

## 1. Fundamental Concepts of Periodic Functions

A **periodic function** is a function with a graph that repeats a regular pattern over a constant interval.

- **Cycle:** One complete repeating portion of the graph.
- **Period:** The change in the independent variable (usually  $x$  or  $t$ ) that corresponds to one cycle.
- **Peak and Trough:** The **peak** is the maximum point on the graph; the **trough** is the minimum point.
- **Equation of the Axis:** The horizontal line halfway between the maximum and minimum values.
- **Amplitude:** The vertical distance from the function's axis to the maximum or minimum value; it is always a positive value.

## 2. Properties of Parent Sinusoidal Functions

Sinusoidal functions are periodic functions that form smooth, symmetrical waves. The two primary parent functions are  $f(x) = \sin x$  and  $f(x) = \cos x$ .

Characteristic	$f(x) = \sin x$	$f(x) = \cos x$
Period	$360^\circ$	$360^\circ$
Amplitude	1	1
Equation of Axis	$y = 0$	$y = 0$
Range	$\{y \in \mathbb{R} \mid -1 \leq y \leq 1\}$	$\{y \in \mathbb{R} \mid -1 \leq y \leq 1\}$
Starting Point ( $x = 0$ )	At the axis $(0, 0)$	At the peak $(0, 1)$

**Key Points for Sketching ( $0^\circ$  to  $360^\circ$ ):**

- **Sine:**  $(0, 0)$ ,  $(90, 1)$ ,  $(180, 0)$ ,  $(270, -1)$ ,  $(360, 0)$ .
- **Cosine:**  $(0, 1)$ ,  $(90, 0)$ ,  $(180, -1)$ ,  $(270, 0)$ ,  $(360, 1)$ .

## 3. Transformations of Sinusoidal Functions

Transformed functions take the form:  $y = a \sin[k(x - d)] + c$  or  $y = a \cos[k(x - d)] + c$ .

- **Amplitude ( $a$ ):** Determined by the vertical stretch/compression. **Amplitude** =  $|a|$ . If  $a < 0$ , the graph is reflected in the x-axis.
- **Period ( $k$ ):** Determined by the horizontal stretch/compression. **Period** =  $\frac{360^\circ}{|k|}$ .
- **Phase Shift ( $d$ ):** The horizontal translation. The graph moves right if  $d > 0$  and left if  $d < 0$ .
- **Vertical Translation ( $c$ ):** Shifts the graph up or down. The **Equation of the Axis** becomes  $y = c$ .

#### 4. Essential Equations and Formulas

- **Equation of the Axis:**  $y = \frac{\text{maximum value} + \text{minimum value}}{2}$ .
- **Amplitude Calculation:**  $a = \frac{\text{maximum value} - \text{minimum value}}{2}$ .
- **Finding  $k$ :**  $k = \frac{360^\circ}{\text{Period}}$ .
- **Circle Coordinates:** Any point  $P(x, y)$  on a circle with radius  $r$  rotated through angle  $\theta$  is  $(r \cos \theta, r \sin \theta)$ .
- **Speed in Circular Motion:** Speed is calculated by dividing the circumference ( $2\pi r$ ) by the period.

#### 5. Modeling Real-World Situations

When modeling phenomena like Ferris wheels or tides, you must connect physical traits to graph features:

- **Radius of the wheel = Amplitude.**
- **Height of the axle = Equation of the Axis** ( $c$  value).
- **Time for one revolution = Period.**
- **Starting Position:** Determines the **Phase Shift ( $d$ )**. For example, if a point starts at its maximum height at  $t = 0$ , use a cosine model with  $d = 0$ .

#### 6. Critical Test Skills

- **Order of Transformations:** Apply stretches, compressions, and reflections (multiplication) before translations (addition/subtraction).
- **Factoring for Phase Shift:** Always ensure the  $k$  value is factored out (e.g.,  $y = \sin(2x + 60^\circ)$  must be rewritten as  $y = \sin[2(x + 30^\circ)]$  to see the  $30^\circ$  left shift).
- **Function Selection:** Use a cosine model if the cycle starts at a maximum or minimum value; use a sine model if it starts at the axis.