Mathematical Economics 1A, Problem Set 2

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Task 1: More Auctions!

(a) Player 1 won't bid over 500, but since the best response for player 2 if player 1 is bidding less then 500 is empty (see problem set 1), player 1 has to bid 500.

$$NE = \left\{ \left(500, \sigma(x_1) = \begin{cases} (x_1, \infty) & x_1 < 500 \\ [0, 500] & x_1 = 500 \\ \mathbb{R}_+ & x_1 > 500 \end{cases} \right) \right\}$$

(b) A SPNE must be perfect in *any* game, even if player 1 bids e.g. 150 Pounds. Player 2 can then not find a best response and therefore there is no SPNE.

(c) The best response of player 1 is

$$BR_1(x_2) = \begin{cases} x_2 + 1 & x_2 < 498 \\ 498, 499 & x_2 = 498 \\ 499 & x_2 = 499 \\ \{0, 1, ..., 500\} & x_2 = 500 \\ \{0, 1, ..., x_2 - 1\} & x_2 > 500 \end{cases}$$

The best response for player 2 is the same. The NE is $NE = \{(498, 498), (499, 499), (500, 500)\}$.

(d) Nash Equilibria:

$$\left(499, \sigma(x_1) = \begin{cases} x_1 + 1 & x_1 < 498 \\ 499 & x_1 = 498 \\ 499 & x_1 = 499 \\ \{0, 1, ..., 500\} & x_1 = 500 \\ \{0, 1, ..., x_1 - 1\} & x_1 > 500 \end{cases} \right)$$

There's a second NE:

$$\left(498, \sigma(x_1) = \begin{cases} x_1 + 1 & x_1 < 498 \\ 498 & x_1 = 498 \\ 499 & x_1 = 499 \\ \{0, 1, ..., 500\} & x_1 = 500 \\ \{0, 1, ..., x_1 - 1\} & x_1 > 500 \end{cases} \right)$$

Task 2: Extensive Form Extravaganza

(a) Normal form:

		Player 3	
		F	M
P 2	F	(2,2,2)	(3,5,1)
	M	(3,1,5)	(4,4,4)

		Player 3	
		M	B
P 2	M	(0,3,2)	(2,1,1)
	B	(5,0,0)	(3,2,3)

Left: Dilemma, Right: Sexes

The only NE is (Dilemma, F, F).

(b) The NE for Dilemma is (F,F) and the NE's for Sexes are (M,M), (B,B) and $\left(\frac{3}{4}M + \frac{1}{4}B, \frac{1}{4}M + \frac{3}{4}B\right)$. The expected payoff for player 1 is

- (F,F) $\rightarrow 2$
- (M,M) $\rightarrow 0$
- (B,B) $\rightarrow 3$
- $\left(\frac{3}{4}M + \frac{1}{4}B, \frac{1}{4}M + \frac{3}{4}B\right) \rightarrow \frac{3}{4} \cdot \frac{3}{4} \cdot 2 + \frac{1}{4} \cdot \frac{1}{4} \cdot 5 + \frac{1}{4} \cdot \frac{3}{4} \cdot 3 = 2$

Player 1 can expect in Dilemma 2 utility and in Sexes $\frac{5}{3}$ so player 1 will choose Dilemma.

Task 3: Repeated Games: Tragedy of the Commons

(a) Normal form:

		Player 3	
		Graze	Gorge
P 2	Graze	(1,1,1)	(-1,-1,3)
	Gorge	(-1,3,-1)	(-3,-1,-1)

		Player 3	
		Graze	Gorge
P 2	Graze	(3,-1,-1)	(-1,-3,-1)
	Gorge	(-1,-1,-3)	(-3,-3,-3)

Left: Graze, Right: Gorge

Gorge is a weakly dominant strategy.