Finanzderivate und Optionen, Übung 10

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Aufgabe 1

Black-Scholes-Modell, europäischer Call:

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) T}{\sigma\sqrt{T}} \\ d_2 &= d_1 - \sigma\sqrt{T} \\ c_0(C) &= S_0\Phi(d_1) - \exp(-rT)\cdot K \cdot \Phi(d_2) \end{aligned}$$

Mittels Put-Call-Parität ergibt sich der Preis eines europäischen Puts:

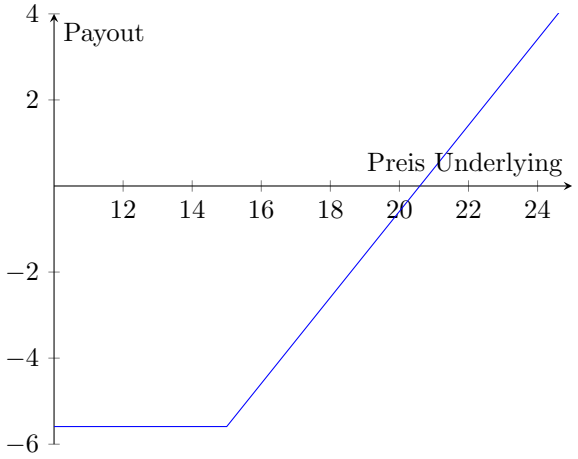
$$c_0(P) = \exp(-rT)\cdot K \cdot \Phi(-d_2) - S_0 \cdot \Phi(-d_1)$$

(a) Einsetzen[1]:

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{20}{15}\right) + \left(0.05 + \frac{0.2^2}{2}\right)\cdot 0.75}{0.2\cdot\sqrt{0.75}} \\ &= 1.9640 \\ d_2 &= 1.9640 - 0.2\cdot\sqrt{0.75} \\ &= 1.7908 \\ c_0(C) &= 20\cdot\Phi(1.9640) - \exp(-0.05\cdot 0.75)\cdot 15\cdot\Phi(1.7908) \\ &= 5.59 \end{aligned}$$

Das Nettoauszahlungsprofil ist

[1] https://www.wolframalpha.com/input?i=black+scholes&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22underlying%22%7D+-%3E%222420%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22div%22%7D+-%3E%220+%25%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22rf%22%7D+-%3E%225+%25%22&assumption=%7B%22FP%22%2C+%22FinancialOption%22%2C+%22OptionName%22%7D+-%3E+%22VanillaEuropean%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22strike%22%7D+-%3E%222415%22&assumption=%7B%22MC%22%2C+%22%22%7D+-%3E+%7B%22Formula%22%2C+%22dflt%22%7D&assumption=%7B%22FP%22%2C+%22FinancialOption%22%2C+%22opttype%22%7D+-%3E+%22Call%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22exptime%22%7D+-%3E%220.75+a%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22vol%22%7D+-%3E%2220+%25%22



Zeitwert: 0.59, innerer Wert: 5

(b) Einsetzen[2]

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{20}{20}\right) + \left(0.05 + \frac{0.2^2}{2}\right) \cdot 0.75}{0.2 \cdot \sqrt{0.75}} \\ &= 0.3031 \\ d_2 &= 0.3031 - 0.2 \cdot \sqrt{0.75} \\ &= 0.1299 \\ c_0(C) &= 20 \cdot \Phi(0.3031) - \exp(-0.05 \cdot 0.75) \cdot 20 \cdot \Phi(0.1299) \\ &= 1.75 \end{aligned}$$

(c) Wir brauchen noch den Preis des Puts[3]:

$$\begin{aligned} c_0(P) &= \exp(-0.05 \cdot 0.75) \cdot 20 \cdot \Phi(-0.1299) - 20 \cdot \Phi(-0.3031) \\ &= 1.01 \end{aligned}$$

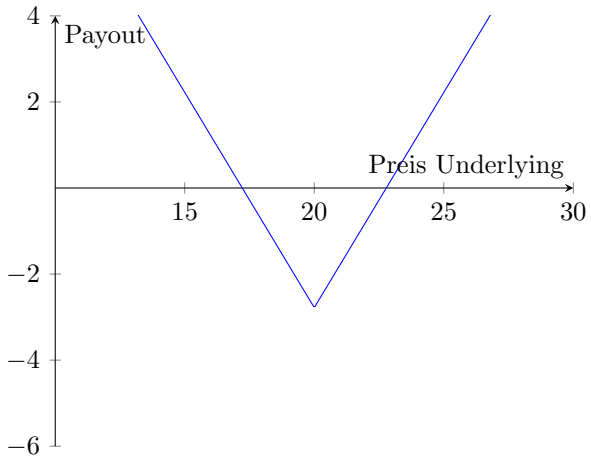
Alternativ

$$c_0(P) = c_0(C) - S_0 - \exp(-rT)K$$

Der Preis der Strategie ist dann $1.01 + 1.75 = 2,76$ und mit dieser Strategie kann man auf steigende Volatilität wetten.

[2] https://www.wolframalpha.com/input?i=black+scholes&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22underlying%22%7D+-%3E%22420%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22div%22%7D+-%3E%220+%25%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22rf%22%7D+-%3E%225+%25%22&assumption=%7B%22FP%22%2C+%22FinancialOption%22%2C+%22OptionName%22%7D+-%3E+%22VanillaEuropean%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22strike%22%7D+-%3E%22420%22&assumption=%7B%22MC%22%2C+%22%22%7D+-%3E+%7B%22Formula%22%2C+%22dflt%22%7D&assumption=%7B%22FP%22%2C+%22FinancialOption%22%2C+%22opttype%22%7D+-%3E+%22Call%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22exptime%22%7D+-%3E%220.75+a%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22vol%22%7D+-%3E%2220+%25%22

[3] https://www.wolframalpha.com/input?i=black+scholes&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22underlying%22%7D+-%3E%22420%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22div%22%7D+-%3E%220+%25%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22rf%22%7D+-%3E%225+%25%22&assumption=%7B%22FP%22%2C+%22FinancialOption%22%2C+%22OptionName%22%7D+-%3E+%22VanillaEuropean%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22strike%22%7D+-%3E%22420%22&assumption=%7B%22MC%22%2C+%22%22%7D+-%3E+%7B%22Formula%22%2C+%22dflt%22%7D&assumption=%7B%22FP%22%2C+%22FinancialOption%22%2C+%22opttype%22%7D+-%3E+%22Put%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22exptime%22%7D+-%3E%220.75+a%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22vol%22%7D+-%3E%2220+%25%22



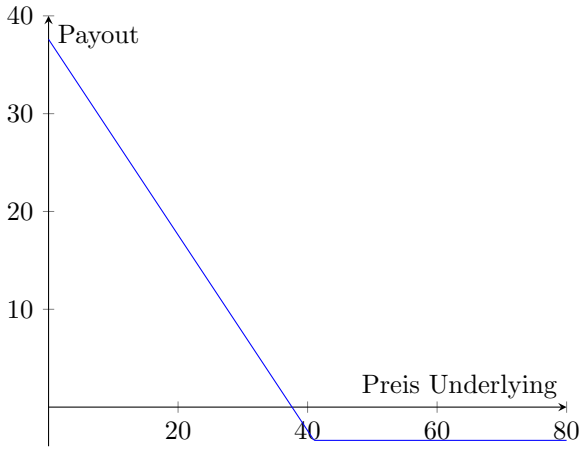
Aufgabe 2

(a) Einsetzen[4]

$$
\begin{aligned}
d_1 &= \frac{\ln\left(\frac{44.50}{41}\right) + \left(0.015 + \frac{0.23^2}{2}\right) \cdot 2}{0.23 \cdot \sqrt{2}} \\
&= 0.5067 \\
d_2 &= 0.5067 - 0.23 \cdot \sqrt{2} \\
&= 0.1814 \\
c_0(P) &= \exp(-0.015 \cdot 2) \cdot 41 \cdot \Phi(-0.1814) - 44.50 \cdot \Phi(-0.5067) \\
&= 3.41
\end{aligned}
$$

Das Nettoauszahlungsprofil ist

[4] https://www.wolframalpha.com/input?i=black+scholes&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22underlying%22%7D+-%3E%22%2444.50%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22div%22%7D+-%3E%220+%25%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22rf%22%7D+-%3E%221.5+%25%22&assumption=%7B%22FP%22%2C+%22FinancialOption%22%2C+%22OptionName%22%7D+-%3E+%22VanillaEuropean%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22strike%22%7D+-%3E%22%2441%22&assumption=%7B%22MC%22%2C+%22%22%7D+-%3E+%7B%22Formula%22%2C+%22dflt%22%7D&assumption=%7B%22FP%22%2C+%22FinancialOption%22%2C+%22opttype%22%7D+-%3E+%22Put%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22exptime%22%7D+-%3E%222+a%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22vol%22%7D+-%3E%2223+%25%22



Zeitwert: 3.41, innerer Wert: 0

(b) Einsetzen[5]

$$
\begin{aligned}
d_1 &= \frac{\ln\left(\frac{44.50}{45}\right) + \left(0.015 + \frac{0.23^2}{2}\right) \cdot 2}{0.23 \cdot \sqrt{2}} \\
&= 0.2205 \\
d_2 &= 0.2205 - 0.23 \cdot \sqrt{2} \\
&= -0.1048 \\
c_0(P) &= \exp(-0.015 \cdot 2) \cdot 45 \cdot \Phi(0.1048) - 44.50 \cdot \Phi(-0.2205) \\
&= 5.29
\end{aligned}
$$

Zeitwert: 4.79, innerer Wert: 0.50. Diese Option ist ITM, sie kostet also auch mehr.

[5] https://www.wolframalpha.com/input?i=black+scholes&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22underlying%22%7D+-%3E%22%2444.50%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22div%22%7D+-%3E%220+%25%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22rf%22%7D+-%3E%221.5+%25%22&assumption=%7B%22FP%22%2C+%22FinancialOption%22%2C+%22OptionName%22%7D+-%3E+%22VanillaEuropean%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22strike%22%7D+-%3E%22%2445%22&assumption=%7B%22MC%22%2C+%22%22%7D+-%3E+%7B%22Formula%22%2C+%22dflt%22%7D&assumption=%7B%22FP%22%2C+%22FinancialOption%22%2C+%22opttype%22%7D+-%3E+%22Put%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22exptime%22%7D+-%3E%222+a%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22vol%22%7D+-%3E%2223+%25%22

(c) Einsetzen[6]

$$
\begin{aligned}
d_1 &= \frac{\ln\left(\frac{40}{41}\right) + \left(0.015 + \frac{0.26^2}{2}\right) \cdot 2}{0.26 \cdot \sqrt{2}} \\
&= 0.1983 \\
d_2 &= 0.1983 - 0.26 \cdot \sqrt{2} \\
&= -0.1694 \\
c_0(P) &= \exp(-0.015 \cdot 2) \cdot 41 \cdot \Phi(0.1694) - 40 \cdot \Phi(-0.1983) \\
&= 5.71
\end{aligned}
$$

Die Option ist jetzt ITM und damit mehr wert. Die Vola steigt auch.

[6] https://www.wolframalpha.com/input?i=black+scholes&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22underlying%22%7D+-%3E%22%2440%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22div%22%7D+-%3E%220+%25%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22rf%22%7D+-%3E%221.5+%25%22&assumption=%7B%22FP%22%2C+%22FinancialOption%22%2C+%22OptionName%22%7D+-%3E+%22VanillaEuropean%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22strike%22%7D+-%3E%22%2441%22&assumption=%7B%22MC%22%2C+%22%22%7D+-%3E+%7B%22Formula%22%2C+%22dflt%22%7D&assumption=%7B%22FP%22%2C+%22FinancialOption%22%2C+%22opttype%22%7D+-%3E+%22Put%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22exptime%22%7D+-%3E%222+a%22&assumption=%7B%22F%22%2C+%22FinancialOption%22%2C+%22vol%22%7D+-%3E%2226+%25%22