

# Chapter 6 Unit Test – Solutions

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## Question 1

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**State  $y = \cos x$  in terms of sine.**

**Answer:**

$$y = \sin(x + 90^\circ)$$

*Reasoning:* The cosine graph is the same as the sine graph shifted to the left by  $90^\circ$ .

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## Question 2

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**For the trig function  $f(x) = -2 \cos(3x - 120) - 1$ , determine:**

First, factor the inner expression to identify the transformations:

$$f(x) = -2 \cos[3(x - 40)] - 1$$

**(a) The phase shift**

Right  $40^\circ$ .

**(b) The amplitude**

$$a = |-2| = 2.$$

**© The period**

$$\text{Period} = \frac{360^\circ}{k} = \frac{360^\circ}{3} = 120^\circ.$$

**(d) The axis**

$$y = -1.$$

**(e) The max value**

Max = Axis + Amplitude =  $-1 + 2 = 1$ .

**(f) The min value**

Min = Axis - Amplitude =  $-1 - 2 = -3$ .

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## Question 3

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**Sketch one period of the following functions.**

(a)  $f(x) = -2 \sin(x) + 2$

*Analysis:*

- Parent:  $\sin x$
- Reflection in x-axis (starts at axis, goes down).
- Amplitude: 2.
- Axis:  $y = 2$  (Vertical shift up 2).
- Period:  $360^\circ$ .

*Key Points (every  $90^\circ$ ):*

- $x = 0^\circ$ : Axis  $\rightarrow y = 2$ .
- $x = 90^\circ$ : Minimum  $\rightarrow y = 2 - 2 = 0$ .
- $x = 180^\circ$ : Axis  $\rightarrow y = 2$ .
- $x = 270^\circ$ : Maximum  $\rightarrow y = 2 + 2 = 4$ .
- $x = 360^\circ$ : Axis  $\rightarrow y = 2$ .

(b)  $g(x) = \cos(2(x + 45))$

Analysis:

- Parent:  $\cos x$
- Period:  $360^\circ / 2 = 180^\circ$ .
- Quarter-points: Every  $45^\circ$ .
- Phase shift: Left  $45^\circ$ .
- Amplitude: 1.
- Axis:  $y = 0$ .

Key Points (Starting at shift  $x = -45^\circ$ ):

- $x = -45^\circ$ : Max  $\rightarrow y = 1$ .
- $x = 0^\circ$ : Axis  $\rightarrow y = 0$ .
- $x = 45^\circ$ : Min  $\rightarrow y = -1$ .
- $x = 90^\circ$ : Axis  $\rightarrow y = 0$ .
- $x = 135^\circ$ : Max  $\rightarrow y = 1$ .

(Note: The sketch should label these coordinates.)

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## Question 4

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**Write the following function in terms of sine:**

$$f(x) = -2 \cos(3x - 120) - 1$$

Steps:

1. Factor the argument:  $f(x) = -2 \cos[3(x - 40)] - 1$ .
2. Use the identity  $\cos \theta = \sin(\theta + 90^\circ)$ .
3. Substitute:  $-2 \sin([3(x - 40)] + 90) - 1$ .
4. Simplify angle:  $3x - 120 + 90 = 3x - 30$ .
5. Factor again:  $3(x - 10)$ .

**Answer:**

$$f(x) = -2 \sin(3(x - 10)) - 1$$

(Alternate Answer using positive sine:  $f(x) = 2 \sin(3(x - 70)) - 1$ )

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## Question 5

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**Analyzed from Graph (Question 5 Image):**

- **Axis (Vertical Shift):** The wave oscillates between  $y = 1$  (Max) and  $y = -3$  (Min).

$$\text{Axis } c = \frac{1+(-3)}{2} = -1.$$

- **Amplitude:**

$$a = \frac{1-(-3)}{2} = 2.$$

- **Period:**

The graph crosses the axis at  $x = 0$  and returns to the axis (completing half a cycle) at  $x = 2$ .

Full Period = 4.

$$k = \frac{360}{4} = 90 \text{ (assuming degrees scaling for consistency).}$$

**(a) State the:**

- (i) **Period:** 4
- (ii) **Amplitude:** 2
- (iii) **Equation of the axis:**  $y = -1$

**(b) State a function in terms of cosine representing this graph:**

The minimum is at  $x = 1$ . A negative cosine function starts at a minimum.

Using no phase shift for negative cosine relative to the minimum at  $x = 1$ :

$$y = -a \cos(k(x - d)) + c$$

$$y = -2 \cos(90(x - 1)) - 1$$

**Answer:**

$$f(x) = -2 \cos(90(x - 1)) - 1$$

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## Question 6

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**The diameter of a car's tire is 60 cm. While the car is being driven, the tire picks up a nail.**

**(a) Graph and Trig Function**

*Setup:*

- **Diameter:** 60 cm → **Radius:** 30 cm.
- **Axis:** The center of the wheel is 30 cm off the ground.  $y = 30$ .
- **Amplitude:** The nail moves from 0 to 60 cm.  $a = 30$ .
- **Start:** Nail is picked up from the ground ( $h = 0$ ). This corresponds to a minimum.
- **Function Type:** Negative cosine (starts at min).

- **Period (Distance):** One rotation = Circumference =  $\pi D = 60\pi \approx 188.5$  cm.

- **k value:**  $k = \frac{360}{60\pi} = \frac{6}{\pi}$  (degrees per cm).

### Function:

$$h(d) = -30 \cos\left(\frac{6}{\pi}d\right) + 30$$

### (b) Height after 1.2 km

*Calculation:*

- $d = 1.2$  km = 1200 m = 120,000 cm.

- Substitute  $d$  into equation:

$$h = -30 \cos\left(\frac{6}{\pi} \times 120,000\right) + 30$$

- Angle  $\theta = \frac{720,000}{\pi}$  degrees.

- Number of full rotations =  $\frac{120,000}{60\pi} \approx 636.61977$ .

- Remainder fraction = 0.61977.

- Angle in current cycle =  $0.61977 \times 360^\circ \approx 223.1^\circ$ .

- $\cos(223.1^\circ) \approx -0.73$ .

- $h \approx -30(-0.73) + 30 = 21.9 + 30 = 51.9$ .

### Answer:

Approximately **51.9 cm** above the ground.

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## Question 7

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**Valve stem problem (Diagram 7).**

- **Wheel:** Outer height 12 cm → Radius 6 cm.
- **Valve Stem:** Indicated at 6 cm height (same as center).
- **Position:** Diagram shows it on the **left** (9 o'clock).
- **Motion:** Rolling right → clockwise rotation.
- **Movement:** From 9 o'clock, clockwise motion moves the valve **UP**.
- **Inner Radius:** Gap at bottom is 3 cm. Valve radius from center =  $6 - 3 = 3$  cm.
- **Amplitude:** 3.
- **Axis:** Center height = 6.

### (a) Function of distance (with graph)

- **Start:** At axis, going up → Positive Sine.
- **Period:** Circumference of tire =  $12\pi$  cm.
- **k:**  $k = \frac{360}{12\pi} = \frac{30}{\pi}$ .

#### Function:

$$h(d) = 3 \sin\left(\frac{30}{\pi}d\right) + 6$$

### (b) Height after rolling 60 cm

- $d = 60$ .
- $\theta = \frac{30}{\pi} \times 60 = \frac{1800}{\pi}$  degrees  $\approx 573^\circ$ .
- $573^\circ - 360^\circ = 213^\circ$  (Quadrant 3).
- $h = 3 \sin(213^\circ) + 6$ .
- $h \approx 3(-0.545) + 6 = -1.63 + 6 = 4.37$ .

#### Answer:

Approximately **4.37 cm.**

### © Function of time

- **Speed:**  $v = 24\pi \text{ cm/s.}$
- Substitute  $d = v \cdot t = 24\pi t.$
- $k_{time} = \frac{30}{\pi} \times 24\pi = 720.$

**Function:**

$$h(t) = 3 \sin(720t) + 6$$