Hough transform

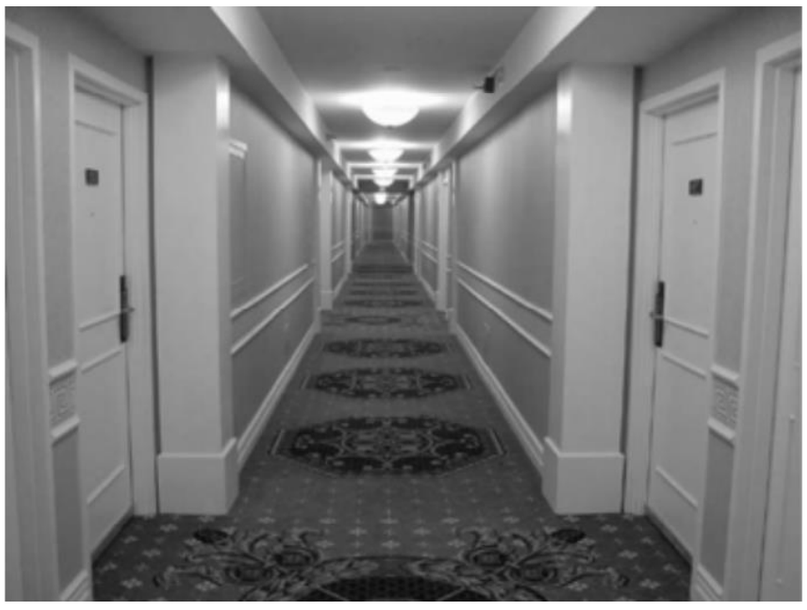
Hough transform

- The Hough transform (HT) can be used to detect lines.
- It was introduced in 1962 (Hough 1962) and first used to find lines in images a decade later (Duda 1972).
- The goal is to find the location of lines in images.
- **Caveat**: Hough transform can detect lines, circles and other structures ONLY if their parametric equation is known.
- It can give robust detection under noise and partial occlusion

Prior to Hough Transform

- Assume that we have performed some edge detection, and a thresholding of the edge magnitude image.

- Thus, we have some pixels that may partially describe the boundary of some objects.



Hough transform

- Generic framework for detecting a parametric model
- Edges don’t have to be connected
- Lines can be occluded
- Key idea: edges **vote** for the possible models

Image and parameter space

variables

$$y = mx + b$$

parameters

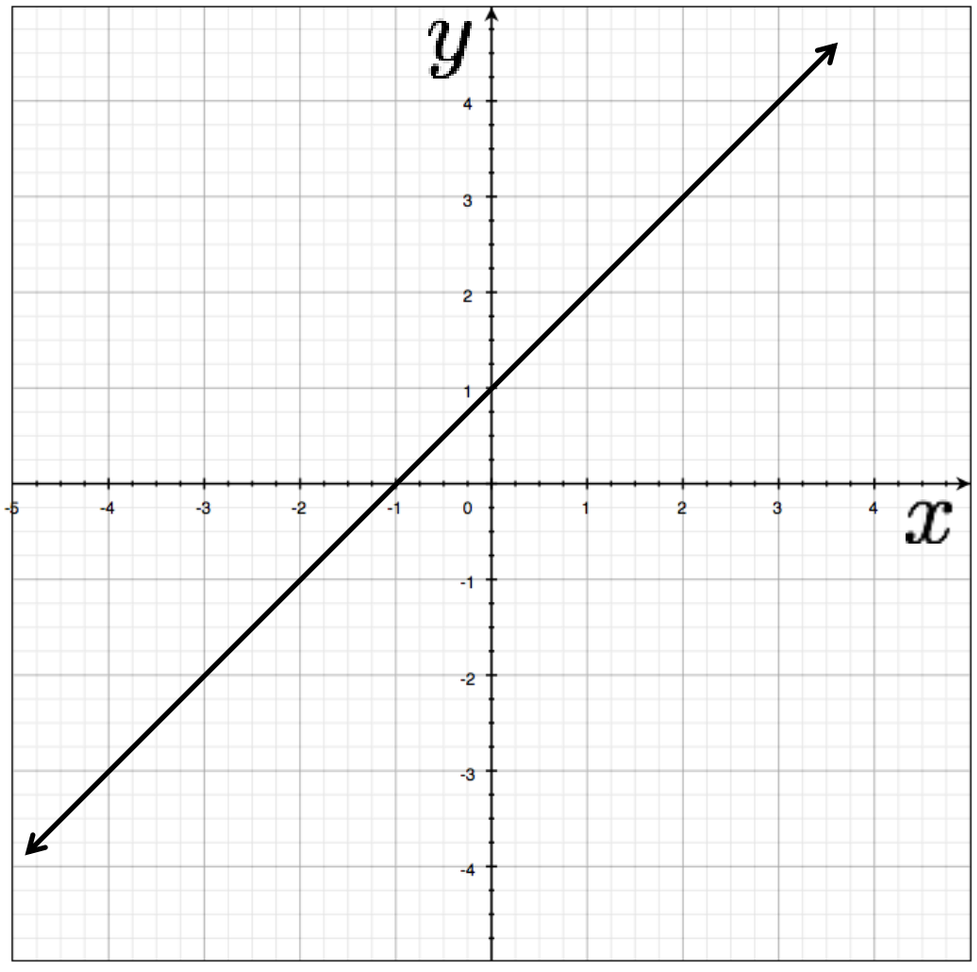


Image space

Image and parameter space

variables

$$y = mx + b$$

parameters

variables

$$y - mx = b$$

parameters

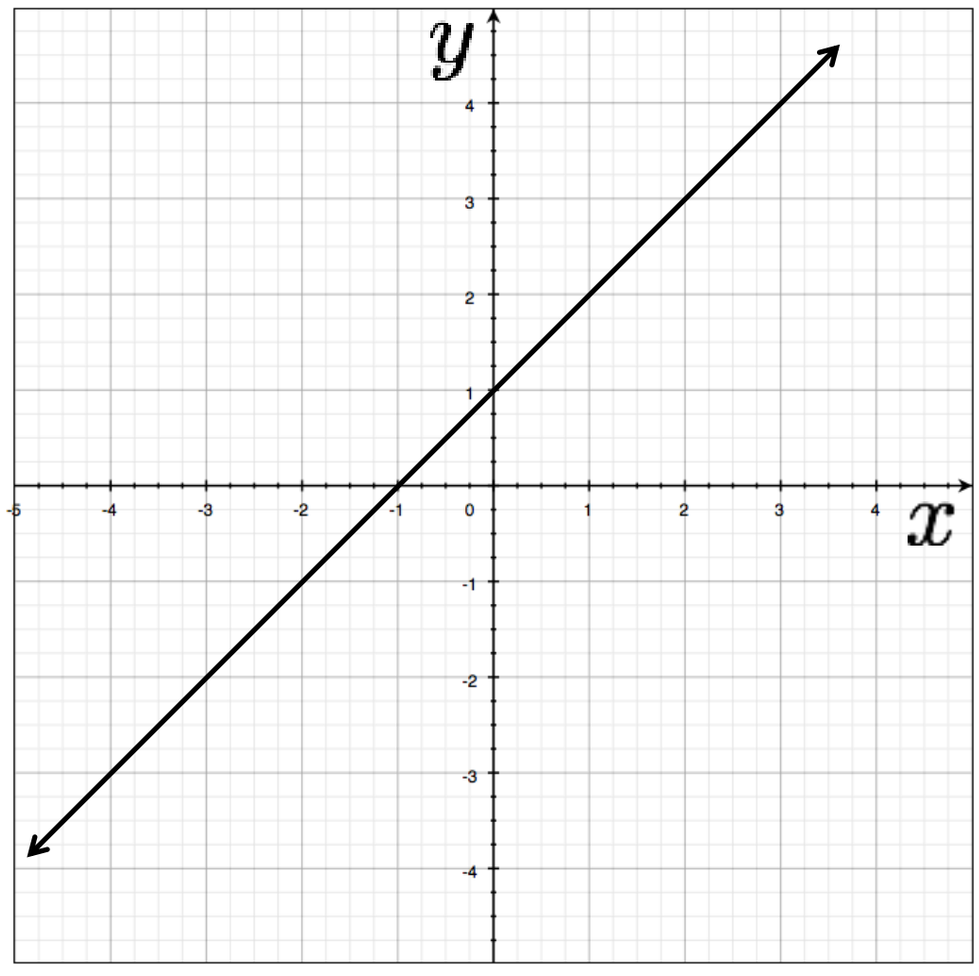
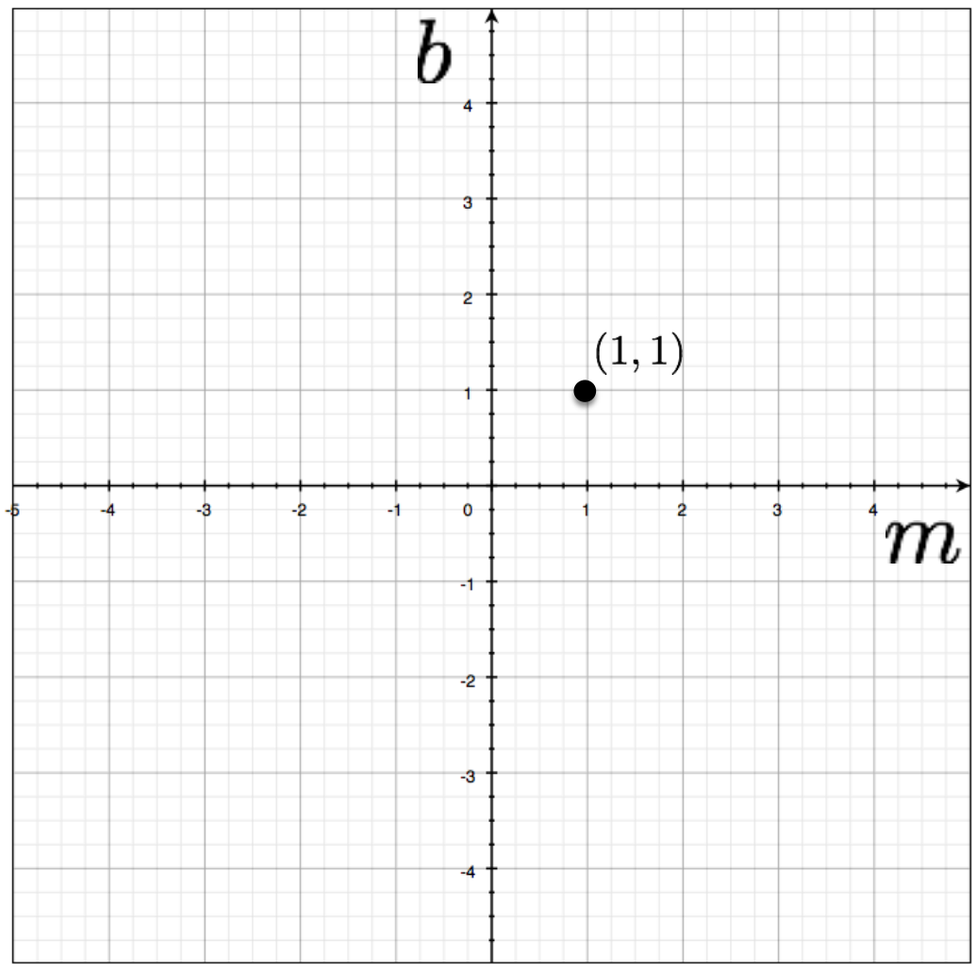


Image space

a line becomes a point



Parameter space

Image and parameter space

variables

$$y = mx + b$$

parameters

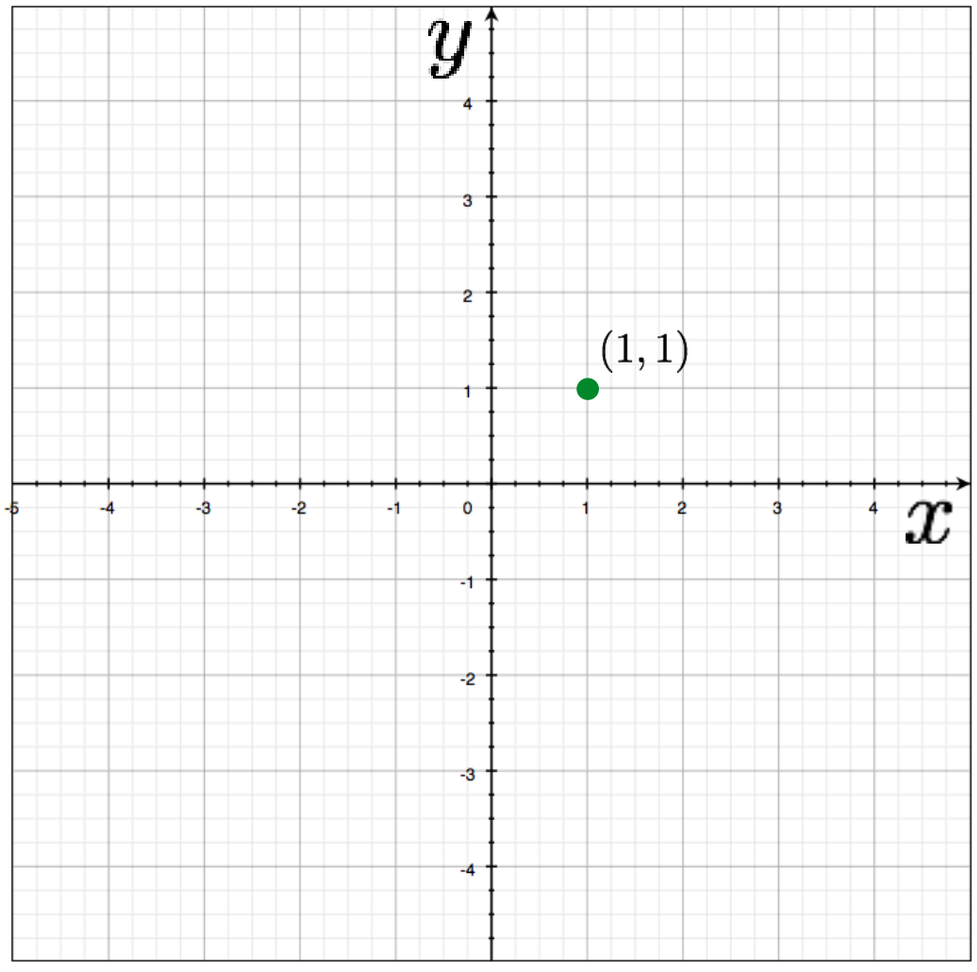


Image space

What would a point in image space become in parameter space?

Image and parameter space

variables

$$y = mx + b$$

parameters

variables

$$y - mx = b$$

parameters

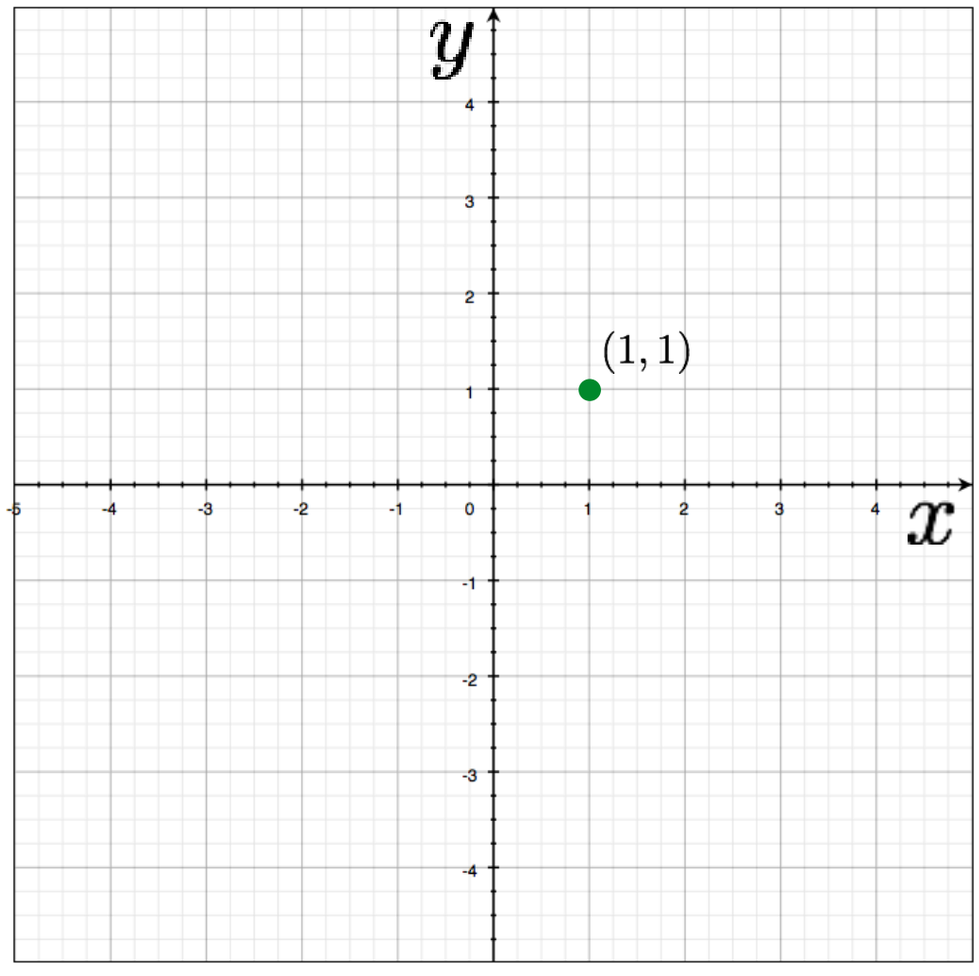
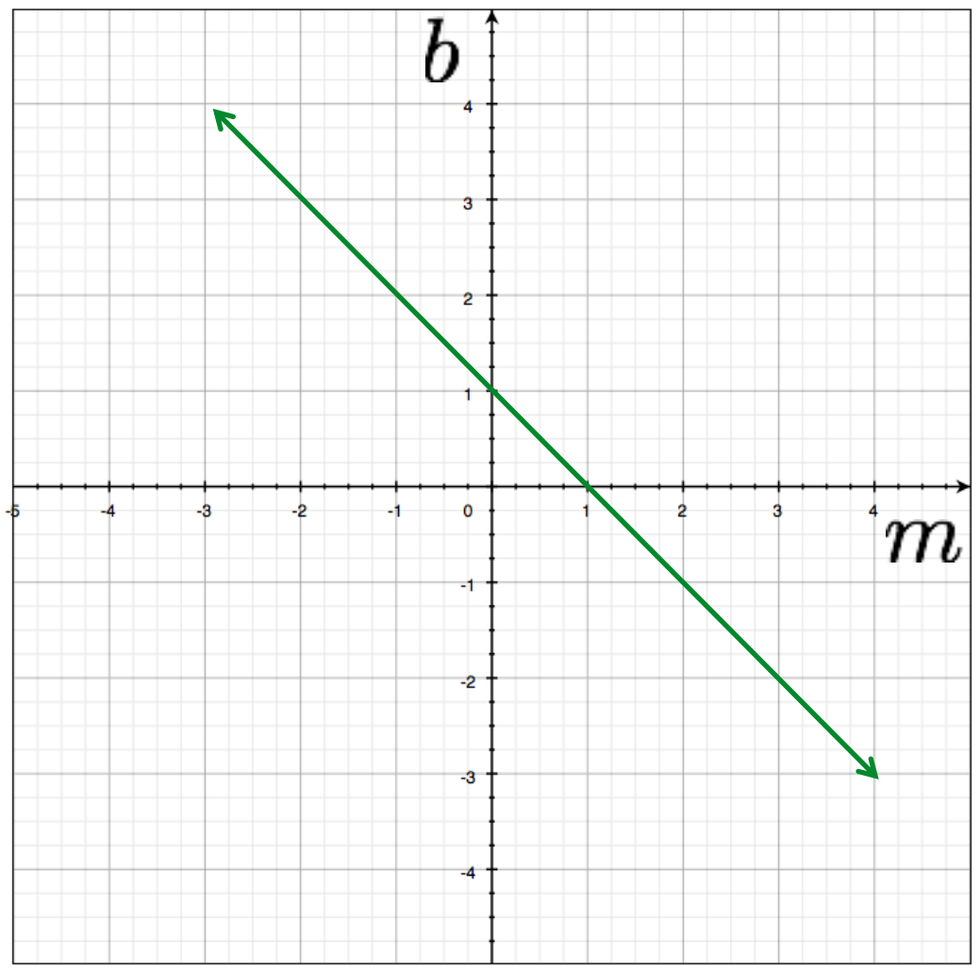


Image space

a point becomes a line



Parameter space

Image and parameter space

variables

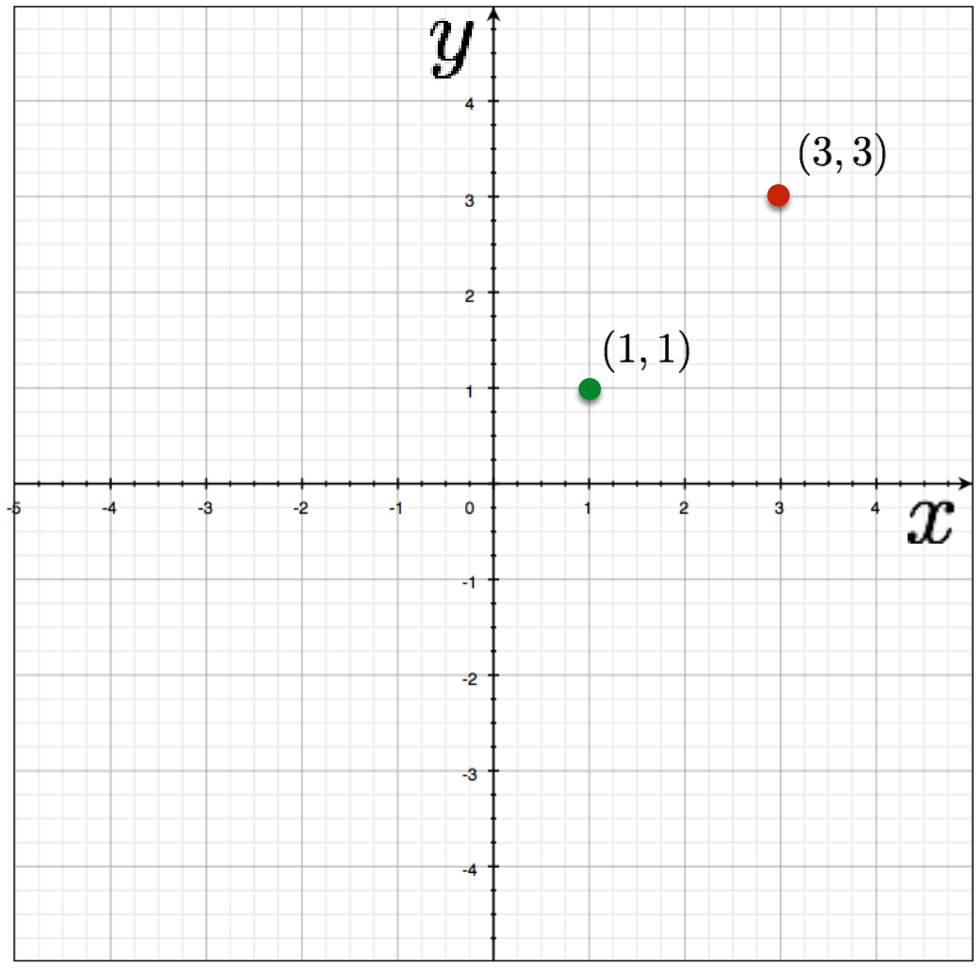
$$y = mx + b$$

parameters

variables

$$y - mx = b$$

parameters



two points become ?

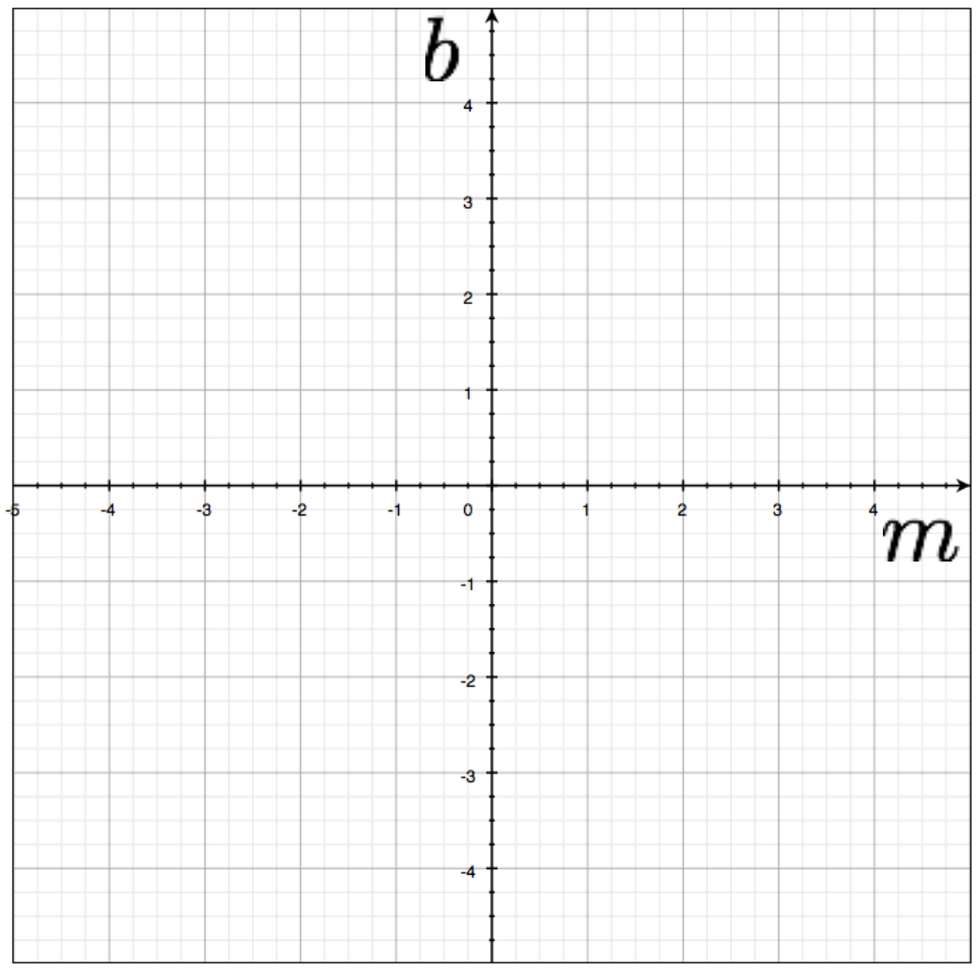


Image space

Parameter space

Image and parameter space

variables

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parameters

variables

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parameters

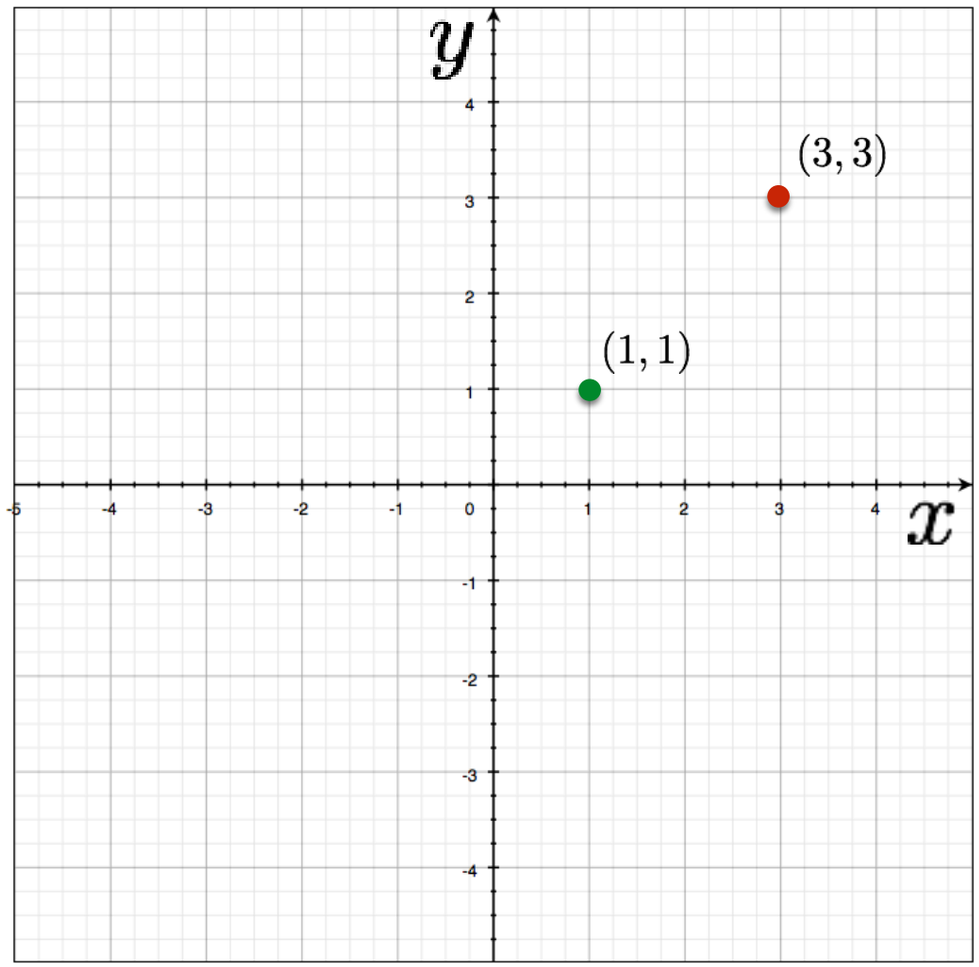
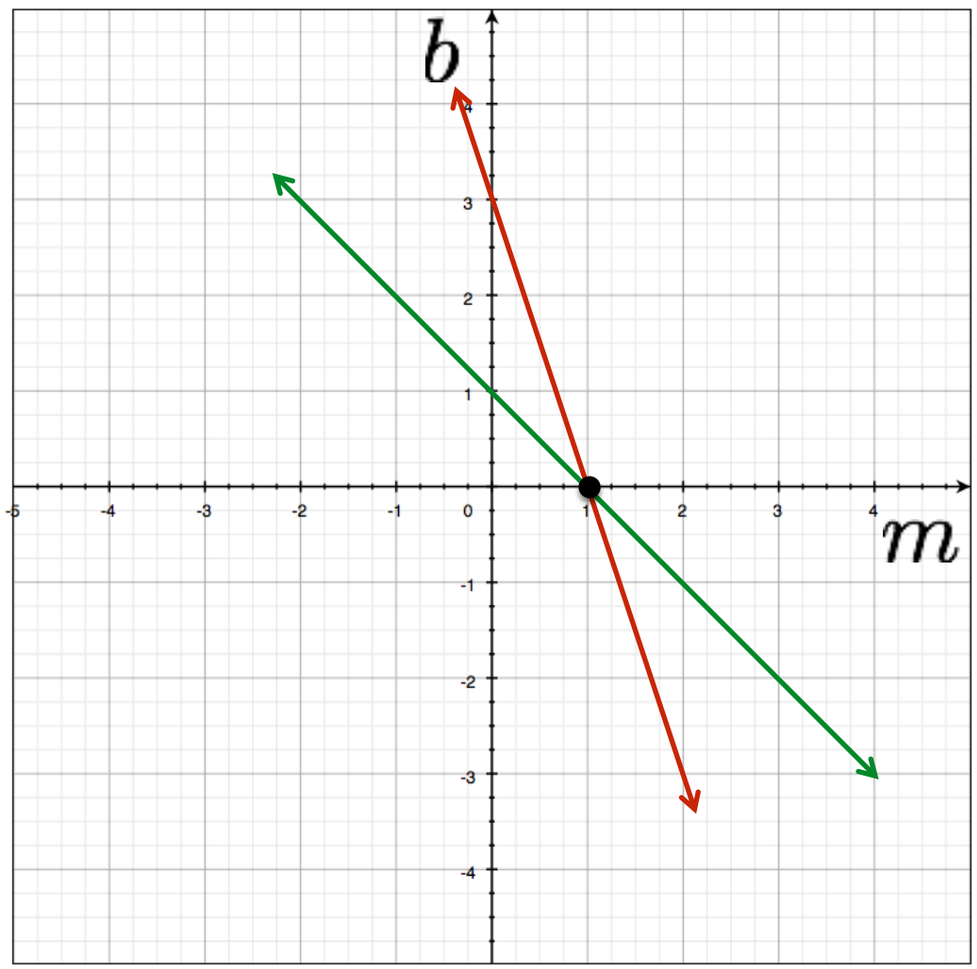


Image space

two points become ?



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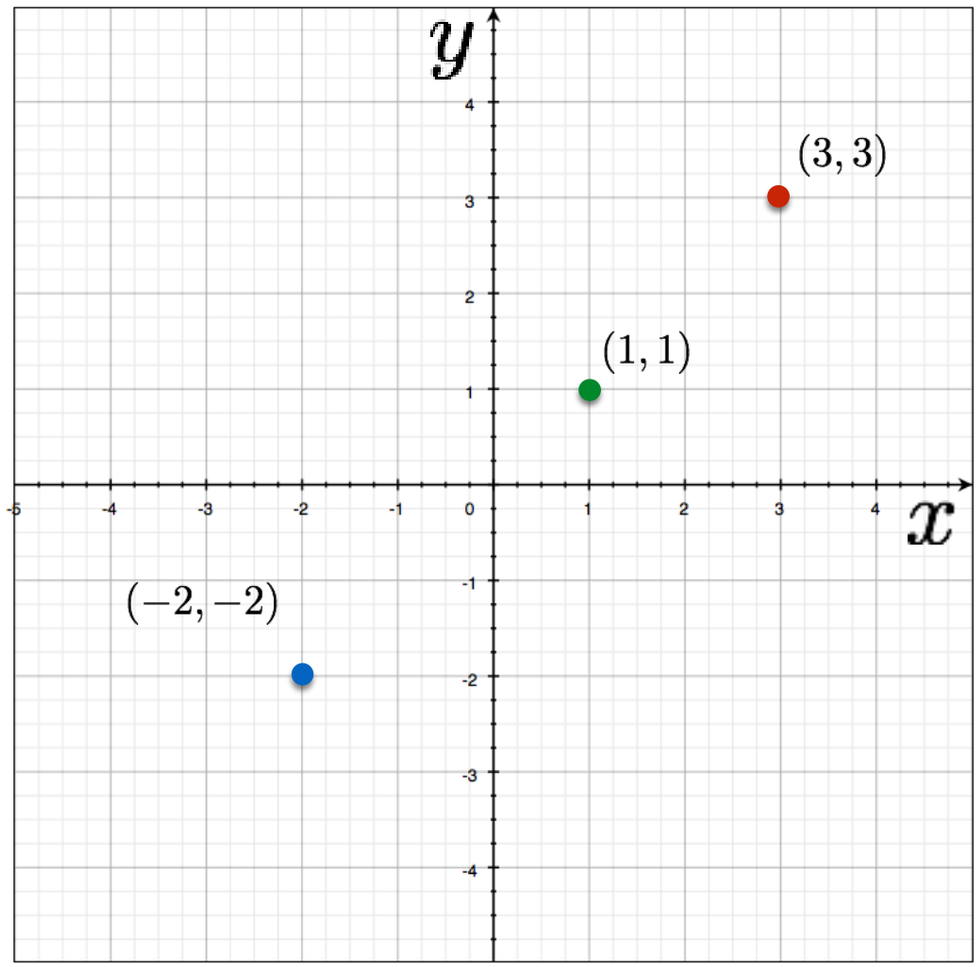
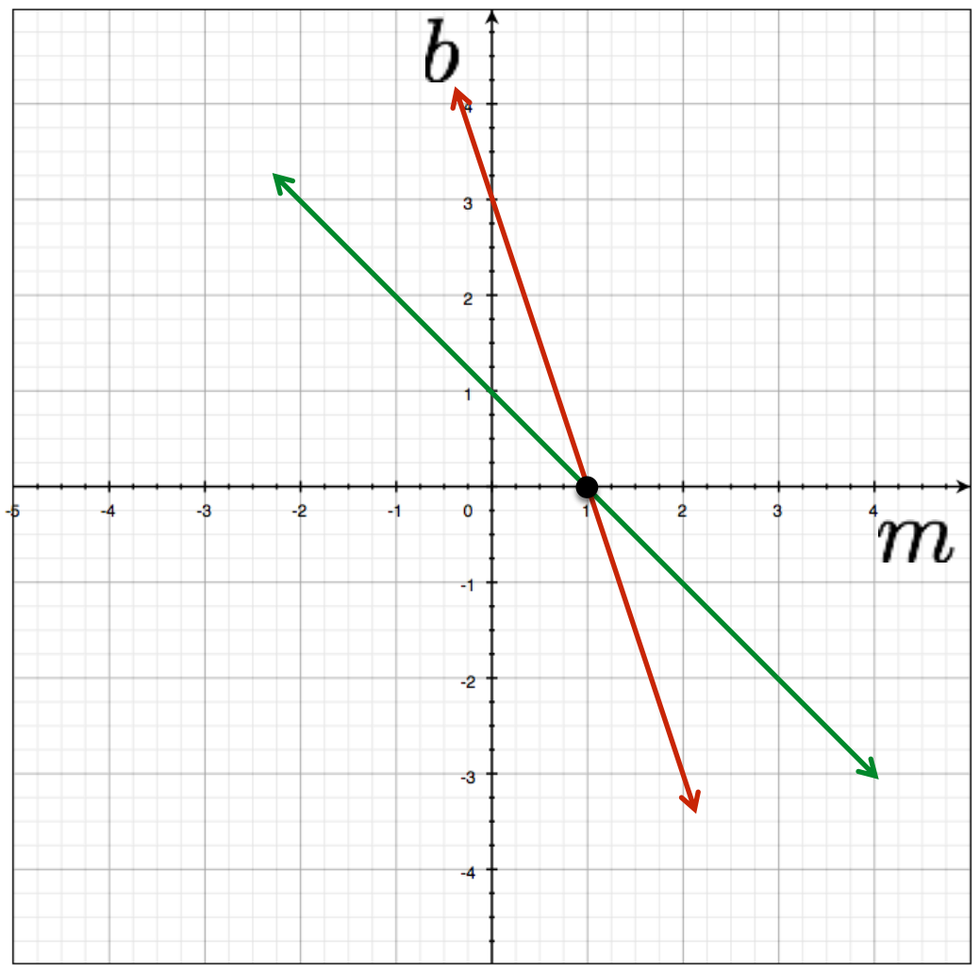


Image space

three points become ?



Parameter space

Image and parameter space

variables

$$y = mx + b$$

parameters

variables

$$y - mx = b$$

parameters

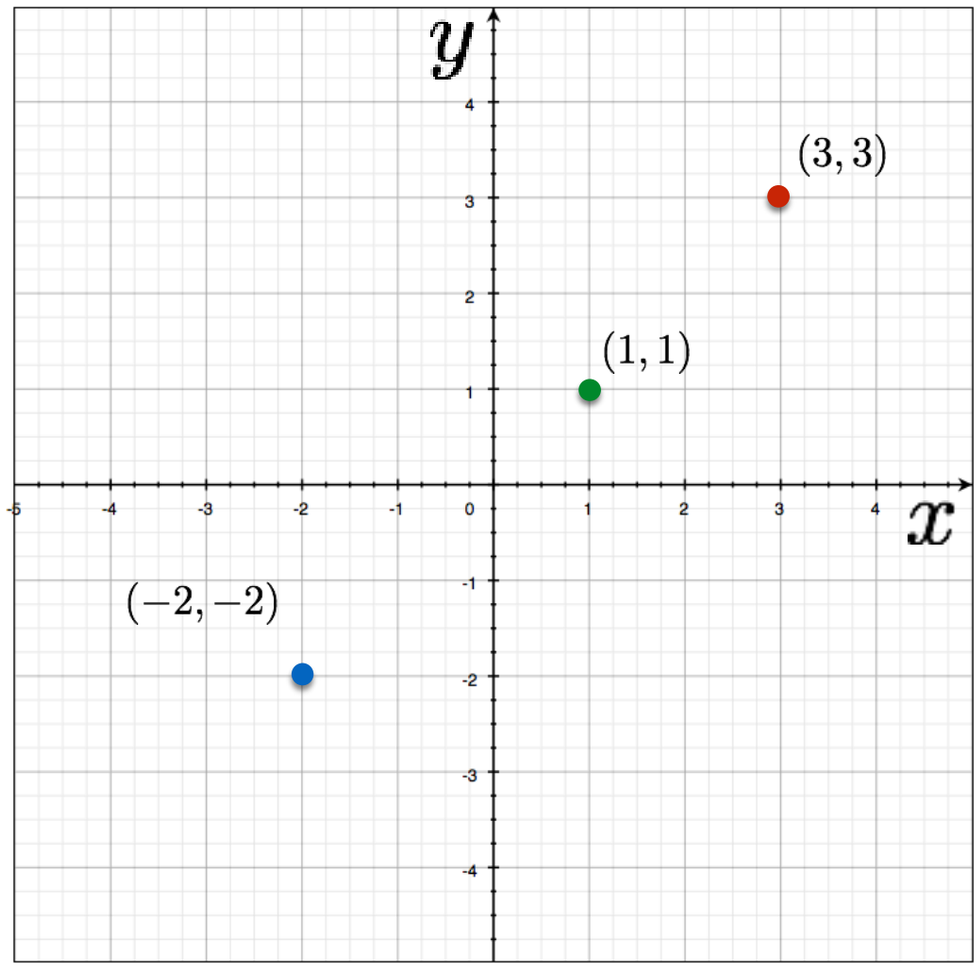
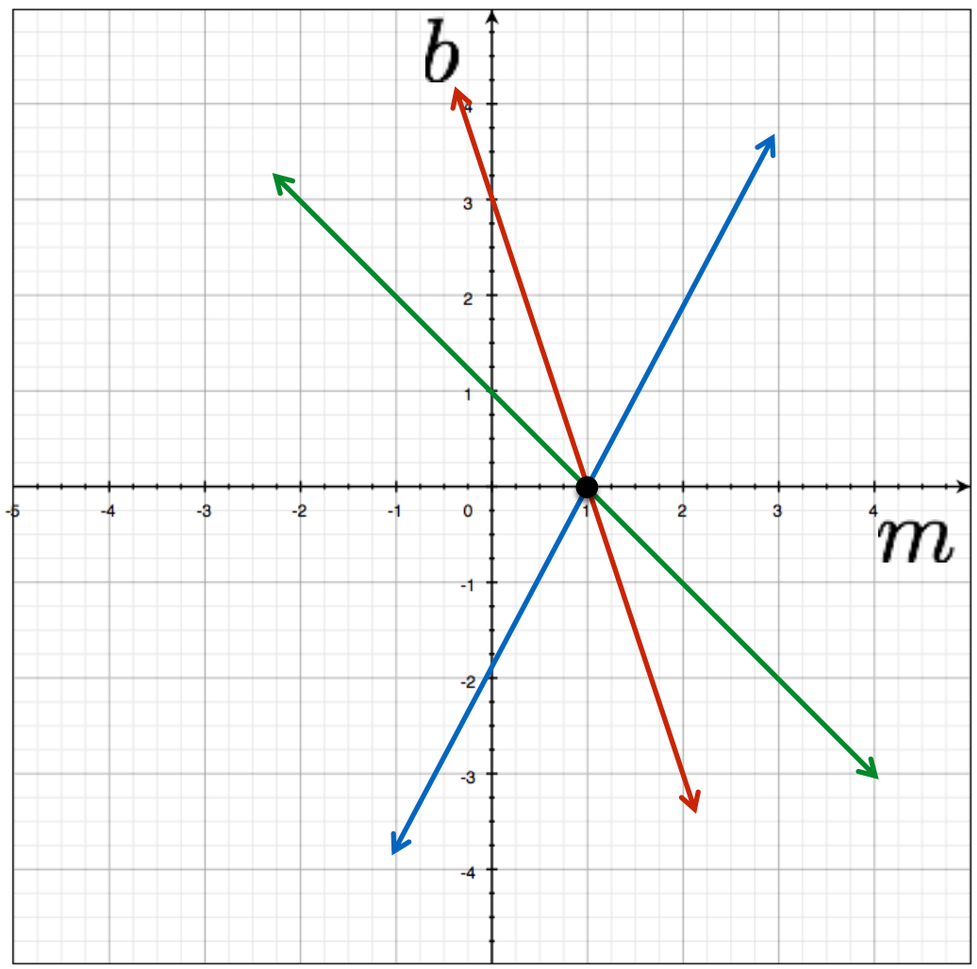


Image space

three points become ?



Parameter space

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variables

$$y - mx = b$$

parameters

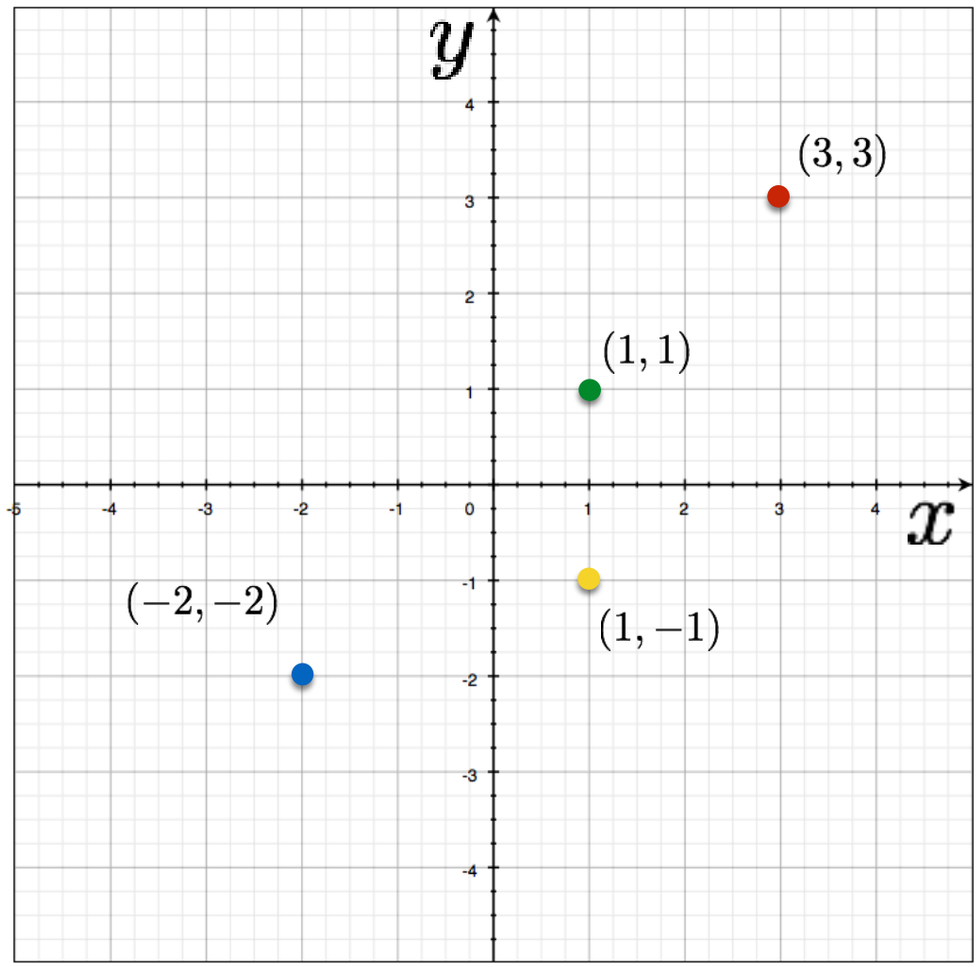
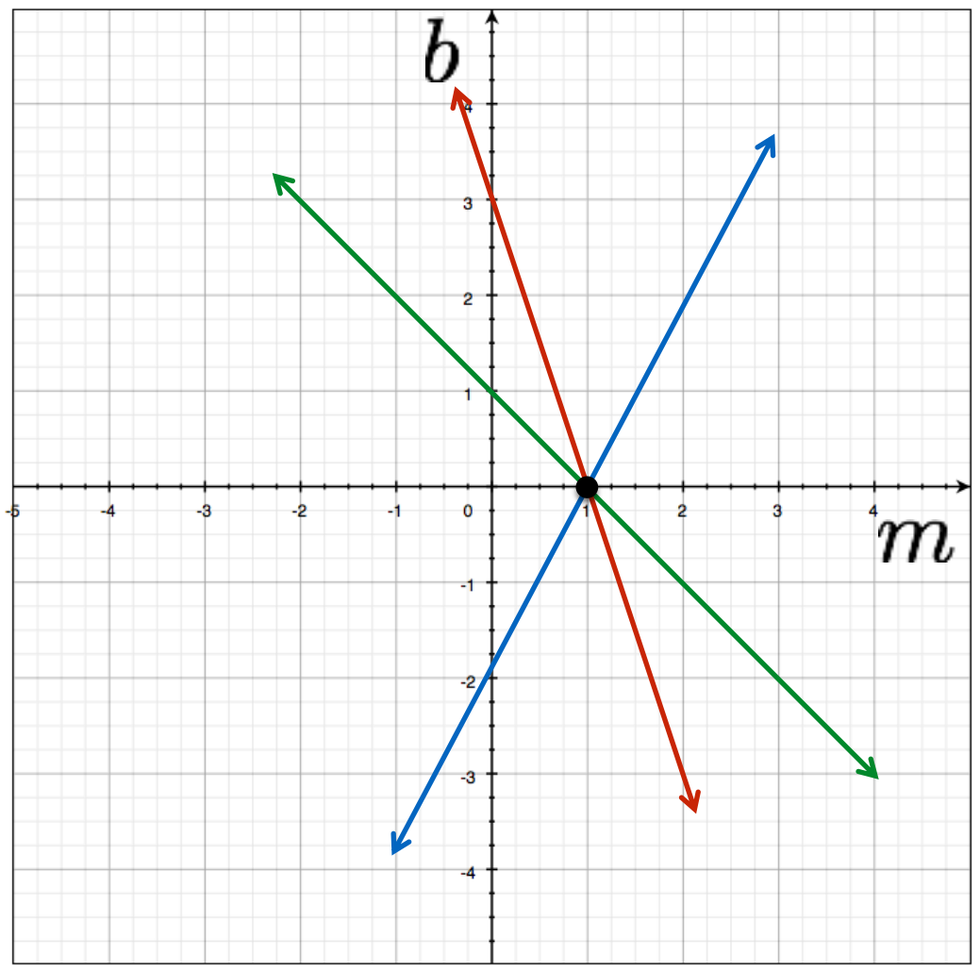


Image space

four points become ?



Parameter space

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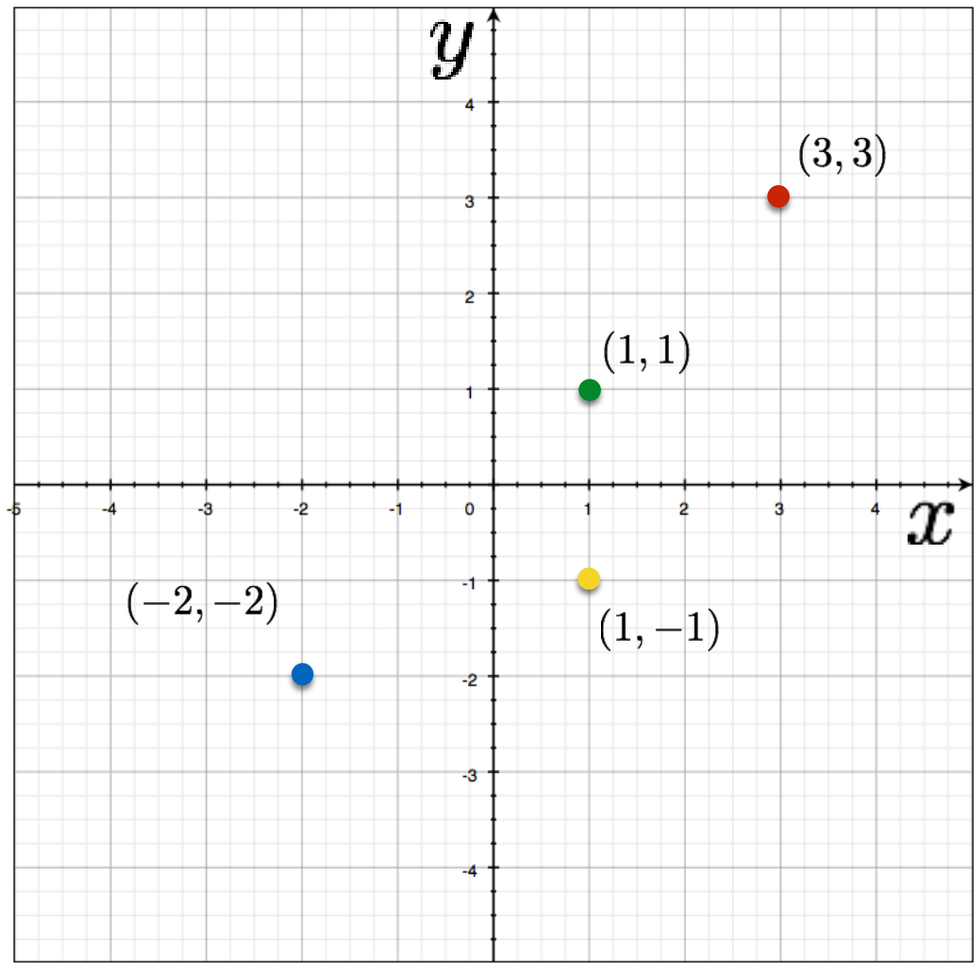


four points become ?

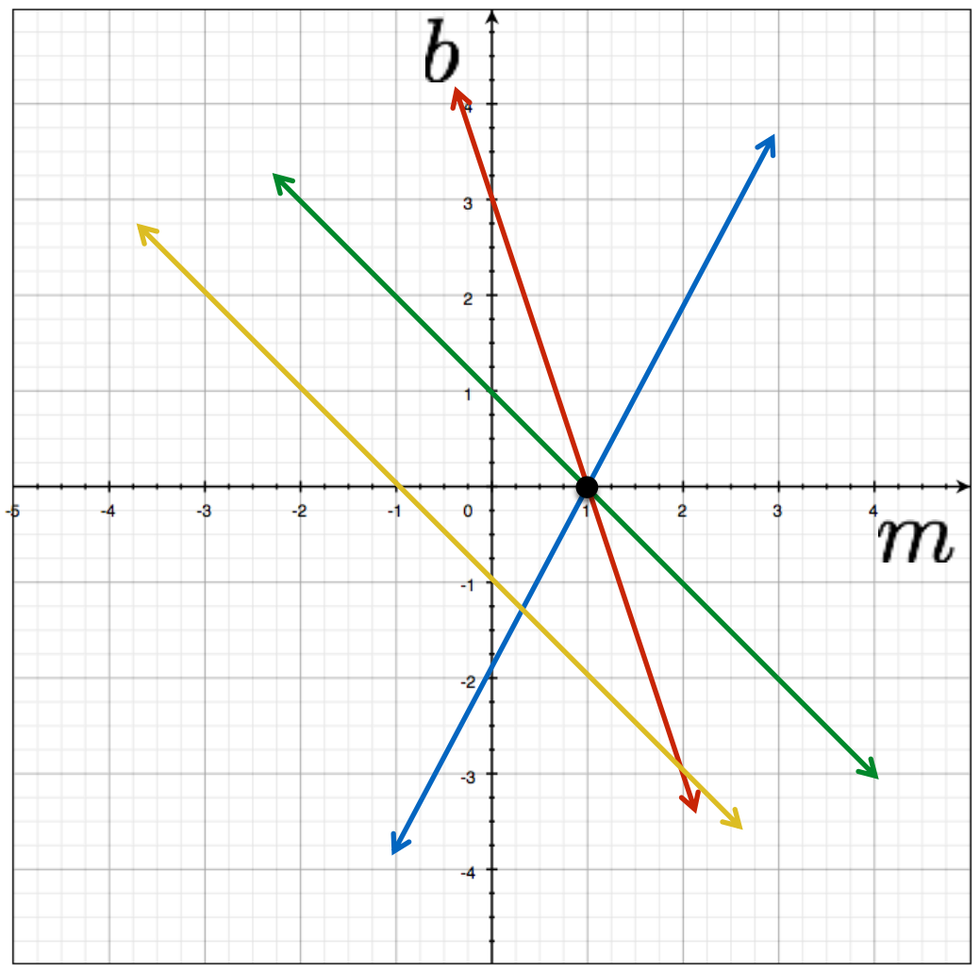


Image space

Parameter space

How would you find the best fitting line?

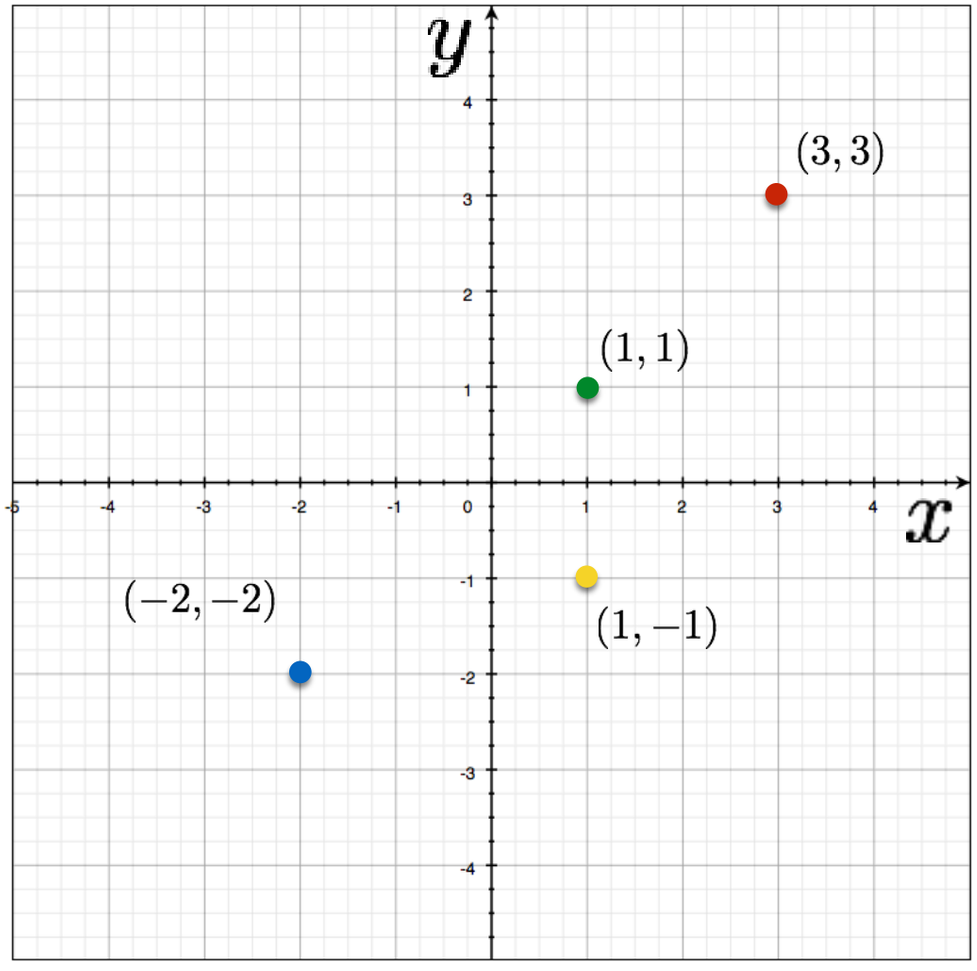
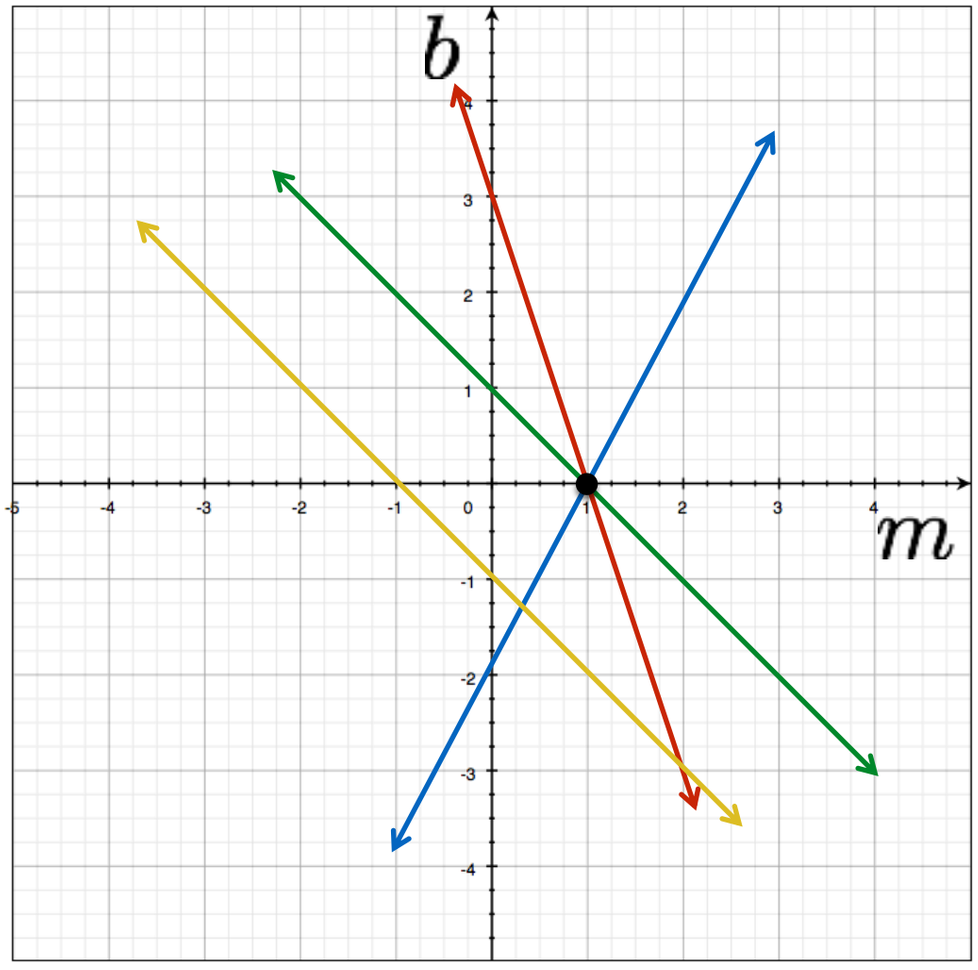


Image space



Parameter space

Is this method robust to measurement noise?

Is this method robust to outliers?

Line Detection by Hough Transform

Algorithm:

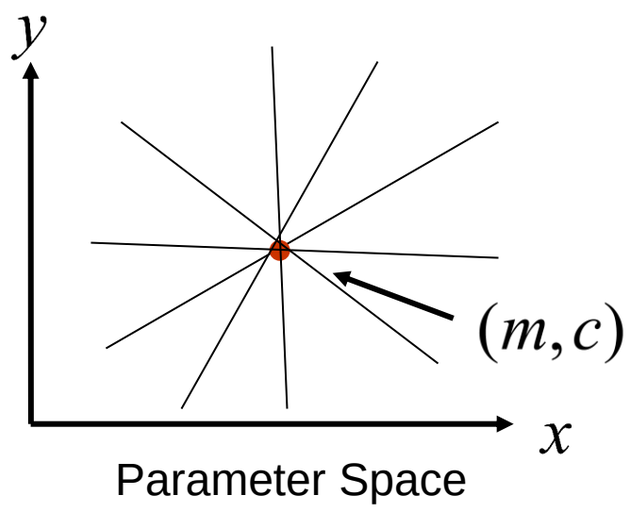
1. Quantize Parameter Space (m,c)

2. Create Accumulator Array $A(m,c)$

3. Set $A(m,c) = 0 \quad \forall m,c$

4. For each image edge (x_i, y_i)
   For each element in $A(m,c)$
   If (m,c) lies on the line: $c = -x_i m + y_i$
   Increment $A(m,c) = A(m,c) + 1$

5. Find local maxima in $A(m,c)$



Parameter Space

$A(m,c)$

	1						1		
		1				1			
			1		1				
				2					
			1		1				
		1				1			
	1						1		

Problems with parameterization

How big does the accumulator need to be for the parameterization (m, c)*?*

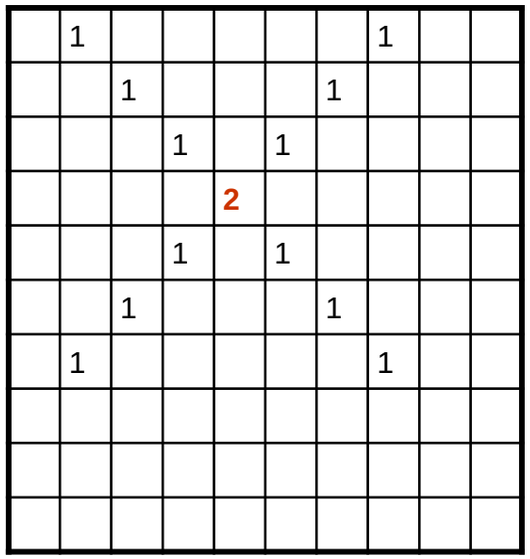
$A(m, c)$

	1						1		
		1				1			
			1		1				
				2					
			1		1				
		1				1			
	1						1		

Problems with parameterization

How big does the accumulator need to be for the parameterization (m, c)*?*

$A(m, c)$



The space of m is huge!

$$-\infty \leq m \leq \infty$$

The space of c is huge!

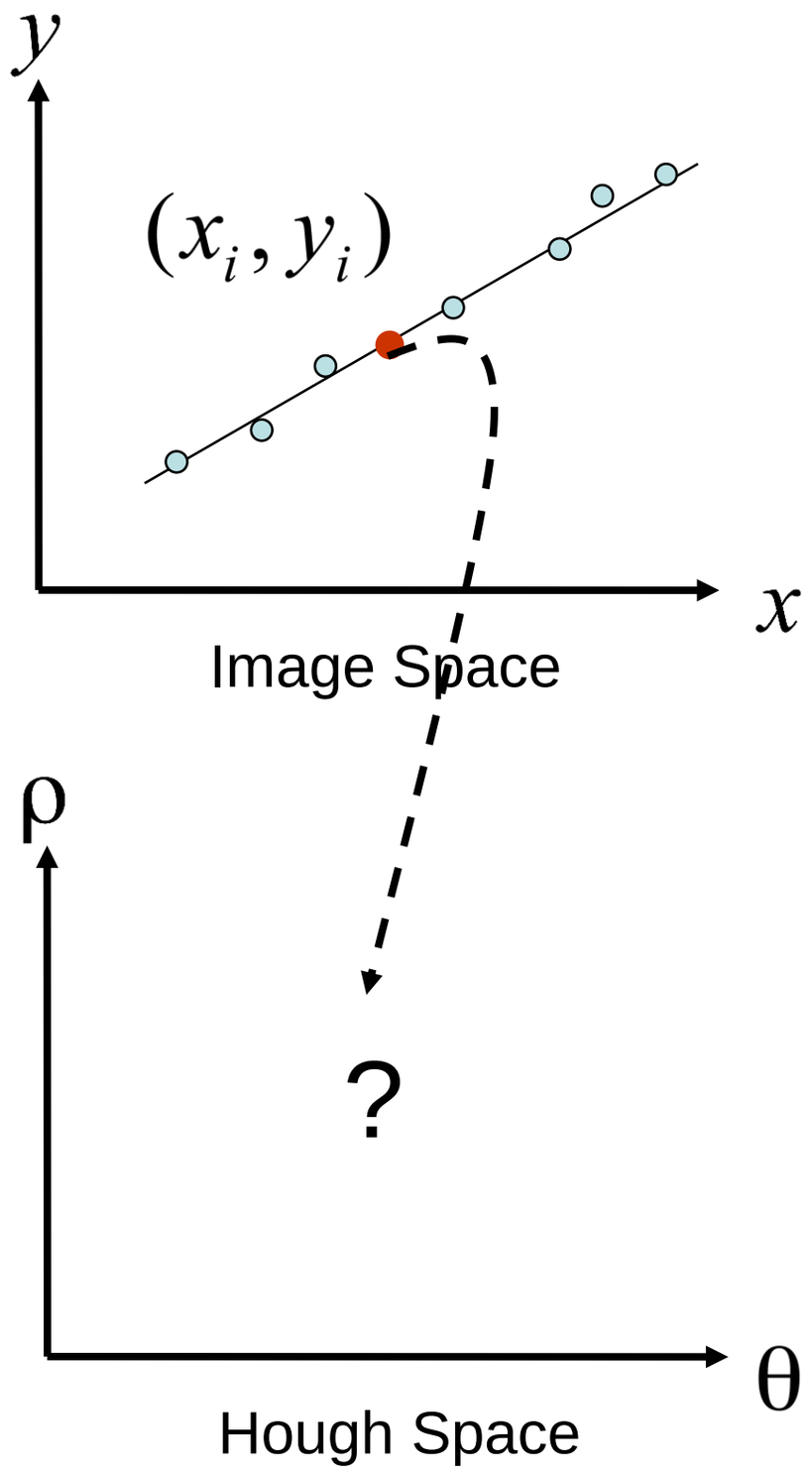
$$-\infty \leq c \leq \infty$$

Better Parameterization

Use normal form:

$$x\cos\theta + y\sin\theta = \rho$$

Given points (x_i, y_i) find (ρ,θ)



Hough Space Sinusoid

$$0 \le \theta \le 2\pi$$

$$0 \le \rho \le \rho_{max}$$

(Finite Accumulator Array Size)

Image and parameter space

variables

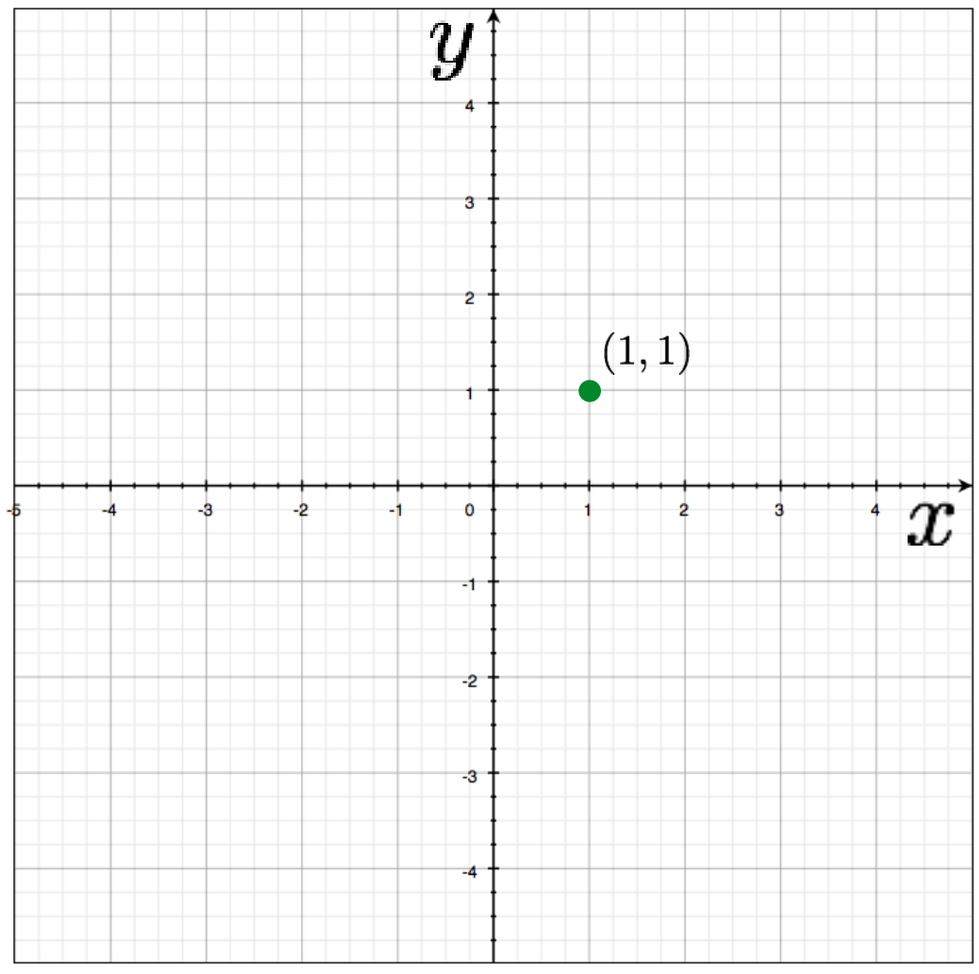
$$y = mx + b$$

parameters

parameters

$$x\cos\theta + y\sin\theta = \rho$$

variables



a point becomes?

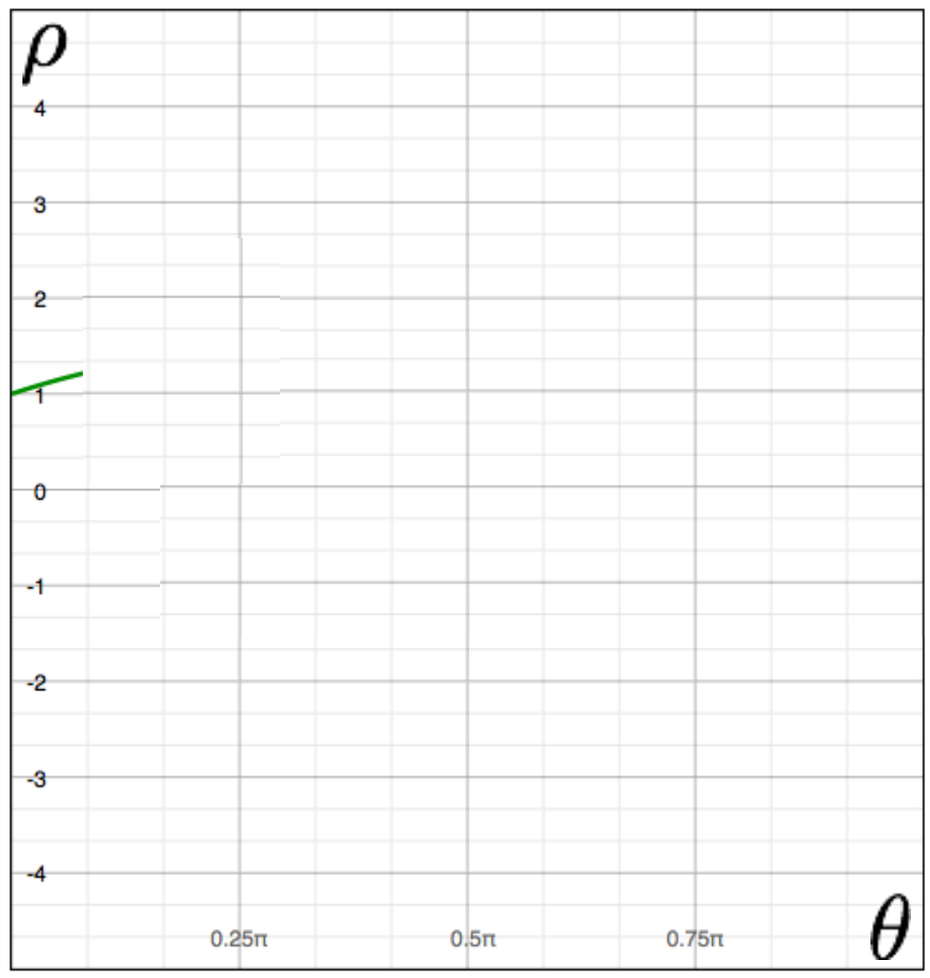


Image space

Parameter space

Image and parameter space

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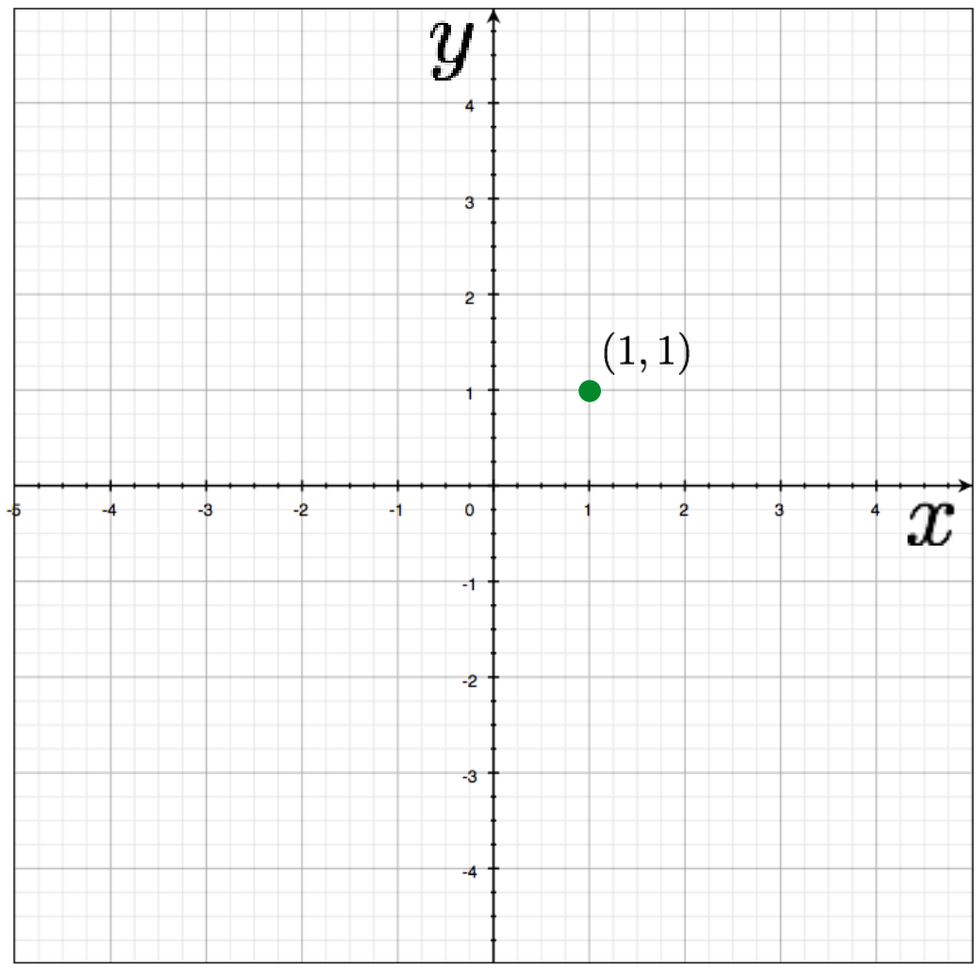
parameters

$$x\cos\theta + y\sin\theta = \rho$$

variables



a point becomes a wave

Image space

Parameter space

Image and parameter space

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$$x\cos\theta + y\sin\theta = \rho$$

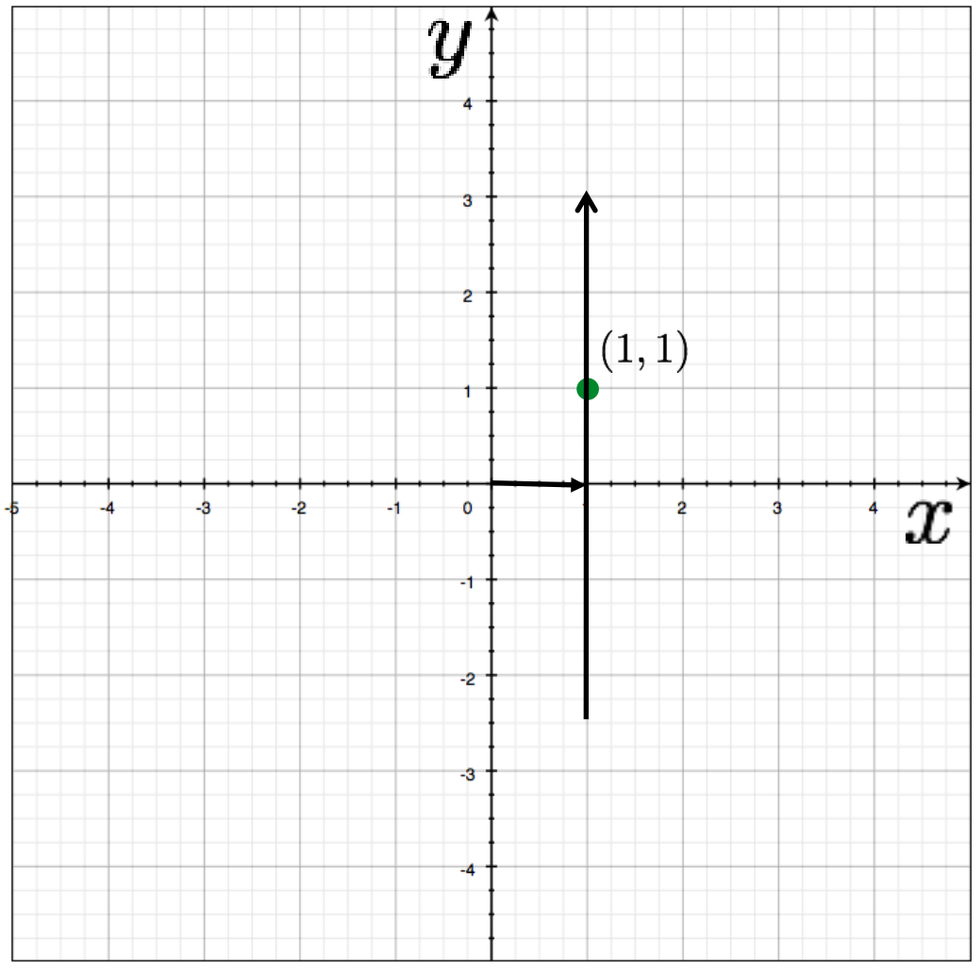
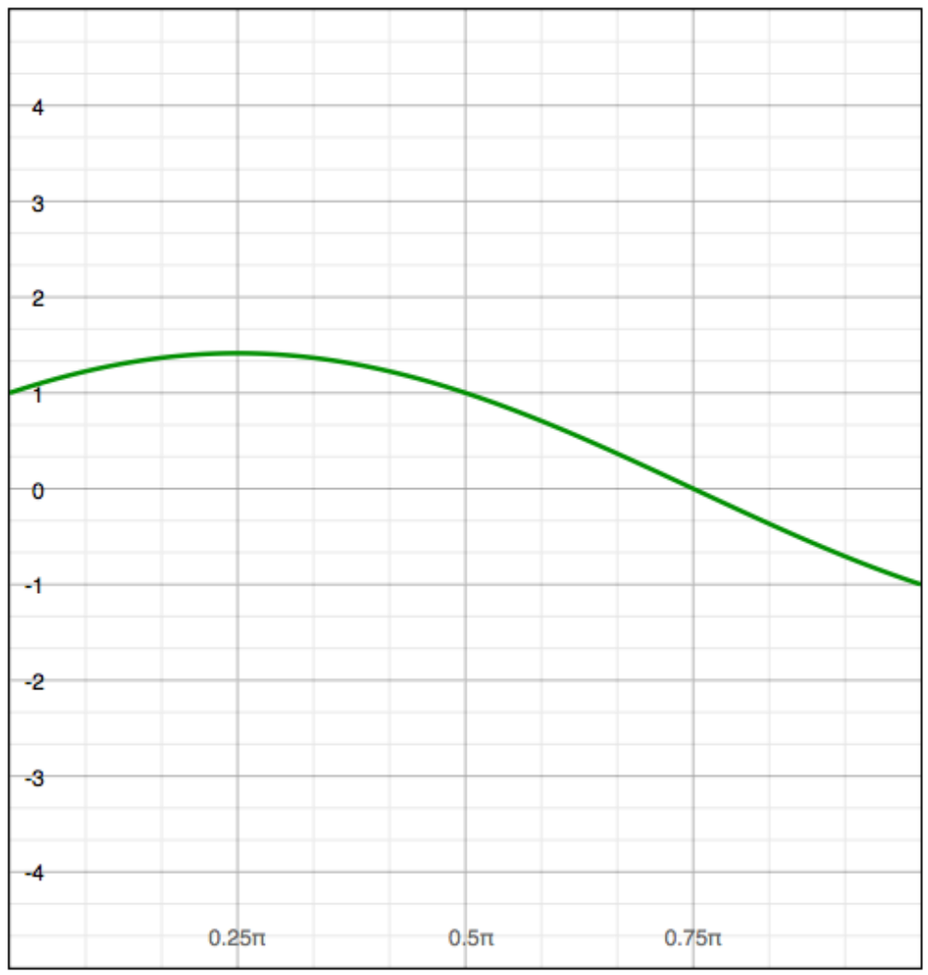


Image space

a line becomes?



Parameter space

Image and parameter space

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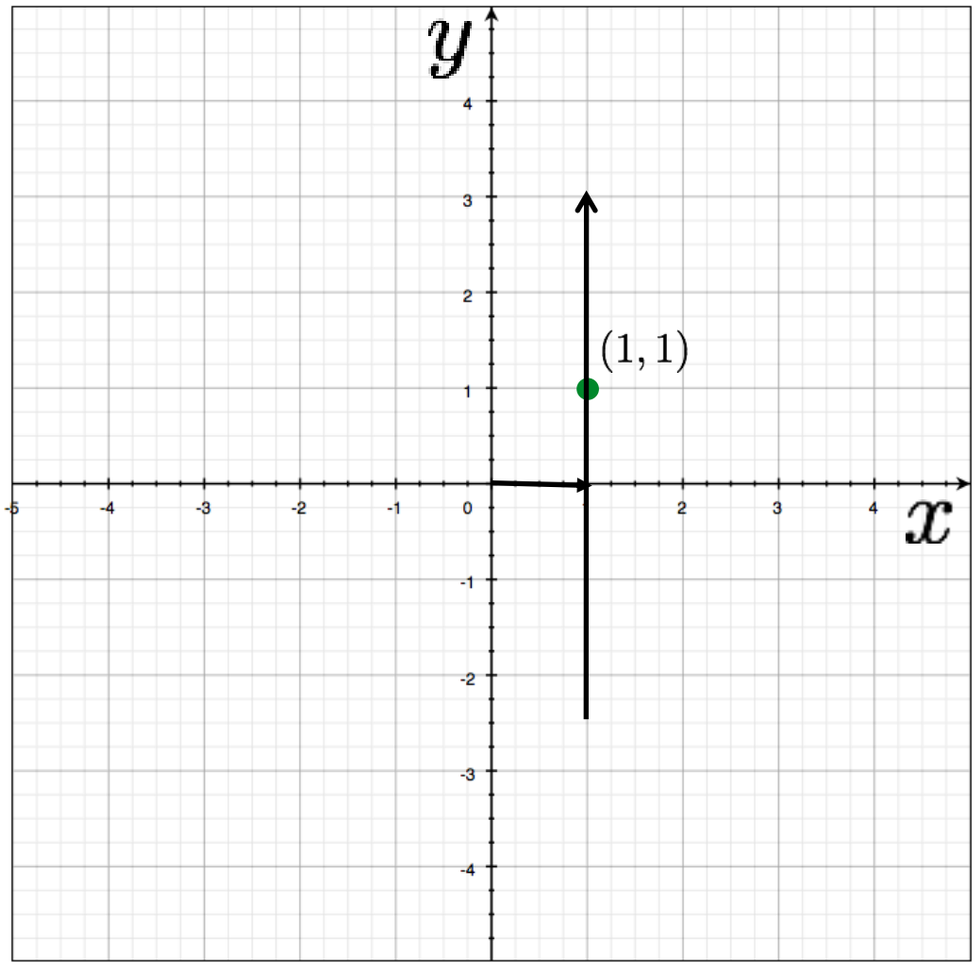
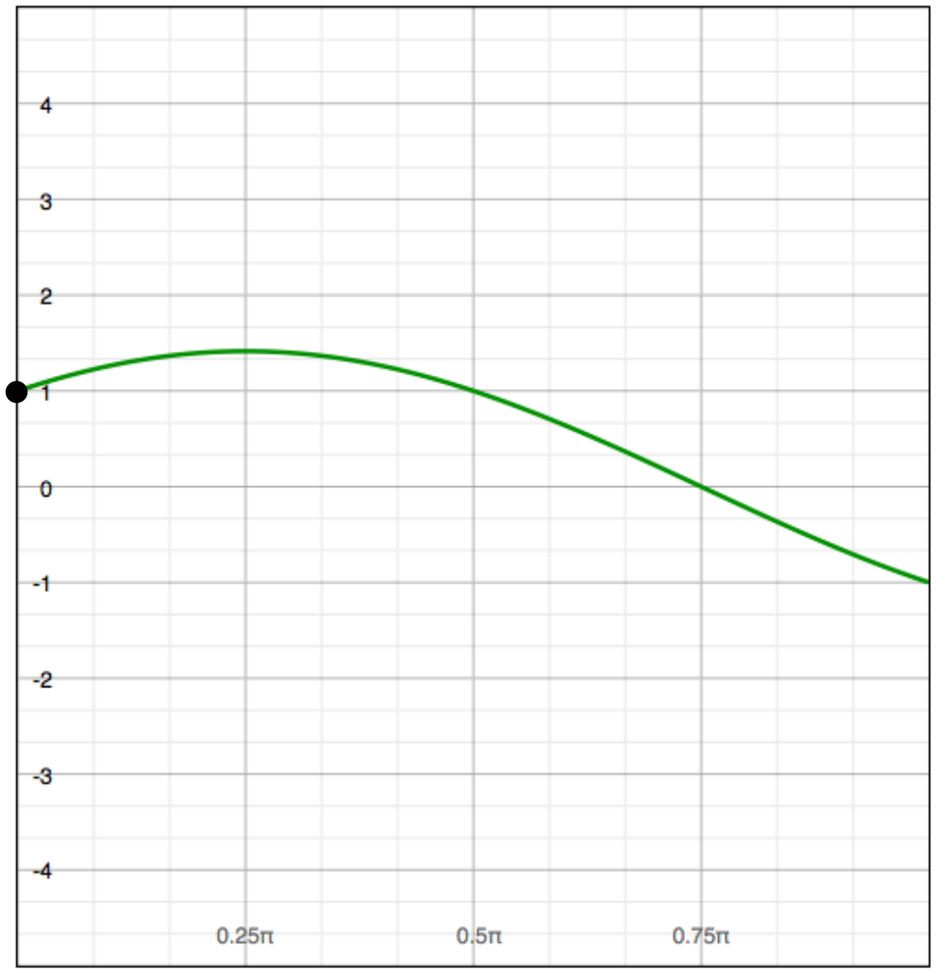


Image space

a line becomes a point



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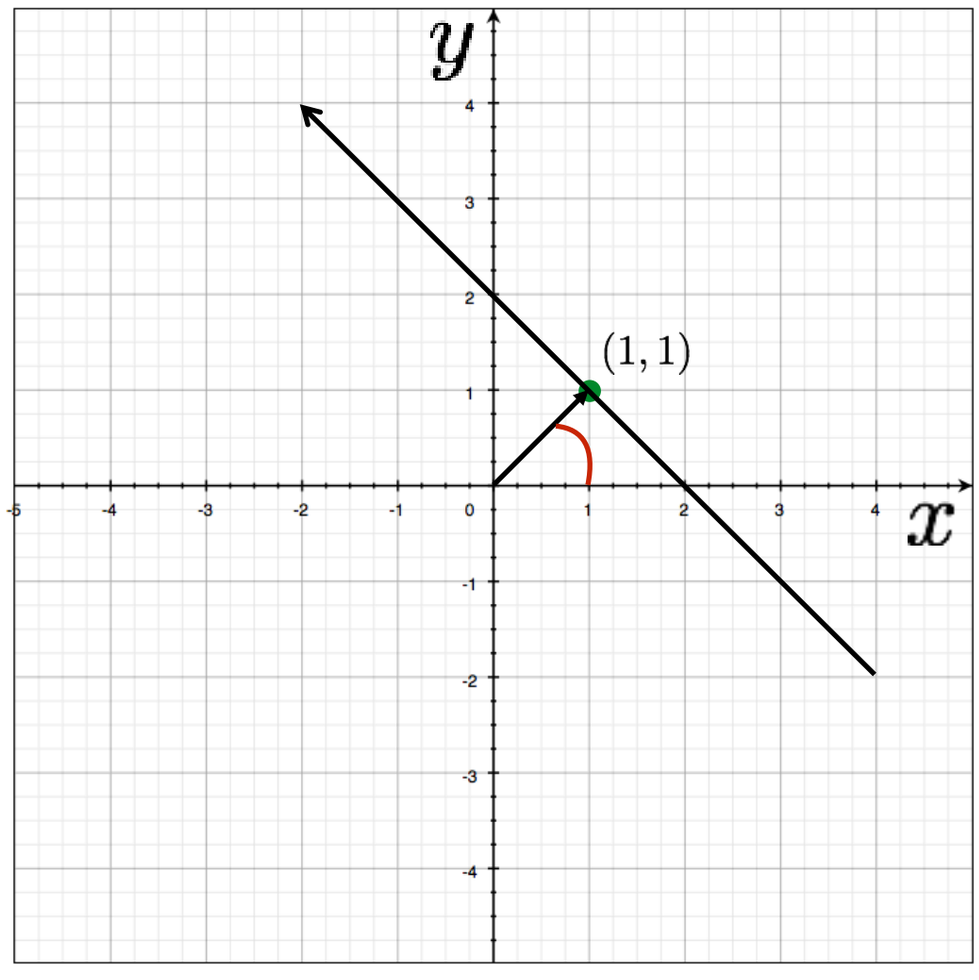
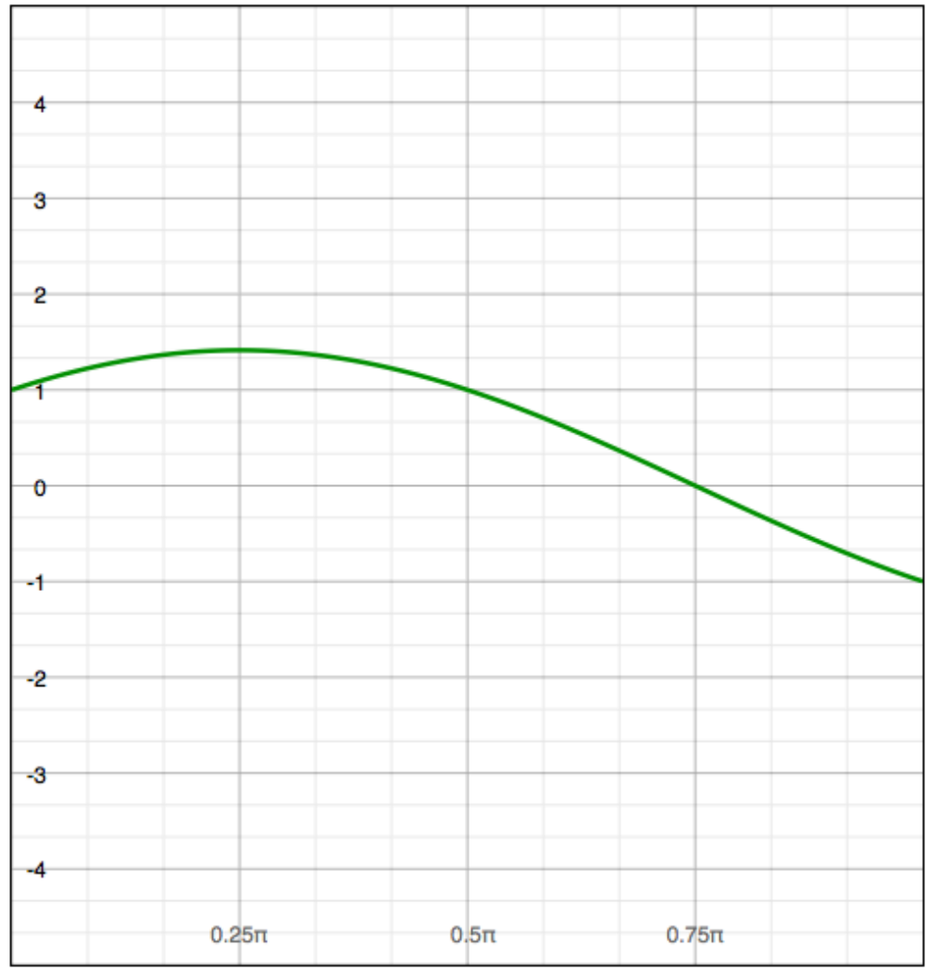


Image space

a line becomes?



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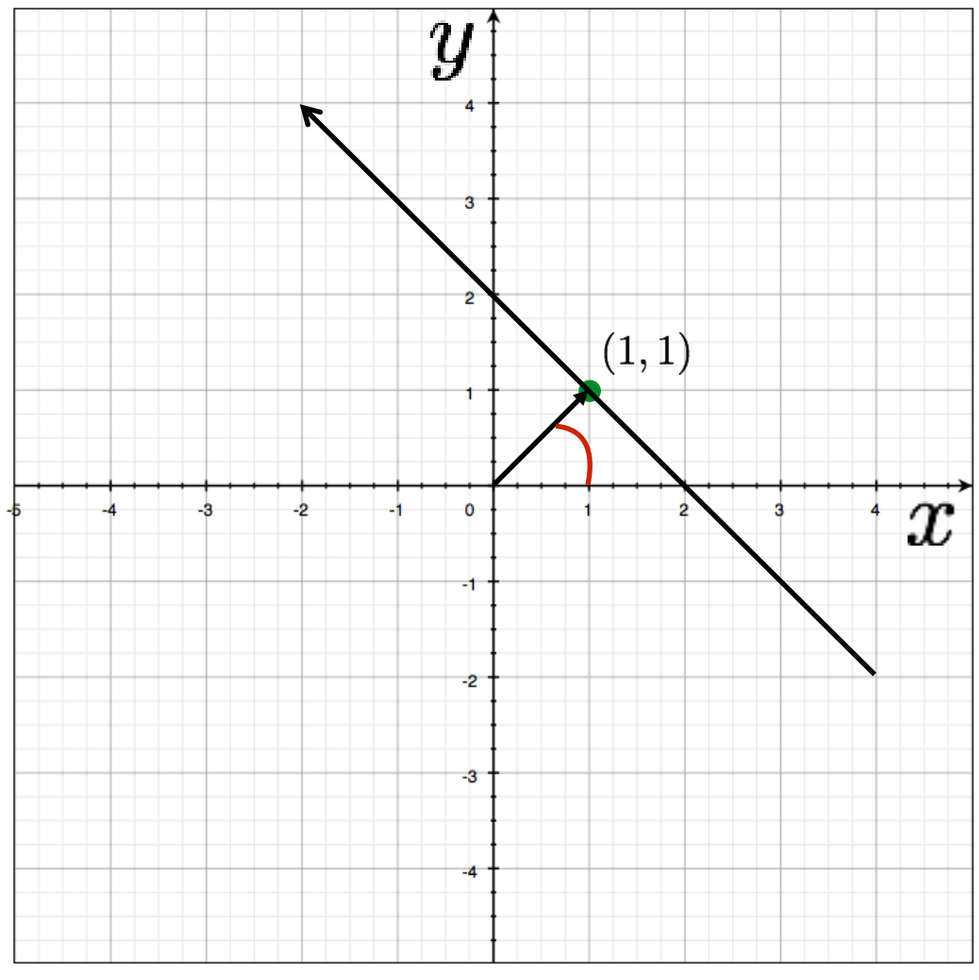

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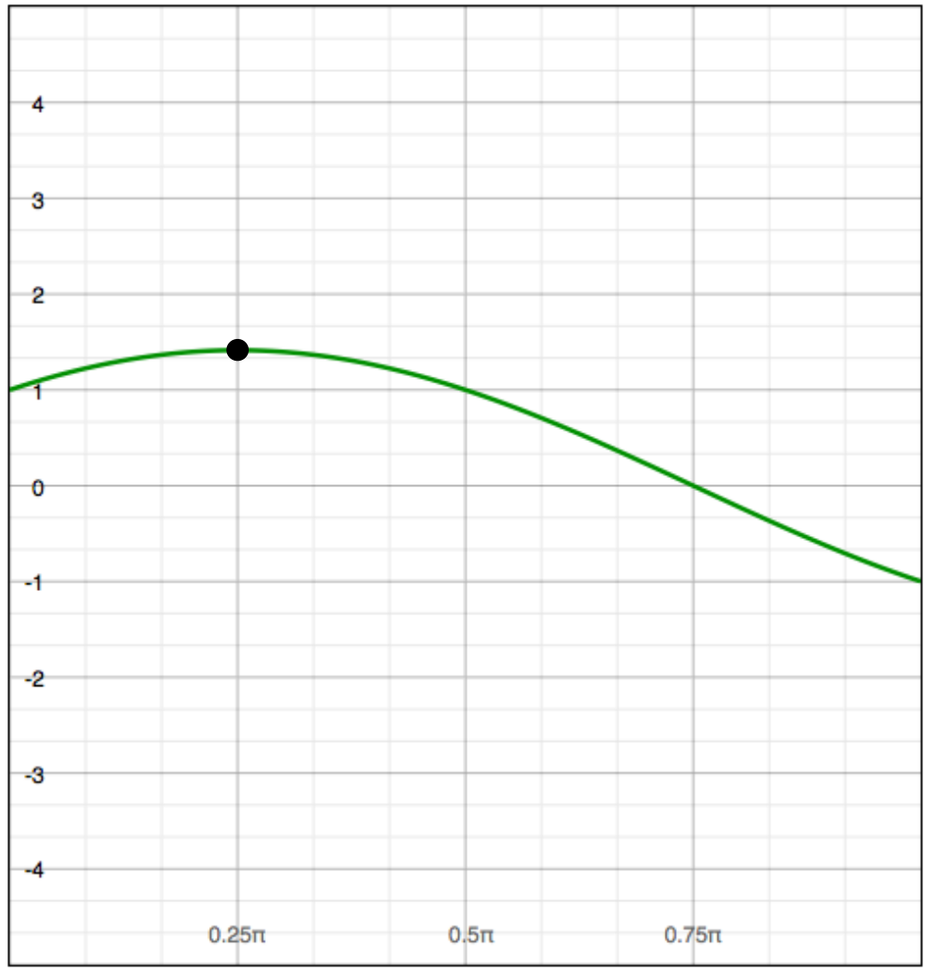

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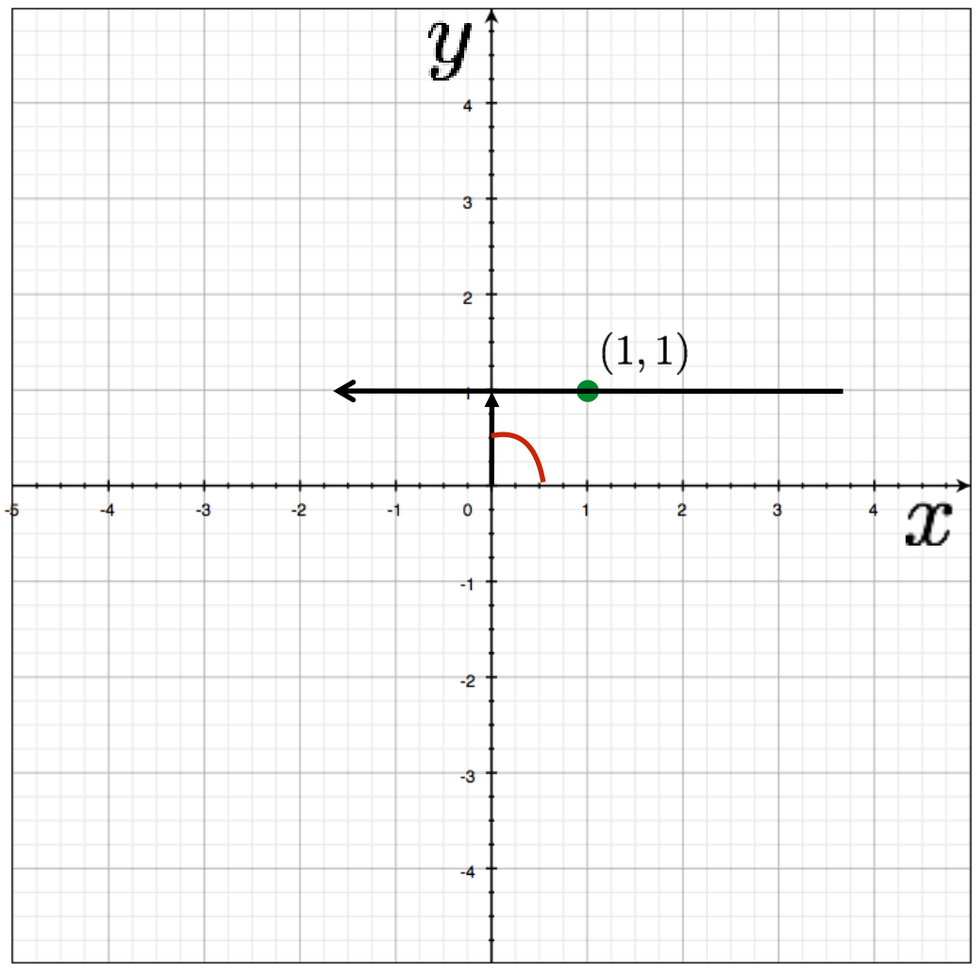

Image space

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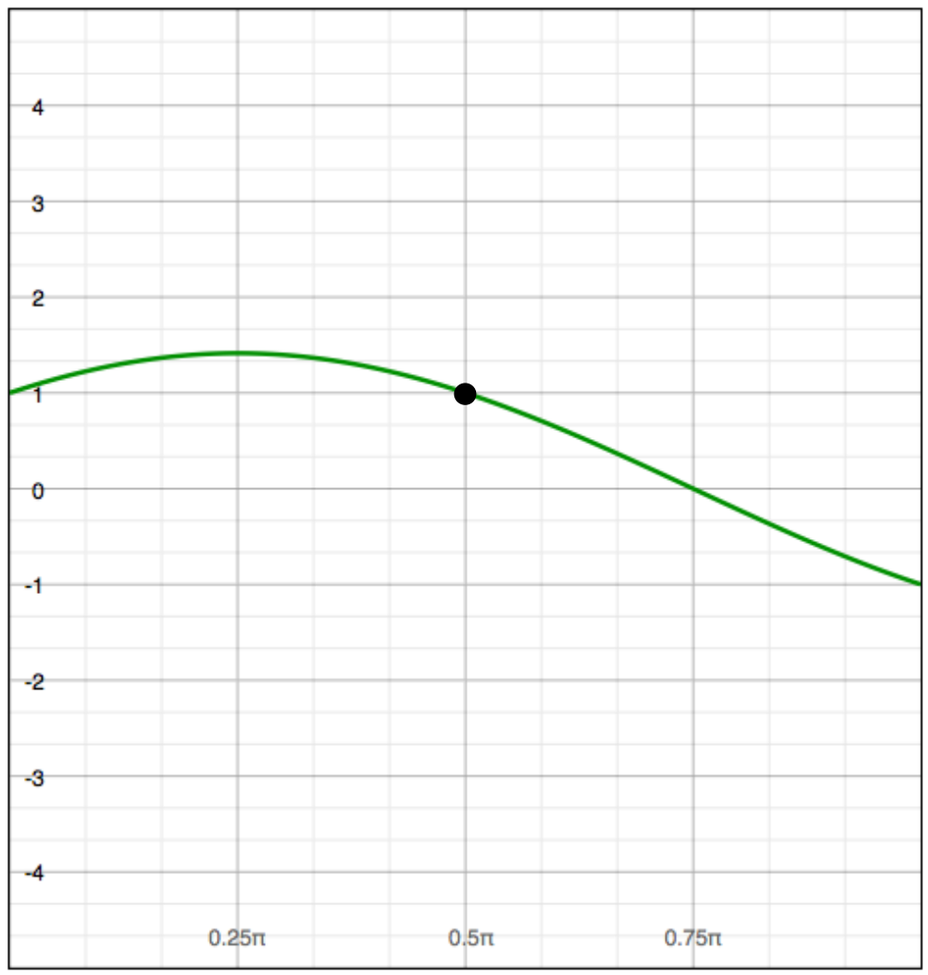

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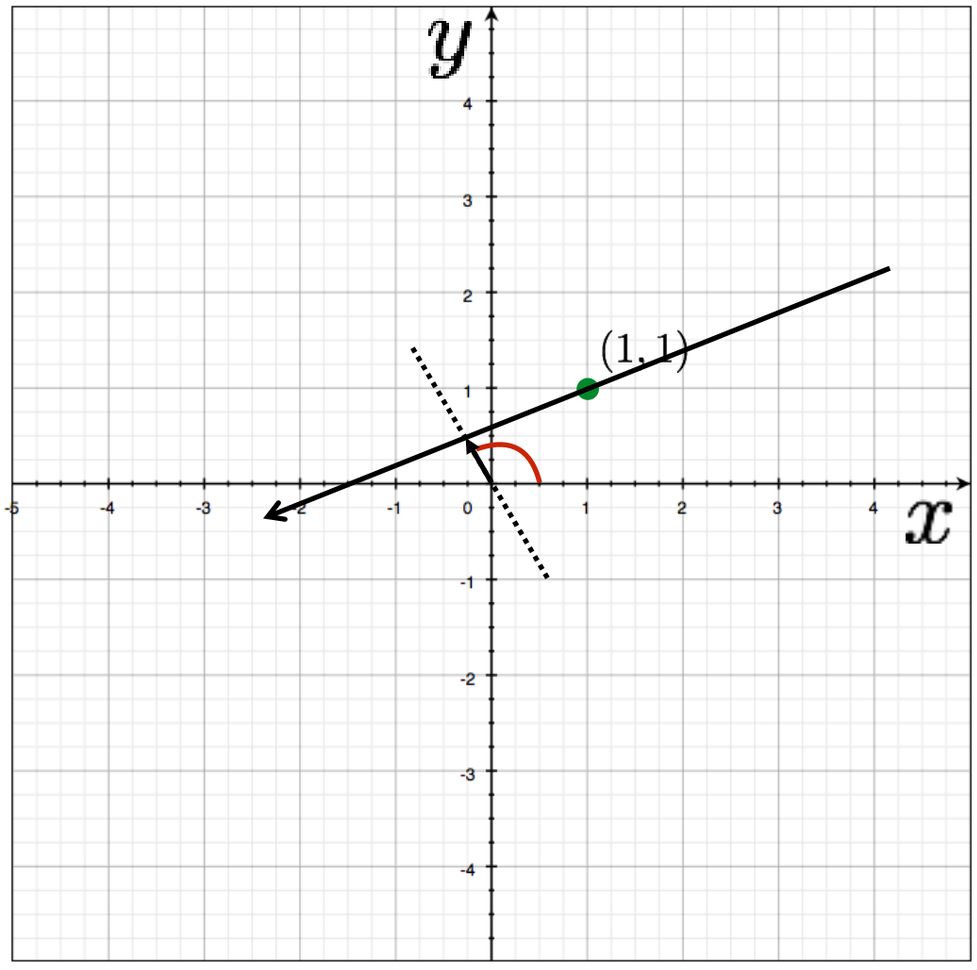

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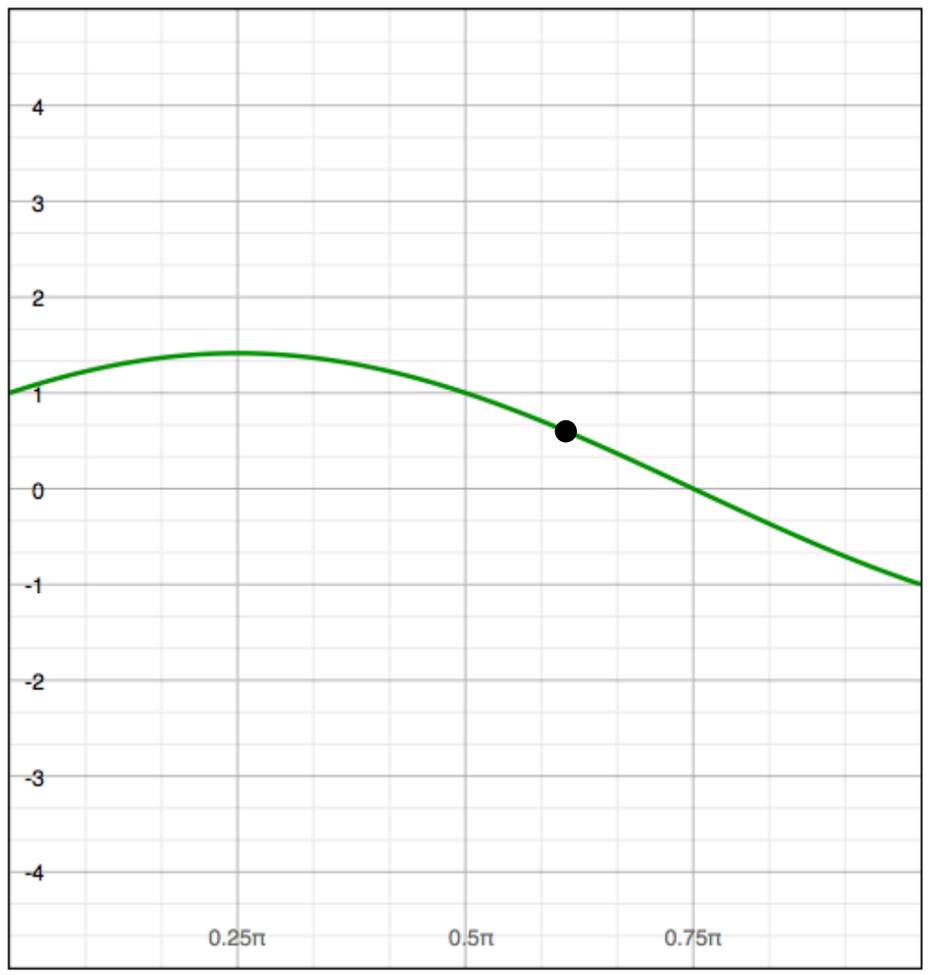

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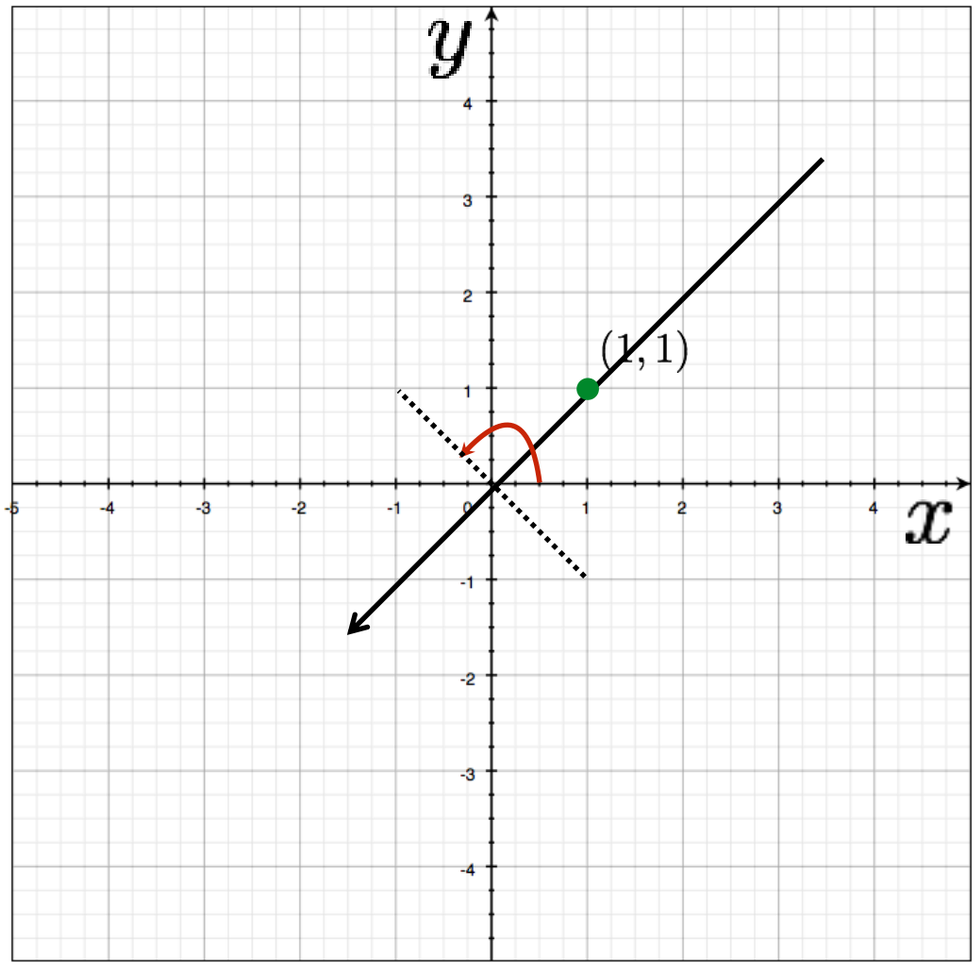

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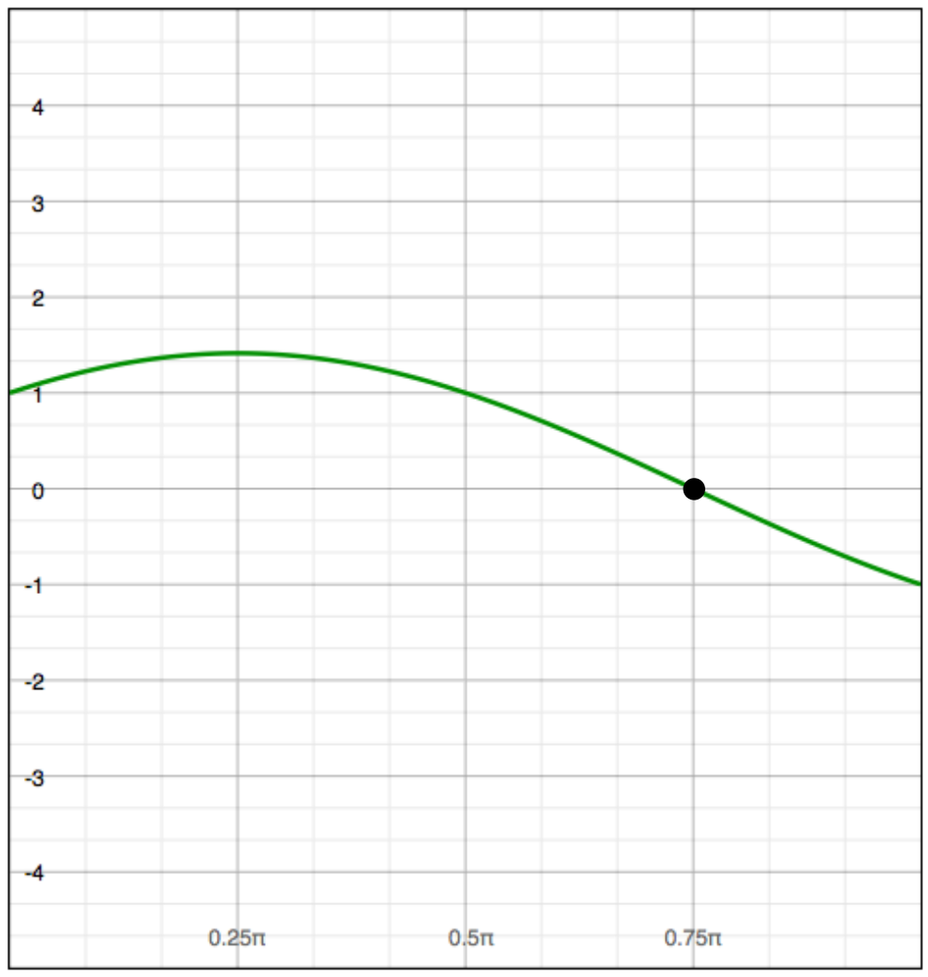

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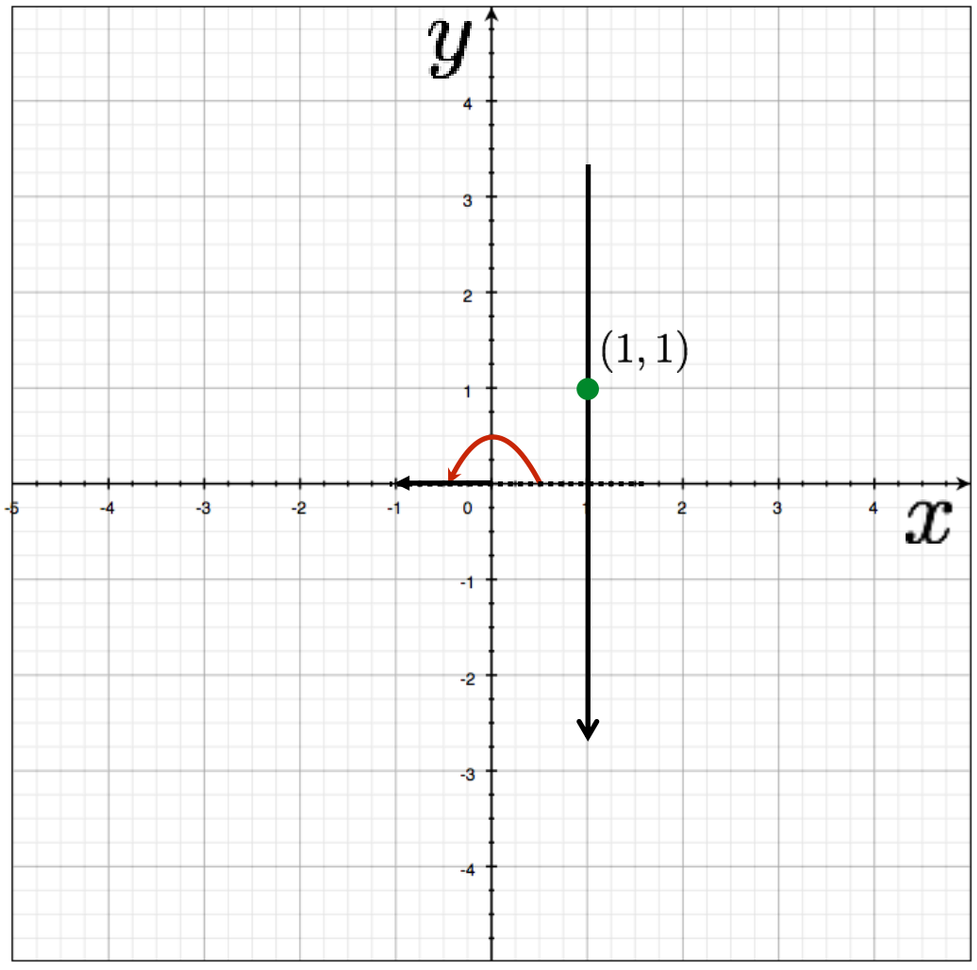

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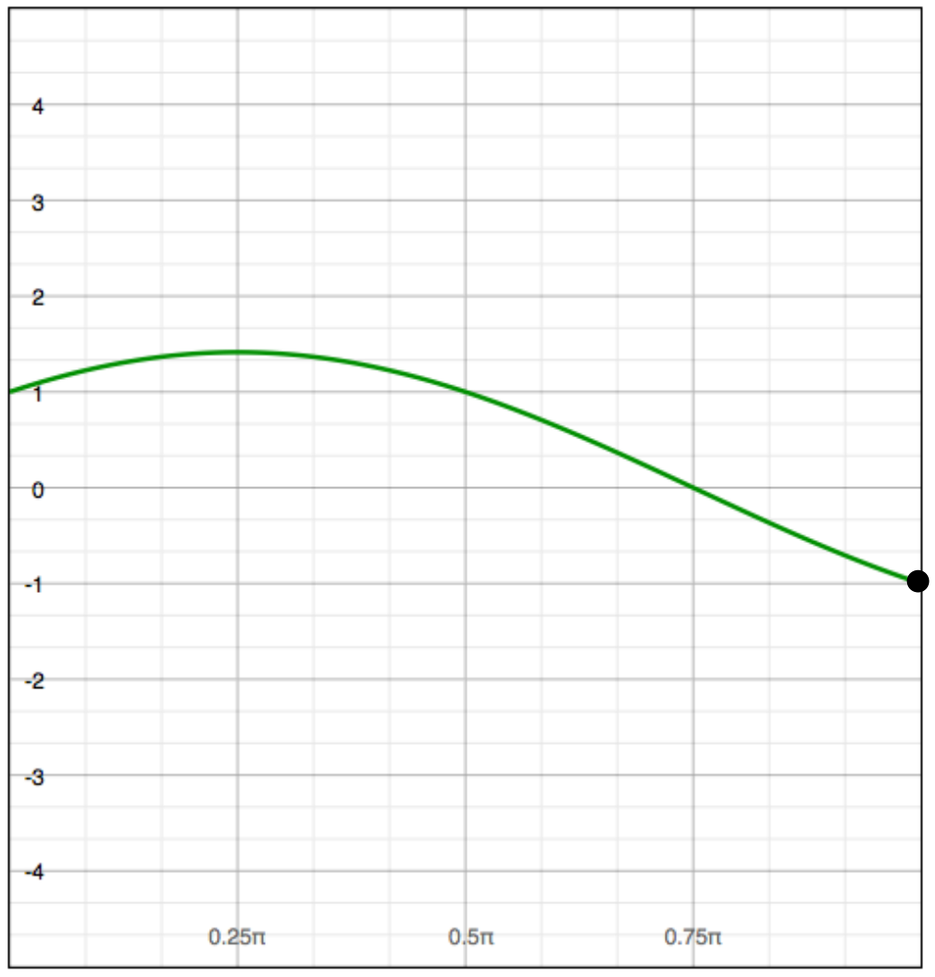

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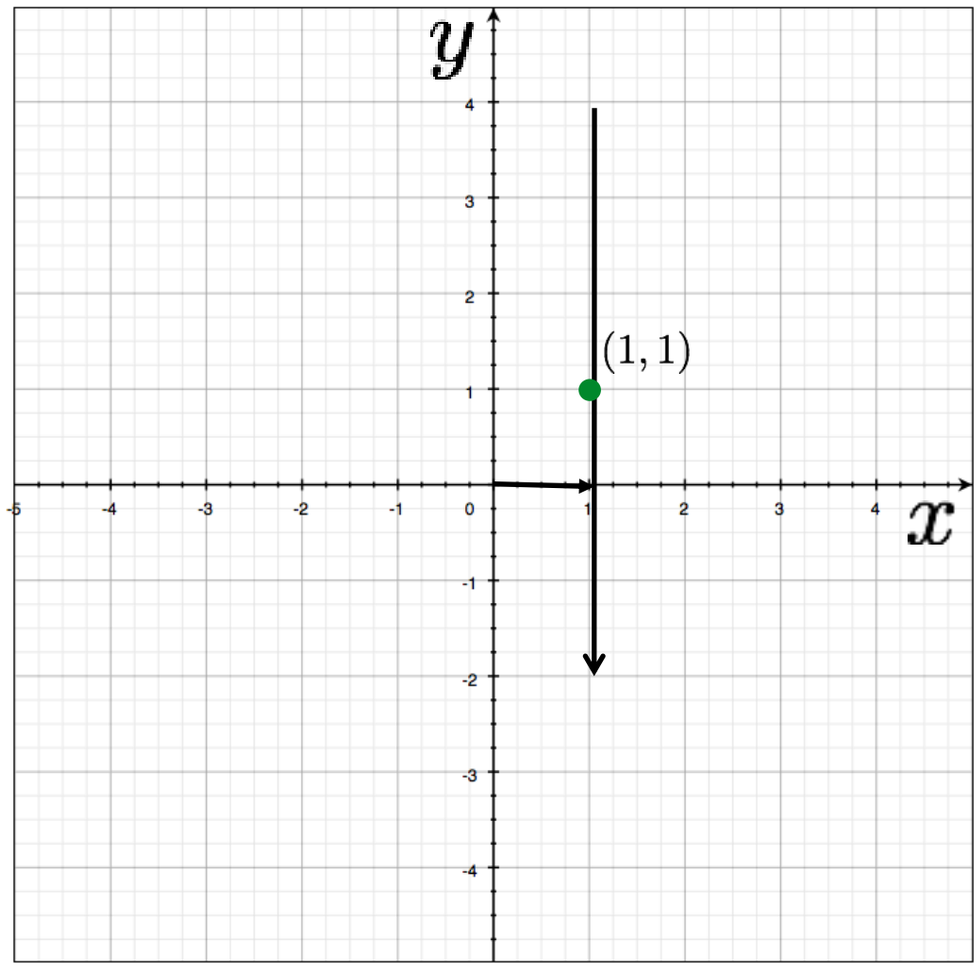

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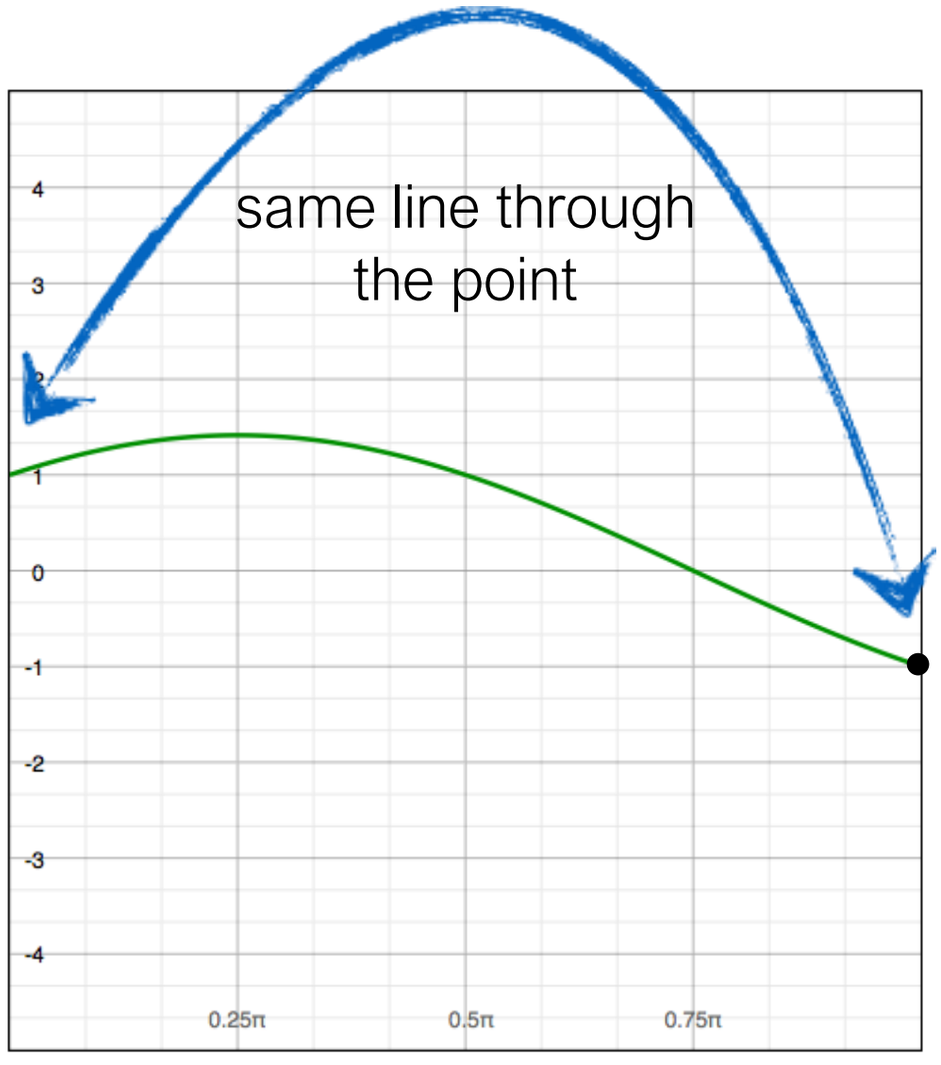

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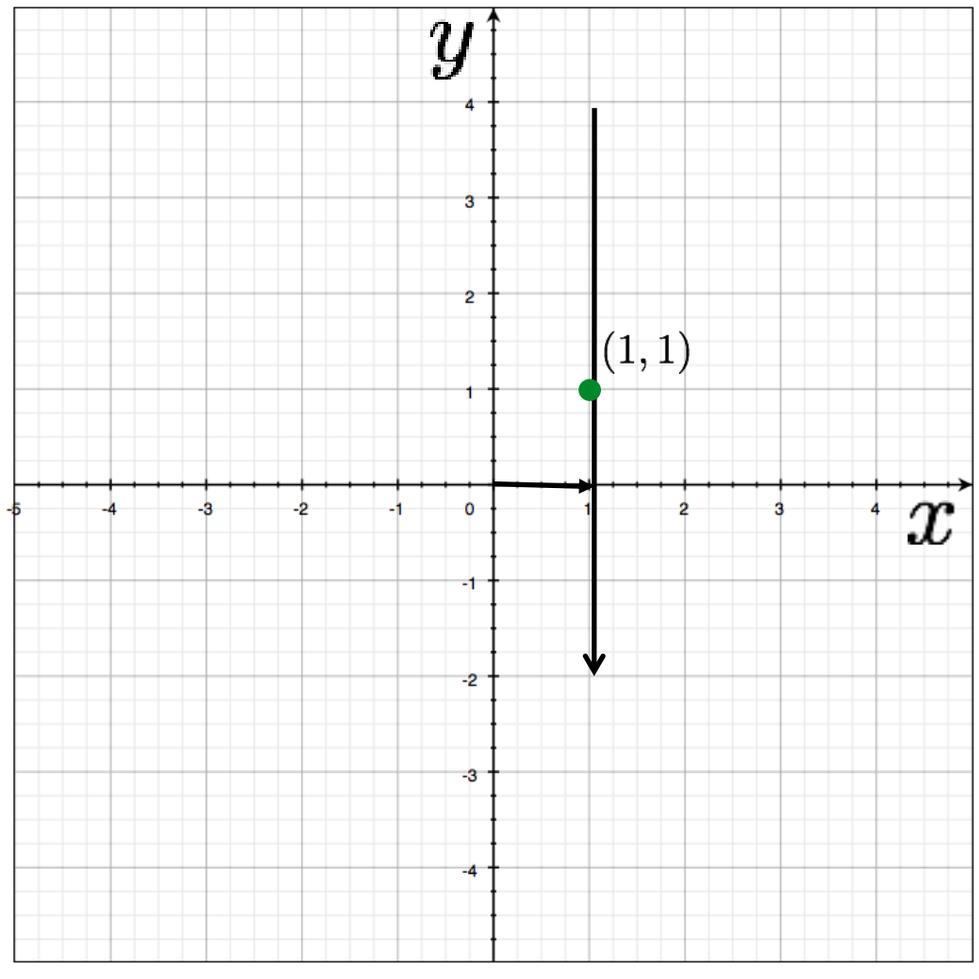

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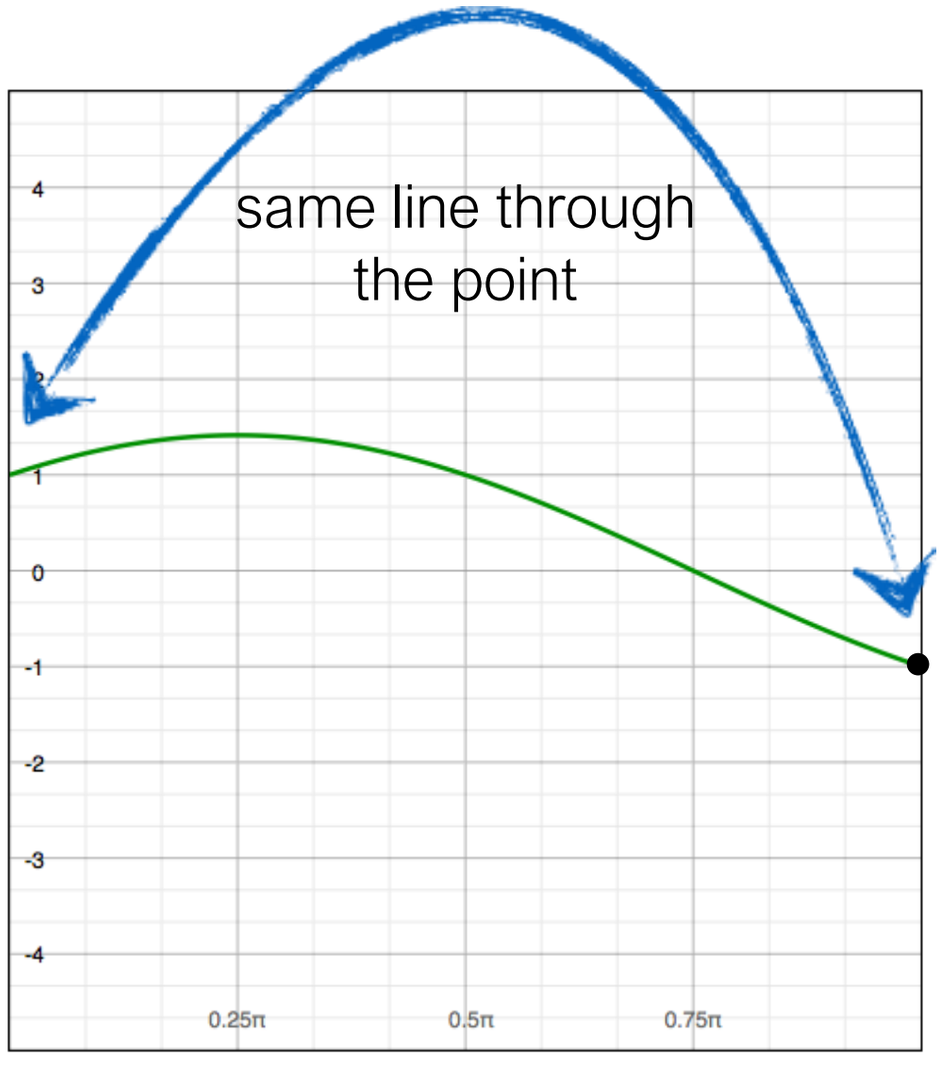

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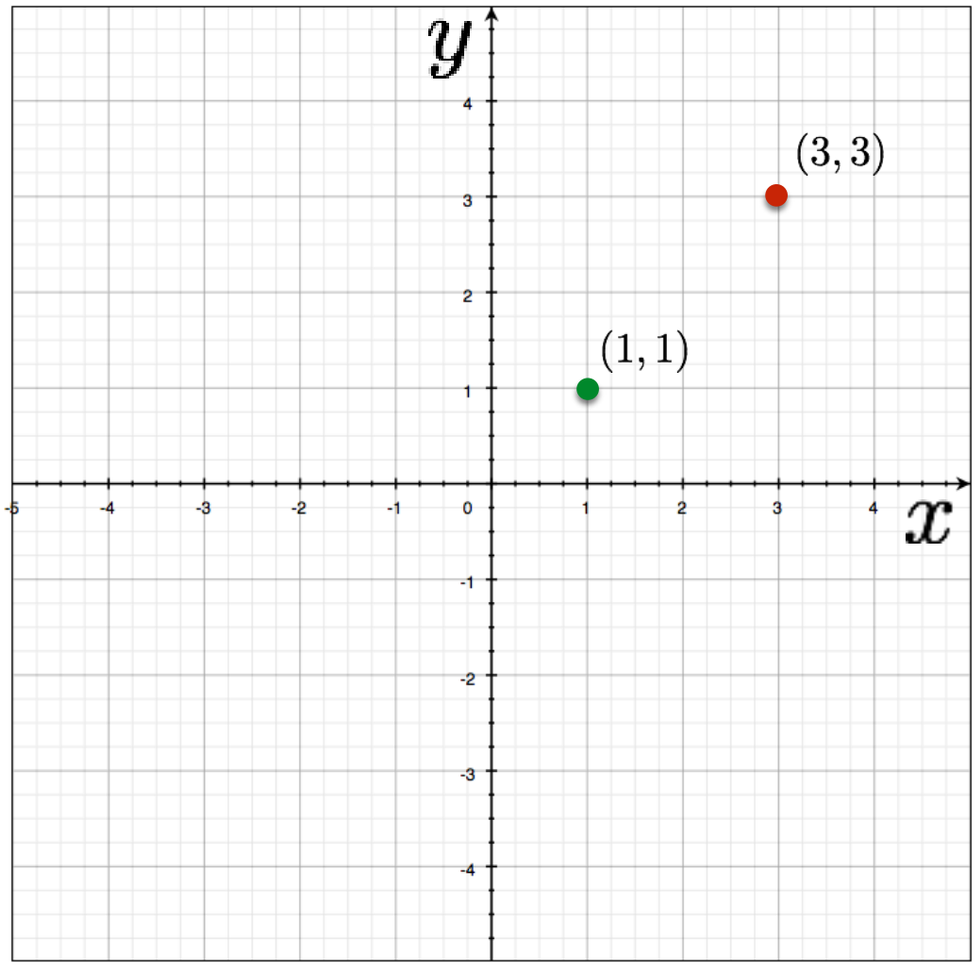
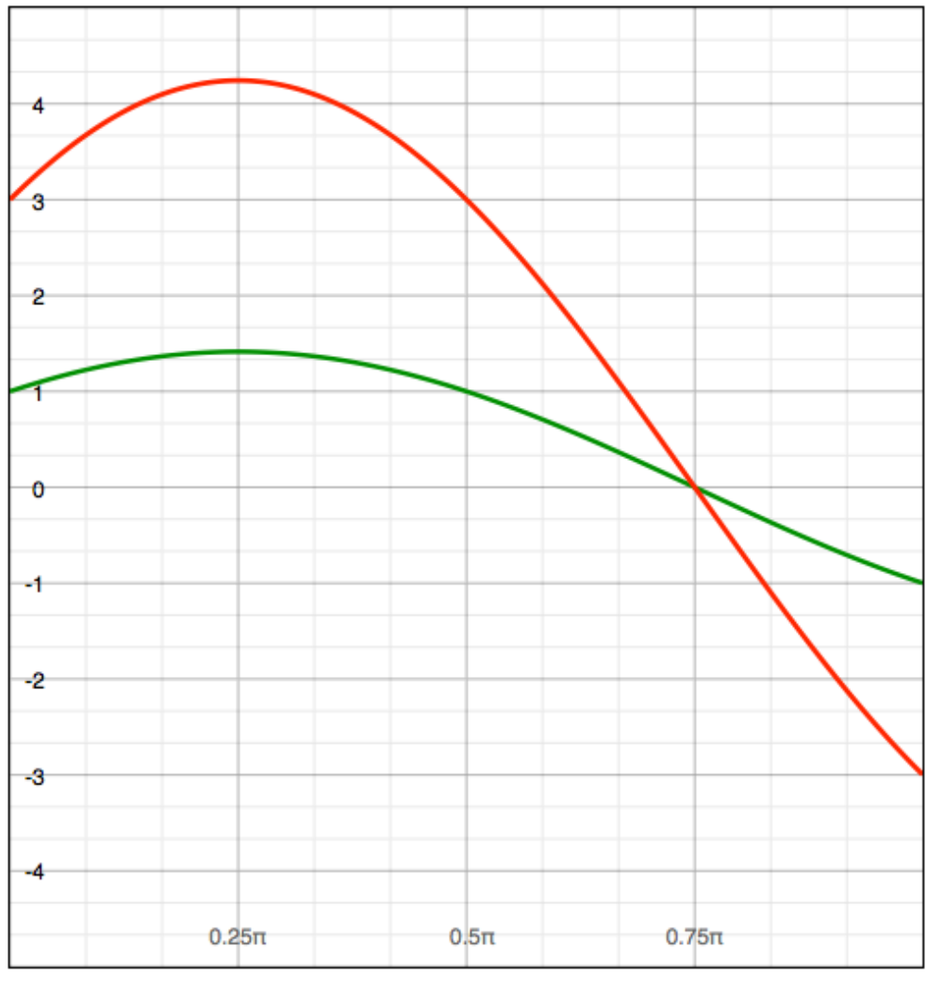


Image space

two points become ?



Parameter space

Image and parameter space

variables

$$y = mx + b$$

parameters

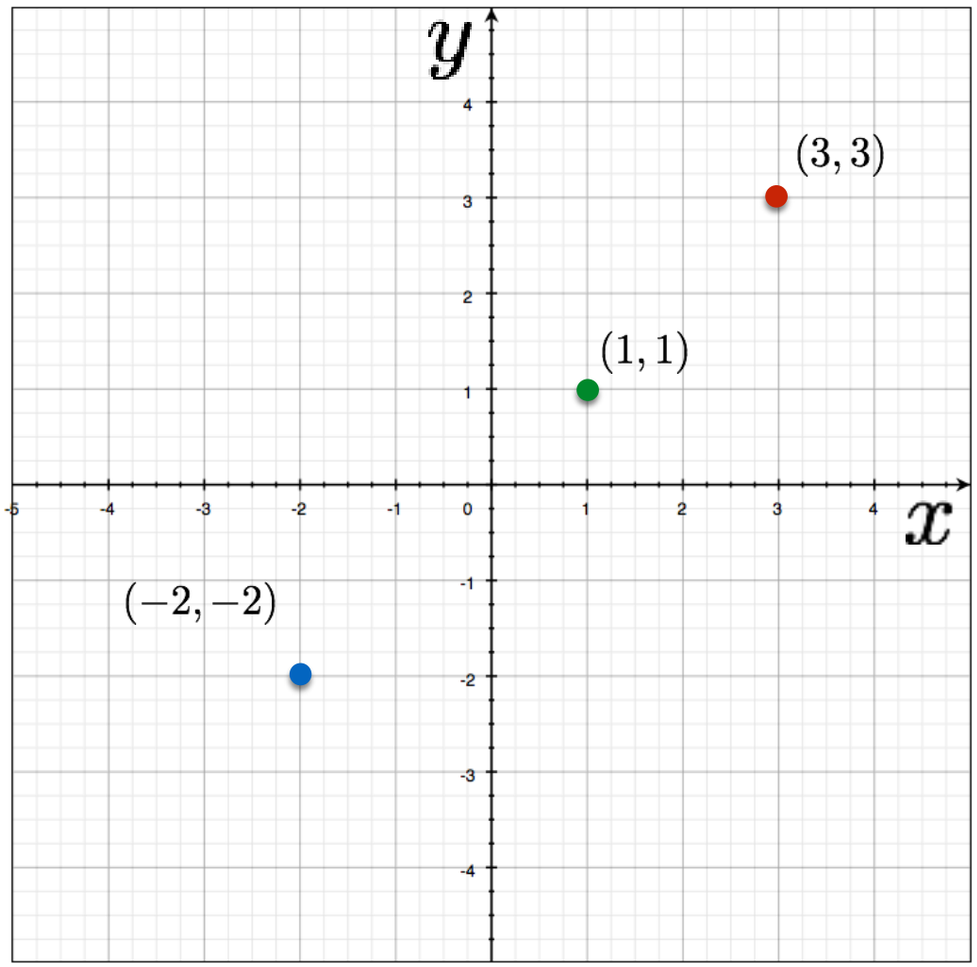
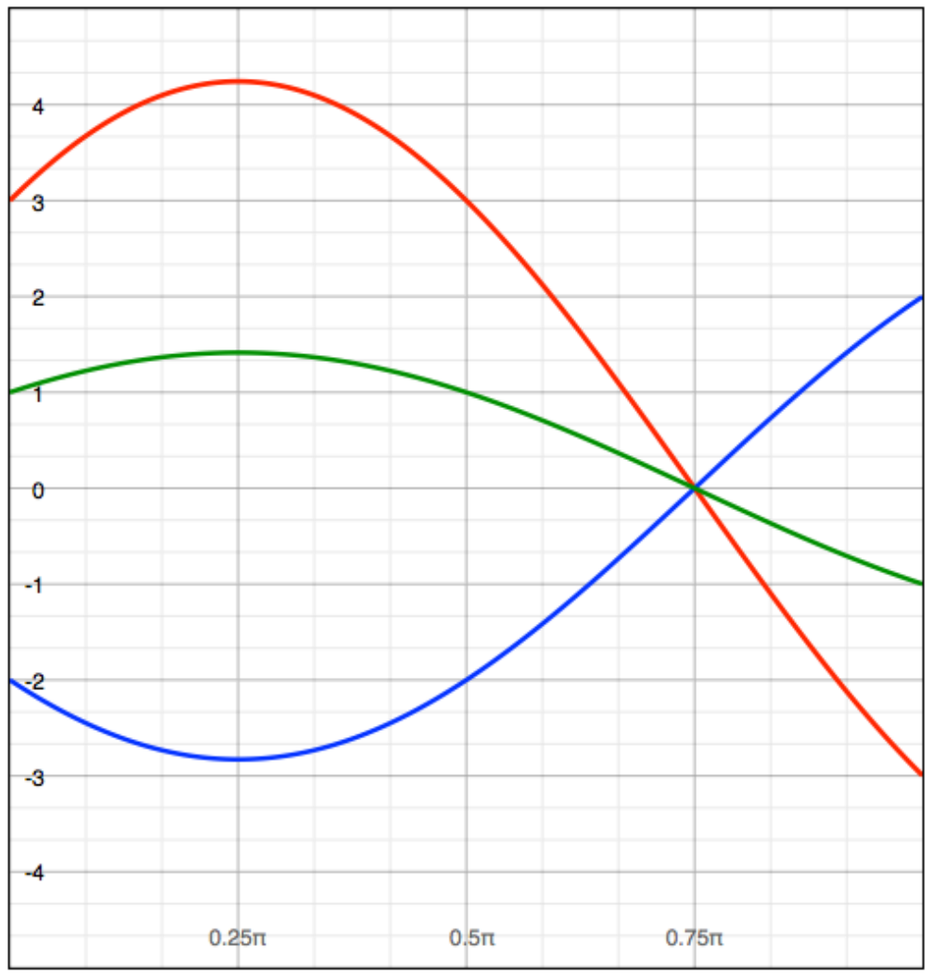


Image space

three points become ?



Parameter space

Image and parameter space

variables

$$y = mx + b$$

parameters

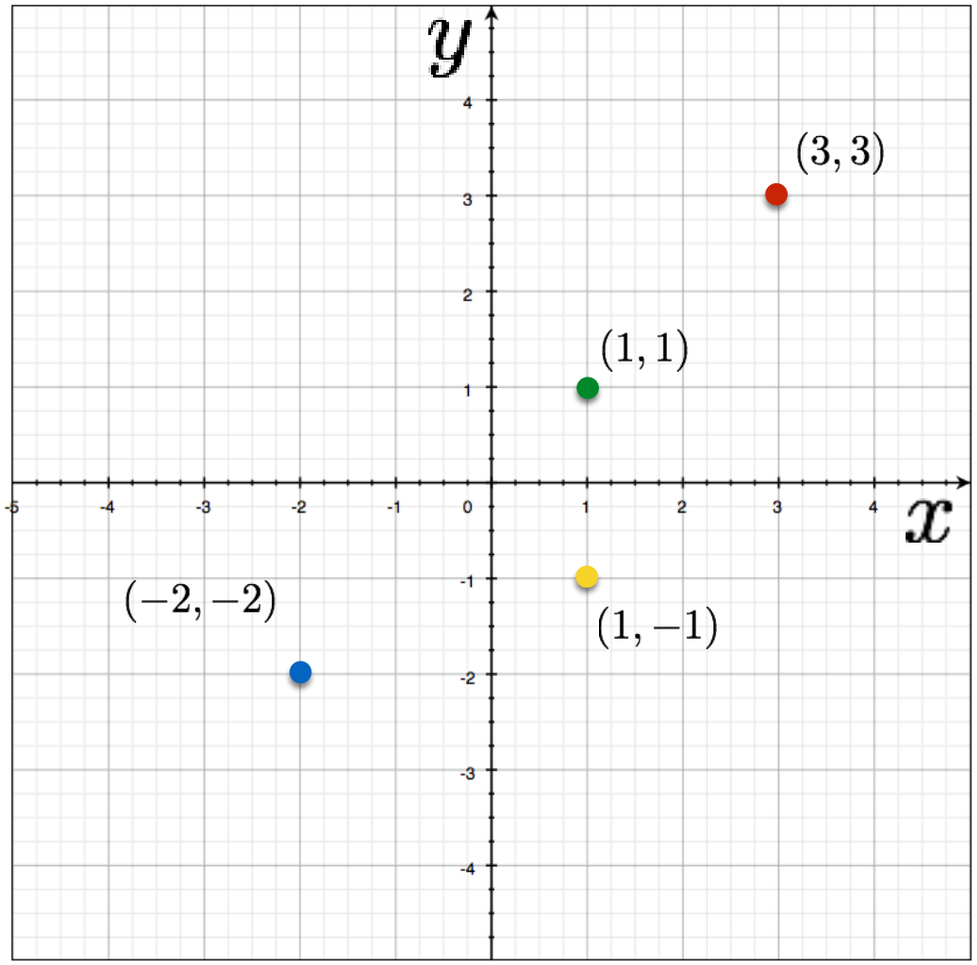
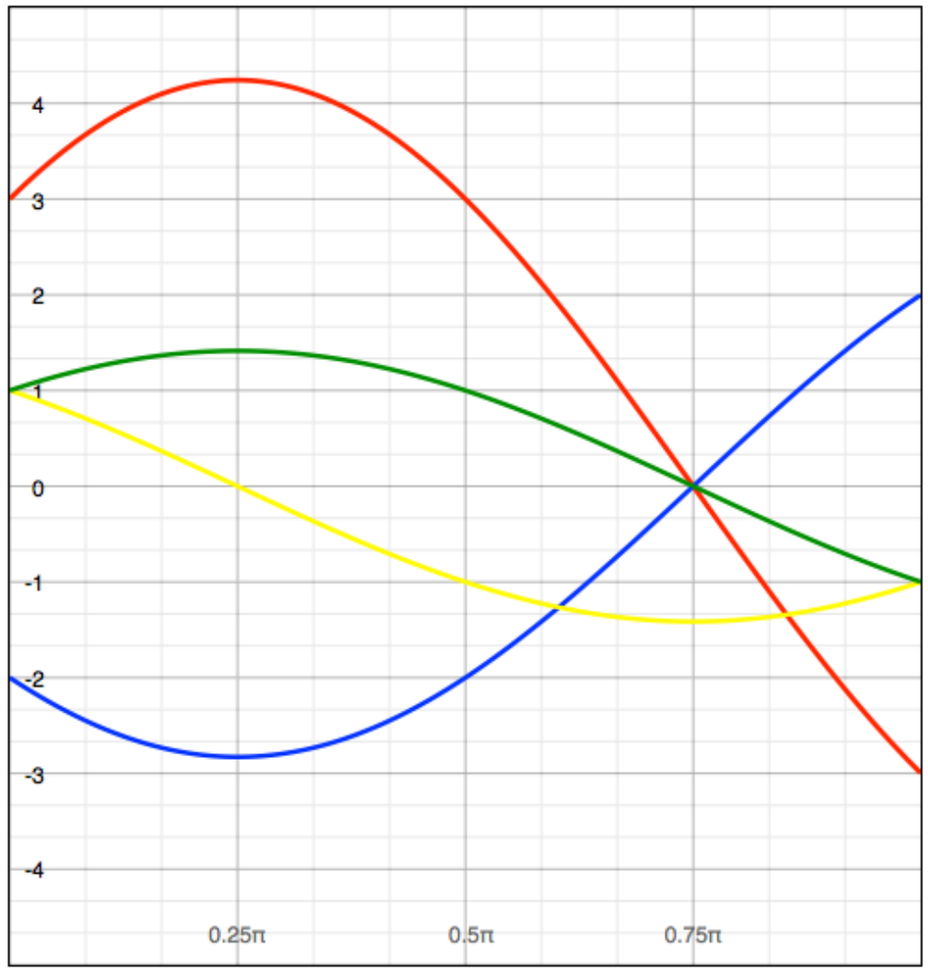


Image space

four points become ?



Parameter space

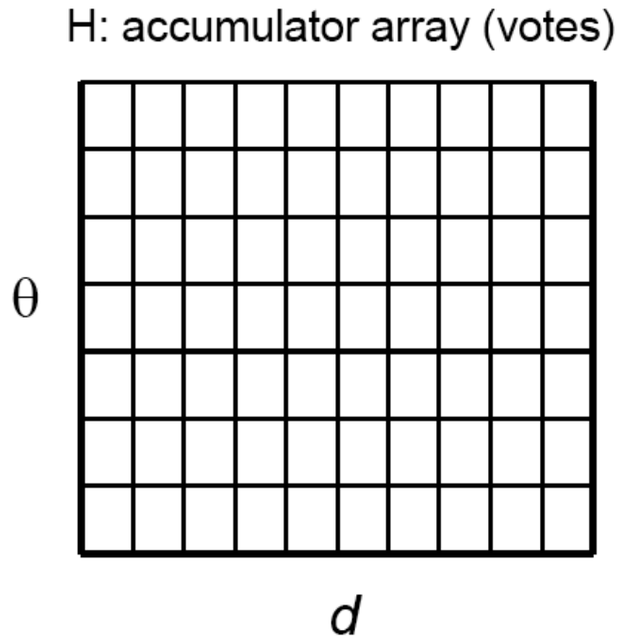
Implementation

1. **Initialize accumulator H to all zeros**

2. **For each edge point (x,y) in the image**
   For θ = 0 to 180
   ρ = x cos θ + y sin θ
   H(θ, ρ) = H(θ, ρ) + 1
   end
   end

3. **Find the value(s) of (θ, ρ) where H(θ, ρ) is a local maximum**

4. **The detected line in the image is given by**
   ρ = x cos θ + y sin θ



NOTE: Watch your coordinates. Image origin is top left!

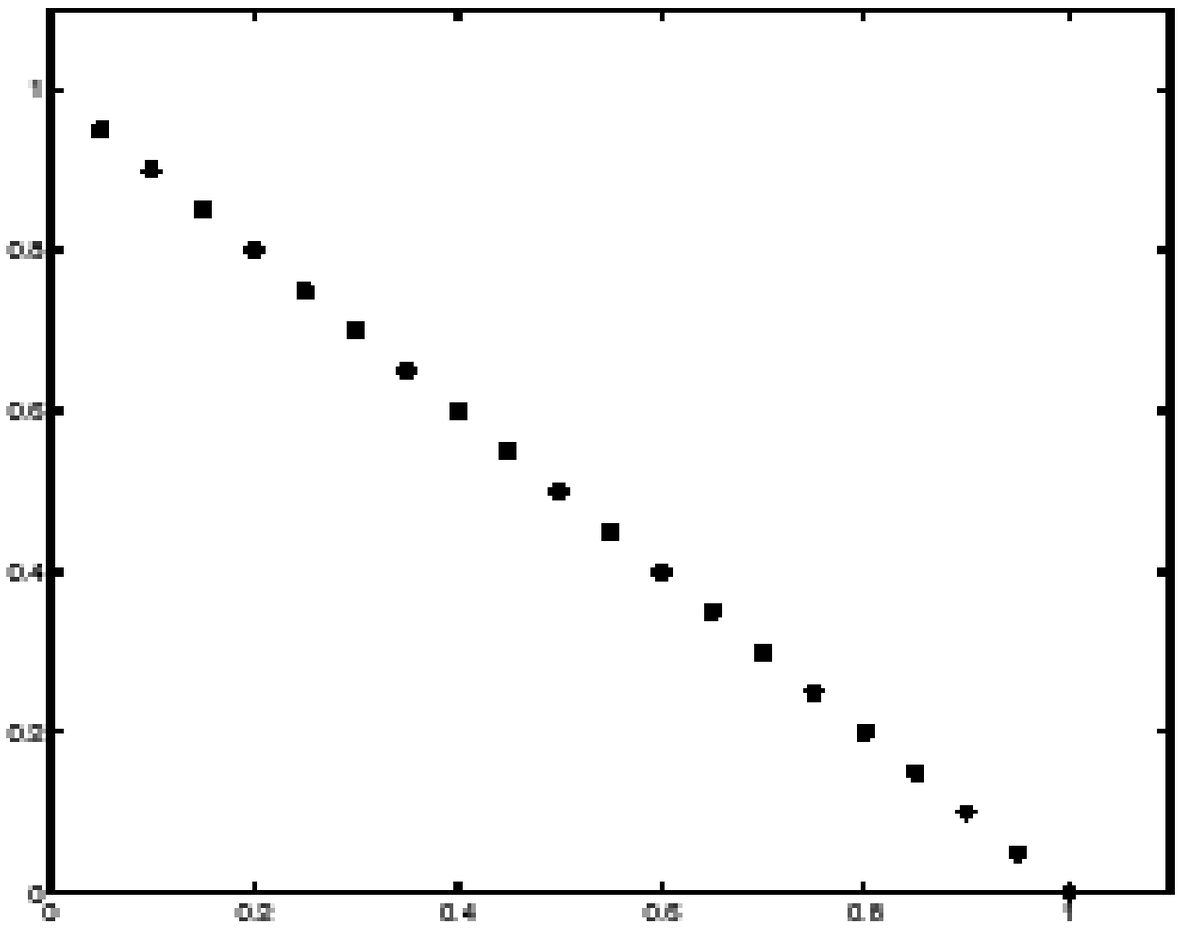
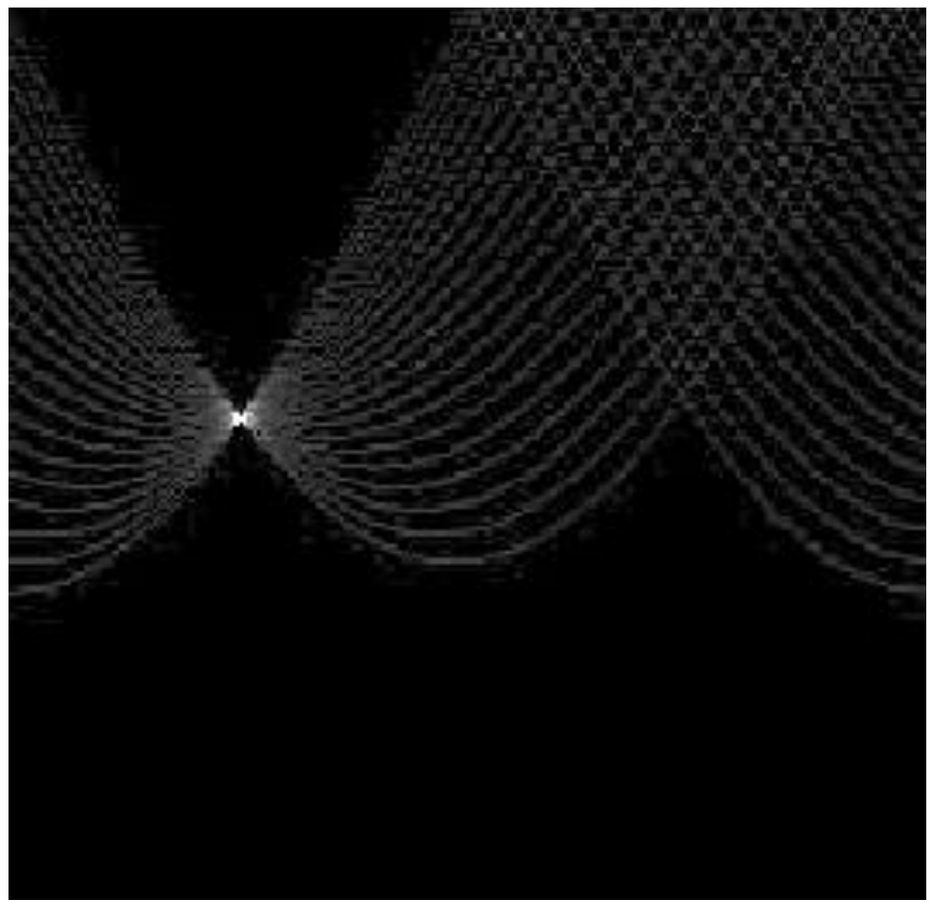


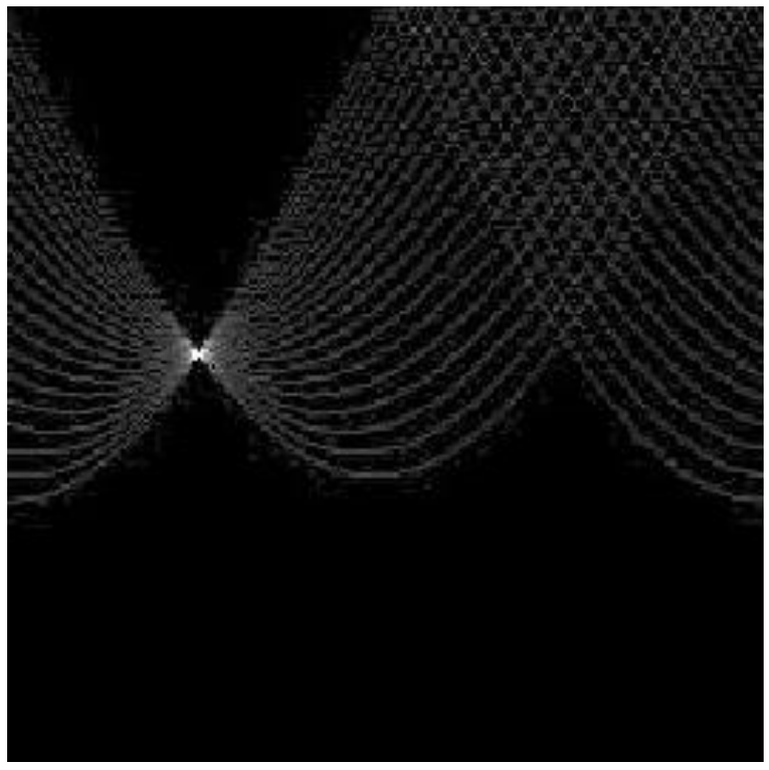
Image space

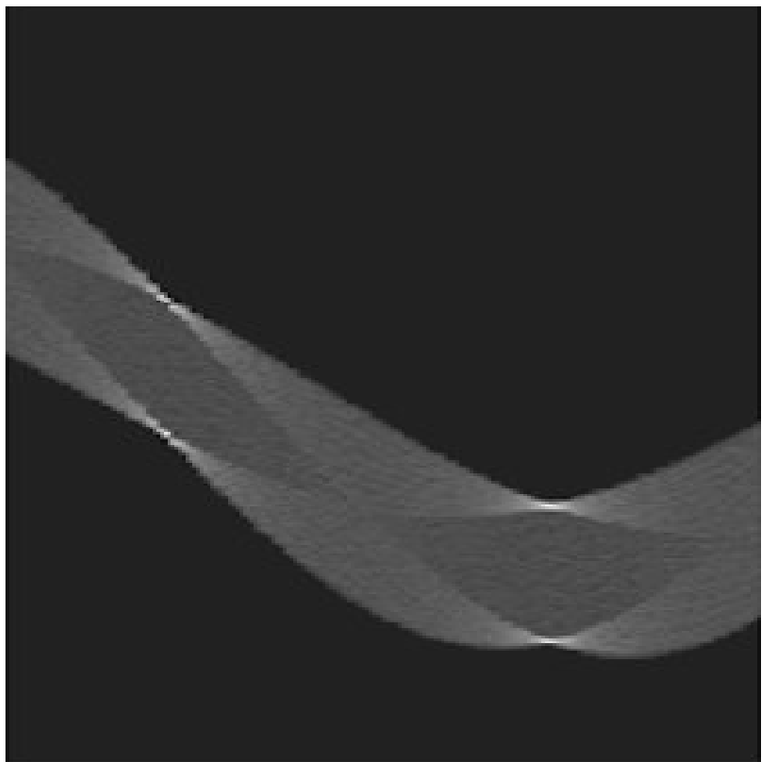


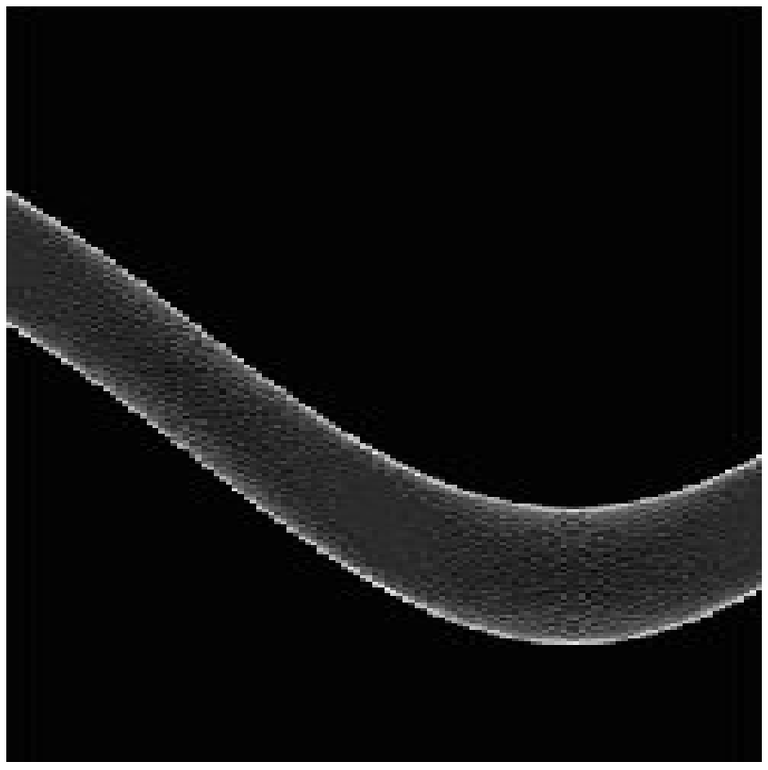
Votes

Basic shapes

(in parameter space)



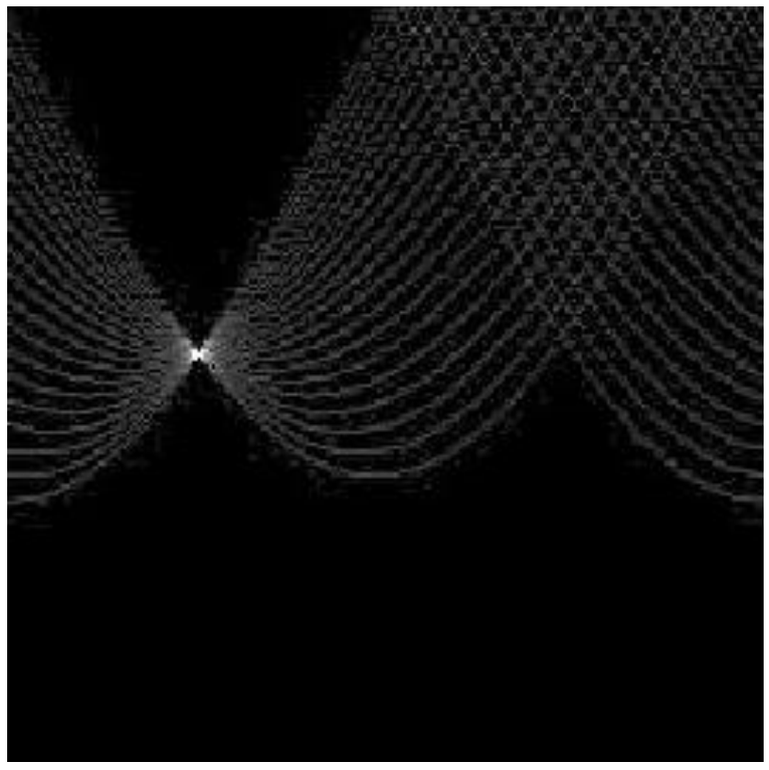




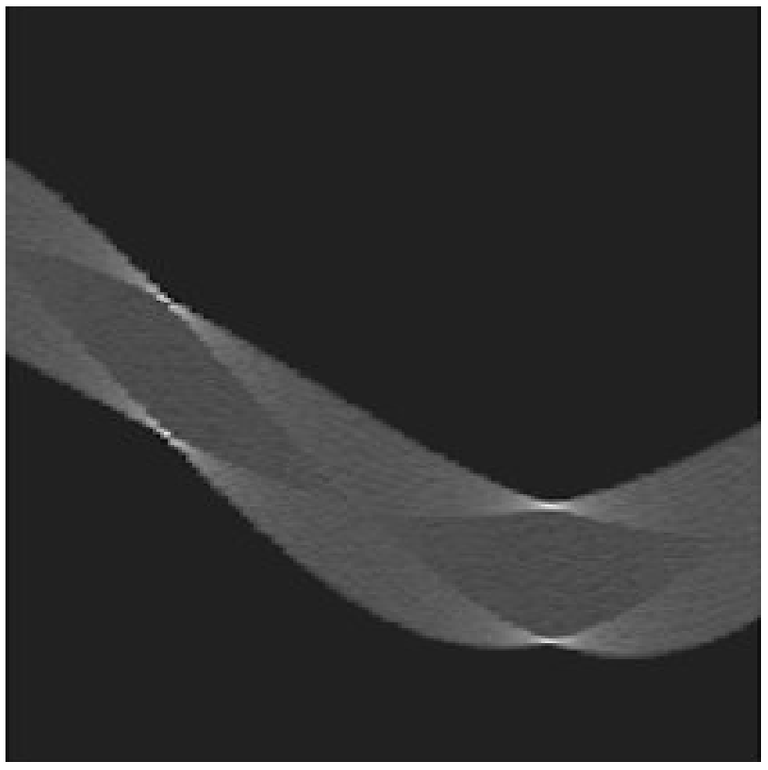
can you guess the shape?

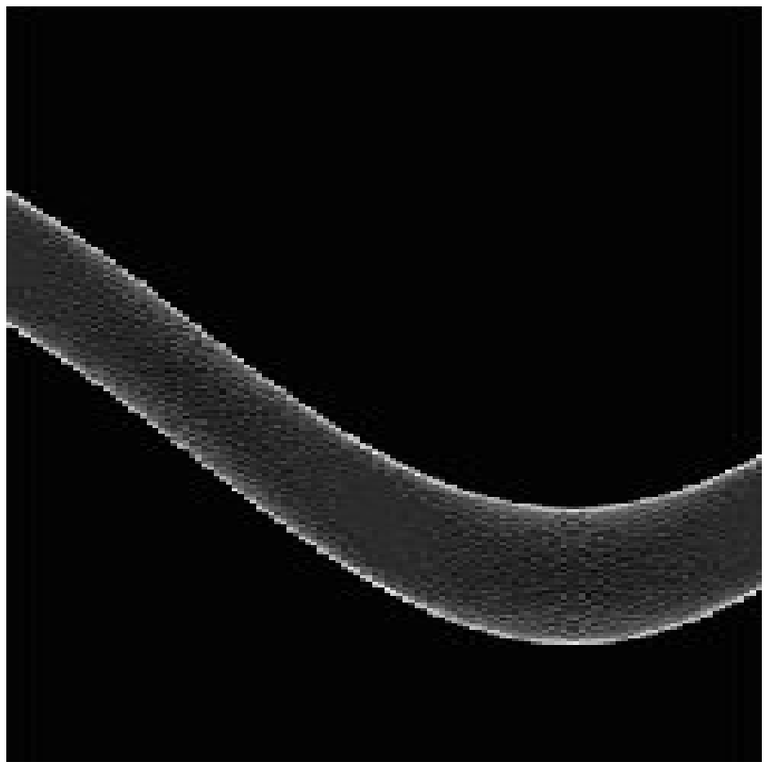
Basic shapes

(in parameter space)



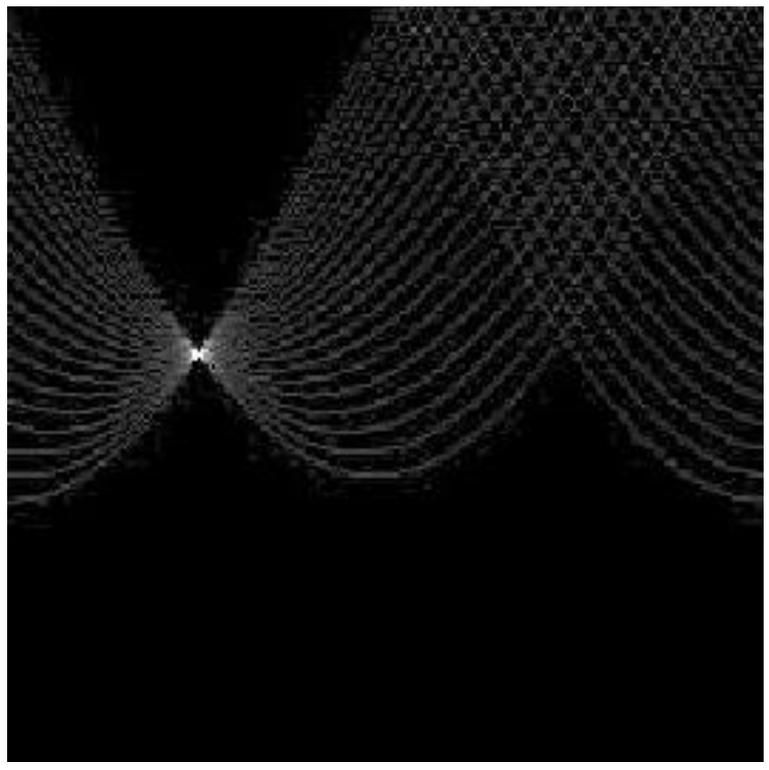
line



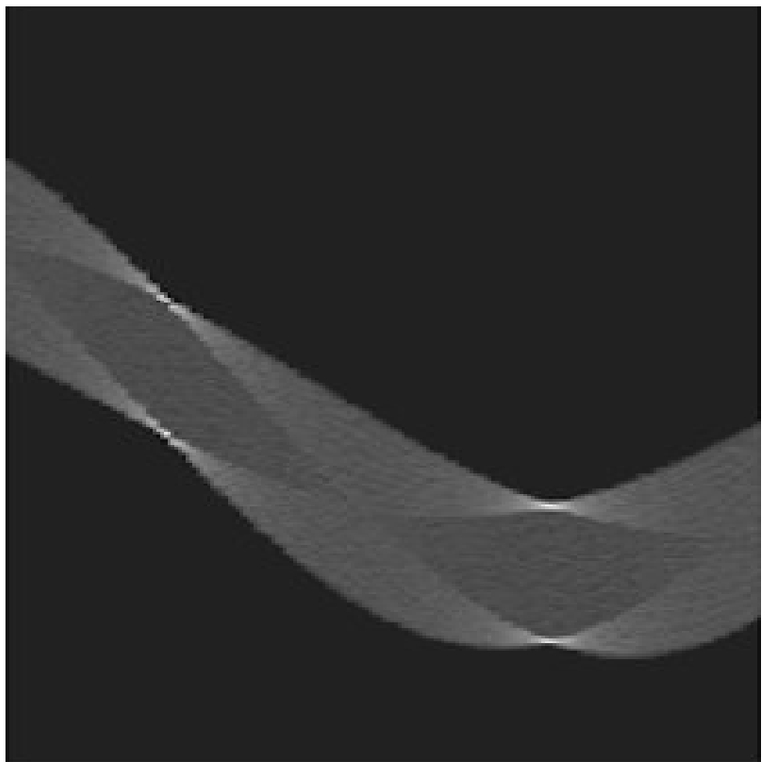


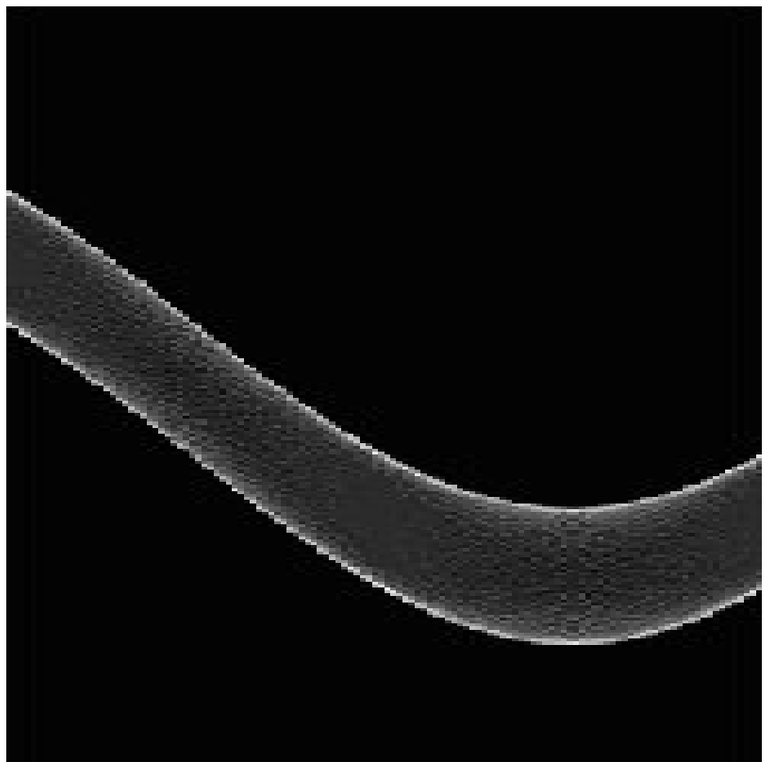
Basic shapes

(in parameter space)



line

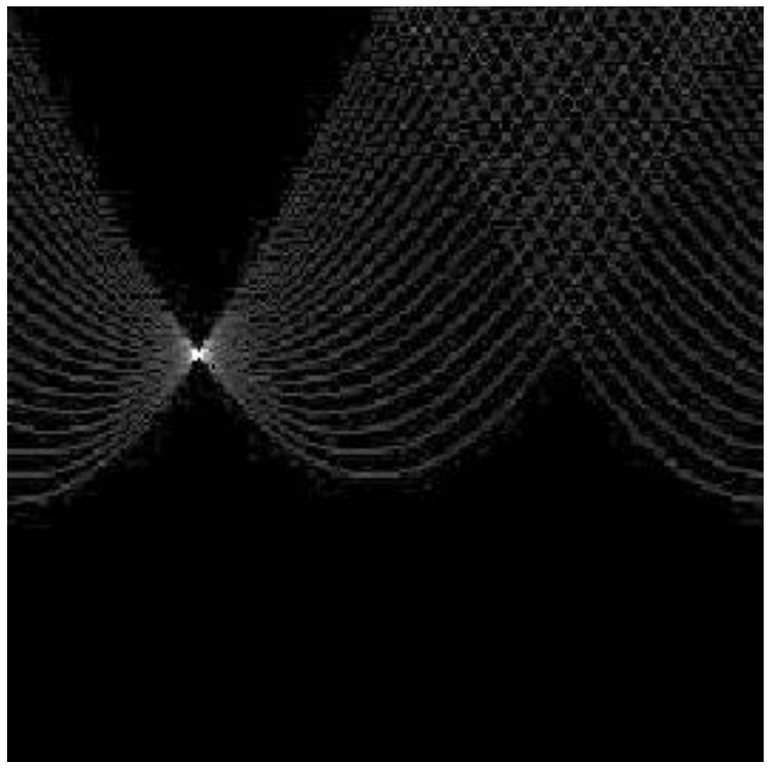




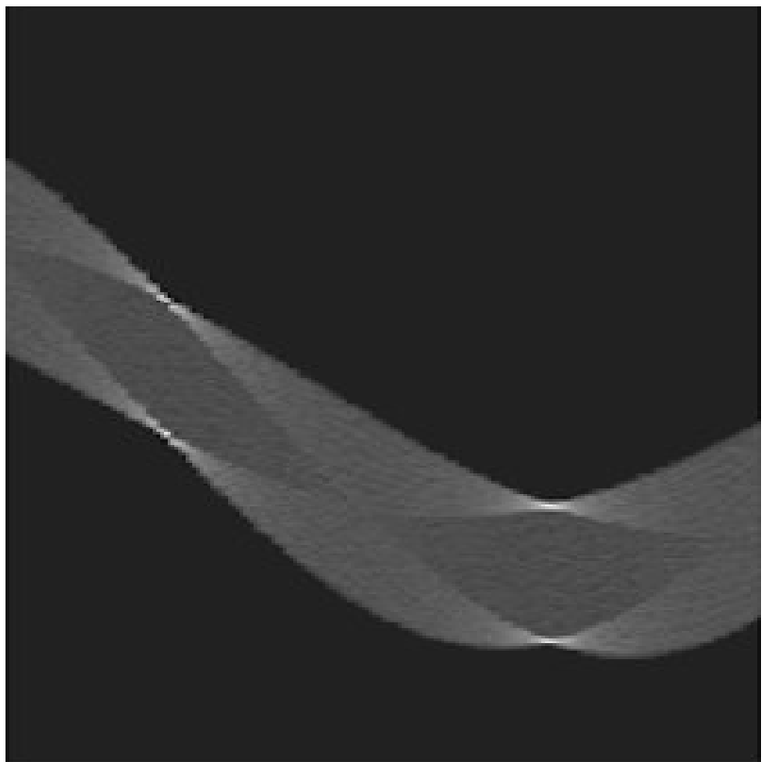
rectangle

Basic shapes

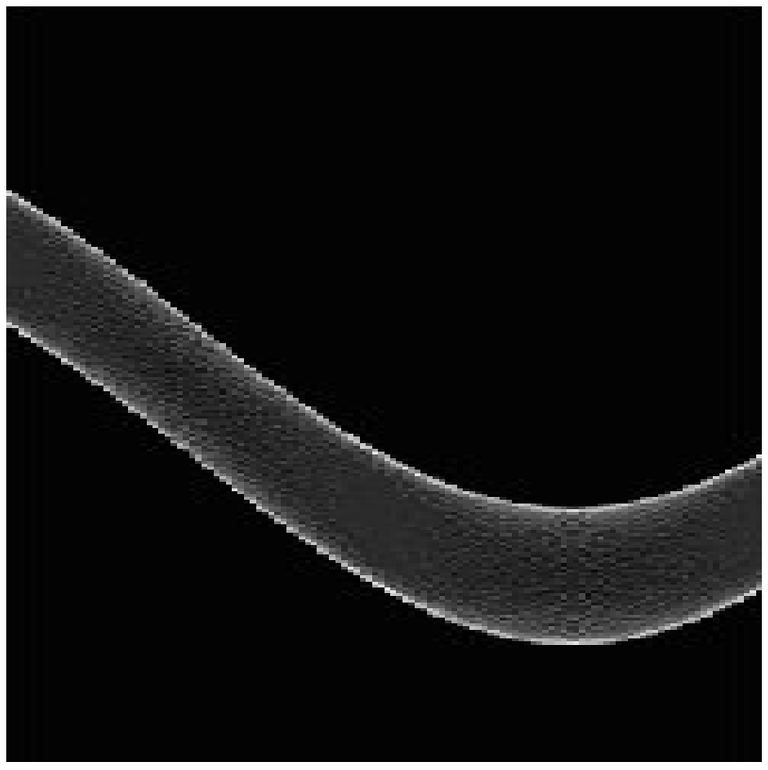
(in parameter space)



line

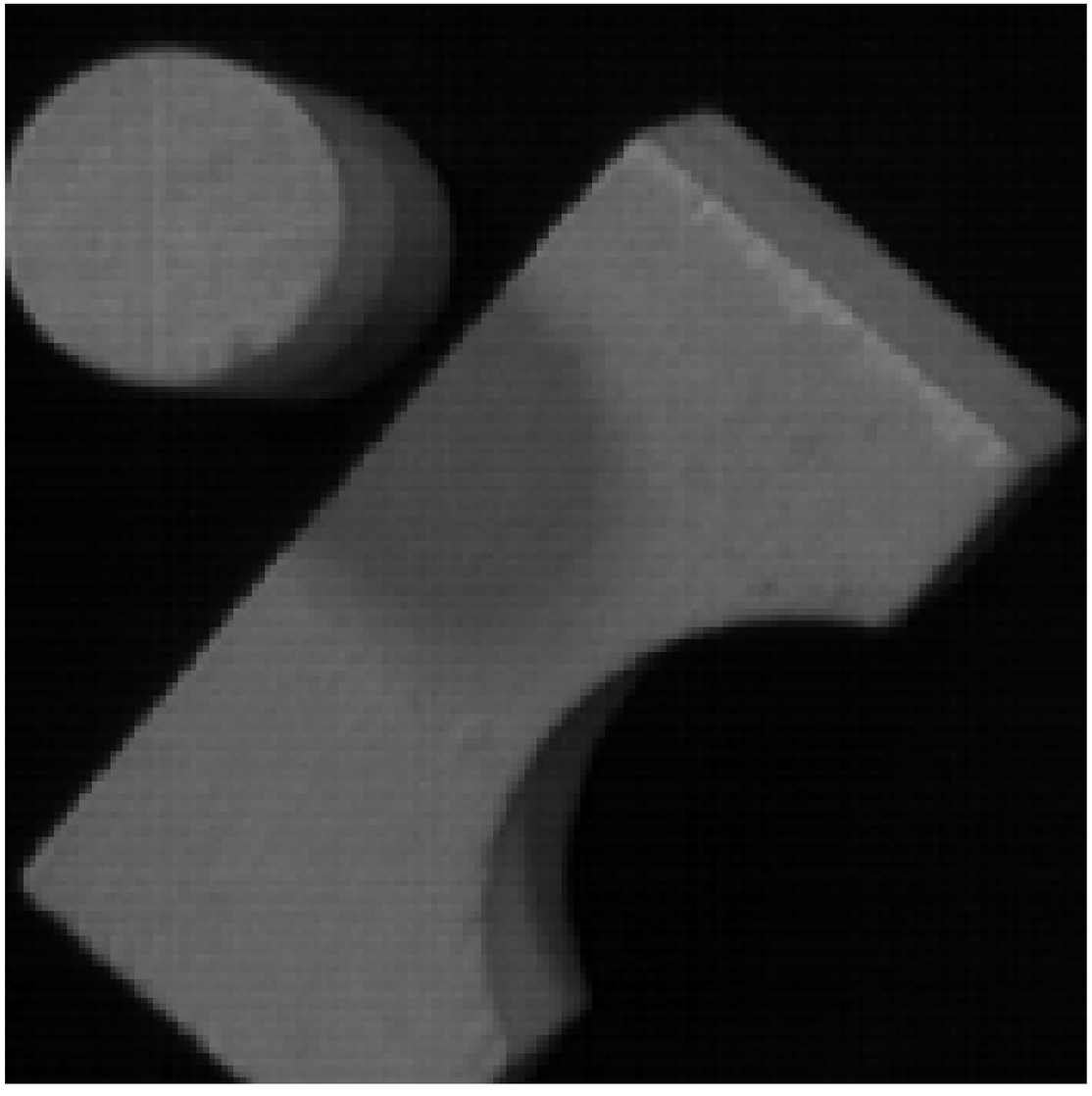


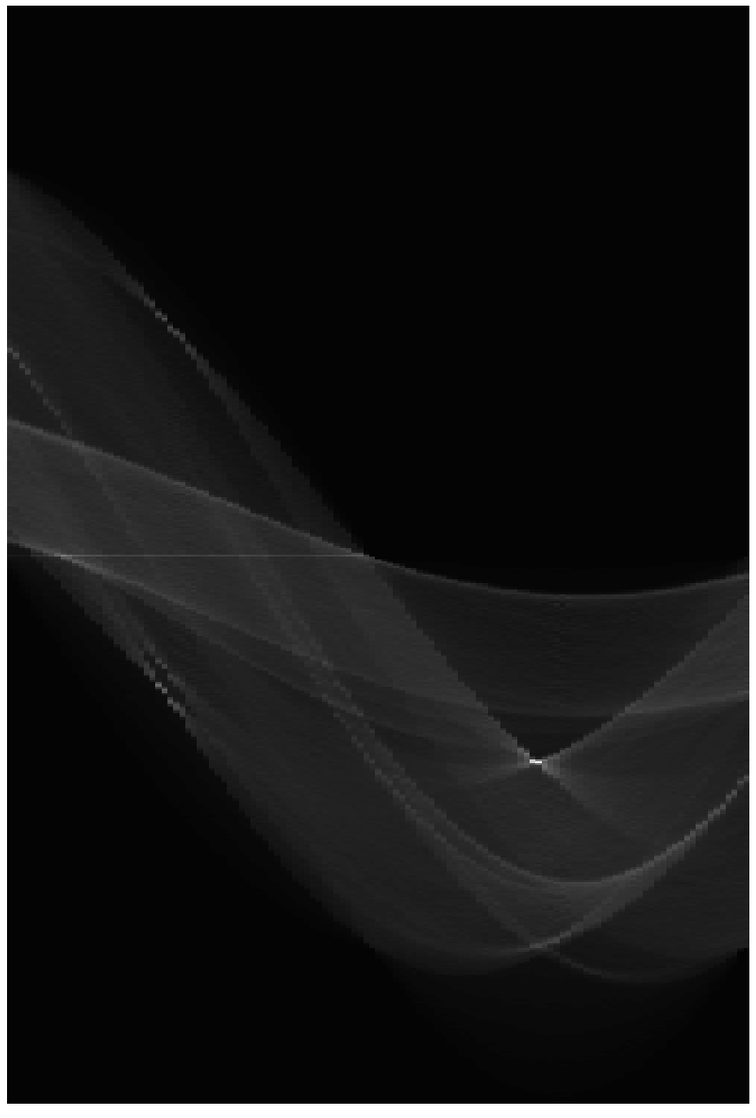
rectangle



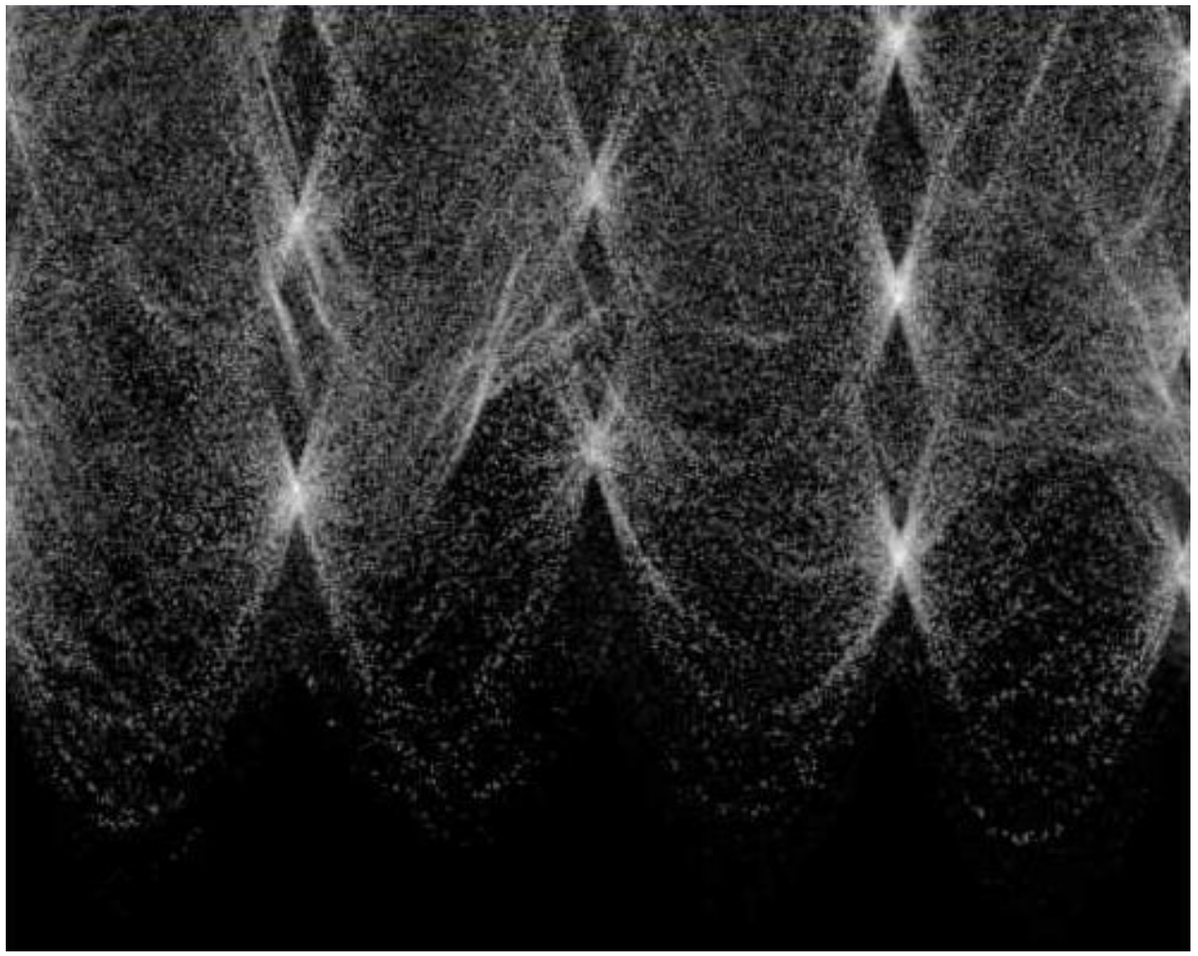
circle

Basic Shapes





More complex image



In practice, measurements are noisy…

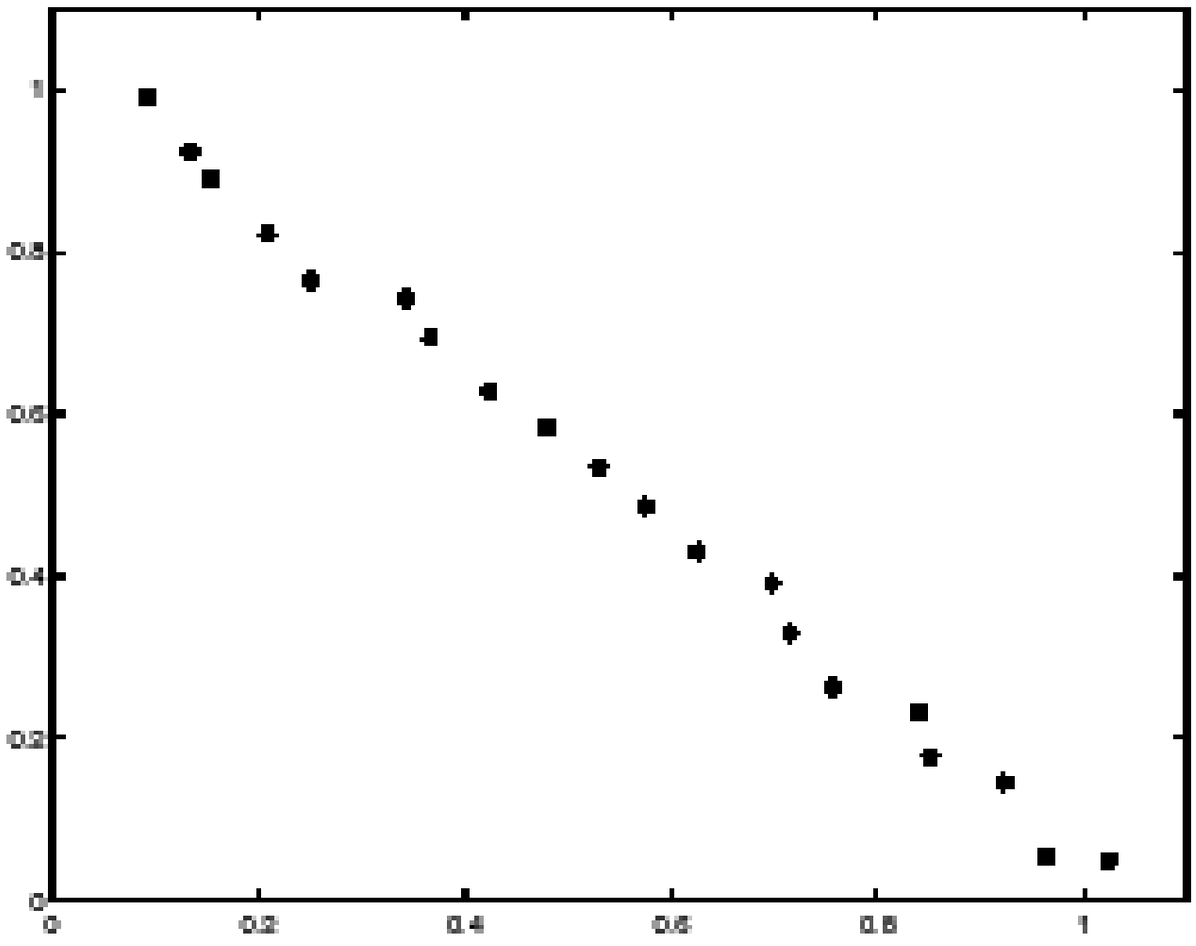
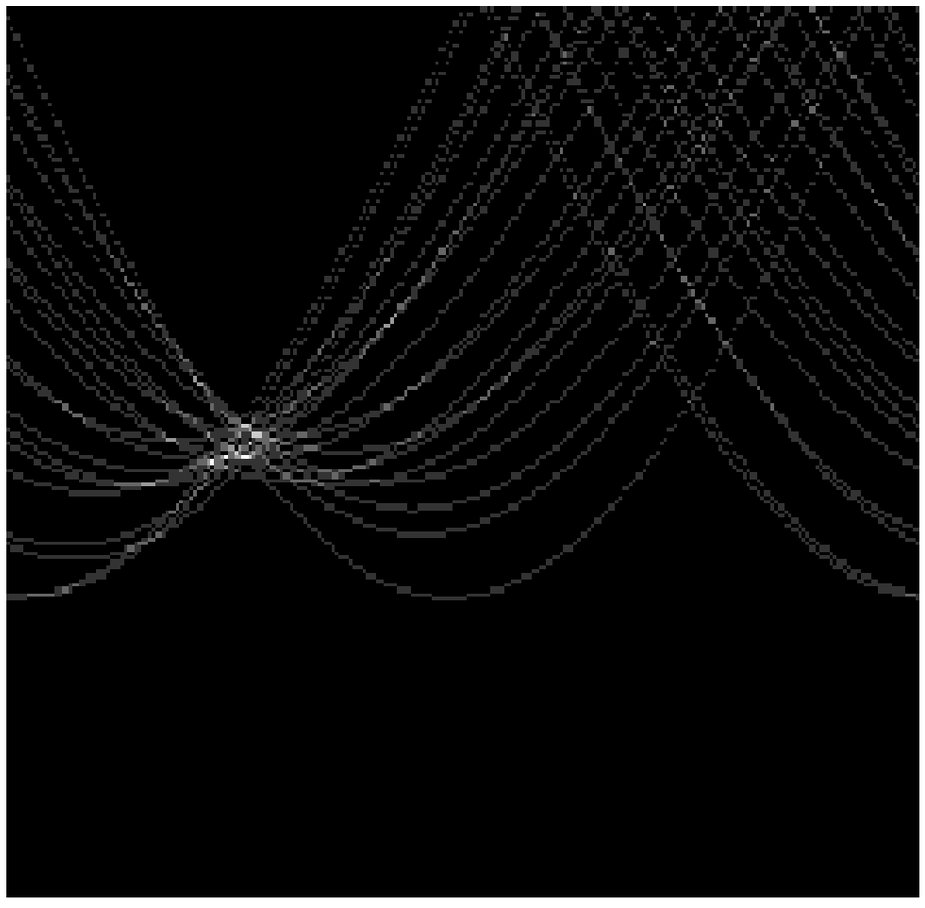


Image space



Votes

Too much noise …

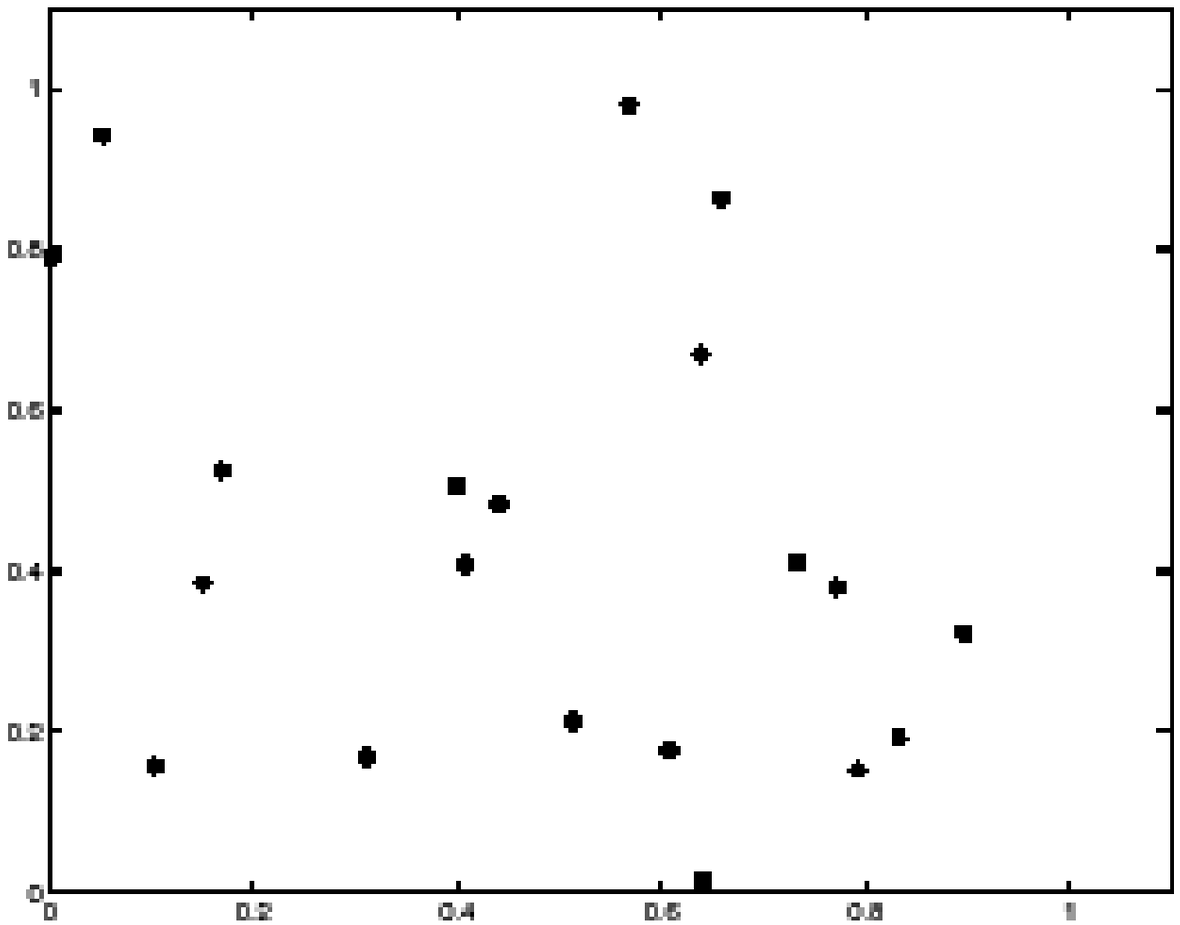
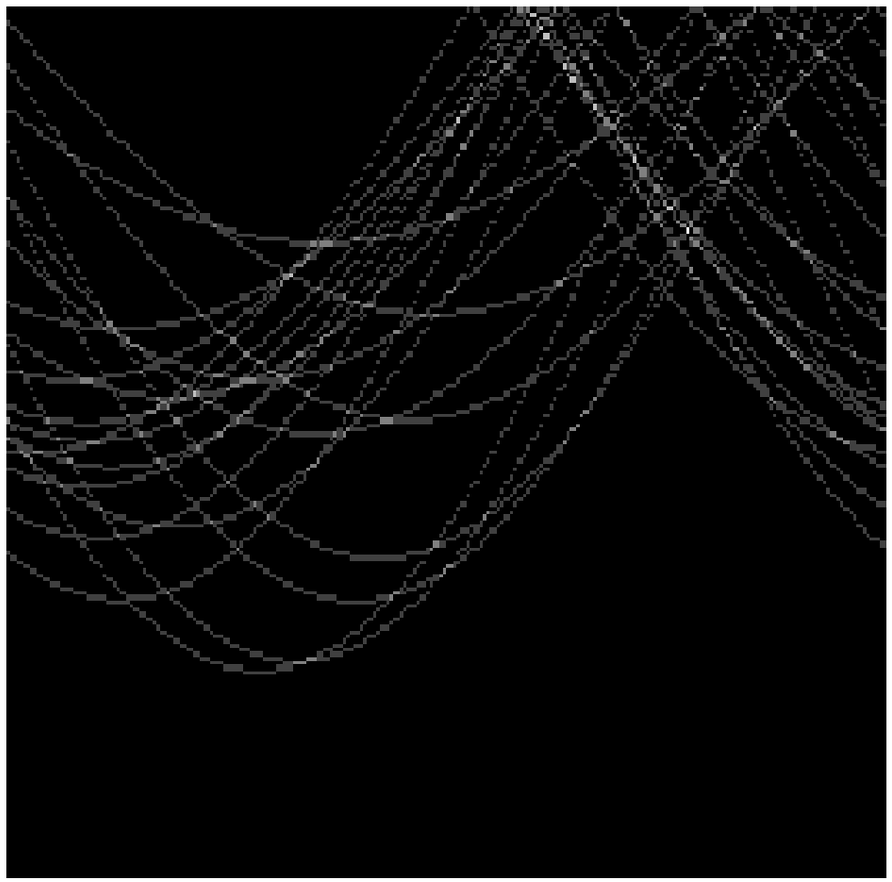
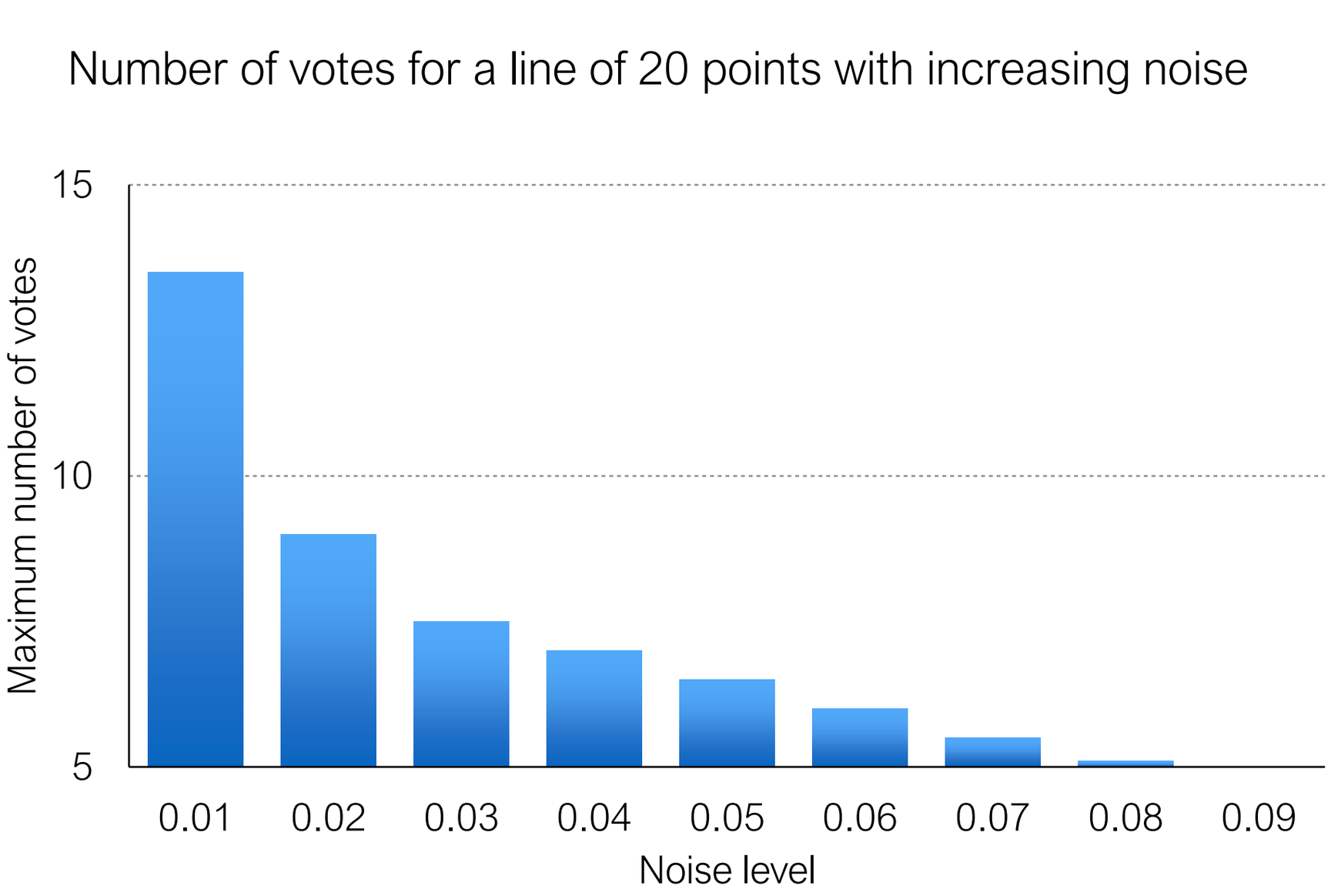


Image space



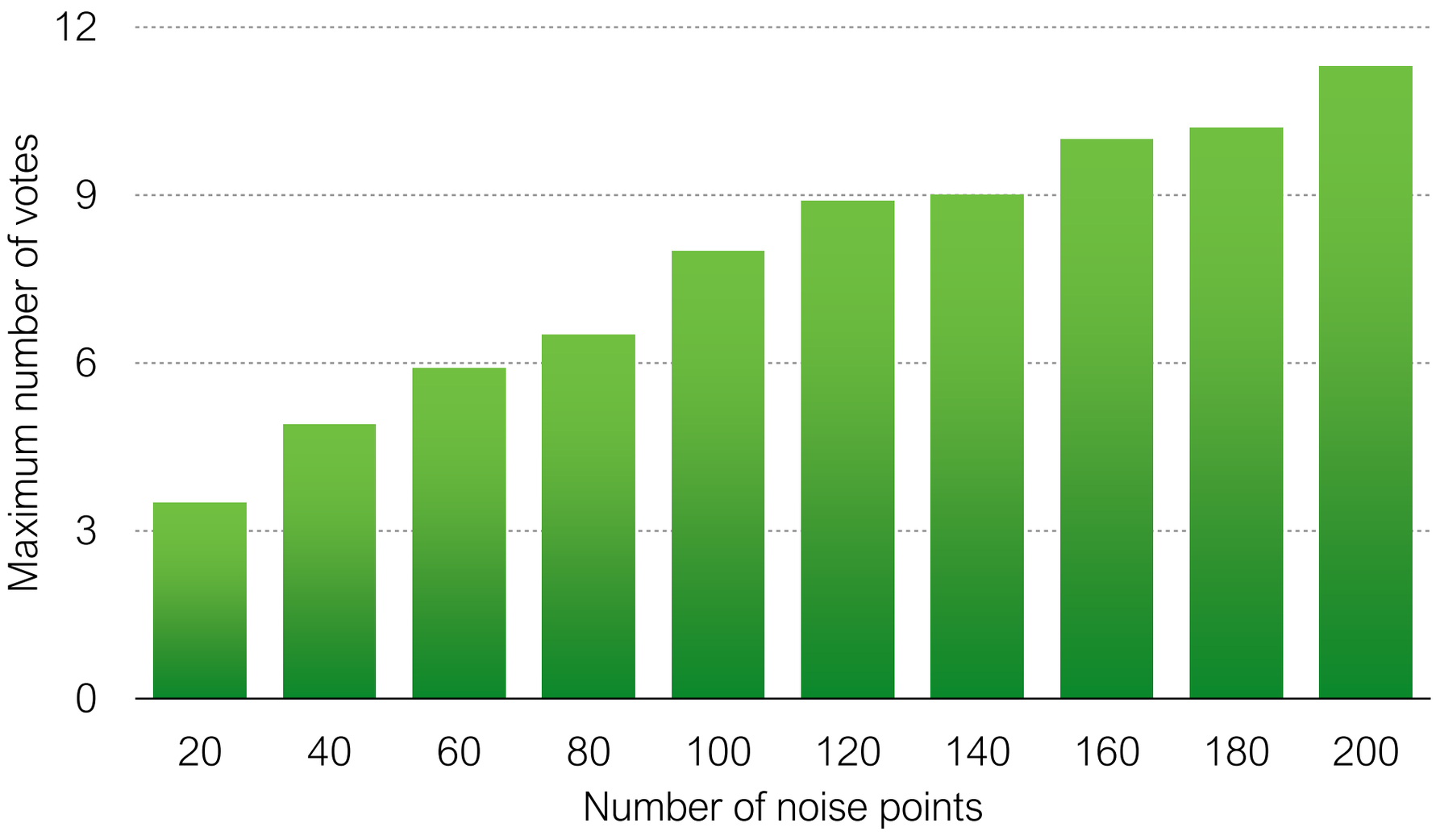
Votes

Effects of noise level



More noise, fewer votes (in the right bin)

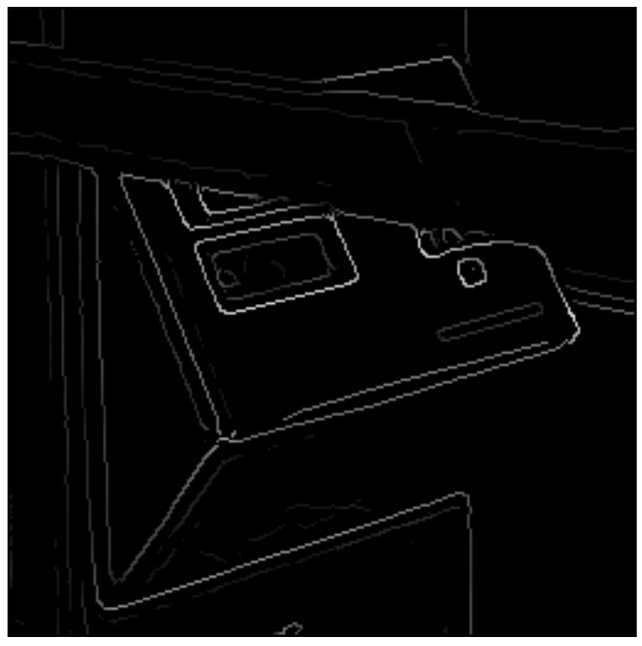
Effect of noise points



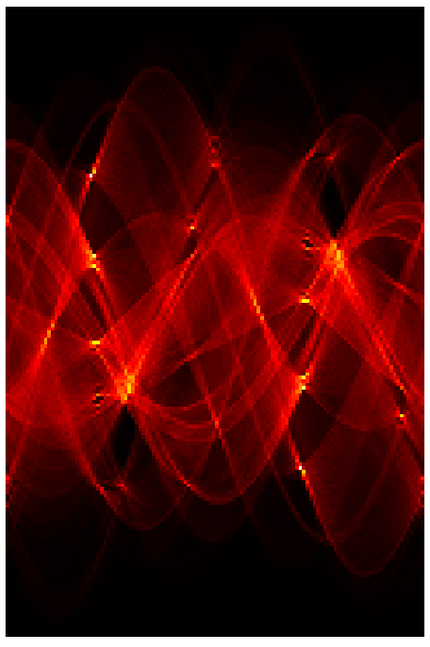
More noise, more votes (in the wrong bin)

Real-world example

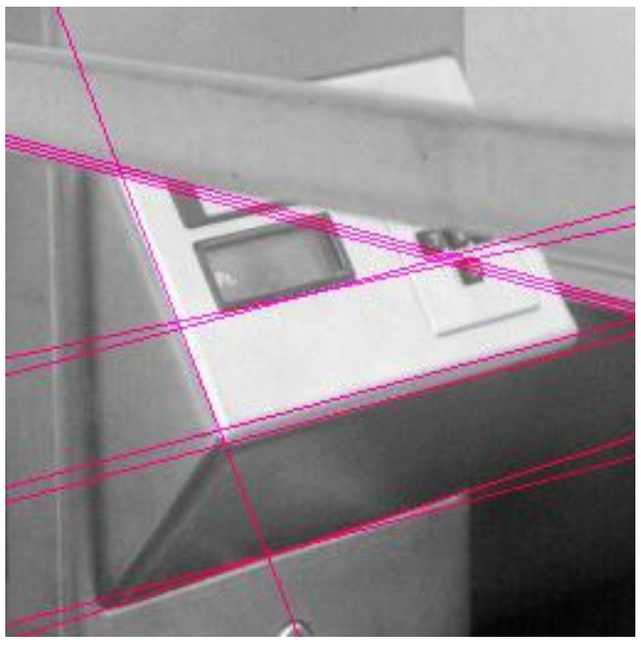
Original



Edges



parameter space



Hough Lines

Hough Circles

Let’s assume radius known

parameters

$$(x-a)^2+(y-b)^2=r^2$$

variables

parameters

$$(x-a)^2+(y-b)^2=r^2$$

variables

What is the dimension of the parameter space?

parameters

$$(x-a)^2+(y-b)^2=r^2$$

variables

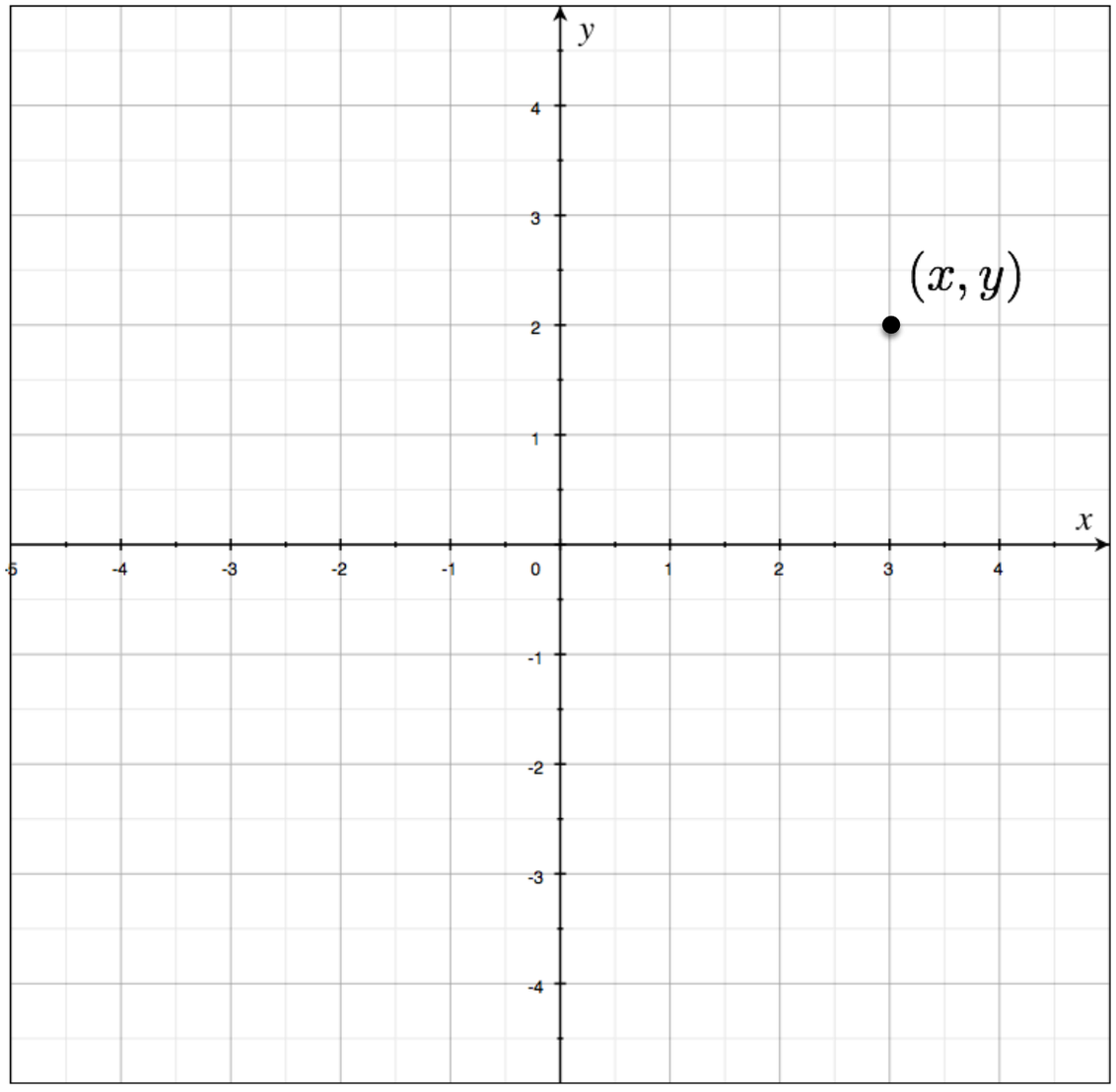
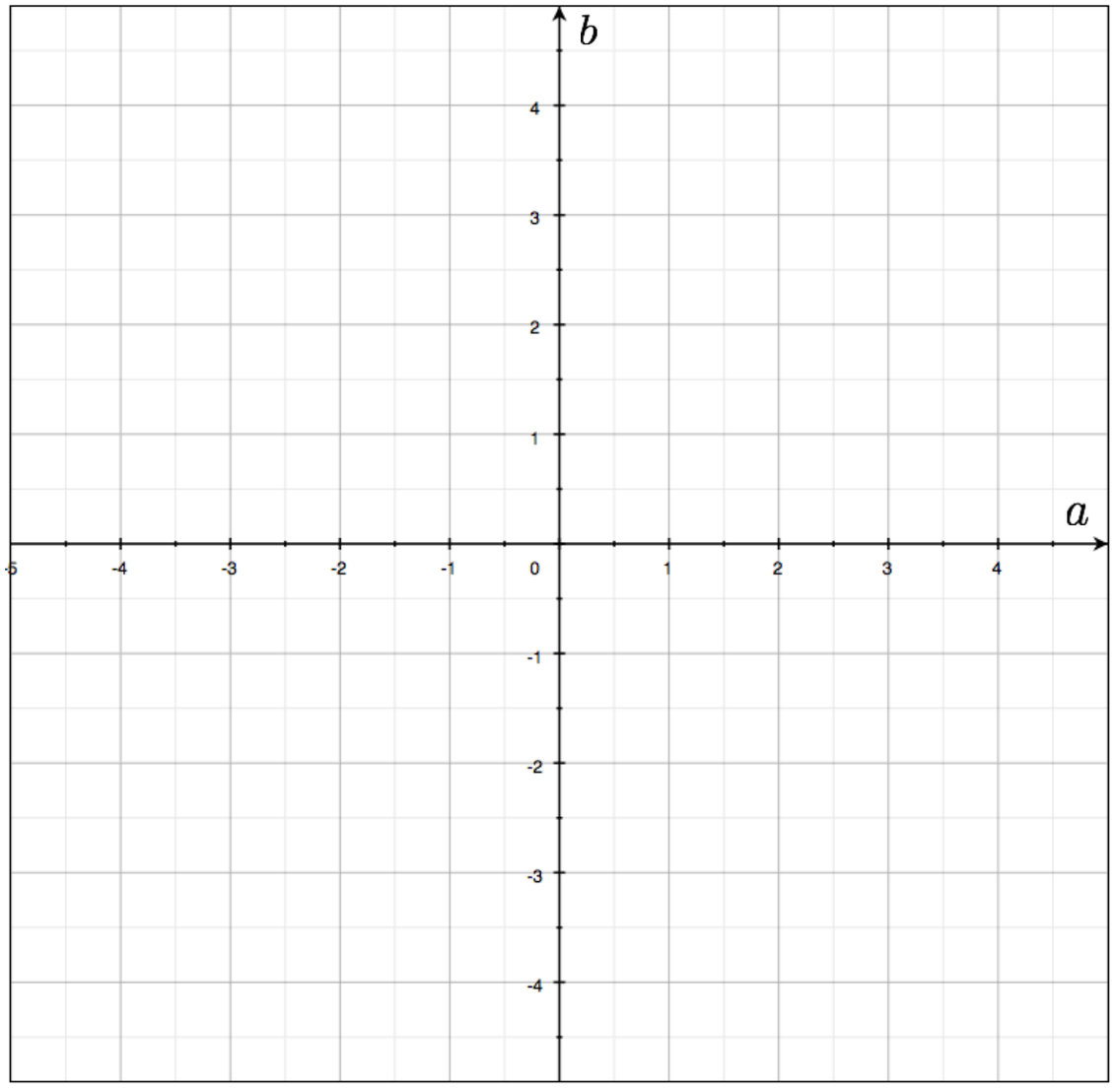


Image space

parameters

$$(x-a)^2+(y-b)^2=r^2$$

variables



Parameter space

What does a point in image space correspond to in parameter space?

parameters

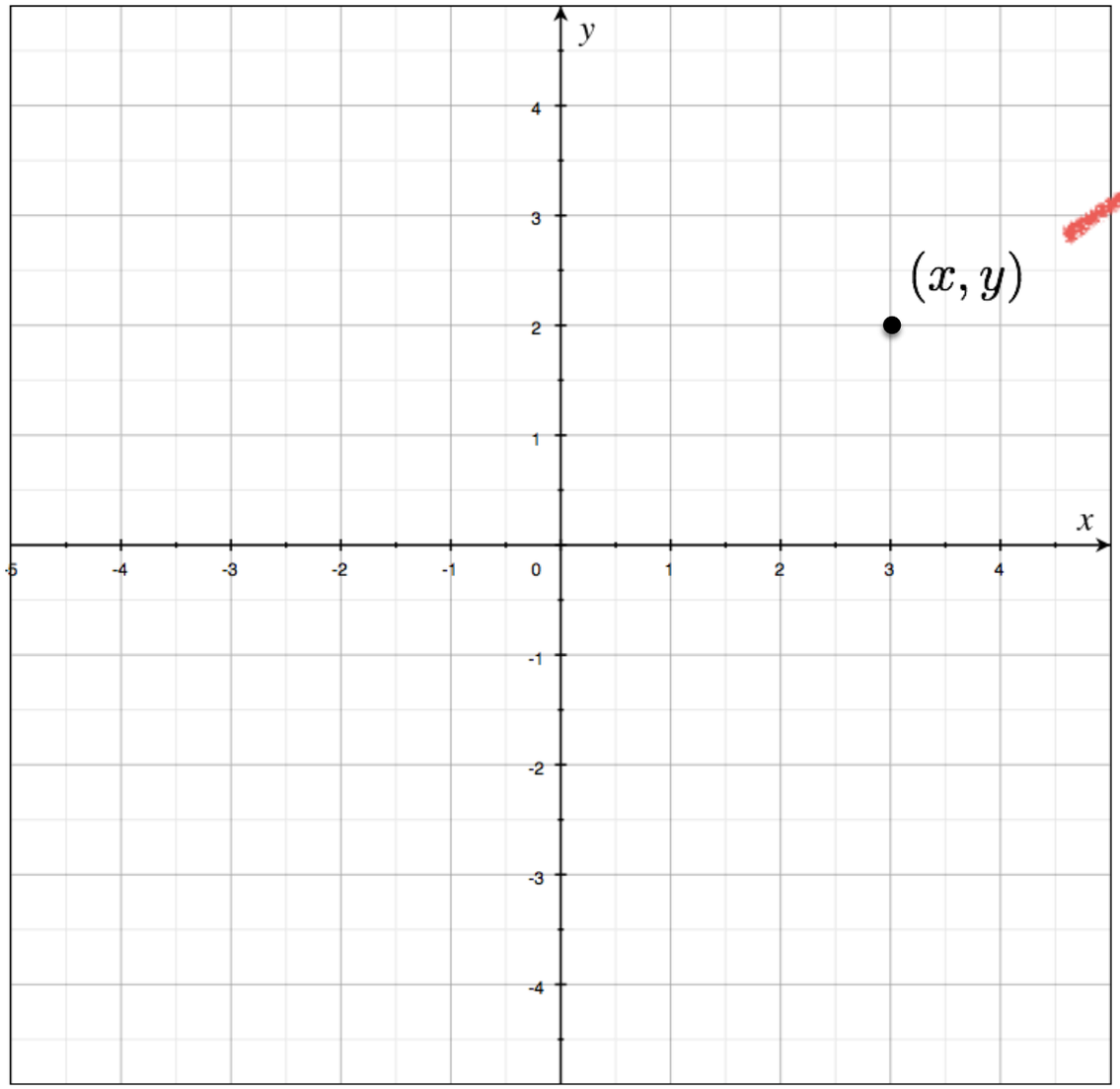
$$(x-a)^2+(y-b)^2=r^2$$

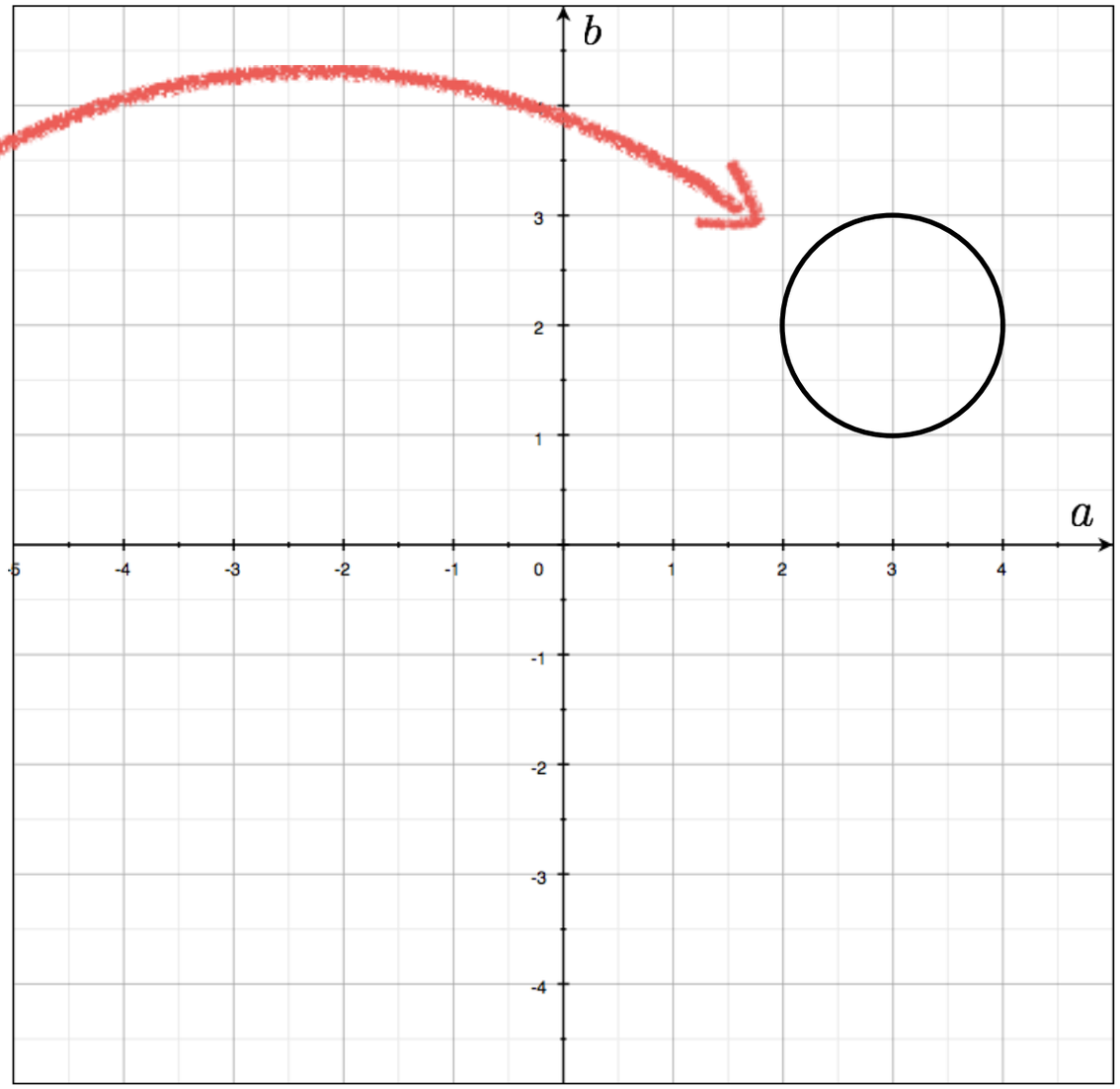
variables

parameters

$$(x-a)^2+(y-b)^2=r^2$$

variables

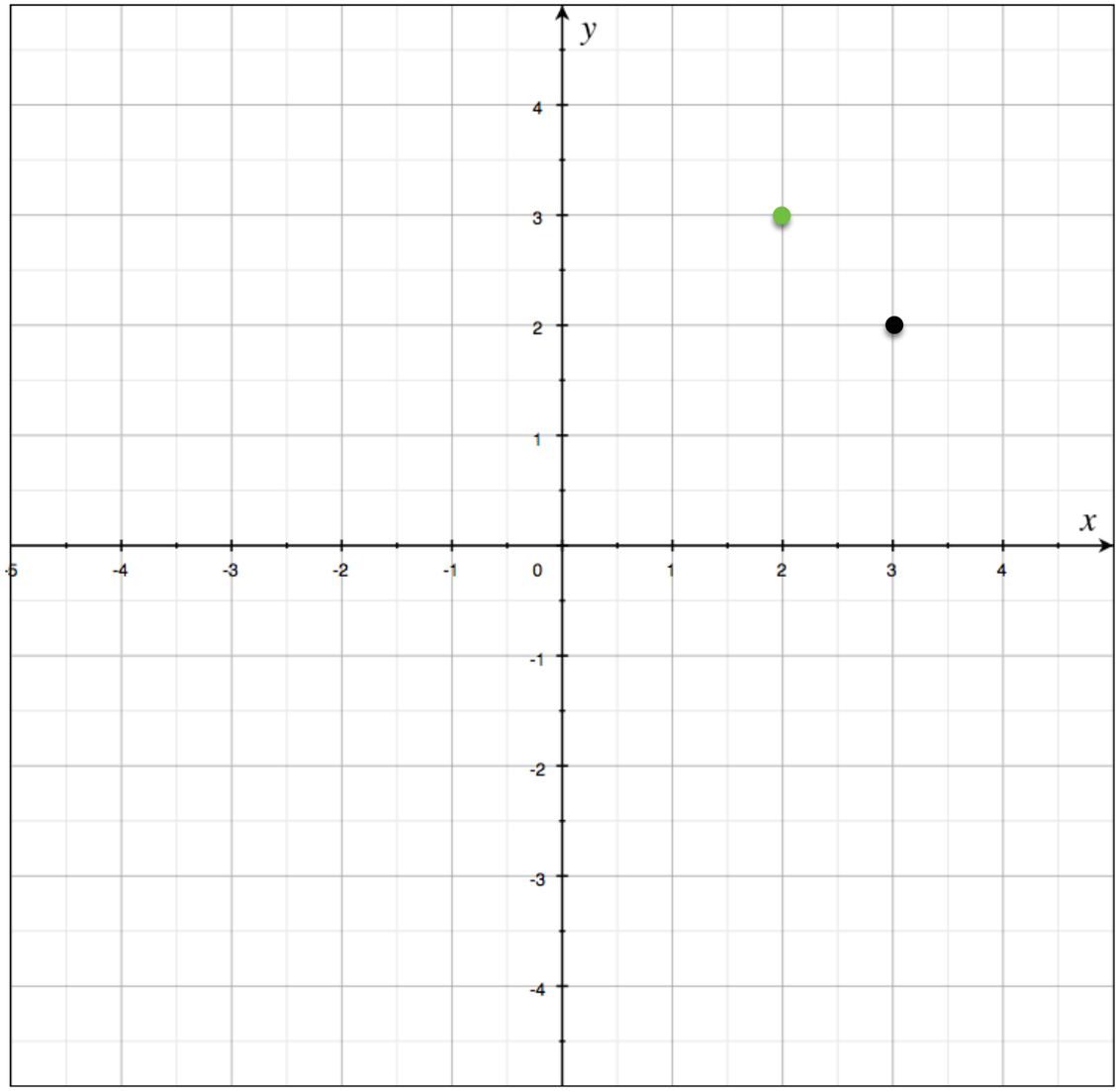




parameters

$$(x-a)^2+(y-b)^2=r^2$$

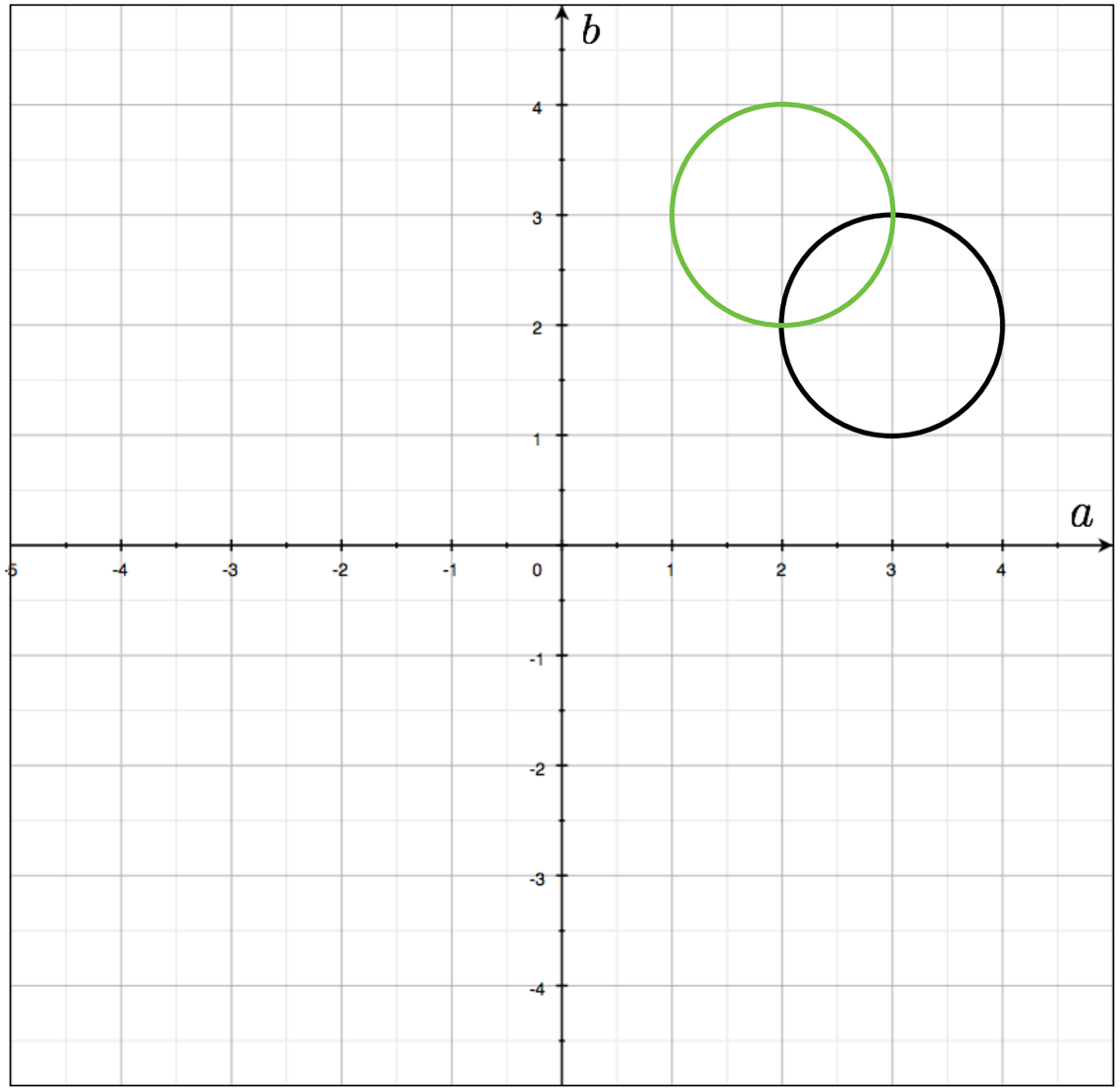
variables



parameters

$$(x-a)^2+(y-b)^2=r^2$$

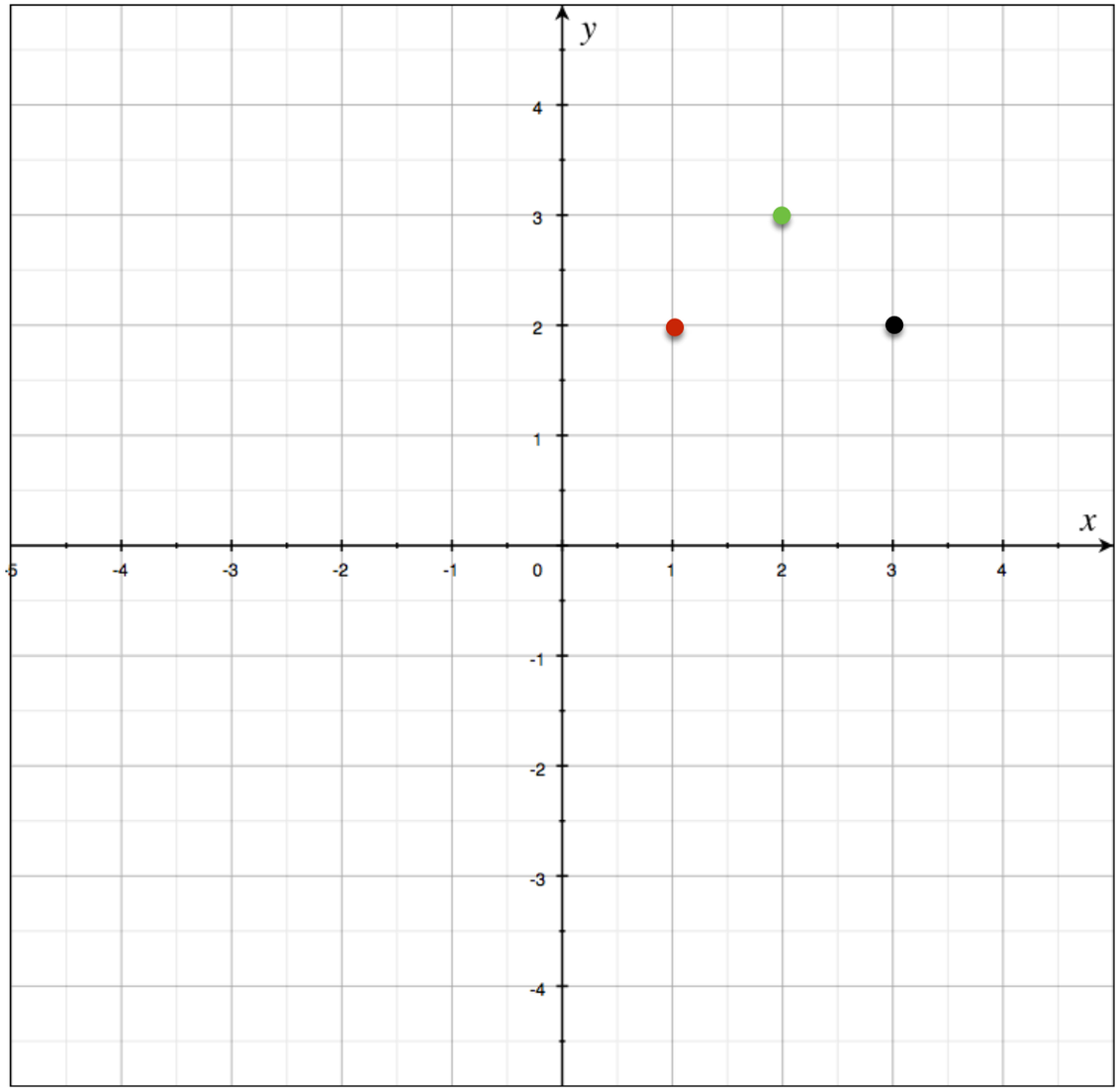
variables



parameters

$$(x-a)^2+(y-b)^2=r^2$$

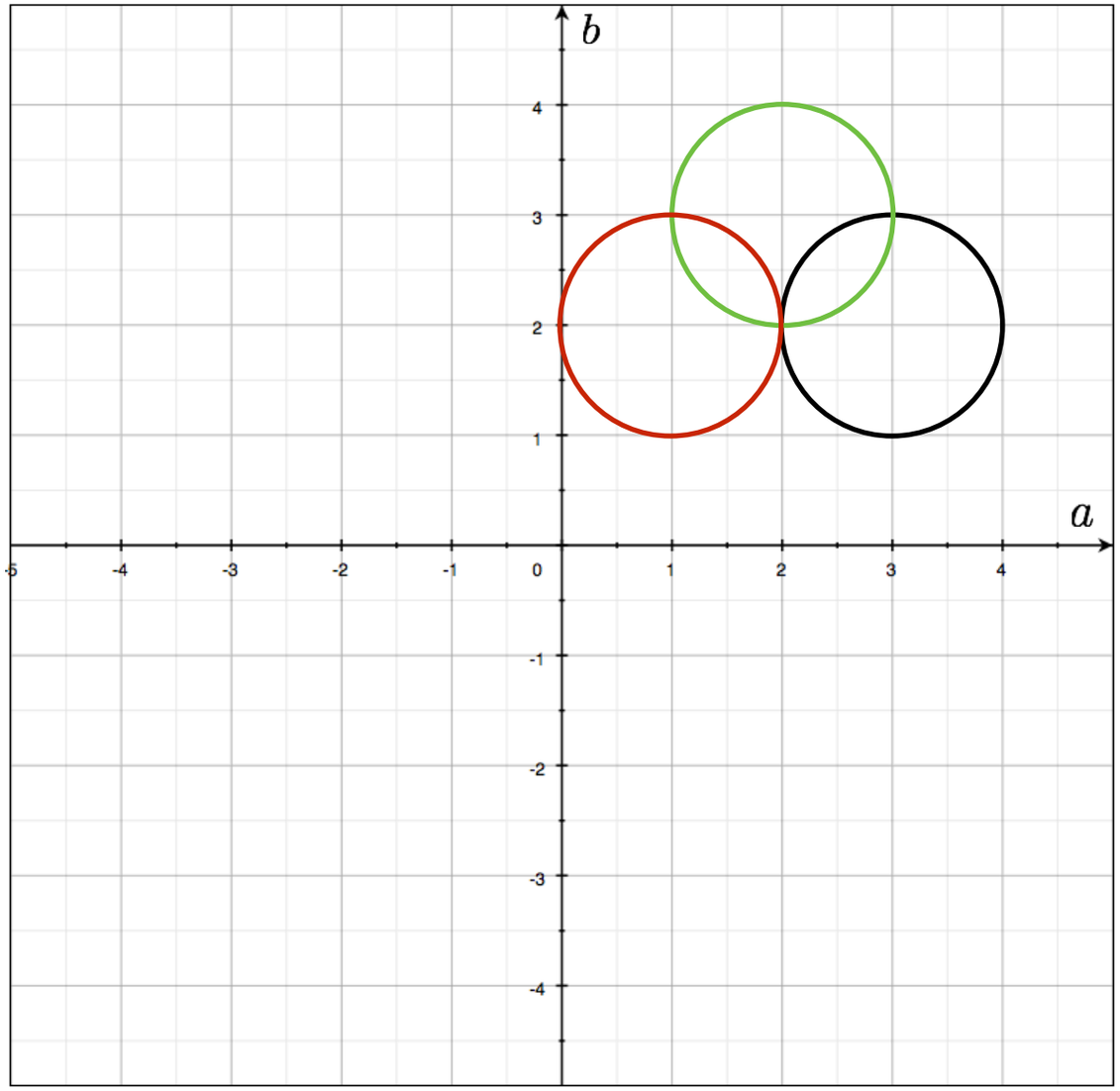
variables



parameters

$$(x-a)^2+(y-b)^2=r^2$$

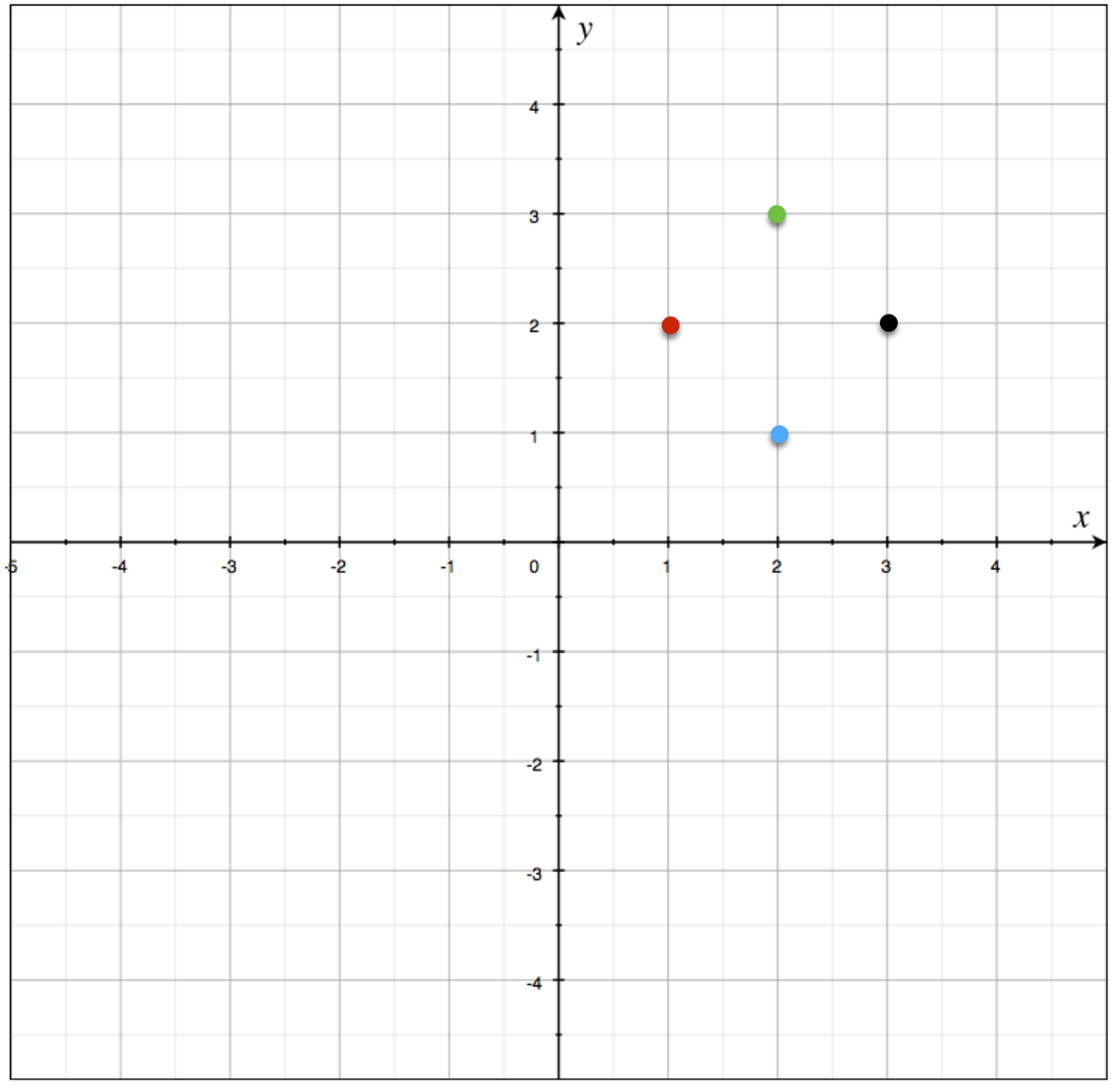
variables



parameters

$$(x-a)^2+(y-b)^2=r^2$$

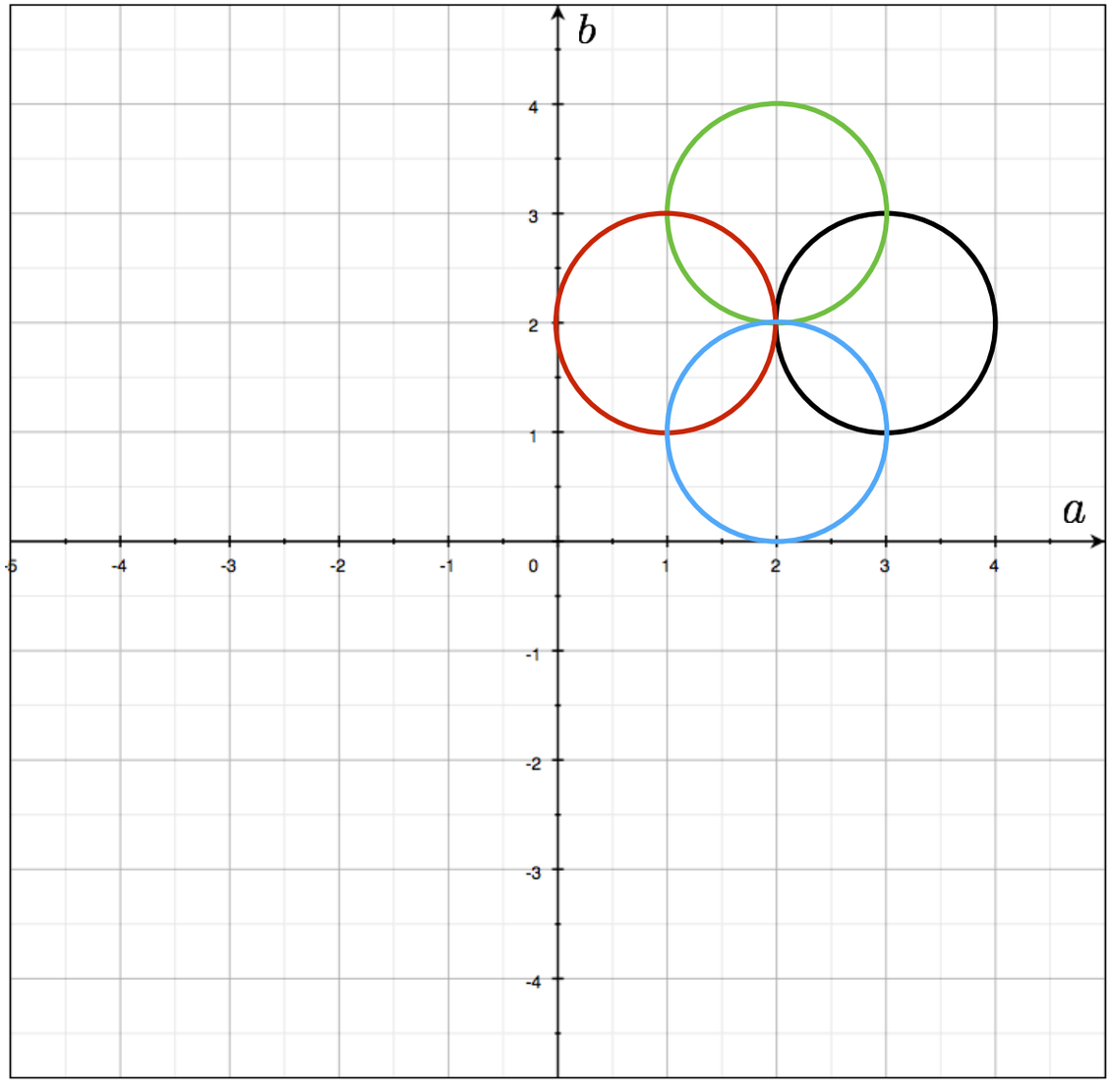
variables



parameters

$$(x-a)^2+(y-b)^2=r^2$$

variables



The Hough transform …

Deals with occlusion well?

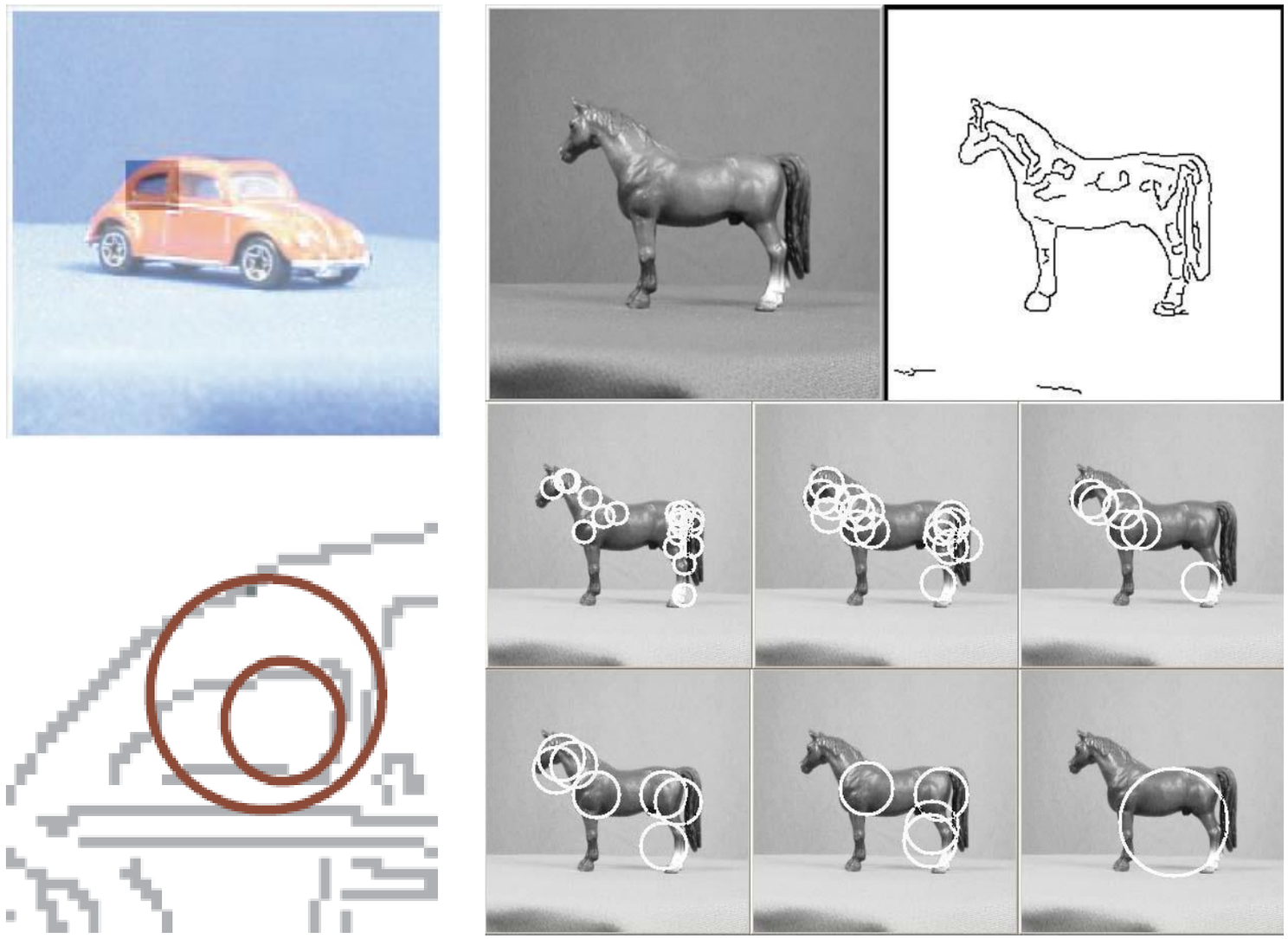
Detects multiple instances?

Robust to noise?

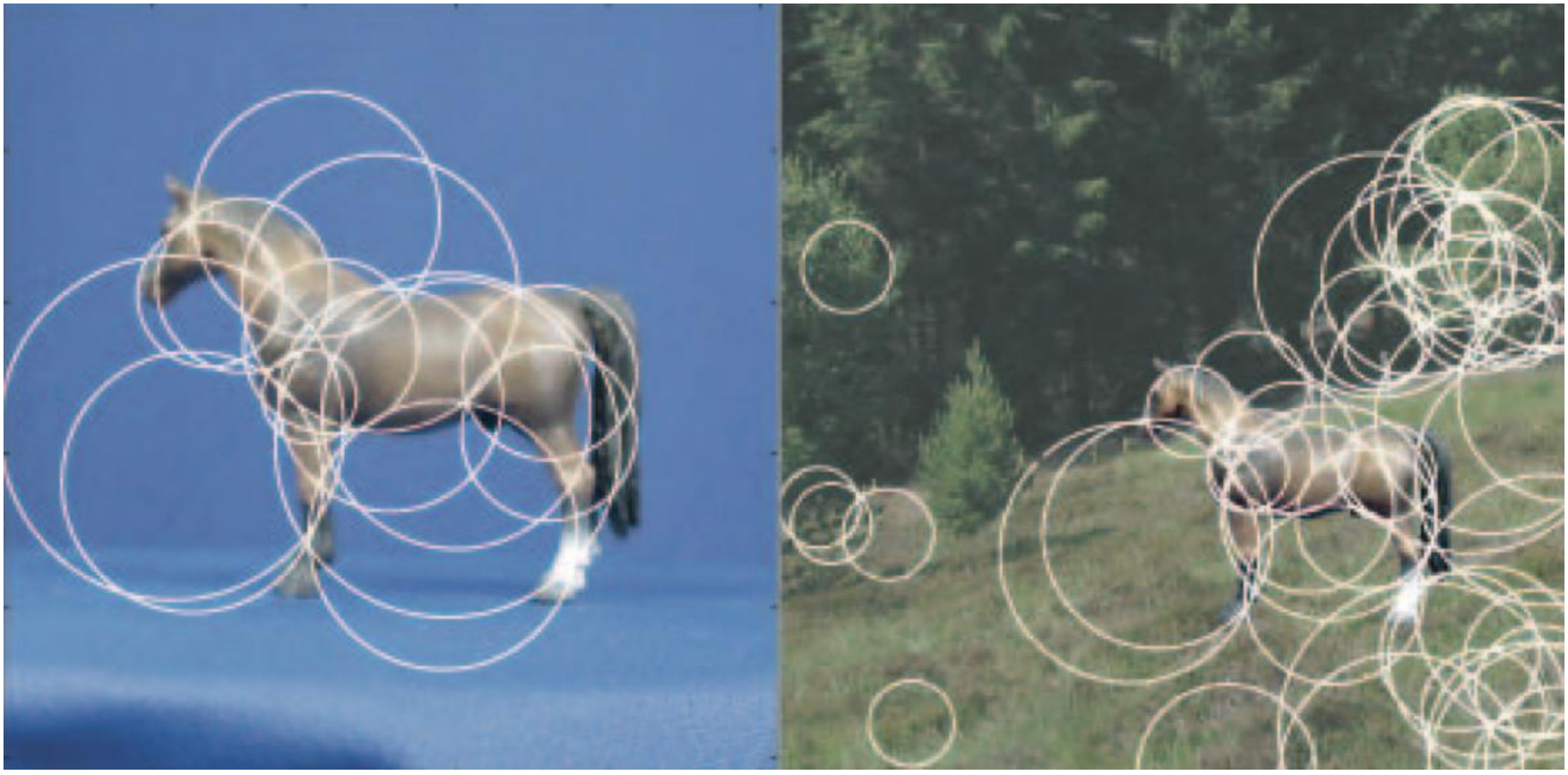
Good computational complexity?

Application of Hough transforms

Detecting shape features



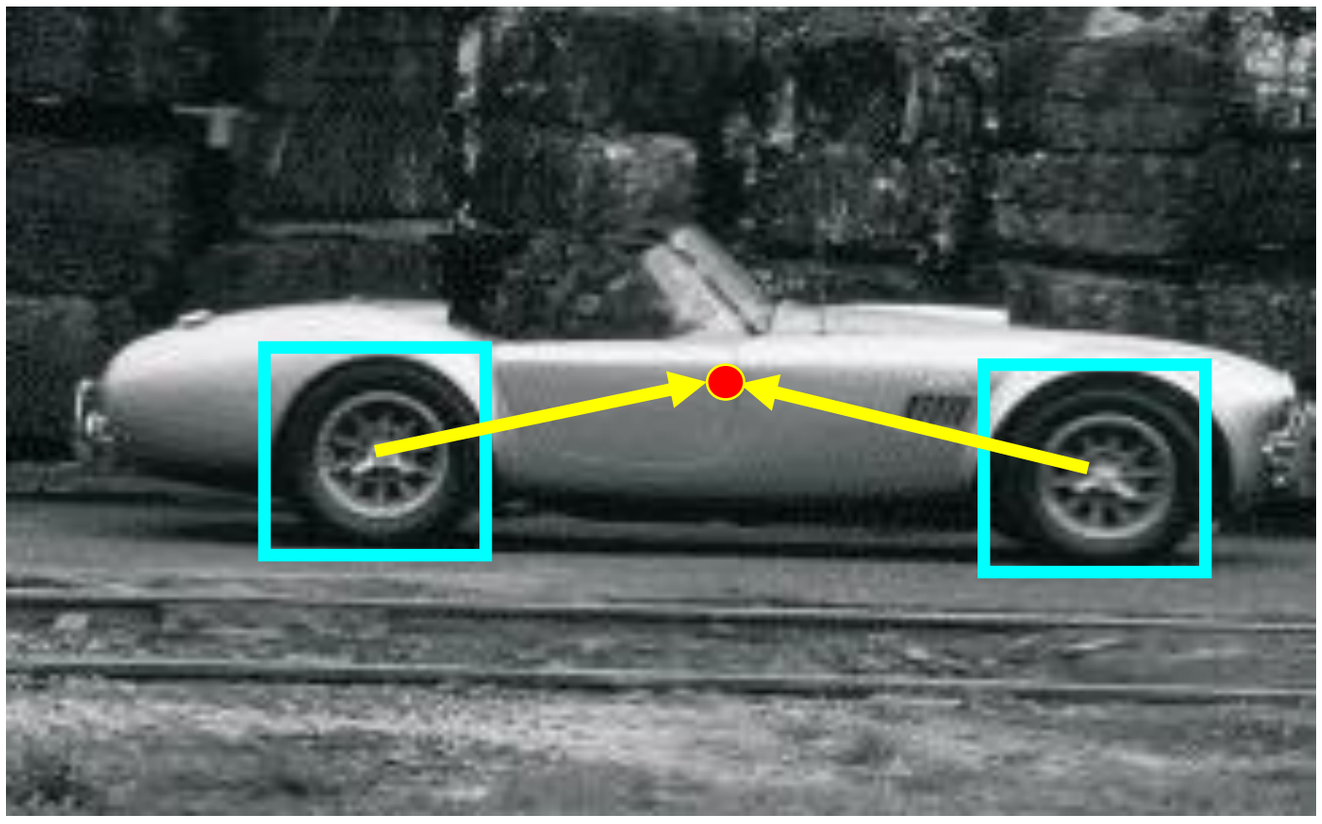
F. Jurie and C. Schmid, Scale-invariant shape features for recognition of object categories, CVPR 2004



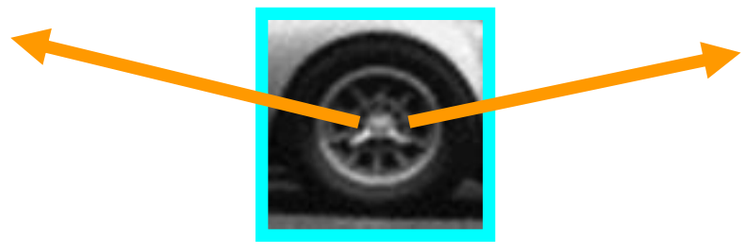
Robustness to scale and clutter

Object detection

Index displacements by “visual codeword”



training image



visual codeword with displacement vectors

B. Leibe, A. Leonardis, and B. Schiele, Combined Object Categorization and Segmentation with an Implicit Shape Model,
ECCV Workshop on Statistical Learning in Computer Vision 2004

References

Basic reading:

- Szeliski textbook, Sections 4.2, 4.3.