Instituto Superior de Engenharia de Coimbra
DEPARTAMENTO DE FÍSICA E MATEMÁTICA

ANÁLISE MATEMÁTICA I - **Engenharia Informática**

TPC nº4 Data limite de entrega: 28/out/2015 (18h)

PRIMITIVAÇÃO IMEDIATA [**C. Aplicação**]

Calcule as primitivas das seguintes funções:

a) $\dfrac{1}{x^3}$; b) $(x^3+1)^4\,x^2$; c) $\dfrac{1}{(x+1)^2}$;

d) $\sec^2 x\tan x$; e) $x^{-1}\ln x$; f) $\sqrt[3]{x^2}$;

g) $x\,\sqrt[3]{x^2}$; h) $\dfrac{\arctan(2x)}{1+4x^2}$; i) e^{5x};

j) $\dfrac{10^{\sqrt{x}}}{\sqrt{x}}$; k) $\dfrac{x+2}{x^2+4x}$; l) $\dfrac{\sin(\ln x)}{x}$;

m) $\dfrac{e^{\frac{1}{x}}}{x^2}$; n) $\dfrac{x}{\sqrt{4-x^2}}$; o) $\dfrac{1}{x\ln x}$;

p) $\dfrac{\cos x}{1+\sin^2 x}$; q) $\dfrac{3x}{\sqrt{1-x^4}}$; r) $\sin(4x)$;

s) $\dfrac{5x}{1+x^4}$; t) $e^{\sin x}\cos x$; u) $\dfrac{1}{x\,(1+\ln^2 x)}$.

Sugestão de resolução:

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a) $$\int \frac{1}{x^3}\,dx = \int \underbrace{x^{-3}\cdot 1}_{R2}\,dx = \frac{x^{-2}}{-2}+c = -\frac{1}{2x^2}+c, \quad c\in\mathbb{R};$$

b) $$\int (x^3+1)^4\,x^2\,dx = \frac{1}{3}\int \underbrace{(x^3+1)^4\,3\,x^2}_{R2}\,dx = \frac{1}{3}\,\frac{(x^3+1)^5}{5}+c = \frac{1}{15}\,(x^3+1)^5+c, \quad c\in\mathbb{R};$$

c) $$\int \frac{1}{(x+1)^2}\,dx = \int \underbrace{(x+1)^{-2}\cdot 1}_{R2}\,dx = \frac{(x+1)^{-1}}{-1}+c = -\frac{1}{x+1}+c, \quad c\in\mathbb{R};$$

d) $$\int \sec^2 x\,\tan x\,dx = \int \underbrace{(\tan x)^1\sec^2 x}_{R2}\,dx = \frac{(\tan x)^2}{2}+c_1 = \frac{1}{2}\,\tan^2 x+c_1, \quad c_1\in\mathbb{R};$$

Alternativa:
$$\int \sec^2 x\,\tan x\,dx = \int \underbrace{(\sec x)^1\sec x\,\tan x}_{R2}\,dx = \frac{(\sec x)^2}{2}+c_2 = \frac{1}{2}\,\sec^2 x+c_2, \quad c_2\in\mathbb{R};$$

e) $$\int x^{-1}\ln x\,dx = \int \underbrace{(\ln x)^1\,\frac{1}{x}}_{R2}\,dx = \frac{(\ln x)^2}{2}+c = \frac{1}{2}\,\ln^2 x+c, \quad c\in\mathbb{R};$$

f) $$\int \sqrt[3]{x^2}\,dx = \int \underbrace{x^{\frac{2}{3}}\cdot 1}_{R2}\,dx = \frac{x^{\frac{5}{3}}}{\frac{5}{3}}+c = \frac{3}{5}\,\sqrt[3]{x^5}+c, \quad c\in\mathbb{R};$$

g) $$\int x\,\sqrt[3]{x^2}\,dx = \int x\,x^{\frac{2}{3}}\,dx = \int x^{\frac{5}{3}}\,dx = \int \underbrace{x^{\frac{5}{3}}\cdot 1}_{R2}\,dx = \frac{x^{\frac{8}{3}}}{\frac{8}{3}}+c = \frac{3}{8}\,\sqrt[3]{x^8}+c, \quad c\in\mathbb{R};$$

h) $$\int \frac{\arctan(2x)}{1+4x^2}\,dx = \int \arctan(2x)\,\frac{1}{1+4x^2}\,dx = \frac{1}{2}\int \underbrace{\arctan(2x)\,\frac{2}{1+4x^2}}_{R2}\,dx$$
$$= \frac{1}{2}\,\frac{\arctan^2(2x)}{2} + c = \frac{1}{4}\,\arctan^2(2x) + c\,,\quad c\in\mathbb{R}\,;$$

i) $$\int e^{5x}\,dx = \frac{1}{5}\int \underbrace{e^{5x}\,5}_{R3}\,dx = \frac{1}{5}\,e^{5x} + c\,,\quad c\in\mathbb{R}\,;$$

j) $$\int \frac{10^{\sqrt{x}}}{\sqrt{x}}\,dx = \int 10^{\sqrt{x}}\,\frac{1}{\sqrt{x}}\,dx = 2\int \underbrace{10^{\sqrt{x}}\,\frac{1}{2\sqrt{x}}}_{R4}\,dx = 2\,\frac{10^{\sqrt{x}}}{\ln 10} + c = \frac{2}{\ln 10}\,10^{\sqrt{x}} + c\,,\quad c\in\mathbb{R}\,;$$

k) $$\int \frac{x+2}{x^2+4x}\,dx = \frac{1}{2}\int \frac{2\,(x+2)}{x^2+4x}\,dx = \frac{1}{2}\int \frac{2x+4}{\underbrace{x^2+4x}_{R5}}\,dx = \frac{1}{2}\,\ln|x^2+4x| + c\,,\quad c\in\mathbb{R}\,;$$

l) $$\int \frac{\sin(\ln x)}{x}\,dx = \int \underbrace{\frac{1}{x}\,\sin(\ln x)}_{R7}\,dx = -\cos(\ln x) + c\,,\quad c\in\mathbb{R}\,;$$

m) $$\int \frac{e^{\frac{1}{x}}}{x^2}\,dx = \int e^{\frac{1}{x}}\,\frac{1}{x^2}\,dx = -\int \underbrace{e^{\frac{1}{x}}\left(-\frac{1}{x^2}\right)}_{R3}\,dx = -e^{\frac{1}{x}} + c\,,\quad c\in\mathbb{R}\,;$$

n) $$\int \frac{x}{\sqrt{4-x^2}}\,dx = \int (4-x^2)^{-\frac{1}{2}}\,x\,dx = -\frac{1}{2}\int \underbrace{(4-x^2)^{-\frac{1}{2}}\,(-2\,x)}_{R2}\,dx = -\frac{1}{2}\,\frac{(4-x^2)^{\frac{1}{2}}}{\frac{1}{2}} + c$$
$$= -\sqrt{4-x^2} + c\,,\quad c\in\mathbb{R}\,;$$

o) $$\int \frac{1}{x\ln x}\,dx = \int \frac{1}{x}\,\frac{1}{\ln x}\,dx = \int \frac{\frac{1}{x}}{\underbrace{\ln x}_{R5}}\,dx = \ln|\ln x| + c\,,\quad c\in\mathbb{R}\,;$$

p) $$\int \frac{\cos x}{\underbrace{1+\sin^2 x}_{R19}}\,dx = \arctan(\sin x) + c\,,\quad c\in\mathbb{R}\,;$$

q) $$\int \frac{3x}{\sqrt{1-x^4}}\,dx = \int \frac{3x}{\sqrt{1-(x^2)^2}}\,dx = \frac{3}{2}\int \frac{2\,x}{\underbrace{\sqrt{1-(x^2)^2}}_{R18}}\,dx = \frac{3}{2}\,\arcsin(x^2) + c\,,\quad c\in\mathbb{R}\,;$$

r) $$\int \sin(4x)\,dx = \frac{1}{4}\int \underbrace{4\,\sin(4x)}_{R7}\,dx = \frac{1}{4}\big(-\cos(4x)\big) + c = -\frac{1}{4}\,\cos(4x) + c\,,\quad c\in\mathbb{R}\,;$$

s) $$\int \frac{5x}{1+x^4}\,dx = 5\int \frac{x}{1+(x^2)^2}\,dx = \frac{5}{2}\int \frac{2\,x}{\underbrace{1+(x^2)^2}_{R19}}\,dx = \frac{5}{2}\,\arctan(x^2) + c\,,\quad c\in\mathbb{R}\,;$$

t) $$\int \underbrace{e^{\sin x}\cos x}_{R3}\,dx = e^{\sin x} + c\,,\quad c\in\mathbb{R}\,;$$

s) $$\int \frac{1}{x\,(1+\ln^2 x)}\,dx = \int \frac{1}{x}\,\frac{1}{1+\ln^2 x}\,dx = \int \frac{\frac{1}{x}}{\underbrace{1+\ln^2 x}_{R19}}\,dx = \arctan(\ln x) + c\,,\quad c\in\mathbb{R}\,.$$