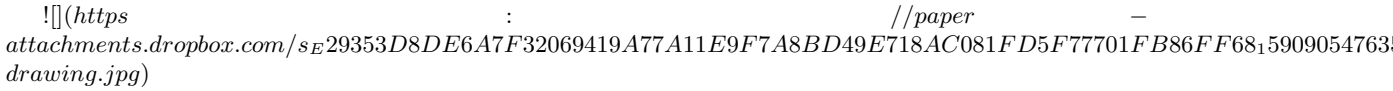
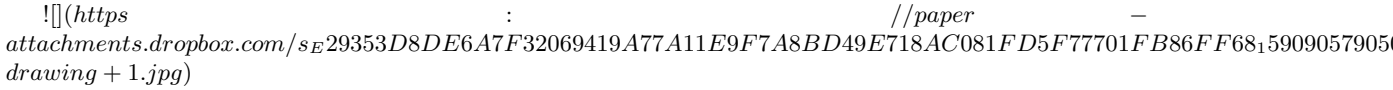
We wish to learn both the parameters and structure of a Bayesian Network.

Parameters

Consider the disease model from the previous lecture:



Cases 1 and 3 have incomplete data, whereas 2 had **complete**.
If every example is complete, the data set is called complete.
If complete, the maximum likelihood parameters are unique.
We find the likelihood of a parameter set like so: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Param $\{S_1\} \rightarrow BN_1 \rightarrow Pr_1(.)$
Param $\{S_2\} \rightarrow BN_2 \rightarrow Pr_2(.)$
.
.
.
We pick the set of parameters that maximizes the probability,
so score $S_i = \prod Pr_i(e_i)$ is the likelihood of parameters

Parameter Estimation With Complete Data

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$drawing + 2.jpg)$
The large table is our empirical distribution .
This is formed by collapsing our distribution.
$\theta_{\{}\neg S|H\} = \frac{Pr(\neg s,h)}{Pr(h)} = \frac{5}{6}$.
This is our maximum likelihood parameter estimate!

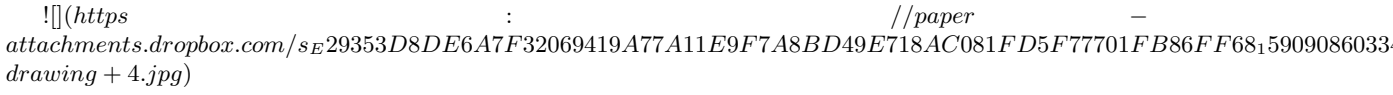
Parameter Estimation With Incomplete Data

This uses an iterative algorithm called **expected maximum**.
Say we wish to fill the row <h, s, ?>.
We start with an arbitrary guess at the CPT.
$\text{CPT}_1 \rightarrow \text{BN}_1 \rightarrow Pr_1(.)$.
We must choose from {TFT, TFF}, so we assign probabilities according to the probability function
x = Pr1(e|h, ¬s)
y = Pr(¬e|h, ¬s)
We then take the assignment with the greater probability, yielding
$\text{CPT}_2 \rightarrow \text{BN}_2 \rightarrow Pr_2(.)$.

We only needed to surmise one term here, but we may need to iterate.
The probabilities Pr1(.) & Pr2(.) are guaranteed to be non-increasing, so this converges.
This is thus a form of local search algorithm, so we know we may have to run repeatedly.
Note: this is the thinking behind the original algorithm, but it isn't how it is performed in practice.

Structure

Consider the following three structures: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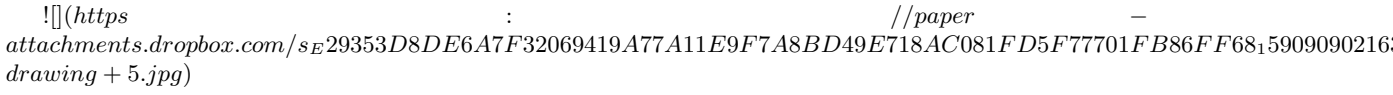
What do I need to optimize to choose between these three structures?
We have learned many of these algorithms, so we won't discuss in detail, but:

1. local search methods (approximate methods)
   → we transform the structure looking for a better score
   We must thus specify movement within the neighborhood structure: → legal operations: add/remove/reverse edge
2. systemic search methods (exact methods)
   → A* is a good example

Why can't we just use maximum likelihood? We face **overfitting**.
Say we are using likelihood to compare; then C > B > A.
We will thus end up with a complete DAG, no matter what.
Thus we need a model that balances structure complexity with likelihood.
Why? Consider: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The data is clearly a linear fit, but if we estimate to the fourth degree, we get bad data!
There is no fixed answer to the problem; we clearly need some function of the form f(likelihood) - g(complexity).
A common one is called MDK, but we need not discuss details.

Model-Oriented Vs. Query-Oriented Learning

This can also be referred to as (unsupervised vs supervised) or (unlabeled vs labeled).
We have done model learning now; we move on to query based.

A model-based approach might give us the following:

$$

Say we promise only to infer upward:

Then we don't 'model' the system and instead prepare for a specific 'query'.

Labeling of correct responses is done by humans, and this is done to 'train'.

If we want to talk about supervised learning, we need to introduce

Arithmetic Circuits

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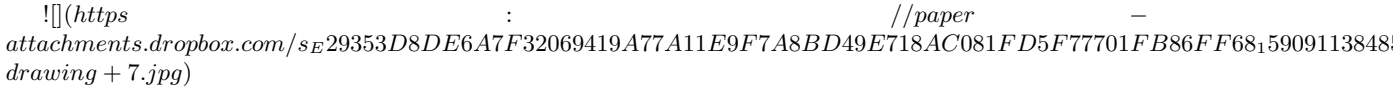
The (+) symbols represent OR gates
The (*) symbols represent AND gates
The θ are the parameters
The λ are the evidence

When given a query, we can perform weighted model counting on the equivalent arithmetic circuit.
If A=True, $\lambda_A = 1$; if A=False, $\lambda_{\neg A} = 1$; if unknown, both are 1
Thus we can evaluate this in linear time;u nfortunately converting to an arithmetic circuit is $O(nd^w)$

BUT what do we do if we don't have the parameters? We use the labeled data: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Cross Entropy gives us a measure of the disagreement between the two.

Therefore, we often seek to minimize it across a structure.
We generally use it to perform gradient descent (nth dimensional hill climbing).
Consider the example of recognizing shapes:

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Though pixel is functional in general, we allow it to be probabilistic to permit noise.
It will thus be inferred to be very close to 0/1.
A lot of heights will be zeroed as well depending on the column.
We can thus see that we have a lot of **background knowledge** The simplest case of this is $A \iff B$, for which we could simply substitute and reduce cases.