Portfolio Optimization

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ADSP 32013 - Team 3

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Agenda

Problem Statement

Background: Financial portfolio construction faces the challenge of balancing risk and return. The complexity of optimizing such portfolios necessitates sophisticated analytical methods.

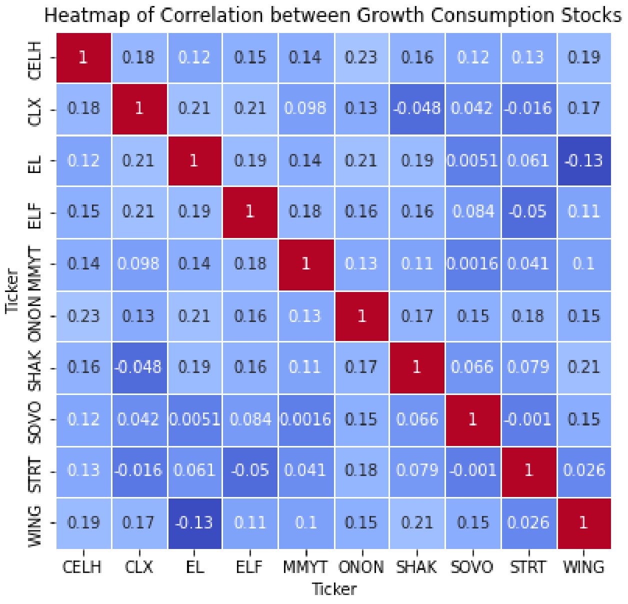
Goal: The aim is to create a financial model using quadratic programming to minimize risk while meeting specific return targets. This model will not only optimize financial portfolios within the Technology, Healthcare, and Consumption sectors but also facilitate a comparative analysis of outcomes between these sectors and the two stock types (value and growth). The objective is to derive actionable insights that inform investment strategies and focus on sectoral performance.

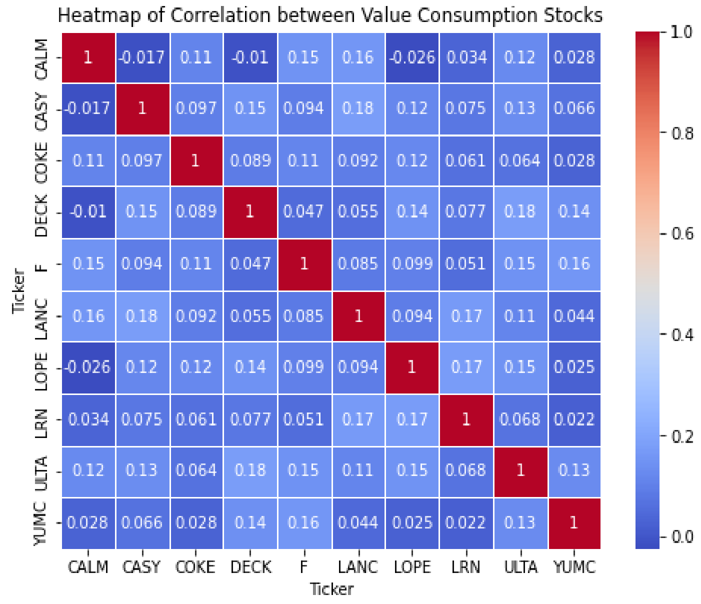
Stock Selection: Value Stock vs Growth Stock

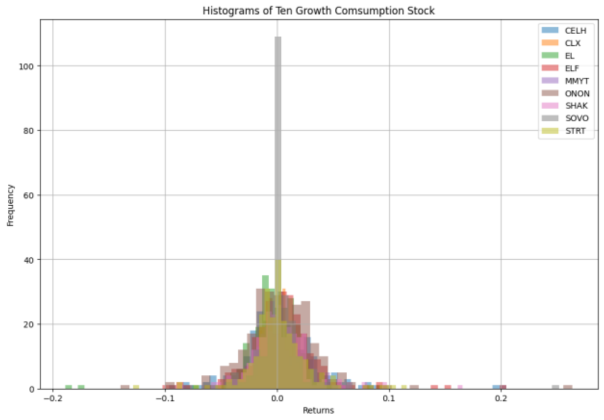
- Value Stock
  - Low P/B ratio and Low P/E ratio
  - The stock is viewed as undervalued
  - The company is usually well established
  - Since the company is well established, the stock is less risky
  - The stock often yields high dividends because the company needs less cash flow
  - It is suitable for investors seeking a low-risk and steady return
  - **P/E ratio** lower than **23.7,** which is the average of the S&P 500

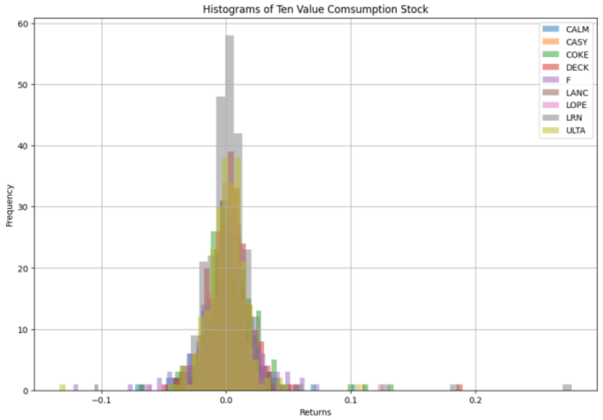
- Growth stock
  - High P/E ratio
  - The company expected to grow sales and earnings at a faster rate than the market average
  - However, it has the risk of dramatic decline
  - It normally doesn’t pay dividends
  - It is suitable for investors seeking high returns and willing to tolerate high risks
  - **P/E ratio** greater than **50**

Consumption - Correlation Heatmap and Daily Return









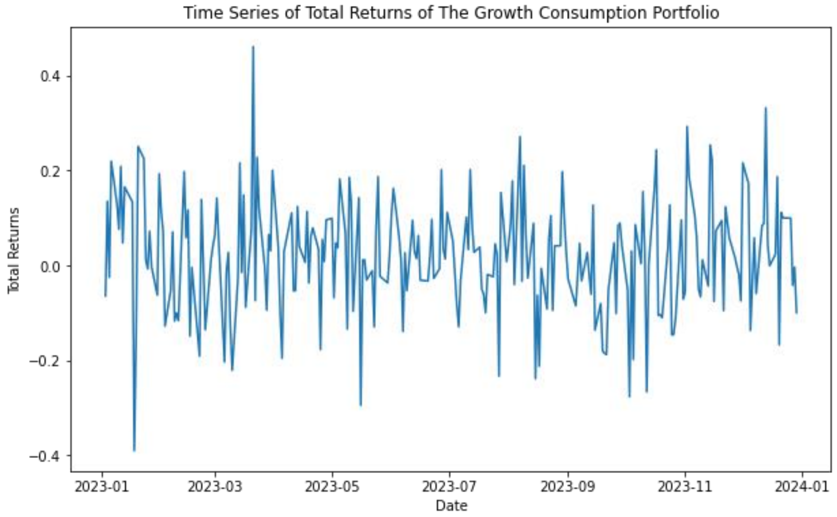
Correlation Analysis:

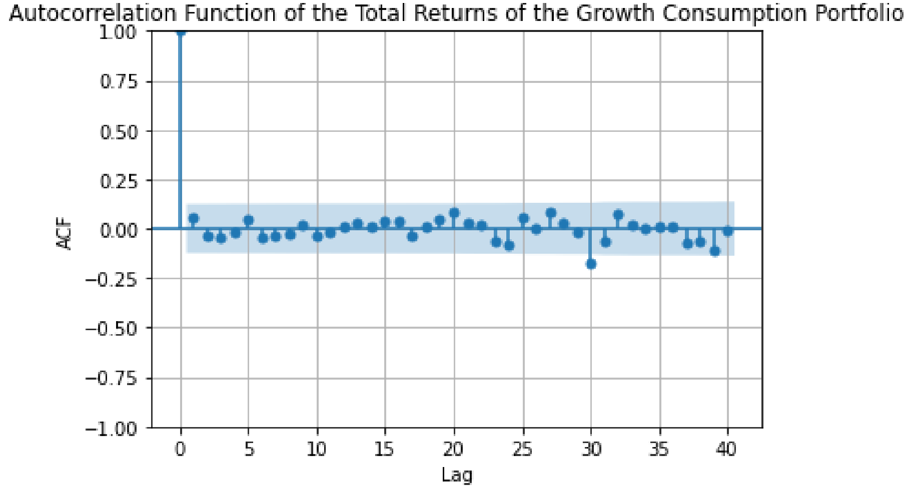
- Both value consumption stocks and growth consumption stocks are weakly correlated to each other
- Weak correlations imply that our portfolio is diversified
- Diversified portfolio usually has low volatility

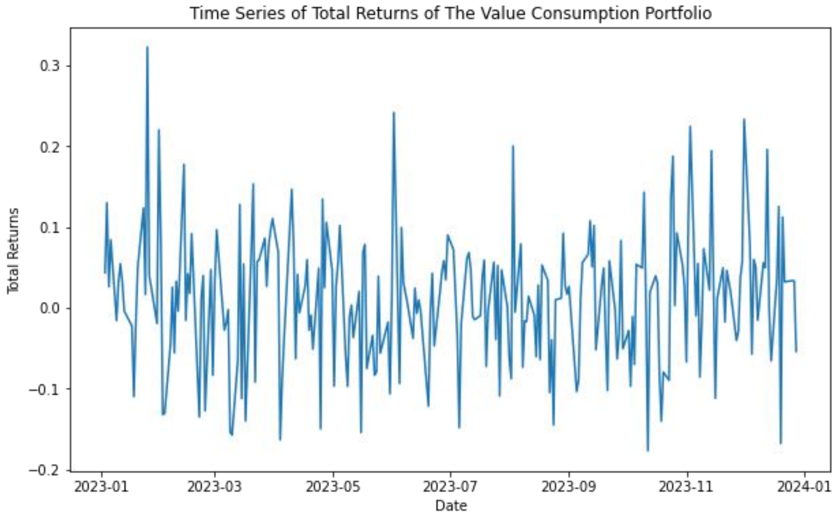
Histogram of Daily Returns:

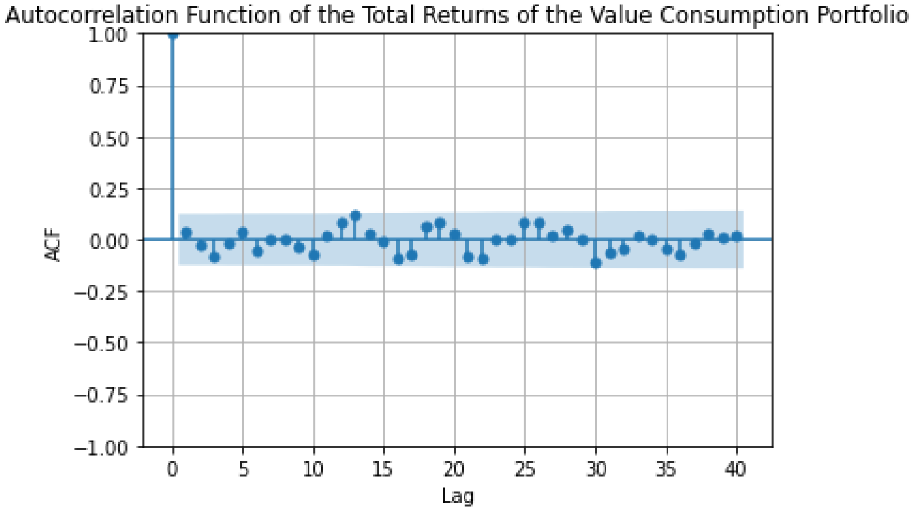
- The distributions are centered at zero
- The distributions are roughly symmetric with a long right tail

Consumption - Time series and ACF





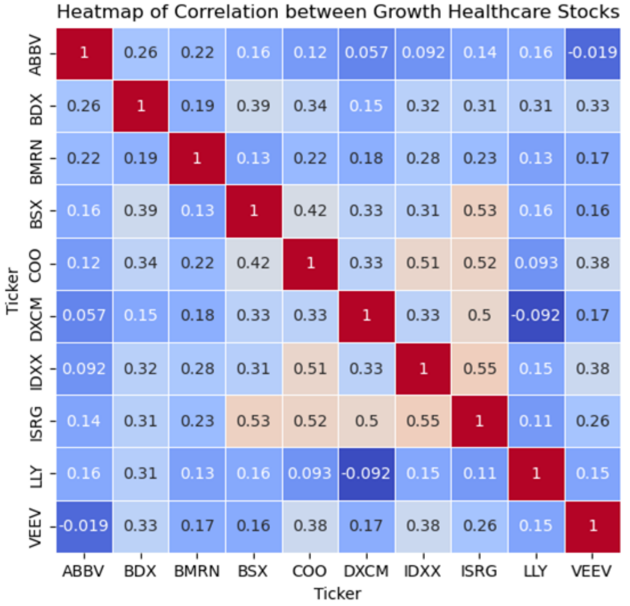


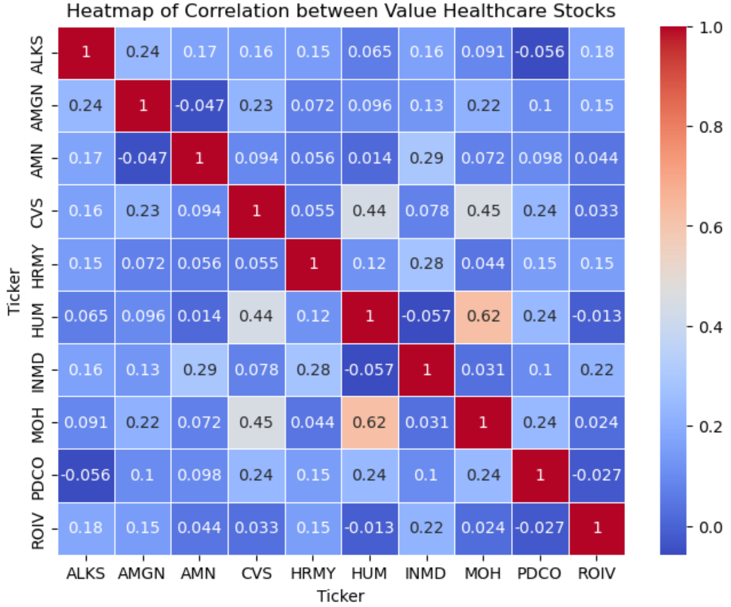


- Time series of both portfolios are basically white noise without obvious trends and seasonality

- The ACFs of both portfolios show that the return of the portfolio is not self-correlated

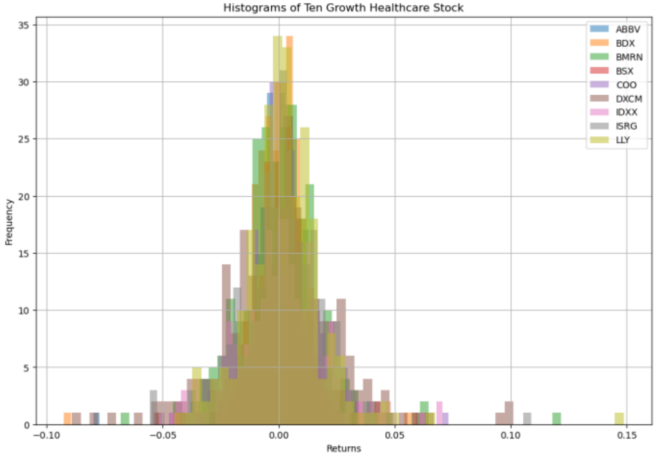
Healthcare - Correlation Heatmap and Daily Return

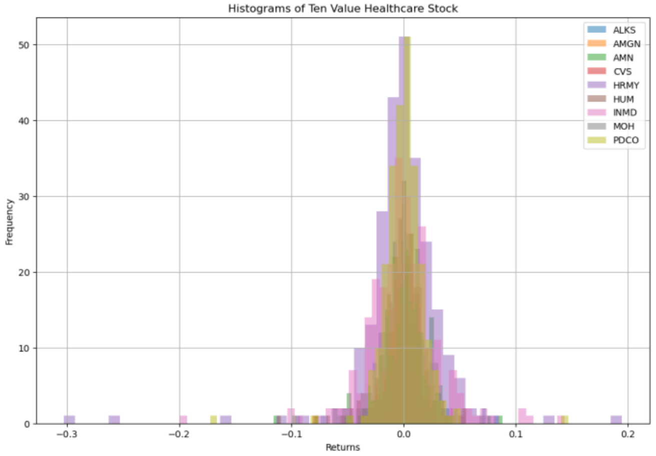




Correlation Analysis:

- Most of the growth healthcare stocks and value healthcare stocks are weakly correlated
- The presence of moderate correlations in both stocks could potentially pose challenges for effectively controlling volatility

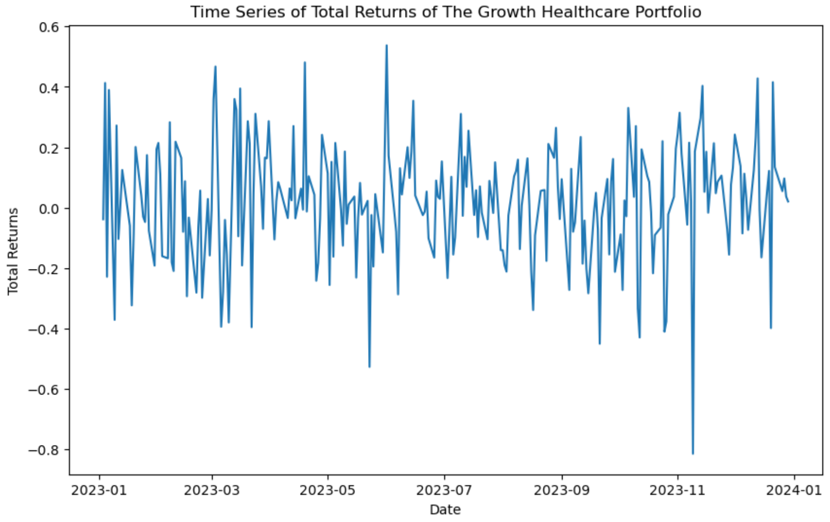


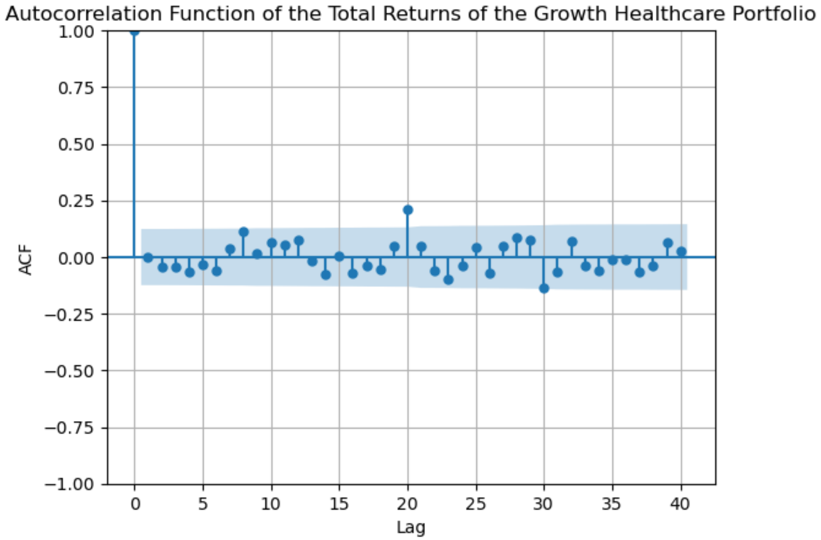


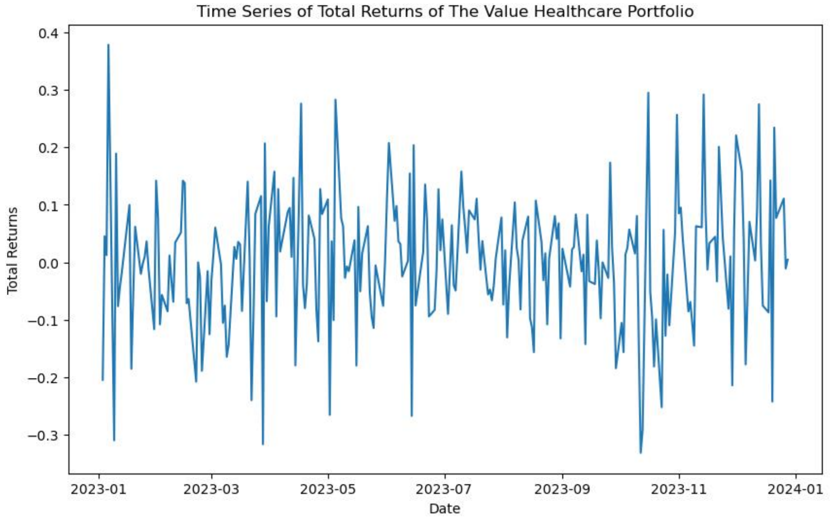
Histogram of Daily Returns:

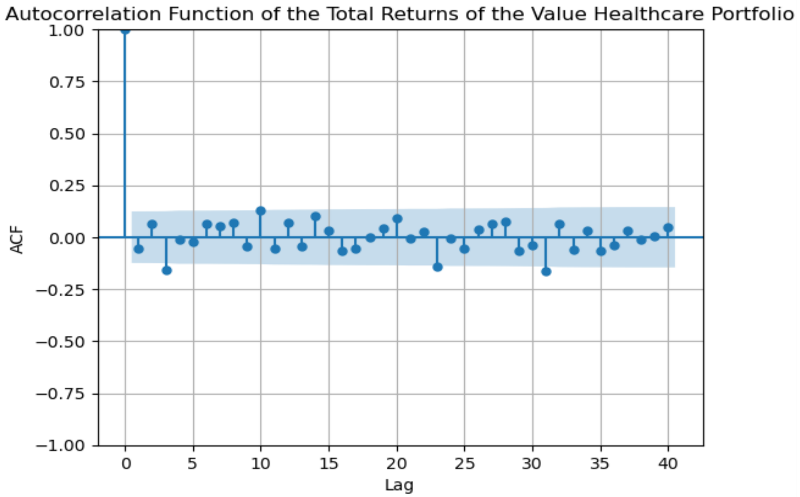
- Both distributions are centered at 0
- Distribution of growth stocks is roughly symmetric with a right long tail
- Distribution of value stocks is roughly symmetric with a left long tail

Healthcare - Time series and ACF









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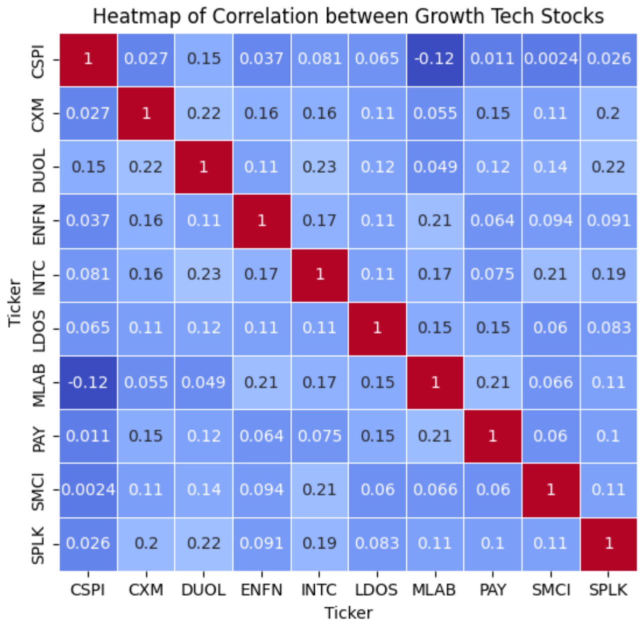
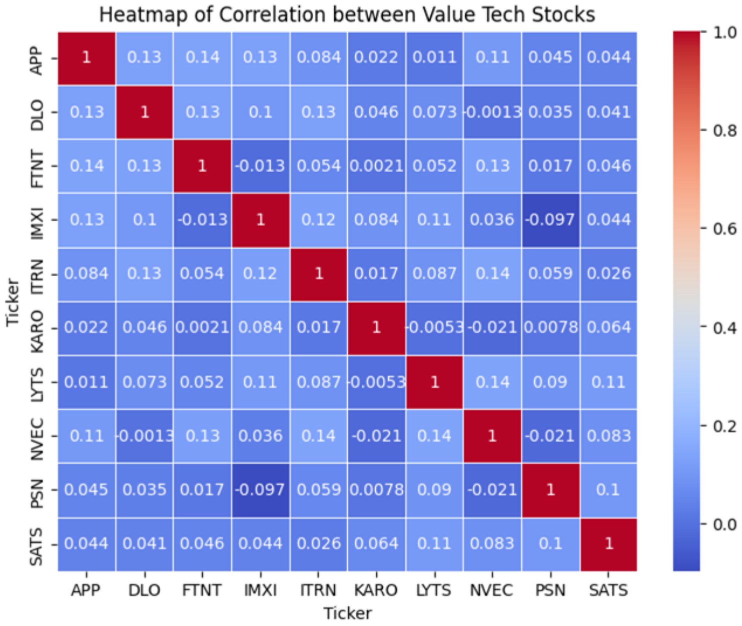
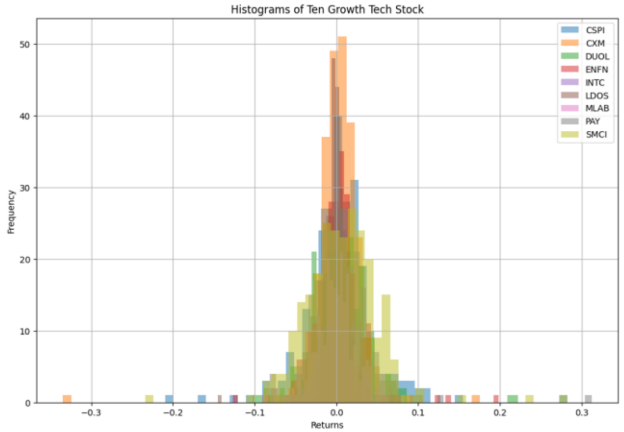
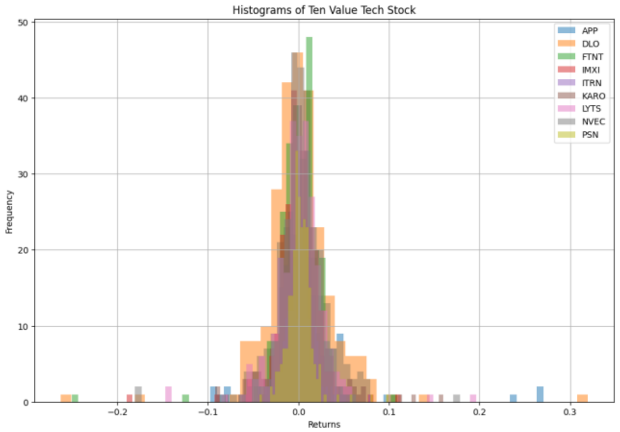
Tech - Correlation Heatmap and Daily Return









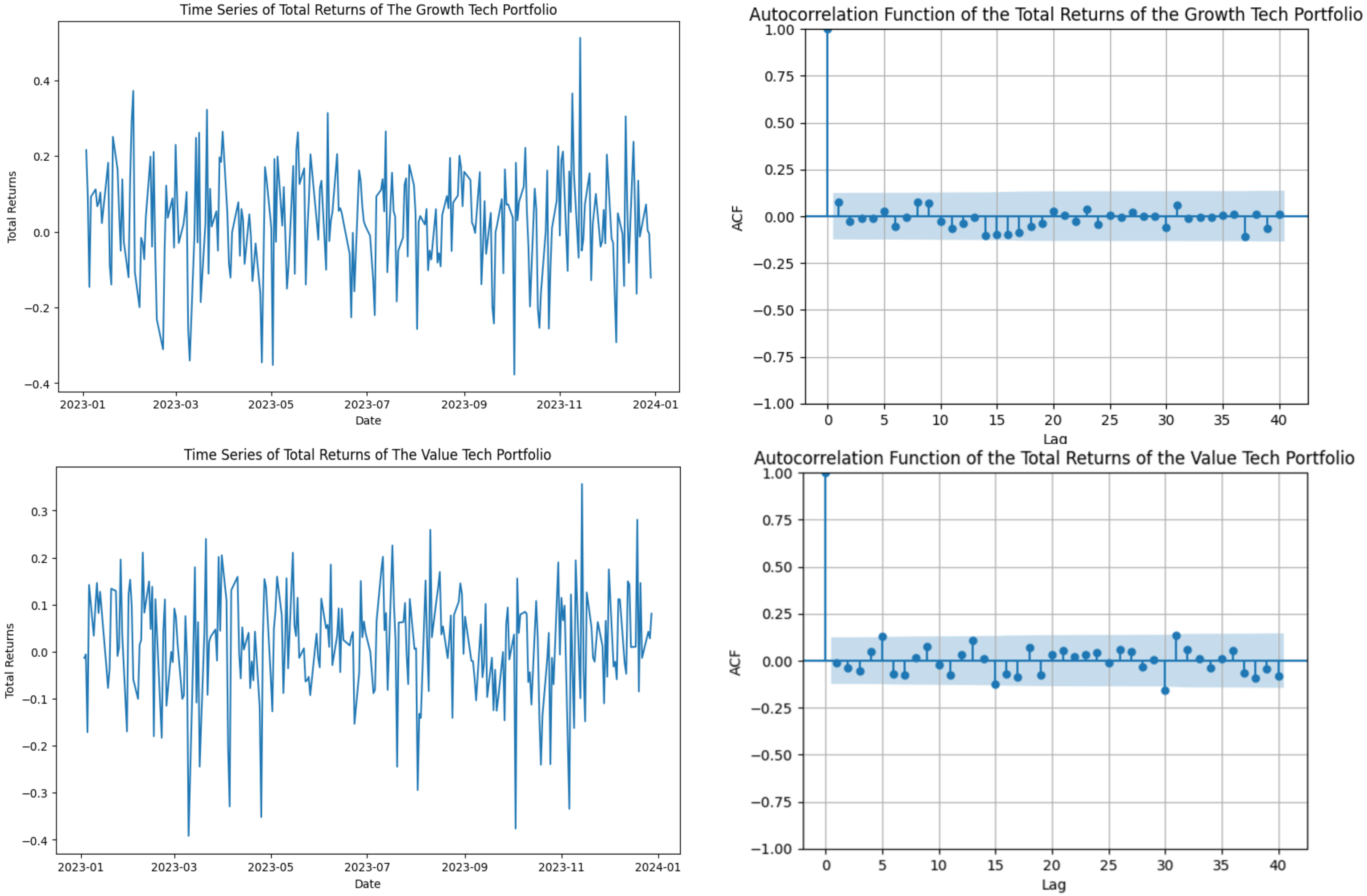
Correlation Analysis:

- Both value tech stocks and growth tech stocks are weakly correlated to each other
- Weak correlations imply that our portfolio is diversified
- Diversified portfolio usually has low volatility

Histogram of Daily Returns:

- Both distributions are centered at 0
- Both distributions are roughly symmetric without obvious tail

Tech - Time Series and ACF



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Quadratic Programming

Quadratic Programming Definition: Quadratic programming entails *maximizing* or *minimizing* a quadratic function subject to linear equalities or inequalities.

Why Choosing Quadratic Programming for Optimizing Financial Portfolio:

- Quadratic Objective Function:
  - Objective: Minimize the risk of the portfolio's return
  - Risk (variance) is a quadratic function of stock weights
- Linear Constraints: The return, and weight constraints are linear
- Convexity:
  - Definition: A convex function has a shape that curves upwards, ensuring a unique global minimum
  - Portfolio Context: The variance of returns is a convex function when the covariance matrix is positive semi-definite
  - Benefit: Guarantees that the solution is globally optimal, avoiding local minima

Quadratic Programming: Mathematical Formula

Variables:

- ER: Vector of expected returns in percent for each stock
- C: Covariance matrix of stock returns
- n: Number of stocks. In this case, n=10
- ERp: Target expected return for the portfolio
- x: Vector of asset weights in the portfolio

Objective:

$$\min \sum_{i=1}^{n}\sum_{j=1}^{n} x_i C_{ij} x_j$$

Constraints:

- Expected Return Constraint:

$$\sum_{i=1}^{n} ER_i \cdot x_i = ER_p$$

- Sum of Weights Constraint:

$$\sum_{i=1}^{n} x_i = 1$$

- Non-negativity Weights Constraint (without short):

$$x_i \geq 0, \forall i \in \{1, 2, \ldots, n\}$$

Julia Ipopt: Primal-Dual Interior Point Method

1. Formulate the **Lagrangian**: Incorporate the equality constraints with Lagrange multipliers λ for the return constraint and γ for the sum-to-one constraint:

$$L(x,\lambda,\gamma) = x^T Cx - \lambda(ER^T x - ER_p) - \gamma(\sum_{i=1}^{n} x_i - 1)$$

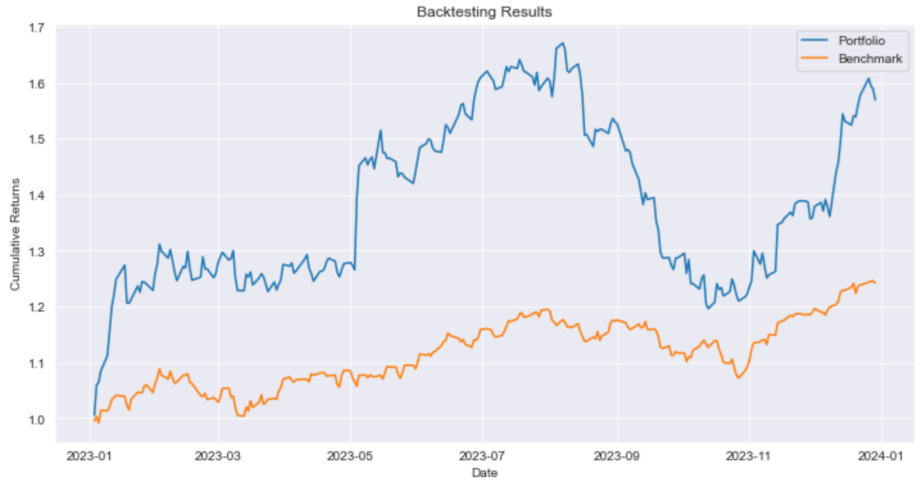
2. Derive **Karush-Kuhn-Tucker(KKT)** Conditions
   - Gradient w.r.t. x : $2Cx - \lambda ER - \gamma\mathbf{1} = 0$
   - Constraint for expected return: $ER^T x = ER_p$
   - Constraint for weights sum: $\sum_{i=1}^{n} x_i = 1$

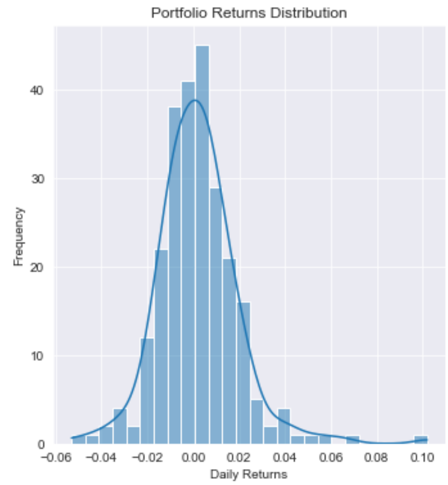
3. Apply a **Barrier Function**: $-\sum_{i=1}^{n} \ln(x_i)$ (penalty for movements toward non-feasible regions)

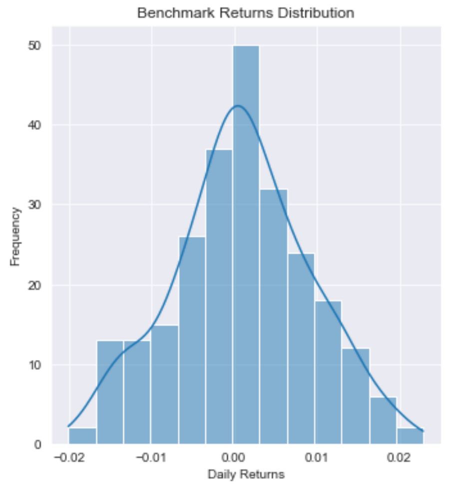
4. Iterative Solution with **Newton's Method**

Genetic Algorithm

1. **Initialization:** Create a random population of portfolio weights
2. **Fitness Function:** Evaluate how ‘good’ a solution is (use both objective function and expected return constraint)
3. **Selection:** Select the fittest individuals for reproduction (tournament selection)
4. **Crossover & Mutation:** Produce new offspring of the combined pairs to increase genetic diversity. Adjust the portfolio weights and normalize them
5. **Replacement:** Replace the least fit individuals with the new offspring
6. **Termination:** Repeat the process for many generations until the termination criteria are met (convergence or maximum number of generations)







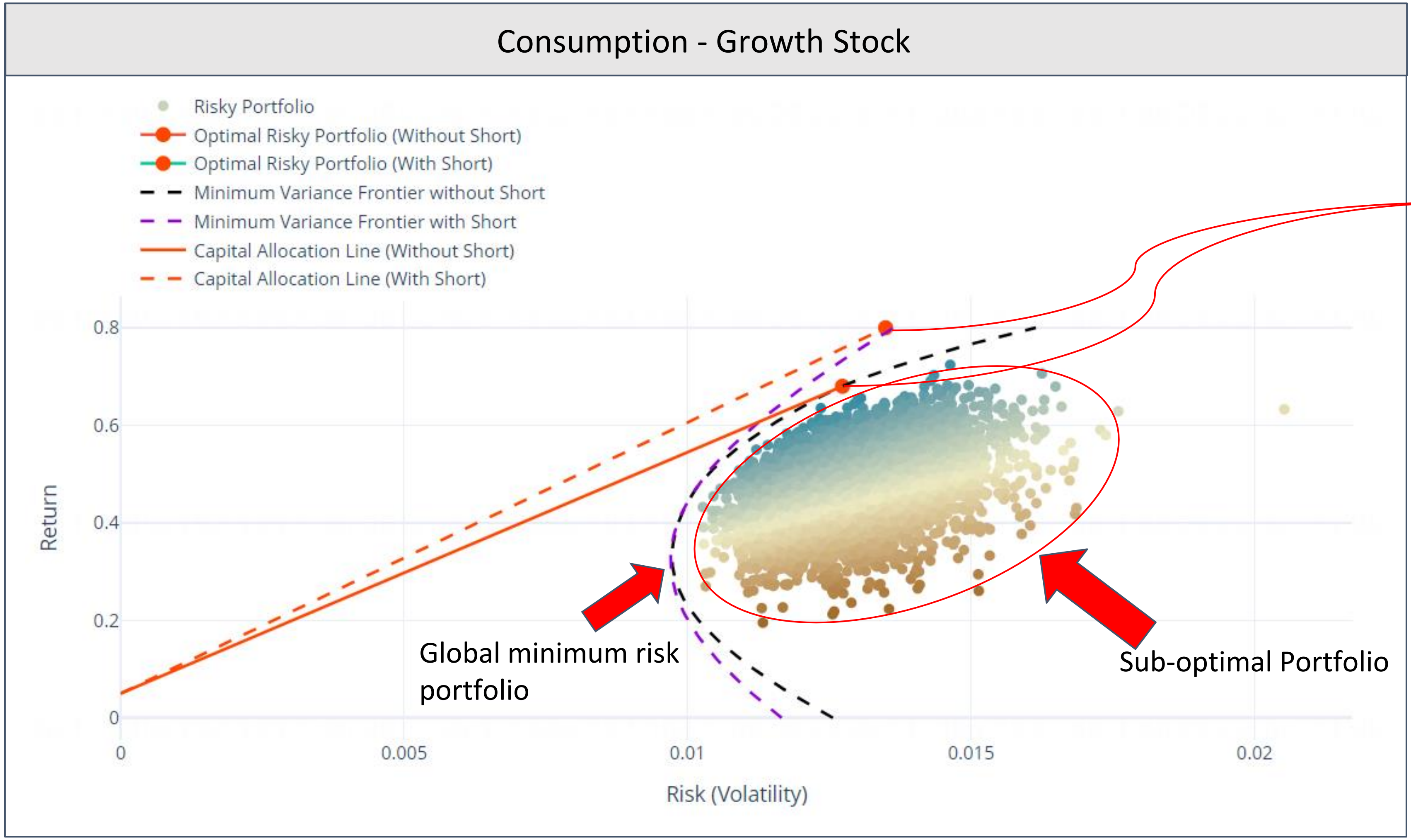
Comparison between Two Algorithms

Find Optimal Risky Portfolio: $Sharpe\ Ratio = \frac{E[R_p - R_f]}{\sigma_p}$

Algorithm	Julia Ipopt: Primal-Dual Interior Point Method	Genetic Algorithm
Expected Return (Annual)*	0.68	0.504
Minimum Volatility*	0.0127	0.0175
Maximum Sharpe Ratio*	49.48	25.94
Strengths	Efficient & Accurate; Strong Theoretical Guarantees	Global Exploration; Handling Nonlinearities
Drawbacks	Requirement for Gradients	Results from Stochastic Processes

** Take the growth stocks from the consumption sector as an example*

Optimal Risky Portfolio, Efficient Frontier & CAL



- Efficient Frontier
- Capital Allocation Line
  - Combined **risk-free rate** with **risky assets**
- Optimal Risky Portfolio
  - Portfolio that has the **highest return per unit risk** (**Sharpe Ratio**)

Short result in a high Sharpe Ratio
Possible reasons:

- More diverse strategy
- Short involves a form of leveraging
- Short can serve as a hedge against risk

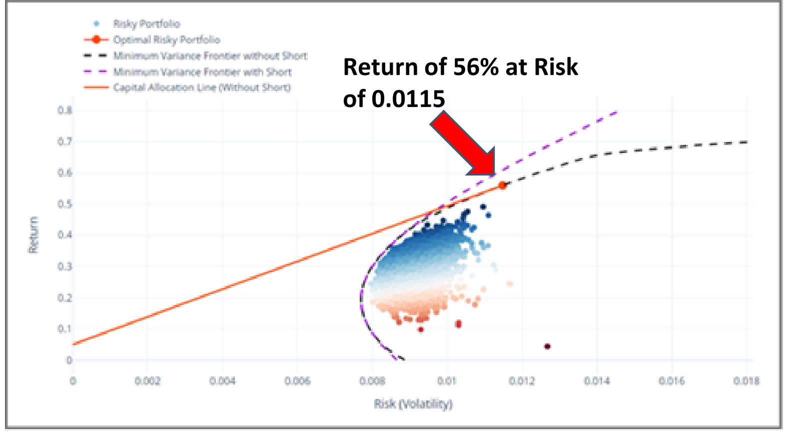
Optimization Results Comparison

Consumption - Growth Stocks

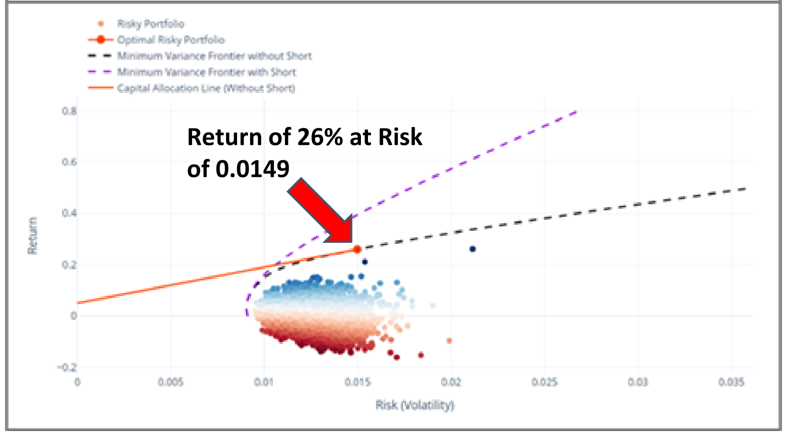
Healthcare – Growth Stocks

Tech – Growth Stocks

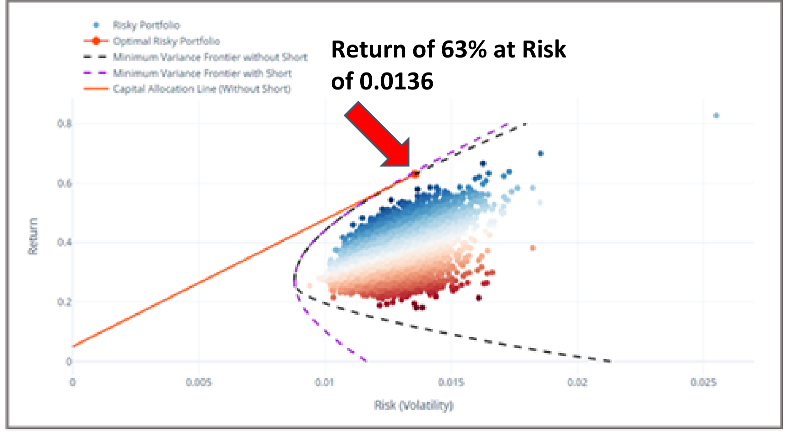
Consumption – Value Stocks



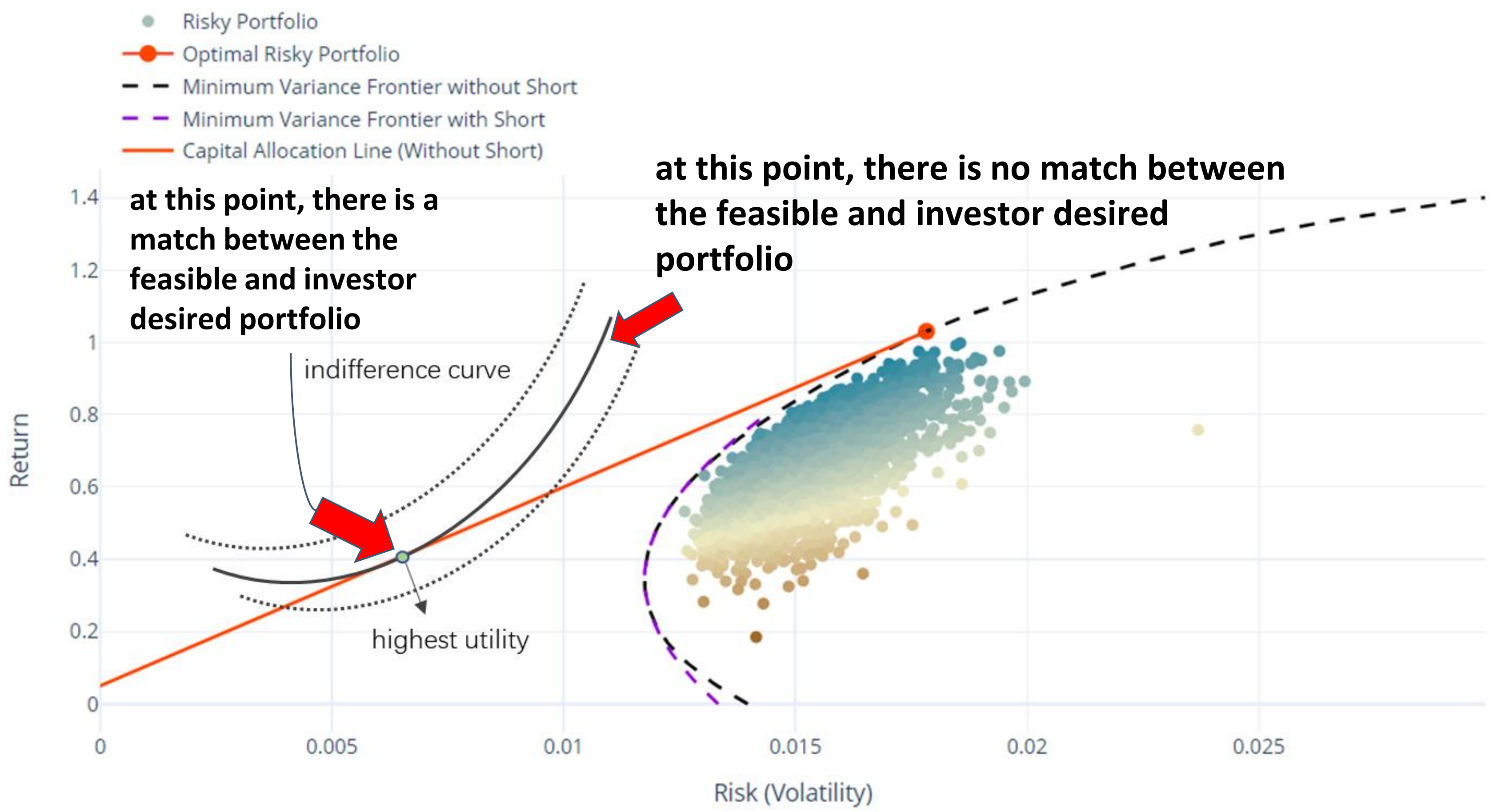
Healthcare – Value Stocks



Tech – Value Stocks



Indifference Curve



Indifference Curve:

- Convex curve
- Represents **trade-off** between expected return and risk based on **investor's risk tolerance**
- When crossed the efficient frontier, it means **there is a match between supply and demand**

Conclusion

- Model construed to **minimize risk** of a portfolio on a **given return**
- Portfolio stocks were assembled based on **low self-correlated Value stocks** and **Growth stocks** determined by **average P/E ratio**
- **Computation algorithm** in Ipopt solver performed **better** than **Genetic Algorithm**
- **Tech** Industry generates the **highest return** on investment retaining the same level of risk

Appendix

- Data Source: Yahoo Finance (download through Python API yfinance)

- Stocks selected: (01/01/2023-12/31/2023)
  - Healthcare Sector Value Stocks
    HUM, ROIV, HRMY, INMD, CVS, PDCO, MOH, AMN, ALKS, AMGN
  - Healthcare Sector Growth Stocks
    LLY, ABBV, PFE, ISRG, BSX, BDX, DXCM, IDXX, VEEV, COO
  - Technology Sector Value Stocks
    APP, DLO, FTNT, IMXI, ITRN, KARO, LYTS, NVEC, PSN, SATS
  - Technology Sector Growth Stocks
    CSPI, CXM, DUOL, ENFN, INTC, LDOS, MLAB, PAY, SMCI, SPLK
  - Consumption Sector Value Stocks
    CALM, CASY, COKE, DECK, F, LANC, LOPE, LRN, ULTA, YUMC
  - Consumption Sector Growth Stocks
    CELH, CLX, EL, ELF, MMYT, ONON, SHAK, SOVO, STRT, WING