

The tool we will use to work with probabilistic reasoning is called a **Bayesian Network**. This consists of two major components:

1. Directed Acyclic Graph shows casualty
2. Numbers/Nodes represent probability

Notation:

- Variable — X
- Value — x
- Variable Probability Distribution — $\Pr(x) = \Pr(X=x)$
- Set of Variables — X
- Instantiation — x
- World Probability Distribution — $\Pr(X) = \text{truth table}$

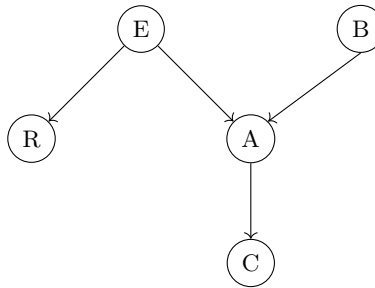
Remember that if $\Pr(\alpha|\beta, \gamma) = \Pr(\alpha|\gamma)$, then α & β are independent given γ

We can apply the same idea to sets of variables:

X and Y are independent given Z if $\Pr(x|y, z) = \Pr(x|z)$

We write this as $I(X, Y, Z)$

Consider the following:



Logic

The alarm is triggered by burglary or earthquake.

A radio report is caused by an earthquake.

A call from the neighbor may come post-alarm.

Independence

R & C are independent given A — $I(R, A, C)$

E & B are independent — $I(E, \phi, B)$

This can be much more efficient than a table.

The independence and parameters uniquely identify the graph.

We introduce a few terms:

- Parents(V) — the nodes pointing through V via an edge
Parents(A) = $\{E, B\}$ & Parents(R) = $\{E\}$
- Descendants(V) — the nodes reachable by V
Descendants(E) = $\{R, A, C\}$ & Descendants(B) = $\{A, C\}$ & Descendants(X) = $\{\}$
- Non-Descendants(V) — the nodes unreachable from V , excluding the self & parents
Non-descendants(A) = $\{R\}$ & Non-descendants(E) = $\{B\}$

We call these independence statements the **Marcovion Assumptions**

\equiv all the defined independence statements of a Bayesian Network

These are of the form $I(V, \text{Parents}(V), \text{Non-Descendants}(V))$

Marcov(G) = $\{ I(C, A, BE), I(R, E, ABC), I(A, BE, R), I(B, \phi, ER), I(E, \phi, B) \}$

We can parametrize the structure by assigning weight to the connections.

Note that $Pr(\neg d, b, \neg c) = \theta_{\{\neg d|b, \neg c\}}$

Consider an arbitrary row abcde

$$Pr(abcde) = \theta_a \theta_{b|a} \theta_{c|a} \theta_{d|bc} \theta_{e|c} = 0.7$$

$$Pr(ab\neg cd\neg e) = \theta_a \theta_{b|a} \theta_{\neg c|a} \theta_{d|b\neg c} \theta_{\neg e|c} = 0.216$$

$$Pr(\neg(abcde)) = \theta_{\neg a} \theta_{\neg b|\neg a} \theta_{\neg c|\neg a} \theta_{\neg d|\neg b\neg c} \theta_{\neg e|\neg c} = 0.09$$

We can evaluate the independence of any two nodes are independent given a third by

Deseperation

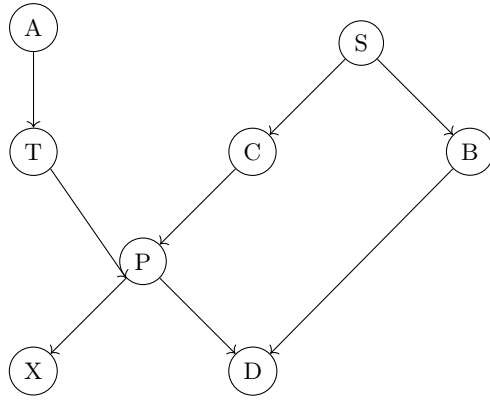
This uses an expansion on Markov's much more efficient than probability theory.

If setting Z blocks paths $x \rightarrow y$, x & y are d-separated given Z

There are three ways for a node to be connected in a path, treated as follows:

- sequential $\equiv \rightarrow w \rightarrow$ is blocked iff $w \in \mathbf{Z}$
- divergent $\equiv \leftarrow w \rightarrow$ is blocked iff $w \in \mathbf{Z}$
- convergent $\equiv \rightarrow w \leftarrow$ is blocked iff $w \& \text{Decendants}(w) \notin \mathbf{Z}$

We can clarify by giving an example:



dsep(C, S, B)?
 \rightarrow open(BSC)? false.
 \rightarrow open(CPDV)?
 \rightarrow open(SPD)? true.
 \rightarrow open(PDS)? false.
 \implies dsep(C, S, B) = true.