That's fantastic! I'm happy to help you with these trigonometry problems. This quiz covers some core concepts: placing angles in standard position, using the Pythagorean theorem to find r, calculating primary trig ratios, and working with exact values using special triangles.

Let's break down how to approach each question step-by-step.

📝 Trigonometry Quiz Solutions and Explanations

Question 1: Point in the Cartesian Plane ($P(-3, -6)$)

(a) Sketching the Diagram and Labeling Angles

1. **Plot the point $P(-3, -6)$:** Start at the origin, move 3 units left (negative x) and 6 units down (negative y).
2. **Draw the Terminal Arm:** Draw a line segment from the origin $(0, 0)$ to the point P.
3. **Identify the Quadrant:** Since both x and y are negative, the point is in **Quadrant III**.
4. **Label the Principal Angle (θ):** θ starts at the positive x-axis (standard position) and rotates counter-clockwise to the terminal arm.
5. **Label the Related Acute Angle (α):** α is the acute angle formed between the terminal arm and the nearest x-axis. It is always positive.

(b) Determining r and Primary Trig Ratios

The coordinates of the point P are $x = -3$ and $y = -6$.

The radius (r) is the distance from the origin to P. We use the Pythagorean theorem: $x^2 + y^2 = r^2$.

$$r^2 = (-3)^2 + (-6)^2$$

$$r^2 = 9 + 36$$

$$r^2 = 45$$

$$r = \sqrt{45}$$

Reduce the radical:

$$r = \sqrt{9 \times 5}$$

$$r = 3\sqrt{5}$$

Now, we use the definitions of the primary trigonometric ratios:

- $\sin(\theta) = \frac{y}{r}$
- $\cos(\theta) = \frac{x}{r}$
- $\tan(\theta) = \frac{y}{x}$

1. $\sin(\theta)$:

$$\sin(\theta) = \frac{-6}{3\sqrt{5}} = \frac{-2}{\sqrt{5}}$$

   Rationalize the denominator:

$$\sin(\theta) = \frac{-2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = -\mathbf{\frac{2\sqrt{5}}{5}}$$

2. $\cos(\theta)$:

$$\cos(\theta) = \frac{-3}{3\sqrt{5}} = \frac{-1}{\sqrt{5}}$$

   Rationalize the denominator:

$$\cos(\theta) = \frac{-1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = -\mathbf{\frac{\sqrt{5}}{5}}$$

3. $\tan(\theta)$:

$$\tan(\theta) = \frac{-6}{-3} = \mathbf{2}$$

© Determining α and θ to 1 Decimal Place

The related acute angle, α, is found using the absolute values of the coordinates (or the tangent ratio found in part (b) before considering the sign).

$$\tan(\alpha) = \left|\frac{y}{x}\right| = \left|\frac{-6}{-3}\right| = 2$$

$$\alpha = \tan^{-1}(2)$$

$$\alpha \approx \mathbf{63.4}^{\circ}$$

The principal angle, θ, is in Quadrant III. In QIII, the formula to find θ is: $\theta = 180^{\circ} + \alpha$.

$$\theta = 180^{\circ} + 63.4^{\circ}$$

$$\theta \approx \mathbf{243.4}^{\circ}$$

Question 2: Exact Value of $\sec(210^{\circ})$

1. Determine the Quadrant and Related Acute Angle (α)

- 210° is between 180° and 270°, so it is in **Quadrant III**.
- The related acute angle is: $\alpha = 210^{\circ} - 180^{\circ} = \mathbf{30}^{\circ}$.

2. Draw the Diagram with the Special Triangle

Draw the $30^{\circ} - 60^{\circ} - 90^{\circ}$ reference triangle in Quadrant III.

The side lengths of a $30^{\circ} - 60^{\circ} - 90^{\circ}$ triangle are in the ratio $1 : \sqrt{3} : 2$.

- Since 210° is in QIII, both x and y coordinates are negative.
    - x (adjacent to 30°) $= -\sqrt{3}$
    - y (opposite to 30°) $= -1$
    - r (hypotenuse) $= 2$

3. Determine $\sec(210^{\circ})$

Secant is the reciprocal of cosine: $\sec(\theta) = \frac{r}{x}$.

$$\sec(210^\circ) = \frac{r}{x} = \frac{2}{-\sqrt{3}}$$

Rationalize the denominator:

$$\sec(210^\circ) = \frac{2}{-\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = -\mathbf{\frac{2\sqrt{3}}{3}}$$

Question 3: Finding Angles from $\cos(A) = -\frac{2}{3}$

We are given $\cos(A) = -\frac{2}{3}$, where $0^\circ \leq A \leq 360^\circ$. Since $\cos(\theta) = \frac{x}{r}$, we know $x = -2$ and $r = 3$.

(i) Sketch the Possible Positions

Since $\cos(A)$ is negative, the terminal arm must lie in **Quadrant II** and **Quadrant III** (where x is negative).

(ii) Determine the Exact Values of $\tan(A)$

First, we must find the exact value of y using $x^2 + y^2 = r^2$:

$$(-2)^2 + y^2 = 3^2$$

$$4 + y^2 = 9$$

$$y^2 = 5$$

$$y = \pm\sqrt{5}$$

Now, we find $\tan(A) = \frac{y}{x}$:

- In Quadrant II (QII): y is positive.

$$\tan(A) = \frac{+\sqrt{5}}{-2} = -\mathbf{\frac{\sqrt{5}}{2}}$$

- In Quadrant III (QIII): y is negative.

$$\tan(A) = \frac{-\sqrt{5}}{-2} = \mathbf{\frac{\sqrt{5}}{2}}$$

(iii) Determine all values for A to 1 decimal place

1. Find the Related Acute Angle (α)

We use the absolute value of $\cos(A)$ to find α:

$$\cos(\alpha) = \left|-\frac{2}{3}\right| = \frac{2}{3}$$

$$\alpha = \cos^{-1}\left(\frac{2}{3}\right)$$

$$\alpha \approx \mathbf{48.2}^\circ$$

2. Find Angle A in QII and QIII

- QII Angle (A_1): $A_1 = 180^\circ - \alpha$

$$A_1 = 180^\circ - 48.2^\circ = \mathbf{131.8}^\circ$$

- QIII Angle (A_2): $A_2 = 180^\circ + \alpha$

$$A_2 = 180^\circ + 48.2^\circ = \mathbf{228.2}^\circ$$

Do you have any questions about these steps, or would you like to practice another trig problem?