🔑 Quiz Solutions and Explanations

1. Sketch both special triangles

- **The** $30^\circ - 60^\circ - 90^\circ$ **Triangle:** Side lengths are in the ratio $1 : \sqrt{3} : 2$. (Opposite 30° is 1, opposite 60° is $\sqrt{3}$, hypotenuse is 2.)
- **The** $45^\circ - 45^\circ - 90^\circ$ **Triangle:** Side lengths are in the ratio $1 : 1 : \sqrt{2}$. (The two legs are 1, hypotenuse is $\sqrt{2}$.)

2. Evaluate using exact values

We use the special triangles and the **CAST rule** to determine the sign.

- (a) $\cos 30^\circ$ (Q1)

$$\cos 30^\circ = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \mathbf{\frac{\sqrt{3}}{2}}$$

- (b) $\sec 45^\circ$ (Q1)

$$\sec 45^\circ = \frac{1}{\cos 45^\circ} = \frac{1}{1/\sqrt{2}} = \mathbf{\sqrt{2}}$$

- **©** $\sin 135^\circ$

1. **Related Acute Angle (α):** $180^\circ - 135^\circ = 45^\circ$.
2. **Quadrant:** 135° is in **Quadrant II**, where $\sin$ is positive (S in CAST).
3. **Value:** $\sin 135^\circ = +\sin 45^\circ = \mathbf{\frac{1}{\sqrt{2}}}$ or $\mathbf{\frac{\sqrt{2}}{2}}$

- **(d)** $\cot 210^\circ$

1. **Related Acute Angle (α):** $210^\circ - 180^\circ = 30^\circ$.
2. **Quadrant:** 210° is in **Quadrant III**, where $\tan$ and $\cot$ are positive (T in CAST).
3. **Value:** $\cot 210^\circ = +\cot 30^\circ = \frac{\cos 30^\circ}{\sin 30^\circ} = \frac{\sqrt{3}/2}{1/2} = \mathbf{\sqrt{3}}$

3. Find all possible angles for $0^\circ \leq x < 360^\circ$

- **(a)** $\sin x = -\frac{1}{\sqrt{2}}$

1. **Related Acute Angle (α):** We recognize $\sin \alpha = \frac{1}{\sqrt{2}}$ from the 45° special triangle. $\alpha = 45^\circ$.

2. **Quadrants:** $\sin x$ is negative in **QIII** and **QIV** (T and C in CAST).

3. **Angles:**

- QIII: $x = 180^\circ + \alpha = 180^\circ + 45^\circ = \mathbf{225}^\circ$

- QIV: $x = 360^\circ - \alpha = 360^\circ - 45^\circ = \mathbf{315}^\circ$

- **(b)** $\cot x = 1$

1. **Related Acute Angle (α):** Since $\cot x = 1$, then $\tan x = 1/1 = 1$. We know $\tan 45^\circ = 1$. $\alpha = 45^\circ$.

2. **Quadrants:** $\cot x$ is positive in **QI** and **QIII** (A and T in CAST).

3. **Angles:**

- QI: $x = \alpha = \mathbf{45}^\circ$

- QIII: $x = 180^\circ + \alpha = 180^\circ + 45^\circ = \mathbf{225}^\circ$

4. Determine all possible values of $\cos A$ for $\sin A = \frac{1}{3}$

We are given $y = 1$ and $r = 3$ (since $\sin A = y/r$). We need to find x.

$$x^2 + y^2 = r^2$$

$$x^2 + 1^2 = 3^2$$

$$x^2 + 1 = 9$$

$$x^2 = 8$$

$$x = \pm\sqrt{8} = \pm 2\sqrt{2}$$

The angle A can be in **QI** (where x is positive) or **QII** (where x is negative), because $\sin A$ is positive.

- **If $x = 2\sqrt{2}$ (QI):** $\cos A = \frac{x}{r} = \mathbf{\frac{2\sqrt{2}}{3}}$
- **If $x = -2\sqrt{2}$ (QII):** $\cos A = \frac{x}{r} = \mathbf{-\frac{2\sqrt{2}}{3}}$

5. Given $(4, -5)$, evaluate θ to the nearest degree.

The point $(4, -5)$ means $x = 4$ and $y = -5$.

1. **Quadrant:** x is positive and y is negative, so the angle is in **Quadrant IV** (C in CAST).

2. Related Acute Angle (α): We use $\tan$ to find α:

$$\tan\alpha = \left|\frac{y}{x}\right| = \left|\frac{-5}{4}\right| = 1.25$$

$$\alpha = \tan^{-1}(1.25) \approx 51.34^\circ$$

3. Principal Angle (θ): In QIV, $\theta = 360^\circ - \alpha$.

$$\theta = 360^\circ - 51.34^\circ \approx 308.66^\circ$$

Rounding to the nearest degree: $\theta \approx \mathbf{309}^\circ$

6. Find all possible angles for $0^\circ \leq x < 360^\circ$ (Calculator Use)

- **(a)** $\cos x = 0.4183$

1. Related Acute Angle (α):

$$\alpha = \cos^{-1}(0.4183) \approx 65.3^\circ$$

2. **Quadrants:** $\cos x$ is positive in **QI** and **QIV** (A and C in CAST).

3. **Angles (1 d.p.):**

- QI: $x_1 = \alpha = \mathbf{65.3}^\circ$
- QIV: $x_2 = 360^\circ - \alpha = 360^\circ - 65.3^\circ = \mathbf{294.7}^\circ$
- **(b)** $\csc x = -2.3151$

1. **Convert to** $\sin$**:** $\sin x = \frac{1}{\csc x} = \frac{1}{-2.3151} \approx -0.43195$
2. Related Acute Angle (α): Use the positive value:

$$\alpha = \sin^{-1}(0.43195) \approx 25.6^\circ$$

3. **Quadrants:** $\sin x$ is negative in **QIII** and **QIV** (T and C in CAST).
4. **Angles (1 d.p.):**

- QIII: $x_1 = 180^\circ + \alpha = 180^\circ + 25.6^\circ = \mathbf{205.6}^\circ$
- QIV: $x_2 = 360^\circ - \alpha = 360^\circ - 25.6^\circ = \mathbf{334.4}^\circ$