📝 Unit Test Study Guide: Trigonometric Ratios

1. Algebraic Tools for Proofs

To prove trigonometric identities effectively, you often need to use standard algebraic expansions and factoring. These patterns allow you to substitute trigonometric ratios (like $\sin\theta$ or $\cos\theta$) into algebraic forms.

Concept	Formula	Trigonometry Example
Difference of Squares	$a^2 - b^2 = (a+b)(a-b)$	$\sin^2 x - \cos^2 x = (\sin x + \cos x)(\sin x - \cos x)$
Perfect Square (Addition)	$(a+b)^2 = a^2 + 2ab + b^2$	$(\sin x + 1)^2 = \sin^2 x + 2\sin x + 1$
Perfect Square (Subtraction)	$(a-b)^2 = a^2 - 2ab + b^2$	$(\tan x - 1)^2 = \tan^2 x - 2\tan x + 1$

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2. Primary and Reciprocal Ratios (Right Triangles)

The reciprocal trigonometric ratios are defined as 1 divided by each of the primary trigonometric ratios.

Ratio Name	Abbreviation	Definition	Relationship
Sine	$\sin\ \theta$	$\frac{\text{opposite}}{\text{hypotenuse}}$	Primary Ratio
Cosine	$\cos\ \theta$	$\frac{\text{adjacent}}{\text{hypotenuse}}$	Primary Ratio
Tangent	$\tan\ \theta$	$\frac{\text{opposite}}{\text{adjacent}}$	Primary Ratio
Cosecant	$\csc\ \theta$	$\frac{\text{hypotenuse}}{\text{opposite}}$	$\csc\ \theta = \frac{1}{\sin\ \theta}$
Secant	$\sec\ \theta$	$\frac{\text{hypotenuse}}{\text{adjacent}}$	$\sec\ \theta = \frac{1}{\cos\ \theta}$

Ratio Name	Abbreviation	Definition	Relationship
Cotangent	$\cot\ \theta$	$\frac{\text{adjacent}}{\text{opposite}}$	$\cot\ \theta = \frac{1}{\tan\ \theta}$

Important Notes for Right Triangles:

- **Calculators:** Most calculators do not have buttons for $\csc, \sec, \cot$. To evaluate them, use the reciprocal of the primary ratio (e.g., for $\sec\ 20^\circ$, calculate $1 \div \cos\ 20^\circ$).
- **Inverse Functions:** Use $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$ to determine the angle associated with a specific ratio.

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3. Trigonometric Identities

A trigonometric identity is an equation involving trigonometric ratios that is true for all values of the variable.

A. Quotient Identities

- $\tan\ \theta = \frac{\sin\ \theta}{\cos\ \theta}$ (where $\cos\ \theta \neq 0$)
- $\cot\ \theta = \frac{\cos\ \theta}{\sin\ \theta}$ (where $\sin\ \theta \neq 0$)

B. Reciprocal Identities

- $\csc\ \theta = \frac{1}{\sin\ \theta}$ (where $\sin\ \theta \neq 0$)
- $\sec\ \theta = \frac{1}{\cos\ \theta}$ (where $\cos\ \theta \neq 0$)
- $\cot\ \theta = \frac{1}{\tan\ \theta}$ (where $\tan\ \theta \neq 0$)

C. Pythagorean Identities

- $\sin^2\ \theta + \cos^2\ \theta = 1$
- $1 + \tan^2\ \theta = \sec^2\ \theta$
- $1 + \cot^2\ \theta = \csc^2\ \theta$

Strategy for Proving Identities

1. **Separate sides:** Work with the Left Side (L.S.) and Right Side (R.S.) independently.
2. **Simplify:** Start with the more complicated side.
3. **Rewrite:** Express tangent and reciprocal ratios in terms of sine and cosine.
4. **Operations:** Use common denominators, factoring, or the Pythagorean identity to simplify expressions.

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4. Solving Oblique (Non-Right) Triangles

A. The Sine Law

In any triangle $\triangle ABC$, the ratios of each side to the sine of its opposite angle are equal.

$$\frac{a}{\sin\ A} = \frac{b}{\sin\ B} = \frac{c}{\sin\ C}$$

- **When to Use:**
  - Two angles and any side (**AAS** or **ASA**).
  - Two sides and one angle opposite a given side (**SSA**).
- **The Ambiguous Case (SSA):** When given two sides and an angle opposite one of them (SSA), 0, 1, or 2 triangles may be possible depending on the side lengths and the angle size (acute vs. obtuse).

B. The Cosine Law

- **To find a side (e.g., a):**

$$a^2 = b^2 + c^2 - 2bc\cos\ A$$

- **To find an angle (e.g., $\angle A$):**

$$\cos\ A = \frac{b^2 + c^2 - a^2}{2bc}$$

- **When to Use:**
  - Two sides and the contained angle (**SAS**).
  - All three sides (**SSS**).

Quick Selection Guide

Given Information	Law to Use
SSA (Side-Side-Angle)	**Sine Law** (Check for Ambiguous Case)
ASA or **AAS** (Two Angles, One Side)	**Sine Law**
SAS (Side-Angle-Side)	**Cosine Law**
SSS (Side-Side-Side)	**Cosine Law**