Mathematical Economics 1A, Problem Set 1

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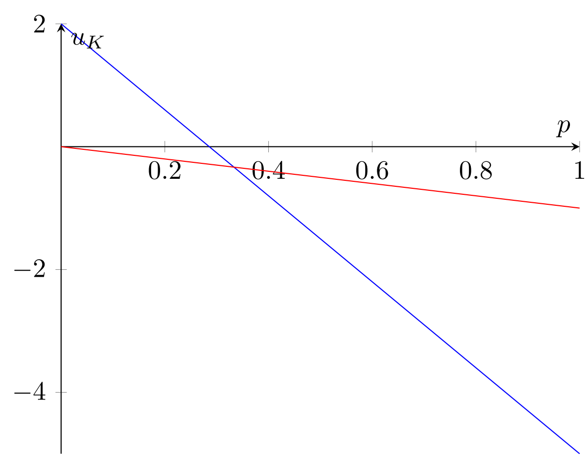
Task 1: Ye olde game of Chicken

(a) The normal form of this game is

		Kate	
		swerve	not swerve
Jane	swerve	(0,0)	(-1,2)
	not swerve	(2,-1)	(-5,-5)

(b) Jane's strategy is $p \cdot \text{not swerve} + (1-p) \cdot \text{swerve}$. Then

- $u_K(p \cdot \text{not swerve} + (1-p) \cdot \text{swerve}, \text{not swerve}) = p(-5) + (1-p)2 = 2 - 7p$
- $u_K(p \cdot \text{not swerve} + (1-p) \cdot \text{swerve}, \text{swerve}) = p(-1) + (1-p)0 = -p$



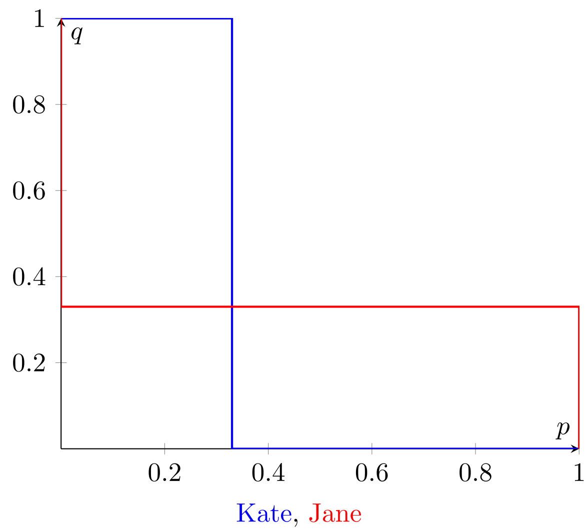
$2 - 7p$, $-p$

(c) Not swerve is the best reponse if

$$\begin{aligned} u_K(p \cdot \text{not swerve} + (1-p) \cdot \text{swerve}, \text{not swerve}) &\geq u_K(p \cdot \text{not swerve} + (1-p) \cdot \text{swerve}, \text{swerve}) \\ 2 - 7p &\geq -p \\ p &\leq \frac{1}{3} \end{aligned}$$

This leads to

$$BR_K(p) = \begin{cases} \text{not swerve} & p < \frac{1}{3} \\ \text{choose random} & p = \frac{1}{3} \\ \text{swerve} & p > \frac{1}{3} \end{cases}$$



(d) You can see that (swerve, not swerve) and (not swerve, swerve) are pure strategy Nash Equilibria. If we assign probabilities to each strategy we can find a NE in mixed strategies.

		Kate		
		swerve	not swerve	
Jane	swerve	(0,0)	(-1,2)	p
	not swerve	(2,-1)	(-5,-5)	$1-p$
		q	$1-q$	

Then the outcome for each player is

$$u_K(p,q) = 2p(1-q) - q(1-p) - 5(1-p)(1-q) = 7p - 6pq + 4q - 5$$
$$u_J(p,q) = -p(1-q) + 2q(1-p) - 5(1-p)(1-q) = 7q - 6pq + 4p - 5$$

Maximizing

$$\frac{\partial u_K}{\partial q} = -6p + 4 \stackrel{!}{=} 0$$
$$p = \frac{2}{3}$$
$$\frac{\partial u_J}{\partial p} = -6q + 4 \stackrel{!}{=} 0$$
$$q = \frac{2}{3}$$

So the third NE is $\left(\frac{2}{3}\text{swerve} + \frac{1}{3}\text{not swerve}, \frac{2}{3}\text{swerve} + \frac{1}{3}\text{not swerve}\right)$.

(e) With the underline-approach you find that (not swerve, swerve) is definitely a NE. If Kate played not swerve and Jane has do decide what she plays, the question is $x - 5 > -1$ (we want her to choose not swerve so that (swerve, not swerve) doesn't become a NE). The equation above leads to $x > 4$ and then the only NE is (not swerve, swerve).

Task 2: Auctions!

(a) Die utility function for player 1 with bid b_i is

$$u_1(x_1, x_2) = \begin{cases} 500 - x_1 & x_1 > x_2 \\ \frac{1}{2}(500 - x_1) & x_1 = x_2 \\ 0 & x_1 < x_2 \end{cases}$$

(b) The best response for player 1 is

$$BR_1(x_2) = \begin{cases} [0, x_2) & x_2 > 500 \\ [0, 500] & x_2 = 500 \\ \emptyset & x_2 < 500 \end{cases}$$

(c)

Task 3: Cournot Price Competition

(a) The utility functions are

$$\begin{aligned} u_1(q_1, q_2) &= q_1 \cdot (1 - q_1 - q_2) \\ u_2(q_1, q_2) &= q_2 \cdot (1 - q_1 - q_2) \end{aligned}$$

(b) Maximizing

$$\begin{aligned} \frac{\partial u_1}{\partial q_1} &= 1 - 2q_1 - q_2 \overset{!}{=} 0 \\ q_1 &= \frac{1 - q_2}{2} \qquad (1) \\ \frac{\partial u_2}{\partial q_2} &= 1 - 2q_2 - q_1 \overset{!}{=} 0 \\ q_2 &= \frac{1 - q_1}{2} \qquad (2) \end{aligned}$$

(c) Insert (2) in (1):

$$\begin{aligned} q_1 &= \frac{1 - \frac{1 - q_1}{2}}{2} \\ &= \frac{1}{2} - \frac{1 - q_1}{4} \\ 4q_1 &= 2 - (1 - q_1) \\ 3q_1 &= 1 \\ q_1 &= \frac{1}{3} \\ \Rightarrow q_2 &= \frac{1}{3} \end{aligned}$$

(d)