Defn: Vertex v is dominated by u if v and u are adjacent and every neighbor of v is also a neighbor of u

Cop or Robber first? Probably

A graph is cop win <=> you can remove dominated vertices down to a single vertex <-> one vert.

* Does order of dismantling effect things?

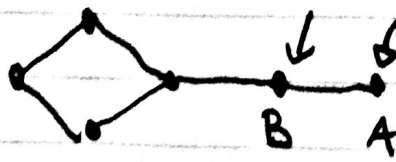
minimal core - graph with no dominated vertices

* google isomorphism in graph theory

Algorithms

How do you compute a minimal core

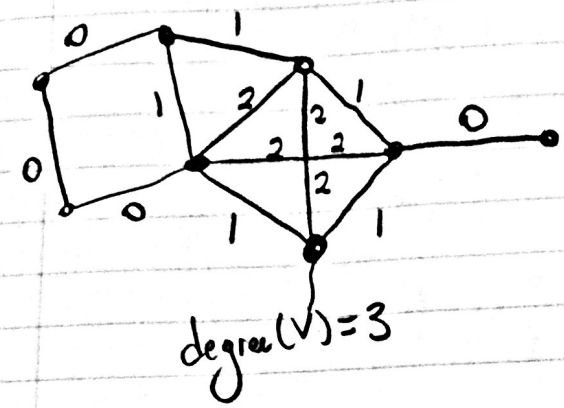
what remains after all dominated vertices are removed



Removing A makes vertex B a dominated vertex

"Simple" algorithm

2 steps: 1) find all triangles
2) Count # edges each triangle participates in



repeatedly find a vertex V with label degree V−1
=> V is dominated by the vertex on the other end.

remove V & update triangle counts.

Google implimentations

Buzzwords
- Polymake
- Sage
- Code Ourselves

Unit Disc Graph

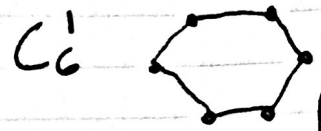
Consider $\mathbb{R}^2$, let $u, v \in \mathbb{R}^2$ if $dist(u,v) \leq 1$, create an edge $\overline{uv}$

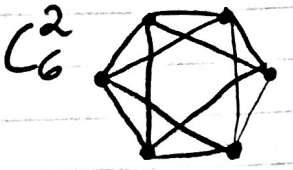
Main Question
- Are the minimal cores for unit disc graphs in $\mathbb{R}^2$ "simple combinations" of minimal cores for unit disc graphs in the circle or are there "wildly new possibilities".

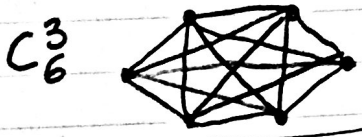
Minimal Cores of Points on a circle

Fact: given any set of points on the circle, a minimal core for its unit disc graph is of the form C_n^k for $k \leq \frac{n}{2}$

(n = # of points; k = # of neighbors each has both directions)





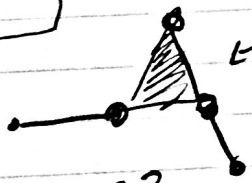


Circle 1D sphere (line locally)
2 sphere 2D sphere (plane locally)
3 sphere 4D sphere ($\mathbb{R}^3$ locally)

Topological Conjecture:

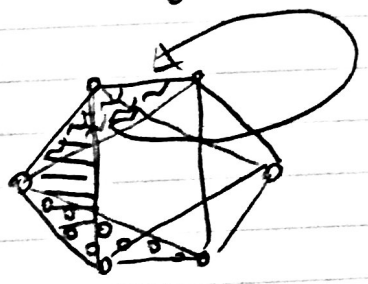
~~If you~~ simple combinations of spheres

~~Spaces associated with graphs~~



← fill triangles or 4 points or 5 points etc

6 triangles on C_6^2 and Middle triangles



Existing research on attatchments between minimal cores?

We can focus entirely on simple connected graphs?

Papers on the connection to topology?
Mathematical defn of the spaces

Defn of Simple Combinations ← to be defined

Write Q's on overleaf (blue note environment)