Based on **Chapter 4 (Exponential Functions)** from your textbook, I have created a comprehensive review guide to help you prepare for your unit test.

This guide breaks down the chapter into its core concepts, formulas, and the specific skills you need to master.

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1. Exponent Laws (Sections 4.2 & 4.3)

Before tackling functions, you must master the mechanics of exponents.

The Basics:

- **Product Rule:** $x^a \cdot x^b = x^{a+b}$
- **Quotient Rule:** $\frac{x^a}{x^b} = x^{a-b}$
- **Power of a Power:** $(x^a)^b = x^{a \cdot b}$

The "Tricky" Ones (Need to Know):

- **Zero Exponent:** $x^0 = 1$ (provided $x \neq 0$).
- **Negative Exponent:** $x^{-n} = \frac{1}{x^n}$.
  - *Tip:* If you have a fraction, flip it: $(\frac{a}{b})^{-n} = (\frac{b}{a})^n$.
- **Rational (Fraction) Exponents:**
  - $x^{\frac{1}{n}} = \sqrt[n]{x}$
  - $x^{\frac{m}{n}} = (\sqrt[n]{x})^m$ or $\sqrt[n]{x^m}$
  - *Memory Aid:* Flower Power — The **roots** are at the bottom (denominator), the **power** is at the top (numerator).

Common Test Task:

- Simplify an algebraic expression containing radicals and negative exponents and express the final answer with **positive exponents**.

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2. Characteristics of Exponential Functions (Section 4.5)

You need to recognize what an exponential function looks like in an equation, a table, and a graph.

The Parent Function: $f(x) = b^x$

- **Base (b):** Must be positive ($b > 0$) and not equal to 1.
- **Domain:** $\{x \in \mathbb{R}\}$ (All real numbers).
- **Range:** $\{y \in \mathbb{R} \mid y > 0\}$ (y is always positive, never touches 0).
- **Asymptote:** There is a horizontal asymptote at $y = 0$ (the x-axis).
- **Y-Intercept:** Always $(0, 1)$ because $b^0 = 1$.

Growth vs. Decay:

- **Exponential Growth:** $b > 1$ (Graph goes up to the right).
- **Exponential Decay:** $0 < b < 1$ (Graph goes down to the right).

Recognizing Data (Table of Values):

- **Linear:** First differences are constant (add/subtract same amount).
- **Quadratic:** Second differences are constant.
- **Exponential:** The **ratio** of consecutive y-values is constant (you multiply by the same number to get the next term).

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3. Transformations (Section 4.6)

You must be able to graph and analyze the transformed equation:

$$g(x) = a(b)^{k(x-d)} + c$$

$$g(x) = 2(2)^{1/3(x+4)} + 1$$

What each letter does:

- c **(Vertical Translation):** Moves the graph Up/Down.
  - *Crucial*: This determines your **Horizontal Asymptote**. The new asymptote is $y = c$.
- d **(Horizontal Translation):** Moves the graph Left/Right.
  - Remember to flip the sign: $(x - 3)$ moves Right 3; $(x + 3)$ moves Left 3.

- a **(Vertical Stretch/Reflection):**
  - If $a < 0$: Reflection in the **x-axis**.
  - $|a| > 1$: Vertical Stretch.
  - $0 < |a| < 1$: Vertical Compression.
- k **(Horizontal Stretch/Reflection):**
  - If $k < 0$: Reflection in the **y-axis**.
  - Note: The factor is $\frac{1}{k}$. (e.g., if $k = 2$, it is a horizontal compression by $\frac{1}{2}$).

Test Task:

- Given an equation like $y = -2(3)^{x+1} + 4$, state the domain, range, asymptote, and sketch the graph.
  - *Asymptote:* $y = 4$
  - *Range:* $y < 4$ (because of the negative 'a' reflection).

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4. Real-World Applications (Section 4.7)

Word problems usually fall into two categories: Growth or Decay.

The General Formula:

$$A(t) = A_0(b)^{\frac{t}{P}}$$

- $A(t)$**:** Final amount.
- A_0**:** Initial amount (at $t = 0$).
- b**:** Growth/Decay factor.
- t**:** Total time elapsed.
- P**:** The period (how long it takes for the growth/decay to happen once).

Scenario 1: Doubling or Half-Life

- **Doubling:** $A(t) = A_0(2)^{\frac{t}{d}}$ (where d is doubling time).
- **Half-Life:** $A(t) = A_0(\frac{1}{2})^{\frac{t}{h}}$ (where h is half-life).

Scenario 2: Percentage Growth/Decay

- **Growth:** $b = 1 + r$ (e.g., grows by 5% $\rightarrow$ $b = 1.05$).

- **Decay:** $b = 1 - r$ (e.g., depreciates by 12% $\rightarrow b = 0.88$).
- **Formula:** $A(t) = A_0(1 \pm r)^n$

Test Task:

- "A bacteria culture starts with 500 cells and doubles every 3 hours. How many are there after 10 hours?"
  - $y = 500(2)^{\frac{10}{3}}$

Study Checklist: Can you...?

- [ ] Evaluate $27^{-2/3}$ without a calculator? (Answer: $1/9$)
- [ ] Simplify $\frac{(2x^3y^{-2})^2}{8x^{-1}y^3}$?
- [ ] Sketch $y = 3^x$ and $y = (\frac{1}{3})^x$ on the same grid?
- [ ] Identify the asymptote and range of $y = 5(2)^x - 3$? (Asymptote: $y = -3$, Range: $y > -3$)
- [ ] Solve for x if $2^x = 32$ and if $9^{x+1} = 27^{2x}$? (Common base method).
- [ ] Write an equation for a car purchased for $$20,000$ that loses 15% of its value every year? ($V = 20000(0.85)^t$).

Common Pitfall to Avoid:

- **Order of Operations:** In $y = -2(3)^x$, do **NOT** multiply -2×3 to get $(-6)^x$. The exponent applies *only* to the base (3). The -2 is a vertical stretch applied *after* the exponent.