PRIMITIVAÇÃO IMEDIATA E POR DECOMPOSIÇÃO

A.Conhecimento

Reproduza a seguinte regra:

$$\int f' f^p = \frac{f^{p+1}}{p+1} + C, C \in \Re$$

1. $\int x^{10} dx$

Nota: $\int af = a \int f, \; a \in \Re$

2. $\int 3x^{10} dx$

3. $\int (2x)^{10} dx$

4. $\int (x+1)^2 dx$

5. $\int (2x+1)^3 dx$

6. $\int (1+x)^{-2} dx$

7. $\int 3(2x+1)^4 dx$

Nota: $\sqrt[q]{f^p} = f^{p/q}$

8. $\int \sqrt{(2x-5)} dx$

9. $\int \sqrt[3]{3x-2} dx$

10. $\int \sqrt[3]{(x-2)^2} dx$

Nota: $\dfrac{1}{f^p} = f^{-p}$

11. $\int \dfrac{1}{(x-3)^2} dx$

12. $\int \dfrac{1}{(2x-1)^{-2}} dx$

13. $\int \dfrac{1}{\sqrt{x-1}} dx$

14. $\int \dfrac{1}{\sqrt[3]{1-2x}} dx$

15. $\int \dfrac{\pi}{\sqrt[3]{(1-2x)^5}} dx$

Nota: $\left(f^p\right)' = p f' f^{p-1}$

16. $\int x(x^2-3)^4 dx$

17. $\int x^2 (x^3+1)^3 dx$

18. $\int 2x^3 (3-x^4)^5 dx$

19. $\int x \sqrt[5]{x^2-2} dx$

20. $\int (x^3) \sqrt[4]{x^4-2} dx$

21. $\int x \sqrt[5]{(x^2-2)^3} dx$

22. $\int x \sqrt[4]{(1-x^2)^{-3}} dx$

23. $\int \dfrac{2x^2}{(x^3-3)^2} dx$

24. $\int \dfrac{2x^3}{(3-x^4)^{-3}} dx$

25. $\int \dfrac{x}{\sqrt[3]{x^2-2}} dx$

26. $\int \dfrac{2x^2}{\sqrt[5]{2-x^3}} dx$

27. $\int \dfrac{2x}{\sqrt[5]{(2-x^2)^3}} dx$

Nota: $\left(e^f\right)' = f' e^f$

28. $\int e^x (1+e^x)^{-2} dx$

29. $\int \dfrac{e^x}{(2-e^x)^2} dx$

30. $\int e^{2x} \sqrt{9+e^{2x}} dx$

31. $\int \dfrac{e^{-x}}{\sqrt{1-2e^{-x}}} dx$

32. $\int \dfrac{e^{x+2}}{\sqrt{1-e^x}}dx$

33. $\int \dfrac{e^{x-1}}{\sqrt{\left(1-e^x\right)^3}}dx$

34. $\int \dfrac{e^{\sqrt{x}}(1+e^{\sqrt{x}})^{-2}}{\sqrt{x}}dx$

Nota: $\left(ln(f)\right)' = \dfrac{f'}{f}$

35. $\int \dfrac{1}{x}(ln(x))^2\,dx$

36. $\int \dfrac{(ln(x+1))^3}{x+1}dx$

37. $\int \dfrac{1}{xln^4(x)}dx$

38. $\int \dfrac{1}{xln^4(x^2)}dx$

39. $\int \dfrac{1}{xln^{-2}(\sqrt{x})}dx$

40. $\int \dfrac{x}{ln^4(2e^{x^2})}dx$

41. $\int \dfrac{1}{x\sqrt{ln(x^2)+1}}dx$

42. $\int \dfrac{1}{x\sqrt{\left(ln(x)+1\right)^3}}dx$

Nota: $\left(sen(f)\right)' = f'cos(f)$

$\quad\left(cos(f)\right)' = -f'sen(f)$

43. $\int cos(x)sen^2(x)dx$

44. $\int cos(2x)sen(2x)dx$

45. $\int xcos(x^2)sen^4(x^2)dx$

46. $\int \dfrac{cos(x)}{sen^2(x)}dx$

47. $\int \dfrac{cos(3x)}{sen^4(3x)}dx$

48. $\int \dfrac{sen(x)}{(cos(x)-2)^2}dx$

49. $\int cos(\dfrac{x}{2})\sqrt[3]{sen(\dfrac{x}{2})}dx$

50. $\int sen(\dfrac{x}{3})\sqrt[3]{cos(\dfrac{x}{3})}dx$

51. $\int sen(3x)\sqrt[5]{2+cos(3x)}dx$

52. $\int xcos(x^2)\sqrt[5]{\left(sen(x^2)\right)^3}dx$

53. $\int xsen(x^2)\left(\sqrt{cos(x^2)+\pi}\right)^3 dx$

54. $\int \dfrac{cos(3x)}{\sqrt{sen(3x)}}dx$

55. $\int \dfrac{3cos(x+1)}{\sqrt{sen(x+1)}}dx$

56. $\int \dfrac{xsen(x^2+1)}{\sqrt[3]{\left(cos(x^2+1)\right)^2}}dx$

57. $\int \dfrac{cos(\sqrt{x})sen^3(\sqrt{x})}{\sqrt{x}}dx$

58. $\int e^x cos(e^x)sen^3(e^x)dx$

59. $\int \dfrac{e^{2x}cos(e^{2x})}{sen^3(e^{2x})}dx$

60. $\int \dfrac{e^x sen(e^x)}{\left(cos(e^x)+3\right)^2}dx$

61. $\int \dfrac{1}{x}cos(lnx)sen^3(lnx)dx$

62. $\int \dfrac{cos(lnx)}{xsen^3(lnx)}dx$

63. $\int \dfrac{cos(lnx)}{x\sqrt{sen(lnx)}}dx$

64. $\int \dfrac{cos(lnx)}{x\sqrt{sen^3(lnx)}}dx$

Nota: $sec(f) = \dfrac{1}{cos(f)}$

$\quad cosec(f) = \dfrac{1}{sen(f)}$

65. $\int cos(x)cosec^2(x)dx$

66. $\int \dfrac{sen(2x)}{sec^3(2x)}dx$

67. $\int \dfrac{sen(2x)}{\sqrt{sec^3(2x)}}dx$

68. $\int \dfrac{cos(3x)}{\sqrt[3]{cosec^2(3x)}}dx$

69. $\int 3xcos(x^2)\sqrt[3]{cosec^2(x^2)}dx$

70. $\int e^{x+1}sen(e^x)\sqrt{sec^3(e^x)}dx$

71. $\int \dfrac{3cos(lnx)}{xcosec^2(lnx)}dx$

72. $\int \dfrac{sen(\sqrt{x})}{\sqrt{xsec^3(\sqrt{x})}}dx$

Nota: $\left(tg(f)\right)' = f'sec^2(f)$

$\left(cotg(f)\right)' = -f'cosec^2(f)$

73. $\int cosec^2(\dfrac{x}{2})cotg^4(\dfrac{x}{2})dx$

74. $\int sec^2(3x)tg^3(3x)dx$

75. $\int sec^2(2x)(tg(2x)+1)^2dx$

76. $\int \dfrac{sec^2(x)}{tg^3(x)}dx$

77. $\int \dfrac{tg^3(2x)}{cos^2(2x)}dx$

78. $\int \dfrac{\pi}{cos^2(2x)tg^3(2x)}dx$

79. $\int \dfrac{e^{x+2}}{sen^2(e^x)cotg^3(e^x)}dx$

80. $\int \dfrac{sec^2(ln(x))}{xcotg^4(ln(x))}dx$

81. $\int \dfrac{cosec^2(\sqrt{x})}{\sqrt{x}\,tg^5(\sqrt{x})}dx$

Nota: $\left(arctg(f)\right)' = \dfrac{f'}{1+f^2}$

$\left(arcsen(f)\right)' = \dfrac{f'}{\sqrt{1-f^2}}$

82. $\int \dfrac{2}{1+x^2}(arctg(x))^3dx$

83. $\int \dfrac{(arcsen(2x))^2}{\sqrt{1-4x^2}}dx$

84. $\int \dfrac{2e^x}{1+e^{2x}}arctg(e^x)dx$

85. $\int \dfrac{e^x\sqrt[3]{arcsen(e^x)}}{\sqrt{1-e^{2x}}}dx$

86. $\int \dfrac{x}{(1+x^4)\sqrt{arctg(x^2)}}dx$

87. $\int \dfrac{x}{\sqrt{1-x^4}}(arcsen(x^2))^3dx$

88. $\int \dfrac{arcsen(ln(x))}{x\sqrt{1-ln^2(x)}}dx$

89. $\int \dfrac{4}{\sqrt{arcsen(x)}(1-x^2)}dx$

90. $\int \dfrac{\sqrt{x}}{(x+x^2)\sqrt[3]{arctg(\sqrt{x})+1}}dx$

91. $\int \dfrac{3}{x(1+ln^2(x))\sqrt[3]{arctg(ln(x))+\pi}}dx$

Nota: *exercícios variados*

92. $\int \dfrac{sen(2x)}{(sen^2(x)-2)^2}dx$

93. $\int \dfrac{e^x sen(e^x)}{\sqrt[3]{(cos(e^x)+3)^2}}dx$

94. $\int cosxe^{senx}(1+e^{senx})^2dx$

95. $\int \dfrac{cosxe^{2senx}}{\sqrt[3]{1-e^{2senx}}}dx$

96. $\int \dfrac{tg(x)}{ln^2(cosx)}dx$

97. $\int 2cotg(x)(ln(sen(x))+1)^{-3}dx$

98. $\int \dfrac{2cotg(x)}{\sqrt[3]{1-ln(sen(x))}}dx$

99. $\int e^{x+1}cosec^2(e^x)cotg(e^x)dx$

Reproduza as seguintes regras:

$$\int f' e^f = e^f + C, C \in \Re$$

$$\int f' a^f = \frac{a^f}{lna} + C, C \in \Re$$

1. $\int e^{2x-3} dx$

2. $\int x e^{x^2} dx$

3. $\int x^2 4^{x^3+2} dx$

4. $\int \frac{x^2}{e^{x^3}} dx$

5. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

6. $\int \frac{10^{\frac{1}{x}}}{x^2} dx$

7. $\int sen(x) e^{2cos(x)} dx$

8. $\int \frac{sen(x)}{e^{cos(x)}} dx$

9. $\int sec^2(2x) e^{tg(2x)} dx$

10. $\int \frac{e^{cotg(2x)}}{sen^2(2x)} dx$

11. $\int \frac{2^{arcsen(x)}}{\sqrt{1-x^2}} dx$

12. $\int \frac{e^{arctg(x)}}{1+x^2} dx$

13. $\int \frac{e^{arcotg(2x)}}{1+4x^2} dx$

14. $\int \frac{e^{arctg(ln(x))}}{x(1+(lnx)^2)} dx$

$$\int \frac{f'}{f} = ln|f| + C, C \in \Re$$

1. $\int \frac{2}{1-x} dx$

2. $\int \frac{2x^3+x}{x^4+x^2} dx$

3. $\int \frac{e^x}{e^x+1} dx$

4. $\int \frac{e^{x+1}}{e^x+1} dx$

5. $\int \frac{3}{xln(x)} dx$

6. $\int \frac{3}{xln(\sqrt{x})} dx$

7. $\int \frac{cos(3x)}{sen(3x)} dx$

8. $\int \frac{sen(x)}{cos(x)+\pi} dx$

9. $\int \frac{sen(2x)}{sen^2(x)+4} dx$

10. $\int \frac{e^x cos(e^x)}{sen(e^x)} dx$

11. $\int \frac{sec^2(2x)}{tg(2x)} dx$

12. $\int \frac{\pi}{sen^2(x)cotg(x)} dx$

13. $\int cosec^2(x) tg(x) dx$

14. $\int \frac{cotg(2x)}{cos^2(2x)} dx$

15. $\int \frac{tg(\sqrt{x})}{\sqrt{x}\, sen^2(\sqrt{x})} dx$

16. $\int \dfrac{3}{\sqrt{1-x^2}\,arcsen(x)}\,dx$

17. $\int \dfrac{\pi cos(x)}{(\sqrt{1-sen^2(x)})arccos(sen(x))}\,dx$

18. $\int \dfrac{2x}{(1+x^4)arctg(x^2)}\,dx$

19. $\int \dfrac{2e^x}{(1+e^{2x})arctg(e^x)}\,dx$

20. $\int \dfrac{3}{x(1+ln^2(x))arctg(ln(x))}\,dx$

$$\boxed{\begin{array}{c} \int f'senf = -cosf + C, C \in \Re \\ \int f'cosf = senf + C, C \in \Re \end{array}}$$

1. $\int sen(2x)dx$

2. $\int cos(3x+1)dx$

3. $\int xsen(x^2)dx$

4. $\int \dfrac{sen(\sqrt{x})}{\sqrt{x}}\,dx$

5. $\int \dfrac{xsen(\sqrt{x^2+1})}{\sqrt{x^2+1}}\,dx$

6. $\int e^x cos(e^x)dx$

7. $\int \dfrac{sen(ln(x))}{2x}\,dx$

8. $\int \dfrac{sen(ln(x^2))}{x}\,dx$

9. $\int \sqrt{e^x}\,sen(\sqrt{e^x})dx$

10. $\int sen(x)sen(cos(x))dx$

11. $\int sen(2x)cos(cos^2(x))dx$

12. $\int \dfrac{sen(x)}{\sqrt{cos(x)}}\,sen(\sqrt{cos(x)})dx$

13. $\int tg(2x)sen(ln(cos(2x))dx$

14. $\int \dfrac{4}{\sqrt{1-4x^2}}\,sen(arccos(2x))dx$

15. $\int \dfrac{e^{x+1}}{\sqrt{1-e^{2x}}}\,cos(arcsen(e^x))dx$

16. $\int \dfrac{cos(x)}{2-cos^2(x)}\,cos(arctg(sen(x))dx$

$$\boxed{\begin{array}{c} \int f'sec^2 f = tgf + C, C \in \Re \\ \int f'cosec^2 f = -cotgf + C, C \in \Re \end{array}}$$

1. $\int sec^2(x+1)dx$

2. $\int x^2 cosec^2(x^3)dx$

3. $\int \dfrac{cosec^2(\sqrt[3]{x^2})}{\sqrt[3]{x}}\,dx$

4. $\int \dfrac{cosec^2(e^{x+2})}{e^{-x}}\,dx$

5. $\int \dfrac{sec^2(ln(x))}{x}\,dx$

6. $\int \dfrac{cosec^2(sen(3x))}{sec(3x)}\,dx$

7. $\int \dfrac{e^x cosec^2(cos(e^x))}{cosec(e^x)}\,dx$

8. $\int \dfrac{1}{4+x^2}\,sec^2(arctg(\dfrac{x}{2}))dx$

$$\boxed{\begin{array}{c} \int f'cosecf = ln|cosecf - cotgf| + C, C \in \Re \\ \int f'secf = ln|secf + tgf| + C, C \in \Re \end{array}}$$

1. $\int cosec(3x+2)dx$

2. $\int 3xcosec(x^2+1)dx$

3. $\int \dfrac{sec(\sqrt{x+2})}{3\sqrt{x+2}}\,dx$

4. $\int e^x sec(e^{x+1})dx$

5. $\int \dfrac{cosec(ln(\sqrt{x}))}{x}\,dx$

6. $\int cos(2x)sec(sen(2x))dx$

7. $\int e^{2x} cosec(e^{2x+1}))dx$

8. $\int cotg(2x)cosec(ln(sen(2x)))dx$

$$\int \frac{f'}{\sqrt{1-f^2}} = arcsen(f)+C, C \in \Re$$

$$\int \frac{f'}{1+f^2} = artg(f)+C, C \in \Re$$

1. $\int \dfrac{1}{\sqrt{1-(2x)^2}} dx$

2. $\int \dfrac{1}{\sqrt{1-4x^2}} dx$

3. $\int \dfrac{1}{\sqrt{4-4x^2}} dx$

4. $\int \dfrac{x^2}{\sqrt{9-x^6}} dx$

5. $\int \dfrac{x^3}{\sqrt{4-9x^8}} dx$

6. $\int \dfrac{e^x}{\sqrt{1-9e^{2x}}} dx$

7. $\int \dfrac{sen(x)}{\sqrt{1-cos^2(x)}} dx$

8. $\int \dfrac{3}{sec(2x)\sqrt{1-sen^2(2x)}} dx$

9. $\int \dfrac{4}{x\sqrt{1-ln^2(x)}} dx$

10. $\int \dfrac{4cotg(x)}{\sqrt{1-ln^2(sen(x))}} dx$

11. $\int \dfrac{4sec^2(2x)}{\sqrt{1-4tg^2(2x)}} dx$

12. $\int \dfrac{4}{\sqrt{x}\sqrt{1-4x}} dx$

13. $\int \dfrac{1}{1+4x^2} dx$

14. $\int \dfrac{1}{4+4x^2} dx$

15. $\int \dfrac{x}{9+4x^4} dx$

16. $\int \dfrac{x^2}{4+9x^6} dx$

17. $\int \dfrac{e^x}{1+4e^{2x}} dx$

18. $\int \dfrac{sen(x)}{1+cos^2(x)} dx$

19. $\int \dfrac{4}{x(1+ln^2(x))} dx$

20. $\int \dfrac{4cotg(2x)}{(1+ln^2(sen(2x)))} dx$

21. $\int \dfrac{4}{x(1+ln^2(x^2))} dx$

22. $\int \dfrac{xcos(x^2)}{1+sen^2(x^2)} dx$

23. $\int \dfrac{sen(ln\,x)}{x(1+cos^2(ln\,x))} dx$

24. $\int \dfrac{4}{sec(x)(1+sen^2(x))} dx$

25. $\int \dfrac{3tg(2x)}{1+ln^2(cos(2x))} dx$

26. $\int \dfrac{6}{\sqrt{x}(1+x)} dx$

27. $\int \dfrac{e^x cosec^2(e^x)}{1+9tg^2(e^x)} dx$

B.Compreensão

1-Faça as correspondências entre a função e a respetiva primitiva:

função($f(x)$)	**Primitiva**($F(x)$)

BLOCO I	
a) $f(x) = 2x$	1. $F(x) = 3x^3$
b) $f(x) = 9x^2$	2. $F(x) = \dfrac{(x^4+5)^3}{6}$
c) $f(x) = 2(x+1)$	3. $F(x) = (x^2+1)^2$
d) $f(x) = 2x^3(x^4+5)^2$	4. $F(x) = x^2$
e) $f(x) = 4x(x^2+1)$	5. $F(x) = (x+1)^2$
BLOCO II	
f) $f(x) = \dfrac{1}{2\sqrt{x+1}}$	6. $F(x) = \sqrt{2ln(x^2)}$
g) $f(x) = \dfrac{1}{\sqrt[3]{(x+1)^2}}$	7. $F(x) = 3\sqrt[3]{x+1}$
h) $f(x) = \dfrac{x}{\sqrt{x^2+1}}$	8. $F(x) = \sqrt{x+1}$
i) $f(x) = \dfrac{1}{x\sqrt{ln(x)}}$	9. $F(x) = \sqrt{ln(x)}$
j) $f(x) = \dfrac{1}{2x\sqrt{ln(x)}}$	10. $F(x) = \sqrt{x^2+1}$
BLOCO III	
k) $f(x) = \dfrac{e^{\sqrt{x+1}}}{2\sqrt{x}}$	11. $F(x) = e^{sen(x)}$
l) $f(x) = 2xe^{x^2}$	12. $F(x) = \dfrac{e^{arccos(2x)}}{2}$
m) $f(x) = \dfrac{-e^{arccos2x}}{\sqrt{1-4x^2}}$	13. $F(x) = e^{\sqrt{x}}$
n) $f(x) = \dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$	14. $F(x) = e^{x^2}$
o) $f(x) = cos(x)e^{sen(x)}$	15. $F(x) = e^{\sqrt{x+1}}$

BLOCO IV	
p) $f(x) = 2tg(2x)$	16. $F(x) = ln(x^2 + 1)$
q) $f(x) = 2cotg(2x)$	17. $F(x) = ln(\sqrt{x})$
r) $f(x) = \dfrac{2x}{x^2 + 1}$	18. $F(x) = -ln(cos(2x))$
s) $f(x) = \dfrac{2}{x}$	19. $F(x) = ln(sen(2x))$
t) $f(x) = \dfrac{1}{2x}$	20. $F(x) = ln(x^2)$
BLOCO V	
u) $f(x) = sen(x)$	21. $F(x) = tg(2x)$
v) $f(x) = -2xcos(x^2)$	22. $F(x) = -tg(x^2)$
w) $f(x) = 2sec^2(2x)$	23. $F(x) = -cos(x)$
x) $f(x) = -2xsec^2(x^2)$	24. $F(x) = -sen(x^2)$
y) $f(x) = -3x^2 sen(x^3)$	25. $F(x) = cos(x^3)$
BLOCO VI	
z) $f(x) = \dfrac{-1}{9 + x^2}$	26. $F(x) = arctg(2x)$
aa) $f(x) = \dfrac{3}{\sqrt{1 - 9x^2}}$	27. $F(x) = arcsen(\dfrac{x}{2})$
bb) $f(x) = \dfrac{2}{1 + 4x^2}$	28. $F(x) = arcsen(2x)$
cc) $f(x) = \dfrac{1}{\sqrt{4 - x^2}}$	29. $F(x) = \dfrac{1}{3} arcotg(\dfrac{x}{3})$
dd) $f(x) = \dfrac{2}{\sqrt{1 - 4x^2}}$	30. $F(x) = -arccos(3x)$

2-Considere as primitivas abaixo identificadas e complete justificando, o espaço assinalado com $[\ ]$ por forma a que possam ser aplicadas as regras da primitivação imediata (Tabelas de Matemática, página 3) indicadas:

I. $\int [\ \ ] sin(x)dx$

a.R1   b.R2   c.R5   d.R7   e.R18   f.R19

II. $\int [\ \ ] e^{2x}dx$

a.R1   b.R2   c.R3   d.R5   e.R18   f.R19

III. $\int [\ \ ] sec(x)tg(x)dx$

a.R1   b.R2   c.R5   d.R10   e.R12

3-Identifique em cada uma das seguintes primitivas a(s) expressão(ões) em falta marcadas com $[\ ]$ por forma a que possam ser aplicadas as regras indicadas:

1. $\int \dfrac{[\ ]}{(1+e^{2x})^2}\,dx$ (Regra 2)

2. $\int [\ ]\sqrt[3]{ln^2(x)}\,dx$ (Regra 2)

3. $\int [\ ]\sqrt[3]{1+ln(2x)}\,dx$ (Regra 2)

4. $\int \dfrac{sen[\ ]}{[\ ]cos^3(\sqrt{x+1})}\,dx$ (Regra 2)

5. $\int [\ ]\sqrt{1+cos(3x)}\,dx$ (Regra 2)

6. $\int \dfrac{[\ ]}{sec^3(x)}\,dx$ (Regra 2)

7. $\int \dfrac{[\ ]}{cotg^5(\sqrt{x})}\,dx$ (Regra 2)

8. $\int \dfrac{sen[\ ]cos^3(ln(x))}{[\ ]}\,dx$ (Regra 2)

9. $\int [\ ]e^{\sqrt{x+1}}\,dx$ (Regra 3)

10. $\int [\ ]e^{2sen(x)}\,dx$ (Regra 3)

11. $\int [\ ]e^{cos(\sqrt{x})}\,dx$ (Regra 3)

12. $\int \dfrac{[\ ]}{sen(e^x+1)}\,dx$ (Regra 5)

13. $\int \dfrac{[\ ]}{cos(ln(x))+1}\,dx$ (Regra 5)

14. $\int \dfrac{[\ ]}{[\ ]arctg(x^2)}\,dx$ (Regra 5)

15. $\int [\ ]cotg(e^x)\,dx$ (Regra 5)

16. $\int [\ ]cos\sqrt{e^{2x}}\,dx$ (Regra 6)

17. $\int [\ ]cos\sqrt{x^2+1}\,dx$ (Regra 6)

18. $\int [\ ]sen(e^{2x})\,dx$ (Regra 7)

19. $\int [\ ]sen(ln(x))\,dx$ (Regra 7)

20. $\int \dfrac{[\ ]}{\sqrt{4-9x^2}}\,dx$ (Regra 18)

21. $\int \dfrac{[\ ]}{\sqrt{1-e^{4x}}}\,dx$ (Regra 18)

22. $\int \dfrac{[\ ]}{\sqrt{4-sen^2(2x)}}\,dx$ (Regra 18)

23. $\int \dfrac{[\ ]}{9+4x^2}\,dx$ (Regra 19)

24. $\int \dfrac{[\ ]}{1+e^{6x}}\,dx$ (Regra 19)

25. $\int \dfrac{[\ ]}{4+cos^2(3x)}\,dx$ (Regra 19)

4-Identifique em cada uma das seguintes primitivas o(s) erro(s) cometido(s) por forma a que possam ser aplicadas as regras indicadas:

1. $\int e^x cos(x)\sqrt[3]{sen(e^x)}\,dx$ (Regra 2)

2. $\int sen(\sqrt{x})\sqrt{cos(\sqrt{x})}\,dx$ (Regra 2)

3. $\int \dfrac{cos(x)}{ln^{-2}(sen(2x))}\,dx$ (Regra 2)

4. $\int \dfrac{cos(x)}{ln(sen(x))}\,dx$ (Regra 5)

5. $\int \dfrac{e^x}{1+e^{2x}}\,dx$ (Regra 5)

6. $\int \dfrac{3}{1+ln(x)}\,dx$ (Regra 5)

7. $\int \dfrac{sen(x)}{cos(x)}sen(cos(x))\,dx$ (Regra 7)

8. $\int \dfrac{x^7}{\sqrt{4-9x^8}}\,dx$ (Regra 18)

9. $\int \dfrac{e^{2x}}{\sqrt{9-e^{2x}}}\,dx$ (Regra 18)

10. $\int \dfrac{xe^x}{9+e^{2x}}\,dx$ (Regra 19)

11. $\int \dfrac{xcos(x)}{1+sen^2(x^2)}\,dx$ (Regra 19)

5-Faça as correspondências entre a função e a respetiva primitiva:

Função($f(x)$)	Primitiva($F(x)$)
a) $f(x) = \dfrac{-\,sen\sqrt{2x}}{\sqrt{2x}}$	1. $F(x) = arccos(e^x) + C$
b) $f(x) = \dfrac{sen(x)cos(x)}{\sqrt{sen^2(x)+1}}$	2. $F(x) = ln(x) + C$
c) $f(x) = \dfrac{1}{\sqrt{4-x^2}}$	3. $F(x) = arctg(\dfrac{x}{3}) + C$
d) $f(x) = \dfrac{1}{x}$	4. $F(x) = ln(x^2) + C$
e) $f(x) = 3x^2cos(x^3+1)$	5. $F(x) = cos(e^x) + C$
f) $f(x) = \dfrac{2}{x}$	6. $F(x) = e^{arctg(x)} + C$
g) $f(x) = \dfrac{-\,3e^{3x}}{2\sqrt{1-e^{3x}}}$	7. $F(x) = cos(\sqrt{2x}) + C$
h) $f(x) = \dfrac{3}{9+x^2}$	8. $F(x) = \sqrt{sen^2(x)+1} + C$
i) $f(x) = -e^x sen(e^x)$	9. $F(x) = arcsen(\dfrac{x}{2}) + C$
j) $f(x) = \dfrac{-1}{(x+1)^2}$	10. $F(x) = \sqrt{1-e^{3x}} + C$
k) $f(x) = cotg(x)$	11. $F(x) = \dfrac{1}{\sqrt{x}} + C$
l) $f(x) = \dfrac{cos(x)}{2-cos^2(x)}$	12. $F(x) = ln(tg(x)) + C$
m) $f(x) = \dfrac{e^{arctg(x)}}{1+x^2}$	13. $F(x) = \sqrt{x^2+1} + C$
n) $f(x) = \dfrac{2sen(x)}{(cos(x)+1)^2}$	14. $F(x) = ln(sen(x)) + C$
o) $f(x) = \dfrac{-\,e^x}{\sqrt{1-e^{2x}}}$	15. $F(x) = cotg(ln(x)) + C$
p) $f(x) = \dfrac{e^x}{e^x+2}$	16. $F(x) = \dfrac{2}{cos(x)+1} + C$
q) $f(x) = \dfrac{x}{\sqrt{x^2+1}} \backslash$	17. $F(x) = \dfrac{1}{x+1} + C$
r) $f(x) = \dfrac{2}{sen(2x)}$	18. $F(x) = ln(e^x+2) + C$
s) $f(x) = \dfrac{-1}{xsen^2(ln(x))}$	19. $F(x) = sen(x^3+1) + C$
t) $f(x) = \dfrac{-1}{2\sqrt{x^3}}$	20. $F(x) = arctg(sen(x)) + C$

6-Faça as correspondências entre a função e a respetiva primitiva:

$f(x)$	$F(x) = \int f(x)dx$
a) $\int e^{x+1}dx$	1. $F(x) = e^{sen(x)} + C$
b) $\int \sqrt{(x+2)}dx$	2. $F(x) = -\dfrac{1}{2}arctg(\dfrac{cos(x)}{2}) + C$
c) $\int xsen(x^2)dx$	3. $F(x) = arcsen(\dfrac{e^x}{2}) + C$
d) $\int \dfrac{x}{x^2+1}dx$	4. $F(x) = e^{x+1} + C$
e) $\int cos(x)e^{senx}dx$	5. $F(x) = -\dfrac{1}{2}cos(x^2) + C$
f) $\int \dfrac{e^{arctg(x)}}{1+x^2}dx$	6. $F(x) = \dfrac{2(x+2)^{3/2}}{3} + C$
g) $\int cos(x)sen^2(x)dx$	7. $F(x) = \dfrac{\sqrt[3]{(x^3+1)^2}}{2} + C$
h) $\int cotg(3x)dx$	8. $F(x) = -\dfrac{1}{2e^{sen(x)}} + C$
i) $\int cos(x)\sqrt[3]{2+sen(x)}dx$	9. $F(x) = \dfrac{1}{2}ln(x^2+1) + C$
j) $\int sec^2(x)(tg(x)+1)^2 dx$	10. $F(x) = e^{arctg(x)} + C$
k) $\int e^x\sqrt{4+e^x}dx$	11. $F(x) = -\dfrac{5}{2ln^2(x+1)} + C$
l) $\int \dfrac{cos(2x)}{sen^2(2x)}dx$	12. $F(x) = \dfrac{ln^2(sen(x))}{2} + C$
m) $\int \dfrac{cos(2x)}{e^{sen(2x)}}dx$	13. $F(x) = -\sqrt{1-2e^x} + C$
n) $\int \dfrac{x}{\sqrt{1-x^4}}dx$	14. $F(x) = \dfrac{3}{4}(2+sen(x))^{4/3} + C$
o) $\int \dfrac{e^x}{\sqrt{1-2e^x}}dx$	15. $F(x) = \dfrac{1}{3}ln|sen(3x)| + C$
p) $\int \dfrac{x^2}{\sqrt[3]{(x^3+1)}}dx$	16. $F(x) = \dfrac{sen^3(x)}{3} + C$
q) $\int \dfrac{sen(x)}{cos^2(x)+4}dx$	17. $F(x) = \dfrac{(tg(x)+1)^3}{3} + C$
r) $\int \dfrac{e^x}{\sqrt{4-e^{2x}}}dx$	18. $F(x) = \dfrac{2(4+e^x)^{3/2}}{3} + C$
s) $\int cotg(x)ln(sen(x))dx$	19. $F(x) = -\dfrac{1}{2}cosec(2x) + C$
t) $\int \dfrac{5}{(x+1)ln^3(x+1)}dx$	20. $\dfrac{1}{2}arcsen(x^2) + C$

Resultados da Aprendizagem
Primitivação imediata

C.Aplicação

Calcule as primitivas das seguintes funções:

a) $\dfrac{1}{x^3}$ b) $(x^3+1)^4 x^2$ c) $\dfrac{1}{(x+1)^2}$ d) $sec^2(x)\,tg(x)$

e) $x^{-1}.ln(x)$ f) $\sqrt[3]{x^2}$ g) $x\sqrt[3]{x^2}$ h) $\dfrac{arctg(2x)}{1+4x^2}$

i) e^{5x} j) $\dfrac{10^{\sqrt{x}}}{\sqrt{x}}$ k) $\dfrac{x+2}{x^2+4x}$ l) $\dfrac{sen(lnx)}{x}$

m) $\dfrac{e^{\frac{1}{x}}}{x^2}$ n) $\dfrac{x}{\sqrt{4-x^2}}$ o) $\dfrac{1}{x\,ln\,x}$ p) $\dfrac{cos\,x}{1+sen^2 x}$

q) $\dfrac{3x}{\sqrt{1-x^4}}$ r) $sen(4x)$ s) $\dfrac{5x}{1+x^4}$ t) $e^{senx}cosx$

u) $\dfrac{1}{x(1+ln^2 x)}$ v) $\dfrac{5x}{(1+x^2)\sqrt{1-ln^2(1+x^2)}}$ w) $cosec^2(kx)\,cos(cotg\,kx)$

D.Análise

1.Identifique as funções que têm primitiva imediata:

a) $\dfrac{1}{sen^3(x)}$ b) $x^3(x^4+1)^5$ c) $\dfrac{4x}{(2x^2+2)^3}$ d) $\dfrac{6x^5}{\sqrt{(4-x^2)^3}}$

e) xe^{5x^2} f) $\dfrac{2x}{x^2+2x+1}$ g) $x^2(\sqrt[4]{x^3})$ h) $\dfrac{arcotg(2x)}{1+4x^2}$

i) xe^{5x} j) $\dfrac{4}{(2x^2+2)^3}$ k) $\dfrac{cos(x)}{sen^3(x)}$ l) $\dfrac{x+1}{x^2+2x+1}$

m) $x^3(x^5+1)^5$ n) $\dfrac{-2}{x^2+2x+1}$ o) $\dfrac{sen(lnx^2)}{x^2}$ p) $\dfrac{x}{\sqrt{(4-x^2)^3}}$

2. Justificando convenientemente a sua escolha, identifique em cada uma das seguintes primitivas a(s) expressão(ões) em falta marcadas com [ ]por forma a que possam ser aplicadas as regras da primitivação imediata:

a. $\displaystyle\int \dfrac{e^{x+1}}{\sqrt{4-2e^{[\ ]}}}dx$ b. $\displaystyle\int \dfrac{ln(x^2)}{x+[\ ]^4}dx$ c. $\displaystyle\int \dfrac{[\ ]e^{[\ ]}}{1+9e^{2cos(x)}}dx$

E.Sintese

1.Determine as seguintes primitivas:

a) $\displaystyle\int \sqrt{1-\dfrac{3}{4-sen^2(x)}}dx$ b) $\displaystyle\int \dfrac{\sqrt{4+x}}{\sqrt{16-x^2}}dx$ c) $\displaystyle\int \dfrac{-2}{x^3-3x^2+3x-1}dx$

2.Calcule as primitivas das seguintes funções, utilizando a técnica da primitivação por decomposição $\int (f+g) = \int f + \int g$:

a) $x^3 - 5x^2 + 2x + 1$
b) $\dfrac{1 + cos(2x)}{2}$
c) $\dfrac{x^3}{4} + \dfrac{tgx}{cos^2 x}$
d) $\dfrac{senx + cos x}{cos x}$

e) $\left(1 + \sqrt{x}\right)^3$
f) $\left(cos x + 3\right)^2$
g) $\dfrac{x^2 + 2\sqrt[3]{x}}{\sqrt{x}}$
h) $\left(e^{2x} + e^{-x}\right)^2$

3.Determine a primitiva da função $f(x) = \dfrac{2x}{x^2 + 2x + 1}$.

F.Avaliação

1.Determine a função f que satisfaz as seguintes condições $f''(x) = x + 1$, $f(0) = 1$ e $f'(0) = 0$.

2. Sem recorrer à definição de primitiva, comente a proposição $\int \dfrac{2}{x+1} dx = ln(3(x+1)^2)$.