**First Order Differential equations**

A first order differential equation is of the form:

isplaymath137

**Linear Equations:**

isplaymath139

The general general solution is given by

isplaymath141

where

isplaymath143

is called the **integrating factor**.

**Separable Equations:**

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**(1)**

Solve the equation *g*(*y*) = 0 which gives the constant solutions.

**(2)**

The non-constant solutions are given by

isplaymath149

**Bernoulli Equations:**

isplaymath151

**(1)**

Consider the new function ex2html_wrap_inline153 .

**(2)**

The new equation satisfied by *v* is

isplaymath157

**(3)**

Solve the new linear equation to find *v*.

**(4)**

Back to the old function *y* through the substitution ex2html_wrap_inline163 .

**(5)**

If *n* > 1, add the solution *y*=0 to the ones you got in (4).

**Homogenous Equations:**

isplaymath137

is **homogeneous** if the function *f*(*x*,*y*) is homogeneous, that is

isplaymath173

By substitution, we consider the new function

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The new differential equation satisfied by *z* is

isplaymath179

which is a separable equation. The solutions are the constant ones *f*(1,*z*) - *z* =0 and the non-constant ones given by

isplaymath183

Do not forget to go back to the old function *y* = *xz*.

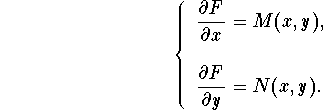
**Exact Equations:**

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is **exact** if

isplaymath189

The condition of exactness insures the existence of a function *F*(*x*,*y*) such that



All the solutions are given by the implicit equation

isplaymath195

**Second Order Differential equations**

**Homogeneous Linear Equations with constant coefficients:**

isplaymath197

Write down the **characteristic equation**

isplaymath199

**(1)**

If ex2html_wrap_inline201 and ex2html_wrap_inline203 are distinct real numbers (this happens if ex2html_wrap_inline205 ), then the general solution is

isplaymath207

**(2)**

If ex2html_wrap_inline209 (which happens if ex2html_wrap_inline211 ), then the general solution is

isplaymath213

**(3)**

If ex2html_wrap_inline201 and ex2html_wrap_inline203 are complex numbers (which happens if ex2html_wrap_inline219 ), then the general solution is

isplaymath221

where

isplaymath223

that is

isplaymath225

**Non Homogeneous Linear Equations:**

isplaymath227

The general solution is given by

isplaymath229

where ex2html_wrap_inline231 is a particular solution and ex2html_wrap_inline233 is the general solution of the associated homogeneous equation

isplaymath235

In order to find ex2html_wrap_inline237 two major techniques were developed.

**Method of undetermined coefficients or Guessing Method**

This method works for the equation

isplaymath239

where *a*, *b*, and *c* are constant and

isplaymath247

where ex2html_wrap_inline249 is a polynomial function with degree *n*. In this case, we have

isplaymath253

where

isplaymath255

The constants ex2html_wrap_inline257 and ex2html_wrap_inline259 have to be determined. The power *s* is equal to 0 if ex2html_wrap_inline265 is not a root of the characteristic equation. If ex2html_wrap_inline265 is a simple root, then *s*=1 and *s*=2 if it is a double root.  
**Remark.** If the nonhomogeneous term *g*(*x*) satisfies the following

isplaymath275

where ex2html_wrap_inline277 are of the forms cited above, then we split the original equation into *N* equations

isplaymath281

then find a particular solution ex2html_wrap_inline283 . A particular solution to the original equation is given by

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**Method of Variation of Parameters**

This method works as long as we know two linearly independent solutions ex2html_wrap_inline287 of the homogeneous equation

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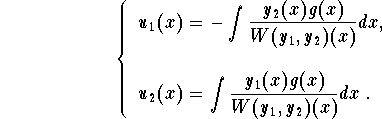
Note that this method works regardless if the coefficients are constant or not. a particular solution as

isplaymath291

where ex2html_wrap_inline293 and ex2html_wrap_inline295 are functions. From this, the method got its name.  
The functions ex2html_wrap_inline293 and ex2html_wrap_inline295 are solutions to the system:

isplaymath301

which implies



Therefore, we have

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**Euler-Cauchy Equations:**

isplaymath307

where *b* and *c* are constant numbers. By substitution, set

isplaymath313

then the new equation satisfied by *y*(*t*) is

isplaymath317

which is a second order differential equation with constant coefficients.

**(1)**

Write down the characteristic equation

isplaymath129

**(2)**

If the roots ex2html_wrap_inline201 and ex2html_wrap_inline203 are distinct real numbers, then the general solution is given by

isplaymath130

**(2)**

If the roots ex2html_wrap_inline201 and ex2html_wrap_inline203 are equal ( ex2html_wrap_inline209 ), then the general solution is

isplaymath131

**(3)**

If the roots ex2html_wrap_inline201 and ex2html_wrap_inline203 are complex numbers, then the general solution is

isplaymath132

where ex2html_wrap_inline339 and ex2html_wrap_inline341 .