**Transformation Mapping**

# Comprehensive Guide to Transformation Mapping

**Applicable to Ontario Curriculum (MCR3U / MHF4U)**

## 1. The General Theory

Any parent function $f(x)$ can be transformed using the general equation:

$$y = af[k(x - d)] + c$$

### The Pointwise Mapping Formula

To find the new coordinates of any point from the parent function, apply the following rule:

$$(x, y) \to \left(\frac{x}{k} + d,\ ay + c\right)$$

### Parameter Breakdown

| Parameter | Transformation Type | Mathematical Action |
|---|---|---|
| $a$ | Vertical Stretch / Compression / Reflection | Multiply $y$ by $a$ |
| $k$ | Horizontal Stretch / Compression / Reflection | Divide $x$ by $k$ |
| $d$ | Horizontal Translation (Phase Shift) | Add $d$ to $x$ |
| $c$ | Vertical Translation | Add $c$ to $y$ |

# 2. Quadratic Functions

**Parent Function:** $f(x) = x^2$

**Key Points:** $(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)$

**Example:** $y = -2(x + 4)^2 - 3$

- **Identify:** $a = -2, k = 1, d = -4, c = -3$
- **Mapping Rule:** $(x, y) \to (x - 4, -2y - 3)$

| Parent Point | Calculation | Transformed Point |
|---|---|---|
| $(0, 0)$ | $(0 - 4, -2(0) - 3)$ | $(-4, -3)$ (Vertex) |
| $(1, 1)$ | $(1 - 4, -2(1) - 3)$ | $(-3, -5)$ |
| $(2, 4)$ | $(2 - 4, -2(4) - 3)$ | $(-2, -11)$ |

# 3. Exponential Functions

**Parent Function:** $f(x) = b^x$ (e.g., $2^x$)

**Key Points:** $(-1, 0.5), (0, 1), (1, 2)$

**Horizontal Asymptote (HA):** $y = 0$

**Example:** $y = 3(2)^{2x} + 1$

- **Identify:** $a = 3, k = 2, d = 0, c = 1$
- **Mapping Rule:** $(x, y) \to (\frac{x}{2}, 3y + 1)$
- **New Asymptote:** $y = c \to \mathbf{y = 1}$

| Parent Point | Calculation | Transformed Point |
|---|---|---|
| $(0, 1)$ | $(0/2, 3(1) + 1)$ | $(0, 4)$ ($y$-intercept) |
| $(1, 2)$ | $(1/2, 3(2) + 1)$ | $(0.5, 7)$ |

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# 4. Trigonometric Functions

**Parent Function:** $f(x) = \cos(x)$

**Key Points:** $(0°, 1), (90°, 0), (180°, -1), (270°, 0), (360°, 1)$

### Example:

- **Identify:** $y = 0.5\cos[3(x - 60°)] + 2$
- **Mapping Rule:** $(x, y) \to (\frac{x}{3} + 60°, 0.5y + 2)$

| Parent Point | Calculation | Transformed Point |
|---|---|---|
| $(0°, 1)$ | $(0/3 + 60, 0.5(1) + 2)$ | $(60°, 2.5)$ (Max) |
| $(90°, 0)$ | $(90/3 + 60, 0.5(0) + 2)$ | $(90°, 2)$ (Midline) |
| $(180°, -1)$ | $(180/3 + 60, 0.5(-1) + 2)$ | $(120°, 1.5)$ (Min) |

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# 5. Critical Pro-Tips

1. **Always Factor :** If the equation looks like , you **must** rewrite it as before identifying .
2. **Order of Operations:** The mapping formula automatically handles the correct order (Stretches/Reflections before Translations).
3. **The -Logic:** Remember that and always perform the **inverse** operation of what is written in the bracket. If it says , you add . If it says , you divide by .