# Mathematical Economics 1A, Problem Set 3

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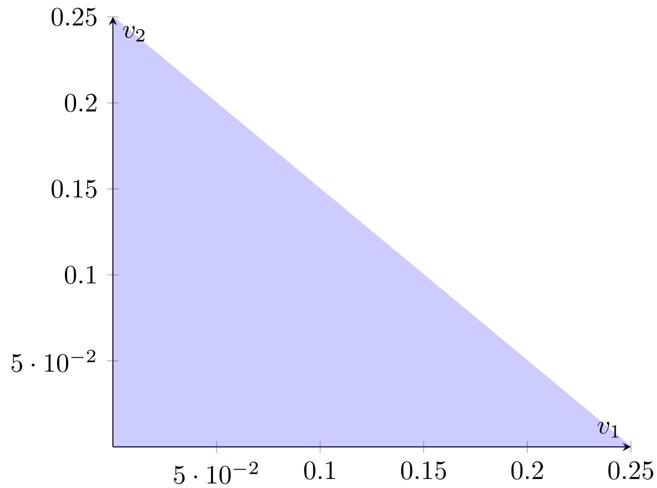
## Task 1: Repeated Games: Cournot

(a) Maximizing $(1 - q_1 - q_2)q_1 - cq_1$ gives $\left(\frac{1}{3}, \frac{1}{3}\right)$. Payoffs are $\left(\frac{1}{9}, \frac{1}{9}\right)$.

(b) Total payoff = monopoly case $\Rightarrow$ $\max_{q_1,q_2} \{(1 - q_1 - q_2)(q_1 + q_2)\}$ gives $q_1 + q_2 = \frac{1}{2}$ and $\Pi = \frac{1}{4}$. Feasible set




(c) Solve $(1 - q_1 - q_2)q_1 = v_1$ and $(1 - q_1 - q_2)q_2 = v_2$ gives

$$q_1 = \frac{v_1 - v_1\sqrt{1 - 4(v_1 + v_2)}}{2(v_1 + v_2)}$$
$$q_2 = \frac{v_2 - v_2\sqrt{1 - 4(v_1 + v_2)}}{2(v_1 + v_2)}$$

(d) Nash reversion:

$$s_i(h) \begin{cases} q_i(v) & \text{no deviation in } h \\ \frac{1}{3} & \text{other} \end{cases}$$

The upper bound for deviation is $\frac{1}{4}$. This leads to

$$(1 - \delta)\frac{1}{4} + (1 - \delta)\sum_{t=1}^{\infty} \delta^t \frac{1}{9} \leq (1 - \delta)\sum_{t=0}^{\infty} \delta^t v_i$$
$$\delta \geq \frac{\frac{1}{4} - v_1}{\frac{1}{4} - \frac{1}{9}}$$

(e) Yes because both $v_1$ are below $\frac{1}{4}$.

(f) No because $\frac{1}{12} < \frac{1}{3}$.

## Task 2: Cournot with Incomplete Information about Costs

(a) Maximizing $(1 - q_1 - q_2)q_1 - cq_1$ gives

$$q_1 = \frac{1}{3} - \frac{2c_1}{3} + \frac{c_2}{3}$$
$$q_2 = \frac{1}{3} - \frac{2c_2}{3} + \frac{c_1}{3}$$

This results in $\left(\frac{1}{3}, \frac{1}{3}\right)$.

(b) $\left(\frac{1}{4}, \frac{1}{4}\right)$

(c) $\left(\frac{1}{6}, \frac{5}{12}\right)$

(d) Maximizing expected utility gives

$$q_1(0) = \frac{1 - pq_2(0) - (1-p)q_2(0.25)}{2}$$
$$q_1(0.25) = \frac{\frac{3}{4} - pq_2(0) - (1-p)q_2(0.25)}{2}$$

Same for player 2.

$$q(0) = \frac{9-p}{24}$$
$$q(0.25) = \frac{6-p}{24}$$

(e) Beliefs are correlated, so $\mathbb{P}(c_2 = 0.25 \mid c_1 = 0.25) = \mathbb{P}(c_2 = 0 \mid c_1 = 0) = 2p$ and $\mathbb{P}(c_2 = 0.25 \mid c_1 = 0) = \mathbb{P}(c_2 = 0 \mid c_1 = 0.25) = 1 - 2p$. This leads to

$$q(0) = \frac{14p + 5}{12(4p+1)}$$
$$q(0.25) = \frac{7p + 1}{6(4p+1)}$$

(f) If $p = 0.5$ we know the production costs of our opponent.