中山大學
SUN YAT-SEN UNIVERSITY

Lecture 4.
RANSAC and feature detectors

Pattern Recognition and Computer Vision

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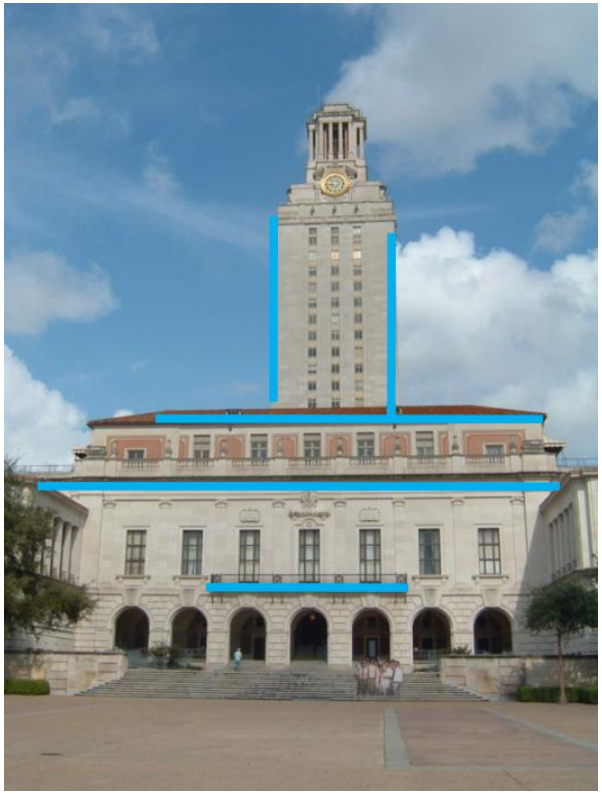
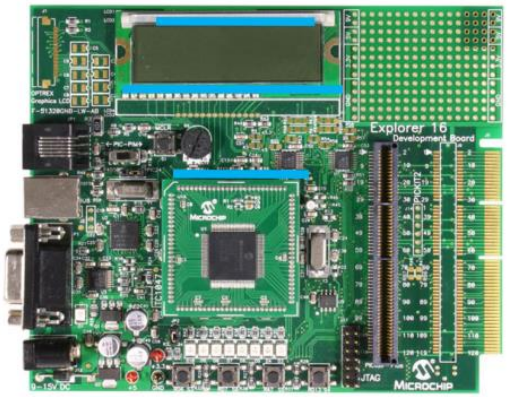
What we will learn today?

- A model fitting method for line detection
  - RANSAC
- Local invariant features
  - Motivation
  - Requirements, invariances
- Keypoint localization
  - Harris corner detector

Example: Line Fitting

- Why fit lines?
  - Many objects characterized by presence of straight lines

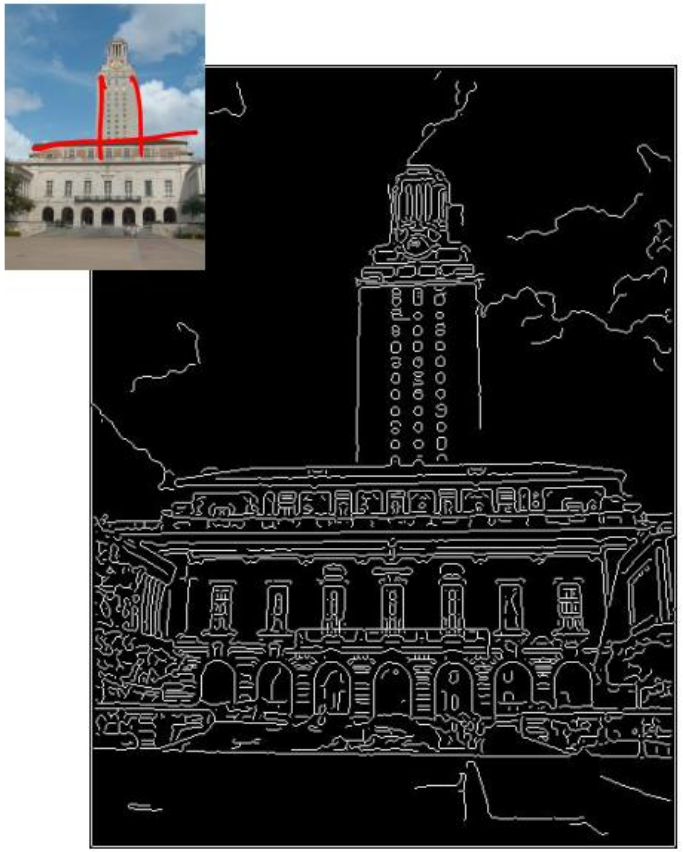




• Wait, why aren't we done just by running edge detection?

Difficulty of Line Fitting

- Extra edge points (clutter), multiple models:
  - Which points go with which line, if any?
- Only some parts of each line detected, and some parts are missing:
  - How to find a line that bridges missing evidence?
- Noise in measured edge points orientations:
  - How to detect true underlying parameters?



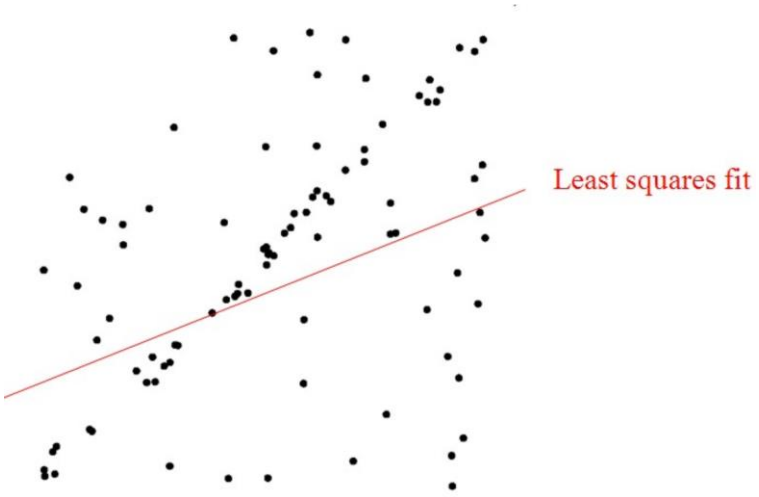
Voting

- It's not feasible to check all combinations of features by fitting a model to each possible subset.
- **Voting** is a general technique where we let the features vote for all models that are compatible with it.
  - Cycle through features, cast votes for model parameters.
  - Look for model parameters that receive a lot of votes.
- Noise & clutter features will cast votes too, but typically their votes should be inconsistent with the majority of "good" features.
- Ok if some features not observed, as model can span multiple fragments.

RANSAC [Fischler & Bolles 1981]

- RANdom SAmple Consensus

- **Approach:**
- we want to avoid the impact of outliers, so let's look for "inliers", and use only those.



- **Intuition:**
- if an outlier is chosen to compute the current fit, then the resulting line won't have much support from rest of the points.

RANSAC [Fischler & Bolles 1981]

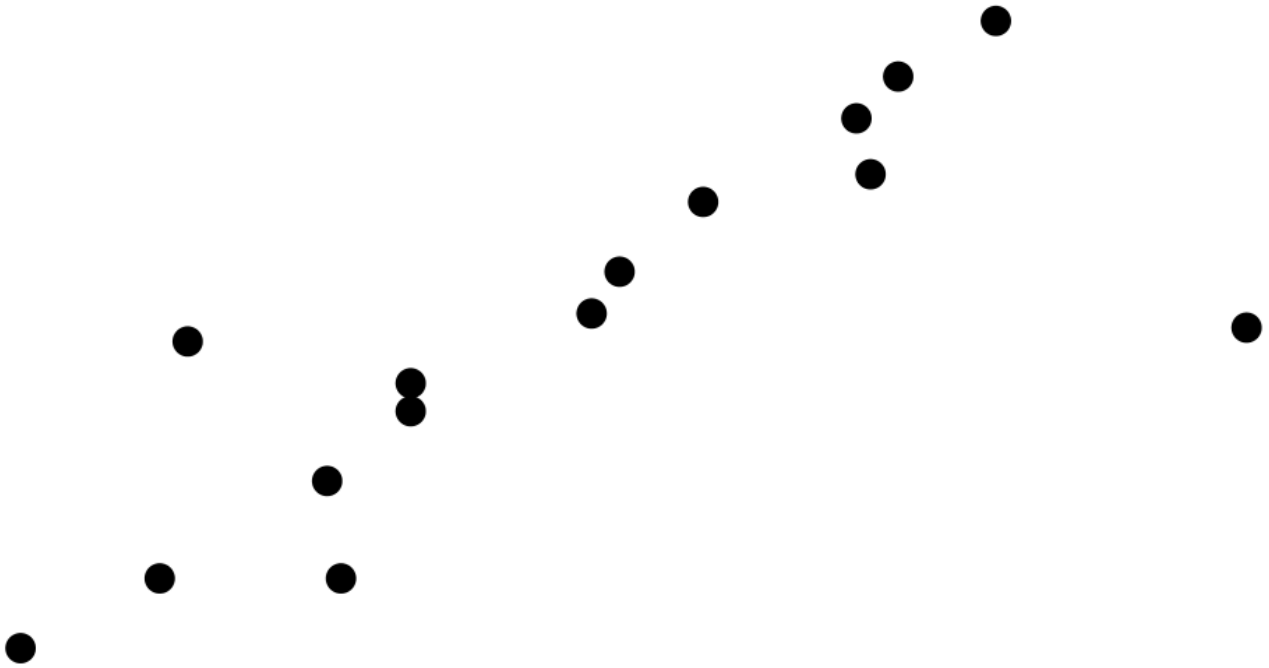
- RANSAC loop:

- 1. Randomly select a seed group of points on which to base transformation estimate (e.g., a group of matches)
- 2. Compute transformation from seed group
- 3. Find inliers to this transformation
- 4. If the number of inliers is sufficiently large, re-compute least-squares estimate of transformation on all of the inliers

- Keep the transformation with the largest number of inliers

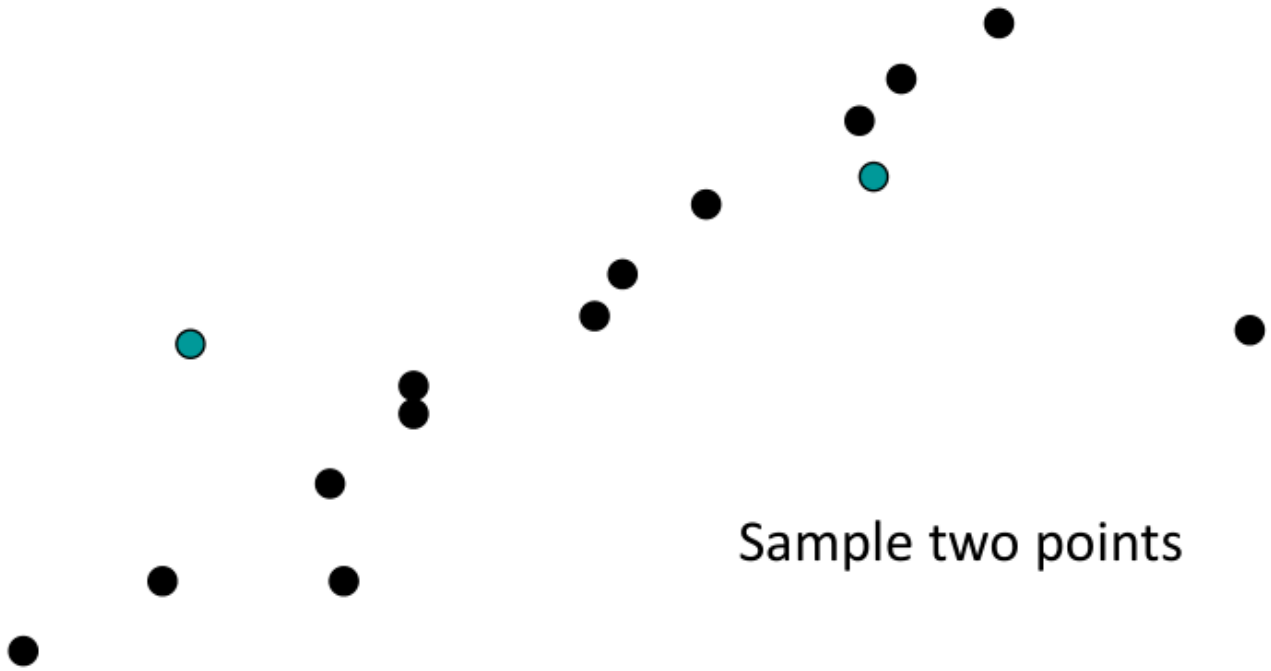
RANSAC Line Fitting Example

- **Task:** Estimate the best line
  - *How many points do we need to estimate the line?*



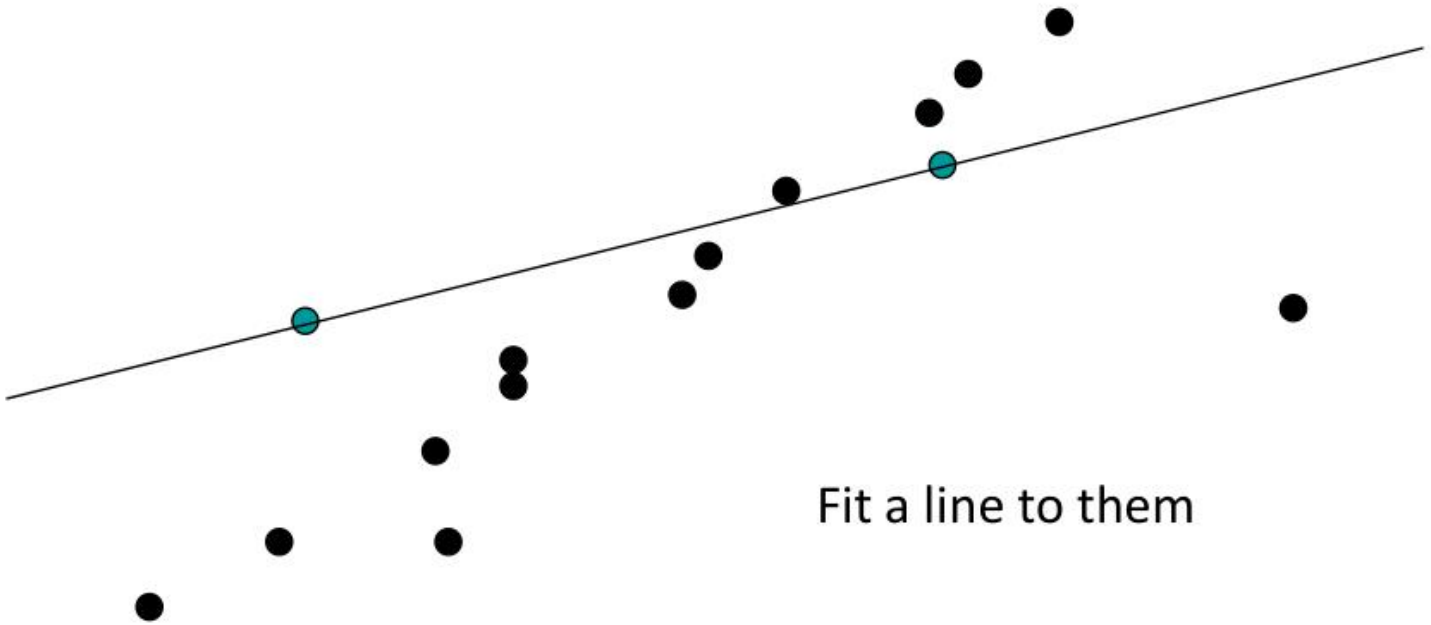
RANSAC Line Fitting Example

- **Task:** Estimate the best line



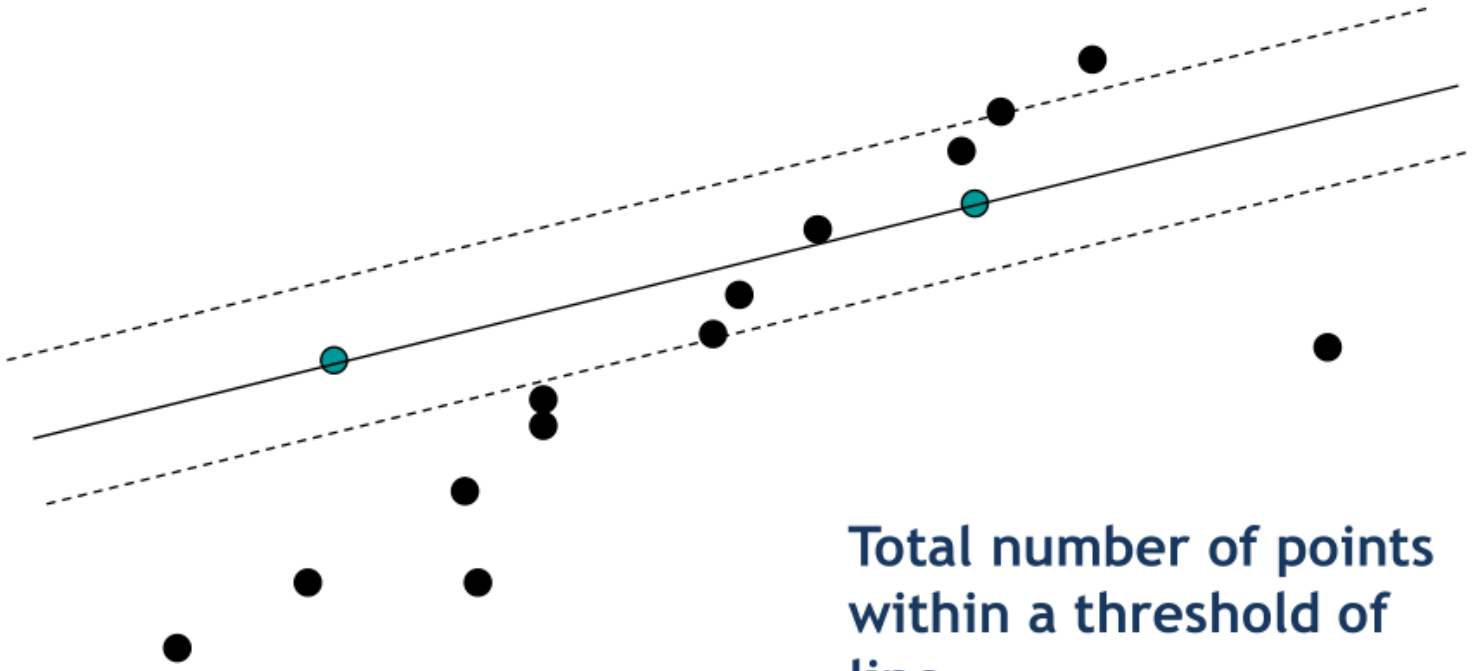
RANSAC Line Fitting Example

- **Task:** Estimate the best line



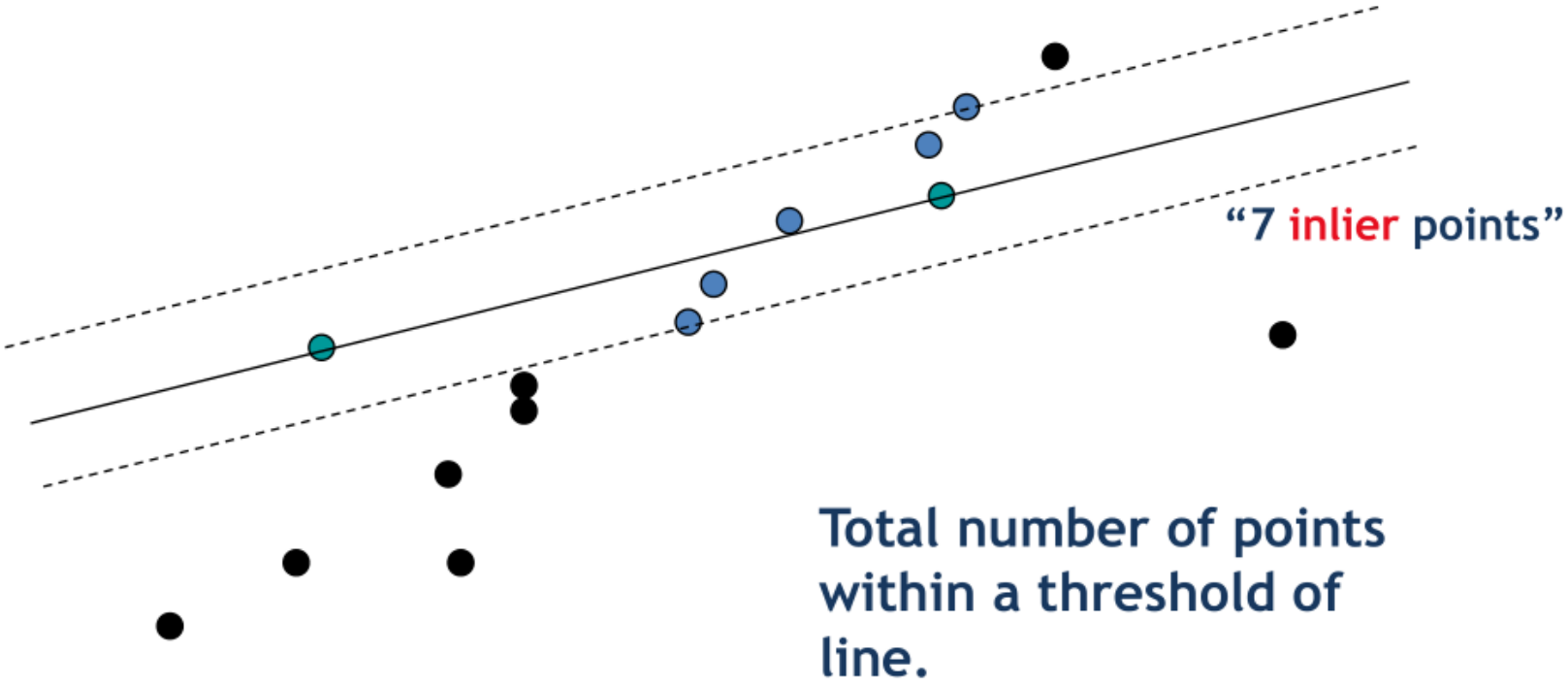
RANSAC Line Fitting Example

- **Task:** Estimate the best line



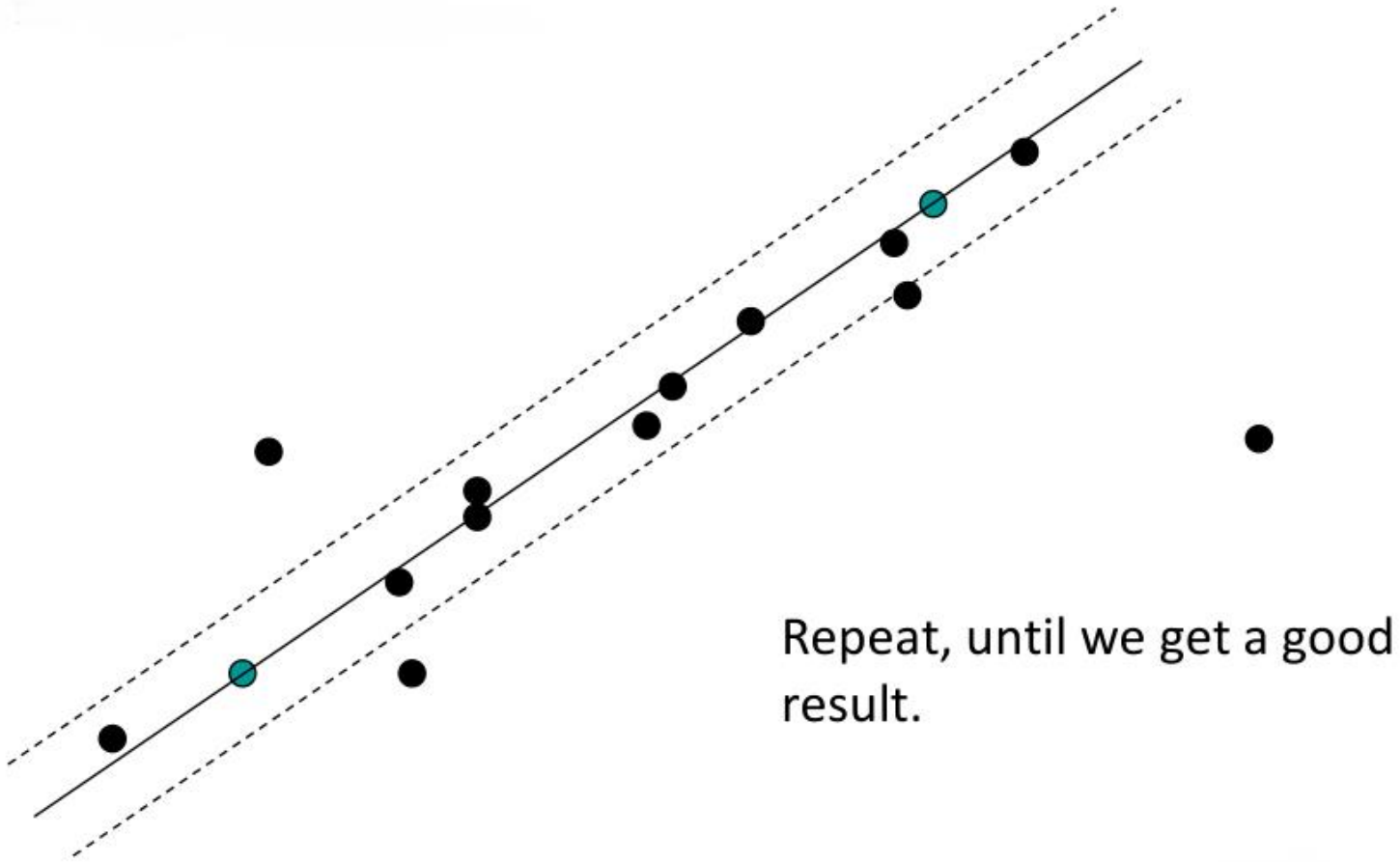
RANSAC Line Fitting Example

- **Task:** Estimate the best line



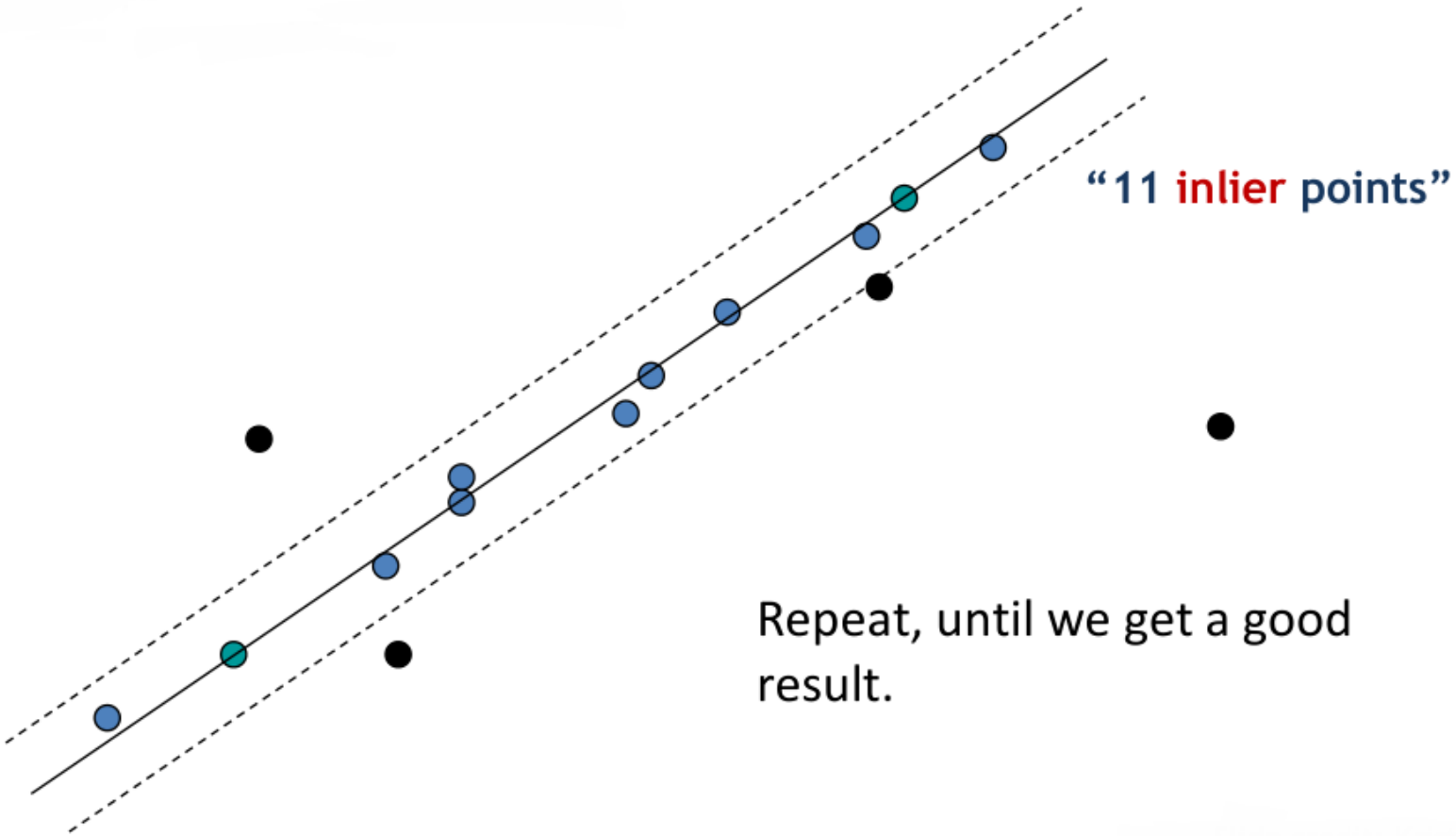
RANSAC Line Fitting Example

- **Task:** Estimate the best line



RANSAC Line Fitting Example

- **Task:** Estimate the best line



RANSAC: Algorithm

Algorithm 15.4: RANSAC: fitting lines using random sample consensus

Determine:
    n — the smallest number of points required
    k — the number of iterations required
    t — the threshold used to identify a point that fits well
    d — the number of nearby points required
      to assert a model fits well
Until k iterations have occurred
    Draw a sample of n points from the data
      uniformly and at random
    Fit to that set of n points
    For each data point outside the sample
      Test the distance from the point to the line
        against t; if the distance from the point to the line
        is less than t, the point is close
    end
    If there are d or more points close to the line
      then there is a good fit. Refit the line using all
      these points.
end
Use the best fit from this collection, using the
  fitting error as a criterion

RANSAC: How many samples?

- How many samples are needed?
  - Suppose *w* is fraction of inliers (points from line).
  - *n* points needed to define hypothesis (2 for lines)
  - *k* samples chosen.

- Prob. that a single sample of *n* points is correct: w^n

- Prob. that all *k* samples fail is: $(1 - w^n)^k$

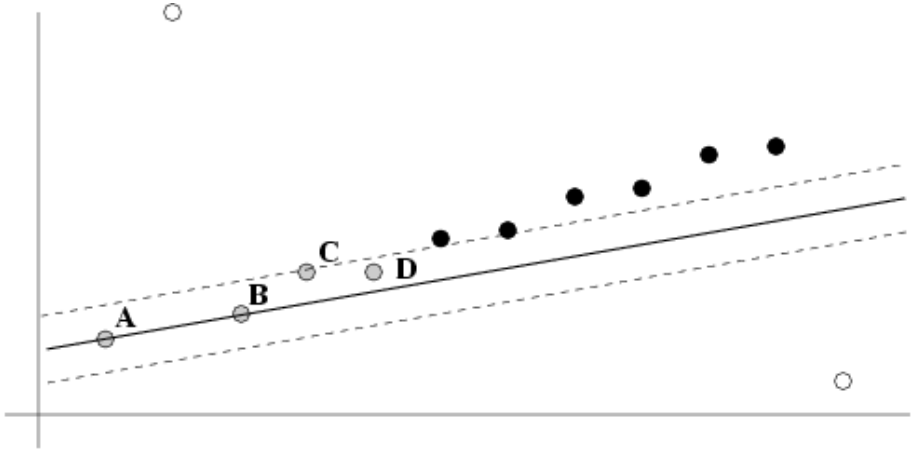
⇨ Choose *k* high enough to keep this below desired failure rate.

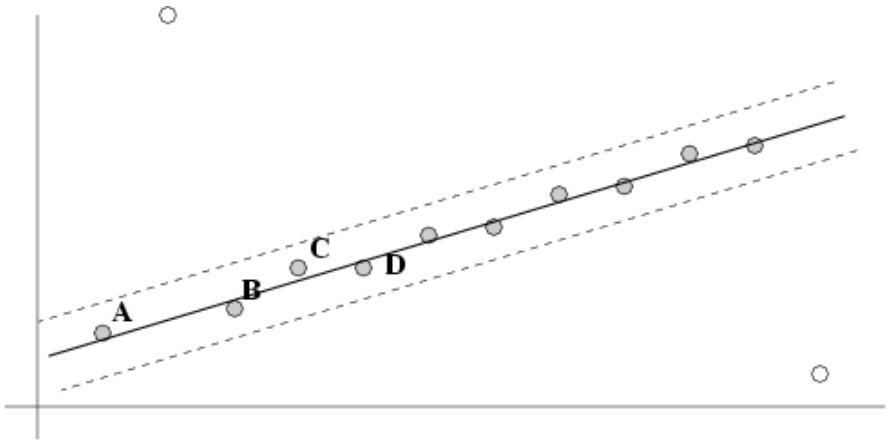
RANSAC: Computed k (p=0.99)

Sample size n	Proportion of outliers						
	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

After RANSAC

- RANSAC divides data into inliers and outliers and yields estimate computed from minimal set of inliers.
- Improve this initial estimate with estimation over all inliers (e.g. with standard least-squares minimization).
- But this may change inliers, so alternate fitting with re-classification as inlier/outlier.





RANSAC: Pros and Cons

- **Pros:**
  - General method suited for a **wide range** of model fitting problems
  - Easy to **implement** and easy to **calculate** its failure rate

- **Cons:**
  - Only handles a **moderate** percentage of outliers without cost blowing up
  - Many real problems have **high rate** of outliers (but sometimes selective choice of random subsets can help)

- A voting strategy, The Hough transform, can handle high percentage of outliers

What we will learn today?

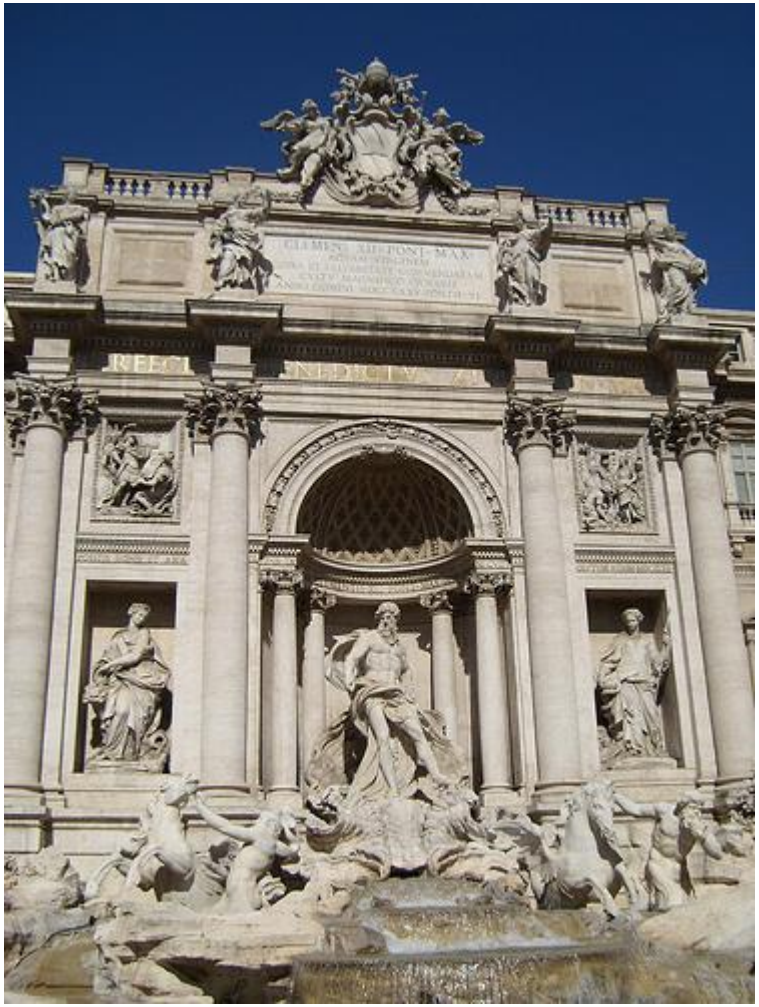
- A model fitting method for line detection
  - RANSAC
- Local invariant features
  - Motivation
  - Requirements, invariances
- Keypoint localization
  - Harris corner detector

- Some background reading:

  Rick Szeliski, Chapter 4.1.1; David Lowe, IJCV 2004

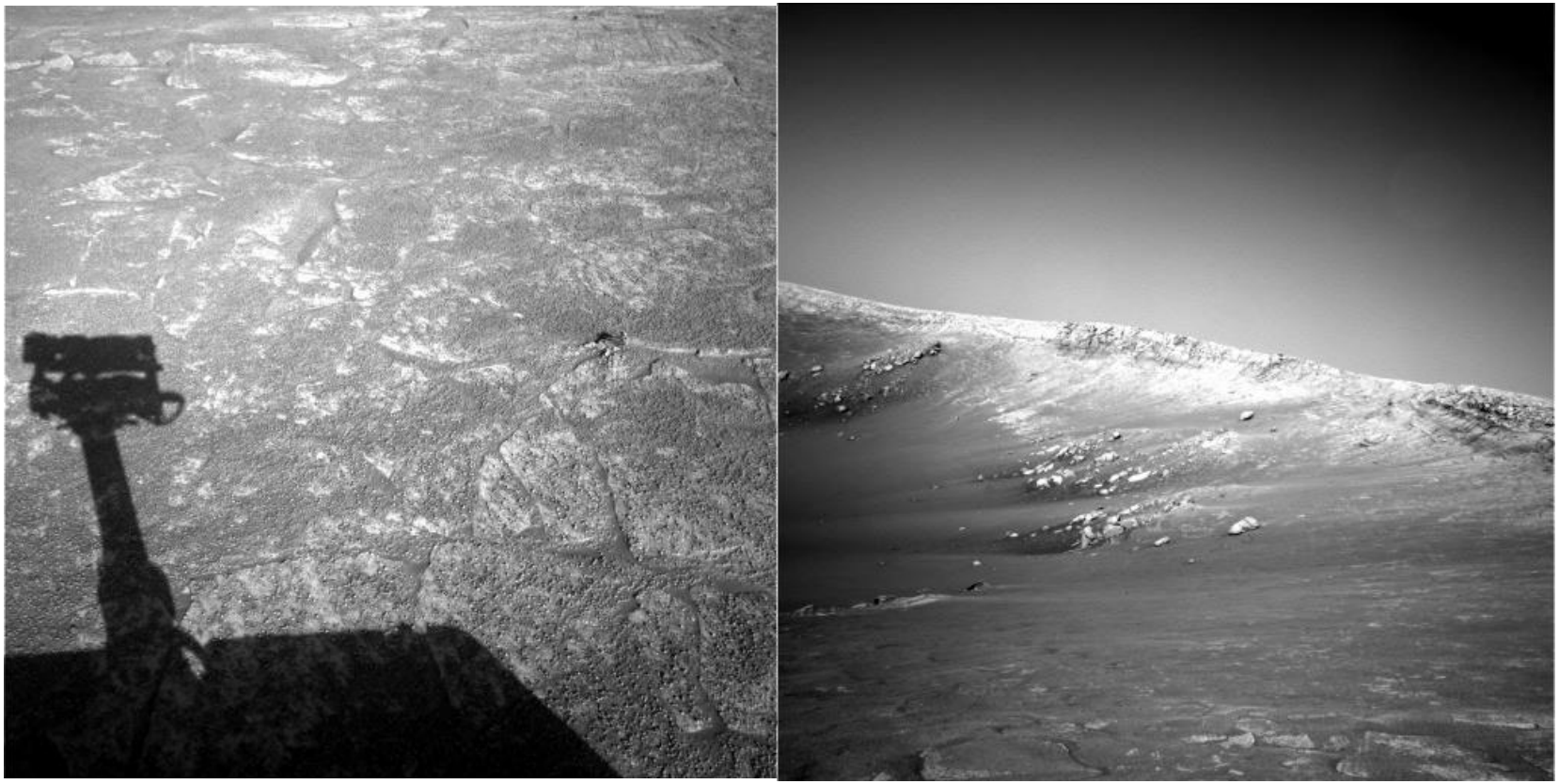
Image matching: a challenging problem

Image matching: a challenging problem



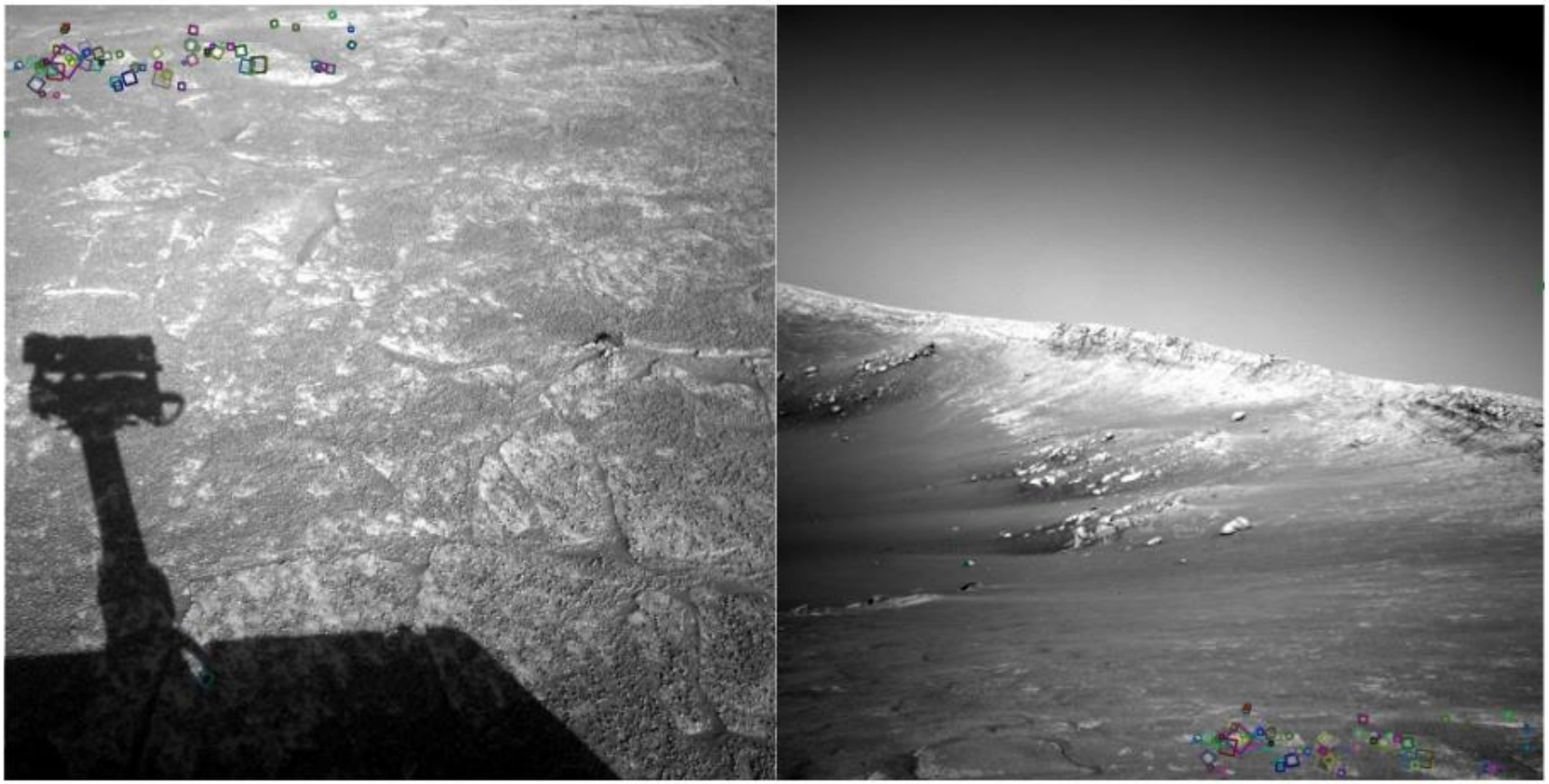
Harder Case

Harder Still?



NASA Mars Rover images

Answer Below (Look for tiny colored squares)



NASA Mars Rover images with SIFT feature matches
(Figure by Noah Snavely)

Applications

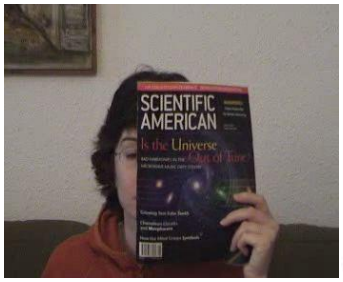
Comparison Between 2 or more images

- Image alignment
- Image Stitching
- 3D reconstruction
- Object recognition (matching)
- Indexing and database retrieval
- Object tracking
- Robot navigation

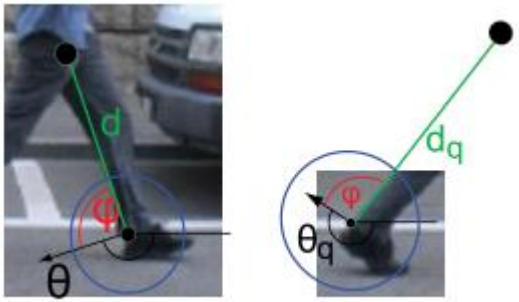
The following slides are borrowed from Course ME5286 (Saad J Bedros)

Motivation for using local features

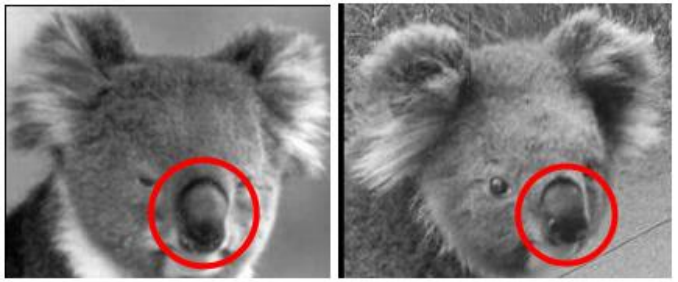
- Global representations have major limitations
- Instead, describe and match only local regions
- Increased robustness to
  - Occlusions



  - Articulation

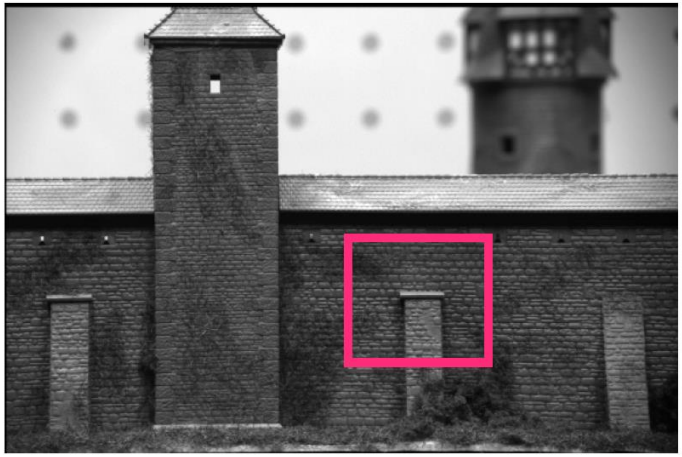


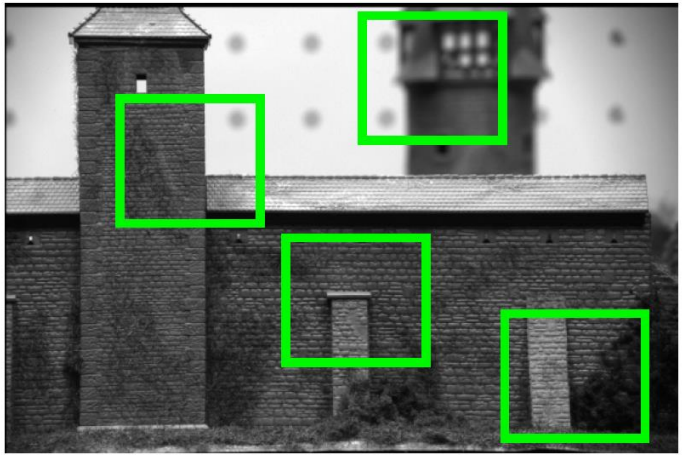
  - Intra-category variations



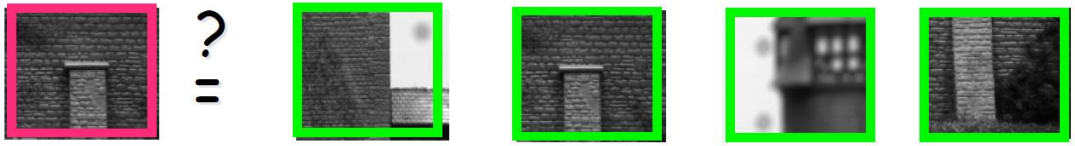
Motivation: Patch Matching

Elements to be matched are image patches of fixed size

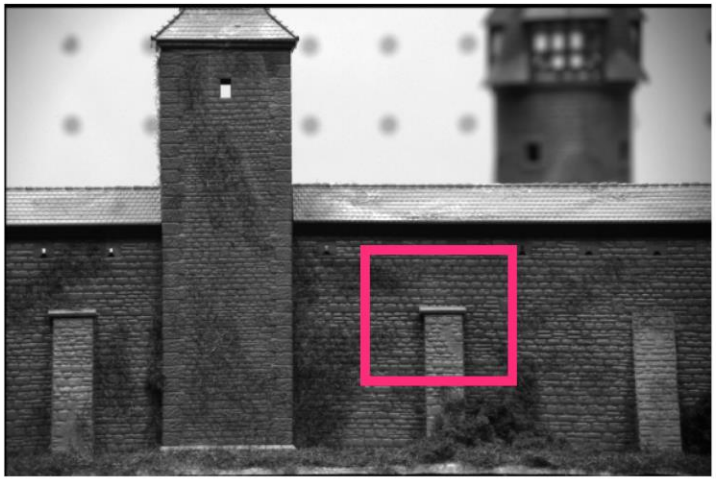


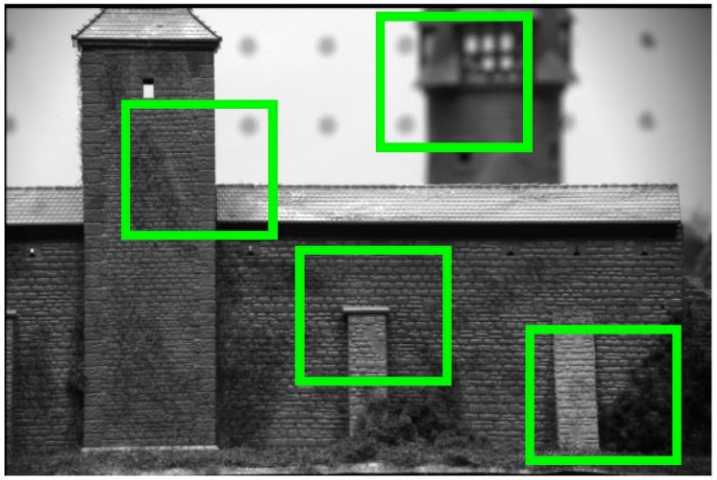


Task: find the best (most similar) patch in a second image

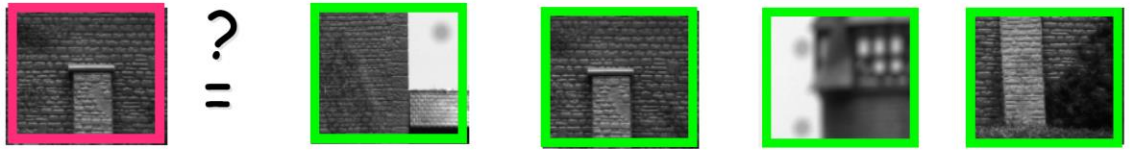


Not all Patches are Created Equal!

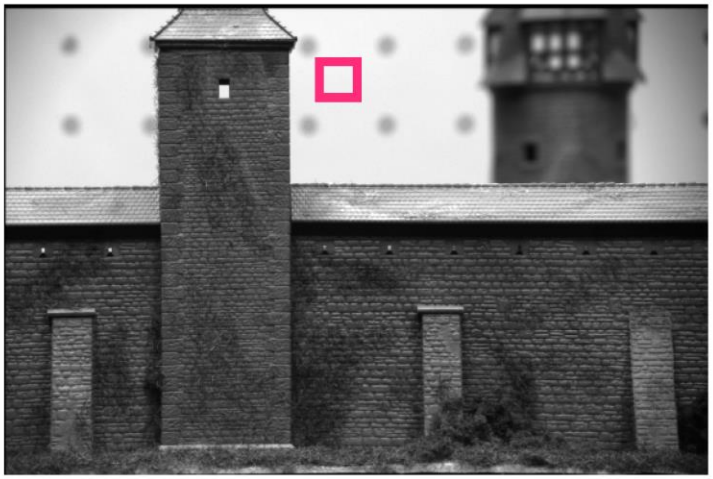




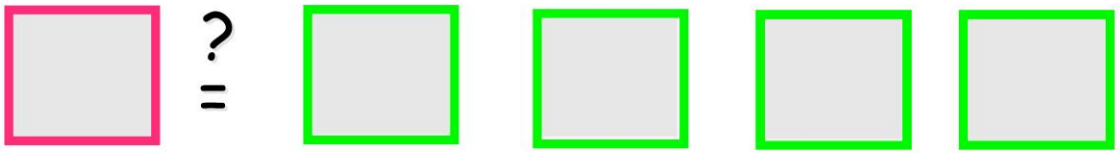
Inituition: this would be a good patch for matching, since it is very distinctive (there is only one patch in the second frame that looks similar).



Not all Patches are Created Equal!



Inituition: this would be a BAD patch for matching, since it is not very distinctive (there are many similar patches in the second frame)

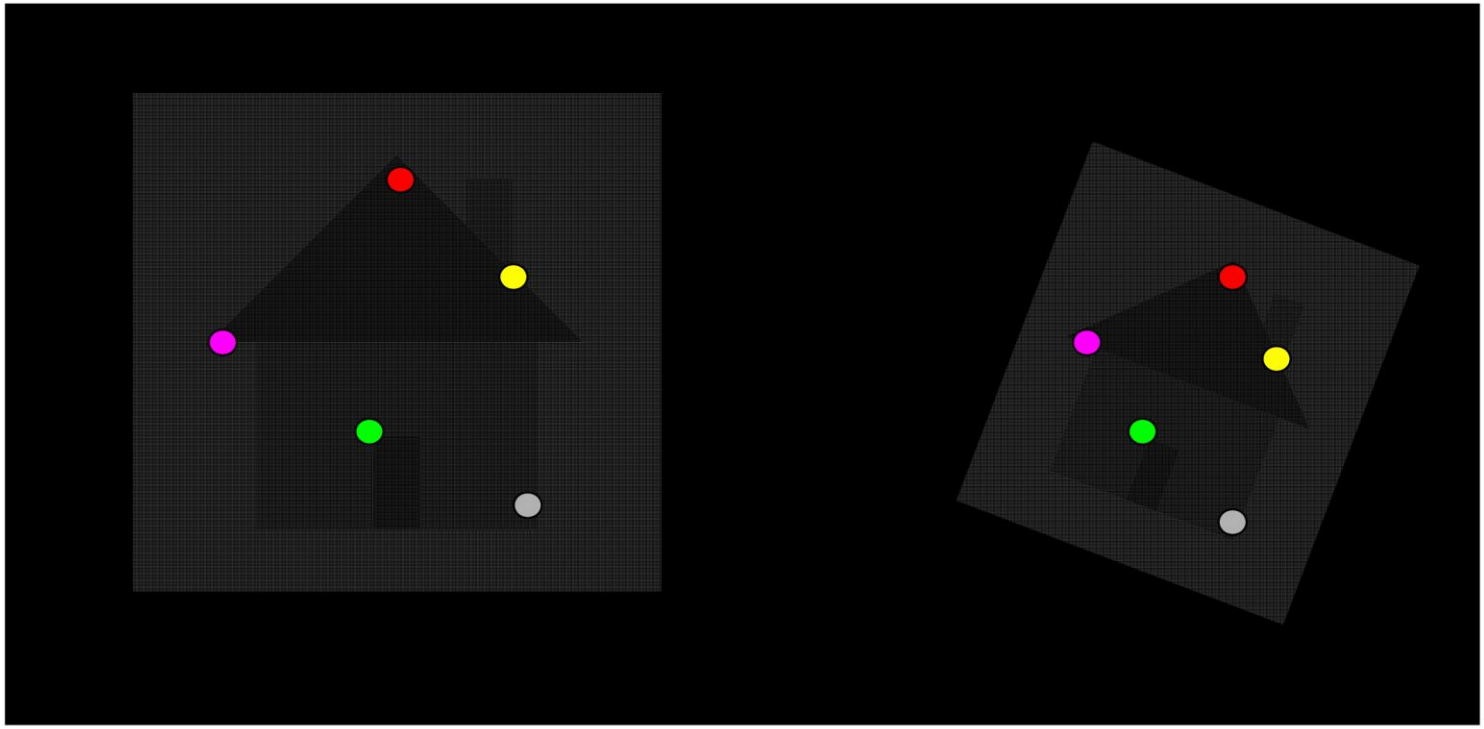


Interest Points (Key Points)

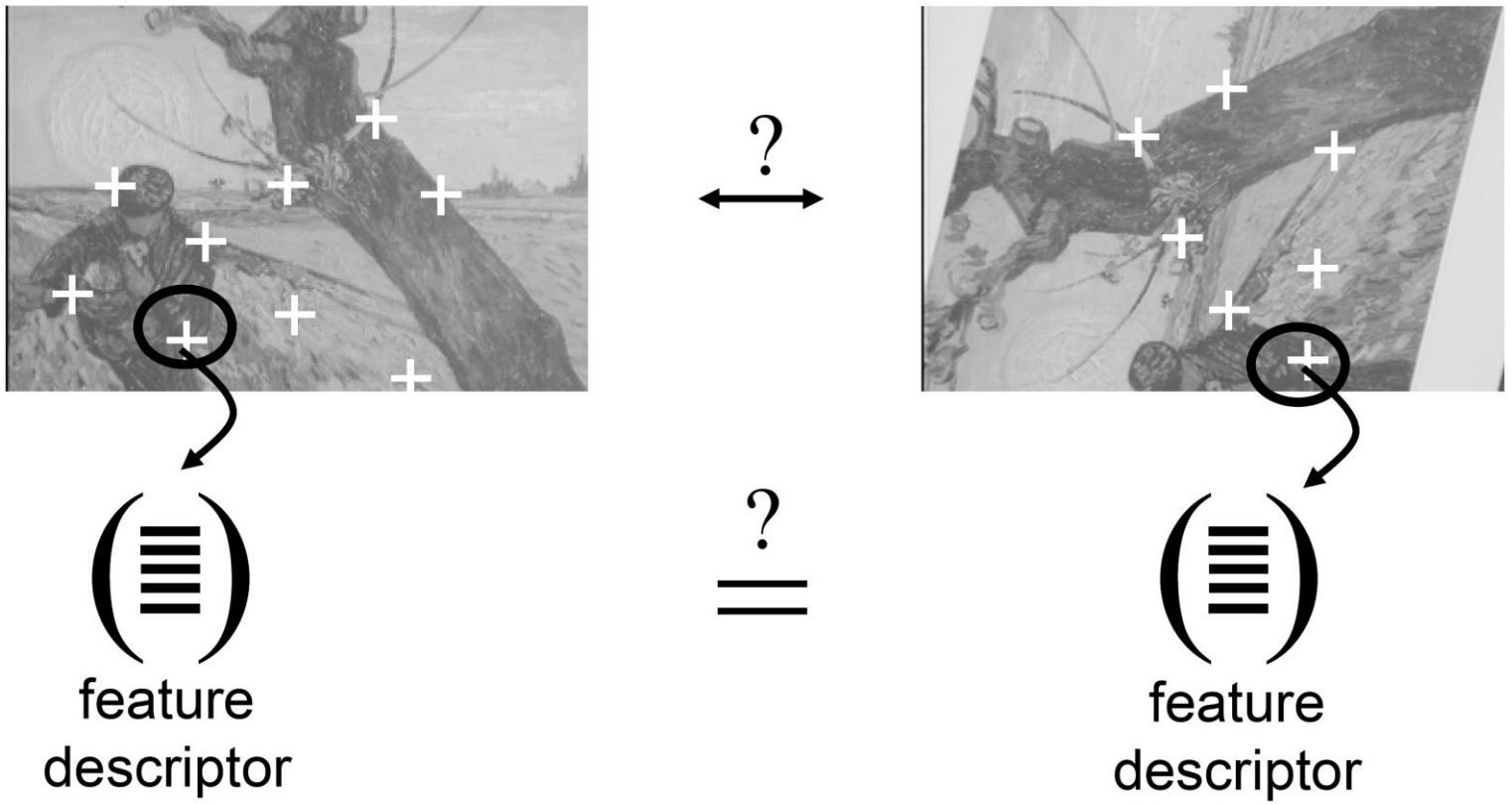
- Local features associated with a significant change of an image property of several properties simultaneously (e.g., intensity, color, texture).

Why Extract Interest Points?

- Corresponding points (or features) between images enable the estimation of parameters describing geometric transforms between the images.



What if we don't know the correspondences?



- Need to compare *feature descriptors* of local patches surrounding interest points

Properties of good features

- **Local:** features are local, robust to occlusion and clutter (no prior segmentation!).
- **Accurate:** precise localization.

- **Invariant / Covariant**
- **Robust:** noise, blur, compression, etc. do not have a big impact on the feature.

} **Repeatable**

- **Distinctive:** individual features can be matched to a large database of objects.
- **Efficient:** close to real-time performance.

Invariant / Covariant

- A function f( ) is **invariant** under some transformation T( ) if its value does change when the transformation is applied to its argument:

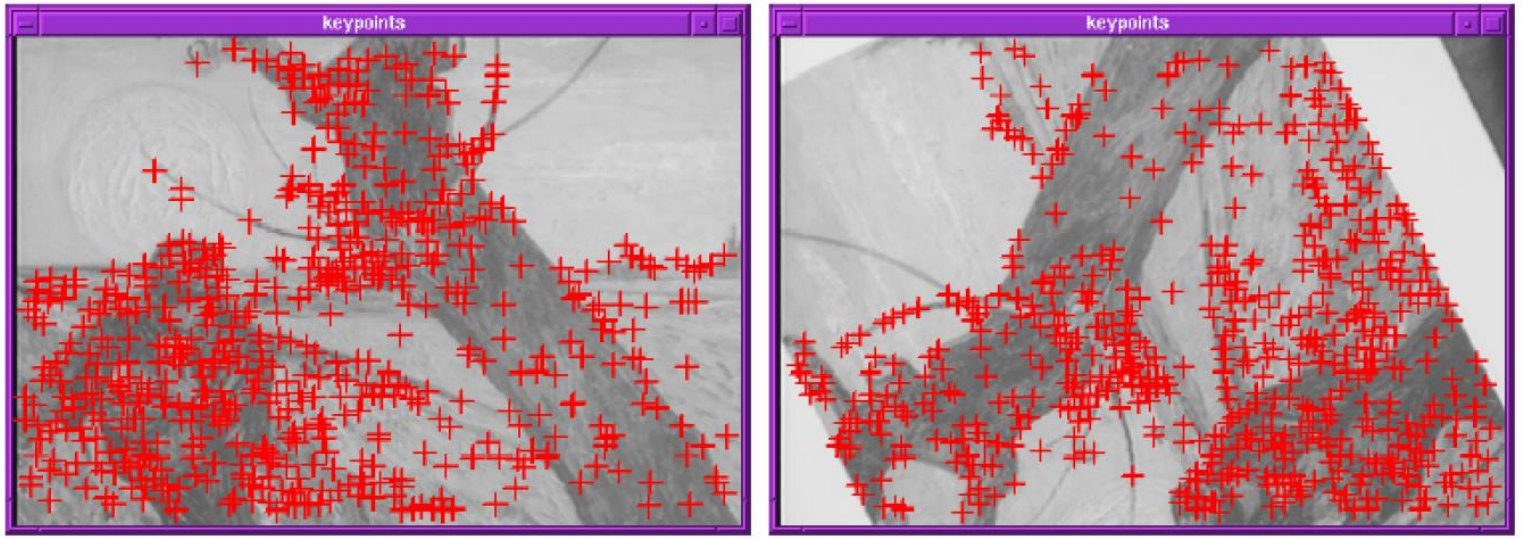
$$\text{if } f(x) = y \text{ then } f(T(x))=y$$

- A function f( ) is **covariant** when it commutes with the transformation T( ):

$$\text{if } f(x) = y \text{ then } f(T(x))=T(f(x))=T(y)$$

Invariance

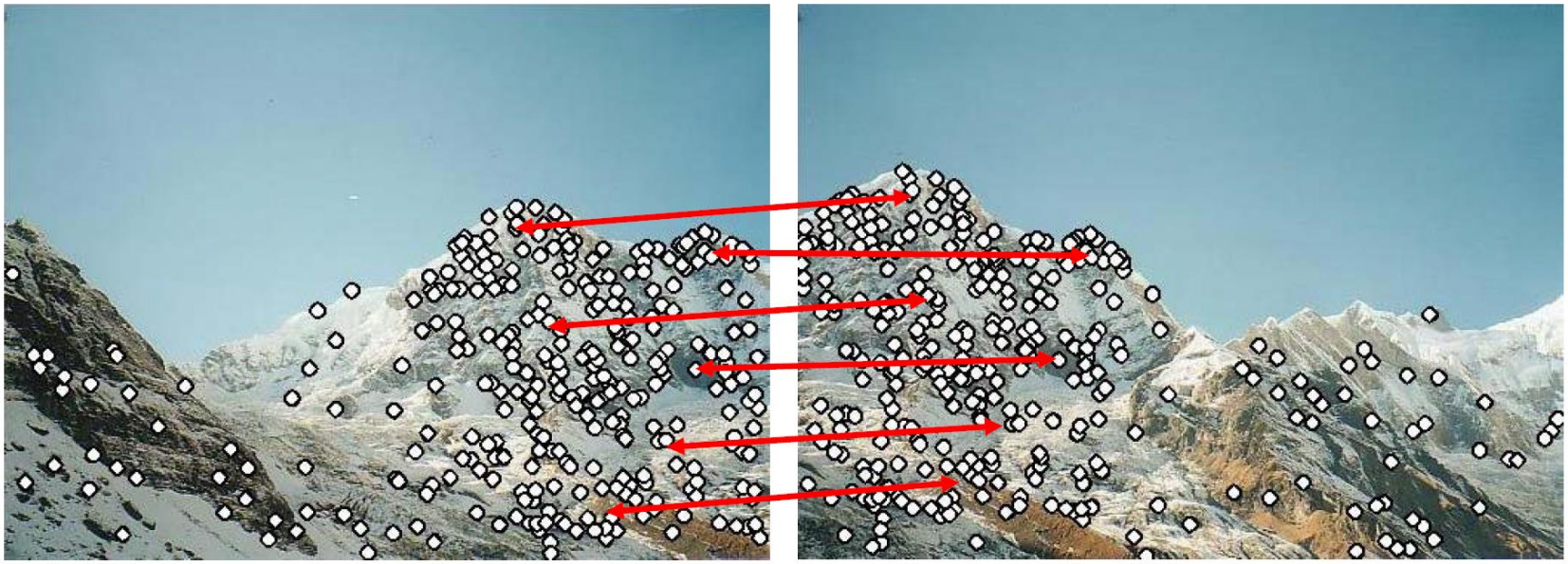
- Features should be detected despite geometric or photometric changes in the image.
- Given two transformed versions of the same image, features should be detected in corresponding locations.



Example: Panorama Stitching

- How do we combine these two images?

Example: Panorama Stitching



Step 1: extract features

Step 2: match features

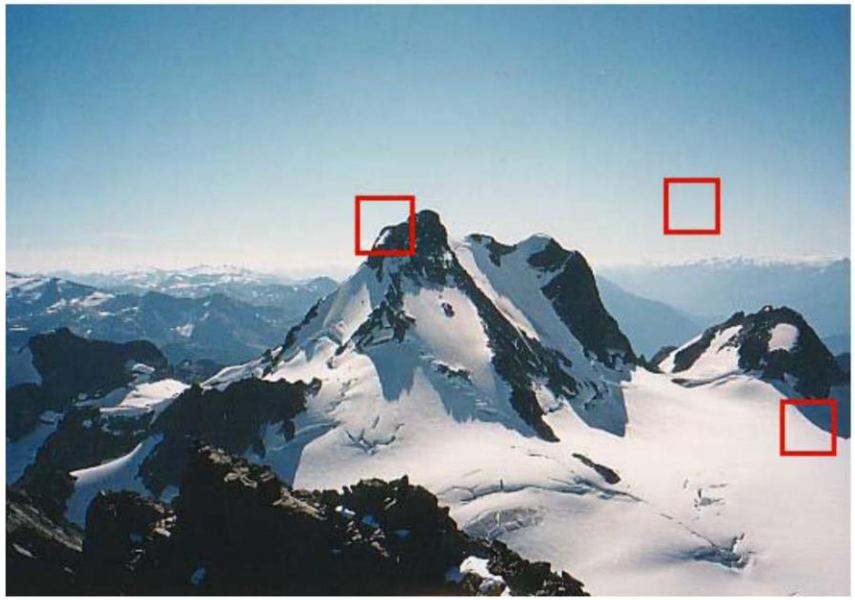
Example: Panorama Stitching

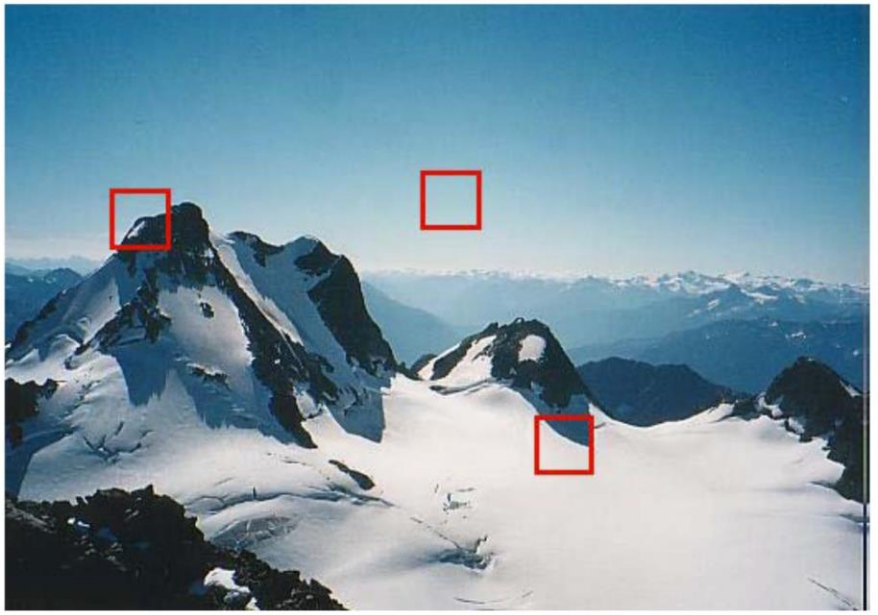
Step 1: extract features

Step 2: match features

Step 3: align images

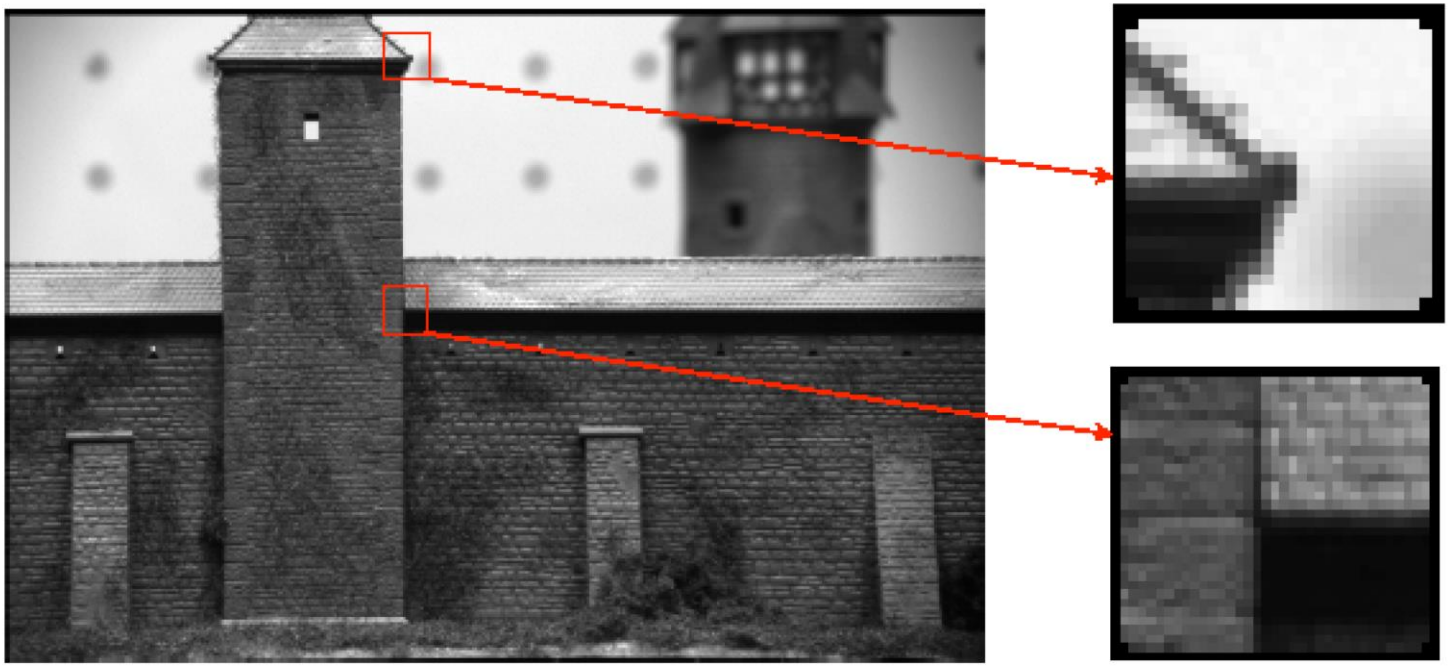
What features should we use?





Use features with gradients in at least two (significantly) different orientations → patches ? Corners ?

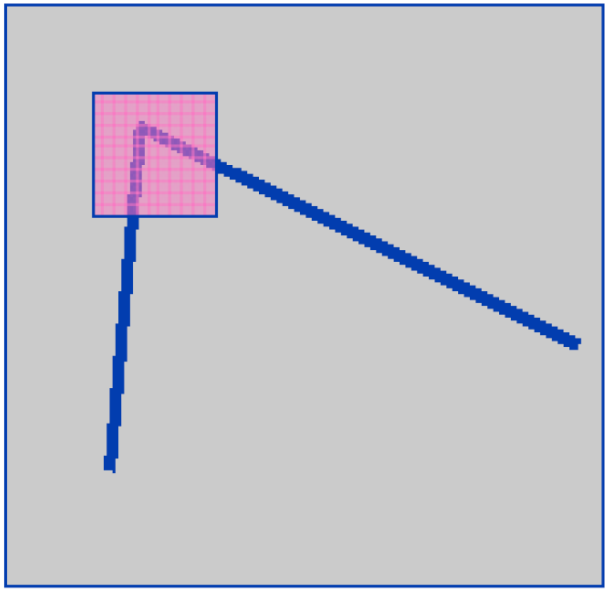
What are Corners?



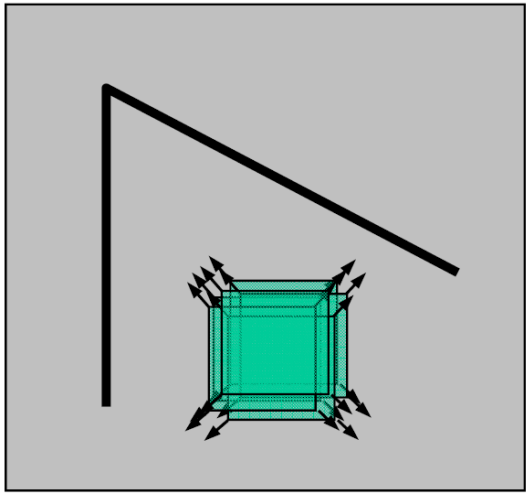
- Intuitively, junctions of contours.
- Generally more stable features over changes of viewpoint
- Intuitively, large variations in the neighborhood of the point in all directions
- They are good features to match!

Corner Points: Basic Idea

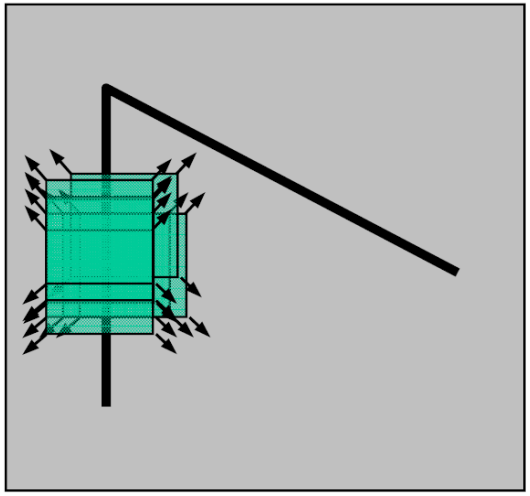
- We should easily recognize the point by looking at intensity values within a small window
- Shifting the window in *any direction* should yield a *large change* in appearance.



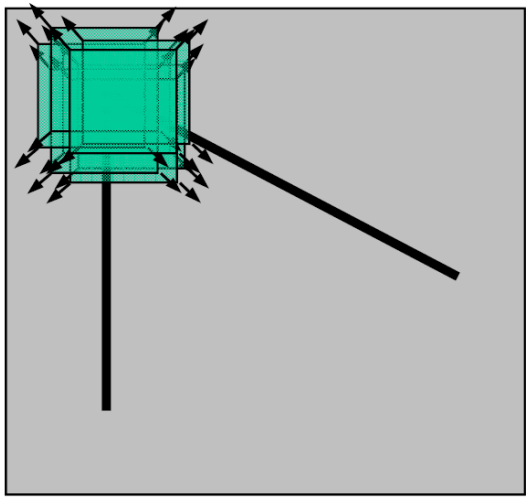
Corner Detector: Basic Idea



"flat" region:
no change in
all directions



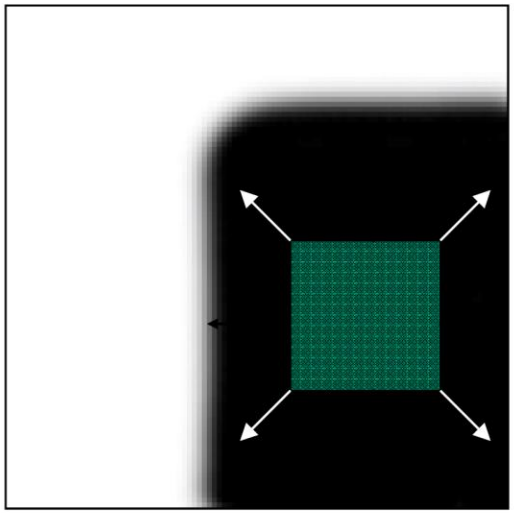
"edge":
no change along
the edge direction



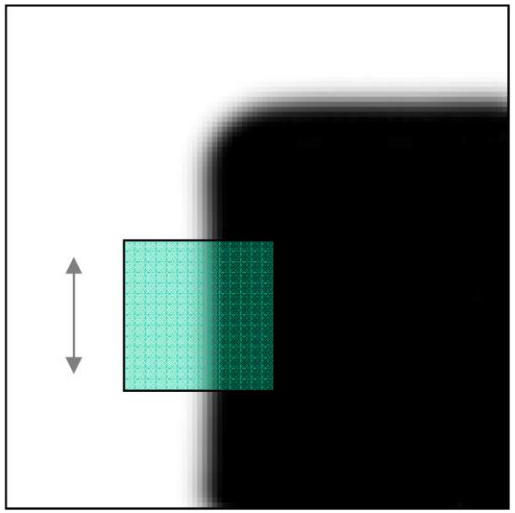
"corner":
significant change
in all directions

Corner Detection Using Intensity: Basic Idea

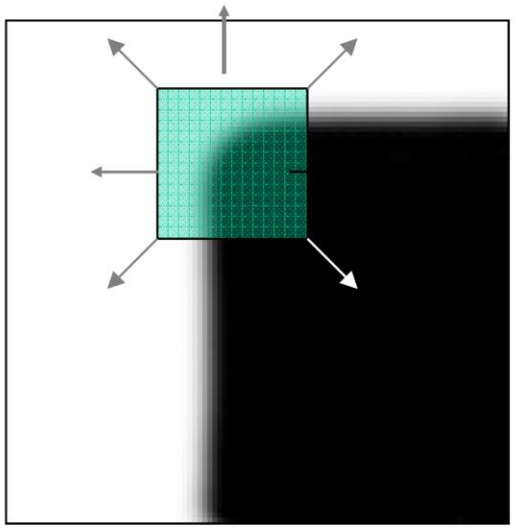
- Image gradient has two or more dominant directions near a corner.
- Shifting a window in *any direction* should give *a large change* in intensity.



"flat" region:
no change in all directions



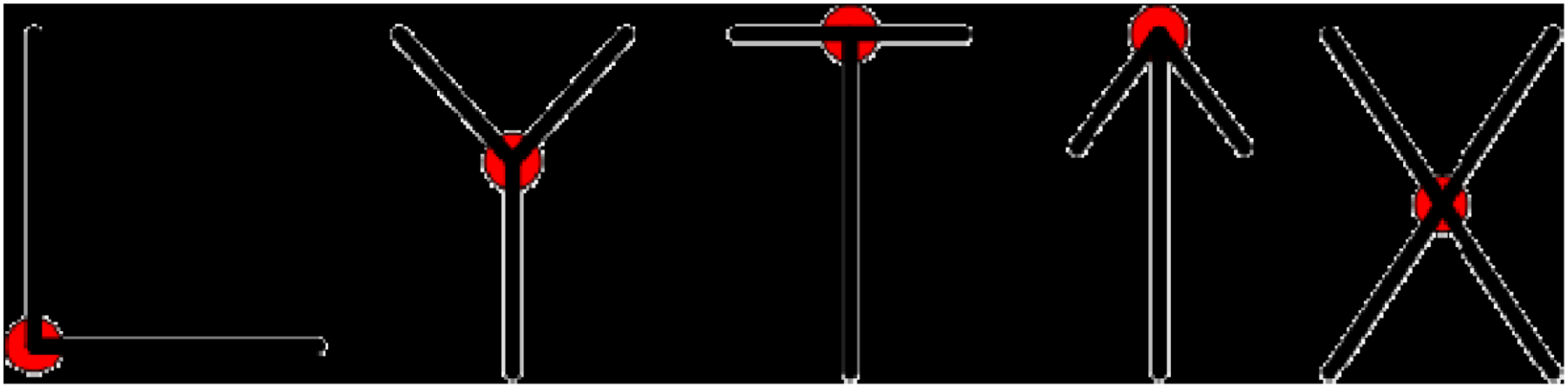
"edge": no change along the edge direction



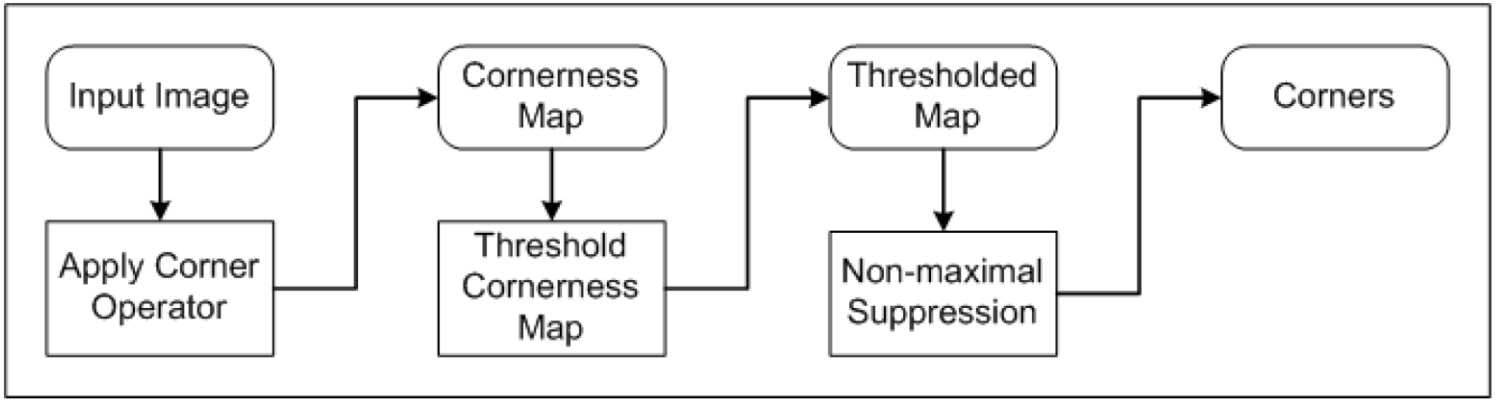
"corner": significant change in all directions

Corner Types

Example of L-junction, Y-junction, T-junction, Arrow-junction, and X-junction corner types

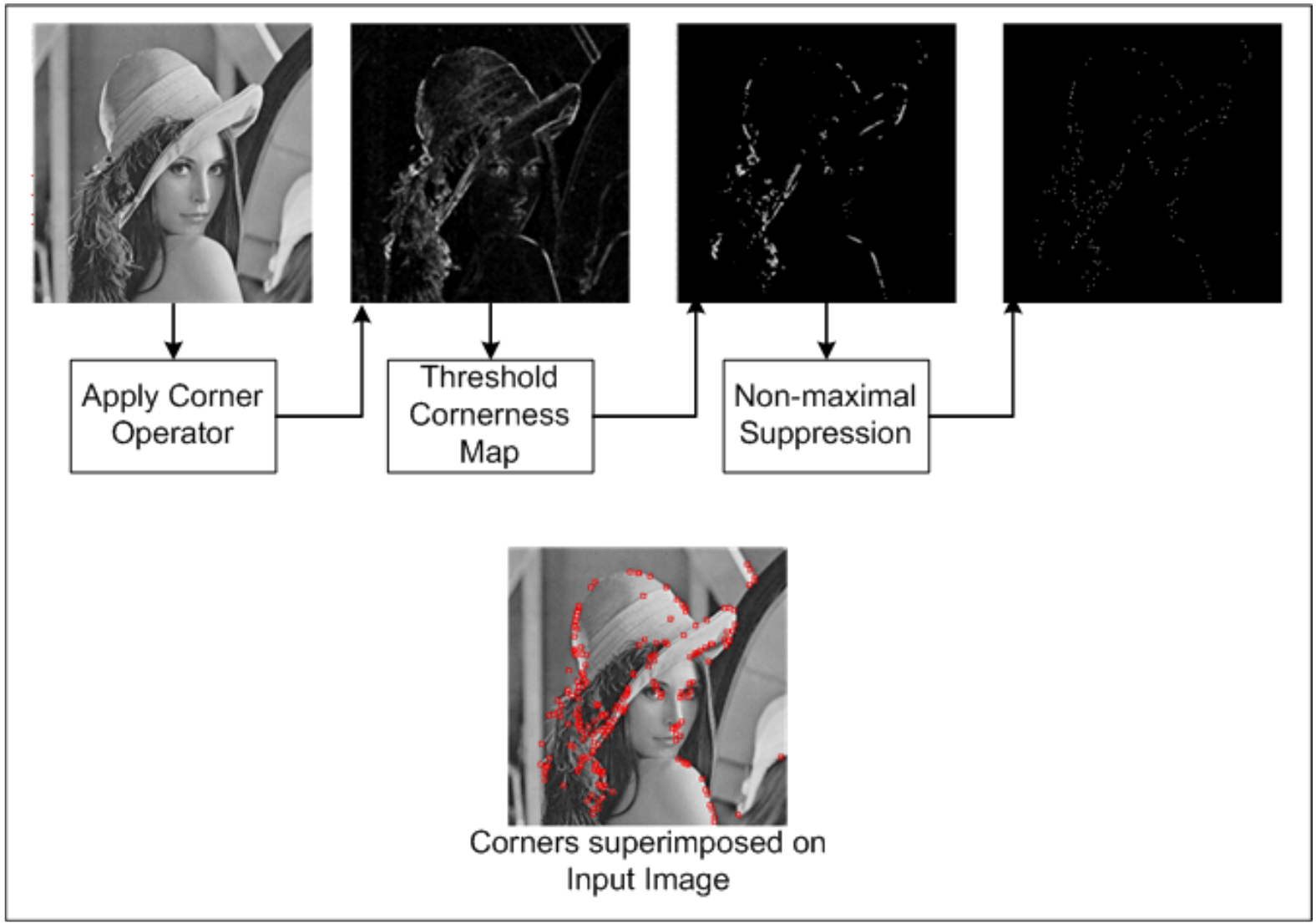


Mains Steps in Corner Detection



1. For each pixel in the input image, the corner operator is applied to obtain a *cornerness* measure for this pixel.
2. Threshold cornerness map to eliminate weak corners.
3. Apply non-maximal suppression to eliminate points whose cornerness measure is not larger than the cornerness values of all points within a certain distance.

Mains Steps in Corner Detection

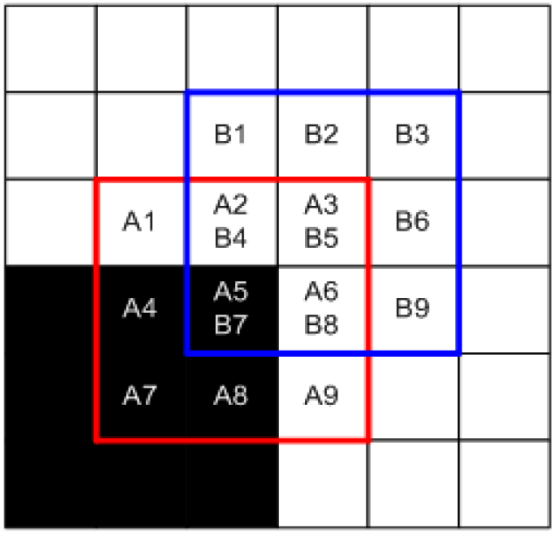


Moravec Detector (1977)

Measure intensity variation at (x,y) by shifting a small window (3x3 or 5x5) by one pixel in each of the **eight** principle directions (horizontally, vertically, and four diagonals).

Moravec Detector (1977)

Calculate intensity variation by taking the sum of squares of intensity differences of corresponding pixels in these two windows.



$$S_W(\Delta x, \Delta y) = \sum_{x_i \in W} \sum_{y_i \in W} \left(f(x_i, y_i) - f(x_i - \Delta x, y_i - \Delta y)\right)^2 .$$

8 directions

$\Delta x, \Delta y$ in {-1,0,1}

$S_W(-1,-1), S_W(-1,0), \ldots S_W(1,1)$

Moravec Detector (1977)

- The "cornerness" of a pixel is the minimum intensity variation found over the eight shift directions:

$$\text{Cornerness}(x,y) = \min\{S_W(-1,-1), S_W(-1,0), \ldots S_W(1,1)\}$$

X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
X	X	0	0	0	0	0	0	0	0	0	1	1	1	X	X
X	X	0	0	0	0	0	1	1	0	0	1	2	1	X	X
X	X	0	0	0	0	0	2	1	0	0	1	1	1	X	X
X	X	0	0	0	0	0	0	0	0	0	0	0	0	X	X
X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X

Cornerness Map (normalized)

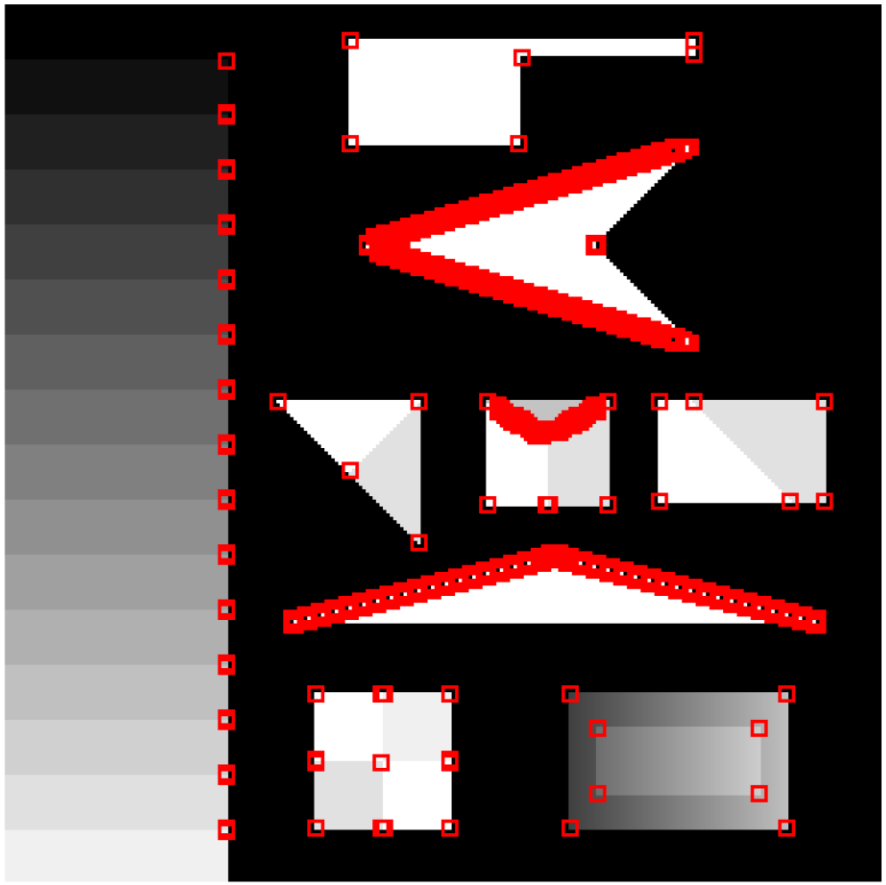
Note response to isolated points!

Moravec Detector (1977)

- Use Non-maximal suppression will yield the final corners. Process of Zero out all pixels that are not the maximum along the direction of the gradient (look at 1 pixel on each side)

X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
X	X	0	0	0	0	0	0	0	0	0	1	1	1	X	X
X	X	0	0	0	0	0	1	1	0	0	1	2	1	X	X
X	X	0	0	0	0	0	2	1	0	0	1	1	1	X	X
X	X	0	0	0	0	0	0	0	0	0	0	0	0	X	X
X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X

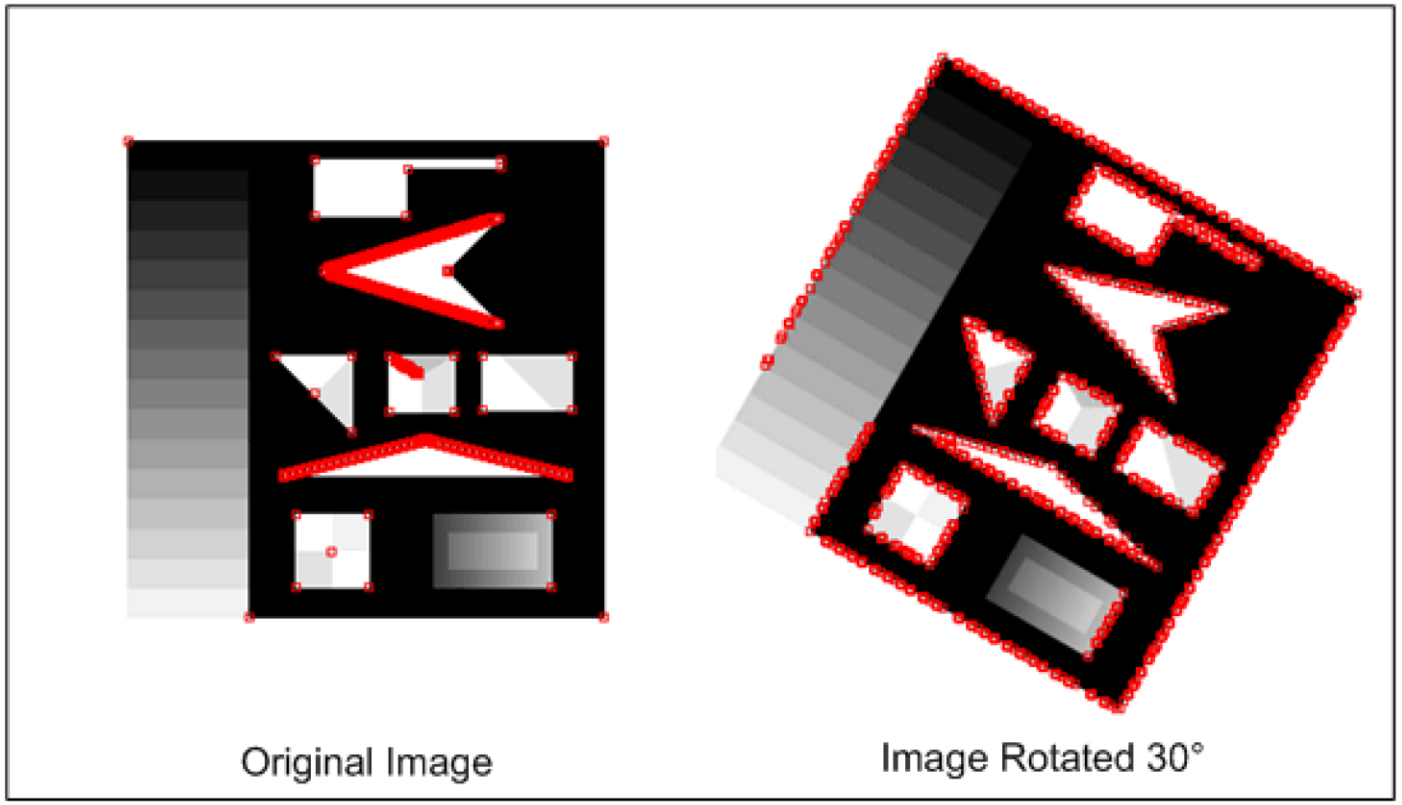
Moravec Detector (1977)



- Does a reasonable job in finding the majority of true corners.
- Edge points not in one of the eight principle directions will be assigned a relatively large cornerness value.

Moravec Detector (1977)

- The response is anisotropic (directionally sensitive) as the intensity variation is only calculated at a discrete set of shifts (i.e., not rotationally invariant)



What we will learn today?

- A model fitting method for line detection
  - RANSAC
- Local invariant features
  - Motivation
  - Requirements, invariances
- Keypoint localization
  - Harris corner detector

- Some background reading:

Rick Szeliski, Chapter 4.1.1; David Lowe, IJCV 2004

Harris Detector: Mathematics

Change of intensity for the shift $[u,v]$:

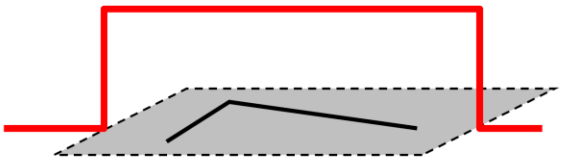
$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

Window function

Shifted intensity

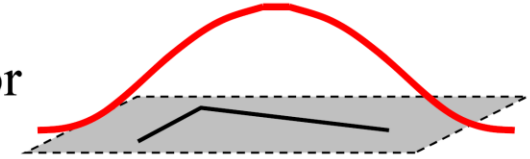
Intensity

Window function $w(x,y)$ =



1 in window, 0 outside

or



Gaussian

Harris Detector

Change of intensity for the shift $[u, v]$:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u, y+v) - I(x,y)\right]^2$$

Window function

Shifted intensity

Intensity

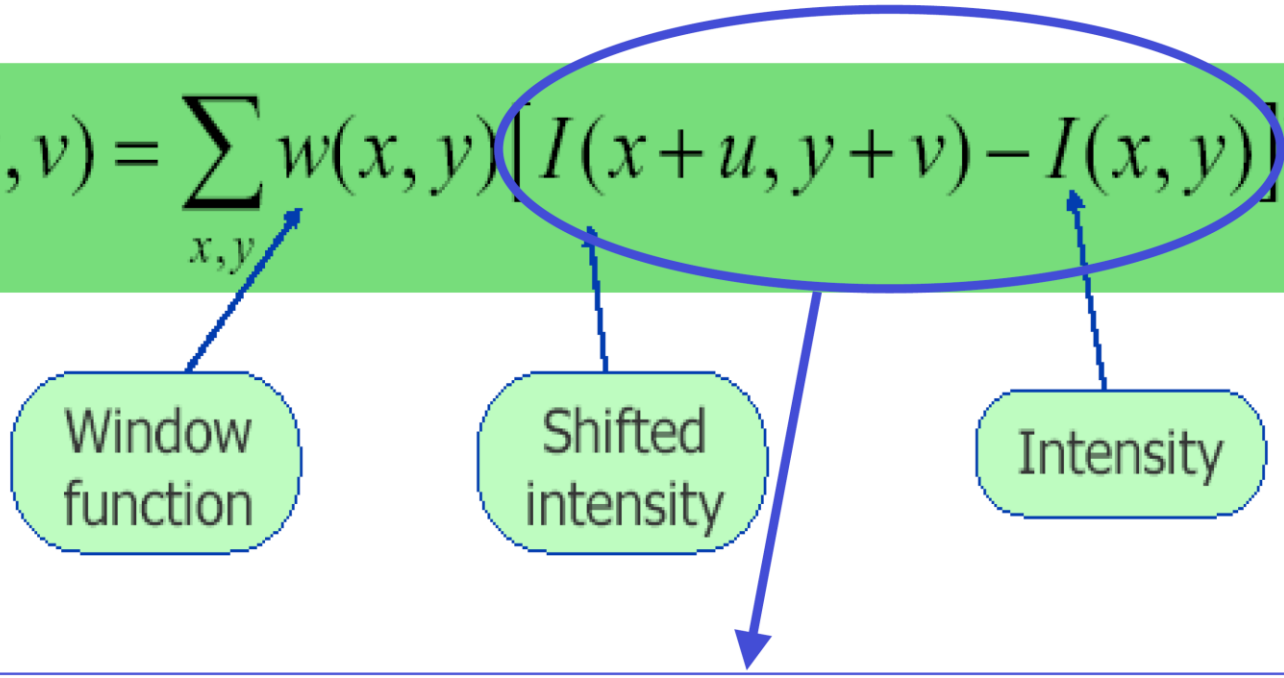
Window function $w(x,y)$ =

- Improves the Moravec operator by avoiding the use of discrete directions and discrete shifts.
- Uses a Gaussian window instead of a square window.

Harris Detector: Intuition

Change of intensity for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$



For nearly constant patches, this will be near 0.
For very distinctive patches, this will be larger.
Hence... we want patches where E(u,v) is LARGE.

Taylor Series for 2D Functions

- 一元函数在点x_k处的泰勒展开式为：

$$f(x) = f(x_k) + (x - x_k)f'(x_k) + \frac{1}{2!}(x - x_k)^2 f''(x_k) + o^n$$

- 二元函数在点(x_k, y_k)处的泰勒展开式为：

$$\begin{aligned} f(x,y) = {} & f(x_k,y_k) + (x - x_k)f'_x(x_k,y_k) + (y - y_k)f'_y(x_k,y_k) \\ & + \frac{1}{2!}(x - x_k)^2 f''_{xx}(x_k,y_k) + \frac{1}{2!}(x - x_k)(y - y_k)f''_{xy}(x_k,y_k) \\ & + \frac{1}{2!}(x - x_k)(y - y_k)f''_{yx}(x_k,y_k) + \frac{1}{2!}(y - y_k)^2 f''_{yy}(x_k,y_k) \\ & + o^n \end{aligned}$$

Taylor Series for 2D Functions

$$f(x+u,y+v) = f(x,y) + uf_x(x,y) + vf_y(x,y) +$$

First partial derivatives

$$\frac{1}{2!}\left[u^2 f_{xx}(x,y) + uv f_{xy}x,y + v^2 f_{yy}(x,y)\right] +$$

Second partial derivatives

$$\frac{1}{3!}\left[u^3 f_{xxx}(x,y) + u^2 v f_{xxy}(x,y) + uv^2 f_{xyy}(x,y) + v^3 f_{yyy}(x,y)\right]$$

Third partial derivatives

$+ \ldots$ (Higher order terms)

First order approx

$$f(x+u,y+v) \approx f(x,y) + uf_x(x,y) + vf_y(x,y)$$

Harris Corner Derivation

$$\sum [I(x+u, y+v) - I(x,y)]^2$$

$$\approx \sum [I(x,y) + uI_x + vI_y - I(x,y)]^2$$ First order approx

$$= \sum u^2 I_x^2 + 2uvI_xI_y + v^2 I_y^2$$

$$= \sum \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} I_x^2 & I_xI_y \\ I_xI_y & I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$ Rewrite as matrix equation

$$= \begin{bmatrix} u & v \end{bmatrix} \left(\sum \begin{bmatrix} I_x^2 & I_xI_y \\ I_xI_y & I_y^2 \end{bmatrix} \right) \begin{bmatrix} u \\ v \end{bmatrix}$$

Harris Detector: Mathematics

For small shifts $[u,v]$ we have a *bilinear* approximation:

$$E(u,v) \cong [u,v] \quad M \quad \begin{bmatrix} u \\ v \end{bmatrix}$$

where M is a 2×2 matrix computed from image derivatives:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Windowing function - computing a weighted sum (simplest case, w=1)

Note: these are just products of components of the gradient, Ix, Iy

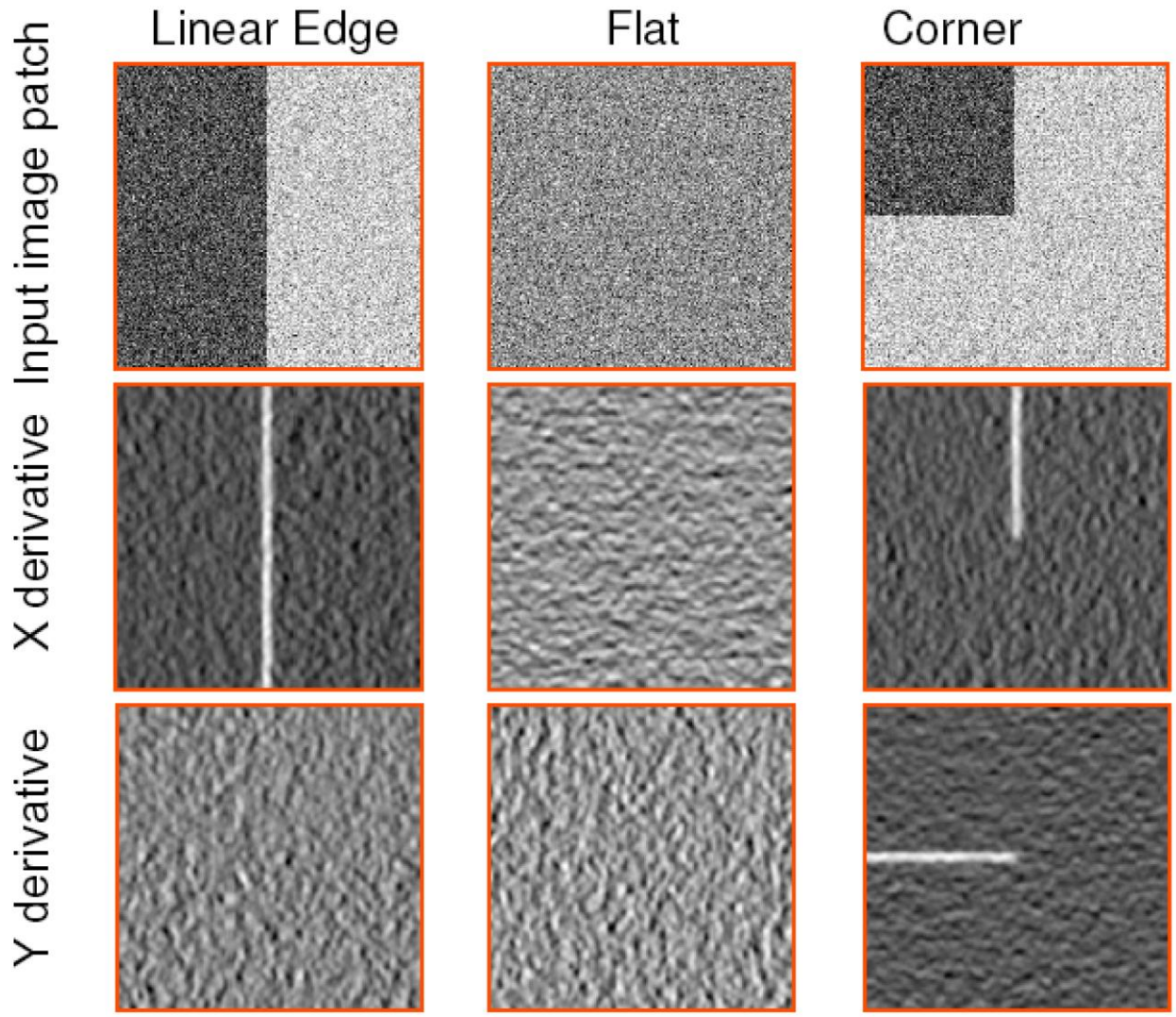
Intuitive Way to Understand Harris

Treat gradient vectors as a set of (dx,dy) points with a center of mass defined as being at (0,0).

Fit an ellipse to that set of points via scatter matrix

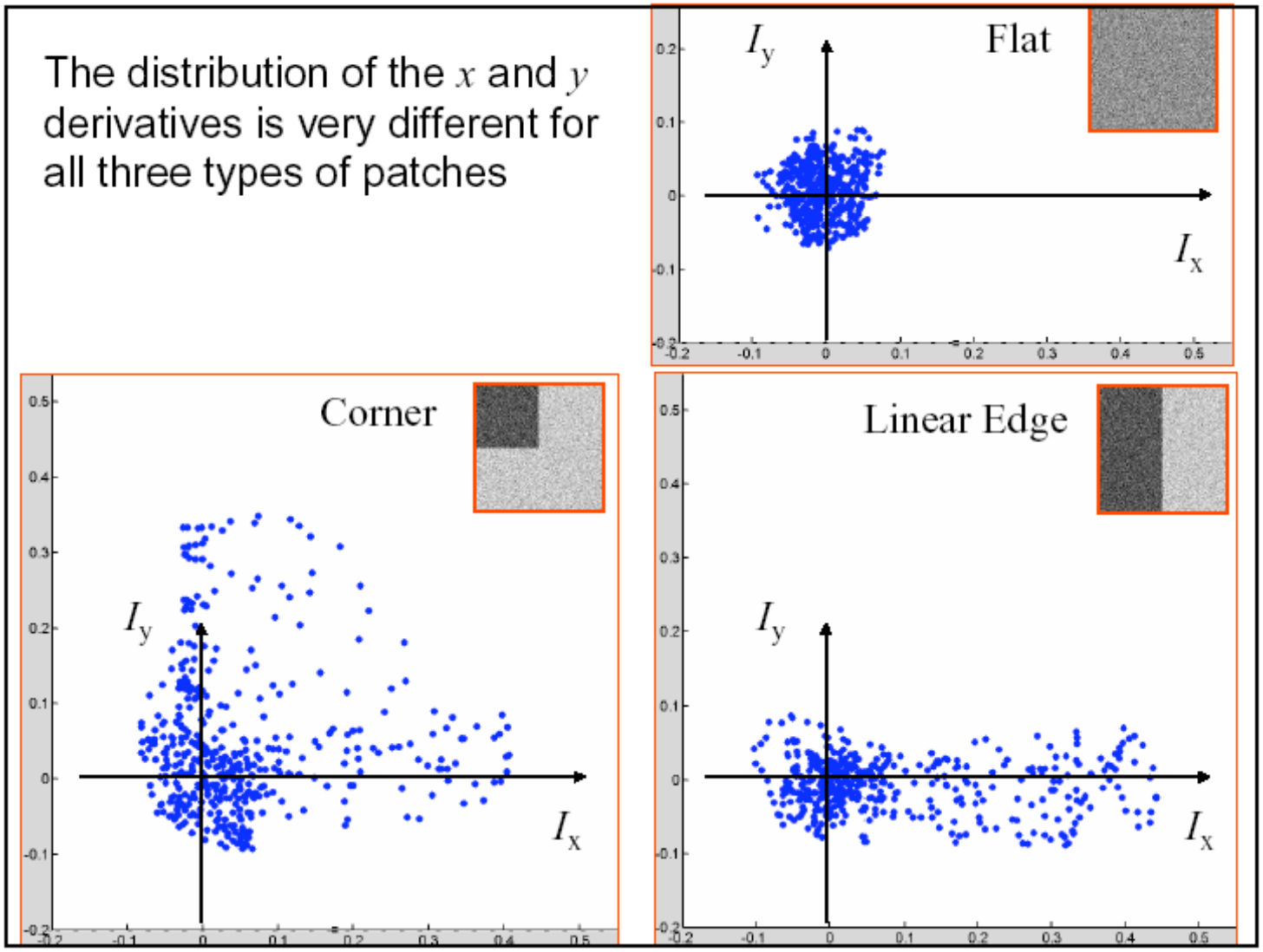
Analyze ellipse parameters for varying cases…

Example: Cases and 2D Derivatives



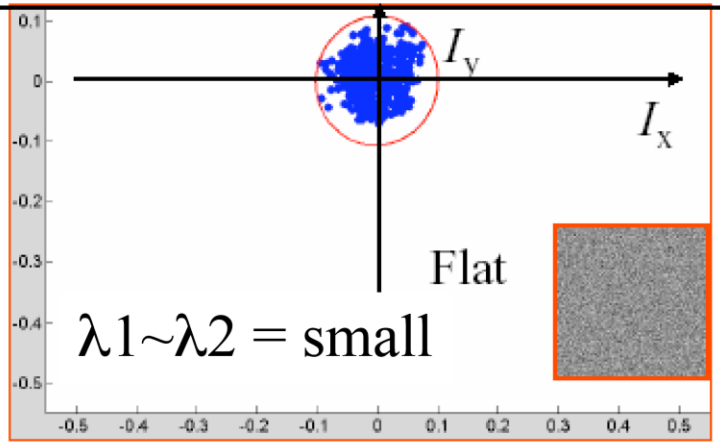
Plotting Derivatives as 2D Points

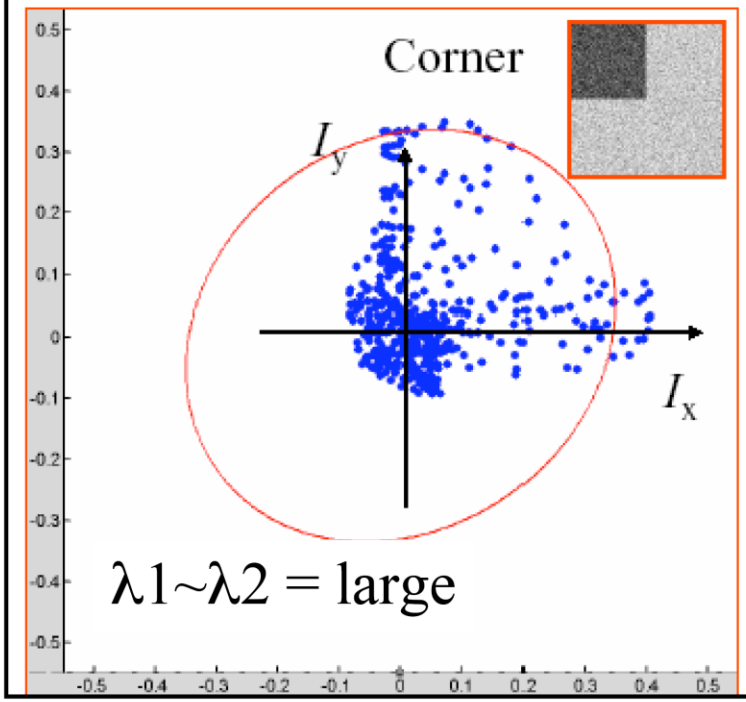
The distribution of the x and y derivatives is very different for all three types of patches

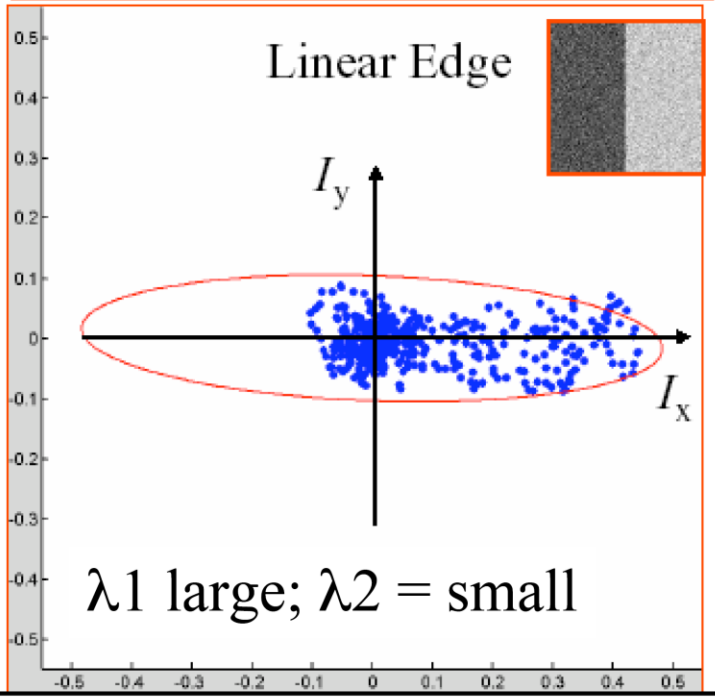


Fitting Ellipse to each Set of Points

The distribution of x and y derivatives can be characterized by the shape and size of the principal component ellipse



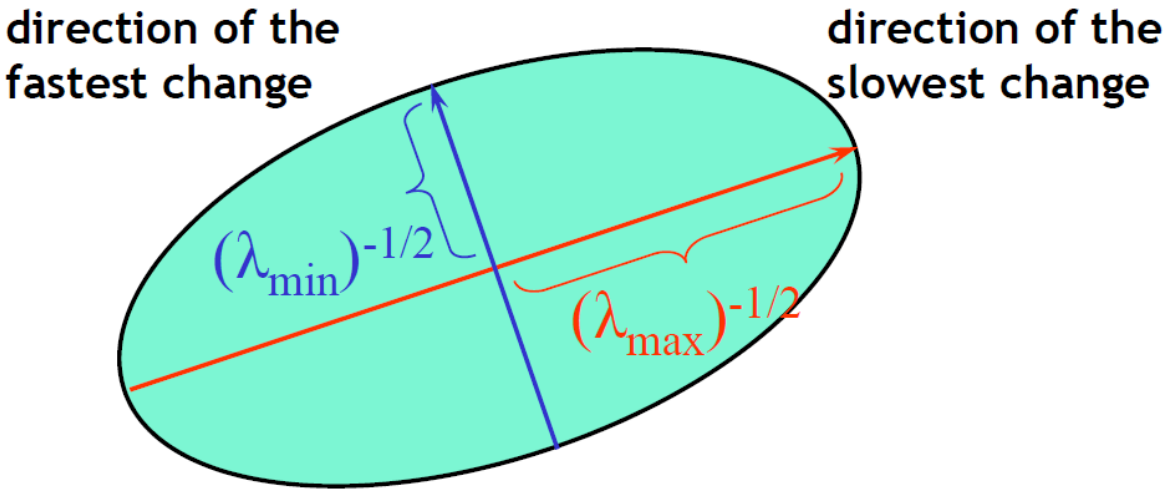




Harris Detector

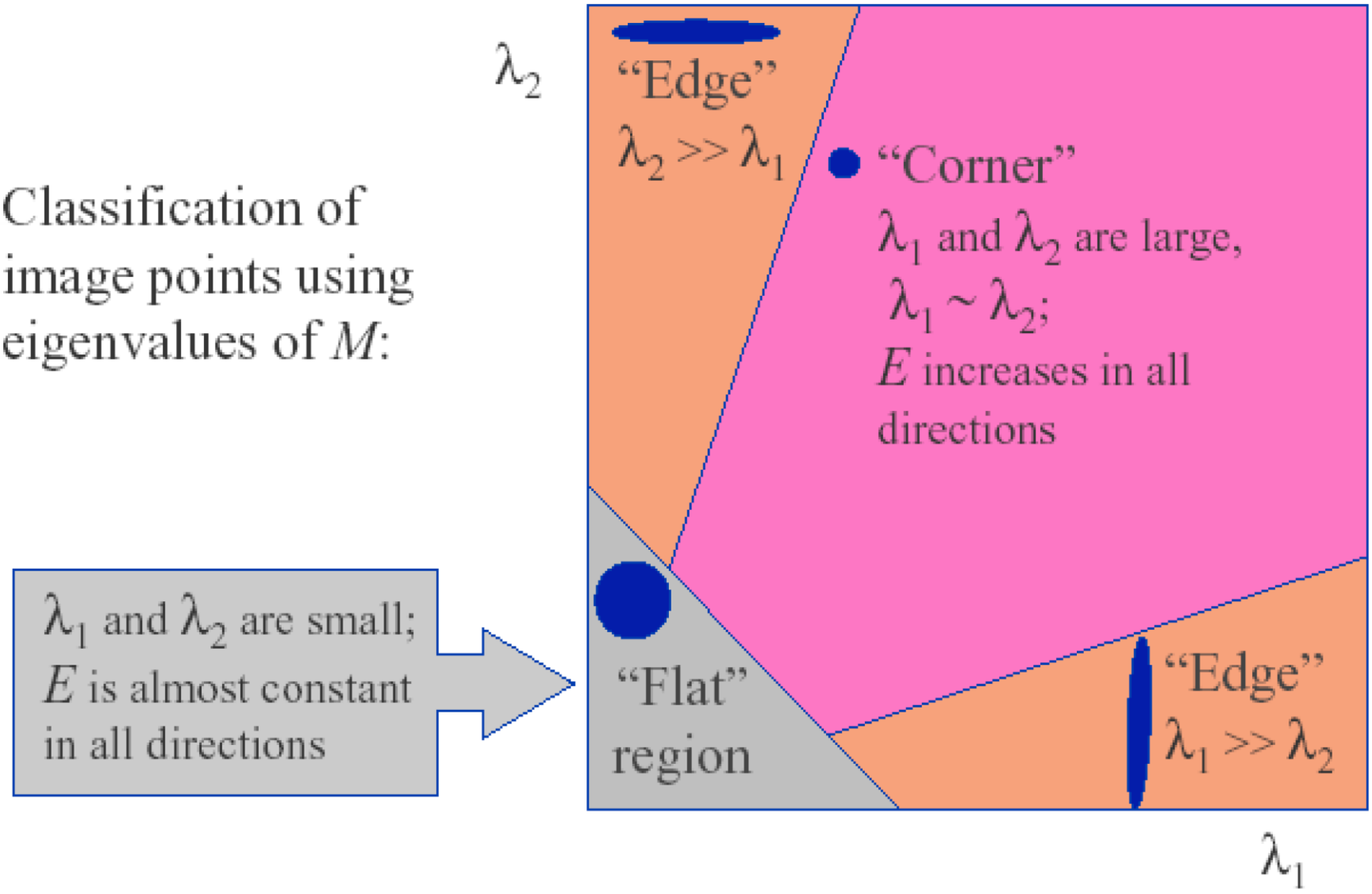
Since M is symmetric, we have: $M = R^{-1}\begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{bmatrix}R$

We can visualize M as an ellipse with axis lengths determined by the eigenvalues and orientation determined by R



Classification via Eigenvalues

Classification of image points using eigenvalues of M:



Corner Response Measure

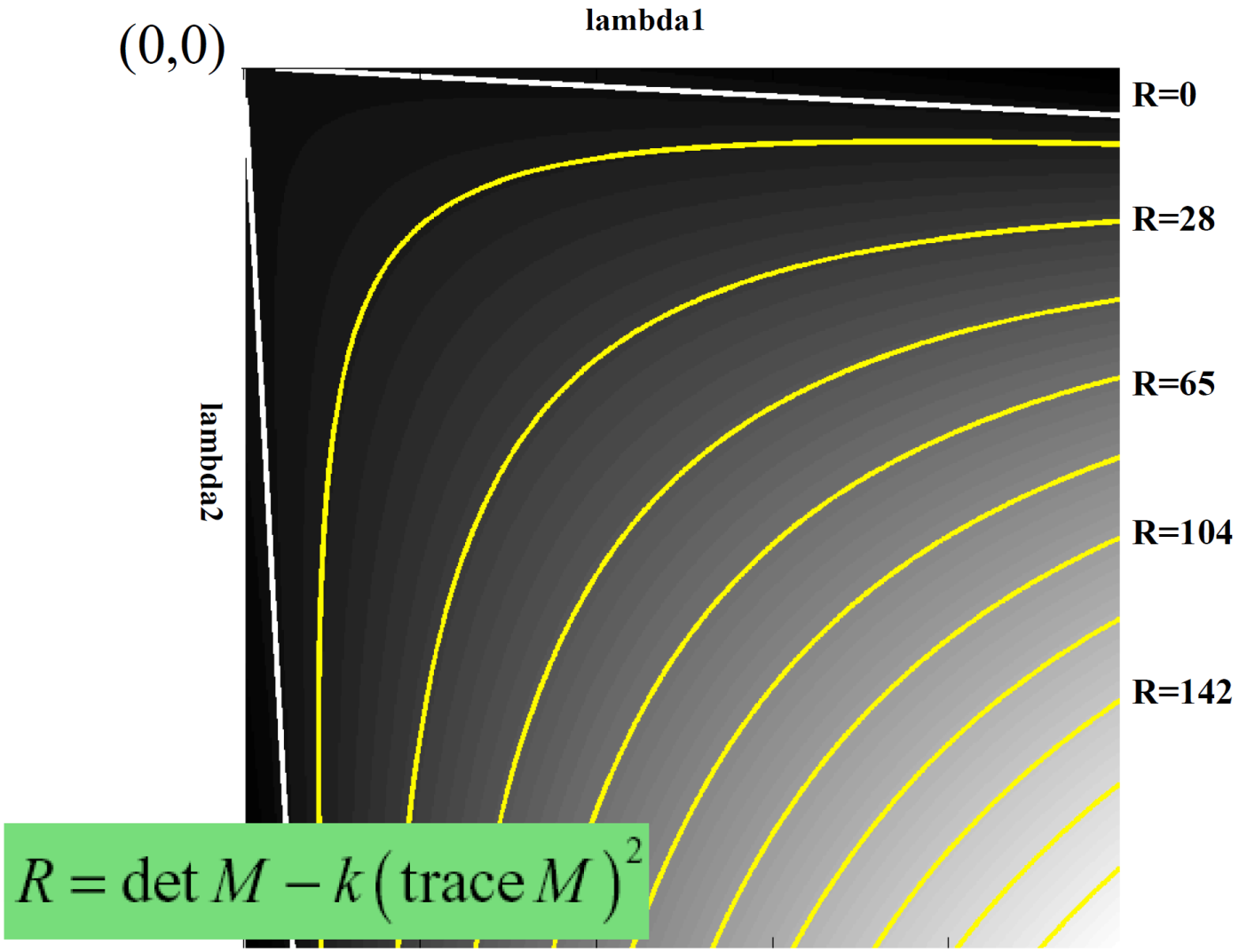
Measure of corner response:

$$R = \det M - k\left(\text{trace}\, M\right)^2$$

$$\det M = \lambda_1 \lambda_2$$
$$\text{trace}\, M = \lambda_1 + \lambda_2$$

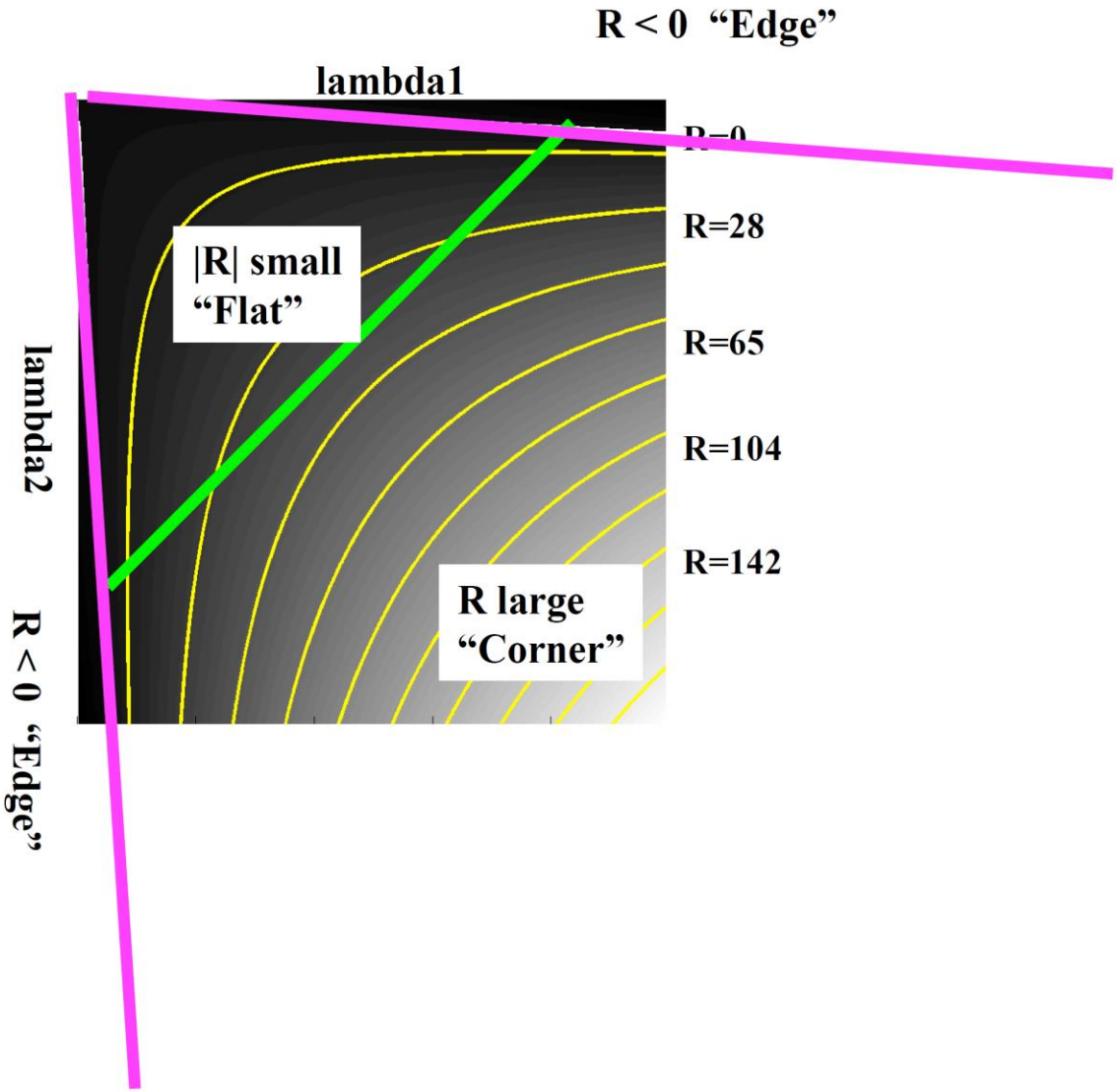
(k is an empirically determined constant; $k = 0.04$ - 0.06)

Corner Response Map

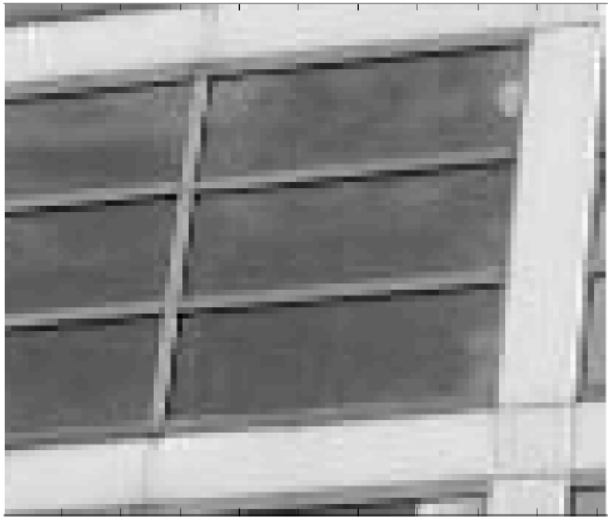


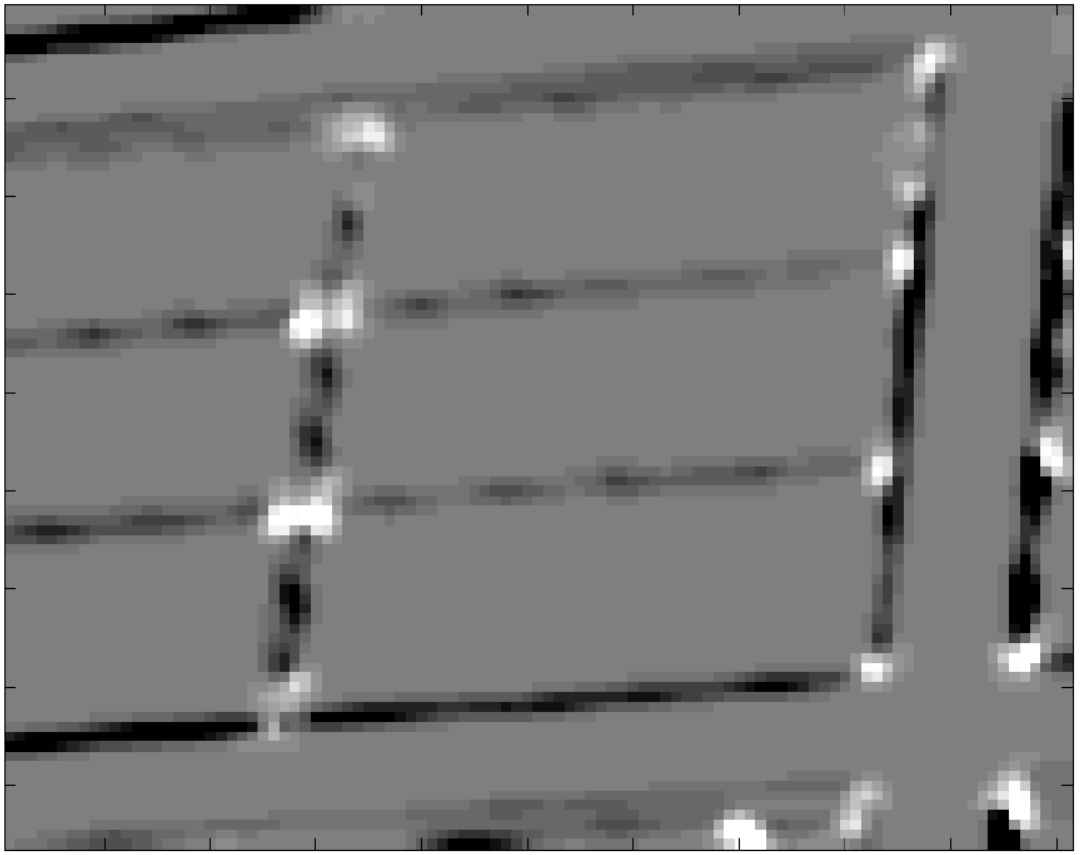
Corner Response Map

- R depends only on eigenvalues of M
- R is large for a corner
- R is negative with large magnitude for an edge
- $|R|$ is small for a flat region



Corner Response Example



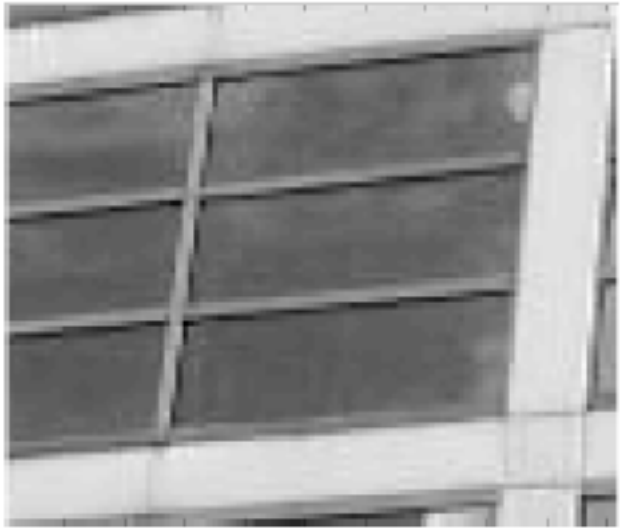


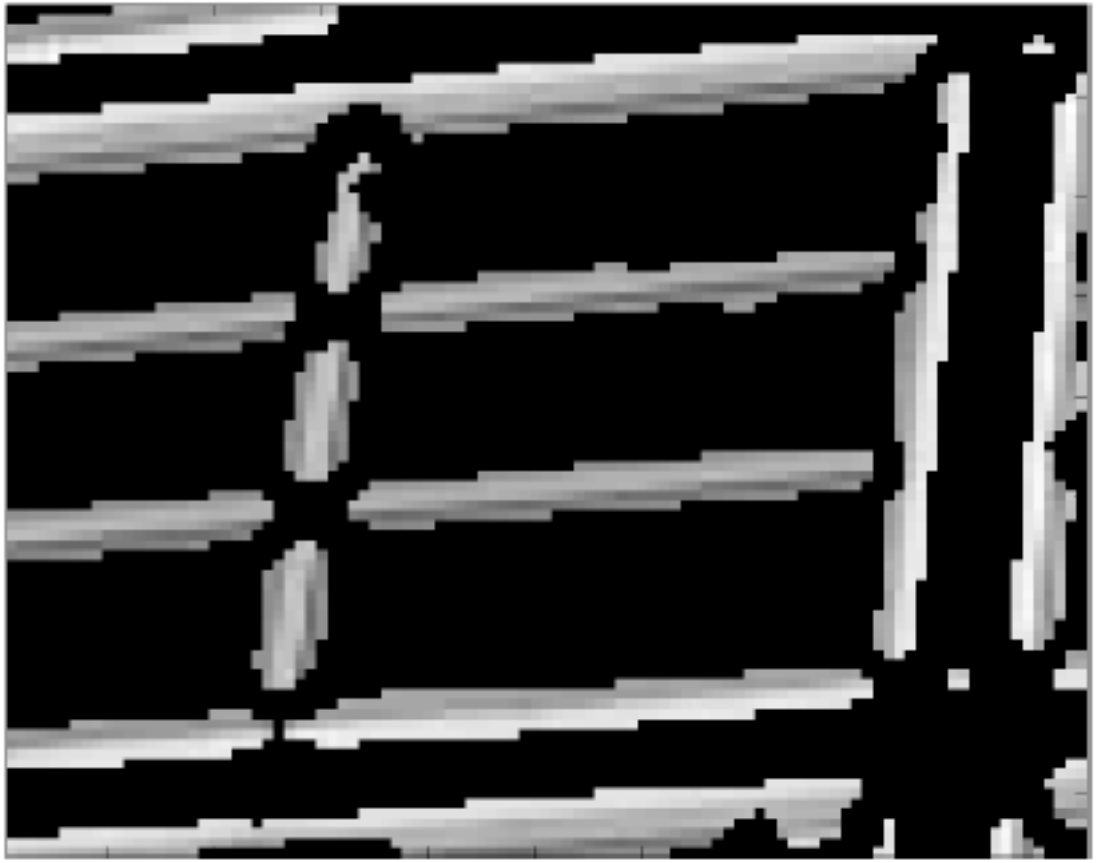
Harris R score.

Ix, Iy computed using Sobel operator

Windowing function w = Gaussian, sigma=1

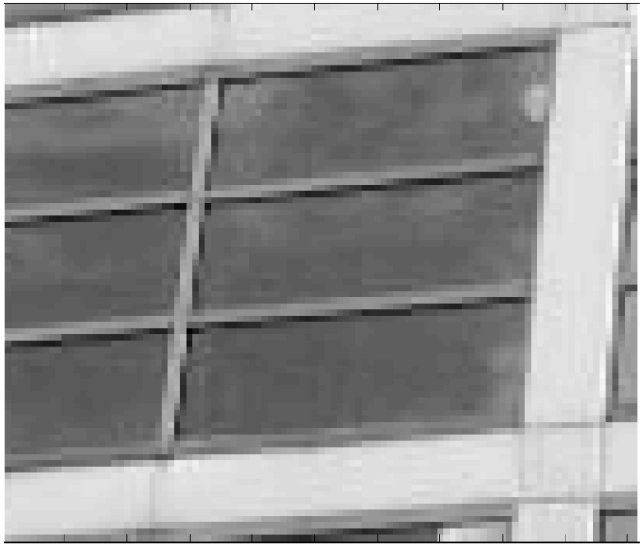
Corner Response Example

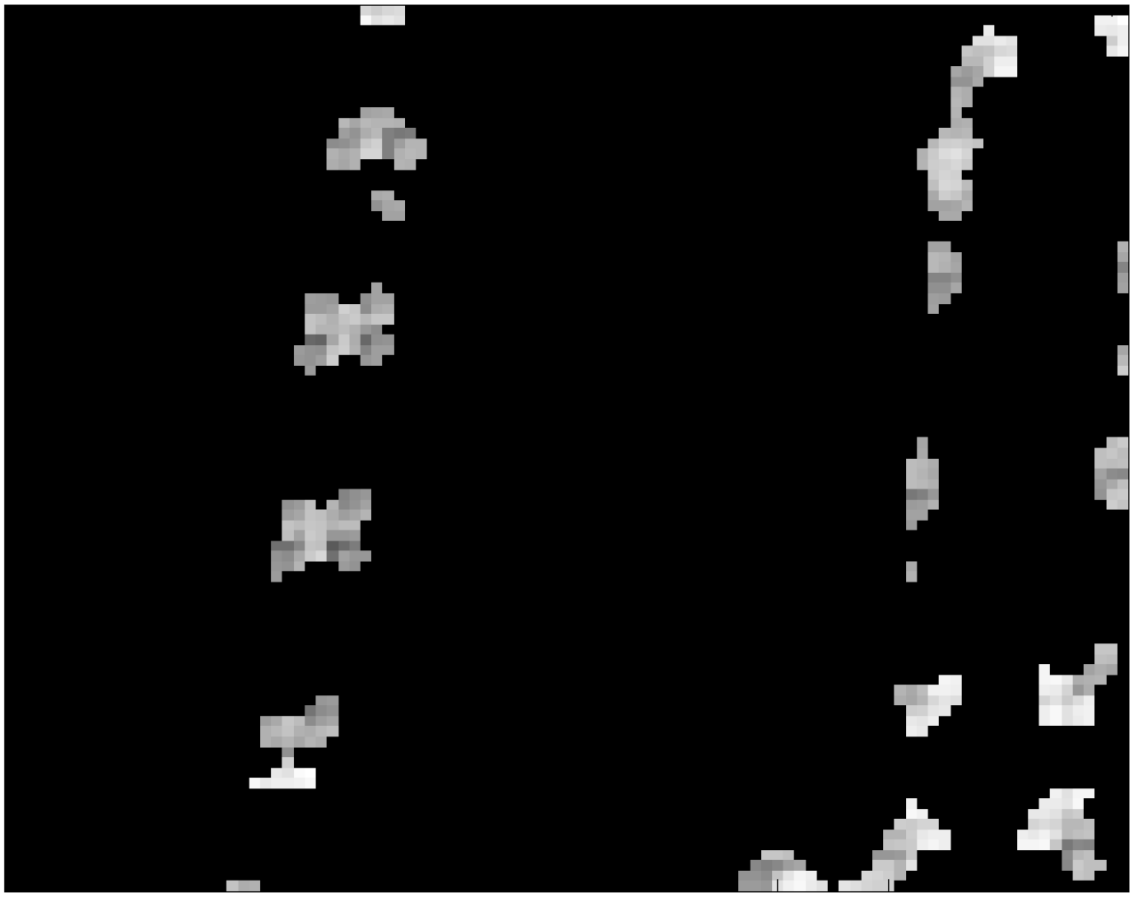




Threshold: $R < -10000$
(edges)

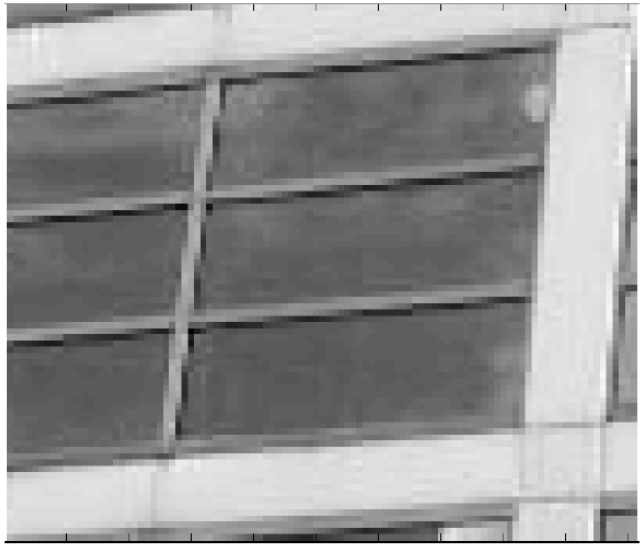
Corner Response Example

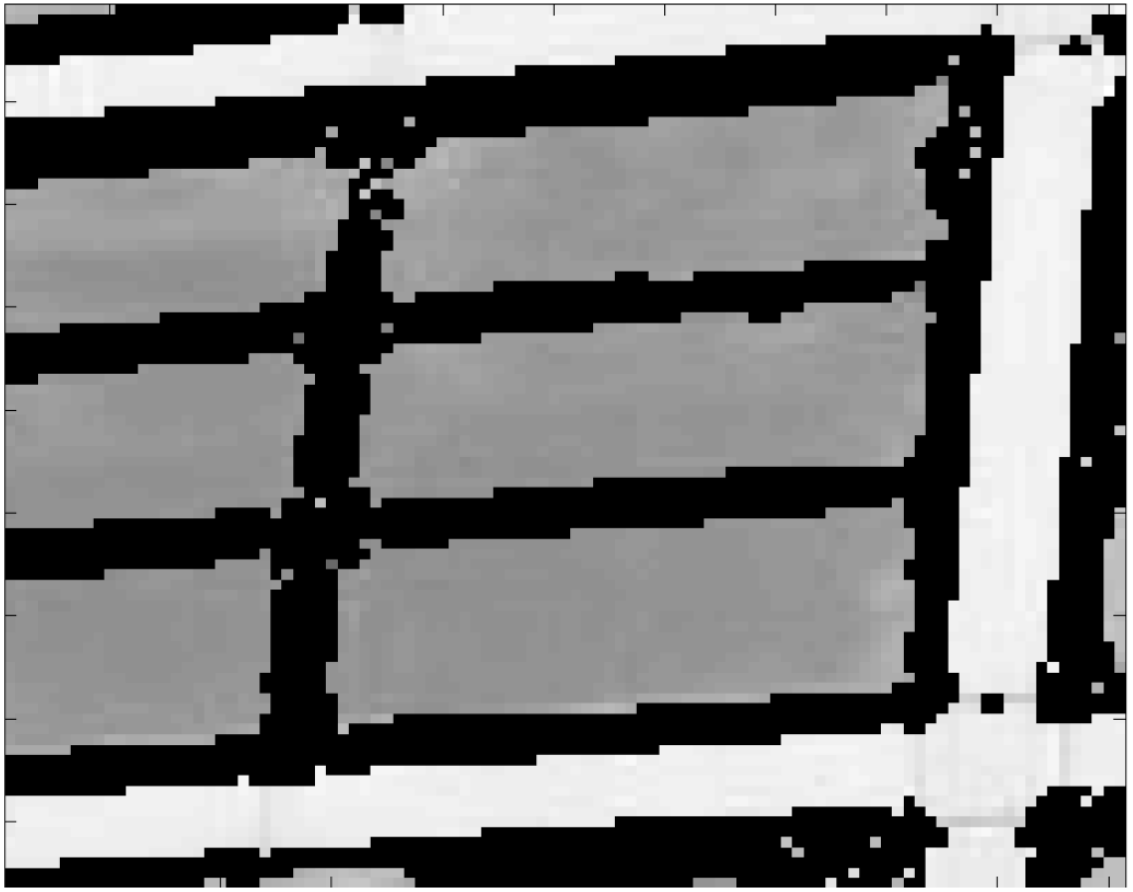




Threshold: > 10000
(corners)

Corner Response Example





Threshold: $-10000 < R < 10000$
(neither edges nor corners)

Harris Corner Detection Algorithm

1. Compute x and y derivatives of image

$$I_x = G_\sigma^x * I \quad I_y = G_\sigma^y * I$$

2. Compute products of derivatives at every pixel

$$I_{x2} = I_x.I_x \quad I_{y2} = I_y.I_y \quad I_{xy} = I_x.I_y$$

3. Compute the sums of the products of derivatives at each pixel

$$S_{x2} = G_{\sigma\prime} * I_{x2} \quad S_{y2} = G_{\sigma\prime} * I_{y2} \quad S_{xy} = G_{\sigma\prime} * I_{xy}$$

4. Define at each pixel (x, y) the matrix

$$H(x,y) = \begin{bmatrix} S_{x2}(x,y) & S_{xy}(x,y) \\ S_{xy}(x,y) & S_{y2}(x,y) \end{bmatrix}$$

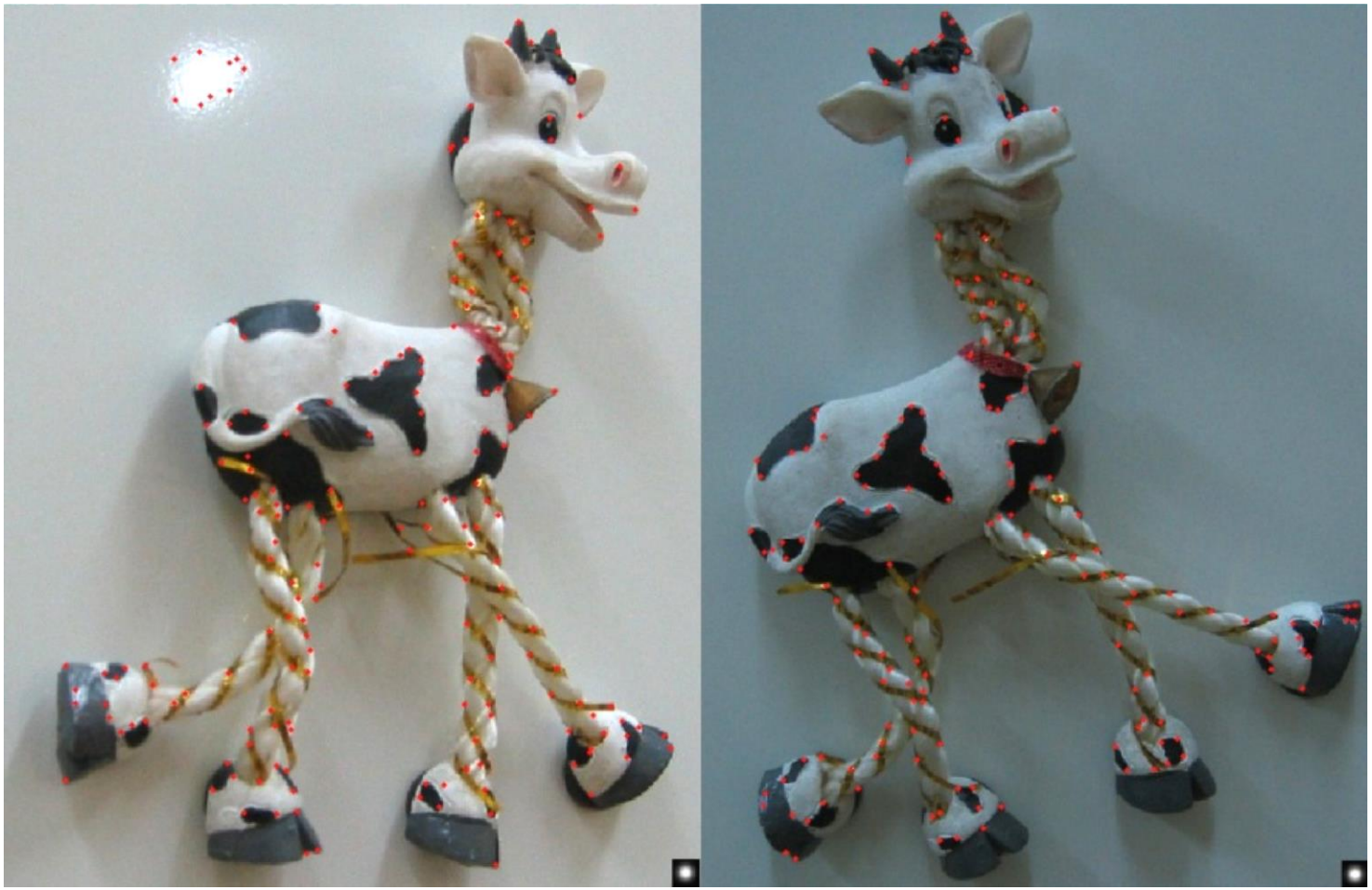
5. Compute the response of the detector at each pixel

$$R = Det(H) - k(Trace(H))^2$$

6. Threshold on value of R. Compute nonmax suppression.

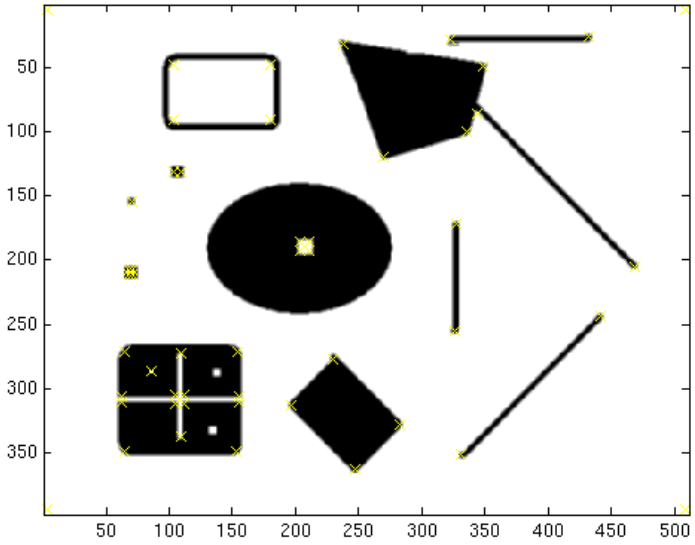
Harris Detector: Workflow

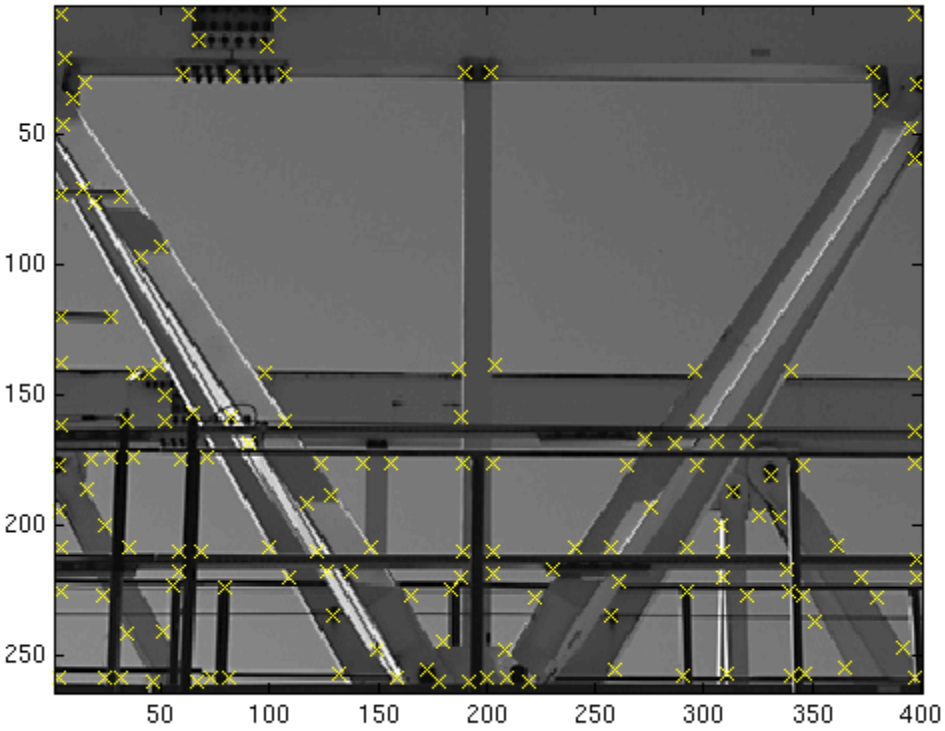
- **Resulting Harris points**



Harris Detector – Responses [Harris88]

- Effect: A very precise corner detector.





Harris Detector – Responses [Harris88]

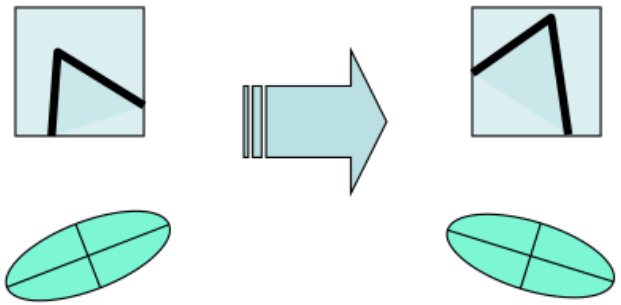
Harris Detector – Responses [Harris88]

- Results are well suited for finding stereo correspondences

Harris Detector: Properties

- **Translation invariance**

- **Rotation invariance**



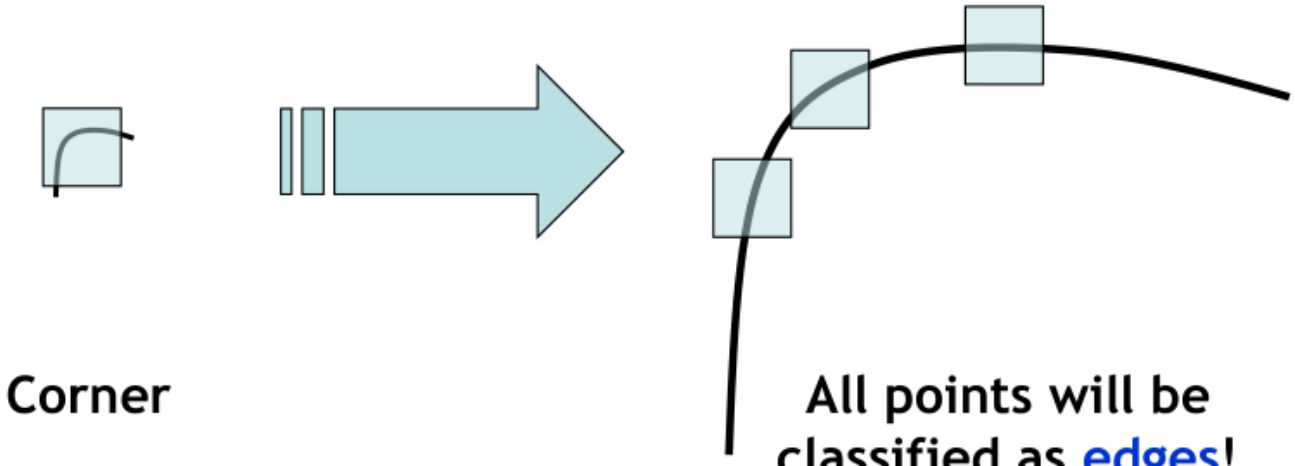
Ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner response θ is invariant to image rotation

Harris Detector: Properties

- Translation invariance
- Rotation invariance

- **Scale invariance**



Not invariant to image scale!

What we are learned today?

- A model fitting method for line detection
  - RANSAC
- Local invariant features
  - Motivation
  - Requirements, invariances
- Keypoint localization
  - Harris corner detector