Mathematical Economics 1A, Problem Set 3

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Task 1: Adverse Selection

(a) for everyone

(b) types $x \geq p$

(c) consumer accepts any $p \leq x$, rejects otherwise. Insurer offers $p = x$.

(d) consumer accepts any $p \leq x$, rejects otherwise.

$$\begin{aligned}
u_I(p,x) &= \begin{cases} 0 & x < p \\ p - \frac{x}{2} & x \geq p \end{cases} \\
\mathbb{E}(u_I(p,x)) &= \int_0^1 p(x) u_I(p,x) dx \\
&= \int_0^p 0 dx + \int_p^1 \left(p - \frac{x}{2}\right) dx \\
&= -\frac{3}{4}p^2 + p - \frac{1}{4} \to \max \\
p^* &= \frac{2}{3}
\end{aligned}$$

PBE: Insurer offers $p = \frac{2}{3}$. Consumers buys if $x \geq p$. Insurer's beliefs are the prior beliefs.

(e) all consumers with $x \geq \frac{2}{3}$

(f) Utility:

$$u_i(p_i, p_j, x) = \begin{cases} p & p_i > x \vee p_i > p_j \\ p_i - \frac{x}{2} & p_i \leq x \wedge p_i < p_j \\ \frac{p_i - \frac{x}{2}}{2} & p_i = p_j \leq x \end{cases}$$

(g) $p_1^* = p_2^*$ and $\Pi_1^* = \Pi_2^* = 0$ for insurer

$$\begin{aligned}
\Pi_1 = \Pi_2 &= \int_0^1 u_I(p,p,x) dx \\
&= \int_0^p 0 dx + \int_p^1 \frac{p_i - \frac{x}{2}}{2} dx \overset{!}{=} 0 \\
0 &= -\frac{3}{4}p^2 + p - \frac{1}{4} \\
p^* &= \frac{1}{3}
\end{aligned}$$

The other root at $p = 1$ is not a equilibrium as reducing the price a bit increases sales.

(h) all consumers with $x \geq \frac{1}{3}$. More competition is not helpful since we are already at zero profits.

(i) p is lowers price

$$\mathbb{E}(\Pi_i) = \int_0^1 p - \frac{x}{2} dx \stackrel{!}{=} 0$$
$$p^* = \frac{1}{4}$$

$\Rightarrow x - p^* \geq -k$ for every customer, especially for $x = 0 \Rightarrow k = \frac{1}{4}$

Task 2: Perfect Bayesian Equilibrium in Stackelberg with informed first-mover

(a) Payoffs for $x = 3$:

- LL: $\Pi_1 = 1$, $\Pi_2 = 1$
- LH: $\Pi_1 = 0$, $\Pi_2 = 0$
- HL: $\Pi_1 = 0$, $\Pi_2 = 0$
- HH: $\Pi_1 = 0$, $\Pi_2 = 0$

Payoffs for $x = 6$:

- LL: $\Pi_1 = 4$, $\Pi_2 = 4$
- LH: $\Pi_1 = 3$, $\Pi_2 = 6$
- HL: $\Pi_1 = 6$, $\Pi_2 = 3$
- HH: $\Pi_1 = 4$, $\Pi_2 = 4$

(b)