

③

FIRST P.C.

$$S\vec{\phi}_1 = \lambda_1 \vec{\phi}_1 \rightarrow \vec{\phi}_1 = \begin{bmatrix} \phi_{11} \\ \phi_{21} \\ \vdots \\ \phi_{p1} \end{bmatrix}$$

$$\vec{z}_1 = \phi_{11} \begin{bmatrix} \vec{x}_1 \end{bmatrix} + \phi_{21} \begin{bmatrix} \vec{x}_2 \end{bmatrix} + \dots + \phi_{p1} \begin{bmatrix} \vec{x}_p \end{bmatrix}$$

$$= \begin{bmatrix} \vec{x}_1 & \vec{x}_2 & \dots & \vec{x}_p \end{bmatrix} \begin{bmatrix} \phi_{11} \\ \phi_{21} \\ \vdots \\ \phi_{p1} \end{bmatrix} = X \vec{\phi}_1$$

(line 164 of code)

④

reconstituting
the image \check{z} (1 pc)

($\check{\phi}_1, \check{\phi}_2, \dots, \check{\phi}_P$)

$\check{\phi}_1, \check{\phi}_2, \dots, \check{\phi}_P$

$\check{z}_1, \check{z}_2, \dots, \check{z}_P$

$\check{\phi}_1 : 200 \times 1$

$\check{z}_1 : 267 \times 1$

new image :

$\check{z}_1, \check{\phi}_1^T$

$267 \times 1 \quad 1 \times 200$

267×200

$$\begin{bmatrix} z_{11} \\ z_{21} \\ \vdots \\ z_{n1} \end{bmatrix}$$

$$\begin{bmatrix} \phi_{11} & \phi_{21} & \dots & \phi_{P1} \end{bmatrix}$$

267

200

$\check{\phi}_1^T$

5

reconstituting
(2 PC)

$$\begin{bmatrix} z_{11} \\ z_{21} \\ \vdots \\ z_{n1} \end{bmatrix} \xrightarrow{\vec{z}_1} \begin{bmatrix} \varphi_{11} & \varphi_{21} & \dots & \varphi_{p1} \end{bmatrix} \begin{bmatrix} z_{11} \varphi_{11} & z_{11} \varphi_{12} & \dots \end{bmatrix}$$

267

$$+ \begin{bmatrix} z_{12} \\ z_{22} \\ \vdots \\ z_{n2} \end{bmatrix} \xrightarrow{\vec{z}_2} \begin{bmatrix} \varphi_{12} & \varphi_{22} & \dots & \varphi_{p2} \end{bmatrix} \begin{bmatrix} z_{12} \varphi_{12} & z_{12} \varphi_{12} & \dots \end{bmatrix}$$

267

200

new image

$$\vec{z}_1 \vec{\varphi}_1^\top + \vec{z}_2 \vec{\varphi}_2^\top$$

3PC

$$\begin{bmatrix} \vec{z}_1 & \vec{z}_2 & \vec{z}_3 \\ z_{11} & z_{12} & z_{13} \\ z_{21} & z_{22} & z_{23} \\ \vdots & \vdots & \vdots \\ z_{n1} & z_{n2} & z_{n3} \end{bmatrix} \quad \begin{bmatrix} \vec{\phi}_1^T \\ \vec{\phi}_2^T \\ \vec{\phi}_3^T \end{bmatrix} \quad \begin{bmatrix} \varphi_{11} & \varphi_{21} & \dots & \varphi_{p1} \\ \varphi_{12} & \varphi_{22} & \dots & \varphi_{p2} \\ \varphi_{13} & \varphi_{23} & \dots & \varphi_{p3} \end{bmatrix}$$

first pixel

$$n \times 3 \quad 3 \times p$$

\vec{z}

$$\begin{bmatrix} z_{11}\varphi_{11} + z_{12}\varphi_{12} + z_{13}\varphi_{13} \\ z_{21}\varphi_{11} + z_{22}\varphi_{12} + z_{23}\varphi_{13} \\ \vdots \\ z_{n1}\varphi_{11} + z_{n2}\varphi_{12} + z_{n3}\varphi_{13} \end{bmatrix}$$