

Dynamic Bayesian Network for Probabilistic Modeling of Tunnel Excavation Processes

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Abstract: *A dynamic Bayesian network (DBN) model for probabilistic assessment of tunnel construction performance is introduced. It facilitates the quantification of uncertainties in the construction process and of the risk from extraordinary events that cause severe delays and damages. Stochastic dependencies resulting from the influence of human factors and other external factors are addressed in the model. An efficient algorithm for evaluating the DBN model is presented, which is a modification of the so-called Frontier algorithm. The proposed model and algorithm are applied to an illustrative case study, the excavation of a road tunnel by means of the New Austrian Tunneling Method.*

1 INTRODUCTION

Estimations of time and cost of tunnel construction projects are subject to major uncertainties, which are caused by uncertain geotechnical conditions, varying performance of the used excavation technologies, and human and organizational factors. Despite these uncertainties, at present a majority of stakeholders (project owners, contractors, public, etc.) rely on deterministic estimates of project time and cost, which are based on expert judgment. In addition to these deterministic estimates, the risk is often analyzed by means of semiquantitative or qualitative methods, which are also based on

expert judgments; applications of this approach are presented in Sturk et al. (1996), Shahriar et al. (2008), and Hong et al. (2009). The approach is also recommended in “Guidelines for tunnelling risk management: International Tunnelling Association” (Eskesen et al., 2004).

Because time and cost estimates are fundamental parameters for decision making in all phases of tunnel construction projects, it should be evident that a more realistic assessment of the associated uncertainties is crucial. The need of probabilistic prediction of construction time and costs and their communication with the stakeholders has been discussed in the tunneling community in recent years (Lombardi, 2001; Reilly, 2005; Grasso et al., 2006). Several models for such predictions have been developed, as summarized in the following.

In Ruwanpura and Ariaratnam (2007), tools for simulation of the tunnel drilling process are presented, which include Monte Carlo Simulation (MCS) for the evaluation of uncertainties in predicting construction time and costs. Isaksson and Stille (2005) suggest an analytical solution for probabilistic estimation of tunnel construction time and cost considering both normal variations of the performance and extraordinary events. In Chung et al. (2006) and in Benardos and Kaliampakos (2004), observed advance rates are used for updating the predictions of advance rates and resulting excavation time for the remaining part of the tunnel, by means of Bayesian analysis and artificial neural networks, respectively.

At present, the most advanced method for probabilistic modeling of uncertainties in the tunnel construction processes available in the literature is the Decision Aids for Tunneling (DAT), developed by the group of

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Prof. Einstein at MIT. It has been applied to several projects, an overview of which is given in Min (2008). DAT uses MCS for probabilistic prediction of construction time, costs, and consumption of resources. It takes into account the geotechnical uncertainties, which are modeled by means of a Markov process (Chan, 1981), as well as the uncertainties in the construction process. In the applications, the coefficients of variation of the total construction time and cost estimated by DAT are typically less than 5%. This computed uncertainty is too low when compared to the one observed in practice, e.g., in Flyvbjerg et al. (2004).

The risk of extraordinary events, such as tunnel collapse, tunnel flooding, or legislative and political obstruction, is not included in the original DAT model. A model to include the risk of a tunnel collapse has been presented in Sousa and Einstein (2012). Therein, a dynamic Bayesian networks (DBN) model and decision graphs were used to identify the optimal construction method based on a comparison of expected utilities of different methods (whereby utilities correspond to costs). Uncertainty in the costs is not considered and no probabilistic estimate of the overall construction cost and time is presented.

None of the models known to the authors fulfills all requirements that are deemed important for a realistic estimation of construction time and costs. A tunnel construction model should ideally provide the following: (1) It should correctly model common factors that systematically influence the construction process, such as human and organizational factors. These factors lead to stochastic dependence among the random variables describing performance at different phases of the excavation. The significant influence of such dependencies on construction time estimates is shown for example in Yang (2007) and Moret and Einstein (2011). (2) The model should consider the risk of extraordinary events (e.g., tunnel collapse, tunnel flooding). These events, even if they have relatively small probabilities, cannot be neglected as they often lead to huge delays and damages (IMIA, 2006; Špačková et al., 2010; Sousa and Einstein, 2012). (3) The model should allow for making full use of data available from previous projects, such as advance rates and costs recorded during excavation of tunnels under similar conditions. In this way, the know-how can be systematically managed. (4) The methodology should facilitate the easy updating of predictions when new information on the analyzed project (e.g., geotechnical investigations, performance rates and costs observed after commencement of excavation) is available. (5) The model assumptions and involved simplifications must be properly understood and described. This is important in probabilistic modeling, where results are difficult to

validate by experiments and must therefore be well reasoned.

The requirements stated above motivate the development of a methodology based on DBN. Although many of the requirements could be satisfied by means of the commonly used MCS approach, we propose DBNs, because they are more efficient in updating of the predictions based on additional observations (requirement 4) and because the graphical nature of DBN strongly facilitates the representation and communication of the model assumptions (requirement 5), in particular when dependencies among random variables are present (requirement 1). The DBN presented in this article also includes extraordinary events and the influence of human and organizational factors (requirements 1 and 2). The learning of model parameters from data (requirement 3) is not addressed in detail in this article. However, the DBN framework facilitates such learning.

The proposed DBN is applied to the case study originally presented in Min (2003) and Min et al. (2003). The proposed DBN is here limited to estimating construction time, but adaptation of the model to predict construction cost is straightforward by simply replacing the variable time with variable cost.

The article starts out with a general introduction to Bayesian networks (BNs) and to the Frontier algorithm applied for evaluating DBNs (Section 2). Thereafter, the structure of the proposed DBN model for tunnel excavation is presented in Section 3. In Section 4, the application of the frontier algorithm for evaluating the tunnel DBN is explained in detail. A modification of the algorithm is presented, which significantly increases its efficiency for this application. Finally, the DBN model is applied to a case study, allowing investigation of the effect of extraordinary events and the influence of human and organizational factors on the excavation time estimates.

2 BAYESIAN NETWORKS—BASIC PRINCIPLES

BNs are directed acyclic graphical models for representation of a set of random variables. Random variables are symbolized by the nodes of the BN, the dependencies between them are depicted by directed links. The set of random variables X_1, X_2, \dots, X_N is fully described by the graphical structure and the conditional probability distribution of each node X_i given its parent nodes $pa(X_i)$. Parent nodes are all nodes with links pointing toward X_i . The joint probability mass function (PMF) of X_1, X_2, \dots, X_N is expressed using the chain rule as

$$p(x_1, x_2, \dots, x_N) = \prod_{i=1}^N p(x_i | pa(x_i)) \quad (1)$$

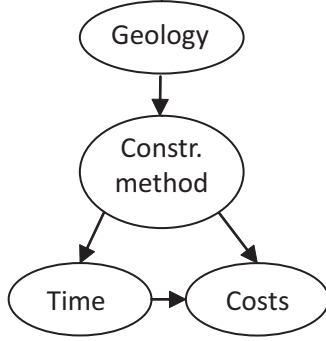


Fig. 1. Example of a Bayesian network.

where $p(x_i|pa(x_i))$ is the conditional PMF of variable X_i given its parent variables. The notation used here applies to discrete random variables, which is the case for the model proposed in this article. Whenever no ambiguity arises we use $p(x_1, x_2, \dots, x_N)$ as the short notation for $p_{X_1, X_2, \dots, X_N}(x_1, x_2, \dots, x_N)$ and similarly $p(x_i|pa(x_i))$ for $p_{X_i|pa(X_i)}(x_i|pa(x_i))$.

The efficiency of the BN stems from the decomposition of the joint probability distribution into local conditional probability distributions according to Equation (1). This decomposition is made possible, because the graphical structure of the BN encodes information about dependence among random variables. From the BN graph, one can directly infer which random variables are statistically independent of each other (d-separated in BN terminology). The statistical dependencies change when states of one or more nodes in the network are fixed (e.g., when evidence is available). For a given set of nodes A , it is possible to identify another set of nodes, which, when fixed, d-separate A from the rest of the network. This set is called the Markov blanket of A . For a more detailed introduction to BN we refer to Jensen and Nielsen (2007).

An example of a simple BN is depicted in Figure 1. This BN contains four random variables: geology G , construction method M , excavation time T , and construction costs C . The construction method M is defined conditionally on geology G (i.e., G is a parent node of M and, correspondingly, M is a child node of G), the excavation time T is defined conditionally on the construction method M , and costs C are defined conditionally on both M and T .

Following Equation (1), the joint PMF of this BN is

$$p(g, m, t, c) = p(g)p(m|g)p(t|m)p(c|m, t) \quad (2)$$

where $p(g)$ is the PMF of G and $p(m|g)$, $p(t|m)$, and $p(c|m, t)$ are conditional PMFs of M , T , and C . The values of the conditional PMFs are conveniently organized in conditional probability tables (CPTs). An ex-

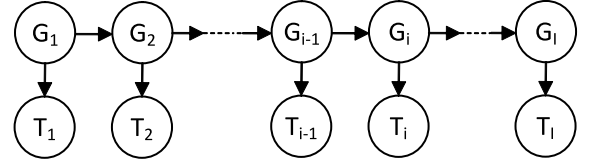


Fig. 2. Example of a dynamic Bayesian network (DBN).

ample CPT is provided in the application presented in Section 5.1.

Assumptions concerning dependencies among the random variables are made when constructing the BN in Figure 1. According to this model, if the construction method is known (fixed), any information about geology does not alter the probability distribution of time and costs, i.e., T and C are statistically independent of G if M is fixed.

DBNs are a special case of BNs used for modeling of random processes. An example is depicted in Figure 2. The i th slice of the DBN represents the state of the system in time/position i . Because the state of the system in slice i has only its state in slice $(i - 1)$ as its parent, this DBN represents a Markov chain. In a Markov chain, the future states are independent of the past states when the present state is known.

In the example of Figure 2, each slice i consists of two random variables: geology G_i and unit excavation time T_i . The joint probability of G_i and T_i is obtained as

$$p(g_i, t_i) = \sum_{g_{i-1}} p(g_{i-1})p(g_i|g_{i-1})p(t_i|g_i) \quad (3)$$

where $p(g_{i-1})$ is the marginal probability distribution of random variable G_{i-1} in slice $(i - 1)$, $p(g_i|g_{i-1})$ is the conditional probability describing changes of geology between neighboring slices and $p(t_i|g_i)$ is the conditional probability of T_i in slice i defined conditionally on geology in the same slice.

The use of BNs and DBNs in engineering applications has grown significantly in recent years (Weber et al., 2010). One reason is their graphical nature that facilitates communication of the model assumptions. Second, the BNs allow to easily update the prediction when additional information becomes available. Finally, the BN allows decomposing large models into local probabilistic dependences. Therefore, they are especially suitable for engineering applications, where statistical data is often sparse, but where conditional probability distributions of variables can be modeled by means of engineering models, expert judgment, or other known relations. Applications of BNs and DBNs in engineering problems can be found for example in Faber et al. (2002), Grêt-Regamey and Straub (2006), Neil et al. (2008), Droguett et al. (2008), Straub (2009), or Straub and Der Kiureghian (2010b).

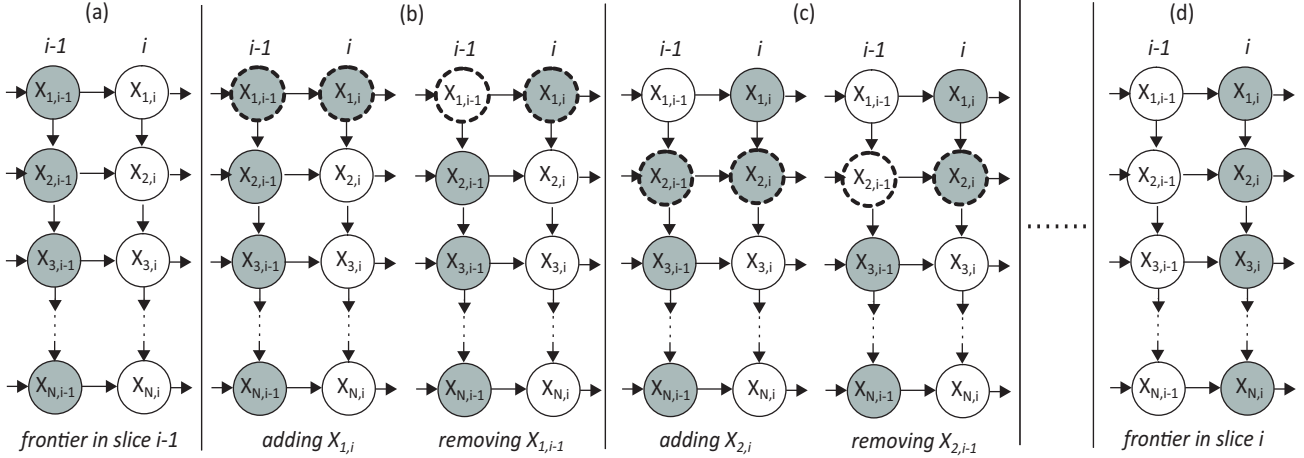


Fig. 3. Graphical representation of one cycle of the Frontier algorithm for an example DBN. The grey nodes are those included in the frontier at a given step.

2.1 Evaluation of the DBN—basic principles of the Frontier algorithm

For the evaluation of the DBN presented in this article, a procedure based on the so-called Frontier algorithm (Murphy, 2002) is used. It enables one to compute the marginal probability distribution of all random variables. The algorithm belongs to the group of exact inference methods and is applicable to DBNs with discrete nodes.

The Frontier algorithm uses the fact that in the DBN one can identify sets of nodes, which, if fixed, d-separate the nodes on their left side from the nodes on their right side. (These sets of nodes are Markov blankets of the sets of nodes on either their left or their right side.) These sets are called frontiers (or frontier sets). To give an example, all variables in slice i of the DBN in Figure 2 create a frontier. They d-separate variables in the slices $j > i$, representing future states of the process (right side of the DBN), from the variables in the slices $j < i$, representing past states of the process (left side of the DBN).

For the evaluation of the DBN, the frontier is moved slice by slice along the network. We can add a variable to the frontier if all its parents are already included in the frontier. We can remove a variable from the frontier if all its children variables are included in the frontier.

In the following, the Frontier algorithm is illustrated on a DBN containing N variables in each slice. One cycle of the algorithm moving frontier from slice $i - 1$ to slice i is shown in Figure 3. The variables marked with grey are those included in the frontier at a particular step. At the beginning of the cycle (Figure 3a), the frontier contains variables $X_{1,i-1}, X_{2,i-1}, \dots, X_{N,i-1}$, and $p(x_{1,i-1}, \dots, x_{N,i-1})$ is the known joint PMF of these variables.

In step (b), a variable $X_{1,i}$ is added to the frontier and variable $X_{1,i-1}$ is removed from the frontier. Adding $X_{1,i}$ corresponds to calculating the joint PMF $p(x_{1,i-1}, \dots, x_{N,i-1}, x_{1,i})$ as

$$p(x_{1,i-1}, \dots, x_{N,i-1}, x_{1,i}) = p(x_{1,i-1}, \dots, x_{N,i-1})p(x_{1,i}|pa(x_{1,i})) \quad (4)$$

Removing $X_{1,i-1}$ is performed by summing the joint PMF over all states of $X_{1,i-1}$. This operation is called marginalization in BN terminology.

$$p(x_{2,i-1} \dots x_{N,i-1}, x_{1,i}) = \sum_{X_{1,i-1}} p(x_{1,i-1}, x_{2,i-1} \dots x_{N,i-1}, x_{1,i}) \quad (5)$$

The above steps are repeated for other nodes until the frontier consists only of nodes of slice i as shown in Figure 3d. The cycle is then repeated for the next slice $i + 1$ and so on. The marginal distribution of any variable $X_{j,i}$ can be obtained from the joint distribution of any frontier that includes this variable, through elimination of all other variables in that frontier.

As seen from Equations (4) and (5) above, in each step the algorithm requires only the joint probability of the variables in the frontier, which reduces the computational demand significantly. In every step, the frontier should include as few variables as possible. We therefore add a new variable to the frontier as late as possible and we remove variables from the frontier as soon as possible.

Evidence (observations of random variables) can be efficiently included in the DBN. Consider observation of node $X_{j,i} = x$. The frontier algorithm proceeds until a frontier including $X_{j,i}$ is reached. The observation is then included by setting the probability of all outcome states with $X_{j,i} \neq x$ equal to zero and normalizing the

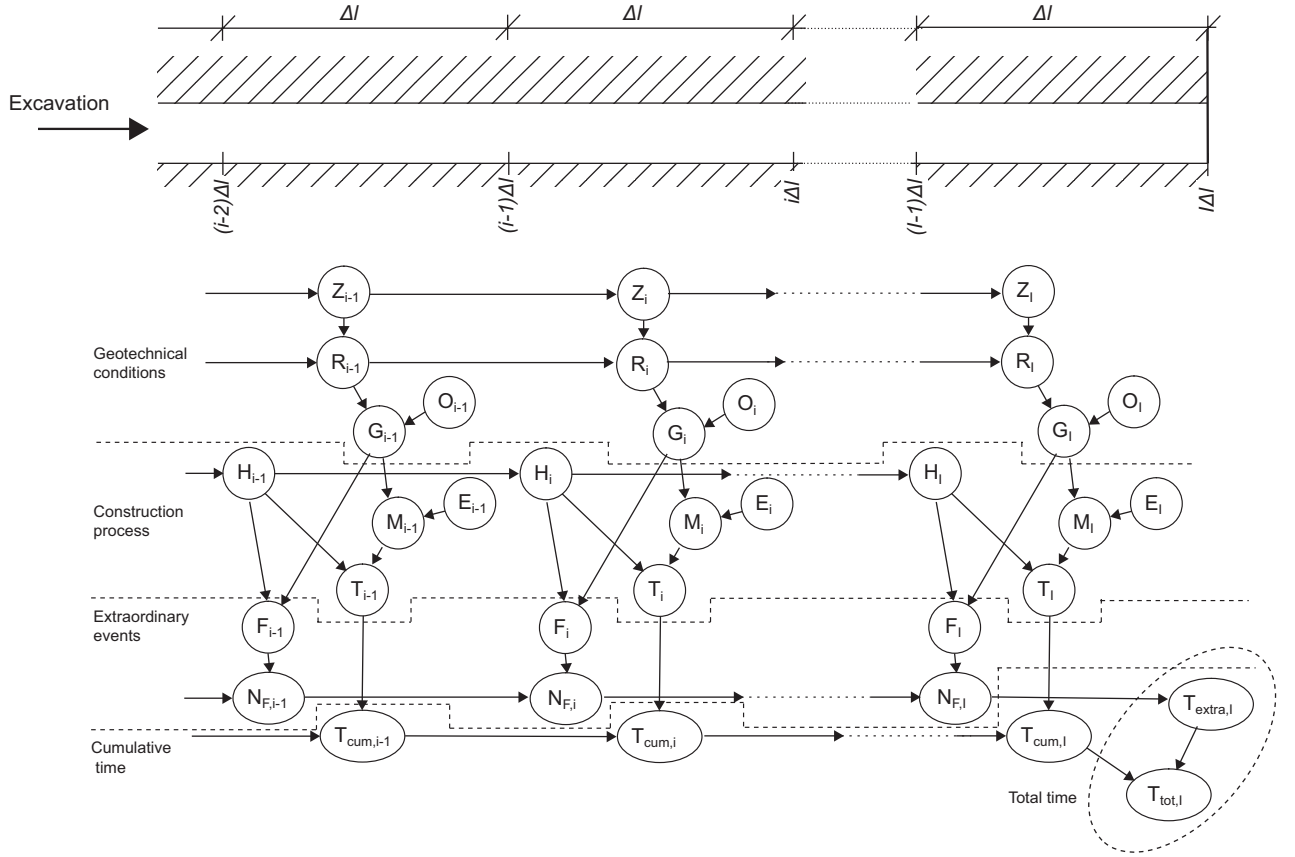


Fig. 4. DBN for tunnel excavation. (The variables are explained in Table 1.)

probabilities of the remaining outcome states. With this procedure, the probability distribution of a variable in slice i is updated with the evidence from all slices $k \leq i$. The evidence in slices $k > i$ is not included. To include such evidence, the above algorithm must be extended by a backward computation, in which the frontier moves from right to left. This case is not considered in this article. Details on the algorithm can be found in Murphy (2002). Straub (2009) presents an application of the algorithm to modeling the effect of inspection and monitoring of deteriorating structures.

3 MODELING TUNNEL EXCAVATION PROCESS VIA DBN

A DBN for modeling uncertainties connected with tunnel excavation processes is presented in this section. The DBN is shown in Figure 4. The excavation process is described by random variables representing (1) geotechnical conditions, (2) the construction process, (3) extraordinary events, and (4) overall excavation time. Excavation cost is not included here for ease of presentation, but its modeling is analogous to

excavation time. Each slice in the DBN represents a tunnel segment of length Δl , i.e., a segment from position $(i-1)\Delta l$ to position $i\Delta l$ along the tunnel axis. Within one slice, all variables are modeled as constant. Table 1 provides an overview of the definition of the variables as they are used in the case study presented in Section 5. The variables are introduced in more detail in the following subsections.

3.1 Geotechnical conditions

The variables to be used for describing the geotechnical conditions vary depending on the specifics of the tunneling project. The variables selected here follow the description in Min (2003). Within a segment i of the tunnel, the geotechnical conditions are described by the random variables zone Z_i , rock class R_i , height of the overburden O_i , and ground class G_i .

Along the tunnel axis, quasi-homogenous geotechnical zones are identified. The positions of the boundaries of these quasi-homogenous geotechnical zones are not known with certainty; therefore, the zone Z_i to which segment i belongs is modeled as a random variable. The definition of this variable is in detail described in

Table 1
Overview of the variables in the DBN

<i>Id.</i>	<i>Variable</i>	<i>Type</i>	<i>States of the variable</i>
<i>Z</i>	Zone	Random/discrete	1, 2, ..., 8
<i>R</i>	Rock class	Random/discrete	I, II, III, IV, V
<i>O</i>	Overburden	Determ./discrete	Low, medium, high
<i>G</i>	Ground class	Random/discrete	L-I, L-II, L-III, L-IV, L-V, M-I, M-II, M-III, M-IV, M-V, H-I, H-II, H-III, H-IV, H-V
<i>H</i>	Human factor	Random/discrete	Favorable, neutral, unfavorable
<i>E</i>	Geometry	Determ./discrete	1 (begin/end), 2 (typical), 4 (chemical plant), 5 (Emergency Parking Places [EPP])
<i>M</i>	Construction method	Random/discrete	P.1, P.2, P.3, P.4, P.5, P.6, P.2-1, P.2-2, P.2-3, P.EPP
<i>T</i>	Unit time	Random/discretized	0, t_{int} , $2t_{\text{int}}$, ..., 15 (days)*
<i>F</i>	Failure mode	Random/discrete	Failure, no failure
N_F	Number of failures	Random/discrete	0, 1, 2, 3, 4, >5
T_{cum}	Cumulative time	Random/discretized	0, t_{int} , $2t_{\text{int}}$, ..., 1,830 (days)**
T_{extra}	Delays caused by failures	Random/discretized	15, t_{int} , $2t_{\text{int}}$, ..., $t_{\text{extra},99.9}$ (days)***
T_{tot}	Total time	Random/discretized	0, t_{int} , $2t_{\text{int}}$, ..., $(1,830 + t_{\text{extra},99.9})$ (days)

* t_{int} is the discretization interval of time variables. In the application example it is $t_{\text{int}} = 0.5$ days.

**Upper bound of cumulative time = $122 \times 15 = (\text{number of segments}) \times (\text{upper bound of unit time})$.

*** $t_{\text{extra},99.9}$ is the 99.9 percentiles of T_{extra} .

Appendix A. The rock class R within a zone is modeled as a homogeneous Markov process. The suitability of Markov processes for modeling of geotechnical conditions (such as rock class, degree of jointing) along the tunnel axis was shown already in Chan (1981). The parameters of the Markov process can be estimated by experts. The description of the parameters of the Markov process and definition of the variable R_i in the DBN are given in Appendix A.

The height of overburden O_i is modeled deterministically. The ground class G_i is defined deterministically for given R_i and O_i . As evident from Table 1, each G_i corresponds to a specific combination of R_i and O_i , e.g., ground class L-I stands for rock class I with low overburden, M-II for rock class II with medium overburden.

3.2 Construction process

The construction performance in segment i of the tunnel is described by the variables cross-section geometry E_i , construction method M_i , human factor H_i , and unit excavation time T_i .

The deterministic variable geometry E_i enables one to consider the different cross-sections along the tunnel (typical cross-section vs. extended cross-section for emergency parking places EPP) and it is also used to consider the special conditions at the beginning and end of the tunnel and at the location where the tunnel passes an existing chemical plant.

The construction method M_i describes the excavation type and the related support pattern applied in the i th segment and is determined conditional on the ground

class G_i and tunnel geometry E_i . The modeling of M_i follows Min (2003), where the details of the construction methods are described.

The variable human factor H_i represents the uncertain quality of design and construction works and other external factors (legislative, political, etc.) affecting the construction process. The human factor H_i is in one of the three states “unfavorable,” “neutral,” or “favorable” throughout the entire tunnel construction, i.e., the H_i s are fully dependent from one slice to the next and the conditional probability matrix $p(h_i|h_{i-1})$ in each slice is the 3×3 identity matrix. This simple model reflects the fact that the influence of human factor cannot be directly measured and can only be deduced from the average performance over long sections of the tunnel project (Špačková et al., 2010). The uncertainty in the human factor introduces dependence among the performance in each segment of the tunnel, and thus increases the variability of the estimated total construction time. H_i can also be interpreted as a random variable describing a model class, which reflects uncertainty on the selection of the appropriate probabilistic model of variables that are defined conditionally on H_i , i.e., unit time and failure rate. Before construction, the probability distribution of unit time and the failure rate are not known with certainty. Several probabilistic models are thus applied. During the construction, it becomes apparent which of the models is the most appropriate. This learning process is automated through the updating of the probability distribution of H_i . The concept of model classes is often applied in the context of structural identification (e.g., Cheung and Beck, 2010).

For every construction method M_i and human factor H_i , the unit time T_i is defined by a conditional cumulative probability distribution function (CDF) $F(t_i|m_i, h_i)$. To facilitate the application of the exact inference algorithm presented earlier, the variable T_i is discretized, as described in Section 4.1.

3.3 Extraordinary events

Extraordinary events are defined as events that stop the advance of the excavation (progress of the tunnel heading) for longer than a threshold value (here chosen as 15 days); these events can be considered a failure of the construction process. They are, e.g., tunnel collapses, tunnel flooding, or major legislative or organizational problems. They are characterized by the variables failure mode F_i and number of failures $N_{F,i}$.

The failure mode F_i describes the occurrence of failure in segment i . Different failure modes can be taken into account (cave-in collapse, tunnel flooding, fire). In the presented application, only modes “failure” and “no failure” were considered. F_i is defined conditionally on ground class G_i and human factor H_i .

The random variable $N_{F,i}$ represents the total number of failures from the beginning of the tunnel to location $i\Delta l$. It is defined conditionally on the number of failures in the previous slice, $N_{F,i-1}$, and the failure mode F_i in the i th segment. Definition of the variables F_i and $N_{F,i}$ is described in Appendix A.

3.4 Excavation time

The main output of the model is the total excavation time T_{tot} . In the DBN, it is computed as the sum of excavation time excluding extraordinary events, T_{cum} , and the time delay caused by extraordinary events, T_{extra} .

The cumulative time $T_{\text{cum},i}$ is the time for the excavation of the tunnel up to location $i\Delta l$. It is defined as the sum of $T_{\text{cum},i-1}$ and the unit time in segment i , T_i : $T_{\text{cum},i} = T_{\text{cum},i-1} + T_i$.

$T_{\text{extra},i}$ is the time delay because of occurrences of failures (extraordinary events) in the tunnel construction up to segment i . The distribution of $T_{\text{extra},i}$ for given number of failures $N_{F,i}$ can be derived from the statistics of observed delays, which provide the probability density function (PDF) of the delay caused by one failure event, $f_D(d)$. With D_k being the delay caused by the k th failure, the total delay because of $N_{F,i}$ failures is computed as the sum of the individual delays.

$$T_{\text{extra},i} = \sum_{k=1}^{N_{F,i}} D_k \quad (6)$$

We compute the PDF of $T_{\text{extra},i}$ for given $N_{F,i}$ by assuming that all delays D_k are independent and have

identical PDF $f_D(d_k)$. This implies the assumption that the expected delay caused by a failure is independent of the position where it occurs.

Assessment of the total excavation time is in most cases of interest for the tunnel as a whole or for a section of the tunnel. Therefore, its distribution is computed only at selected positions, as illustrated in Figure 4, where T_{tot} is assessed for the whole tunnel.

3.5 Length of segment represented by a slice of DBN

We expect that the changes of conditions can only occur at the boundaries of the tunnel segments represented by slices of the DBN. We thus assume that the random variables are fully dependent within each segment. The optimal segment length Δl corresponds (in case of cyclic excavation methods) approximately to the length of the excavation cycle. This topic is discussed in more detail in Špačková and Straub (2011).

The definition of random variables, i.e., the determination of their conditional probabilities, also depends on Δl . Calculation of conditional PMFs of zone Z_i , rock class R_i , and failure mode F_i for given Δl is described in Appendix A. The conditional PDFs of unit time T_i should be ideally determined directly from data for a given Δl .

The human factor H_i is supposed to be fully dependent throughout the entire excavation, therefore its CPT does not change if Δl changes. Deterministic variables and variables defined by deterministic functions (ground class G_i , construction method M_i , number of failures $N_{F,i}$, and cumulative time $T_{\text{cum},i}$) are not influenced by the segment length.

4 EVALUATION OF THE DBN

The evaluation of the DBN proceeds in three steps. First, all continuous variables are discretized. Second, some of the nodes are eliminated from the DBN to simplify the computations in the modified Frontier algorithm. Third, the modified Frontier algorithm, which is outlined in Section 4.3, is applied. The three steps are presented in the following. In Section 4.4, the procedure for updating of the prediction based on observed geotechnical conditions and performance is shown.

4.1 Discretization of random variables

Random variables defined in a continuous space (i.e., variables describing unit time T_i and delay $T_{\text{extra},i}$) are transformed into random variables defined in a discrete space. The discretization is described below for T_i ; an analogous procedure is used for discretizing $T_{\text{extra},i}$.

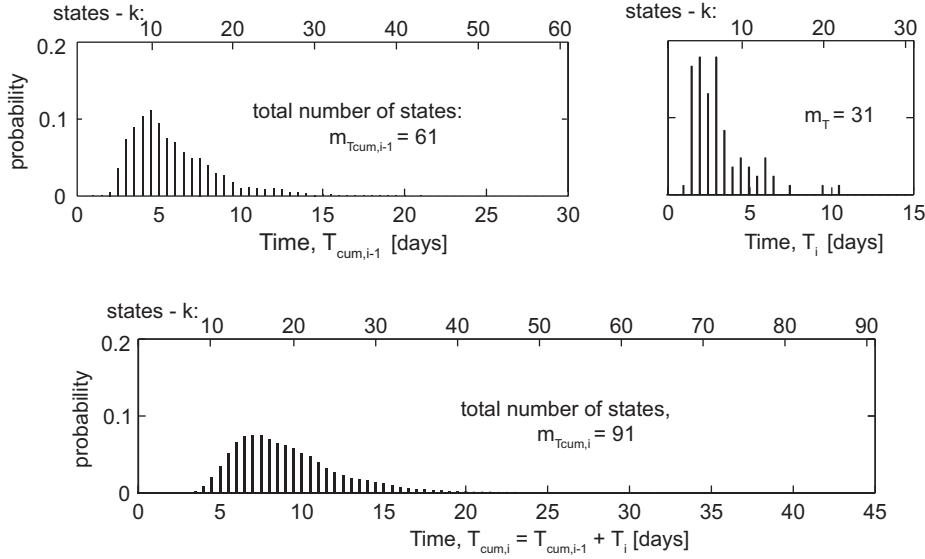


Fig. 5. Illustration of the summation of two discretized random variables: PMF of cumulative time $T_{cum,i-1}$, unit time T_i , and cumulative time $T_{cum,i}$ for tunnel segment $i = 3$, zone $Z_i = 1$, human factor $H_i = neutral$, and rock class $R_i = III$. The PMF of $T_{cum,i}$ is obtained through convolution of $T_{cum,i-1}$ and T_i . Discretization interval $t_{int} = 0.5$ day.

Let \tilde{T}_i be the original continuous random variable with parent variables $pa(\tilde{T}_i)$, which is defined by a CDF $F(\tilde{t}_i | Pa(\tilde{T}_i))$. Let T_i be the corresponding discrete random variable whose m_T states are denoted by $t_i^{(k)}$, where $k = 1, \dots, m_T$. Let state $t_i^{(k)}$ represent an interval $\langle \tilde{t}_i^{(k-1)}, \tilde{t}_i^{(k)} \rangle$ in the original continuous space, $\tilde{t}_i^{(k)}$ is the upper bound of the interval corresponding to state $t_i^{(k)}$. The conditional PMF of T_i then equals

$$p(t_i^{(k)} | Pa(\tilde{T}_i)) = F(\tilde{t}_i^{(k)} | Pa(\tilde{T}_i)) - F(\tilde{t}_i^{(k-1)} | Pa(\tilde{T}_i)) \quad (7)$$

The intervals are defined so that they have an equal length t_{int} . Each state $t_i^{(k)}$ is represented by the central value of corresponding interval, i.e., $t_i^{(k)} = \tilde{t}_i^{(k)} - \frac{t_{int}}{2}$.

For computational purposes, which will become clear later in the article (Section 4.3), it is beneficial to define the representative values as integer multiplications of t_{int} .

In addition to T_i and $T_{extra,i}$, also $T_{cum,i}$ must be discretized. Because $T_{cum,i}$ is defined as a sum of $T_{cum,i-1}$ and T_i , it is convenient to use the same discretization interval length t_{int} for all three variables and to define the representative values of their states as integer multiplications of t_{int} . This, however, implies that the number of states of $T_{cum,i}$ increases with every slice of the DBN as is illustrated in Figure 5. If m_T is the number of states of T_i , $i = 1, \dots, I$, then the number of states of $T_{cum,i}$ is $m_{T_{cum,i}} = i(m_T - 1) + 1$. To deal with the resulting large number of states of $T_{cum,i}$, a

modification to the Frontier algorithm is proposed in Section 4.3.

4.2 Elimination of nodes

Before the application of the Frontier algorithm, it is computationally beneficial to eliminate some nodes from the DBN. Such an elimination of nodes can be considered a preprocessing of the DBN. In the presented DBN, we eliminate ground class G_i , overburden O_i , cross-section geometry E_i , and construction method M_i . This operation can be performed generically for all slices in the DBN. The resulting DBN is shown in Figure 6, following the procedure for graphical elimination of nodes explained in Straub and Der Kiureghian (2010a). When eliminating nodes from the network, additional links must be added to the remaining nodes, to ensure that their joint probability distribution is not altered. New links are introduced from R_i to F_i and T_i , and from F_i to T_i . This new definition includes all the information from the eliminated nodes, which ensures that the reduced DBN gives the same results as the original DBN.

In principle, one could directly define this reduced DBN instead of the original DBN. However, because the effect of variables such as overburden or ground class is only implicit in this reduced model, the direct determination of the conditional probabilities in the reduced DBN is not straightforward.

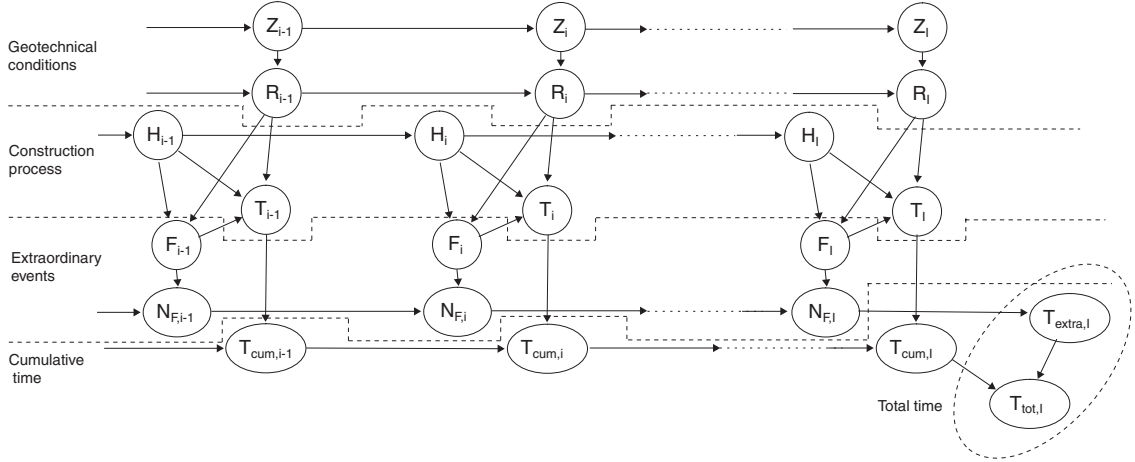


Fig. 6. DBN after elimination of nodes.

For the resulting network, because of the new links introduced in the elimination process, it becomes necessary to compute the conditional PMFs $p(f_i|q_i, r_i)$ and $p(t_i|q_i, r_i, f_i)$. The conditional PMF of failure mode F_i can be calculated as (compare with Figure 4)

$$\begin{aligned} p(f_i|r_i, h_i) &= \sum_{G_i} p(f_i, g_i|r_i, h_i) \\ &= \sum_{G_i} p(f_i|g_i, h_i) \sum_{O_i} p(g_i|o_i, r_i) p(o_i) \end{aligned} \quad (8)$$

The conditional PMF of unit time T_i is obtained as

$$\begin{aligned} p(t_i|r_i, h_i, f_i) &= \frac{1}{p(f_i)} \sum_{G_i} p(f_i, g_i|r_i, h_i) \sum_{M_i} p(t_i|m_i, h_i) \\ &\quad \times \sum_{E_i} p(m_i|e_i, g_i) p(e_i) \end{aligned} \quad (9)$$

4.3 Modified Frontier algorithm

A modified version of the Frontier algorithm is used for evaluating the DBN. The new algorithm is investigated and validated on an academic example in Appendix B. Because some random variables in the DBN have large numbers of states, direct application of the Frontier algorithm is inefficient. We therefore propose two modifications to the algorithm, which avoid defining large CPT: (1) the frontier is optimized by excluding some of the variables; this modification was originally proposed by Murphy (2002) under the name “interface algorithm”; (2) some steps of the original algorithm are replaced by computations of convolutions of conditional PMFs; to our knowledge, this modification has not been previously published. The new algorithm is computationally efficient; computations shown here were performed in Matlab and take in the order

of 80 CPU seconds on a MacBook Pro with a 2.53 GHz Intel Core 2 Duo Processor, 4 GB 1,067 MHz DDR3 RAM, and Mac OS X v. 10.6.8. The computational efficiency in comparison with the original Frontier algorithm is presented in Appendix B.

In the following, we present one cycle of the modified Frontier algorithm, which advances the frontier from slice $i-1$, with corresponding joint PMF $p(z_{i-1}, r_{i-1}, t_{cum,i-1}, n_{F,i-1}, h_{i-1})$, to slice i , with corresponding joint PMF $p(z_i, r_i, t_{cum,i}, n_{F,i}, h_i)$. As outlined in Section 2.1, the frontier is moved from slice $i-1$ to slice i by sequentially adding nodes from slice i , and removing nodes from slice $i-1$ in the frontier. The individual steps are graphically documented in Figure 7 and are described in the following.

At the beginning of the cycle, the joint PMF $p(z_{i-1}, r_{i-1}, t_{cum,i-1}, n_{F,i-1}, h_{i-1})$ is available from the previous cycle. In the first step (a) of the cycle, the node Z_i (the geotechnical zone in segment i) is added and node Z_{i-1} is removed, as indicated in Figure 7a. The corresponding computation is

$$\begin{aligned} p(z_i, r_{i-1}, t_{cum,i-1}, n_{F,i-1}, h_{i-1}) \\ = \sum_{Z_{i-1}} p(z_{i-1}, r_{i-1}, t_{cum,i-1}, n_{F,i-1}, h_{i-1}) p(z_i|z_{i-1}) \end{aligned} \quad (10)$$

where $p(z_i|z_{i-1})$ is obtained as described in Appendix A.

In the second step (b) of the cycle, the random variable rock class R_i is added to the frontier and R_{i-1} is removed, as depicted in Figure 7b. The corresponding computation is

$$\begin{aligned} p(z_i, r_i, t_{cum,i-1}, n_{F,i-1}, h_{i-1}) \\ = \sum_{R_{i-1}} p(z_i, r_{i-1}, t_{cum,i-1}, n_{F,i-1}, h_{i-1}) p(r_i|r_{i-1}, z_i) \end{aligned} \quad (11)$$

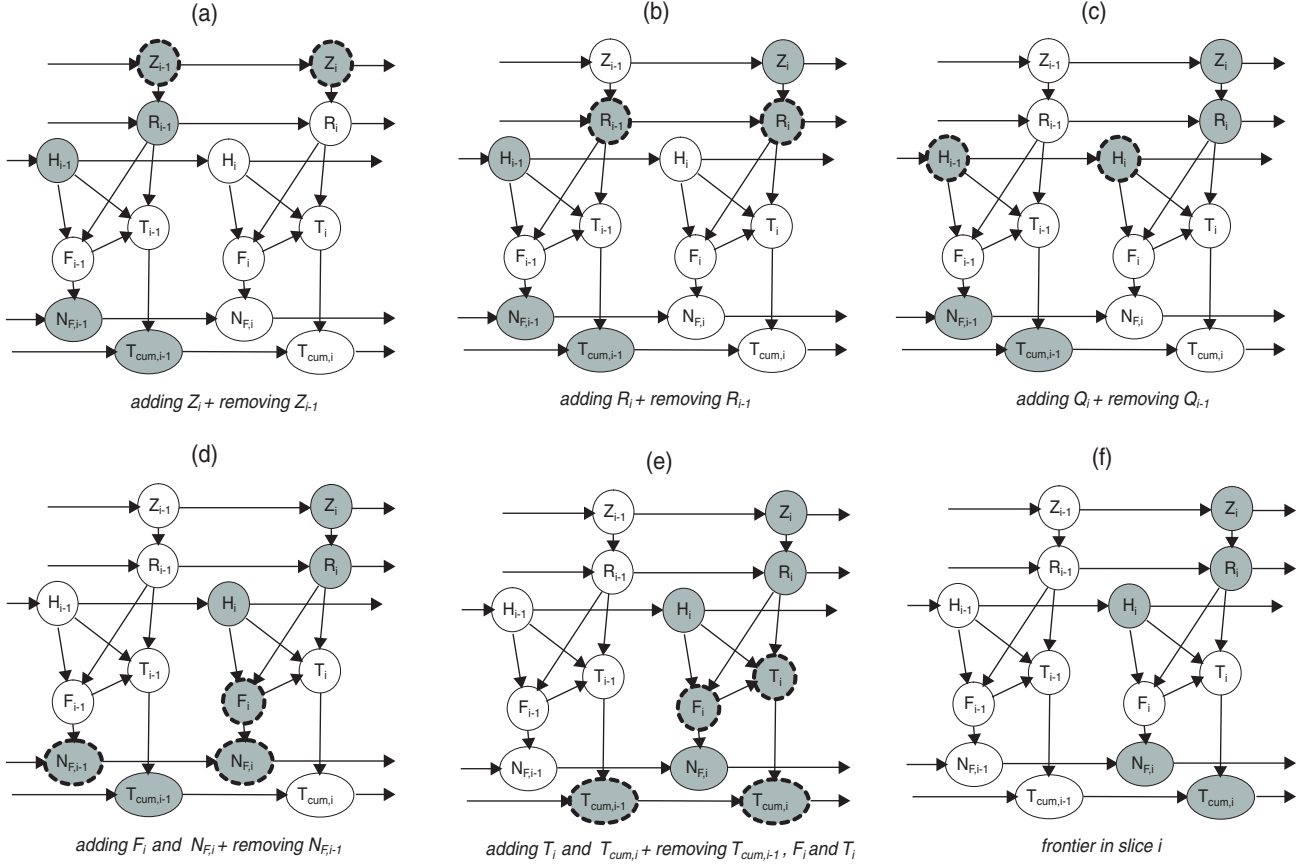


Fig. 7. Graphical documentation of one cycle of the Frontier algorithm for evaluation of the DBN of tunnel excavation processes. The grey nodes are those included in the frontier at a given step. Grey nodes with dashed line indicate the nodes that are operated in a particular step (i.e., nodes which are added or removed from the frontier in this step).

where $p(r_i|r_{i-1}, z_i)$ is obtained as described in Appendix A.

In the third step (c) of the cycle, the random variable human factor H_i is added to the frontier and H_{i-1} is eliminated, as shown in Figure 7c. The corresponding computation is

$$p(z_i, r_i, t_{cum,i-1}, n_{F,i-1}, h_i) = \sum_{H_{i-1}} p(z_i, r_i, t_{cum,i-1}, n_{F,i-1}, h_{i-1}) p(h_i|h_{i-1}) \quad (12)$$

The conditional probability $p(h_i|h_{i-1})$ is defined by an identity matrix (Section 3.2). Because of this definition, the calculation from Equation (12) can be skipped and the joint PMF can be obtained simply by replacing h_{i-1} with h_i in the known joint PMF $p(z_i, r_i, t_{cum,i-1}, n_{F,i-1}, h_{i-1})$.

In the fourth step (d) of the cycle, the random variable $N_{F,i}$, representing the number of failures, is added to the frontier, and $N_{F,i-1}$ is removed. Because $N_{F,i}$ is defined conditional on the failure mode F_i , this random

variable is also added to the frontier. The step is shown in Figure 7d and the corresponding computation is

$$p(z_i, r_i, t_{cum,i-1}, n_{F,i}, h_i, f_i) = \sum_{N_{F,i-1}} p(z_i, r_i, t_{cum,i-1}, n_{F,i-1}, h_i) \times p(n_{F,i}|n_{F,i-1}, f_i) p(f_i|r_i, h_i) \quad (13)$$

where $p(n_{F,i}|n_{F,i-1}, f_i)$ is computed as described in Appendix A and $p(f_i|r_i, h_i)$ is obtained after the elimination of nodes according to Section 4.2.

To complete the cycle, one could, in principle, perform the following two operations (corresponding to the fifth step shown in Figure 7e). First, the random variable T_i , representing unit time, could be added and F_i removed.

$$p(z_i, r_i, t_{cum,i-1}, n_{F,i}, h_i, t_i) = \sum_{F_i} p(z_i, r_i, t_{cum,i-1}, n_{F,i}, h_i, f_i) p(t_i|q_i, r_i, f_i) \quad (14)$$

Second, the cumulative costs $T_{\text{cum},i}$ could be added, whereas $T_{\text{cum},i-1}$ and T_i could be eliminated.

$$p(z_i, r_i, t_{\text{cum},i}, n_{F,i}, h_i) = \sum_{T_{\text{cum},i-1}} \sum_{T_i} p(z_i, r_i, t_{\text{cum},i-1}, n_{F,i}, h_i, t_i) \times p(t_{\text{cum},i} | t_{\text{cum},i-1}, t_i) \quad (15)$$

Because random variables $T_{\text{cum},i-1}$ and $T_{\text{cum},i}$ can have large numbers of states, computation of Equation (15) puts high demands on computer memory, which can make exact computations infeasible. For this reason, an alternative solution that avoids this computation is developed in the following.

We exploit the fact that the cumulative time in segment i is obtained as the sum $T_{\text{cum},i} = T_{\text{cum},i-1} + T_i$, by using the convolution function to compute the distribution function of $T_{\text{cum},i}$. If $T_{\text{cum},i-1}$ and T_i were independent random variables, the PMF of $T_{\text{cum},i}$ could be computed as

$$p_{T_{\text{cum},i}}(t) = \sum_{\tau} p_{T_{\text{cum},i-1}}(t - \tau) p_{T_i}(\tau) \quad (16)$$

where the summation is over all states τ of T_i . This is the convolution function (illustrated in Figure 5), which is written in short notation as

$$p_{T_{\text{cum},i}}(t) = p_{T_{\text{cum},i-1}} * p_{T_i}(t) \quad (17)$$

$T_{\text{cum},i-1}$ and T_i are dependent and direct application of Equation (17) is not possible. However, from the graphical structure of the DBN, it can be inferred that $T_{\text{cum},i-1}$ and T_i are independent for given values of Z_i , R_i , Q_i , and F_i . (This follows from the d-separation properties of the BN.) Making use of this conditional independence, we can write

$$p_{T_{\text{cum},i} | Z_i, R_i, H_i, F_i}(t) = p_{T_{\text{cum},i-1} | Z_i, R_i, H_i, F_i} * p_{T_i | R_i, H_i, F_i}(t) \quad (18)$$

The conditional PMF of T_i , $p(t_i | r_i, q_i, f_i)$, is known from Equation (9). Furthermore, from the joint PMF of step (d), Equation (13), we obtain

$$p(t_{\text{cum},i-1} | z_i, r_i, h_i, f_i) = \frac{\sum_{N_{F,i}} p(z_i, r_i, t_{\text{cum},i-1}, n_{F,i}, h_i, f_i)}{\sum_{T_{\text{cum},i-1}} \sum_{N_{F,i}} p(z_i, r_i, t_{\text{cum},i-1}, n_{F,i}, h_i, f_i)} \quad (19)$$

The convolution operation in Equation (18) is numerically efficient because it avoids the summation over the states of $T_{\text{cum},i-1}$, which is necessary in the conventional approach (Equation 15). This reduces the number of necessary operations by a factor corresponding to the number of states of $T_{\text{cum},i-1}$. In addition, standard software like Matlab has optimized algorithms for computing the convolution function based on fast Fourier

transform. The computation times of both algorithms are compared in Appendix B.

With $p(t_{\text{cum},i} | z_i, r_i, h_i, f_i)$ of Equation (18), the final frontier shown in Figure 7f is calculated from

$$p(z_i, r_i, t_{\text{cum},i}, n_{F,i}, h_i) = \sum_{F_i} p(t_{\text{cum},i} | z_i, r_i, h_i, f_i) p(z_i, r_i, n_{F,i}, h_i, f_i) \quad (20)$$

with

$$p(z_i, r_i, n_{F,i}, h_i, f_i) = \sum_{T_{\text{cum},i-1}} p(z_i, r_i, t_{\text{cum},i-1}, n_{F,i}, h_i, f_i), \quad (21)$$

where $p(z_i, r_i, t_{\text{cum},i-1}, n_{F,i}, h_i, f_i)$ is the joint PMF of step (d), Equation (13).

The full DBN is evaluated by repeatedly applying the cycle described above, starting at $i = 2$ and ending at the last slice $i = I$. To initiate the calculation, the frontier in slice 1 must be known. It is

$$p(z_1, r_1, t_1, n_{F,1}, h_1) = p(z_1) p(r_1 | z_1) p(h_1) \sum_{F_1} p(f_1 | r_1, h_1) \times p(n_{F,1} | f_1) p(t_1 | r_1, f_1, h_1) \quad (22)$$

Because $T_{\text{cum},1} = T_1$, the joint PMF of the initial frontier is obtained simply by replacing t_1 with $t_{\text{cum},1}$ in the above expression.

4.4 Updating

If observations of the tunnel construction performance are available, the predictions can be updated according to the description in Section 2.1. Commonly, the rock class, cumulative time, and number of failures for individual segments can be directly observed as the construction proceeds. The observations in segment i are denoted as $R_i = r_{\text{obs},i}$, $T_{\text{cum},i} = t_{\text{obs},i}$, and $N_{F,i} = n_{\text{obs},i}$. To include the evidence in the Frontier algorithm, the joint PMF computed according to Equation (20), $p(z_i, r_i, t_{\text{cum},i}, n_{F,i}, h_i)$, is replaced by the conditional PMF

$$p(z_i, r_i, t_{\text{cum},i}, n_{F,i}, h_i | r_{\text{obs},i}, t_{\text{obs},i}, n_{\text{obs},i}) = \begin{cases} \alpha \cdot p(z_i, r_i, t_{\text{cum},i}, n_{F,i}, h_i), & \text{for } r_i = r_{\text{obs},i}, \\ & t_{\text{cum},i} = t_{\text{obs},i}, n_{F,i} = n_{\text{obs},i} \\ 0, & \text{else} \end{cases} \quad (23)$$

where α is a normalization constant to ensure that the sum over all states of $p(z_i, r_i, t_{\text{cum},i}, n_{F,i}, h_i | r_{\text{obs},i}, t_{\text{obs},i}, n_{\text{obs},i})$ is equal to one. This conditional PMF is then used as the input for the next cycle of the Frontier algorithm.

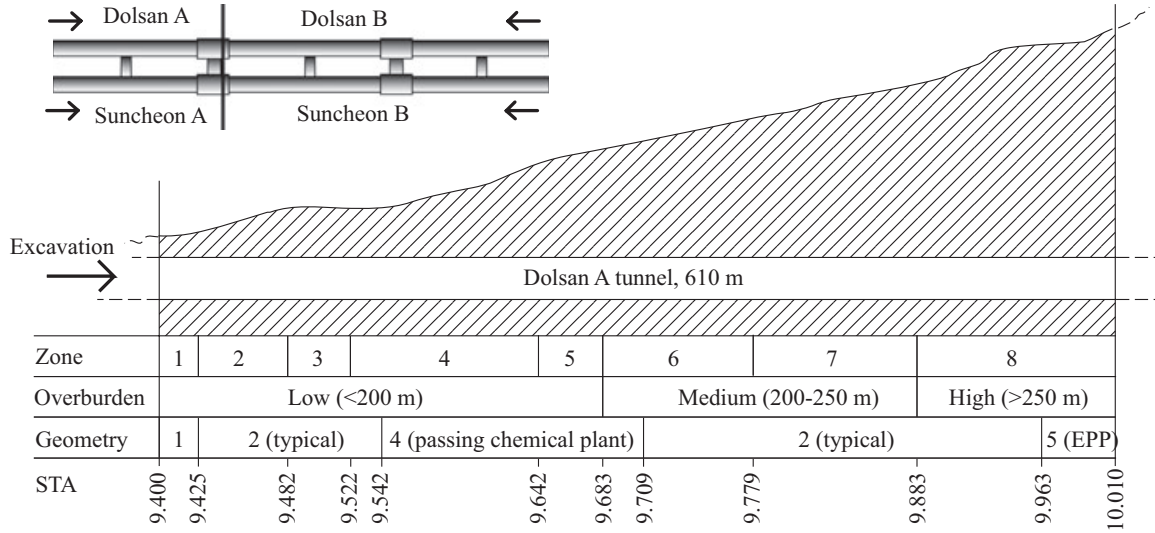


Fig. 8. Scheme of the modeled tunnel excavation.

4.5 Calculation of total time

The total time $T_{\text{tot},i}$ is the sum of the cumulative time $T_{\text{cum},i}$ and delays caused by extraordinary events $T_{\text{extra},i}$: $T_{\text{tot},i} = T_{\text{cum},i} + T_{\text{extra},i}$. For given value of $N_{F,i}$, $T_{\text{cum},i}$, and $T_{\text{extra},i}$ are independent. Therefore, the distribution of $T_{\text{tot},i}$ can be computed via the convolution function as

$$p_{T_{\text{tot},i}|N_{F,i}}(t) = p_{T_{\text{cum},i}|N_{F,i}} * p_{T_{\text{extra},i}|N_{F,i}}(t) \quad (24)$$

The conditional PMF $p_{T_{\text{cum},i}|N_{F,i}}$ is obtained from the joint PMF $p(z_i, r_i, t_{\text{cum},i}, n_{F,i}, q_i)$, which results from the Frontier algorithm, as follows:

$$\begin{aligned}
p(t_{\text{cum},i} | n_{F,i}) &= \frac{p(t_{\text{cum},i}, n_{F,i})}{p(n_{F,i})} \\
&= \frac{\sum_{Q_i} \sum_{Z_i} \sum_{R_i} p(z_i, r_i, t_{\text{cum},i}, n_{F,i}, h_i)}{\sum_{T_{\text{cum},i}} \sum_{Q_i} \sum_{Z_i} \sum_{R_i} p(z_i, r_i, t_{\text{cum},i}, n_{F,i}, h_i)}. \quad (25)
\end{aligned}$$

$p_{T_{\text{extra},i}|N_{F,i}}$ is evaluated following Equation (6).

5 NUMERICAL EXAMPLE

The DBN model is applied to the excavation of a section of the Suncheon–Dolsan road tunnel in South Korea. The case study was originally presented in Min (2003) and Min et al. (2003), where the DAT model was applied for probabilistic prediction of construction time and costs.

The modeled tunnel section is a 610-m-long tunnel tube with two lanes, which is excavated with a conven-

tional tunneling method (ITA, 2009). A scheme of the tunnel tube is depicted in Figure 8.

A simplified version of the proposed DBN model, without consideration of extraordinary events, human factor H_i and zone Z_i , was previously applied to this tunnel in Špačková and Straub (2011), to validate the DBN model by comparing its results with those of the DAT model (Min, 2003). In Špačková and Straub (2011), the variable human factor H_i is called quality Q_i , its meaning and modeling is however the same.

5.1 Parameters of the DBN model

For the application of the DBN model, the tunnel is discretized in segments of length $\Delta l = 5$ m.

The description of the geotechnical conditions (zone Z_i , rock class R_i , overburden O_i , and ground class G_i) as well as of some variables describing the construction process (geometry E_i and construction method M_i) are taken from Min (2003). An example of a CPT, describing the rock class in slice i , R_i , is shown in Table 2. The rock class definition is based on the classification used for the individual tunnel and it combines electrical resistivity, Rock Mass Rating (RMR) and Q-value. For further details on geotechnical classification we refer to Singh and Goel (1999).

The conditional probability distributions $p(t_i|m_i, h_i)$ of unit time T_i were determined based on data recorded during excavation of a Czech tunnel. An example of the used nonparametric distributions of T_i for selected construction methods M_i and human factor $H_i = \text{“neutral”}$ is shown in Figure 9. The means and standard deviations of T_i conditional on human factor H_i and construction

Table 2

Conditional probability table of rock class in zone 2 for a DBN with slice length $\Delta l = 5$ m. For example, the conditional probability of rock class in slice i being $R_i = III$, given that the rock class in slice $i - 1$ is $R_{i-1} = I$ and the zone in slice i is $Z_i = 2$, is $\Pr(R_i = III | R_{i-1} = I, Z_i = 2) = 0.134$

R_i	$Z_i = 2$				
	$R_{i-1} = I$	$R_{i-1} = II$	$R_{i-1} = III$	$R_{i-1} = IV$	$R_{i-1} = V$
I	0.606	0.260	0.215	1	1
II	0.260	0.606	0.417	0	0
III	0.134	0.134	0.368	0	0
IV	0	0	0	0	0
V	0	0	0	0	0

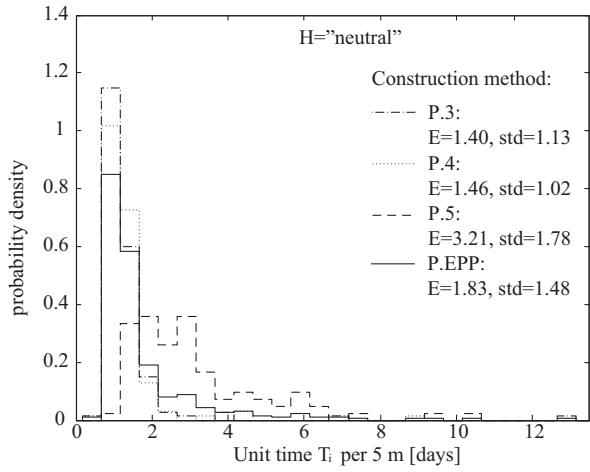


Fig. 9. PDF of unit time T_i for excavation of a segment with length of $\Delta l = 5$ m, on the condition of “neutral” human factor H_i and for selected construction methods M_i .

method M_i , as applied in the numerical example, are summarized in Table 3.

The discretization interval for T_i and $T_{cum,i}$, as discussed in Section 4.1, is selected as $t_{int} = 0.5$ days.

The conditional probability of an extraordinary event (a failure) in segment i , $p(f_i | g_i, h_i)$, is estimated based on experience from the Czech Republic, where the number of tunnel collapses is known (e.g., Aldorf, 2010) and can be related to the total length of constructed tunnels (Barták, 2007). The rate of failures is dependent on human factor H_i and ground class G_i , and the estimated values [in number of failures per m] are assessed to range from $2.21E^{-4}$ to $6.71E^{-3}$ for unfavorable influence of human factor, from $1.11E^{-4}$ to $3.31E^{-3}$ for neutral influence of human factor, and from $5.61E^{-5}$ to $1.71E^{-3}$ for favorable influence of human factor. The estimates were not validated with data from other countries and they should not be taken as generally applicable.

The determination of the probability distribution of H_i will generally be based on a subjective assessment. For the purpose of this study, we apply two alternative prior distributions, to investigate the influence of this choice. The used probabilistic models are

$$H(a) : \Pr(H_i = \text{“unfavorable”}) = 0.3,$$

$$\Pr(H_i = \text{“neutral”}) = 0.6 \quad \Pr(H_i = \text{“favorable”}) = 0.1$$

$$H(b) : \Pr(H_i = \text{“unfavorable”}) = 0.33,$$

$$\Pr(H_i = \text{“neutral”}) = 0.33, \quad \Pr(H_i = \text{“favorable”}) = 0.33$$

The probability distribution of delay caused by one extraordinary event $p(t_{extra,i} | N_{F,i} = 1)$ was derived from (Sousa, 2010). A shifted exponential distribution with minimum at 15 days, mean value 175 days, and

Table 3

Means and standard deviations of unit time T_i (in days) for different human factors and construction methods, for slices of length $\Delta l = 5$ m. The description of construction methods is taken from Min et al. (2008)

Construction method	Excavation type	Bolts: length/horizontal interval/vertical interval [m]	Unfavorable human factor		Neutral human factor		Favorable human factor	
			Mean	SD	Mean	SD	Mean	SD
P.1	Full face	3/>3.5/>2	1.56	1.36	1.40	1.13	1.24	0.89
P.2	Full face	3/3.5/2	1.56	1.36	1.40	1.13	1.24	0.89
P.3	Full face	3/2/1.8	1.56	1.36	1.40	1.13	1.24	0.89
P.4	Bench cut	4/1.5/1.5	1.62	1.35	1.46	1.02	1.35	0.83
P.5	Bench cut	4/1.2/1.5	3.49	2.35	3.21	1.78	2.83	1.44
P.6	Bench cut	4/1.0/1.5	3.49	2.35	3.21	1.78	2.83	1.44
P.2-1	Full face	–	2.02	1.88	1.83	1.48	1.65	1.20
P.2-2	Full face	–	2.02	1.88	1.83	1.48	1.65	1.20
P.2-3	Full face	–	2.02	1.88	1.83	1.48	1.65	1.20
P.EPP	–	–	2.02	1.88	1.83	1.48	1.65	1.20

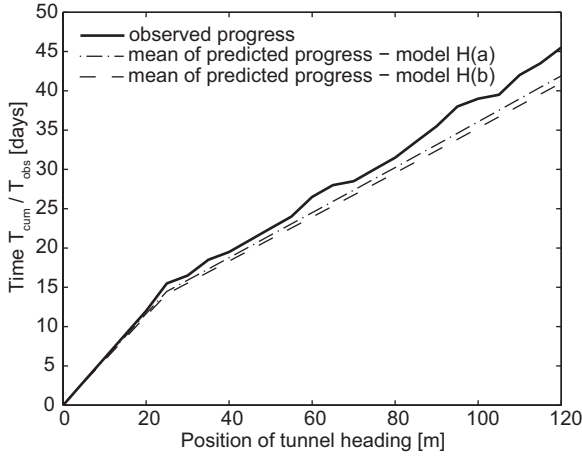


Fig. 10. Performance data used for updating the predictions: observed excavation time in the first 120 m of the tunnel, together with the predicted excavation time.

standard deviation 160 days was fitted to the data given in that reference and, in discretized form, applied in the example.

5.2 Bayesian updating of the probabilistic estimate with performance data

To demonstrate the ability of the DBN for updating the prediction as the construction proceeds, we introduce hypothetical performance data. These are from the first 120 m of the tunnel and include observed rock class R_i , number of failures $F_{cum,i}$, and cumulative time $T_{cum,i}$ at each segment.

It is assumed that no failure occurs in this section and that the cumulative time is slightly higher (up to 10%) than the mean prior prediction. The predicted and observed cumulative time $T_{cum,i}$ is shown in Figure 10. Rock class II is found in the first 49 m, rock class III in the next 41 m, and rock class IV in the last 30 m of the section.

5.3 Results without observations

The resulting probabilistic estimates of construction time for the whole tunnel are presented in Figures 11 and 12. Figure 11 shows the prediction without consideration of extraordinary events, Figure 12 includes the extraordinary events. In both figures, results are shown separately for the two *a priori* probabilistic models of human factor H_i . In addition, results for a fixed human factor $H_i = \text{"neutral"}$ are shown, which correspond to a model that does not consider human factor as a random variable, i.e., which neglects the uncertainty in the model class. By comparing the results for fixed and un-

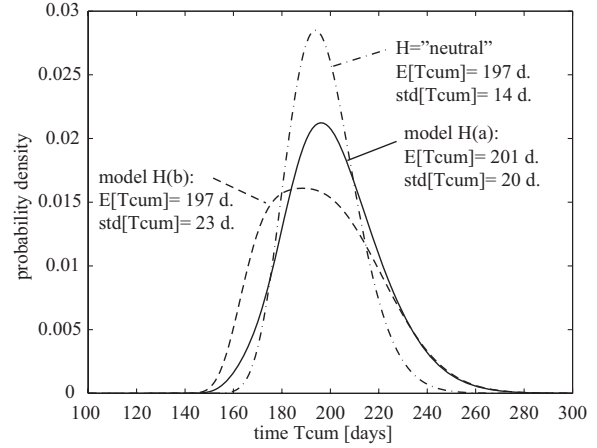


Fig. 11. Prediction of excavation time T_{cum} without consideration of extraordinary events, for two *a priori* models of human factor and for the case of fixed human factor.

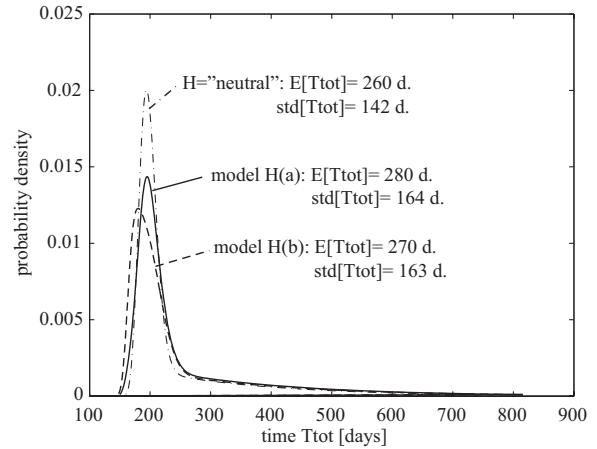


Fig. 12. Prediction of total excavation time T_{tot} with consideration of extraordinary events, for two *a priori* models of human factor and for the case of fixed human factor.

certain human factor, the effect of introducing H_i can be observed: The standard deviation of the construction time estimate increases because of the uncertainty in H_i . If the average performance (e.g., advance rate) is uncertain, the overall uncertainty of the total construction time is higher than in the case when this average value is known, and only the variability of the performance is considered (which is the case of fixed human factor $H_i = \text{"neutral"}$).

By comparing Figures 11 and 12, the significant impact of extraordinary events on the expected construction time and, in particular, on the uncertainty in the construction time are evident. The resulting distributions of total excavation time are strongly skewed toward larger construction times.

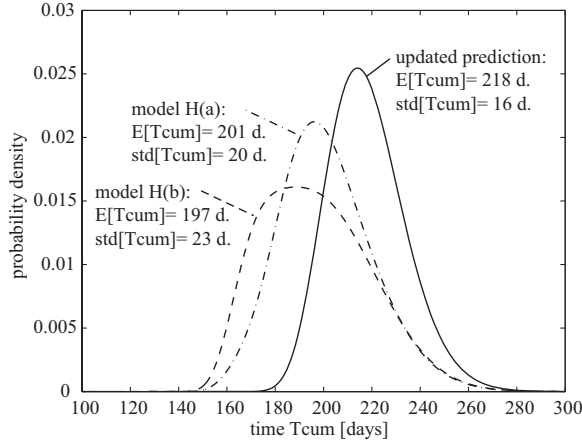


Fig. 13. Updated prediction of excavation time T_{cum} (without consideration of extraordinary events) for Dolsan A tunnel based on observations made during excavation of 120 m of the tunnel.

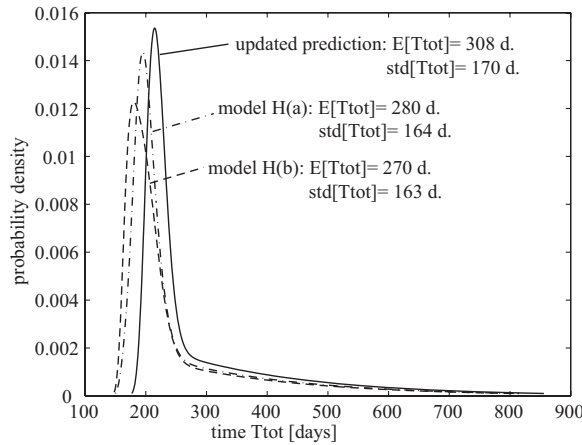


Fig. 14. Updated prediction of excavation time T_{tot} (with consideration of extraordinary events) for Dolsan A tunnel based on observations made during excavation of 120 m of the tunnel.

5.4 Results for the updating with performance data

The updated estimates of cumulative time T_{cum} (excluding extraordinary events) and total time T_{tot} (including extraordinary events) for the whole tunnel, conditional on the observations described in Section 5.2, are shown in Figures 13 and 14. They are obtained by updating the PMF at all slices with observations (segments $i = 1, 2, \dots, 24$), according to Equation (23). For comparison, the estimates computed without the observation data are also provided (corresponding to the results shown in Figures 10 and 11). The updated estimates are identical for the two prior models of human factor H_i , because the observed performance strongly

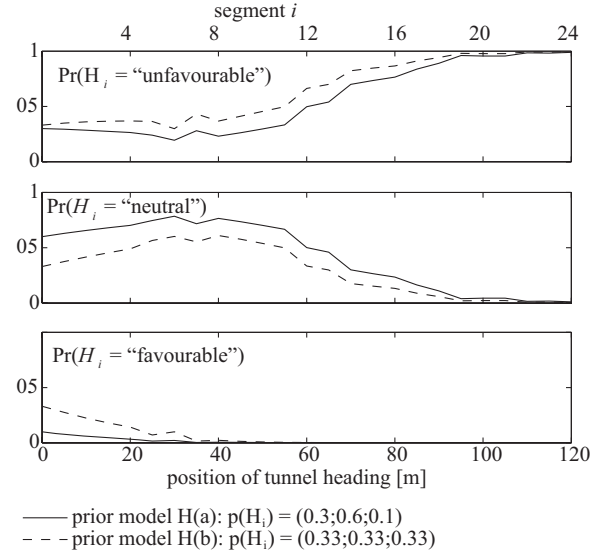


Fig. 15. Updated prediction of human factor H_i for Dolsan A tunnel based on observations made during excavation of 120 m of the tunnel.

indicates that the human factor is $H_i = \text{"unfavorable"}$. This can be observed from Figure 15, which shows the updated probability of H_i as the construction proceeds. This probability is computed by marginalizing all other variables from the conditional joint PMF obtained according to Equation (23).

The updated estimate of T_{cum} (which excludes extraordinary events) shown in Figure 13 exhibits a lower standard deviation, because there is no more uncertainty in H_i , i.e., in the appropriate probabilistic model of unit time T_i . However, the standard deviation of the updated total construction time T_{tot} (including extraordinary events) is higher, because the resulting $H_i = \text{"unfavorable"}$ implies an increased probability of failure (extraordinary events).

6 DISCUSSION

The proposed DBN model and computational algorithm for tunnel excavation processes is a step toward a quantitative assessment of uncertainties that is needed to support the optimization of decisions in infrastructure projects. The significant uncertainty in estimates of construction cost and time observed in practice is not fully reflected in most existing models. In our view, a main reason for this underestimation is the assumption of independence among the performances at different phases of the construction. This observation was also made recently by Yang (2007) and Moret and Einstein (2011). In the proposed DBN model, we

represent correlation among the performance at different phases of the construction through the random variable “human factor,” which is assumed to represent the overall quality of the planning and execution of the construction process and other external factors influencing the entire project. As observed in Figures 1 and 12, the inclusion of this variable leads to an increased variance of the estimate of construction time. As stated earlier, the variable “human factor” can also be interpreted as a model uncertainty, which reflects the fact that the applied probabilistic models of tunnel excavation performance are based on a limited amount of data or expert estimates. This second interpretation has the advantage of not being judgmental and therefore more easily acceptable in practice. As we show in the example application (Figure 15), the DBN model facilitates to update the estimate of the “human factor,” i.e., as the construction proceeds, the observed performance is used in an automated manner to learn the model and to improve the prediction for the remaining construction.

Another main reason for the underestimation of the uncertainty in construction time and cost is that most existing models do not account for possible extraordinary events, which can be considered as failures of the construction process. These events are included in the DBN model. By providing the stakeholders with the full distribution of project time, as in Figure 12, the risk associated with extraordinary events can be more effectively communicated and included in the decision-making process than with the traditional approach of considering only expected values.

The presented Frontier algorithm for evaluating the DBN is computationally efficient and applicable in practice. It is flexible in including observations to update the model. Besides updating the model with performance data and the observed geology of the excavated tunnel sections, as shown in Section 5.2 and Section 5.4, other types of observations, e.g., borehole tests can be used. For some of these observations, an extension of the presented algorithm is required, to include the so-called backward pass (e.g., Murphy, 2002), which allows one to update the probabilistic model at segment i with information from segments $j > i$.

The proposed DBN approach is flexible with regard to changes in the model. One aspect that should be revised in future work is the modeling of the variable M_i to more realistically reflect changes of construction technology during the excavation process. The present model assumes full flexibility in changing the construction patterns based on changes in geology. In reality, the construction pattern is not modified so frequently, as this is connected with additional time and costs. This effect is even more pronounced when mechanized excavation is used.

A second aspect that should be addressed is the modeling of costs. As stated earlier, the variable time can be replaced by the variable cost to obtain a cost estimate. However, for a combined modeling, an extension of the DBN model is needed to account for the dependence of construction costs on construction time.

In probabilistic assessment of construction time and costs, the model parameters (especially advance rates and unit costs) should ideally be based on analyses of data from past projects. Expert estimates are reliable for the assessment of mean values, but not for determining the variances and the probabilistic models. Furthermore, determination of failure probabilities and associated delays should be based on an extensive analysis of information from past projects. The analysis of tunnel construction data is a complex task, because the local conditions and design and construction procedures differ in different countries and projects. Therefore, further research is needed to provide a more robust information basis for probabilistic modeling. The understanding of the benefits of probabilistic modeling among stakeholders should be raised, which should lead them to more systematically manage and statistically analyze data from available projects.

7 CONCLUSION

A model for probabilistic prediction of tunnel construction performance using DBN is introduced. It allows to realistically quantify the uncertainties connected with construction time, by including correlations in the construction process and the risk of extraordinary events (tunnel collapses and other major problems) in the model.

The DBN adopts the modeling of uncertain geotechnical conditions from previous models (DAT model) but modifies the representation of uncertainties inherent in the construction process itself. The dependencies that result from the influence of human, organizational, and external factors are addressed by introducing a random variable “human factor” into the model. The performance variables (unit time, probability of failures) are defined conditionally on the human factor.

An algorithm for the efficient evaluation of the DBN was described. We modified the existing Frontier algorithm to better address specific features of the proposed DBN. The modification enables one to deal with discrete random variables with large numbers of outcomes states that result from the discretization of continuous random variables such as time or cost. We exploited the fact that these variables are defined as cumulative sums of other random variables in the DBN and that the probability distribution of a sum of two random

variables can be efficiently calculated by means of convolution functions.

The DBN model was applied to estimation of the excavation time of a 610-m-long tunnel. Bayesian updating of the estimate with the observed construction performance was also shown. As demonstrated by this application, the DBN model is able to capture the overall uncertainty of estimates of tunnel construction time. Some data from real tunnel projects were used in the presented application, but more extensive analysis of data from past projects should be performed, using the DBN as a model framework.

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APPENDIX A: PROBABILISTIC DEFINITION OF SELECTED NODES IN THE DBN

Zone Z

The variable represents the position of a tunnel segment in quasi-homogenous geotechnical zones along the tunnel axis. The uncertainty in the location of the boundary B_j between zones j and $j + 1$ is described by the CDF of the location of the boundary, $F_{B_j}(x)$. To establish the conditional PMF of Z_i , let $\Pr(Z_i = j)$ denote the prob-

ability that segment i is part of zone j , and $\Pr(Z_{i-1} = j)$ the probability that segment $i - 1$ lies in zone j . Assuming that a segment i can only be in either zone j or zone $j + 1$, the probability of the i th segment being in zone j is calculated as

$$\Pr(Z_i = j) \approx 1 - F_{B_j}\left(i\Delta l - \frac{\Delta l}{2}\right) \quad (\text{A.1})$$

where Δl is the length of the segment represented by one slice of the DBN.

The conditional probabilities defining the variable Z_i (i.e., the values in the CPT) are

$$\begin{aligned} \Pr(Z_i = j | Z_{i-1} = j) &= \frac{\Pr(Z_i = j \cap Z_{i-1} = j)}{\Pr(Z_{i-1} = j)} \\ &= \frac{\Pr(Z_i = j)}{\Pr(Z_{i-1} = j)} \end{aligned} \quad (\text{A.2})$$

$$\begin{aligned} \Pr(Z_i = j + 1 | Z_{i-1} = j) &= 1 - \Pr(Z_i = j | Z_{i-1} = j) \\ &= 1 - \frac{\Pr(Z_i = j)}{\Pr(Z_{i-1} = j)} \end{aligned} \quad (\text{A.3})$$

$$\Pr(Z_i = j | Z_{i-1} = j + 1) = 0 \quad (\text{A.4})$$

The second equality in Equation (A.2) as well as Equation (A.4) follow from the fact that segment i can only be in zone j if segment $i - 1$ is also in zone j .

If the segment i can be in more than two different zones (e.g., in zone $j - 1$, j and $j + 1$), Equations (A.1)–(A.4) must be extended accordingly.

Rock class R

The rock class describes the geotechnical conditions along the tunnel axis. In a zone $Z_i = j$ it is modeled as a Markov process. Parameters of the Markov process are obtained from experts in form of the average length $l_k^{(j)}$ for which the rock class remains in state k and transition probabilities $p_{km}^{(j)}$, i.e., probability that, in case of a change, rock class k is followed by rock class m (Chan, 1981).

In the DBN model, the Markov process is discretized into a Markov chain, i.e., it is transformed to a discrete space represented by slices of the DBN corresponding to segments of length Δl . Assuming that changes in rock class occur as a Poisson process, the conditional probabilities of rock class in segment i , R_i , are derived from the parameters of the continuous Markov process as follows:

$$\Pr(R_i = k | R_{i-1} = k, Z_i = j) = \exp\left(-\frac{\Delta l}{l_k^{(j)}}\right) \quad (\text{A.5})$$

$$\begin{aligned} \Pr(R_i = m | R_{i-1} = k, Z_i = j) \\ = p_{km}^{(j)} \left[1 - \exp\left(-\frac{\Delta l}{l_k^{(j)}}\right) \right], k \neq m \end{aligned} \quad (\text{A.6})$$

Note that because of the dependence introduced through the parent variables Z_i , the rock class is not a Markov process. (It is a Markov process only conditional on zone Z_i .)

Failure mode F

Variable failure mode F_i represents the possible occurrence of an extraordinary event in segment i . In the presented application it can be in one of two states: “failure” or “no failure.” It is defined conditionally on ground class G_i and human factor H_i . The conditional failure rate $\lambda_{F_i|G_i, H_i}$ for given ground class and human factor can be determined from historic data. Assuming that failures occur as a Poisson process for given H_i and G_i , the conditional probability of failure mode F_i within a section of length Δl can be approximated by

$$\Pr(F_i = \text{“no failure”} | G_i, H_i) \approx \exp(-\lambda_{F_i|G_i, H_i} \Delta l) \quad (\text{A.7})$$

$$\Pr(F_i = \text{“failure”} | G_i, H_i) \approx 1 - \exp(-\lambda_{F_i|G_i, H_i} \Delta l) \quad (\text{A.8})$$

Number of failures N_F

Number of failures $N_{F,i}$ represents the total number of failures from the beginning of the tunnel up to the segment i . With m_F being the maximal number of failures to be considered (where state m_F represents m_F or more failures), the conditional probabilities are

$$\begin{aligned} \Pr(N_{F,i} = j | N_{F,i-1} = j-1, F_i = \text{“failure”}) = 1, \\ \text{for } j = \{1, \dots, m_F\} \end{aligned} \quad (\text{A.9})$$

$$\begin{aligned} \Pr(N_{F,i} = j | N_{F,i-1} = j, F_i = \text{“no failure”}) = 1, \\ \text{for } j = \{0, \dots, m_F\} \end{aligned} \quad (\text{A.10})$$

$$\Pr(N_{F,i} = m_F | N_{F,i-1} = m_F, F_i = \text{“failure”}) = 1 \quad (\text{A.11})$$

For all other conditional probabilities it holds

$$p(n_{F,i} | n_{F,i-1}, f_i) = 0 \quad (\text{A.12})$$

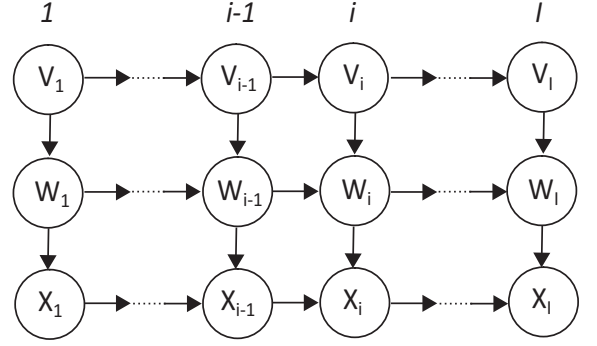


Fig. B.1. Sample DBN calculated with FA and MFA.

APPENDIX B: VALIDATION OF MODIFIED FRONTIER ALGORITHM

In this appendix, a simple DBN is evaluated using the original Frontier algorithm (FA) and the modified Frontier algorithm (MFA). The example is applied to validate the proposed MFA and to compare its computational performance with that of the original FA. The used sample DBN is depicted in Figure B.1. Each slice of the DBN consists of three random variables. Variable V_i has two states, $m_V = 2$, and is defined conditionally on V_{i-1} . The CPT of this random variable is shown in Table B.1.

The variable W_i is defined as a Normal distributed random variable conditional on V_i with parameters as given in Table B.2. For the application of the FA and MFA, the variable W_i must be discretized according to the procedure described in Section 4.1. Here, the variable is discretized into $m_W = 13$ states: 0, 1, ..., 12.

The variable X_i is defined as the sum of X_{i-1} and W_i . The interest is in calculating the PDF of variable $X_I = \sum_{i=1}^I W_i$, where I is the number of slices in the DBN.

Table B.1		
Conditional probability table (CPT) of random variable V_i		
V_i	$V_{i-1} = I$	$V_{i-1} = II$
I	0.3	0.6
II	0.7	0.4

Table B.2		
Parameters of the Normal distributed variable W_i for given V_i		
W_i	$V_{i-1} = I$	$V_{i-1} = II$
Mean	4	6
St. dev.	1.5	2.5

Table B.3

Comparison of X_I for $I = 10$ and $I = 100$ computed with FA and MFA with exact analytical solution

	X_{10}		X_{100}	
	FA/MFA	Anal.	FA/MFA	Anal.
Mean	50.78	50.77	507.76	507.69
St. dev.	7.04	7.05	22.19	22.23

Frontier algorithm

Before the application of the FA, we eliminate the variables W_i , because they do not have links to nodes in neighboring slices. Elimination of these nodes can be understood as a preprocessing of the DBN, reducing the computational demand during application of the FA. The elimination of W_i is performed for the whole DBN at once, variable X_i is then defined directly on V_i and on X_{i-1}

$$p(x_i|v_i, x_{i-1}) = \sum_{W_i} p(w_i|v_i) p(x_i|w_i, x_{i-1}) \quad (\text{B.1})$$

where $p(w_i|v_i)$ is known from the discretization process of W_i and $p(x_i|w_i, x_{i-1}) = \Pr(X_i = x_i | W_i = w_i, X_{i-1} = x_{i-1})$ takes value 1 for $x_i = x_{i-1} + w_i$ and value 0 otherwise. The number of states of X_i is $m_X = I(m_W - 1) + 1$.

One cycle of the FA, i.e., moving the frontier from slice $i - 1$ to slice i , is shown in the following. First, the variable V_i is added and V_{i-1} removed from the frontier

$$p(v_i, x_{i-1}) = \sum_{V_{i-1}} p(v_i|v_{i-1}) p(v_{i-1}, x_{i-1}) \quad (\text{B.2})$$

where $p(v_i|v_{i-1})$ is defined in Table B.1 and $p(v_{i-1}, x_{i-1})$ is the joint PMF known from previous cycle of the FA.

Second, the variable X_i is added and X_{i-1} removed from the frontier

$$p(v_i, x_i) = \sum_{X_{i-1}} p(x_i|v_i, x_{i-1}) p(v_i, x_{i-1}) \quad (\text{B.3})$$

where $p(x_i|v_i, x_{i-1})$ and $p(v_i, x_{i-1})$ are known from Equation (B.1) and (B.2), respectively. Equation (B.3) represents the most demanding computational step in the algorithm. The computation of this equation of the DBN requires $O(m_X^2 \times m_V)$ time in the i th slice. The evaluation of the whole DBN with I slices therefore requires $O(I \times m_X^2 \times m_V) = O(I \times [I(m_W - 1) + 1]^2 \times m_V)$ time. It is evident that the computation time increases exponentially with the number of states of the variable W , m_W , and with the number of slices of the DBN, I .

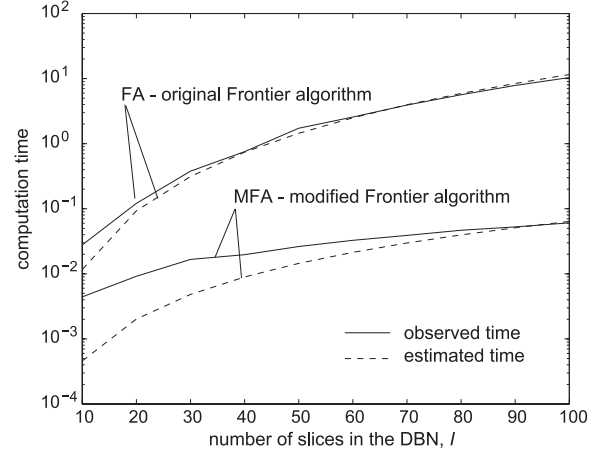


Fig. B.2. Computation time for evaluation of the sample DBN with different number of slices, I : comparison of FA and MFA.

Modified Frontier algorithm

One cycle of the MFA is presented in the following. First, variable V_i is added and V_{i-1} is removed from the frontier according to Equation (B.2). Second, the PMF of X_i is calculated using convolution (analogously to Equation 16 and 18)

$$\begin{aligned} p_{X_i|V_i}(x) &= p_{X_{i-1}|V_i} * p_{W_i|V_i}(x) \\ &= \sum_{\tau} p_{X_{i-1}|V_i}(x - \tau) p_{W_i|V_i}(\tau) \end{aligned} \quad (\text{B.4})$$

where $p_{X_{i-1}|V_i} = p(x_{i-1}|v_i) = p(v_i, x_{i-1})/p(v_i)$, $p(w_i|v_i)$ is known from the discretization of variable W_i and the summation is over all states τ of W_i . Finally, the joint PMF describing the frontier in slice i is calculated as $p(v_i, x_i) = p(x_i|v_i)p(v_i)$.

The number of states of X_i is increasing in each slice of the DBN; it is $m_{X,i} = i(m_W - 1)$ for $i = 1, 2, \dots, I$. The most demanding computational step of the MFA is the calculation of Equation (B.4), which in the i th slice of the DBN requires $O(m_V \times m_{X,i} \times m_W)$ time. For computation of the convolution, the fast Fourier transform (FFT) is commonly used (Walker, 1996). With FFT, the calculation of the i th slice of the DBN requires $O(m_V \times m_{X,i} \log(m_{X,i}))$ time and evaluation of the whole DBN with I slices requires $O(m_V \sum_{i=1}^I m_{X,i} \log m_{X,i}) = O(m_V \sum_{i=1}^I i(m_W - 1) \log[i(m_W - 1)])$ time (Walker, 1996).

Results

Computations are performed for the DBN with varying number of slices I . The computation times depicted in Figure B.2 show the theoretical computation time estimated based on the number of performed operations as

presented above and the observed time of computations performed in Matlab on the computer specified in Section 4.3. The time needed for evaluation of the DBN with only $I = 100$ slices is almost 1,000 times higher with the original FA than with the MFA. The observed increase in computation time with the number of cycles is lower than the one estimated above. The likely reason for this is that the FFT algorithm implemented in

Matlab is more efficient than the estimate given above (which represents a general upper bound).

A comparison of the mean and standard deviation of X_I computed with FA and MFA with the exact analytical solution is given in Table B.3. FA and MFA give exactly the same results, which differ slightly from the analytical results because of the small discretization errors.

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