

Here are the solutions for finding the two possible values of  $\theta$  for each trigonometric ratio in the interval  $0^\circ \leq \theta \leq 360^\circ$ .

The process involves finding the **reference angle ( $\alpha$ )** and then using the sign of the ratio to determine the correct two **quadrants** for the principal angles ( $\theta$ ).

## Solutions and Principal Angles

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a)  $\cos \theta = 0.6951$

1. Reference Angle ( $\alpha$ ):

$$\alpha = \arccos(0.6951) \approx 46.0^\circ$$

2. **Quadrants:** Cosine is **positive**, so  $\theta$  is in **Quadrant I (A)** and **Quadrant IV (C)**.

3. **Angles ( $\theta$ ):**

- **QI:**  $\theta_1 = \alpha \approx 46.0^\circ$
- **QIV:**  $\theta_2 = 360^\circ - \alpha \approx 360^\circ - 46.0^\circ = 314.0^\circ$

4. **Sketch:**

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b)  $\tan \theta = -0.7571$

1. Reference Angle ( $\alpha$ ):

$$\alpha = \arctan(| -0.7571 |) \approx 37.1^\circ$$

2. **Quadrants:** Tangent is **negative**, so  $\theta$  is in **Quadrant II (S)** and **Quadrant IV (C)**.

3. **Angles ( $\theta$ ):**

- **QII:**  $\theta_1 = 180^\circ - \alpha \approx 180^\circ - 37.1^\circ = 142.9^\circ$

- **QIV:**  $\theta_2 = 360^\circ - \alpha \approx 360^\circ - 37.1^\circ = 322.9^\circ$

**4. Sketch:**

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c)  $\sin \theta = 0.3154$

1. Reference Angle ( $\alpha$ ):

$$\alpha = \arcsin(0.3154) \approx 18.4^\circ$$

2. **Quadrants:** Sine is **positive**, so  $\theta$  is in **Quadrant I** (A) and **Quadrant II** (S).

3. **Angles ( $\theta$ ):**

- **QI:**  $\theta_1 = \alpha \approx 18.4^\circ$
- **QII:**  $\theta_2 = 180^\circ - \alpha \approx 180^\circ - 18.4^\circ = 161.6^\circ$

**4. Sketch:**

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d)  $\cos \theta = -0.2882$

1. Reference Angle ( $\alpha$ ):

$$\alpha = \arccos(|-0.2882|) \approx 73.2^\circ$$

2. **Quadrants:** Cosine is **negative**, so  $\theta$  is in **Quadrant II** (S) and **Quadrant III** (T).

3. **Angles ( $\theta$ ):**

- **QII:**  $\theta_1 = 180^\circ - \alpha \approx 180^\circ - 73.2^\circ = 106.8^\circ$
- **QIII:**  $\theta_2 = 180^\circ + \alpha \approx 180^\circ + 73.2^\circ = 253.2^\circ$

**4. Sketch:**

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e)  $\tan \theta = 2.3151$

1. Reference Angle ( $\alpha$ ):

$$\alpha = \arctan(2.3151) \approx 66.6^\circ$$

2. **Quadrants:** Tangent is **positive**, so  $\theta$  is in **Quadrant I** (A) and **Quadrant III** (T).

3. **Angles ( $\theta$ ):**

- **QI:**  $\theta_1 = \alpha \approx 66.6^\circ$
- **QIII:**  $\theta_2 = 180^\circ + \alpha \approx 180^\circ + 66.6^\circ = 246.6^\circ$

4. **Sketch:**

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f)  $\sin \theta = -0.7503$

1. Reference Angle ( $\alpha$ ):

$$\alpha = \arcsin(|-0.7503|) \approx 48.6^\circ$$

2. **Quadrants:** Sine is **negative**, so  $\theta$  is in **Quadrant III** (T) and **Quadrant IV** (C).

3. **Angles ( $\theta$ ):**

- **QIII:**  $\theta_1 = 180^\circ + \alpha \approx 180^\circ + 48.6^\circ = 228.6^\circ$
- **QIV:**  $\theta_2 = 360^\circ - \alpha \approx 360^\circ - 48.6^\circ = 311.4^\circ$

4. **Sketch:**

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Do you have any questions about how to use the reference angle to find the principal angles in the different quadrants?