Let's motivate our study:

Early AI was almost entirely logic-based, but in the 70s, a crisis emerged.
Classical logic is **monotonic**, but human reasoning is not
  ($\equiv$ things that are true remain true with the introduction of new information)
  $((\Delta \models \alpha) \implies (\Delta \vee \beta \models \alpha))$

Consider the following example:
  Say you are told that Tweety is a bird ($\Delta$)
  Say you are asked "Does Tweety fly?" You would say yes ($\alpha$)
  But now say I tell you Tweety is a penguin ($\Delta$)
  Now say you are asked "Does Tweety fly?" You would say no ($\neg\alpha$)
If we were to build this with first-order logic:
  $\Delta$ : $\forall$ x bird(x) $\implies$ flies (x)
    good: $\Delta\wedge$ bird(Tweety) $\models$ flies(Tweety)
    bad: $\Delta\wedge$ bird(Tweety) $\wedge\neg$flies(Tweety) (CONTRADICTION)
We may thus conclude our information $\Delta$ was bad and do
  $\Delta$ : $\forall$ x bird(x) $\wedge$ abnormal(x) $\implies$ flies (x)
    good: $\Delta\wedge$ bird(Tweety) $\models$ flies(Tweety)
    bad: $\Delta \wedge \neg$lies(Tweety) $\models_?$ flies(Tweety)
We therefore must assume abnormal is false unless we hear otherwise.
This is not classical logic! It involves assumptions!
Another abnormal logic type:
  $\Delta$ : Quaker(x) $\wedge\neg$ab(x) $\implies$ Pacifist(x)
    Republican(x) $\wedge\neg$ab(x) $\implies$ $\neg$Pacifist(x)
But Nixon is a Quaker Republican, he can't both be and not be a pacifist!
Resolving this turns out to be complicated, and we will not address it; instead we introduce a new model:

## Belief Revision

We introduce the idea of degrees of belief in [0, 1]
We will use the same example as with logic:

| W | E | B | A | Pr(w) |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 0.0190 |
| 2 | 1 | 1 | 0 | 0.0010 |
| 3 | 1 | 0 | 1 | 0.0560 |
| 4 | 1 | 0 | 0 | 0.0240 |
| 5 | 0 | 1 | 1 | 0.1620 |
| 6 | 0 | 1 | 0 | 0.0180 |
| 7 | 0 | 0 | 1 | 0.0072 |
| 8 | 0 | 0 | 0 | 0.7128 |

Instead of ruling worlds in or out by entailment, we represent the chance of each world occurring.
f: sentences $\rightarrow$ degree of certainty; $Pr(\alpha) = \sum Pr(w)$ for $w \models \alpha$
Therefore

- Pr(E) = 0.1
- $Pf(\neg E) = 1 - Pr(E) = 0.9$
- Pr(B) = 0.12
- $Pr(B \vee \neg E) = 1 - Pr(\neg B \wedge E) = 0.92$

We have a few properties that are required for probability to work:

1. $0 \leq Pr(\alpha) \leq 1$
2. $\alpha$ is inconsistent $\iff Pr(\alpha) = 0$
3. $\alpha$ is valid $\iff Pr(\alpha) = 1$
4. $Pr(\alpha) + Pr(\neg\alpha) = 1$
5. $Pr(\alpha \vee B) = Pr(A) + Pr(B) - Pr(\alpha \wedge B)$
6. $If\ \alpha$ & $\beta$ are mutually exclusive, $Pr(\alpha \vee \beta) = Pr(\alpha) + Pr(\beta)$

Now how can we change beliefs with the introduction of new information?
Clearly, we must zero out any worlds that are invalid
P($\alpha|\beta$) =

$$\left\{ \begin{array}{ll} 0 & \text{if } w \models \neg\beta \\ Pr(\alpha)/Pr(\beta) & \text{if } w \models \beta \end{array} \right\}$$

We can thus get **Bayes Condition**

$$Pr(\alpha|\beta) = \frac{Pr(\alpha \wedge \beta)}{Pr(\beta)}$$

We apply this to the earlier example:
  Pr(B) = 0.2, Pr(B|A) = 0.741
  Pr(E) = 0.1, Pr(E|A) = 0.307
  Pr(B) = 0.2, Pr(B|E) = 0.2
  Pr(E) = 0.1, Pr(E|B) = 0.1

We say two variables are **independent** iff P(E) = P(E|B) && P(B) = Pr(B|E)
Therefore we can see that B and E are independent in this system of belief.

How does this gel when introducing a third?
Pr(B|A,E) = 0.253 < Pr(B|A)
Pr(B|A,¬E) = 0.957 > Pr(B|A)
While inapplicable this graph, Pr finds A **conditionally independent** iff $P(\alpha|B \wedge C) = P(\alpha|C)$.
This is equivalent to saying "B gives C".

Thus using these properties I can define a model of the world.
We have a few equations that can calculate all we need to know:

1. **Chain Rule** $\equiv Pr(e_1 \wedge e_2 \wedge ... \wedge e_n) = Pr(e_1|e_2 \wedge e_3 \wedge ... \wedge e_n) + ... + Pr(en)$
2. **Case Analysis** $\equiv Pr(\alpha) = \sum Pr(\alpha \wedge \beta_i) = \sum Pr(\alpha|\beta_i)Pr(\alpha)$ where $\{\beta_i\}$
3. **Bayes' Rule** $\equiv Pr(\alpha|\beta) = \frac{Pr(\alpha)}{Pr(\beta)} Pr(\beta|\alpha)$

This last rule is just a rework of Bayes' Condition, so why did he get credit for another rule? The rule turns out to be incredibly useful; consider:
Say a patient gets a positive test for a disease (D)
Say we know P(D) = 1/1000
Say we know the false positive rate P(T|¬D) = 2%
Say we know the false negative rate P(D|¬T) =5%
We can use this to estimate the odds of the person having the disease!
P(D|T) = (P(D)/P(T)) P(T+D) = (P(T|D)P(D))/(P(T∧¬B)P(¬D) * (1-P(D|¬T))P(D)) = 4.5%

We have thus build a probability calculus on top of probability logic:
To move on, we need to consider the fact that a variable can have multiple values
We can't use our previous rules; this leads us to **equality** $\equiv (x = grn) \vee (x = blue) \implies P(y = large)$