20. Jun. 2014

Fug. 2. AnI - Informática

① a) $\sum_{n=1}^{+\infty} \frac{2n}{\sqrt[3]{8n^7}} = \sum_{n=1}^{+\infty} \frac{2n}{\sqrt[3]{8}\cdot n^{7/3}} = \sum_{n=1}^{+\infty} \frac{1}{n^{4/3}}$ Dirichlet, conv ($\alpha = \frac{4}{3}$) ✓

b) $\sum_{n=1}^{+\infty} e^{-n}(1-e^{-1}) = \sum_{n=1}^{+\infty}\left(\underbrace{e^{-n}}_{a_n} - \underbrace{e^{-(n+1)}}_{a_{n+1}}\right) = (e^{-1}-e^{-2}) + (e^{-2}-e^{-3}) + (e^{-3}-e^{-4}) + \cdots$

Trata-se de uma série de Mengoli com $u_n = a_n - a_{n+1}$, $a_n = e^{-n}$

Como $\lim a_n = \lim e^{-n} = \lim \frac{1}{e^n} = 0$ (finito) então a série é convergente

e tem-se Soma $= e^{-1} - 1.\lim a_n = \frac{1}{e}$ ✓

② a) $\sum_{n=1}^{+\infty} \frac{3^{n-1}}{2^{2n+1}}$ $\qquad \frac{u_{n+1}}{u_n} = \frac{\frac{3^{(n+1)-1}}{2^{2(n+1)+1}}}{\frac{3^{n-1}}{2^{2n+1}}} = \frac{3^n \cdot 2^{2n+1}}{2^{2n+3}\cdot 3^{n-1}} = \frac{3}{2^2} = \frac{3}{4} = ct.$

$= \sum_{n=1}^{+\infty} \frac{3^n}{2^{2n}}\cdot\frac{3^{-1}}{2}$

$= \sum_{n=1}^{+\infty} \left(\frac{3}{4}\right)^n \cdot \frac{1}{6}$

R

a série é geométrica como $|R|<1$ então a série é conv.

b) $\sum_{n=1}^{+\infty} \frac{\sqrt{9n^2+1}}{n} = \sum_{n=1}^{+\infty} \frac{3n+1}{n} = \sum_{n=1}^{+\infty}\left(3+\frac{1}{n}\right) = 4 + \left(3+\frac{1}{2}\right) + \left(3+\frac{1}{3}\right) + \cdots$

Como $\lim u_n = \lim\left(3+\frac{1}{n}\right) = 3 \neq 0$ então a série é divergente (C.N.C).

③ $\int \frac{e^{6x+1}+e^{3x}}{\sqrt{25-e^{6x}}}\,dx = \int \frac{e^{6x}\cdot e}{\sqrt{25-e^{6x}}}\,dx + \int \frac{e^{3x}}{\sqrt{25}\sqrt{1-\frac{e^{6x}}{25}}}\,dx$

$= e\cdot\int e^{6x}\cdot(25-e^{6x})^{-1/2}\,dx + \frac{1}{5}\cdot\int \frac{e^{3x}}{\sqrt{1-\left(\frac{e^{3x}}{5}\right)^2}}\,dx$

$= \frac{e}{-6}\cdot\int -6e^{6x}(25-e^{6x})^{-1/2}\,dx + \frac{1}{5}\cdot\frac{5}{3}\int \frac{\frac{3e^{3x}}{5}}{\sqrt{1-\left(\frac{e^{3x}}{5}\right)^2}}\,dx$

$= -\frac{e}{6}\,\frac{(25-e^{6x})^{1/2}}{1/2} + \frac{1}{3}\arcsin\left(\frac{e^{3x}}{5}\right) + C,\ C\in\mathbb{R}$

$= -\frac{e}{3}\sqrt{25-e^{6x}} + \frac{1}{3}\arcsin\left(\frac{e^{3x}}{5}\right) + C,\ C\in\mathbb{R}$

61

4)

$$\int x^3 \cdot e^{x^2}\,dx = \int \underbrace{x^2}_{D} \cdot \underbrace{x e^{x^2}}_{P}\,dx$$

- $\int x e^{x^2}dx = \frac{1}{2}\int 2x e^{x^2}dx = \frac{1}{2}e^{x^2}$
- $(x^2)' = 2x$

$$= x^2 \cdot \frac{1}{2}e^{x^2} - \int 2x \cdot \frac{1}{2}e^{x^2}\,dx$$

$$= \frac{x^2}{2}\cdot e^{x^2} - \frac{1}{2}\int 2x \cdot e^{x^2}\,dx$$

$$= \frac{x^2}{2}\cdot e^{x^2} - \frac{1}{2}e^{x^2} + C, \ C \in \mathbb{R}$$

5)

a) $$\left(\operatorname{arctg}(2\sqrt{x})+C\right)' = \frac{(2\sqrt{x})'}{1+(2\sqrt{x})^2} + C' = \frac{2\cdot\frac{1}{2\sqrt{x}}}{1+4x} + 0 = \frac{\frac{1}{\sqrt{x}}}{1+4x} = \frac{1}{\sqrt{x}\,(1+4x)}$$

b) $$\int \frac{1}{\sqrt{x}\,(4x+1)}\,dx = \int \frac{\frac{1}{\sqrt{x}}}{4x+1}\,dx = 2\int \frac{\frac{1}{2\sqrt{x}}}{1+(2\sqrt{x})^2}\,dx = 2\cdot\operatorname{arctg}(2\sqrt{x}) + C, \ C \in \mathbb{R}$$

c) $$\int \frac{1}{\sqrt{x}\,(4x+1)}\,dx$$ $\underline{\text{m.v.}}$: $x = t^2 \longrightarrow x' = 2t$

$$= \int \frac{1}{\sqrt{t^2}\,(4t^2+1)}\cdot 2t\,dt = \int \frac{2t}{t\,(4t^2+1)}\,dt = \int \frac{2}{1+(2t)^2}\,dt$$

$$= 1\cdot\operatorname{arctg}(2t) + C, \ C \in \mathbb{R} = \operatorname{arctg}(2\sqrt{x}) + C, \ C \in \mathbb{R}$$

6) a) <u>Resolução 1</u>

$$\int \operatorname{sen} x \cdot \operatorname{sen}(2x)\,dx = \int \operatorname{sen} x \cdot 2\operatorname{sen} x \cdot \cos x\,dx = 2\int \cos x \cdot \operatorname{sen}^2 x\,dx$$

$$= 2\cdot\frac{\operatorname{sen}^3 x}{3} + C, \ C \in \mathbb{R}$$

<u>Resolução 2</u>

$$\int \operatorname{sen} x \cdot \operatorname{sen}(2x)\,dx = \int \frac{1}{2}\left(\cos(-x) - \cos(3x)\right)dx$$

$$= -\frac{1}{2}\int -\cos(-x)\,dx - \frac{1}{2}\cdot\frac{1}{3}\int 3\cos(3x)\,dx$$

$$= -\frac{1}{2}\operatorname{sen}(-x) - \frac{1}{6}\operatorname{sen}(3x) + C, \ C \in \mathbb{R}$$

Resolução 3

$$\int \underbrace{\operatorname{sen} x}_{P} \cdot \underbrace{\operatorname{sen}(2x)}_{D}\, dx = -\cos x \cdot \operatorname{sen}(2x) - \int -\cos x \cdot 2\cos(2x)\, dx$$

$$= -\cos x \cdot \operatorname{sen}(2x) + 2\int \underbrace{\cos x}_{P} \cdot \underbrace{\cos(2x)}_{D}\, dx$$

$$= -\cos x \cdot \operatorname{sen}(2x) + 2\left(\operatorname{sen} x \cdot \cos(2x) - \int \operatorname{sen} x \cdot (-2\operatorname{sen}(2x))\, dx\right.$$

$$= -\cos x \cdot \operatorname{sen}(2x) + 2\operatorname{sen} x \cdot \cos(2x) + 4\boxed{\int \operatorname{sen} x \cdot \operatorname{sen}(2x)\, dx}$$

$$\Rightarrow -3\int \operatorname{sen} x \cdot \operatorname{sen}(2x)\, dx = -\cos x \cdot \operatorname{sen}(2x) + 2\operatorname{sen} x \cdot \cos(2x)$$

$$\Rightarrow \int \operatorname{sen} x \cdot \operatorname{sen}(2x)\, dx = \frac{1}{3}\cos x \cdot \operatorname{sen}(2x) - \frac{2}{3}\operatorname{sen} x \cdot \cos(2x) + C, \quad C \in \mathbb{R}$$

b) $$\int \underbrace{x}_{D} \cdot \underbrace{\sqrt{x+1}}_{P}\, dx = x \cdot \frac{(x+1)^{3/2}}{3/2} - \int 1 \cdot \frac{(x+1)^{3/2}}{3/2}\, dx$$

$$= \frac{2}{3}x \cdot (x+1)^{3/2} - \frac{2}{3} \cdot \frac{(x+1)^{5/2}}{5/2} + C, \quad C \in \mathbb{R}$$

$$= \frac{2x}{3}\sqrt{(x+1)^3} - \frac{4}{15}\sqrt{(x+1)^5} + C, \quad C \in \mathbb{R}$$

c)
$$\begin{array}{l|l} 3x^3 - 6x^2 + 5x + 1 & x^3 - 2x^2 + x \\ \hline -(3x^3 - 6x^2 + 3x) & 3 \\ \hline \qquad\qquad 2x + 1 & \end{array}$$

$$\Rightarrow \frac{3x^3 - 6x^2 + 5x + 1}{x^3 - 2x^2 + x} = 3 + \frac{2x+1}{x^3 - 2x^2 + x}$$

$$\downarrow$$

$$x^3 - 2x^2 + x = 0$$
$$\Leftrightarrow x(x^2 - 2x + 1) = 0$$
$$\Leftrightarrow x = 0 \lor x = 1 \lor x = 1$$

$$\frac{\boxed{2x+1}}{x^3 - 2x^2 + x} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} = \frac{\boxed{A(x-1)^2 + B(x-1)\cdot x + Cx}}{x^3 - 2x^2 + x}$$

$x = 0$: $1 = A$

$x = 1$: $3 = C$

$x = 2$: $5 = A + 2B + 2C \longrightarrow 5 = 1 + 2B + 6 \Rightarrow 2B = -2 \Rightarrow B = -1$

$$\int \frac{3x^3 - 6x^2 + 5x + 1}{x^3 - 2x^2 + x}\, dx = \int 3 + \frac{2x+1}{x^3 - 2x^2 + x}\, dx = \int 3 + \frac{1}{x} + \frac{-1}{x-1} + \frac{3}{(x-1)^2}\, dx$$

$$= 3x + \ln|x| - \ln|x-1| + 3\int (x-1)^{-2}\, dx = 3x + \ln|x| - \ln|x-1| + 3 \cdot \frac{(x-1)^{-1}}{-1} + C$$

$$= 3x + \ln|x| - \ln|x-1| - 3/(x-1) + C, \quad C \in \mathbb{R}$$

d) $\int \ln 3 + \frac{\sqrt[5]{x} + \sqrt[3]{x}}{\sqrt{x}}\, dx = \int \ln 3 \, dx + \int \frac{x^{1/5}}{x^{1/2}}\, dx + \int \frac{x^{1/3}}{x^{1/2}}\, dx$

$= (\ln 3)\, x + \int x^{1/5 - 1/2}\, dx + \int x^{1/3 - 1/2}\, dx$

$= \ln 3 \cdot x + \int x^{3/10}\, dx + \int x^{-1/6}\, dx$

$= \ln 3 \cdot x + \frac{x^{13/10}}{13/10} + \frac{x^{5/6}}{5/6} + C,\ C \in \mathbb{R}$

$= \ln 3 \cdot x + \frac{10}{13} \cdot \sqrt[10]{x^{13}} + \frac{6}{5} \cdot \sqrt[6]{x^5} + C,\ C \in \mathbb{R}$