# Answers to Unit Test 4: Exponential Functions

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Here are the step-by-step solutions, explained simply!

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## 1. Simplify

### c) $\left(\frac{3x^2}{y^{-3}}\right)^{-5}$

#### Step 1: The "Shower" Rule

Imagine the exponent $-5$ outside the brackets is like water from a shower. It has to sprinkle onto *everything* inside the house (the brackets).

$\rightarrow$ The $3$ gets a $-5$.

$\rightarrow$ The $x^2$ gets a $-5$.

$\rightarrow$ The $y^{-3}$ gets a $-5$.

#### Step 2: Multiply the Exponents

When an exponent meets another exponent across a bracket (like $(x^2)^{-5}$), they multiply.

- For x: $2 \times -5 = -10$
- For y: $-3 \times -5 = +15$ (Two negatives make a positive!)

So now we have:

$$\frac{3^{-5}x^{-10}}{y^{15}}$$

#### Step 3: Fix the Negative Exponents

Negative exponents are like unhappy people. To make them happy (positive), you have to move them to the other floor.

- $3^{-5}$ is unhappy upstairs, so move it downstairs: $3^5$.
- $x^{-10}$ is unhappy upstairs, so move it downstairs: $x^{10}$.
- $y^{15}$ is already happy (+15) downstairs, so it stays there.

**Final Answer:**

$$\frac{1}{3^5x^{10}y^{15}}$$

*(Note: $3^5 = 243$)*

$$\frac{1}{243x^{10}y^{15}}$$

**d)** $\left[\left(2x^2\right)^{-2}\right]^{-3}$

**Step 1: Simplify the Outside First**

We have a $-3$ outside and a $-2$ inside. Let's combine them first by multiplying.

$-2 \times -3 = 6$.

So the problem becomes just: $(2x^2)^6$

**Step 2: Share the Power**

Give the power of 6 to everyone inside.

- The number $2$ gets the power of 6 $\rightarrow$ $2^6$.
- The $x^2$ gets the power of 6 $\rightarrow$ $(x^2)^6$.

**Step 3: Calculate**

- $2^6 = 64$

- $x^{2 \times 6} = x^{12}$

**Final Answer:**

$$64x^{12}$$

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## 2. Function Properties

Equation: $y = 2\left(3^{-\frac{1}{2}(x-2)}\right) - 4$

#### i) Base Function

Strip away all the extra numbers and shifts. What is the core engine? It is the base number with an exponent.

**Answer:** $f(x) = 3^x$

#### ii) Asymptote

Exponential functions normally flatten out at $y = 0$ (the floor). But look at the $-4$ at the very end. That pulls the whole building down 4 floors.

**Answer:** $y = -4$

#### iii) Y-intercept

The y-intercept is where the graph crosses the vertical line. This happens when $x$ is 0.

Let's plug in 0 for $x$:

1. Look at the exponent: $-\frac{1}{2}(0-2) = -\frac{1}{2}(-2) = 1$.
2. So we have $3^1$, which is just 3.
3. Multiply by the front number 2: $2 \times 3 = 6$.
4. Subtract 4: $6 - 4 = 2$.

**Answer:** $(0, 2)$

#### iv) Transformation Form

We just need to match the parts to the template $y = a \cdot f(b(x - c)) + d$.

- $a = 2$ (Vertical stretch)
- $b = -1/2$ (Horizontal reflection and stretch)
- $c = 2$ (Shift right 2)
- $d = -4$ (Shift down 4)

**Answer:** $y = 2f(-0.5(x - 2)) - 4$

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## 3. Graphing

#### The Mapping Formula (Pointwise)

We take points $(x, y)$ from the simple parent function $y = 3^x$ and transform them.

**New x:** Divide old x by $b$, then add $c$. $\rightarrow \frac{x}{-0.5} + 2 \rightarrow -2x + 2$

**New y:** Multiply old y by $a$, then add $d$. $\rightarrow 2y - 4$

**Let's transform 3 key points:**

1. **Point (-1, 1/3)**

- New x: $-2(-1) + 2 = 4$
- New y: $2(1/3) - 4 = -3.33$
- **Plot:** $(4, -3.33)$

2. **Point (0, 1)**

- New x: $-2(0) + 2 = 2$
- New y: $2(1) - 4 = -2$
- **Plot:** $(2, -2)$

3. **Point (1, 3)**

- New x: $-2(1) + 2 = 0$
- New y: $2(3) - 4 = 2$
- **Plot:** $(0, 2)$ (The y-intercept!)

**Don't forget the Asymptote:** Draw a dashed line at $y = -4$. The curve will get closer and closer to this line but never touch it.

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## 4. Computer Value Application

Model: $V(m) = 1500(0.95)^m$

#### (a) Initial Value

"Initial" means at the very start, when time is zero ($m = 0$).

Anything to the power of 0 is 1.

So, $1500 \times 1 = 1500$.

**Answer:** The computer cost **$1500**.

#### (b) Rate of Depreciation

Look at the multiplier $0.95$.

This means every month, the computer keeps 95% of its value.

If it keeps 95%, how much did it lose?

$100\% - 95\% = 5\%.$

**Answer:** It loses **5%** value per month.

**© Value after 2 years**

Be careful with units! The formula uses months ($m$), but the question says 2 *years*.

2 years = 24 months.

Plug 24 into the machine:

$V = 1500(0.95)^{24}$

Using a calculator: $0.95^{24} \approx 0.292$

$1500 \times 0.292 \approx 438.$

**Answer:** The value is approximately **$438.00**.

**(d) When does it fall below $900?**

We need to find when the value hits 900.

$1500(0.95)^m = 900$

1. Divide by 1500: $0.95^m = 0.6$
2. Use “Trial and Error” or Logarithms.

- Try $m = 9$: $1500(0.95)^9 \approx $945$ (Still too high)
- Try $m = 10$: $1500(0.95)^{10} \approx $898$ (Dropped below!)

Since it drops below 900 between month 9 and month 10, it happens **during the 10th month**.

**Answer:** In the **10th month**.