Prerequisite Knowledge Test

import statements

In [ ]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

question 1

1. load the data set in x.csv, into a pandas dataframe.

In [ ]:
```
data = pd.read_csv('x.csv', header=None)
data.columns = data.iloc[0]
data = data.drop(0, axis=0)
data = data.astype(float)
print(data)
```

```
0            V1         V2        V3        V4        V5        V6        V7  
1     -2.162804 -10.961029 -0.621498 -2.803665 -0.580941 -0.580710 -0.581963   \
2     -1.291995 -10.105274  1.395344 -2.410507 -0.528001 -0.529797 -0.528555   
3     -1.134455 -10.443600 -2.955879 -1.418108 -1.181266 -1.181214 -1.179092   
4      0.852331 -11.353038  0.875886 -1.845476  1.072317  1.073025  1.072928   
5     -0.333579 -10.635619 -0.603898 -2.034150 -1.402213 -1.400313 -1.401911   
...         ...        ...       ...       ...       ...       ...       ...   
2044  0.023132  -9.311094 -2.131136 -4.087130  0.209641  0.208257  0.208621   
2045 -2.299781  -9.695048 -0.227320 -0.836117  0.939085  0.937953  0.937301   
2046 -0.664126  -9.201309  1.720920 -2.341774  1.418704  1.418204  1.418829   
2047 -1.498205  -8.976333  1.204337 -3.933410  0.146557  0.144868  0.144880   
2048 -1.150925 -10.830732  1.676142 -1.400922 -1.490902 -1.490875 -1.489613   

0             V8        V9       V10  ...       V23       V24       V25  
1      -8.690998 -7.927330  0.022250  ... -4.417787 -5.380623  6.612514  \
2      -8.809485 -8.043681 -1.639488  ... -4.848208 -6.879187  5.396009   
3      -8.858426 -7.816160 -1.939337  ... -3.209146 -6.435492  5.157129   
4      -8.138915 -9.236082 -2.015748  ... -4.444997 -5.518425  5.429050   
5     -10.368267 -7.118578 -1.972175  ... -3.276372 -2.320216  5.034461   
...          ...       ...       ...  ...       ...       ...       ...   
2044 -10.653908 -7.435042 -1.247361  ... -3.675190 -4.338944  6.818249   
2045  -8.359275 -7.822782 -0.463678  ... -4.505038 -6.118302  5.073941   
2046  -8.021046 -7.477500 -1.415199  ... -6.729110 -4.498188  7.022160   
2047 -10.073287 -8.133478 -1.328082  ... -4.993660 -4.792246  5.345860   
2048  -9.285309 -8.191487 -3.346882  ... -3.853121 -5.511999  5.786886   

0           V26        V27       V28       V29       V30       V31       V32  
1     -0.143761   6.794164  2.556425 -0.580325 -0.621283 -0.580868 -0.621093  
2      0.417580   8.425074  1.923890 -0.527368  1.395489 -0.528136  1.393603  
3      2.560825   8.580052  2.286159 -1.180627 -2.954022 -1.181911 -2.956870  
4      0.927708   9.548430  2.443501  1.071856  0.876422  1.070787  0.875737  
5      0.723761   9.760236 -0.656288 -1.404291 -0.602963 -1.401789 -0.601442  
...         ...        ...       ...       ...       ...       ...       ...  
2044   2.178004   9.797912  1.309617  0.210725 -2.130185  0.210002 -2.128867  
2045   2.229203  10.642326  1.853915  0.939801 -0.225879  0.938923 -0.226167  
2046   0.570507   9.995046  3.943661  1.421253  1.720424  1.420025  1.722045  
2047   2.560915  10.594102  3.090243  0.144264  1.206424  0.142533  1.207139  
2048  -0.730443   8.813910  1.256484 -1.492809  1.673976 -1.490043  1.675111  

[2048 rows x 32 columns]
```

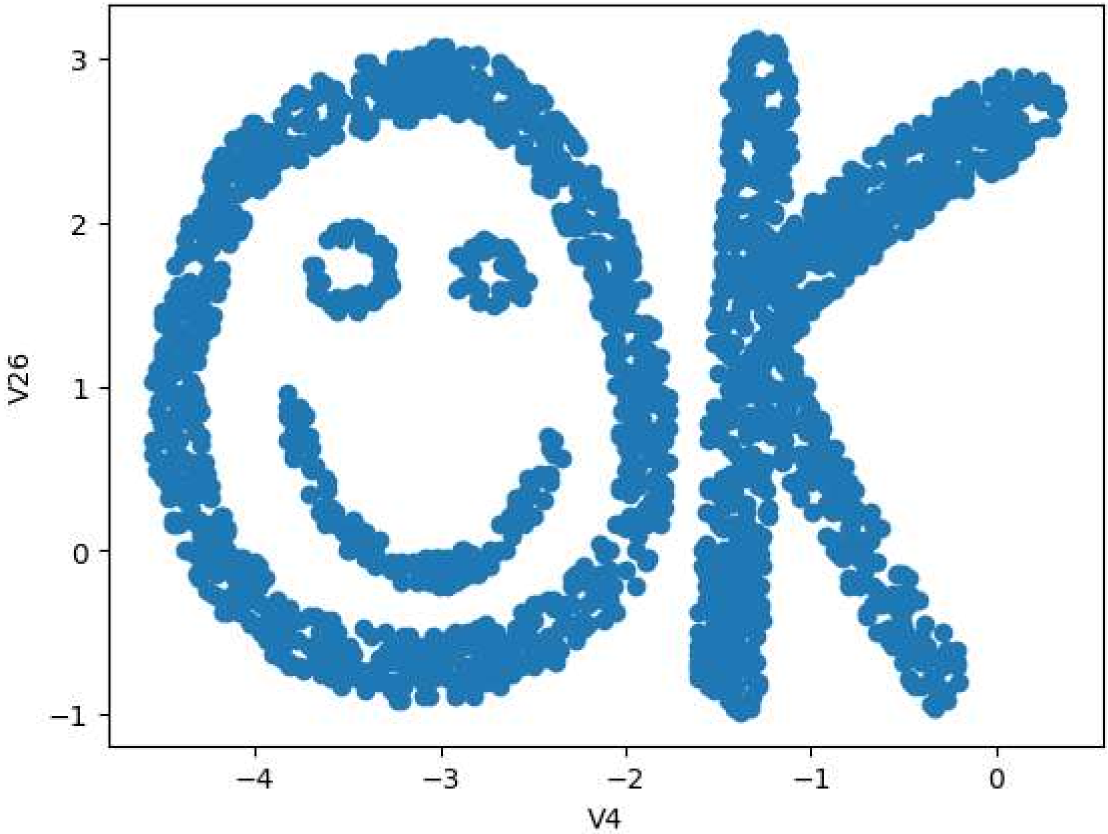
2. find the two variables with the largest variances

In [ ]:
```python
varience = data.var().sort_values(ascending=False)
sorted_data = data[varience.index]
highest_variences = sorted_data.iloc[:, 0:2]
print(f"largest var: {highest_variences.columns[0]}")
print(f"Second largest var: {highest_variences.columns[1]}")
```

```
largest var: V4
Second largest var: V26
```

3. make a scatterplot of the data items using these two variables found in question 2

In [ ]:
```python
plt.scatter(highest_variences.iloc[:, 0], highest_variences.iloc[:, 1])
plt.xlabel(highest_variences.columns[0])
plt.ylabel(highest_variences.columns[1])
plt.show()
```



Question 2

1. Solve numerically and report the eigenvalues λi and column eigenvectors xi, where i ∈ {1, 2}. Normalise the eigenvectors to unit length (if necessary).

In [ ]:
```
A = np.array([[1, 2],
              [2, 3.14159]])
eigenvalues, eigenvectors = np.linalg.eig(A)
print("Question 2.1")
print("------------")
print("eigenvalues:")
print(eigenvalues)
print("\neigenvectors:")
print(np.matrix(eigenvectors))
print()
```

```
Question 2.1
------------
eigenvalues:
[-0.19781734  4.33940734]

eigenvectors:
[[-0.85790571 -0.51380716]
 [ 0.51380716 -0.85790571]]
```

2. verify that the eigenvectors are orthogonal

In [ ]:
```
orthogonal = np.dot(eigenvectors[:, 0], eigenvectors[:, 1])
norms = np.linalg.norm(eigenvectors, axis=0)
print("Question 2.2")
```

```
print("------------")
print("dot product rounded to 5th digit (orthogonality check)", orthogonal.round
print("Norms (normalization check):", norms)
print()
```

```
Question 2.2
------------
dot product rounded to 5th digit (orthogonality check) -0.0
Norms (normalization check): [1. 1.]
```

3. Show, by performing the numerical matrix computation, that A satisfies the equation

In [ ]:
```
new_A =np.zeros((2, 2))
for i in range(len(A)):
    eigenvalue_matrix = eigenvalues[i] * np.identity(2)
    new_A += np.dot(eigenvalue_matrix,
                    eigenvectors[:, i].reshape(2, 1) * eigenvectors[:, i].reshap
print("Question 2.3")
print("------------")
print("Matrix A:")
print(np.matrix(A))
print()
print("Matrix remade A:")
print(np.matrix(new_A))
```

```
Question 2.3
------------
Matrix A:
[[1.      2.     ]
 [2.      3.14159]]

Matrix remade A:
[[1.      2.     ]
 [2.      3.14159]]
```

Question 3

Task a.

to prove E is a linear operator, we need to show that E holds under:

1. additivy: E[f + g] = E[f] + E[g]
2. multiplicity: E[cf] = cE[f]

for some real valued random variables f, g and scaler c.

1. Addivilty:

   Let f and g be some real valued random variables. we have:

   E[f + g] = Σ (P(ω) * (f(ω) + g(ω))) for all ω in Ω

   ```
       = Σ (P(ω) * (f(ω)) + P(ω) * g(ω))
       = Σ (P(ω) * (f(ω)) + Σ (P(ω) * g(ω)))
       = E[f] + E[g]
   ```

2. Multiplicity:

   let f be some real valued random variable, and c be a scaler. we have:

   cE[f] = c(Σ (P(ω) * (f(ω))))

   ```
   = Σ (c * P(ω) * (f(ω)))
   = Σ (P(ω) * (c * f(ω)))
   = E[cf]
   ```

As E holds under Additivity and Multiplicity, E is a linear operator

Task b.

We know, Var[X] = E[(X - μ)^2]

```
                 = E[(X^2 -2μX + μ^2)]
                 = E[X^2] - E[2μX] + E[μ^2] {using properties of
linear operators}
                 = E[X^2] - 2E[X]E[X] + E[X]^2 {as μ = E[X]}
                 = E[X^2] - 2E[x]^2 + E[X]^2
                 = E[X^2] - E[X]^2
```

thus, we have Var[X] = E[X^2] - E[X]^2

Question 4:

Task a:

we have:

```
    P(Y | X) = P(Y ∧ X)/P(X) {def of conditional probability}
    P(Y | X) = P(X ∧ Y)/P(X) {by properties of union}
    P(Y | X)*P(X) = P(X ∧ Y)
```

Thus we get
[1] P(Y | X)*P(X) = P(X ∧ Y)

Now solving for P(X | Y):

```
    P(X | Y) = P(X ∧ Y)/P(Y)
             = P(Y | X) * P(X) / P(Y) {using [1]}
```

which is Bayes' rule

Task b:

Defining Boolean random variables:

A : A person who is alergic to pollen

A': A person who is not allergit to pollen

T : Test result is positive

T': Test result is negative

we have:

```
P(T |A') = .23
P(T |A ) = 1- .23 = .77
P(T'|A ) = .15
P(T'|A') = 1-.15  = .85
P(A)     = .20
P(A')    = 1-.20  = .80
```

1. first we need to find P(T)

```
P(T) = P(T |A ) ∗ P(A) + P(T |A’) ∗ P(A’)
P(T) = .77 * .20 + .23 * .80
P(T) = 0.354
```

2. now we solve for P(A |T):

```
P(A | T) = P(T |A ) * P(A) / P(T)
P(A | T) = .77 * .20 / 0.354
P(A | T) = 0.480
```

Thus We get:

P(A | T) = 0.480

Question 5

Task a

we can find the value of b S.T. f(b) is minimised using the 0s of the derrivitive of f(b) with respect to b.

First we expand f(b):

f(b) = ∑(i=1, 3) (b^2xi^2 - 2bxiyi + yi^2)

and take its derrivitive:

df/db = ∑(i=1, 3) (2bxi^2 - 2xi*yi)

Now we solve for df/db = 0

df/db = 0

```
0 = ∑(i=1, 3) (2b*xi^2 - 2*xi*yi)
0 = (∑(i=1, 3) (2b*xi^2))  - (∑(i=1, 3) (2*xi*yi))
0 = 2b*(∑(i=1, 3) (xi^2)) - (∑(i=1, 3) (2*xi*yi))
b = (∑(i=1, 3) 2*xi*yi) / 2*(∑(i=1, 3) xi^2)
b = 2*(x1*y1 + x2*y2 + x3*y3) / 2*((x1)^2+(x2)^2+(x3)^2)
```

```
{inputing x and y values}
b = (x1*y1 + x2*y2 + x3*y3) / ((x1)^2+(x2)^2+(x3)^2)
```

- (note as f(b) is a convex polynomial of the 2nd degree, thus b is a local minimum)

thus when we have

b = (x1*y1 + x2*y2 + x3*y3) / ((x1)^2+(x2)^2+(x3)^2)

b is the value that minimises the value of f(b)

task b:

1. **non-zero:**

   if xi = yi = 0, for all i ∈ {1, 2, 3}, then for all values of b, f(b) = 0
2. **non-colinear:**

   if xi = yi where i ∈ {1, 2, 3}, then for all values of b, f(b) = 0
3. **finite:**

   if xi is non-finite or yi is non-finite for any i ∈ {1, 2, 3} f(b) will not have a real solution.

Problem 6:

Task a:

in sudo-code:

Function FibonacciNumbersUpToN(n)

```
    Declare a as Integer := 0
    Declare b as Integer := 1
    Declare fibs as List

    For i from 0 to n-1
        // Update the values of a and b such that:
        // a receives the value of b
        // b receives the value of a + b
        Swap a with b
        b := a + b

        // Add the new value of a to the list
        Add a to fibs

    // Return the list containing the Fibonacci sequence of
numbers to n
    Return fibs
```

in python:

In [ ]:
```python
def fibonacci_to_n(n):
    a = 0
    b = 1
    fibs=[]
```

```
    for i in range(n):
        a, b = b, a+b
        fibs.append(a)
    return fibs

if __name__ == "__main__":
    print("Task A")
    print("------------")
    fibs = fibonacci_to_n(10)
    for i in range(len(fibs)):
        print(f"number: {i+1}, fibonacci number: {fibs[i]}")
```

```
Question 3
------------
number: 1, fibonacci number: 1
number: 2, fibonacci number: 1
number: 3, fibonacci number: 2
number: 4, fibonacci number: 3
number: 5, fibonacci number: 5
number: 6, fibonacci number: 8
number: 7, fibonacci number: 13
number: 8, fibonacci number: 21
number: 9, fibonacci number: 34
number: 10, fibonacci number: 55
```

Task b:

There is one loop that iterates from 1 to n (time complexity of O(n)) the rest of the algorithm has no other loops (time complexity of O(1)), thus the time complexity of this algorithm is O(n).