**Instituto Superior de Engenharia de Coimbra**
DEPARTAMENTO DE FÍSICA E MATEMÁTICA

ANÁLISE MATEMÁTICA I - **Engenharia Informática**

TPC nº9 Data limite de entrega: 2/dez/2016 (18h)

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Primitivação por partes
Resolva as seguintes primitivas, utilizando a técnica de primitivação por partes.

**A. Conhecimento**

9) $\displaystyle\int \arcsin x \, dx\,;$

**C. Aplicação**

5) $\displaystyle\int \ln^2 x \, dx\,;$

6) $\displaystyle\int \frac{x}{\sqrt[3]{x+1}} \, dx\,;$

11) $\displaystyle\int \frac{x^3}{\sqrt[3]{1-x^2}} \, dx\,;$

*Sugestão de resolução:*

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**A. Conhecimento**

9) Tem-se

$$\int \arcsin x \, dx = \int \underbrace{1}_{p} \underbrace{\arcsin x}_{d} \, dx$$

cálculos auxiliares:

$$\boxed{\begin{array}{l} \bullet \displaystyle\int \underbrace{1}_{R1} \, dx = x + c \\ \bullet (\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \end{array}}$$

$$\begin{aligned}
&\overset{PP}{=} x \arcsin x - \int x \frac{1}{\sqrt{1-x^2}} \, dx \\
&= x \arcsin x - \int x \,(1-x^2)^{-\frac{1}{2}} \, dx \\
&= x \arcsin x - \left(-\frac{1}{2}\right) \int \underbrace{-2x\,(1-x^2)^{-\frac{1}{2}}}_{R2} \, dx \\
&= x \arcsin x + \frac{1}{2} \frac{(1-x^2)^{\frac{1}{2}}}{\frac{1}{2}} + c \\
&= x \arcsin x + \sqrt{1-x^2} + c\,, \;\; c \in \mathbb{R}\,.
\end{aligned}$$

## C. Aplicação

5) Tem-se

$$\int \ln^2 x\,dx = \int \underbrace{1}_{p} \cdot \underbrace{\ln^2 x}_{d}\,dx$$

cálculos auxiliares:

- $\int \underbrace{1}_{R1}\,dx = x + c$
- $\left(\ln^2 x\right)' = 2\,(\ln x)\,\frac{x'}{x} = 2\,(\ln x)\,\frac{1}{x}$

$$= x\,\ln^2 x - \int x\,2\,(\ln x)\,\frac{1}{x}\,dx$$

$$= x\,\ln^2 x - \int \underbrace{2}_{p} \cdot \underbrace{(\ln x)}_{d}\,dx$$

cálculos auxiliares:

- $\int \underbrace{2}_{R1}\,dx = 2x + c$
- $\left(\ln x\right)' = \frac{x'}{x} = \frac{1}{x}$

$$= x\,\ln^2 x - \left(2x\,(\ln x) - \int 2x\,\frac{1}{x}\,dx\right)$$

$$= x\,\ln^2 x - 2x\,(\ln x) + \int 2\,dx$$

$$= x\,\ln^2 x - 2x\,(\ln x) + 2x + c\,,\ \ c \in \mathbb{R}\,.$$

6) Tem-se

$$\int \frac{x}{\sqrt[3]{x+1}}\,dx = \int \underbrace{x}_{d}\,\underbrace{(x+1)^{-\frac{1}{3}}}_{p}\,dx$$

cálculos auxiliares:

- $\int \underbrace{(x+1)^{-\frac{1}{3}} \cdot 1}_{R2}\,dx = \frac{(x+1)^{\frac{2}{3}}}{\frac{2}{3}} = \frac{3}{2}\,(x+1)^{\frac{2}{3}} + c$
- $(x)' = 1$

$$\overset{PP}{=} x\,\frac{3}{2}\,(x+1)^{\frac{2}{3}} - \int 1\,\frac{3}{2}\,(x+1)^{\frac{2}{3}}\,dx$$

$$= \frac{3}{2}\,x\,\sqrt[3]{(x+1)^2} - \frac{3}{2}\int \underbrace{(x+1)^{\frac{2}{3}} \cdot 1}_{R2}\,dx$$

$$= \frac{3}{2}\,x\,\sqrt[3]{(x+1)^2} - \frac{3}{2}\,\frac{(x+1)^{\frac{5}{3}}}{\frac{5}{3}} + c$$

$$= \frac{3}{2}\,x\,\sqrt[3]{(x+1)^2} - \frac{9}{10}\,\sqrt[3]{(x+1)^5} + c\,,\ \ c \in \mathbb{R}\,.$$

11) Tem-se

$$
\begin{aligned}
\int \frac{x^3}{\sqrt[3]{1-x^2}}\,dx &= \int x^3\,(1-x^2)^{-\frac{1}{3}}\,dx \\
&= \int x^2\,x\,(1-x^2)^{-\frac{1}{3}}\,dx \\
&= \int \underbrace{x^2}_{d}\,\underbrace{x\,(1-x^2)^{-\frac{1}{3}}}_{p}\,dx
\end{aligned}
$$

cálculos auxiliares:

- $\int x(1-x^2)^{-\frac{1}{3}}\,dx = -\frac{1}{2}\int \underbrace{-2x(1-x^2)^{-\frac{1}{3}}}_{R2}\,dx = -\frac{1}{2}\,\frac{(1-x^2)^{\frac{2}{3}}}{\frac{2}{3}} = -\frac{3}{4}\,(1-x^2)^{\frac{2}{3}} + c$
- $(x^2)' = 2x$

$$
\begin{aligned}
&\overset{PP}{=} x^2\Big(-\frac{3}{4}\,(1-x^2)^{\frac{2}{3}}\Big) - \int 2x\,\Big(-\frac{3}{4}\,(1-x^2)^{\frac{2}{3}}\Big)\,dx \\
&= -\frac{3}{4}\,x^2\,\sqrt[3]{1-x^2} - \frac{3}{4}\int \underbrace{-2x\,(1-x^2)^{\frac{2}{3}}}_{R2}\,dx \\
&= -\frac{3}{4}\,x^2\,\sqrt[3]{1-x^2} - \frac{3}{4}\,\frac{(1-x^2)^{\frac{5}{3}}}{\frac{5}{3}} + c \\
&= -\frac{3}{4}\,x^2\,\sqrt[3]{1-x^2} - \frac{9}{20}\,\sqrt[3]{(1-x^2)^5} + c\,,\ \ c \in \mathbb{R}\,.
\end{aligned}
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