Mathematical Economics 2, Problem Set 3

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Task 1

(a) matrix:

$$\begin{pmatrix} 2,2 & 0,2 & 0,0 \\ 2,0 & 0,0 & 0,3 \\ 0,0 & 3,0 & 4,4 \end{pmatrix}$$

(b) Population space is a triangle

(c) pure $NE = \{e_1, e_3\}$, mixed NE:

- NE of form $(x, 1-x, 0)$: $u(e_1, x) = 2x = u(e_2, x)$, $u(e_3, x) = 3(1-x) \Rightarrow x \geq \frac{3}{5}$
- NE of form $(x, 0, 1-x)$: $u(e_1, x) = 2x = u(e_2, x)$, $u(e_3, x) = 4(1-x) \Rightarrow x = \frac{2}{3}$
- NE of form $(0, x, 1-x)$: e_3 does better than $e_2 \Rightarrow$ no reason to play e_2
- NE of form $(x_1, x_2, 1-x_1-x_2)$: $u(e_1, x) = 2x_1 = u(e_2, x)$, $u(e_3, x) = 3x_2 + 4(1-x_1-x_2) \Rightarrow 6x_1 + x_2 = 4$

(d) e_3 is a strict NE $\Rightarrow$ e_3 is ESS and NSS
$\left(\frac{2}{3}, 0, \frac{1}{3}\right)$ is neither ESS nor NSS (e_3 can invade), same for all points $(x_1, x_2, 1-x_1-x_2)$ on $6x_1+x_2 = 4$
e_1 is NSS because you can't resist e_2 mutants but they can't invade, same for points $(x, 1-x, 0)$ on the line $x \geq \frac{3}{5}$
$\left(\frac{3}{5}, \frac{2}{5}, 0\right)$ is neither ESS nor NSS because e_3 can invade

(e) $u(x, x) = (x_1 + x_2)(2x_1) + (1 - x_1 - x_2)(3x_2 + 4(1 - x_1 - x_2))$
$\dot{x}_1 = x_1[u(e_1, x) - u(x, x)] = x_1(1 - x_1 - x_2)(6x_1 + x_2 - 4)$

(f) Sets

- Stationary: all NE, points between e_1 and e_2
- Lyapunov stable: NSS
- Asymptotically stable: e_3
- Limit states: e_3, points $(x, 1-x, 0)$ with $x \geq \frac{3}{5}$ but not e_1, points $(x_1, x_2, 1-x_1-x_2)$ with $6x_1 + x_2 = 4$