





# Dutnam Seminar

## The Topological Perspective

2-11-19

### Simplicial Complexes

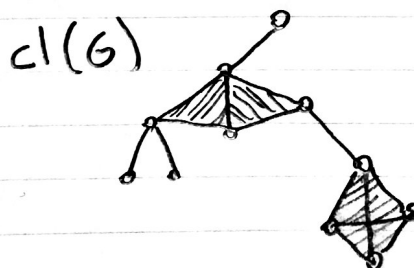
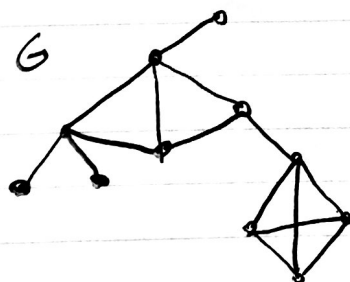




  
 0-simplex 1-simplex 2-simplex 3-simplex  
 + a  $k$  simplex will have  $k+1$  vertices

Defn A simplicial complex  $K$  on a vertex set  $V$  is a set of simplices  $\sigma \subseteq V$  st if

Clique Complex is a simplicial complex based on a graph

"you throw in all the triangles and tetrahedra you can onto a graph" denoted  $cl(G)$

it is the LARGEST simplicial complex you can make on  $G$ .



Fact: if  $v$  is a dominated vertex in  $G$ , then  $cl(G)$  and  $cl(G/v)$  ( $G$  without  $v$ ) "have the same shape."

$$\text{circle with lines} \neq \text{circle with dots} \approx \text{circle} \approx \text{square} \approx \text{triangle}$$

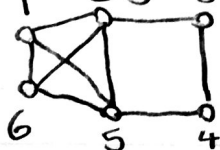
$$\text{circle with dashed line} \approx \text{cube} \neq \text{circle with dot} \approx \text{circle with square inside}$$

Henry Used  
Many Examples of  
Morphing Shapes

$cl(G)$  $\Rightarrow$  $cl(G/u)$ visual representation of  $cl(G) \cong cl(G/u)$ 

Ethan uploaded code (GitHub) to compute minimal cores on Overleaf (python) (3.6)

Adjacency Matrix



Entry = 1 iff there  $\rightarrow$   
is an edge connecting

	1	2	3	4	5	6
1	0	1	0	0	1	1
2	1	0	1	0	1	0
3	0	1	0	1	0	0
4	0	0	1	0	1	0
5	1	1	0	1	0	0
6	1	0	0	0	0	1

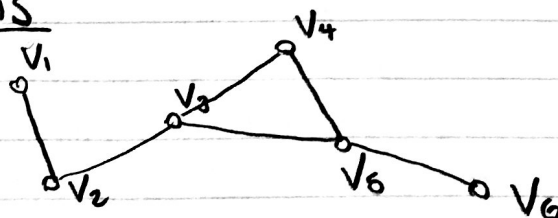
Something about using adjacency lists - over my head  
(stores connections)

exi.)  $\begin{cases} 1: (2, 5, 6, 1), \\ 2: (2, 6, 2, 1), \dots \end{cases}$

saves computations  
& cleans stuff up

Check for dominated vertices  $\rightarrow$  Remove them

Unit Disc Graphs

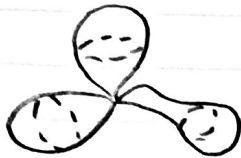
 $V \subseteq \mathbb{R}^2$ 

We have an edge  $v_i v_j$  when  $d(v_i, v_j) \leq 1$   
distance

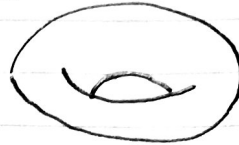
Note: Overleaf LaTeX is forgiving (nothing breaks if use improper syntax)

Conjecture: A clique complex of any unit disc graph "has the same shape" as a wedge of spheres bouquet  $\rightarrow$

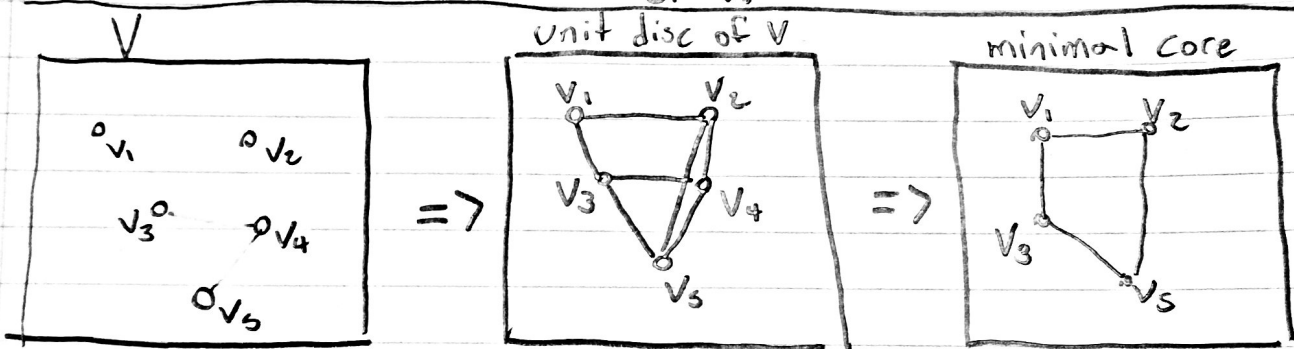
Wedge of Spheres



$\neg$ Wedge of Spheres



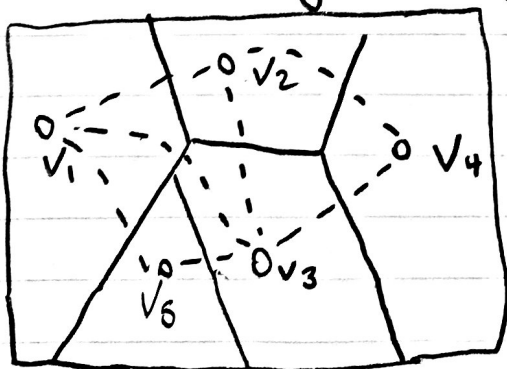
Possible Coding Task: Write code that takes  $V \subseteq \mathbb{R}^2$  as input & outputs unit disc graph of  $V$ .



The goal is to automate this process to speed up searching for patterns

\* Communication is in Overleaf - play with it  
overleaf > email

Voronoi Diagrams



Delaunay Triangulation - - -  
when share Voronoi edge

Could Be Useful