

Here are the solutions for finding the two possible values of θ for each trigonometric ratio in the interval $0^\circ \leq \theta \leq 360^\circ$.

The process involves finding the **reference angle** (α) and then using the sign of the ratio to determine the correct two **quadrants** for the principal angles (θ).

Solutions and Principal Angles

a) $\cos \theta = 0.6951$

1. Reference Angle (α):

$$\alpha = \arccos(0.6951) \approx 46.0^\circ$$

2. **Quadrants:** Cosine is **positive**, so θ is in **Quadrant I** (A) and **Quadrant IV** ©.

3. **Angles** (θ):

- **QI:** $\theta_1 = \alpha \approx 46.0^\circ$

- **QIV:** $\theta_2 = 360^\circ - \alpha \approx 360^\circ - 46.0^\circ = 314.0^\circ$

4. **Sketch:**

b) $\tan \theta = -0.7571$

1. Reference Angle (α):

$$\alpha = \arctan(|-0.7571|) \approx 37.1^\circ$$

2. **Quadrants:** Tangent is **negative**, so θ is in **Quadrant II** (S) and **Quadrant IV** ©.

3. **Angles** (θ):

- **QII:** $\theta_1 = 180^\circ - \alpha \approx 180^\circ - 37.1^\circ = 142.9^\circ$

- **QIV:** $\theta_2 = 360^\circ - \alpha \approx 360^\circ - 37.1^\circ = 322.9^\circ$

4. Sketch:

c) $\sin \theta = 0.3154$

1. Reference Angle (α):

$$\alpha = \arcsin(0.3154) \approx 18.4^\circ$$

2. **Quadrants:** Sine is **positive**, so θ is in **Quadrant I** (A) and **Quadrant II** (S).

3. **Angles (θ):**

- **QI:** $\theta_1 = \alpha \approx 18.4^\circ$

- **QII:** $\theta_2 = 180^\circ - \alpha \approx 180^\circ - 18.4^\circ = 161.6^\circ$

4. Sketch:

d) $\cos \theta = -0.2882$

1. Reference Angle (α):

$$\alpha = \arccos(|-0.2882|) \approx 73.2^\circ$$

2. **Quadrants:** Cosine is **negative**, so θ is in **Quadrant II** (S) and **Quadrant III** (T).

3. **Angles (θ):**

- **QII:** $\theta_1 = 180^\circ - \alpha \approx 180^\circ - 73.2^\circ = 106.8^\circ$

- **QIII:** $\theta_2 = 180^\circ + \alpha \approx 180^\circ + 73.2^\circ = 253.2^\circ$

4. Sketch:

e) $\tan \theta = 2.3151$

1. Reference Angle (α):

$$\alpha = \arctan(2.3151) \approx 66.6^\circ$$

2. **Quadrants:** Tangent is **positive**, so θ is in **Quadrant I** (A) and **Quadrant III** (T).

3. **Angles (θ):**

- **QI:** $\theta_1 = \alpha \approx 66.6^\circ$
- **QIII:** $\theta_2 = 180^\circ + \alpha \approx 180^\circ + 66.6^\circ = 246.6^\circ$

4. **Sketch:**

f) $\sin \theta = -0.7503$

1. Reference Angle (α):

$$\alpha = \arcsin(|-0.7503|) \approx 48.6^\circ$$

2. **Quadrants:** Sine is **negative**, so θ is in **Quadrant III** (T) and **Quadrant IV** ©.

3. **Angles (θ):**

- **QIII:** $\theta_1 = 180^\circ + \alpha \approx 180^\circ + 48.6^\circ = 228.6^\circ$
- **QIV:** $\theta_2 = 360^\circ - \alpha \approx 360^\circ - 48.6^\circ = 311.4^\circ$

4. **Sketch:**

Do you have any questions about how to use the reference angle to find the principal angles in the different quadrants?