H_∞ Fault-Tolerant decentralized Observer-Based PID Formation Tracking Design NCS of LEOSs

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Appendix

$F_{i_1}(t) = -v_{ECI,z_i}\cos\theta_1^i \sin\theta_3^i\dot{\theta}_1^i - \cos\theta_3^i \sin\theta_1^i\dot{\theta}_3^i + \cos\theta_1^i \cos\theta_2^i \cos\theta_3^i\dot{\theta}_2^i - \cos\theta_3^i \sin\theta_1^i \sin\theta_2^i\dot{\theta}_1^i - \cos\theta_1^i \sin\theta_2^i \sin\theta_3^i\dot{\theta}_3^i$
$+\cos\theta_2^i\dot{\theta}_3^i\dot{\theta}_2^i + \cos\theta_3^i \sin\theta_2^i v_{ECI,x_i}\dot{\theta}_2^i + \cos\theta_2^i \sin\theta_3^i v_{ECI,x_i}\dot{\theta}_3^i + \sin\theta_1^i \sin\theta_3^i \dot{v}_{ECI,z_i} - \cos\theta_2^i \cos\theta_3^i \dot{v}_{ECI,x_i} + \cos\theta_1^i \sin\theta_2^i$
$\times\cos\theta_3^i \dot{v}_{ECI,z_i} + \frac{(J_2^i - J_3^i)}{J_1^i}[-\cos\theta_1^i \sin\theta_1^i\dot{\theta}_2^i\dot{\theta}_2^i + \cos\theta_1^i \cos\theta_2^i \cos\theta_1^i\dot{\theta}_2^i\dot{\theta}_3^i + v_{ECI,x_i}\cos\theta_1^i \sin\theta_2^i\dot{\theta}_2^i + v_{ECI,x_i}\cos\theta_2^i \sin\theta_1^i\dot{\theta}_2^i$
$+v_{ECI,x_i}\sin\theta_1^i \cos\theta_2^i \sin\theta_3^i\dot{\theta}_2^i - v_{ECI,x_i}\cos\theta_2^i \cos\theta_2^i \sin\theta_3^i \cos\theta_1^i\dot{\theta}_3^i - v_{ECI,x_i} v_{ECI,x_i}\cos\theta_2^i \sin\theta_3^i \sin\theta_2^i - v_{ECI,x_i}v_{ECI,z_i}$
$\times\cos\theta_2^i \sin\theta_3^i \cos\theta_1^i \cos\theta_2^i + \cos\theta_1^i \cos\theta_1^i \cos\theta_2^i v_{ECI,z_i}\dot{\theta}_2^i - \cos\theta_2^i \sin\theta_1^i \sin\theta_1^i\dot{\theta}_2^i\dot{\theta}_3^i + \cos\theta_2^i \cos\theta_2^i \sin\theta_1^i \cos\theta_1^i\dot{\theta}_3^i\dot{\theta}_3^i$
$+v_{ECI,z_i}\cos\theta_2^i \sin\theta_1^i \cos\theta_1^i \cos\theta_2^i\dot{\theta}_3^i - v_{ECI,z_i}\sin\theta_1^i \sin\theta_1^i \cos\theta_3^i\dot{\theta}_2^i - v_{ECI,z_i}\sin\theta_1^i \cos\theta_1^i \sin\theta_2^i \sin\theta_3^i\dot{\theta}_2^i + v_{ECI,z_i}\sin\theta_1^i$
$\times\cos\theta_3^i \cos\theta_2^i \cos\theta_1^i + v_{ECI,z_i}\cos\theta_1^i \cos\theta_2^i \cos\theta_1^i \sin\theta_2^i \sin\theta_3^i + (v_{ECI,z_i})^2 \times \sin\theta_1^i \cos\theta_3^i \cos\theta_1^i \cos\theta_2^i + (v_{ECI,z_i})^2 \cos\theta_1^i$
$\times\cos\theta_1^i \cos\theta_2^i \sin\theta_2^i \sin\theta_3^i];$

$F_{i_2}(t) = \frac{1}{\cos\theta_1^i}[-v_{ECI,z_i}\cos\theta_1^i \cos\theta_3^i\dot{\theta}_1^i + \sin\theta_1^i \sin\theta_3^i\dot{\theta}_3^i - \cos\theta_1^i \cos\theta_2^i \sin\theta_3^i\dot{\theta}_2^i - \cos\theta_1^i \cos\theta_3^i \sin\theta_2^i\dot{\theta}_3^i + \sin\theta_1^i \sin\theta_2^i \sin\theta_3^i$
$\times\dot{\theta}_1^i + \sin\theta_1^i\dot{\theta}_2^i\dot{\theta}_1^i - \cos\theta_1^i \cos\theta_2^i\dot{\theta}_3^i\dot{\theta}_1^i + \cos\theta_2^i \cos\theta_3^i v_{ECI,x_i}\dot{\theta}_3^i + \sin\theta_1^i \sin\theta_2^i\dot{\theta}_3^i\dot{\theta}_2^i - \sin\theta_2^i \sin\theta_3^i v_{ECI,x_i}\dot{\theta}_2^i + \cos\theta_2^i \sin\theta_3^i\dot{v}_{ECI,x_i}$
$-\sin\theta_1^i \cos\theta_3^i\dot{v}_{ECI,z_i} - \cos\theta_1^i \sin\theta_2^i \sin\theta_3^i\dot{v}_{ECI,z_i} + \frac{(J_1^i - J_3^i)}{J_2^i}(-\sin\theta_1^i\dot{\theta}_1^i\dot{\theta}_2^i + \cos\theta_2^i \cos\theta_1^i\dot{\theta}_1^i\dot{\theta}_3^i + v_{ECI,x_i}\sin\theta_2^i\dot{\theta}_1^i + v_{ECI,z_i}$
$\times\cos\theta_1^i \cos\theta_2^i\dot{\theta}_1^i + \sin\theta_1^i \sin\theta_2^i\dot{\theta}_2^i\dot{\theta}_3^i - \sin\theta_2^i \cos\theta_2^i \cos\theta_1^i\dot{\theta}_3^i\dot{\theta}_3^i - v_{ECI,x_i}\sin\theta_2^i \sin\theta_2^i\dot{\theta}_3^i - v_{ECI,z_i}\sin\theta_2^i \cos\theta_1^i \cos\theta_2^i\dot{\theta}_3^i$
$-v_{ECI,x_i}\sin\theta_1^i \cos\theta_2^i \cos\theta_3^i\dot{\theta}_2^i + v_{ECI,x_i}\cos\theta_1^i \cos\theta_2^i \cos\theta_2^i \cos\theta_3^i + (v_{ECI,x_i})^2 \cos\theta_2^i \cos\theta_3^i \sin\theta_2^i + v_{ECI,x_i}v_{ECI,z_i}\cos\theta_2^i$
$\times\cos\theta_3^i \cos\theta_1^i \cos\theta_2^i - v_{ECI,z_i}\sin\theta_1^i \sin\theta_1^i \sin\theta_3^i\dot{\theta}_2^i + v_{ECI,z_i}\sin\theta_1^i \sin\theta_3^i \cos\theta_2^i \cos\theta_1^i\dot{\theta}_3^i + v_{ECI,x_i}v_{ECI,z_i}\sin\theta_1^i \sin\theta_2^i \sin\theta_3^i$
$+ (v_{ECI,z_i})^2 \sin\theta_1^i \sin\theta_3^i \cos\theta_1^i \cos\theta_2^i + v_{ECI,z_i}\cos\theta_1^i \sin\theta_1^i \sin\theta_2^i \cos\theta_3^i\dot{\theta}_2^i - v_{ECI,z_i}\cos\theta_1^i \sin\theta_2^i \cos\theta_3^i \cos\theta_1^i \cos\theta_2^i$
$- v_{ECI,x_i}v_{ECI,z_i}\cos\theta_1^i \sin\theta_2^i \cos\theta_3^i \sin\theta_2^i - (v_{ECI,z_i})^2 \cos\theta_1^i \cos\theta_1^i \cos\theta_2^i \sin\theta_2^i \cos\theta_3^i)];$

$F_{i_3}(t) = \frac{1}{\cos\theta_1^i \cos\theta_2^i}[\cos\theta_1^i\dot{\theta}_1^i\dot{\theta}_2^i - \cos\theta_2^i v_{ECI,x_i}\dot{\theta}_2^i + \cos\theta_2^i \sin\theta_1^i\dot{\theta}_1^i\dot{\theta}_3^i + \cos\theta_1^i \sin\theta_2^i\dot{\theta}_2^i\dot{\theta}_3^i + \cos\theta_2^i \sin\theta_1^i v_{ECI,z_i}\dot{\theta}_1^i + \cos\theta_1^i$
$\times\sin\theta_2^i v_{ECI,z_i}\dot{\theta}_2^i + \sin\theta_2^i\dot{v}_{ECI,x_i} + \cos\theta_1^i \cos\theta_2^i\ \dot{v}_{ECI,z_i} + \frac{(J_2^i - J_1^i)}{J_3^i}(\cos\theta_1^i\dot{\theta}_1^i\dot{\theta}_2^i + \cos\theta_2^i \sin\theta_1^i - v_{ECI,x_i}\cos\theta_2^i \sin\theta_3^i\dot{\theta}_1^i$
$+v_{ECI,z_i}\sin\theta_1^i \cos\theta_3^i\dot{\theta}_1^i + v_{ECI,z_i}\cos\theta_1^i \sin\theta_2^i \sin\theta_3^i\dot{\theta}_1^i - \sin\theta_2^i \cos\theta_1^i\dot{\theta}_2^i\dot{\theta}_3^i - \sin\theta_2^i \sin\theta_1^i \cos\theta_2^i\dot{\theta}_3^i\dot{\theta}_3^i + v_{ECI,x_i}\sin\theta_2^i \cos\theta_2^i$
$\times\sin\theta_3^i\dot{\theta}_3^i - v_{ECI,z_i}\sin\theta_1^i \cos\theta_3^i \sin\theta_2^i\ \dot{\theta}_3^i - v_{ECI,z_i}\cos\theta_1^i \sin\theta_2^i \sin\theta_3^i \sin\theta_2^i\dot{\theta}_3^i + v_{ECI,x_i}\cos\theta_2^i \cos\theta_3^i \cos\theta_1^i\dot{\theta}_2^i + v_{ECI,x_i}$
$\times\cos\theta_2^i \cos\theta_3^i \cos\theta_2^i \sin\theta_1^i\dot{\theta}_3^i - (v_{ECI,x_i})^2 \cos\theta_2^i \cos\theta_3^i \cos\theta_2^i \sin\theta_3^i + v_{ECI,x_i}v_{ECI,z_i}\cos\theta_2^i \sin\theta_1^i \cos\theta_3^i \cos\theta_3^i + v_{ECI,x_i}$
$\times v_{ECI,z_i}\cos\theta_2^i \cos\theta_3^i \cos\theta_1^i \sin\theta_2^i \sin\theta_3^i - v_{ECI,z_i}\sin\theta_1^i \sin\theta_3^i \cos\theta_1^i\dot{\theta}_2^i - v_{ECI,z_i}\cos\theta_1^i \sin\theta_2^i \cos\theta_3^i \cos\theta_2^i \sin\theta_1^i\dot{\theta}_3^i$
$+v_{ECI,x_i}v_{ECI,z_i}\cos\theta_1^i \sin\theta_2^i \cos\theta_3^i \cos\theta_2^i \sin\theta_3^i - (v_{ECI,z_i})^2 \cos\theta_1^i \sin\theta_2^i \cos\theta_3^i \sin\theta_1^i \cos\theta_3^i - (v_{ECI,z_i})^2 \cos\theta_1^i \sin\theta_2^i$
$\times\cos\theta_3^i \cos\theta_1^i \sin\theta_2^i \sin\theta_3^i)];$