Problem 1

Proof. We want to show that

$$Pr(\alpha_1, ..., \alpha_n) = \prod_{i=1}^{n} Pr(\alpha_i | \alpha_{i+1}, ..., \alpha_n, \beta)$$

We will induct on n.
First we assert our base cases:
We can observe trivially that the identity holds for n = 0 and n = 1.
For n=2, we can observe by Bayes' Conditioning Rule that

$$Pr(\alpha_1, \alpha_2 | \beta) = Pr(\alpha_1 | \alpha_2, \beta) Pr(\alpha_2 | \beta)$$

We will now prove the inductive case:
Assume that

$$Pr(\alpha_1, ..., \alpha_n | \beta) = \prod_{i=1}^{n} Pr(\alpha_i | \alpha_{i+1}, ..., \alpha_n, \beta)$$

Then by Bayes' Conditioning Rule,

$$\begin{aligned}
&Pr(\alpha_1, ..., \alpha_{n+1} | \beta) \\
&\quad = Pr(\alpha_{n+1} | \alpha_1, ..., \alpha_n, \beta) Pr(\alpha_1 | \alpha_2, ..., \alpha_n, \beta) ... Pr(\alpha_n | \beta) \\
&\quad = \frac{Pr(\alpha_{n+1}, \alpha_1 | \alpha_2, ..., \alpha_n, \beta)}{Pr(\alpha_1 | \alpha_2, ..., \alpha_n, \beta)} \Big(Pr(\alpha_1 | \alpha_2, ..., \alpha_n, \beta) ... Pr(\alpha_n | \beta) \Big) \\
&\quad = \frac{Pr(\alpha_1 | \alpha_2, ..., \alpha_{n+1}, \beta) Pr(\alpha_{n+1} | \alpha_2, ..., \alpha_n, \beta)}{Pr(\alpha_1 | \alpha_2, ..., \alpha_n, \beta)} \Big(Pr(\alpha_1 | \alpha_2, ..., \alpha_n, \beta) ... Pr(\alpha_n | \beta) \Big) \\
&\quad = Pr(\alpha_1 | \alpha_2, ..., \alpha_{n+1}, \beta) Pr(\alpha_{n+1} | \alpha_2, ..., \alpha_n, \beta) \Big(Pr(\alpha_2 | \alpha_3, ..., \alpha_n, \beta) ... Pr(\alpha_1, ..., \alpha_n | \beta) \Big) \\
&\quad = Pr(\alpha_1 | \alpha_2, ..., \alpha_n, \beta) Pr(\alpha_2 | \alpha_3, ..., \alpha_n, \beta) Pr(\alpha_{n+1} | \alpha_3, ..., \alpha_n, \beta) \Big(Pr(\alpha_3 | \alpha_4, ..., \alpha_n, \beta) ... Pr(\alpha_n | \beta) \Big)
\end{aligned}$$

We can extrapolate this out to see

$$\begin{aligned}
Pr(\alpha_1, ..., \alpha_{n+1} | \beta) &= \frac{Pr(\alpha_{n+1} | \beta)}{Pr(\alpha_n | \beta)} \Big(Pr(\alpha_1 | \alpha_2, ..., \alpha_{n+1}, \beta) ... Pr(\alpha_{n-1} | \alpha_n, \beta) \Big) Pr(\alpha_n | \beta) \\
&= Pr(\alpha_1 | \alpha_2, ..., \alpha_{n+1}, \beta) ... Pr(\alpha_{n+1} | \beta) \\
&= \prod_{i=1}^{n+1} Pr(\alpha_i | \alpha_{i+1}, ..., \alpha_{n+1}, \beta)
\end{aligned}$$

□

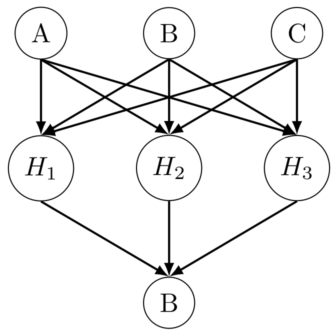
Problem 2

We use the following variables:
o denotes the presence of oil.
g denotes the presence of natural gas.
t denotes a positive geographic test.
Then

$$\begin{aligned}
Pr(o|t) &= \frac{Pr(o)}{Pr(t)} Pr(t|o) \\
&= \frac{Pr(o)Pr(t|o)}{Pr(t|o)Pr(o) + P(t|g)Pr(g) + Pr(t|\bar{o},\bar{g})P(\bar{o},\bar{g})} \\
&= \frac{(0.5)(0.9)}{(0.9)(0.5) + (0.3)(0.2) + (0.1)(0.3)} = 0.833
\end{aligned}$$

Problem 3

We let a positive literal denote a heads on a flip, and a negative denote a tails. We also let H_n denote a heads on the n'th coin flip.



a	0.33
b	0.33
c	0.33

H_n	P(H_n)
a	0.2
b	0.4
c	0.8

H_1	H_2	H_3	P(B)
0	0	0	0
0	0	1	0
...			
1	1	1	0
1	1	1	1

Problem 4

(a) IND(A, ϕ, BE)
IND(B, ϕ, AC)
IND(C, A, BDE)
IND(D, AB, CE)
IND(E, B, ACDFG)
IND(F, CD, ABEH)
IND(G, F, ABCDEH)
IND(H, EF, ABCDG)

(b) d_separated(A, F, E)
→ blocked(ADB)? NO
→ blocked(DBE) NO
= NO

d_separated(G, B, E)
→ blocked(GFH)? NO
→ blocked(FHE)? NO
= NO

d_separated(AB, CDE, GH)
→ blocked(ACF)? YES
→ blocked(ADF)? YES
→ blocked(BDF)? YES
→ blocked(BEH)? YES
= YES

(c)

$$Pr(a,b,c,d,e,f,g,h) = \Theta_a \Theta_b \Theta_{c|a} \Theta_{d|a,b} \Theta_{e|b} \Theta_{f|c,d} \Theta_{h|f,e} \Theta_{g|f}$$

(d) By the statement IND(A, ϕ, BE),

$$\begin{aligned} Pr(A=1, B=1) &= Pr(A=1)Pr(B=1) \\ &= (0.2)(0.7) \\ &= 0.14 \end{aligned}$$

Similarly by IND(A, ϕ, BE) and also the total probability rule,

$$\begin{aligned} Pr(E=0|A=0) &= Pr(E=0) \\ &= Pr(E=0|B=1)Pr(B=1) + Pr(E=0|B=0)Pr(B=0) \\ &= (0.9)(0.7) + (0.1)(0.3) = 0.66 \end{aligned}$$

Problem 5

(a) $M(\alpha) = w_0, w_2, w_3$

(b) $Pr(\alpha) = Pr(A = 0, B = 1) = 1 - Pr(w_1) = 0.8$

(c) $Pr_A, B|\alpha) =$

world	P(world)
w_0	0.375
w_1	0
w_2	0.125
w_3	0.5

(d)

$$
\begin{aligned}
Pr(A \implies \neg B) &= \frac{Pr((A \implies \neg B) \wedge (A \implies B))}{Pr(\alpha)} \\
&= \frac{Pr((\neg A \vee \neg B) \wedge (\neg A \vee B))}{Pr(\alpha)} \\
&= \frac{Pr(\neg A)}{Pr(\alpha)} \\
&= \frac{0.5}{0.8} = 0.625
\end{aligned}
$$