

③ FIRST P.C.

$$S \vec{\phi}_1 = \lambda_1 \vec{\phi}_1 \rightarrow \vec{\phi}_1 = \begin{bmatrix} \phi_{11} \\ \phi_{21} \\ \vdots \\ \phi_{p1} \end{bmatrix}$$

$$\vec{z}_1 = \phi_{11} \begin{bmatrix} \vec{x}_1 \end{bmatrix} + \phi_{21} \begin{bmatrix} \vec{x}_2 \end{bmatrix} + \dots + \phi_{p1} \begin{bmatrix} \vec{x}_p \end{bmatrix}$$

$$= \begin{bmatrix} \vec{x}_1 & \vec{x}_2 & \dots & \vec{x}_p \\ \downarrow & \downarrow & & \downarrow \end{bmatrix} \begin{bmatrix} \phi_{11} \\ \phi_{21} \\ \vdots \\ \phi_{p1} \end{bmatrix} = X \vec{\phi}_1$$

(line 164 of code)

④ reconstituting the image (1 pc)

$$\vec{\phi}_1, \vec{\phi}_2, \dots, \vec{\phi}_p$$

$$\vec{z}_1, \vec{z}_2, \dots, \vec{z}_p$$

$$\vec{\phi}_1 : 200 \times 1$$

$$\vec{z}_1 : 267 \times 1$$

new image :

$$\underbrace{\vec{z}_1 \vec{\phi}_1^T}_{267 \times 1 \quad 1 \times 200} = 267 \times 200$$

\vec{z}_1

$$\begin{bmatrix} z_{11} \\ z_{21} \\ \vdots \\ z_{n1} \end{bmatrix}$$

$$\begin{bmatrix} \phi_{11} & \phi_{21} & \dots & \phi_{p1} \\ z_{11}\phi_{11} & z_{11}\phi_{21} & \dots & z_{11}\phi_{p1} \\ \vdots & \vdots & \ddots & \vdots \\ z_{n1}\phi_{11} & z_{n1}\phi_{21} & \dots & z_{n1}\phi_{p1} \end{bmatrix}$$

267

200

$\vec{\phi}_1^T$

5 reconstituting
(2 PC)

$$\begin{array}{c} \vec{z}_1 \end{array} \begin{bmatrix} z_{11} \\ z_{21} \\ \vdots \\ z_{n1} \end{bmatrix} + \begin{array}{c} \vec{\phi}_1^T \end{array} \begin{bmatrix} \phi_{11} & \phi_{12} & \dots & \phi_{1p} \\ z_{11}\phi_{11} & z_{11}\phi_{12} & \dots & \\ \vdots & \vdots & \ddots & \vdots \\ z_{n1}\phi_{11} & z_{n1}\phi_{12} & \dots & \end{bmatrix} \begin{array}{c} 267 \end{array} + \begin{array}{c} \vec{z}_2 \end{array} \begin{bmatrix} z_{12} \\ z_{22} \\ \vdots \\ z_{n2} \end{bmatrix} + \begin{array}{c} \vec{\phi}_2^T \end{array} \begin{bmatrix} \phi_{12} & \phi_{22} & \dots & \phi_{p2} \\ z_{12}\phi_{12} & & & \\ \vdots & \vdots & \ddots & \vdots \\ z_{n2}\phi_{12} & & & \end{bmatrix} \begin{array}{c} 267 \\ 200 \end{array}$$

new image : $\vec{z}_1 \vec{\phi}_1^T + \vec{z}_2 \vec{\phi}_2^T$

3PC

$$\begin{matrix} \vec{z}_1 & \vec{z}_2 & \vec{z}_3 \\ \left[\begin{array}{ccc} z_{11} & z_{12} & z_{13} \\ z_{21} & z_{22} & z_{23} \\ \vdots & \vdots & \vdots \\ z_{n1} & z_{n2} & z_{n3} \end{array} \right] & \begin{matrix} \vec{\phi}_1^T \\ \vec{\phi}_2^T \\ \vec{\phi}_3^T \end{matrix} & \left[\begin{array}{cccc} \phi_{11} & \phi_{21} & \dots & \phi_{p1} \\ \phi_{12} & \phi_{22} & \dots & \phi_{p2} \\ \phi_{13} & \phi_{23} & \dots & \phi_{p3} \end{array} \right] \end{matrix}$$

$n \times 3$ $3 \times p$

first
pixel

$n \times p$

$$= \left[\begin{array}{l} z_{11} \phi_{11} + z_{12} \phi_{12} + z_{13} \phi_{13} \\ z_{21} \phi_{11} + z_{22} \phi_{12} + z_{23} \phi_{13} \\ \vdots \\ \vdots \end{array} \right]$$