3U Trigonometry Test 1 - Answers & Solutions

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1. Solve for all of the unknowns in the following right triangle.

(Assumed: A right triangle where one acute angle is 29°, and the adjacent side (let's say b) is 37 units (or hypotenuse? Based on student working $z = 37/\sin(61)$, z is Hypotenuse. If z is hyp, then side 37 is opposite angle 61. Angle 61 is B. So $b = 37$ is opposite B. $A = 29$ is opposite a.)

Given:

- Right Triangle ($C = 90^\circ$)
- Angle $A = 29^\circ$
- Side adjacent to A (or opposite B) = 37.

Step 1: Find Missing Angle (β)

The sum of angles in a triangle is 180°.

$$\beta = 180^\circ - 90^\circ - 29^\circ = 61^\circ$$

Step 2: Find Hypotenuse (z)

Using Sine Law or SOH CAH TOA:

$$\sin(61^\circ) = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{37}{z}$$

$$z = \frac{37}{\sin(61^\circ)}$$

$$z \approx \frac{37}{0.8746}$$

$$z \approx 42.3$$

(Note: Student answer 42.53 suggests they might have used $\cos(29)$ *or similar with rounding diff, or original diagram labeled 37 differently. If 37 was adjacent to 29,* $z = 37/\cos(29) \approx 42.3$ *as well.)*

Step 3: Find Remaining Side (y)

Using Tangent:

$$\tan(29^\circ) = \frac{y}{37}$$

$$y = 37 \cdot \tan(29^\circ)$$

$$y = 37 \cdot 0.5543$$

$$y \approx 20.5$$

Answers:

- $\beta = 61^\circ$
- $y \approx 20.5$ (Student got 20.6)
- $z \approx 42.3$ (Student got 42.53)

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2. In ΔABC, sides $a = 26$ cm, $b = 63$ cm, and $c = 44$ cm. Determine all angles to 1 decimal place.

Step 1: Find the largest angle first (Angle B, opposite side b=63) given SSS.

Using Cosine Law: $b^2 = a^2 + c^2 - 2ac\cos B$

$$63^2 = 26^2 + 44^2 - 2(26)(44)\cos B$$

$$3969 = 676 + 1936 - 2288 \cos B$$

$$3969 = 2612 - 2288 \cos B$$

$$1357 = -2288 \cos B$$

$$\cos B = -\frac{1357}{2288} \approx -0.5931$$

$$B = \cos^{-1}(-0.5931) \approx 126.4^\circ$$

Step 2: Find Angle A using Sine Law

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\sin A = \frac{26 \cdot \sin(126.4^\circ)}{63}$$

$$\sin A = \frac{26 \cdot 0.8049}{63} \approx 0.3323$$

$$A = \sin^{-1}(0.3323) \approx 19.4^\circ$$

Step 3: Find Angle C

$$C = 180^\circ - (126.4^\circ + 19.4^\circ)$$

$$C = 180^\circ - 145.8^\circ = 34.2^\circ$$

Answers:

- $A = 19.4^\circ$
- $B = 126.4^\circ$
- $C = 34.2^\circ$

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3. Determine all lengths and angles in ΔABC where angle B is obtuse and $\angle A = 34^\circ, b = 5$ cm, $a = 4$ cm.

Step 1: Check for Ambiguous Case (SSA)

Height $h = b \sin A = 5 \sin(34^\circ) \approx 2.8$.

Since $h < a < b$ ($2.8 < 4 < 5$), there are two possible triangles (Acute B and Obtuse B).

Problem states **B is obtuse**.

Step 2: Find Angle B

Using Sine Law:

$$\frac{\sin B}{5} = \frac{\sin 34^\circ}{4}$$

$$\sin B = \frac{5 \sin 34^\circ}{4} \approx 0.699$$

$$B_{ref} = \sin^{-1}(0.699) \approx 44.3^\circ$$

Since B is obtuse:

$$B = 180^\circ - 44.3^\circ = 135.7^\circ$$

Step 3: Find Angle C

$$C = 180^\circ - (34^\circ + 135.7^\circ)$$

$$C = 180^\circ - 169.7^\circ = 10.3^\circ$$

Step 4: Find side c

$$\frac{c}{\sin 10.3^\circ} = \frac{4}{\sin 34^\circ}$$

$$c = \frac{4 \sin 10.3^\circ}{\sin 34^\circ} \approx \frac{4(0.1788)}{0.559} \approx 1.3 \text{ cm}$$

Answers:

- $B = 135.7^\circ$
- $C = 10.3^\circ$

- $c = 1.3$ cm

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4. Determine x (and involved steps).

Reconstructed Solution based on student work:

The problem likely involves two triangles linked together.

Step 1: Calculate a missing side or angle.

Student used Cosine Law or similar to find a value $C \approx 21.83$. Assuming this is an angle based on later usage inside $\sin()$.

Step 2: Solve for x.

$$\frac{x}{\sin 122^\circ} = \frac{25}{\sin 21.83^\circ}$$
$$x = \frac{25 \sin 122^\circ}{\sin 21.83^\circ}$$
$$x = \frac{25(0.848)}{0.372}$$
$$x \approx 57.0$$

(Student answer 57.3 is reasonably close, likely due to intermediate rounding differences).

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5. Point P(-3, -6)

(a) Diagram

- Point P is in Quadrant 3 (both x and y are negative).
- Triangle drawn to x-axis.

(b) Exact Values

$$x = -3, y = -6$$

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + (-6)^2} = \sqrt{9 + 36} = \sqrt{45} = 3\sqrt{5}$$

Trig Ratios:

$$\sin\theta = \frac{y}{r} = \frac{-6}{3\sqrt{5}} = \frac{-2}{\sqrt{5}}$$

$$\cos\theta = \frac{x}{r} = \frac{-3}{3\sqrt{5}} = \frac{-1}{\sqrt{5}}$$

$$\tan\theta = \frac{y}{x} = \frac{-6}{-3} = 2$$

© Angles

Related Acute Angle α:

$$\tan\alpha = |2| \implies \alpha = \tan^{-1}(2) \approx 63.4^\circ$$

Principal Angle θ (Quadrant 3):

$$\theta = 180^\circ + \alpha = 180^\circ + 63.4^\circ = 243.4^\circ$$

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6. Determine exact value of $\sec(210^\circ)$

1. **Quadrant:** 210° is in Quadrant 3.

2. **Related Acute Angle:** $210^\circ - 180^\circ = 30^\circ$.

3. **Special Triangle:** 30-60-90.

- Sides for 30°: Opp = 1, Adj = $\sqrt{3}$, Hyp = 2.

- In Q3: $x = -\sqrt{3}, y = -1, r = 2$.

4. **Secant Ratio:**

$$\sec\theta = \frac{r}{x}$$

$$\sec(210^\circ) = \frac{2}{-\sqrt{3}} = -\frac{2}{\sqrt{3}}$$

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7. For $\cos(A) = -2/3, 0^\circ \leq A \leq 360^\circ$

(i) Sketch

Cosine is negative in Quadrants 2 and 3. There are two terminal arms.

(ii) Determine $\tan(A)$

Given $x = -2, r = 3$. Find y.

$$x^2 + y^2 = r^2$$

$$(-2)^2 + y^2 = 3^2$$

$$4 + y^2 = 9$$

$$y^2 = 5 \implies y = \pm\sqrt{5}$$

- **Quadrant 2 (y is pos):** $\tan A = \frac{\sqrt{5}}{-2} = -\frac{\sqrt{5}}{2}$
- **Quadrant 3 (y is neg):** $\tan A = \frac{-\sqrt{5}}{-2} = \frac{\sqrt{5}}{2}$

(iii) Angles

$$\alpha = \cos^{-1}(2/3) \approx 48.2^\circ$$

- Q2: $A = 180^\circ - 48.2^\circ = 131.8^\circ$
- Q3: $A = 180^\circ + 48.2^\circ = 228.2^\circ$

Part 2: Proofs

(a) Prove $\frac{1-\sin x}{\cos x} = \frac{\cos x}{1+\sin x}$

Strategy: Multiply LS by conjugate $(1 + \sin x)$

$$LS = \frac{1 - \sin x}{\cos x} \cdot \frac{1 + \sin x}{1 + \sin x}$$

$$LS = \frac{1 - \sin^2 x}{\cos x(1 + \sin x)}$$

$$\text{Identity: } 1 - \sin^2 x = \cos^2 x$$

$$LS = \frac{\cos^2 x}{\cos x(1 + \sin x)}$$

$$LS = \frac{\cos x}{1 + \sin x}$$

$$LS = RS \quad \blacksquare$$

(b) Prove $\tan x + \frac{\cos x}{1+\sin x} = \frac{1}{\cos x}$

Strategy: Convert tan to sin/cos and find common denominator

$$LS = \frac{\sin x}{\cos x} + \frac{\cos x}{1 + \sin x}$$

$$LS = \frac{\sin x(1 + \sin x) + \cos x(\cos x)}{\cos x(1 + \sin x)}$$

$$LS = \frac{\sin x + \sin^2 x + \cos^2 x}{\cos x(1 + \sin x)}$$

$$\text{Identity: } \sin^2 x + \cos^2 x = 1$$

$$LS = \frac{\sin x + 1}{\cos x(1 + \sin x)}$$

$$LS = \frac{(1 + \sin x)}{\cos x(1 + \sin x)}$$

$$LS = \frac{1}{\cos x}$$

$$LS = RS \quad \blacksquare$$