

Chapter 6 Unit Test – Solutions

Question 1

State $y = \cos x$ in terms of sine.

Answer:

$$y = \sin(x + 90^\circ)$$

Reasoning: The cosine graph is the same as the sine graph shifted to the left by 90° .

Question 2

For the trig function $f(x) = -2 \cos(3x - 120) - 1$, determine:

First, factor the inner expression to identify the transformations:

$$f(x) = -2 \cos[3(x - 40)] - 1$$

(a) The phase shift

Right 40° .

(b) The amplitude

$$a = |-2| = 2.$$

(c) The period

$$\text{Period} = \frac{360^\circ}{k} = \frac{360^\circ}{3} = 120^\circ.$$

(d) The axis

$$y = -1.$$

(e) The max value

$$\text{Max} = \text{Axis} + \text{Amplitude} = -1 + 2 = 1.$$

(f) The min value

$$\text{Min} = \text{Axis} - \text{Amplitude} = -1 - 2 = -3.$$

Question 3

Sketch one period of the following functions.

(a) $f(x) = -2 \sin(x) + 2$

Analysis:

- Parent: $\sin x$
- Reflection in x-axis (starts at axis, goes down).
- Amplitude: 2.
- Axis: $y = 2$ (Vertical shift up 2).
- Period: 360° .

Key Points (every 90°):

- $x = 0^\circ$: Axis $\rightarrow y = 2$.
- $x = 90^\circ$: Minimum $\rightarrow y = 2 - 2 = 0$.
- $x = 180^\circ$: Axis $\rightarrow y = 2$.
- $x = 270^\circ$: Maximum $\rightarrow y = 2 + 2 = 4$.
- $x = 360^\circ$: Axis $\rightarrow y = 2$.

(b) $g(x) = \cos(2(x + 45))$

Analysis:

- Parent: $\cos x$
- Period: $360^\circ/2 = 180^\circ$.
- Quarter-points: Every 45° .
- Phase shift: Left 45° .
- Amplitude: 1.
- Axis: $y = 0$.

Key Points (Starting at shift $x = -45^\circ$):

- $x = -45^\circ$: Max $\rightarrow y = 1$.
- $x = 0^\circ$: Axis $\rightarrow y = 0$.
- $x = 45^\circ$: Min $\rightarrow y = -1$.
- $x = 90^\circ$: Axis $\rightarrow y = 0$.
- $x = 135^\circ$: Max $\rightarrow y = 1$.

(Note: The sketch should label these coordinates.)

Question 4

Write the following function in terms of sine:

$$f(x) = -2 \cos(3x - 120) - 1$$

Steps:

1. Factor the argument: $f(x) = -2 \cos[3(x - 40)] - 1$.

2. Use the identity $\cos \theta = \sin(\theta + 90^\circ)$.

3. Substitute: $-2 \sin([3(x - 40)] + 90) - 1$.

4. Simplify angle: $3x - 120 + 90 = 3x - 30$.

5. Factor again: $3(x - 10)$.

Answer:

$$f(x) = -2 \sin(3(x - 10)) - 1$$

(Alternate Answer using positive sine: $f(x) = 2 \sin(3(x - 70)) - 1$)

Question 5

Analyzed from Graph (Question 5 Image):

- **Axis (Vertical Shift):** The wave oscillates between $y = 1$ (Max) and $y = -3$ (Min).

$$\text{Axis } c = \frac{1+(-3)}{2} = -1.$$

- **Amplitude:**

$$a = \frac{1-(-3)}{2} = 2.$$

- **Period:**

The graph crosses the axis at $x = 0$ and returns to the axis (completing half a cycle) at $x = 2$.

Full Period = 4.

$$k = \frac{360}{4} = 90 \text{ (assuming degrees scaling for consistency).}$$

(a) State the:

- (i) **Period:** 4
- (ii) **Amplitude:** 2
- (iii) **Equation of the axis:** $y = -1$

(b) State a function in terms of cosine representing this graph:

The minimum is at $x = 1$. A negative cosine function starts at a minimum.

Using no phase shift for negative cosine relative to the minimum at $x = 1$:

$$y = -a \cos(k(x - d)) + c$$

$$y = -2 \cos(90(x - 1)) - 1$$

Answer:

$$f(x) = -2 \cos(90(x - 1)) - 1$$

Question 6

The diameter of a car's tire is 60 cm. While the car is being driven, the tire picks up a nail.

(a) Graph and Trig Function

Setup:

- **Diameter:** 60 cm \rightarrow **Radius:** 30 cm.
- **Axis:** The center of the wheel is 30 cm off the ground. $y = 30$.
- **Amplitude:** The nail moves from 0 to 60 cm. $a = 30$.
- **Start:** Nail is picked up from the ground ($h = 0$). This corresponds to a minimum.
- **Function Type:** Negative cosine (starts at min).

- **Period (Distance):** One rotation = Circumference = $\pi D = 60\pi \approx 188.5$ cm.
- **k value:** $k = \frac{360}{60\pi} = \frac{6}{\pi}$ (degrees per cm).

Function:

$$h(d) = -30 \cos\left(\frac{6}{\pi}d\right) + 30$$

(b) Height after 1.2 km

Calculation:

- $d = 1.2$ km = 1200 m = 120,000 cm.
- Substitute d into equation:

$$h = -30 \cos\left(\frac{6}{\pi} \times 120,000\right) + 30$$

- Angle $\theta = \frac{720,000}{\pi}$ degrees.
- Number of full rotations = $\frac{120,000}{60\pi} \approx 636.61977$.
- Remainder fraction = 0.61977.
- Angle in current cycle = $0.61977 \times 360^\circ \approx 223.1^\circ$.
- $\cos(223.1^\circ) \approx -0.73$.
- $h \approx -30(-0.73) + 30 = 21.9 + 30 = 51.9$.

Answer:

Approximately **51.9 cm** above the ground.

Question 7

Valve stem problem (Diagram 7).

- **Wheel:** Outer height 12 cm \rightarrow Radius 6 cm.
- **Valve Stem:** Indicated at 6 cm height (same as center).
- **Position:** Diagram shows it on the **left** (9 o'clock).
- **Motion:** Rolling right \rightarrow clockwise rotation.
- **Movement:** From 9 o'clock, clockwise motion moves the valve **UP**.
- **Inner Radius:** Gap at bottom is 3 cm. Valve radius from center = $6 - 3 = 3$ cm.
- **Amplitude:** 3.
- **Axis:** Center height = 6.

(a) Function of distance (with graph)

- **Start:** At axis, going up \rightarrow Positive Sine.
- **Period:** Circumference of tire = 12π cm.
- **k:** $k = \frac{360}{12\pi} = \frac{30}{\pi}$.

Function:

$$h(d) = 3 \sin\left(\frac{30}{\pi}d\right) + 6$$

(b) Height after rolling 60 cm

- $d = 60$.
- $\theta = \frac{30}{\pi} \times 60 = \frac{1800}{\pi}$ degrees $\approx 573^\circ$.
- $573^\circ - 360^\circ = 213^\circ$ (Quadrant 3).
- $h = 3 \sin(213^\circ) + 6$.
- $h \approx 3(-0.545) + 6 = -1.63 + 6 = 4.37$.

Answer:

Approximately **4.37 cm**.

© Function of time

- **Speed:** $v = 24\pi$ cm/s.
- Substitute $d = v \cdot t = 24\pi t$.
- $k_{time} = \frac{30}{\pi} \times 24\pi = 720$.

Function:

$$h(t) = 3 \sin(720t) + 6$$