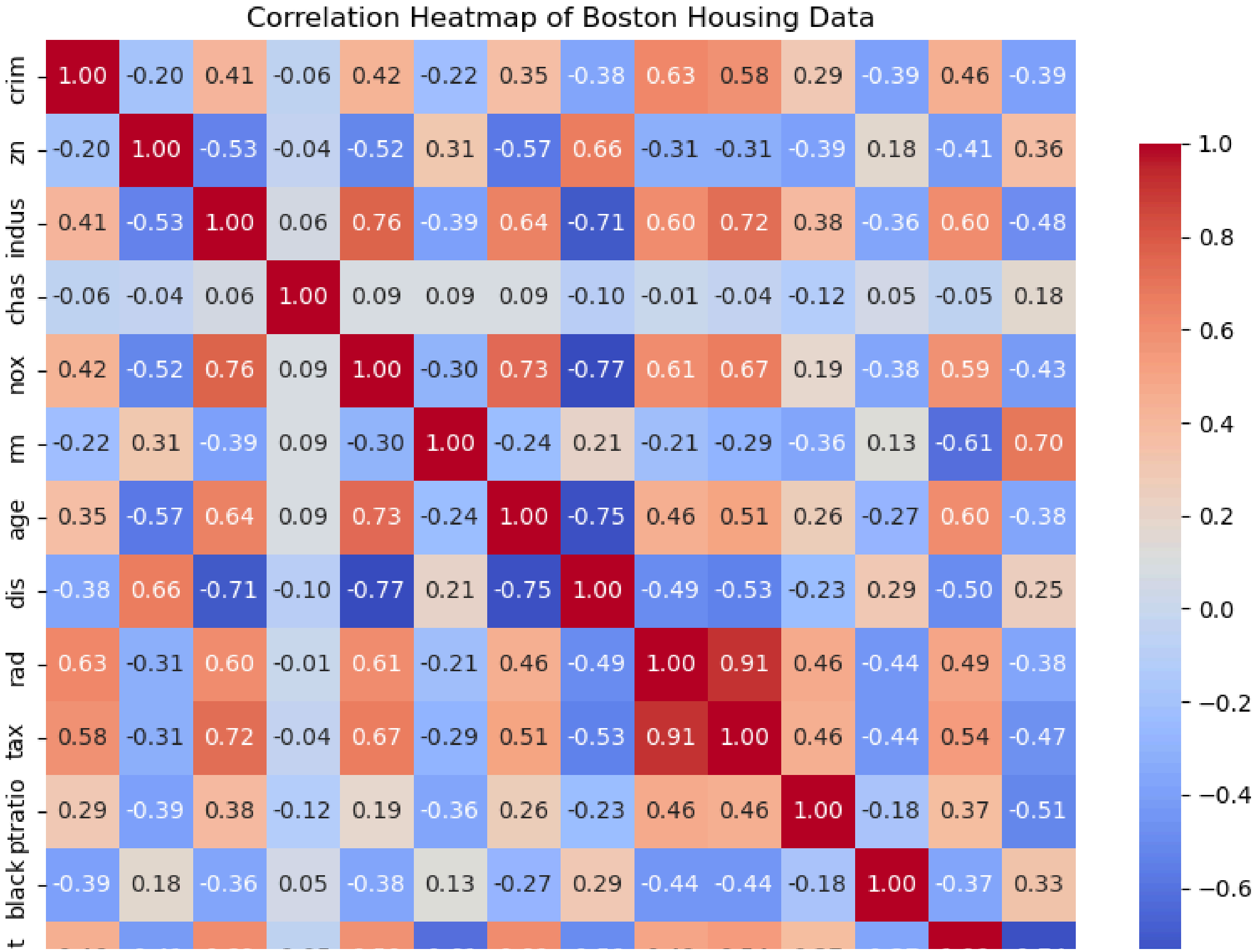

ptratio
black
lstat
medv
crim
zn
indus
chas
nox
rm
age
dis
rad
tax
ptratio
black
lstat
medv

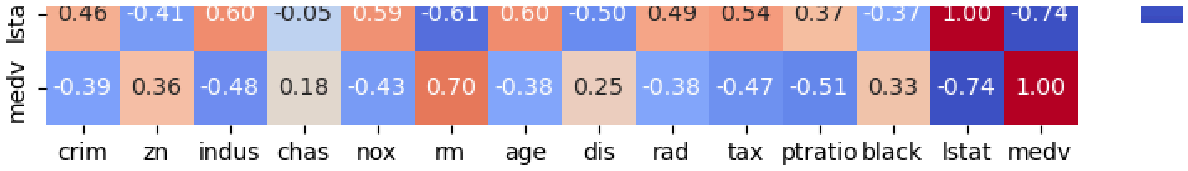
c. Are any ofthe predictors associated with per capita crime rate?

- 與 rad 呈現正相關，意味著 rad 上升時犯罪率也比較容易上升
- 與 tax 呈現正相關，當稅率越高時犯罪率也比較容易上升，可能是因為經濟條件較差的區域的稅收負擔較重，進而影響社區的安全性。
- indus 相關係數為 0.406，顯示與工業區域的比例有一定的正相關，這可能與工業活動引起的社會問題和潛在的犯罪有關。

```
# 計算相關係數矩陣
correlation_matrix = boston.corr()

# 提取 CRIM 列的相關係數
crim_correlation = correlation_matrix['crim']

# 顯示 CRIM 的相關係數
print("相關係數與 CRIM: ")
print(crim_correlation)

# 繪製熱圖
plt.figure(figsize=(10, 8))
sns.heatmap(correlation_matrix, annot=True, fmt='.2f', cmap='coolwarm', square=True, cbar_kws={"shrink": .8})
plt.title('Correlation Heatmap of Boston Housing Data')
plt.show()
```

```
相關係數與 CRIM：
crim       1.000000
zn        -0.200469
indus      0.406583
chas      -0.055892
nox        0.420972
rm        -0.219247
age        0.352734
dis       -0.379670
rad        0.625505
tax        0.582764
ptratio    0.289946
black     -0.385064
lstat      0.455621
medv      -0.388305
Name: crim, dtype: float64
```





d. Do any of the suburbs of Boston appear to haveparticularly high crimerates? Tax rates? Pupil-teacher ratios? Comment on the range of each predictor.

- 由下面兩個圖可以得知，在 tax 高或者 ptratio 高的時候通常比較容易出現高 crim

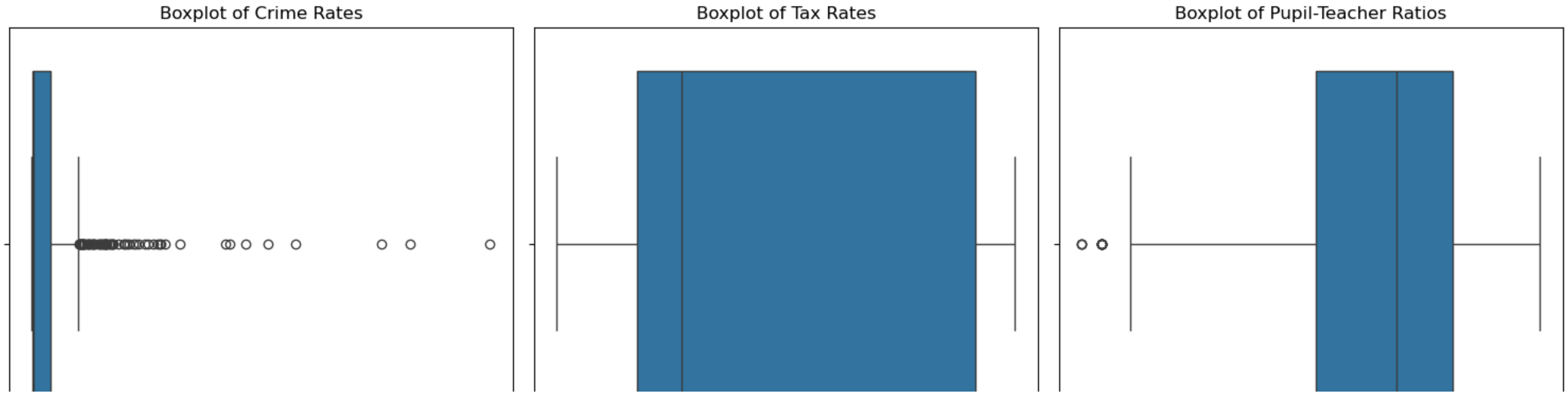
```
# 添加目標變量 'CRIM' (犯罪率)

# 查看犯罪率、稅率和學生與教師比例的範圍
crime_rate_range = boston['crim'].describe()
tax_rate_range = boston['tax'].describe()
pupil_teacher_ratio_range = boston['ptratio'].describe()

# 輸出範圍
print("犯罪率範圍: \n", crime_rate_range)
print("\n稅率範圍: \n", tax_rate_range)
print("\n學生與教師比例範圍: \n", pupil_teacher_ratio_range)
```

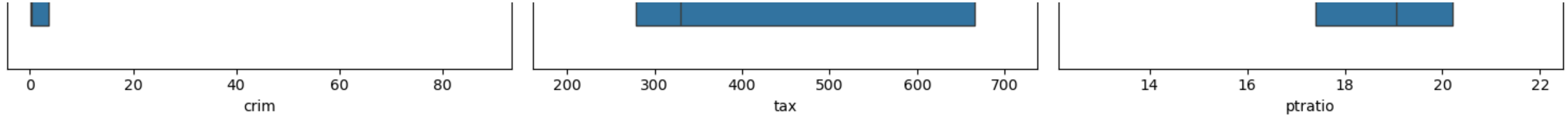
```
犯罪率範圍:
 count    506.000000
mean       3.613524
std        8.601545
min        0.006320
25%        0.082045
50%        0.256510
75%        3.677083
max       88.976200
Name: crim, dtype: float64

稅率範圍:
 count    506.000000
mean     408.237154
std      168.537116
min      187.000000
25%      279.000000
50%      330.000000
```

```
75%      666.000000
max      711.000000
Name: tax, dtype: float64

學生與教師比例範圍:
 count    506.000000
mean      18.455534
std        2.164946
min       12.600000
25%       17.400000
50%       19.050000
75%       20.200000
max       22.000000
Name: ptratio, dtype: float64
```

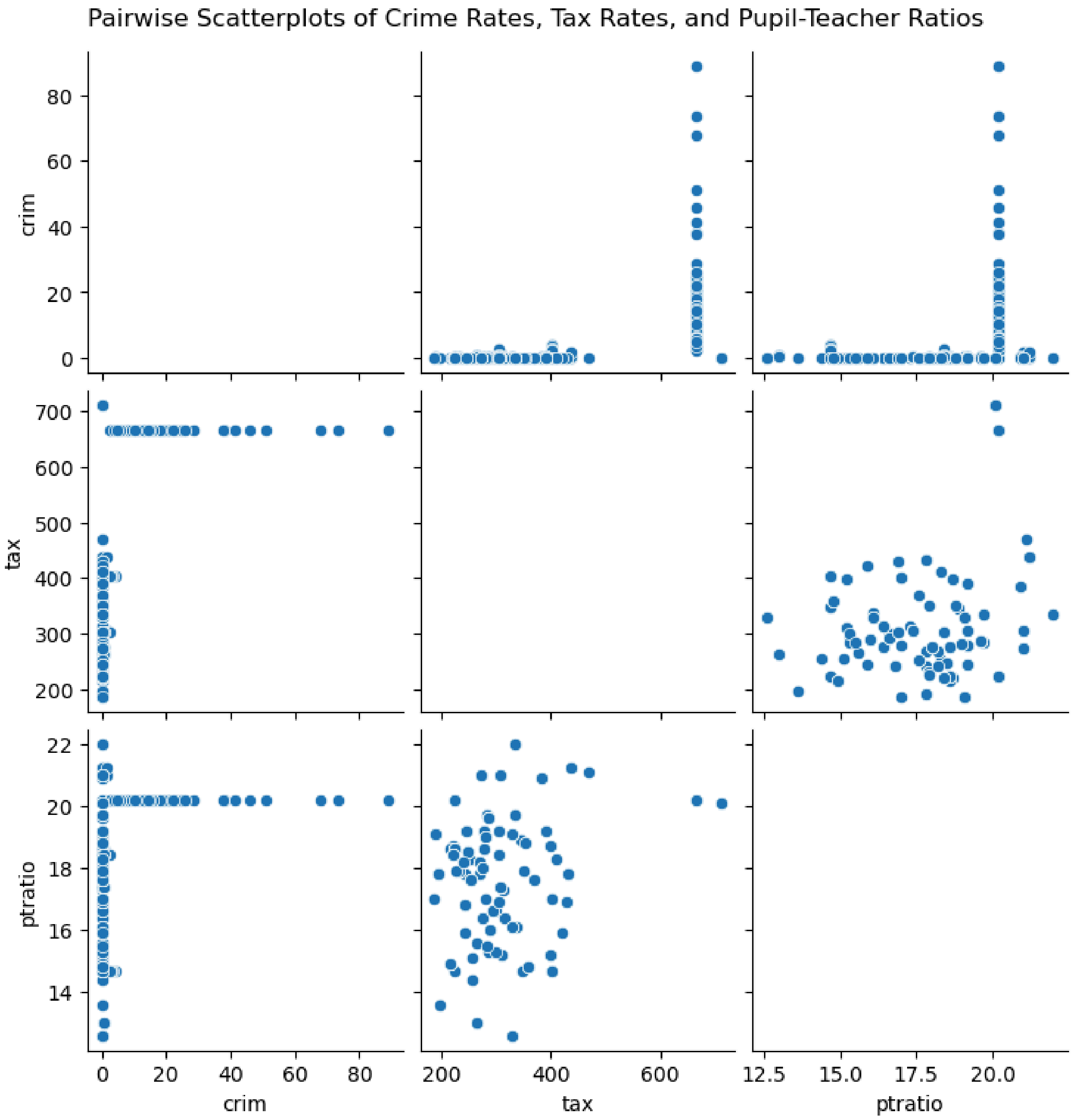
```
# 查看犯罪率、稅率和學生與教師比例的範圍
crime_rate_range = boston['crim'].describe()
tax_rate_range = boston['tax'].describe()
pupil_teacher_ratio_range = boston['ptratio'].describe()

# 輸出範圍
print("犯罪率範圍: \n", crime_rate_range)
print("\n稅率範圍: \n", tax_rate_range)
print("\n學生與教師比例範圍: \n", pupil_teacher_ratio_range)

# 可視化數據
plt.figure(figsize=(15, 5))

plt.subplot(1, 3, 1)
sns.boxplot(x='crim', data=boston)
plt.title('Boxplot of Crime Rates')

plt.subplot(1, 3, 2)
sns.boxplot(x='tax', data=boston)
plt.title('Boxplot of Tax Rates')

plt.subplot(1, 3, 3)
sns.boxplot(x='ptratio', data=boston)
plt.title('Boxplot of Pupil-Teacher Ratios')

plt.tight_layout()
plt.show()
```

```
犯罪率範圍：
 count    506.000000
mean       3.613524
std        8.601545
min        0.006320
25%        0.082045
50%        0.256510
75%        3.677083
max       88.976200
Name: crim, dtype: float64

稅率範圍：
 count    506.000000
mean     408.237154
std      168.537116
min      187.000000
25%      279.000000
50%      330.000000
75%      666.000000
max      711.000000
Name: tax, dtype: float64

學生與教師比例範圍：
 count    506.000000
mean      18.455534
std        2.164946
min       12.600000
25%       17.400000
50%       19.050000
75%       20.200000
max       22.000000
Name: ptratio, dtype: float64
```






```
variables = ['crim', 'tax', 'ptratio']

sns.pairplot(boston[variables], diag_kind='scatter')

plt.suptitle('Pairwise Scatterplots of Crime Rates, Tax Rates, and Pupil-Teacher Ratios', y=1.02)
plt.show()
```


Pairwise Scatterplots of Crime Rates, Tax Rates, and Pupil-Teacher Ratios
crim
tax
ptratio
0
20
40
60
80
200
300
400
500
600
700
14
16
18
20
22
12.5
15.0
17.5
20.0

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```
Auto.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 397 entries, 0 to 396
Data columns (total 9 columns):
 #   Column        Non-Null Count  Dtype
---  ------        --------------  -----
 0   mpg           397 non-null    float64
 1   cylinders     397 non-null    int64
 2   displacement  397 non-null    float64
 3   horsepower    392 non-null    float64
 4   weight        397 non-null    int64
 5   acceleration  397 non-null    float64
 6   year          397 non-null    int64
 7   origin        397 non-null    int64
 8   name          397 non-null    object
dtypes: float64(4), int64(4), object(1)
memory usage: 28.0+ KB
```

- 由於發現 horsepower 有空值所以透過 fillna 的方式去填補他並且填補平均數。

```
import pandas as pd
import statsmodels.api as sm

# 假設你已經有一個 pandas DataFrame，名稱為 'Auto'，包含 'mpg' 和 'horsepower' 欄位
# 讀取數據（如果數據來自一個 CSV 文件）
Auto = pd.read_csv('Auto.csv')
Auto["horsepower"] = Auto["horsepower"].fillna(Auto["horsepower"].mean())

# 自變數 X 和 依變數 y
X = Auto['horsepower']
y = Auto['mpg']


# 添加常數項（截距項）以進行迴歸
X = sm.add_constant(X)

# 構建線性迴歸模型
model = sm.OLS(y, X).fit()
```

```
# 輸出模型摘要
print(model.summary())

# 對結果的評論可以根據輸出的值進行分析
```

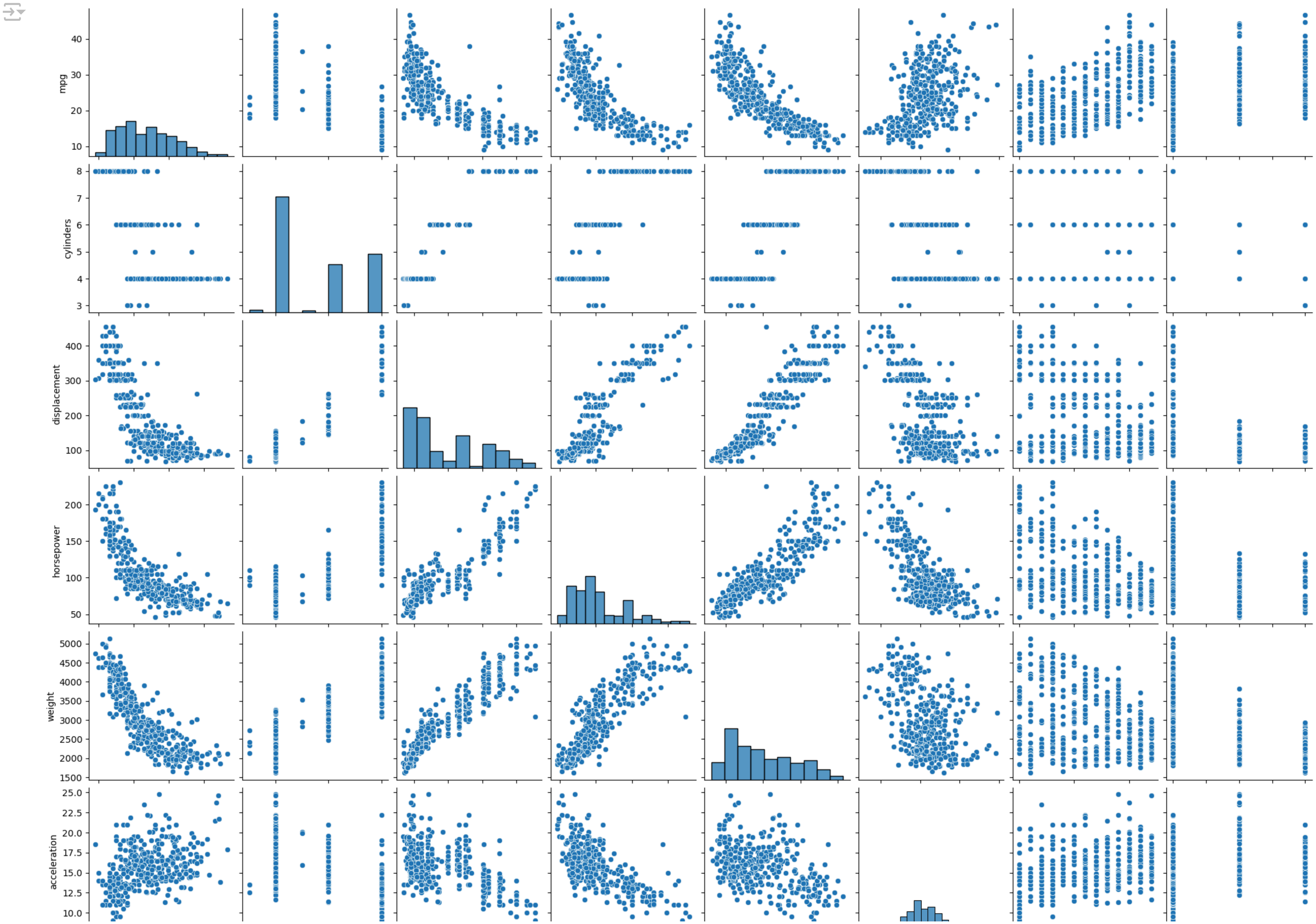
```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.595
Model:                            OLS   Adj. R-squared:                  0.594
Method:                 Least Squares   F-statistic:                     580.6
Date:                Sun, 06 Oct 2024   Prob (F-statistic):           1.45e-79
Time:                        15:26:16   Log-Likelihood:                -1200.1
No. Observations:                 397   AIC:                             2404.
Df Residuals:                     395   BIC:                             2412.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         40.0058      0.729     54.903      0.000      38.573      41.438
horsepower    -0.1578      0.007    -24.096      0.000      -0.171      -0.145
==============================================================================
Omnibus:                       21.884   Durbin-Watson:                   0.902
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               24.108
Skew:                           0.557   Prob(JB):                     5.82e-06
Kurtosis:                       3.464   Cond. No.                         324.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

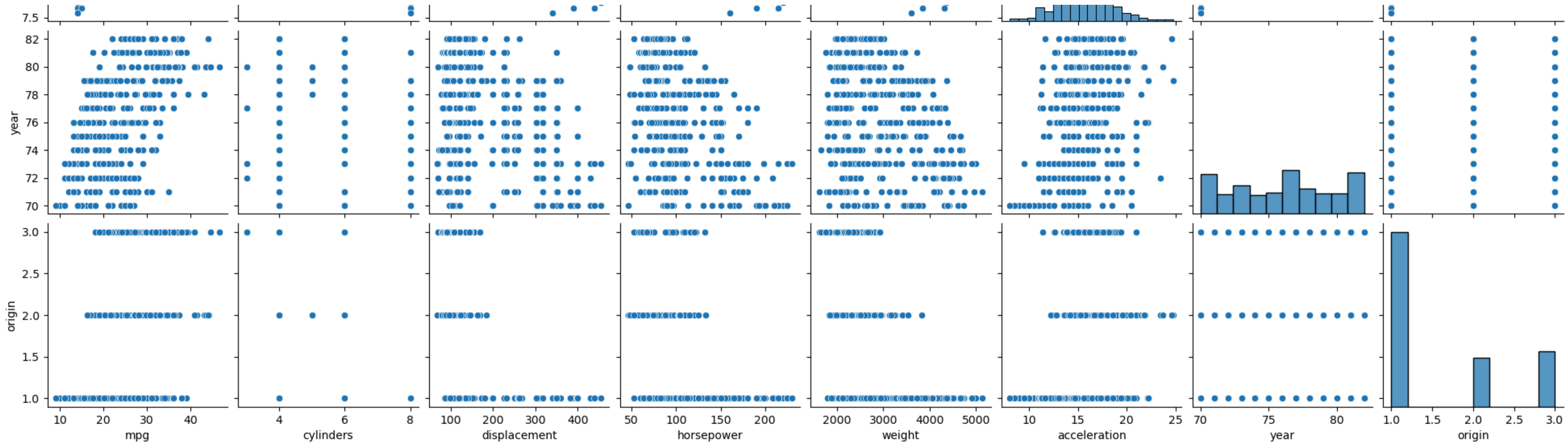
- 在這個訓練好的模型之中我們可以看到 R-squared 約為 0.595 在模型上有一定的解釋力，但整體的預測效果不夠理想。
- const (截距)：40.0058，這表示當 horsepower 為 0 時，mpg 的預測值是 40.0058。
- horsepower (自變數)：-0.1578，這個係數表示 horsepower 每增加一單位，mpg 平均會減少 0.1578。因為係數是負數，說明 horsepower 和 mpg 之間存在負相關。
- P>|t|：對應截距（const）和 horsepower 的 P 值都為 0.000，表示這兩者在 99% 信心水平下都顯著，這意味著這些係數顯著地影響應變數（mpg）。

P122 第九題

a. Produce a scatterplot matrix which includes all of the variables in the data set.

```
import seaborn as sns
import matplotlib.pyplot as plt

# 繪製散佈圖矩陣，包含所有變數
sns.pairplot(Auto)

# 顯示圖表
plt.show()
```


mpg
cylinders
displacement
horsepower
weight
acceleration

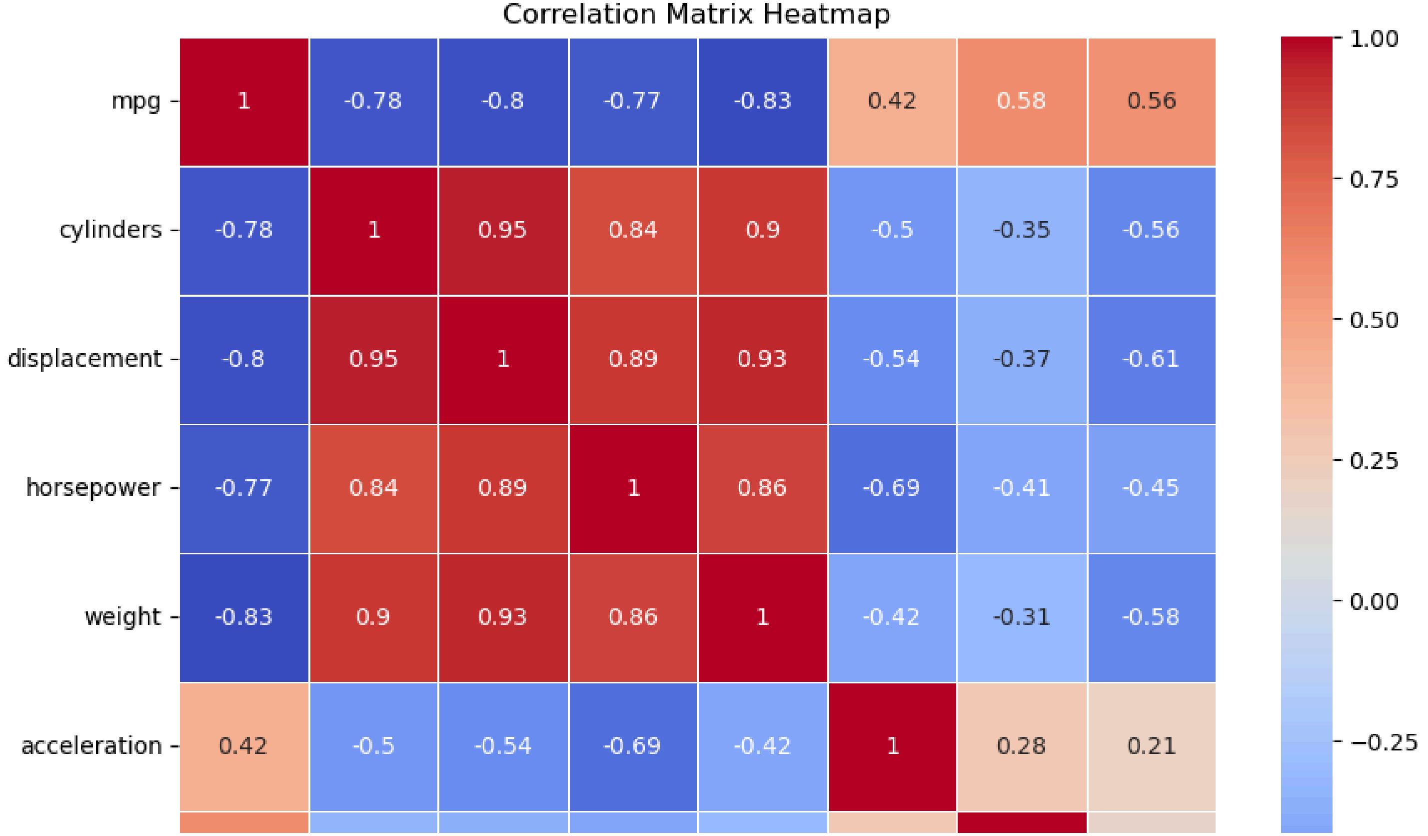


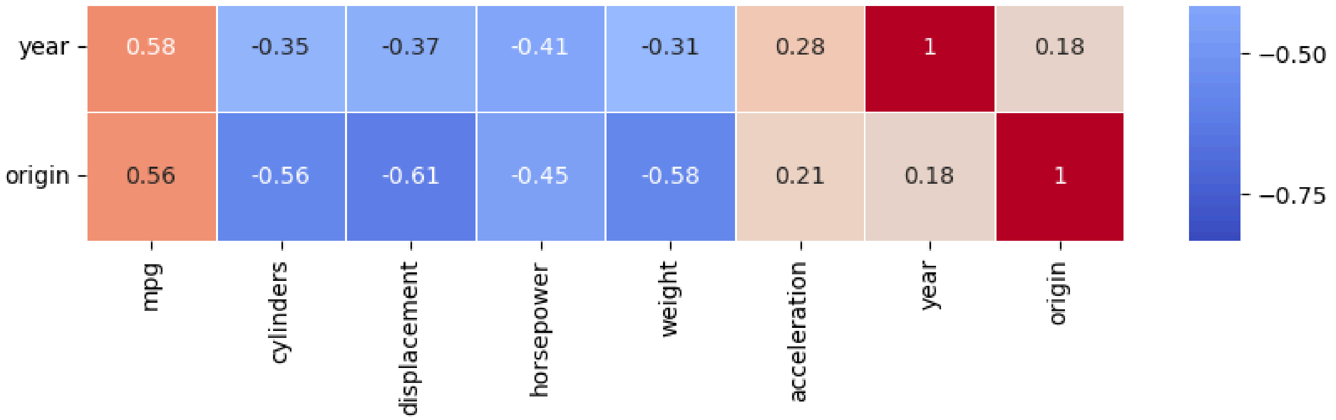
b. Compute the matrix of correlations between the variables using the function cor(). You wil need to exclude the name variable, cor() which is qualitative.

```
import seaborn as sns
import matplotlib.pyplot as plt

# 排除 name 欄位
Auto_numeric = Auto.drop(columns=['name'])

# 計算數值變數之間的相關性矩陣
correlation_matrix = Auto_numeric.corr()

# 繪製相關性矩陣的熱圖
plt.figure(figsize=(10, 8))  # 設置圖表大小
sns.heatmap(correlation_matrix, annot=True, cmap='coolwarm', linewidths=0.5)

print("corr matrix:")
print(correlation_matrix)
```

```
# 顯示圖表
plt.title('Correlation Matrix Heatmap')
plt.show()
```

```
corr matrix:
                   mpg  cylinders  displacement  horsepower    weight  \
mpg           1.000000  -0.776260     -0.804443   -0.771441 -0.831739
cylinders    -0.776260   1.000000      0.950920    0.839715  0.897017
displacement -0.804443   0.950920      1.000000    0.893833  0.933104
horsepower   -0.771441   0.839715      0.893833    1.000000  0.860581
weight       -0.831739   0.897017      0.933104    0.860581  1.000000
acceleration  0.422297  -0.504061     -0.544162   -0.687039 -0.419502
year          0.581469  -0.346717     -0.369804   -0.413022 -0.307900
origin        0.563698  -0.564972     -0.610664   -0.453962 -0.581265

              acceleration      year    origin
mpg               0.422297  0.581469  0.563698
cylinders        -0.504061 -0.346717 -0.564972
displacement     -0.544162 -0.369804 -0.610664
horsepower       -0.687039 -0.413022 -0.453962
weight           -0.419502 -0.307900 -0.581265
acceleration      1.000000  0.282901  0.210084
year              0.282901  1.000000  0.184314
origin            0.210084  0.184314  1.000000
```





c. Use the 1m) function to perform a multiple linear regression with mpg as theresponse and all other variables except name as the predictors. Use the summary () function to print the results. Commento n the output.

```
import pandas as pd
import statsmodels.api as sm

# 將自變數設為除了 'mpg' 和 'name' 的其他變數
X = Auto.drop(columns=['mpg', 'name'])

# 添加常數項
X = sm.add_constant(X)

# 應變數
y = Auto['mpg']

# 進行多元線性回歸
model = sm.OLS(y, X)
results = model.fit()

# 列印回歸結果摘要
print(results.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.822
Model:                            OLS   Adj. R-squared:                  0.818
Method:                 Least Squares   F-statistic:                     256.0
```

```
Date:                Sun, 06 Oct 2024   Prob (F-statistic):          2.41e-141
Time:                        15:26:22   Log-Likelihood:                -1037.4
No. Observations:                 397   AIC:                             2091.
Df Residuals:                     389   BIC:                             2123.
Df Model:                           7
Covariance Type:            nonrobust
================================================================================
                   coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------
const          -18.7116      4.609     -4.060      0.000     -27.773      -9.650
cylinders       -0.4452      0.323     -1.380      0.168      -1.079       0.189
displacement     0.0189      0.007      2.524      0.012       0.004       0.034
horsepower      -0.0094      0.013     -0.709      0.479      -0.035       0.017
weight          -0.0067      0.001    -10.508      0.000      -0.008      -0.005
acceleration     0.1179      0.097      1.217      0.224      -0.073       0.308
year             0.7625      0.051     15.071      0.000       0.663       0.862
origin           1.3968      0.275      5.073      0.000       0.855       1.938
==============================================================================
Omnibus:                       29.782   Durbin-Watson:                   1.291
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               47.819
Skew:                           0.506   Prob(JB):                     4.13e-11
Kurtosis:                       4.366   Cond. No.                     8.53e+04
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 8.53e+04. This might indicate that there are
strong multicollinearity or other numerical problems.
```

```
Auto.head()
```

	mpg	cylinders	displacement	horsepower	weight	acceleration	year	origin	name	x3
0	18.0	8	307.0	130.0	3504	12.0	70	1	chevrolet chevelle malibu	28032
1	15.0	8	350.0	165.0	3693	11.5	70	1	buick skylark 320	29544
2	18.0	8	318.0	150.0	3436	11.0	70	1	plymouth satellite	27488
3	16.0	8	304.0	150.0	3433	12.0	70	1	amc rebel sst	27464
4	17.0	8	302.0	140.0	3449	10.5	70	1	ford torino	27592

1. Is there a relationship between the predictors and the response?

- R-squared 是模型解釋變異的比例。在這個模型中，R-squared 為 0.822，表示自變數解釋了 82.2% 的應變數變異。這表明模型具有很強的解釋力，所以自變數與應變數之間存在明顯的關聯。
- 從摘要中看到一些變數的 p-value 小於 0.05，例如 displacement、weight、year 和 origin，表明這些自變數對 mpg 有顯著的影響。

2. Which predictors appear to have a statistically significant relationship to the response?

- Displacement (p-value = 0.012)：顯著正相關
- Weight (p-value = 0.000)：顯著負相關
- Year (p-value = 0.000)：顯著正相關
- Origin (p-value = 0.000)：顯著正相關

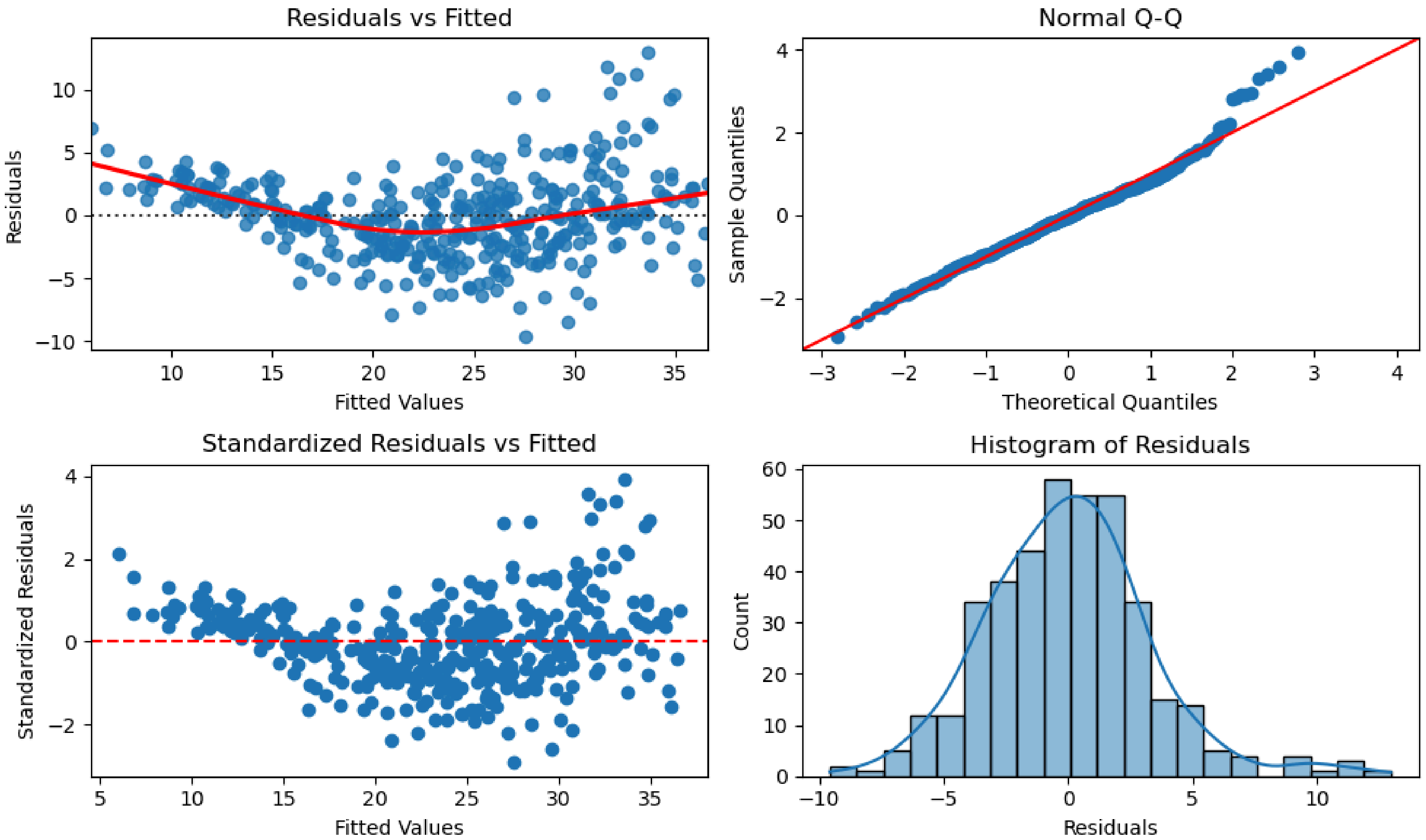
3. What does the coefficient for the year variable suggest?

- Year 變數的係數為 0.7625。這意味著每增加一個年份，車輛的燃油效率 (mpg) 平均會增加約 0.7625 英里每加侖。換句話說，更新的車型相比於較舊的車型，更加省油，這可能與車輛技術的進步或環保法規的改進有關。

d. Use the plot () function to produce diagnostic plotsof the linear regression fit.

```
import numpy as np
# 殘差
residuals = results.resid

# 預測值
fitted = results.fittedvalues

# 1. 殘差 vs 預測值
plt.figure(figsize=(10, 6))
plt.subplot(2, 2, 1)
sns.residplot(x=fitted, y=residuals, lowess=True, line_kws={'color': 'red'})
plt.xlabel('Fitted Values')
plt.ylabel('Residuals')
plt.title('Residuals vs Fitted')

# 2. QQ圖：檢查殘差的正態性
plt.subplot(2, 2, 2)
sm.qqplot(residuals, line='45', fit=True, ax=plt.gca())
plt.title('Normal Q-Q')

# 3. 標準化殘差 vs 標準化預測值
plt.subplot(2, 2, 3)
standardized_residuals = (residuals - np.mean(residuals)) / np.std(residuals)
```

```
plt.scatter(fitted, standardized_residuals)
plt.axhline(y=0, linestyle='--', color='red')
plt.xlabel('Fitted Values')
plt.ylabel('Standardized Residuals')
plt.title('Standardized Residuals vs Fitted')

# 4. 殘差的直方圖：檢查殘差的分佈
plt.subplot(2, 2, 4)
sns.histplot(residuals, kde=True)
plt.xlabel('Residuals')
plt.title('Histogram of Residuals')

plt.tight_layout()
plt.show()
```


Residuals vs Fitted
Residuals
Fitted Values
Normal Q-Q
Sample Quantiles
Theoretical Quantiles
Standardized Residuals vs Fitted
Standardized Residuals
Fitted Values
Histogram of Residuals
Count
Residuals

```
import statsmodels.api as sm
import matplotlib.pyplot as plt

# leverage plot
fig, ax = plt.subplots(figsize=(8, 6))

# 使用 statsmodels 的 influence_plot 生成杠桿圖
sm.graphics.influence_plot(results, ax=ax, criterion="cooks")
```