Question 7a.

State the pointwise formula for the transformation from the parent function in $y = 3^x$ to $y = 2 * 3^{-2(x+1)} - 3$ and use it with at least 3 key points.

Question 8a

State the pointwise formula for the transformation from y = cos(x) to y=-3cos(10(x+9))+2, and use it with at least 3 key points of y = cos(x).

— step-by-step solutions

7a. Exponential Transformation

Function: $y = 2 \cdot 3^{-2(x+1)} - 3$

Parent Function: $y = 3^x$

1. Identify the Parameters

- $a = 2$ (Vertical stretch by 2)
- $k = -2$ (Horizontal reflection and compression by 1/2)
- $d = -1$ (Shift left 1)
- $c = -3$ (Shift down 3)

2. State the Pointwise Formula

Using the general rule $(x, y) \to (\frac{x}{k} + d, ay + c)$, the specific mapping for this function is:

$$(x, y) \to \left(\frac{x}{-2} - 1, 2y - 3\right)$$

3. Apply to 3 Key Points

We will use $x = -1, 0, 1$ for the parent function $y = 3^x$.

Parent Point (x,y)	Mapping Calculation	Transformed Point
$(-1, 0.33)$	$\left(\frac{-1}{-2} - 1, 2(0.33) - 3\right)$	$(-0.5, -2.34)$
$(0, 1)$	$\left(\frac{0}{-2} - 1, 2(1) - 3\right)$	$(-1, -1)$
$(1, 3)$	$\left(\frac{1}{-2} - 1, 2(3) - 3\right)$	$(-1.5, 3)$

Note: The horizontal asymptote also shifts from $y = 0$ to $y = -3$.

8a. Trigonometric Transformation

Function: $y = -3\cos(10(x + 9)) + 2$

Parent Function: $y = \cos(x)$

1. Identify the Parameters

- $a = -3$ (Vertical reflection and stretch by 3)
- $k = 10$ (Horizontal compression by 1/10)
- $d = -9$ (Phase shift left 9)
- $c = 2$ (Vertical shift up 2)

2. State the Pointwise Formula

The specific mapping for this function is:

$$(x, y) \to \left(\frac{x}{10} - 9, -3y + 2\right)$$

3. Apply to 3 Key Points

We will use the standard key points (in degrees) for $y = \cos(x)$.

Parent Point (x,y)	Mapping Calculation	Transformed Point
$(0^\circ, 1)$ (Max)	$\left(\frac{0}{10} - 9, -3(1) + 2\right)$	$(-9^\circ, -1)$
$(90^\circ, 0)$ (Midline)	$\left(\frac{90}{10} - 9, -3(0) + 2\right)$	$(0^\circ, 2)$
$(180^\circ, -1)$ (Min)	$\left(\frac{180}{10} - 9, -3(-1) + 2\right)$	$(9^\circ, 5)$

Observations:

- The original Maximum $(0, 1)$ has become a Minimum $(-9, -1)$ because of the vertical reflection ($a = -3$).
- The original Minimum $(180, -1)$ has become a Maximum $(9, 5)$.