

The tool we will use to work with probabilistic reasoning is called a **Bayesian Network**. This consists of two major components:

1. Directed Acyclic Graph shows casualty
2. Numbers/Nodes represent probability

Notation:

- Variable — X
- Value — x
- Variable Probability Distribution — $\Pr(x) = \Pr(X=x)$
- Set of Variables — X
- Instantiation — x
- World Probability Distribution — $\Pr(X) = \text{truth table}$

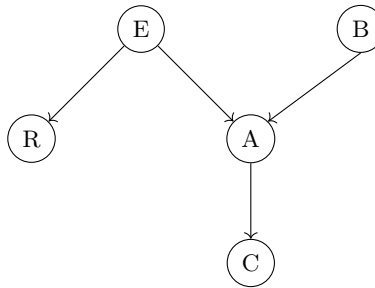
Remember that if $\Pr(\alpha|\beta, \gamma) = \Pr(\alpha|\gamma)$, then α & β are independent given γ

We can apply the same idea to sets of variables:

X and Y are independent given Z if $\Pr(x|y, z) = \Pr(x|z)$

We write this as $I(X, Y, Z)$

Consider the following:



Logic

The alarm is triggered by burglary or earthquake.

A radio report is caused by an earthquake.

A call from the neighbor may come post-alarm.

Independence

R & C are independent given A — $I(R, A, C)$

E & B are independent — $I(E, \phi, B)$

This can be much more efficient than a table.

The independence and parameters uniquely identify the graph.

We introduce a few terms:

- Parents(V) — the nodes pointing through V via an edge
Parents(A) = $\{E, B\}$ & Parents(R) = $\{E\}$
- Descendants(V) — the nodes reachable by V
Descendants(E) = $\{R, A, C\}$ & Descendants(B) = $\{A, C\}$ & Descendants(X) = $\{\}$
- Non-Descendants(V) — the nodes unreachable from V , excluding the self & parents
Non-descendants(A) = $\{R\}$ & Non-descendants(E) = $\{B\}$

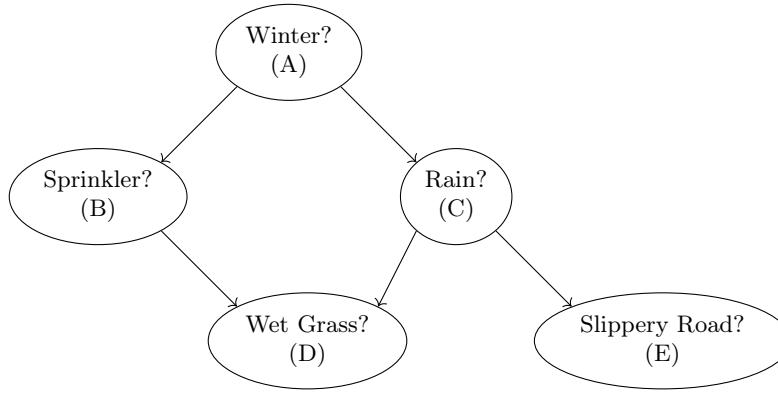
We call these independence statements the **Marcovion Assumptions**

\equiv all the defined independence statements of a Bayesian Network

These are of the form $I(V, \text{Parents}(V), \text{Non-Descendants}(V))$

Marcov(G) = $\{ I(C, A, BE), I(R, E, ABC), I(A, BE, R), I(B, \phi, ER), I(E, \phi, B) \}$

We can parametrize the structure by assigning weight to the connections.



A	θ_A	A	B	$\theta_{B A}$	A	C	$\theta_{C A}$	B	C	D	$\theta_{D B,C}$	C	E	$\theta_{E C}$
1	0.6	1	1	0.2	1	1	0.8	1	1	1	0.95	1	1	0.7
0	0.4	1	0	0.8	1	0	0.2	1	1	0	0.05	1	0	0.3
		0	1	0.75	0	1	0.1	1	0	1	0.9	0	1	0
		0	0	0.25	0	0	0.9	1	0	0	0.1	0	0	1
								0	1	1	0.8			
								0	1	0	0.2			
								0	0	1	0			
								0	0	0	1			

Note that $Pr(\neg d, b, \neg c) = \theta_{\neg d|b, \neg c}$

Consider an arbitrary row abcde

$$Pr(abcde) = \theta_a \theta_{b|a} \theta_{c|a} \theta_{d|bc} \theta_{e|c} = 0.7$$

$$Pr(ab\neg c d \neg e) = \theta_a \theta_{b|a} \theta_{\neg c|a} \theta_{d|b\neg c} \theta_{\neg e|c} = 0.216$$

$$Pr(\neg(abcde)) = \theta_{\neg a} \theta_{\neg b|\neg a} \theta_{\neg c|\neg a} \theta_{\neg d|\neg b\neg c} \theta_{\neg e|\neg c} = 0.09$$

We can evaluate the independence of any two nodes are independent given a third by

Deseperation

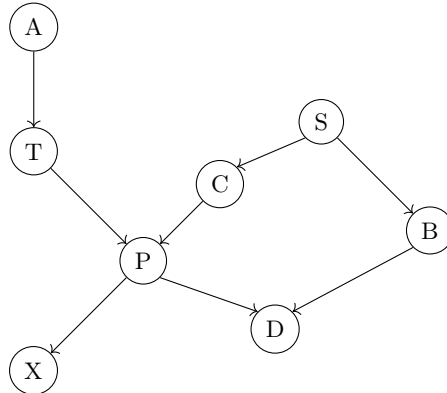
This uses an expansion on Markov's much more efficient than probability theory.

If setting Z blocks paths $x \rightarrow y$, x & y are d-separated given Z

There are three ways for a node to be connected in a path, treated as follows:

- sequential $\equiv \rightarrow w \rightarrow$ is blocked iff $w \in \mathbf{Z}$
- divergent $\equiv \leftarrow w \rightarrow$ is blocked iff $w \in \mathbf{Z}$
- convergent $\equiv \rightarrow w \leftarrow$ is blocked iff $w \notin \mathbf{Z}$ & $\text{Decendants}(w) \notin \mathbf{Z}$

We can clarify by giving an example:



$dsep(C, S, B)?$
 $\rightarrow \text{open}(BSC)? \text{ false.}$
 $\rightarrow \text{open}(CPDV)?$
 $\rightarrow \text{open}(SPD)? \text{ true.}$
 $\rightarrow \text{open}(PDS)? \text{ false.}$
 $\implies dsep(C, S, B) = \text{true.}$