Instituto Superior de Engenharia de Coimbra
DEPARTAMENTO DE FÍSICA E MATEMÁTICA

Análise Matemática I - **Engenharia Informática**

TPC nº3 Data limite de entrega: 21/out/2015 (18h)

Funções trigonométricas inversas [**D. Análise**]

2. Considere a função $f(x) = \pi + 2\arcsin(2x+1)$.

   a) Determine o domínio e o contradomínio de f.

   b) Calcule $f\left(-\dfrac{1}{4}\right)$.

   c) Resolva a equação $f(x) = \dfrac{\pi}{3}$.

   d) Caracterize a função inversa de f indicando o domínio, o contradomínio e a expressão analítica.

Regras de derivação [**A. Conhecimento**]
Reproduza a regra indicada no cálculo da derivada de cada uma das seguintes funções:

página 1 Regra 7: $(f^p)' = p\, f^{p-1}\, f'$ Regra 3: $(c\, f)' = c\, f'$

2. $f(x) = x^9$;

6. $f(x) = \dfrac{1}{2}\, x^9$;

9. $f(x) = (4x)^9$;

17. $f(x) = \dfrac{\sqrt[4]{x^5}}{3}$;

24. $f(x) = \dfrac{1}{(3x)^2}$;

27. $f(x) = \dfrac{2}{\sqrt[3]{x^2}}$;

página 2 Regra 4: $(f+g)' = f' + g'$

3. $f(x) = x^3 + \dfrac{3x}{5} + \dfrac{1}{x^2}$;

7. $f(x) = \sqrt{x^3} - \dfrac{2}{\sqrt[3]{x^2}}$;

12. $f(x) = \dfrac{(x^4-1)^2}{4}$;

página 2 Regra 5: $(f \times g)' = f' \times g + f \times g'$

2. $f(x) = (x^2-2)\left(3 - \dfrac{1}{x^2}\right)$;

página 2 Regra 6: $\left(\dfrac{f}{g}\right)' = \dfrac{f' \times g - f \times g'}{g^2}$

2. $f(x) = \dfrac{3x - 2x^2}{x-1}$;

página 3 Regra 9: $(e^f)' = f'e^f$

4. $f(x) = 5e^{2x} + 3\,;$
8. $f(x) = e^{\sqrt{x}}\,;$
16. $f(x) = e^{\frac{1}{x}}\,;$
17. $f(x) = \dfrac{1}{e^{2x}}\,;$

página 3 Regra 12: $(\ln f)' = \dfrac{f'}{f}$

2. $f(x) = \ln(2x)\,;$
5. $f(x) = \ln(x^2+2)\,;$
12. $f(x) = \ln(x+e^{-x})\,;$
18. $f(x) = \dfrac{\ln^3(x^2+1)}{5}\,;$

página 4 Regra 13: $(\sin f)' = f'\cos f$ Regra 14: $\big(\cos f\big)' = -f'\sin f$

4. $f(x) = \cos(2x^2)\,;$
6. $f(x) = \sin^2 x\,;$
9. $f(x) = \cos(\ln x)\,;$
13. $f(x) = \ln\big(\cos(2x)\big)\,;$

página 4 Regra 15: $(\tan f)' = f'\sec^2 f$ Regra 16: $(\cot f)' = -f'\csc^2 f$

2. $f(x) = \tan(x^3)\,;$
8. $f(x) = 2\tan(e^x)\,;$
10. $f(x) = \tan\big(\ln(x^2+1)\big)\,;$
12. $f(x) = \tan^3\big(\sin(3x)\big)\,;$

página 5 Regra 17: $(\sec f)' = f'\sec f\tan f$ Regra 18: $(\csc f)' = -f'\csc f\cot f$

1. $f(x) =$cosec$(2x^2)\,;$
7. $f(x) = \ln\big(\sec(2x)\big)\,;$

página 5 Regra 19: $(\arcsin f)' = \dfrac{f'}{\sqrt{1-f^2}}$ Regra 20: $(\arccos f)' = -\dfrac{f'}{\sqrt{1-f^2}}$

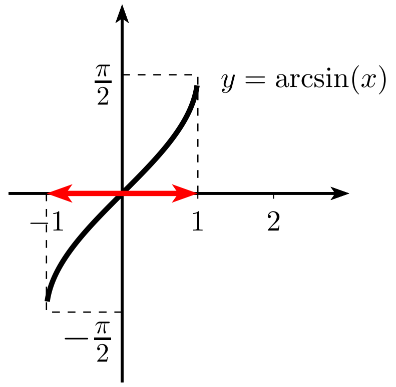
2. $f(x) = \arcsin(\tan x)\,;$
5. $f(x) = \arcsin^3(\ln x)\,;$

página 5 Regra 21: $(\arctan f)' = \dfrac{f'}{1+f^2}$

4. $f(x) = \arctan(\ln x)\,;$
5. $f(x) = \arctan(e^x)\,;$

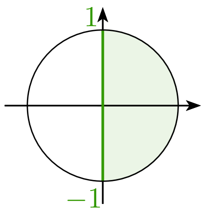
Funções trigonométricas inversas [**D. Análise**]

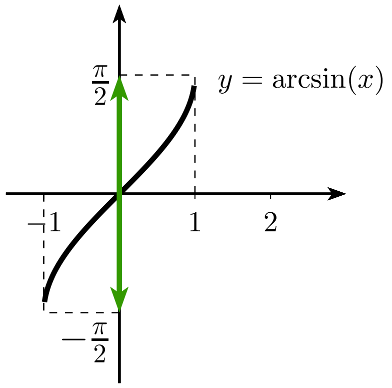
2. a) O domínio de f é definido pelo domínio arco seno (**ver página 10 da Tabelas de Matemática**), pelo que



$$\begin{array}{lcccccc} & -1 & \leq & 2x+1 & \leq & 1 \\ \Leftrightarrow & -2 & \leq & 2x & \leq & 0 \\ \Leftrightarrow & -1 & \leq & x & \leq & 0\,. \end{array}$$

O contradomínio também pode ser determinado tendo por base o contradomínio do arco seno (a restrição principal da função seno - **ver página 10 da Tabelas de Matemática**).





Assim,

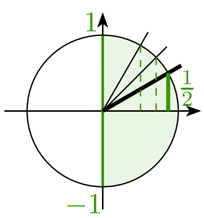
$$\begin{array}{lcccccc} & -\dfrac{\pi}{2} & \leq & \arcsin(2x+1) & \leq & \dfrac{\pi}{2} \\ \Leftrightarrow & -\pi & \leq & 2\arcsin(2x+1) & \leq & \pi \\ \Leftrightarrow & 0 & \leq & \pi + 2\arcsin(2x+1) & \leq & 2\pi\,. \end{array}$$

A função f é então definida por:

$$\begin{array}{ccc} [-1,\, 0] & \xrightarrow{\quad f \quad} & [0,\, 2\pi] \\ x & \longrightarrow & y = \pi + 2\arcsin(2x+1) \end{array}$$

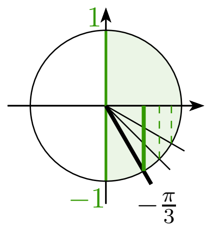
b) Tem-se

$$\begin{aligned} f\Big(-\frac{1}{4}\Big) &= \pi + 2\arcsin\Big(2\,\big(-\frac{1}{4}\big)+1\Big) = \pi + 2\arcsin\Big(-\frac{1}{2}+1\Big) = \pi + 2\arcsin\Big(\frac{1}{2}\Big) \\ &= \pi + 2\,\frac{\pi}{6} = \pi + \frac{\pi}{3} = \frac{4\pi}{3}\,. \end{aligned}$$



c) Tem-se

$$\begin{aligned} f(x) = \frac{\pi}{3} \quad &\Leftrightarrow \quad \pi + 2\arcsin(2x+1) = \frac{\pi}{3} \\ &\Leftrightarrow \quad 2\arcsin(2x+1) = \frac{\pi}{3} - \pi \\ &\Leftrightarrow \quad 2\arcsin(2x+1) = -\frac{2\pi}{3} \\ &\Leftrightarrow \quad \arcsin(2x+1) = -\frac{\pi}{3} \end{aligned}$$



$$\begin{aligned} &\Leftrightarrow \quad 2x+1 = -\frac{\sqrt{3}}{2} \\ &\Leftrightarrow \quad 2x = -\frac{\sqrt{3}}{2} - 1 \\ &\Leftrightarrow \quad x = -\frac{\sqrt{3}}{4} - \frac{1}{2}\,. \end{aligned}$$

d) De acordo da alínea (a) tem-se já

$$\begin{array}{ccc} [-1,\, 0] & \underset{f^{-1}}{\overset{f}{\longleftrightarrow}} & [0,\, 2\pi] \\ ? = {\color{red}x} & \longleftrightarrow & y = \pi + 2\arcsin(2x+1) \end{array}$$

A função é injectiva (e sobrejectiva) pelo que é invertível. O domínio da inversa coincide com o contradomínio de f e o contradomínio da inversa coincide com o domínio de f, pelo que falta apenas determinar a expressão analítica da função inversa f^{-1}. Então,

$$\begin{aligned} y = \pi + 2\arcsin(2x+1) \quad &\Leftrightarrow \quad y - \pi = 2\arcsin(2x+1) \\ &\Leftrightarrow \quad \frac{y-\pi}{2} = \arcsin(2x+1) \\ &\Leftrightarrow \quad \sin\left(\frac{y-\pi}{2}\right) = 2x+1 \end{aligned}$$

Nota: esta equivalência só é válida na restrição principal do seno!

$$\begin{aligned} &\Leftrightarrow \quad \sin\left(\frac{y-\pi}{2}\right) - 1 = 2x \\ &\Leftrightarrow \quad \frac{1}{2}\sin\left(\frac{y-\pi}{2}\right) - \frac{1}{2} = x\,. \end{aligned}$$

Então

$$\begin{array}{ccc} [-1,\, 0] & \underset{f^{-1}}{\overset{f}{\longleftrightarrow}} & [0,\, 2\pi] \\ \frac{1}{2}\sin\left(\frac{y-\pi}{2}\right) - \frac{1}{2} = {\color{red}x} & \longleftrightarrow & y = \pi + 2\arcsin(2x+1) \end{array}$$

Regras de derivação [A. Conhecimento]

página 1 Regras 7 e 3

2. $f'(x) = \underbrace{(x^9)'}_{R7} = 9\,x^8 \underbrace{x'}_{R2} = 9\,x^8 \cdot 1 = 9\,x^8\,;$

6. $f'(x) = \underbrace{\left(\frac{1}{2}\,x^9\right)'}_{R3} = \frac{1}{2}\underbrace{\left(x^9\right)'}_{R7} = \frac{1}{2}\,9\,x^8 \underbrace{x'}_{R2} = \frac{9}{2}\,x^8 \cdot 1 = \frac{9}{2}\,x^8\,;$

9. $f'(x) = \underbrace{\left((4x)^9\right)'}_{R7} = 9\,(4x)^8 \underbrace{(4x)'}_{R3} = 9\,(4x)^8\,4 \underbrace{x'}_{R2} = 36\,(4x)^8 \cdot 1 = 36\,(4x)^8\,;$

17. $f'(x) = \left(\frac{\sqrt[4]{x^5}}{3}\right)' = \underbrace{\left(\frac{1}{3}\,(x^5)^{\frac{1}{4}}\right)'}_{R3} = \frac{1}{3}\underbrace{\left((x^{\frac{5}{4}}\right)'}_{R7} = \frac{1}{3}\,\frac{5}{4}\,x^{\frac{1}{4}} \underbrace{x'}_{R2} = \frac{5}{12}\,\sqrt[4]{x} \cdot 1 = \frac{5}{12}\,\sqrt[4]{x}\,;$

24. $f'(x) = \left(\frac{1}{(3x)^2}\right)' = \underbrace{\left((3x)^{-2}\right)'}_{R7} = -2\,(3x)^{-3} \underbrace{(3x)'}_{R3} = -2\,\frac{1}{(3x)^3} \cdot 3 \underbrace{x'}_{R2} = -6\,\frac{1}{(3x)^3} \cdot 1 = -\frac{6}{(3x)^3}\,;$

27. $f'(x) = \left(\frac{2}{\sqrt[3]{x^2}}\right)' = \left(\frac{2}{x^{\frac{2}{3}}}\right)' = \underbrace{\left(2\,x^{-\frac{2}{3}}\right)'}_{R3} = 2\underbrace{\left(x^{-\frac{2}{3}}\right)'}_{R7} = 2\left(-\frac{2}{3}\right)x^{-\frac{5}{3}} \underbrace{x'}_{R2} = -\frac{4}{3}\,\frac{1}{x^{\frac{5}{3}}} \cdot 1$

$= -\frac{4}{3}\,\frac{1}{\sqrt[3]{x^5}}\,;$

página 2 Regra 4

3. $f'(x) = \underbrace{\left(x^3 + \frac{3x}{5} + \frac{1}{x^2}\right)'}_{R4} = \underbrace{(x^3)'}_{R7} + \underbrace{\left(\frac{3}{5}\,x\right)'}_{R3} + \underbrace{(x^{-2})'}_{R7} = 3\,x^2 \underbrace{x'}_{R2} + \frac{3}{5} \underbrace{x'}_{R2} + (-2)\,x^{-3} \underbrace{x'}_{R2}$

$= 3\,x^2 + \frac{3}{5} - \frac{2}{x^3}\,;$

7. $f'(x) = \underbrace{\left(\sqrt{x^3} - \frac{2}{\sqrt[3]{x^2}}\right)'}_{R4} = \underbrace{\left(x^{\frac{3}{2}}\right)'}_{R7} - \underbrace{\left(2\,x^{-\frac{2}{3}}\right)'}_{R3} = \frac{1}{2}\,x^{\frac{1}{2}} \underbrace{x'}_{R2} - 2\underbrace{\left(x^{-\frac{2}{3}}\right)'}_{R7} = \frac{1}{2}\,\sqrt{x} - 2\left(-\frac{2}{3}\right)x^{-\frac{5}{3}} \underbrace{x'}_{R2}$

$= \frac{1}{2}\,\sqrt{x} + \frac{4}{3}\,\frac{1}{\sqrt[3]{x^5}}\,;$

12. $f'(x) = \left(\frac{(x^4-1)^2}{4}\right)' = \underbrace{\left(\frac{1}{4}\,(x^4-1)^2\right)'}_{R3} = \frac{1}{4}\underbrace{\left((x^4-1)^2\right)'}_{R7} = \frac{1}{4}\,2\,(x^4-1)^1 \underbrace{(x^4-1)'}_{R4}$

$= \frac{2}{4}\,(x^4-1)\left(\underbrace{(x^4)'}_{R7} - \underbrace{(1)'}_{R1}\right) = \frac{1}{2}\,(x^4-1)\,(4\,x^3 \underbrace{x'}_{R2} - 0) = 2\,x^3\,(x^4-1)\,;$

página 2 Regra 5

2. $f'(x) = \underbrace{\left((x^2-2)\left(3 - \frac{1}{x^2}\right)\right)'}_{R5} = \underbrace{((x^2-2)'}_{R4}\left(3 - \frac{1}{x^2}\right) + (x^2-2)\underbrace{\left(3 - x^{-2}\right)'}_{R4}$

$= \left(\underbrace{(x^2)'}_{R7} - \underbrace{2'}_{R1}\right)\left(3 - \frac{1}{x^2}\right) + (x^2-2)\left(\underbrace{3'}_{R1} - \underbrace{\left(x^{-2}\right)'}_{R7}\right)$

$= \left(2\,x \underbrace{x'}_{R2} - 0\right)\left(3 - \frac{1}{x^2}\right) + (x^2-2)\left(0 - (-2)\,x^{-3} \underbrace{x'}_{R2}\right) = 2\,x\left(3 - \frac{1}{x^2}\right) + (x^2-2)\,\frac{2}{x^3}\,;$

página 2 Regra 6

2. $$f'(x) = \underbrace{\left(\frac{3x-2x^2}{x-1}\right)'}_{R6} = \frac{\overbrace{(3x-2x^2)'}^{R4}(x-1)-(3x-2x^2)\overbrace{(x-1)'}^{R4}}{(x-1)^2}$$

$$= \frac{\big(\overbrace{(3x)'}^{R3}-\overbrace{(2x^2)'}^{R3}\big)(x-1)-(3x-2x^2)\big(\underbrace{\overbrace{x'}}_{R2}-\overbrace{1'}^{R1}\big)}{(x-1)^2} = \frac{\big(3\overbrace{x'}^{R2}-2\overbrace{(x^2)'}^{R7}\big)(x-1)-(3x-2x^2)(1-0)}{(x-1)^2}$$

$$= \frac{\big(3-2\cdot 2\,x\overbrace{x'}^{R2}\big)(x-1)-(3x-2x^2)}{(x-1)^2} = \frac{(3-4x)(x-1)-(3x-2x^2)}{(x-1)^2};$$

página 3 Regra 9

4. $$f'(x) = \underbrace{(5e^{2x}+3)'}_{R4} = \underbrace{(5e^{2x})'}_{R3}+\underbrace{3'}_{R1} = 5\underbrace{(e^{2x})'}_{R9}+0 = 5\,e^{2x}\underbrace{(2x)'}_{R3} = 5\,e^{2x}\,2\underbrace{x'}_{R2} = 10\,e^{2x};$$

8. $$f'(x) = \underbrace{(e^{\sqrt{x}})'}_{R9} = e^{\sqrt{x}}(\sqrt{x})' = e^{\sqrt{x}}\underbrace{(x^{\frac{1}{2}})'}_{R7} = e^{\sqrt{x}}\frac{1}{2}\,x^{-\frac{1}{2}}\underbrace{x'}_{R2} = e^{\sqrt{x}}\frac{1}{2}\,\frac{1}{x^{\frac{1}{2}}} = \frac{e^{\sqrt{x}}}{2\,x^{\frac{1}{2}}};$$

16. $$f'(x) = \underbrace{(e^{\frac{1}{x}})'}_{R9} = e^{\frac{1}{x}}\left(\frac{1}{x}\right)' = e^{\frac{1}{x}}\underbrace{(x^{-1})'}_{R7} = e^{\frac{1}{x}}(-1)\,x^{-2}\underbrace{x'}_{R2} = -e^{\frac{1}{x}}\,\frac{1}{x^2};$$

17. $$f'(x) = \left(\frac{1}{e^{2x}}\right)' = \underbrace{(e^{-2x})'}_{R9} = e^{-2x}\underbrace{(-2x)'}_{R3} = e^{-2x}(-2)\underbrace{x'}_{R2} = -2\,e^{-2x};$$

página 3 Regra 12

2. $$f'(x) = \underbrace{\big(\ln(2x)\big)'}_{R12} = \frac{\overbrace{(2x)'}^{R3}}{2x} = \frac{2\overbrace{(x)'}^{R2}}{2x} = \frac{1}{x};$$

5. $$f'(x) = \underbrace{\big(\ln(x^2+2)\big)'}_{R12} = \frac{\overbrace{(x^2+2)'}^{R4}}{x^2+2} = \frac{\overbrace{(x^2)'}^{R7}+\overbrace{2'}^{R1}}{x^2+2} = \frac{2\,x\overbrace{x'}^{R2}+0}{x^2+2} = \frac{2\,x}{x^2+2};$$

12. $$f'(x) = \underbrace{\big(\ln(x+e^{-x})\big)'}_{R12} = \frac{\overbrace{(x+e^{-x})'}^{R4}}{x+e^{-x}} = \frac{\overbrace{(x)'}^{R2}+\overbrace{(e^{-x})'}^{R7}}{x+e^{-x}} = \frac{1+e^{-x}\overbrace{(-x)'}^{R3}}{x+e^{-x}} = \frac{1+e^{-x}(-1)\overbrace{(x)'}^{R2}}{x+e^{-x}}$$

$$= \frac{1-e^{-x}}{x+e^{-x}};$$

18. $$f'(x) = \left(\frac{\ln^3(x^2+1)}{5}\right)' = \underbrace{\left(\frac{1}{5}\ln^3(x^2+1)\right)'}_{R3} = \frac{1}{5}\underbrace{\left(\ln^3(x^2+1)\right)'}_{R7} = \frac{1}{5}\,3\,\ln^2(x^2+1)\underbrace{\left(\ln(x^2+1)\right)'}_{R12}$$

$$= \frac{3}{5}\,\ln^2(x^2+1)\,\frac{\overbrace{(x^2+1)'}^{R4}}{x^2+1} = \frac{3}{5}\,\ln^2(x^2+1)\,\frac{\overbrace{(x^2)'}^{R7}+\overbrace{1'}^{R1}}{x^2+1} = \frac{3}{5}\,\ln^2(x^2+1)\,\frac{2\,x\overbrace{x'}^{R2}+0}{x^2+1}$$

$$= \frac{3}{5}\,\ln^2(x^2+1)\,\frac{2\,x}{x^2+1};$$

página 4 Regras 13 e 14

4. $f'(x) = \underbrace{\Big(\cos(2x^2)\Big)'}_{R14} = -\underbrace{(2x^2)'}_{R13}\sin(2x^2) = -2\underbrace{(x^2)'}_{R7}\sin(2x^2) = -2{\cdot}2\,x\underbrace{(x)'}_{R1}\sin(2x^2) = -4\,x\sin(2x^2)\,;$

6. $f'(x) = \underbrace{\Big(\sin^2 x\Big)'}_{R7} = 2\,\sin x\,\underbrace{(\sin x)'}_{R13} = 2\,\sin x\,\underbrace{(x)'}_{R2}\cos x = 2\,\sin x\,\cos x\,;$

9. $f'(x) = \underbrace{\Big(\cos(\ln x)\Big)'}_{R14} = -\underbrace{(\ln x)'}_{R12}\sin(\ln x) = -\dfrac{\overbrace{(x)'}^{R2}}{x}\sin(\ln x) = -\dfrac{1}{x}\sin(\ln x)\,;$

13. $f'(x) = \underbrace{\Big(\ln\big(\cos(2x)\big)\Big)'}_{R12} = \dfrac{\overbrace{\big(\cos(2x)\big)'}^{R14}}{\cos(2x)} = \dfrac{-\overbrace{(2x)'}^{R3}\sin(2x)}{\cos(2x)} = \dfrac{-2\overbrace{(x)'}^{R2}\sin(2x)}{\cos(2x)} = -2\tan(2x)\,;$

página 4 Regras 15 e 16:

2. $f'(x) = \underbrace{\Big(\tan(x^3)\Big)'}_{R15} = \underbrace{(x^3)'}_{R7}\sec^2(x^3) = 3x^2\underbrace{(x)'}_{R2}\sec^2(x^3) = 3x^2\sec^2(x^3)\,;$

8. $f'(x) = \underbrace{\Big(2\tan(e^x)\Big)'}_{R3} = 2\underbrace{\Big(\tan(e^x)\Big)'}_{R15} = 2\underbrace{(e^x)'}_{R9}\sec^2(e^x) = 2\,e^x\underbrace{(x)'}_{R2}\sec^2(e^x) = 2\,e^x\sec^2(e^x)\,;$

10. $f'(x) = \underbrace{\Big(\tan\big(\ln(x^2+1)\big)\Big)'}_{R15} = \underbrace{(x^2+1)'}_{R4}\sec^2(x^2+1) = \big(\underbrace{(x^2)'}_{R7} + \underbrace{(1)'}_{R1}\big)\sec^2(x^2+1)$

$= \big(2x\underbrace{(x)'}_{R2}+0\big)\sec^2(x^2+1) = 2x\sec^2(x^2+1)\,;$

12. $f'(x) = \underbrace{\Big(\tan^3\big(\sin(3x)\big)\Big)'}_{R7} = 3\tan^2\big(\sin(3x)\big)\underbrace{\Big(\tan\big(\sin(3x)\big)\Big)'}_{R15}$

$= 3\tan^2\big(\sin(3x)\big)\underbrace{\big(\sin(3x)\big)'}_{R13}\sec^2\big(\sin(3x)\big) = 3\tan^2\big(\sin(3x)\big)\underbrace{(3x)'}_{R3}\cos(3x)\sec^2\big(\sin(3x)\big)$

$= 3\tan^2\big(\sin(3x)\big)2\underbrace{(x)'}_{R2}\cos(3x)\sec^2\big(\sin(3x)\big) = 6\tan^2\big(\sin(3x)\big)\cos(3x)\sec^2\big(\sin(3x)\big)\,;$

página 5 Regras 17 e 18

1. $f'(x) = \underbrace{\Big(\csc(2x^2)\Big)'}_{R17} = -\underbrace{(2x^2)'}_{R3}\csc(2x^2)\cot(2x^2) = -2\underbrace{(x^2)'}_{R7}\csc(2x^2)\cot(2x^2)$

$= -2\cdot 2x\underbrace{(x)'}_{R2}\csc(2x^2)\cot(2x^2) = -4x\csc(2x^2)\cot(2x^2)\,;$

7. $f'(x) = \underbrace{\Big(\ln\big(\sec(2x)\big)\Big)'}_{R12} = \dfrac{\overbrace{\big(\sec(2x)\big)'}^{R17}}{\sec(2x)} = \dfrac{\overbrace{(2x)'}^{R3}\sec(2x)\tan(2x)}{\sec(2x)} = 2\overbrace{(x)'}^{R2}\tan(2x) = 2\tan(2x)\,;$

página 5 Regras 19 e 20

2. $f'(x) = \underbrace{\Big(\arcsin(\tan x)\Big)'}_{R12} = \dfrac{\overbrace{(\tan x)'}^{R15}}{\sqrt{1-(\tan x)^2}} = \dfrac{\overbrace{(x)'}^{R2}\sec^2 x}{\sqrt{1-\tan^2 x}} = \dfrac{\sec^2 x}{\sqrt{1-\tan^2 x}}$;

5. $f'(x) = \underbrace{\Big(\arcsin^3(\ln x)\Big)'}_{R7} = 3\arcsin^2(\ln x)\underbrace{\Big(\arcsin(\ln x)\Big)'}_{R19} = 3\arcsin^2(\ln x)\dfrac{\overbrace{(\ln x)'}^{R12}}{\sqrt{1-(\ln x)^2}}$

$$= 3\arcsin^2(\ln x)\frac{\frac{\overbrace{(x)'}^{R2}}{x}}{\sqrt{1-\ln^2 x}} = 3\arcsin^2(\ln x)\frac{\frac{1}{x}}{\sqrt{1-\ln^2 x}};$$

página 5 Regra 21

4. $f'(x) = \underbrace{\Big(\arctan(\ln x)\Big)'}_{R12} = \dfrac{\overbrace{(\ln x)'}^{R12}}{1+(\ln x)^2} = \dfrac{\frac{\overbrace{(x)'}^{R2}}{x}}{1+\ln^2 x} = \dfrac{\frac{1}{x}}{1+\ln^2 x} = \dfrac{1}{x}\,\dfrac{1}{1+\ln^2 x}$;

5. $f'(x) = \underbrace{\Big(\arctan(e^x)\Big)'}_{R12} = \dfrac{\overbrace{(e^x)'}^{R12}}{1+(e^x)^2} = \dfrac{e^x\,\overbrace{(x)'}^{R2}}{1+e^{2x}} = \dfrac{e^x}{1+e^{2x}}$;