Mathematical Economics 2, Problem Set 2

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Task 1

(a) Total Action (TA): contribution to a public good

$$u_i(\sigma_i, \sigma_1, ..., \sigma_n) = f\left(\sigma_i + \alpha \sum_{j \in N_i} \sigma_j\right) - c\sigma_i$$

Best Effort (BE):

$$u_i(\sigma_i, \sigma_1, ..., \sigma_n) = f\left(\max_{j \in N_i \cup \{i\}} \sigma_j\right) - c\sigma_i$$

Average Effort (AE):

$$u_i(\sigma_i, \sigma_1, ..., \sigma_n) = f\left(\sigma_i \frac{\sum_{j \in N_i} \sigma_j}{k_i}\right) - c\sigma_i$$

Weakest Link (WL): connected banks, one goes bankrupt

$$u_i(\sigma_i, \sigma_1, ..., \sigma_n) = f\left(\min_{j \in N_i \cup \{i\}} \sigma_j\right) - c\sigma_i$$

(b) TA: if the function is concave we get substitutes, if the function is convex we get complements. Property A is satisfied.
BE: strategic substitutes. Property A is satisfied.
AE: strategic complements. Property A is not satisfied.
WL: strategic complements. Property A is not always satisfied.

(c) Table

	All 0	**All 1**
TA	$c \geq b$	$b \geq c$
BE	$c \geq b$	never happen when $n > 1$
AE	always	$b \geq c$,
WL	always	$b \geq c$

(Extra) non-decreasing: If your degree is bigger then a certain k then you play 1 otherwise 0.
non-increasing: If your degree is bigger then a certain k then you play 0 otherwise 1.

- TA: function concave: non-increasing, convex: non-decreasing

- BE: non-increasing
- AE: cannot say because it violates property A
- WL: non-increasing/cannot say because it violates property A

Task 2

(a) $u_i(\Gamma) = \sum_{j \neq i} u(d(i, j \mid \Gamma)) - k_i c$

(b) Grand star: 1 hub and $n-1$ spokes.

$$\begin{aligned}
u(\text{hub}) &= (n-1)(u(1)-c) \\
u(\text{spoke}) &= (u(1)-c) + (n-2)u(2) \\
\text{total utility} &= (n-1)[u(1)-c+(n-2)u(2)] + (n-1)[u(1)-c] \\
&= (n-1)[(n-2)u(2) + 2(u(1)-c)]
\end{aligned}$$

Empty network has a utility of 0, when is grand star better?

$$\begin{aligned}
(n-1)[(n-2)u(2) + 2(u(1)-c)] &> 0 \\
\underbrace{2[u(1)-c]}_{\text{negative}} + (n-2)u(2) &> 0
\end{aligned}$$

$$\begin{aligned}
(n-2)u(2) &> 2c - 2u(1) \\
c &< \frac{(n-2)u(2) + 2u(1)}{2}
\end{aligned}$$

(i) efficient network is complete network

(ii) efficient network is grand star

(iii) efficient network is empty network

(c) If $c < u(1) - u(2)$ then a complete network will form. If $c \in (u(1) - u(2), u(1))$ then the hub in a grand star in receiving positive utility and the grand star is pairwise stable. If $c > u(1)$ then the hub will break all links resulting in an empty network.