

1. Fundamental Concepts of Periodic Functions

A **periodic function** is a function with a graph that repeats a regular pattern over a constant interval.

- **Cycle:** One complete repeating portion of the graph.
- **Period:** The change in the independent variable (usually x or t) that corresponds to one cycle.
- **Peak and Trough:** The **peak** is the maximum point on the graph; the **trough** is the minimum point.
- **Equation of the Axis:** The horizontal line halfway between the maximum and minimum values.
- **Amplitude:** The vertical distance from the function's axis to the maximum or minimum value; it is always a positive value.

2. Properties of Parent Sinusoidal Functions

Sinusoidal functions are periodic functions that form smooth, symmetrical waves. The two primary parent functions are $f(x) = \sin x$ and $f(x) = \cos x$.

Characteristic	$f(x) = \sin x$	$f(x) = \cos x$
Period	360°	360°
Amplitude	1	1
Equation of Axis	$y = 0$	$y = 0$
Range	$\{y \in \mathbb{R} \mid -1 \leq y \leq 1\}$	$\{y \in \mathbb{R} \mid -1 \leq y \leq 1\}$
Starting Point ($x = 0$)	At the axis $(0, 0)$	At the peak $(0, 1)$

Key Points for Sketching (0° to 360°):

- **Sine:** $(0, 0), (90, 1), (180, 0), (270, -1), (360, 0)$.
- **Cosine:** $(0, 1), (90, 0), (180, -1), (270, 0), (360, 1)$.

3. Transformations of Sinusoidal Functions

Transformed functions take the form: $y = a \sin[k(x - d)] + c$ or $y = a \cos[k(x - d)] + c$.

- **Amplitude (a):** Determined by the vertical stretch/compression. **Amplitude** = $|a|$. If $a < 0$, the graph is reflected in the x-axis.
- **Period (k):** Determined by the horizontal stretch/compression. **Period** = $\frac{360^\circ}{|k|}$.
- **Phase Shift (d):** The horizontal translation. The graph moves right if $d > 0$ and left if $d < 0$.
- **Vertical Translation (c):** Shifts the graph up or down. The **Equation of the Axis** becomes $y = c$.

4. Essential Equations and Formulas

- **Equation of the Axis:** $y = \frac{\text{maximum value} + \text{minimum value}}{2}$.
- **Amplitude Calculation:** $a = \frac{\text{maximum value} - \text{minimum value}}{2}$.
- **Finding k :** $k = \frac{360^\circ}{\text{Period}}$.
- **Circle Coordinates:** Any point $P(x, y)$ on a circle with radius r rotated through angle θ is $(r \cos \theta, r \sin \theta)$.
- **Speed in Circular Motion:** Speed is calculated by dividing the circumference ($2\pi r$) by the period.

5. Modeling Real-World Situations

When modeling phenomena like Ferris wheels or tides, you must connect physical traits to graph features:

- **Radius of the wheel = Amplitude.**
- **Height of the axle = Equation of the Axis (c value).**
- **Time for one revolution = Period.**
- **Starting Position:** Determines the **Phase Shift (d)**. For example, if a point starts at its maximum height at $t = 0$, use a cosine model with $d = 0$.

6. Critical Test Skills

- **Order of Transformations:** Apply stretches, compressions, and reflections (multiplication) before translations (addition/subtraction).
- **Factoring for Phase Shift:** Always ensure the k value is factored out (e.g., $y = \sin(2x + 60^\circ)$ must be rewritten as $y = \sin[2(x + 30^\circ)]$ to see the 30° left shift).
- **Function Selection:** Use a cosine model if the cycle starts at a maximum or minimum value; use a sine model if it starts at the axis.