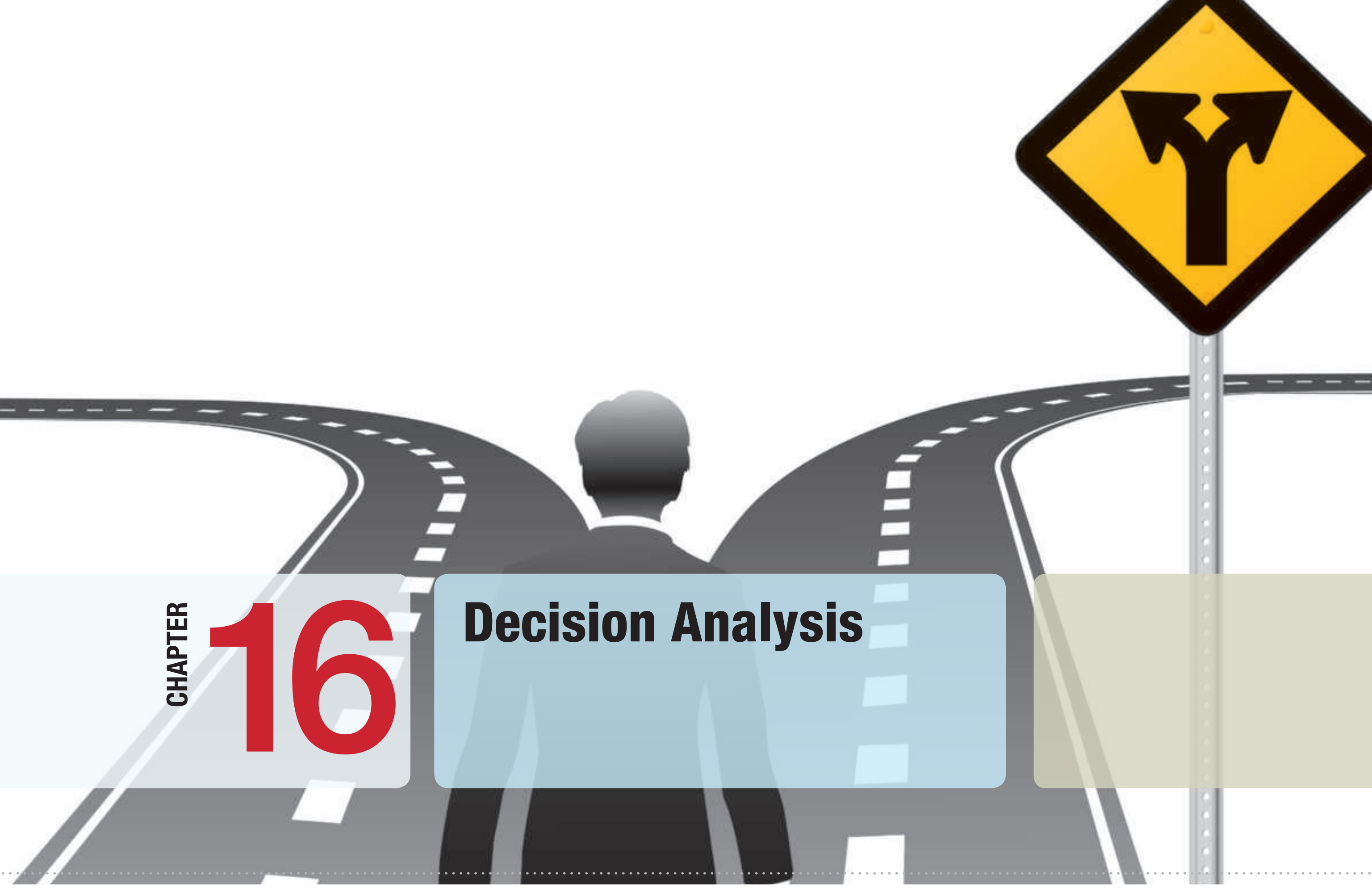

CHAPTER 16 Decision Analysis

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Learning Objectives

After studying this chapter, you will be able to:

- List the three elements needed to characterize decisions with uncertain consequences.
- Construct a payoff table for a decision situation.
- Apply average, aggressive, conservative, and opportunity-loss decision strategies for problems involving minimization and maximization objectives.
- Assess risk in choosing a decision.
- Apply expected values to a decision problem when probabilities of events are known.
- Use *Analytic Solver Platform* to construct decision trees.
- Incorporate Monte Carlo simulation in decision trees.
- Find the risk profile for a decision strategy.
- Compute the expected value of perfect information.
- Incorporate sample information in decision trees and apply Bayes's rule to compute conditional probabilities.
- Construct a utility function and use it to make a decision.
- State the properties of different types of utility functions.

Everybody makes decisions, both personal and professional. Managers are continually faced with decisions involving new products, supply chain configurations, new equipment, downsizing, and many others. The ability to make good decisions is the mark of a successful (and promotable) manager. In today's complex business world, intuition alone is not sufficient. This is where analytics plays an important role.

Throughout this book we have discussed how to analyze data and models using methods of business analytics. Predictive models such as Monte Carlo simulations can provide insight about the impacts of potential decisions, and prescriptive models such as linear optimization provide recommendations as to the best course of action to take. However, the real purpose of such information is to help managers *make decisions*. Their decisions often have significant economic or human resource consequences that cannot always be predicted accurately. For example, in Chapter 12 we analyzed the outsourcing decision model with uncertain demand. Although the results showed that on average, it is better to manufacture than to outsource, Figure 12.9 showed that there was only a 60% chance that this would be the best decision. So what decision should the company make? Similarly, in the Innis Investment example in Chapter 14, we performed a scenario analysis to evaluate the trade-offs between risk and reward (Figure 14.17). How should the client make a trade-off between risk and reward for their portfolio?

Analytic models and analyses provide decision makers with a wealth of information; however, people make the final decision. Good decisions don't simply implement the results of analytic models; they require an assessment of intangible factors and risk attitudes. **Decision making** is the study of how people make decisions, particularly when faced with imperfect or uncertain information, as well as a collection of techniques to support decision choices. Decision analysis differs from other modeling approaches by explicitly considering individual's preferences and attitudes toward risk, and modeling the decision process itself.

Decisions involving uncertainty and risk have been studied for many years. A large body of knowledge has been developed that helps to explain the philosophy associated with making decisions and also provide techniques for incorporating uncertainty and risk in making decisions.

Formulating Decision Problems

Many decisions involve a choice from among a small set of alternatives with uncertain consequences. We may formulate such decision problems by defining three things:

1. the **decision alternatives** that can be chosen,
2. the **uncertain events** that may occur after a decision is made along with their possible **outcomes**, and
3. the consequences associated with each decision and outcome, which are usually expressed as **payoffs**.

The outcomes associated with uncertain events (which are often called **states of nature**), are defined so that one and only one of them will occur. They may be quantitative or qualitative. For instance, in selecting the size of a new factory, the future demand for the product would be an uncertain event. The demand outcomes might be expressed quantitatively in sales units or dollars. On the other hand, suppose that you are planning a spring-break vacation to Florida in January; you might define an uncertain event as the weather; these outcomes might be characterized qualitatively: sunny and warm, sunny and cold, rainy and warm, rainy and cold, and so on. A payoff is a measure of the value of making a decision and having a particular outcome occur. This might be a simple estimate made judgmentally or a value computed from a complex spreadsheet model. Payoffs are often summarized in a **payoff table**, a matrix whose rows correspond to decisions and whose columns correspond to events. The decision maker first selects a decision alternative, after which one of the outcomes of the uncertain event occurs, resulting in the payoff.

Example 16.1 Selecting a Mortgage Instrument

Many young families face the decision of choosing a mortgage instrument. Suppose the Durr family is considering purchasing a new home and would like to finance $150,000. Three mortgage options are available, a 1-year adjusted-rate mortgage (ARM) at a low interest rate, a 3-year ARM at a slightly higher rate, and a 30-year fixed mortgage at the highest rate. However, both ARMs are sensitive to interest rate changes and the rates may change resulting in either higher or lower interest charges; thus, the potential future change in interest rates represents an uncertain event. Because the family anticipates staying in the home for at least 5 years, they want to know the total interest costs they might incur; these represent the payoffs associated with their choice and the future change in interest rates and can easily be calculated using a spreadsheet. The payoff table is as follows:

	Outcome		
Decision	**Rates Rise**	**Rates Stable**	**Rates Fall**
1-year ARM	$61,134	$46,443	$40,161
3-year ARM	$56,901	$51,075	$46,721
30-year fixed	$54,658	$54,658	$54,658

Clearly, no decision is best for each event that may occur. If rates rise, for example, then the 30-year fixed would be the best decision. If rates remain stable or fall, however, then the 1-year ARM is best. Of course, you cannot predict the future outcome with certainty, so the question is how to choose one of the options. Not everyone views risk in the same fashion. Most individuals will weigh their potential losses against potential gains. For example, if they choose the 1-year ARM mortgage instead of the fixed-rate mortgage, they risk losing money if rates rise; however, they would clearly save a lot if rates remain stable or fall. Would the potential savings be worth the risk? Such questions make decision making a difficult task.

Decision Strategies without Outcome Probabilities

We discuss several quantitative approaches that model different risk behaviors for making decisions involving uncertainty when no probabilities can be estimated for the outcomes.

Decision Strategies for a Minimize Objective

Aggressive (Optimistic) Strategy An aggressive decision maker might seek the option that holds the promise of minimizing the potential loss. This type of decision maker would first ask the question, What is the *best* that could result from each decision? and then choose the decision that corresponds to the "best of the best." For a minimization objective, this strategy is also often called a **minimin strategy**; that is, we choose the decision that minimizes the minimum payoff that can occur among all outcomes for each decision. Aggressive decision makers are often called speculators, particularly in financial arenas, because they increase their exposure to risk in hopes of increasing their return; while a few may be lucky, most will not do very well.

Example 16.2 Mortgage Decision with the Aggressive Strategy

For the mortgage-selection example, we find the best payoff—that is, the lowest-cost outcome—for each decision:

	Outcome			
Decision	Rates Rise	Rates Stable	Rates Fall	Best Payoff
1-year ARM	$61,134	$46,443	$40,161	$40,161
3-year ARM	$56,901	$51,075	$46,721	$46,721
30-year fixed	$54,658	$54,658	$54,658	$54,658

Because our goal is to minimize costs, we would choose the 1-year ARM.

Conservative (Pessimistic) Strategy A conservative decision maker, on the other hand, might take a more-pessimistic attitude and ask, "What is the worst thing that might result from my decision?" and then select the decision that represents the "best of the worst." Such a strategy is also known as a **minimax strategy** because we seek the decision that minimizes the largest payoff that can occur among all outcomes for each decision. Conservative decision makers are willing to forgo high returns to avoid undesirable losses. This rule typically models the rational behavior of most individuals.

Example 16.3 Mortgage Decision with the Conservative Strategy

For the mortgage-decision problem, we first find the worst payoff—that is, the largest cost for each option:

	Outcome			
Decision	Rates Rise	Rates Stable	Rates Fall	Worst Payoff
1-year ARM	$61,134	$46,443	$40,161	$61,134
3-year ARM	$56,901	$51,075	$46,721	$56,901
30-year fixed	$54,658	$54,658	$54,658	$54,658

In this case, we want to choose the decision that has the smallest worst payoff, or the 30-year fixed mortgage. Thus, no matter what the future holds, a minimum cost of $54,658 is guaranteed.

Opportunity-Loss Strategy A third approach that underlies decision choices for many individuals is to consider the *opportunity loss* associated with a decision. Opportunity loss represents the "regret" that people often feel after making a nonoptimal decision (I should have bought that stock years ago!). In general, the opportunity loss associated with any decision and event is the absolute difference between the *best* decision for that particular outcome and the payoff for the decision that was chosen. *Opportunity losses can be only nonnegative values.* If you get a negative number, then you made a mistake. Once opportunity losses are computed, the decision strategy is similar to a conservative strategy. The decision maker would select the decision that minimizes the largest opportunity loss among all outcomes for each decision. For these reasons, this is also called a **minimax regret strategy**.

Example 16.4 Mortgage Decision with the Opportunity-Loss Strategy

In our scenario, suppose we chose the 30-year fixed mortgage and later find out that the interest rates had risen. We could not have done any better by selecting a different decision; in this case, the opportunity loss is zero. However, if we had chosen the 3-year ARM, we would have paid \$56,901 instead of \$54,658 with the 30-year fixed instrument, or \$56,901 − \$54,658 = \$2,243 more. This represents the opportunity loss associated with making a nonoptimal decision. Similarly, had we chosen the 1-year ARM, we would have incurred an additional cost (opportunity loss) of \$61,134 − \$54,658 = \$6,476. We repeat this analysis for the other two outcomes and compute the opportunity losses, as summarized here:

	Outcome			
Decision	Rates Rise	Rates Stable	Rates Fall	Max Opportunity Loss
1-year ARM	\$6,476	\$—	\$—	\$6,476
3-year ARM	\$2,243	\$4,632	\$6,560	\$6,560
30-year fixed	\$—	\$8,215	\$14,497	\$14,497

Then, find the maximum opportunity loss that would be incurred for each decision. The best decision is the one with the smallest maximum opportunity loss. Using this strategy, we would choose the 1-year ARM. This ensures that, no matter what outcome occurs, we will never be more than \$6,476 away from the least cost we could have incurred.

Different criteria lead to different decisions; there is no "optimal" answer. Which criterion best reflects your personal values?

Decision Strategies for a Maximize Objective

When the objective is to maximize the payoff, we can still apply aggressive, conservative, and opportunity loss strategies, but we must make some key changes in the analysis.

- For the aggressive strategy, the best payoff for each decision would be the *largest* value among all outcomes, and we would choose the decision corresponding to the largest of these, called a **maximax strategy**.
- For the conservative strategy, the worst payoff for each decision would be the *smallest* value among all outcomes, and we would choose the decision corresponding to the largest of these, called a **maximin strategy**.

- For the opportunity-loss strategy, we need to be careful in calculating the opportunity losses. With a maximize objective, the decision with the largest value for a particular event has an opportunity loss of zero. The opportunity losses associated with other decisions is the absolute difference between their payoff and the largest value. The actual decision is the same as when payoffs are costs: Choose the decision that minimizes the maximum opportunity loss.

Decisions with Conflicting Objectives

Many decisions require some type of tradeoff among conflicting objectives, such as risk versus reward. For example, In the Innis Investment example in Chapter 14, Figure 14.17 showed the results of solving a series of linear optimization models to find the minimum risk that would occur for achieving increasing levels of investment returns. We saw that as the return went up, the risk begins to increase slowly, and then increases at a faster rate once a 6% investment target is achieved. What decision would be best? Another example we saw was the overbooking model. In this case, we can achieve lower costs but incur a loss in customer satisfaction and goodwill because of higher numbers of overbooked customers.

A simple decision rule can be used whenever one wishes to make an optimal tradeoff between any two conflicting objectives, one of which is good, and one of which is bad, that maximizes the ratio of the good objective to the bad (think of this as the "biggest bang for the buck").[1] First, display the tradeoffs on a chart with the "good" objective on the x-axis, and the "bad" objective on the y-axis, making sure to scale the axes properly to display the origin (0,0). Then graph the tangent line to the tradeoff curve that goes through the origin. The point at which the tangent line touches the curve (which represents the smallest slope) represents the best return to risk tradeoff.

EXAMPLE 16.5 Risk-Reward Tradeoff Decision for Innis Investments Example

In Figure 14.17, if we take the ratios of the weighted returns to the minimum risk values in the table, we will find that the largest ratio occurs for the target return of 6%. We can visualize this using the risk-reward tradeoff curve and a tangent line through the origin as shown in Figure 16.1. Note that the tangent line touches the curve at the 6% weighted return value. We can explain this easily from the chart by noting that for any other return, the risk is relatively larger (if all points fell on the tangent line, the risk would increase proportionately with the return).

Many other analytic techniques are available to deal with more complex multiple objective decisions. These include simple scoring models in which each decision is rated for each criterion (which may also be weighted to reflect the relative importance in comparison with other criteria). The ratings are summed over all criteria to rank the decision

[1]This rule was explained by Dr. Leonard Kleinrock at a lecture at the University of Cincinnati in 2011.

alternatives. Other techniques include variations of linear optimization known as *goal programming*, and a pairwise comparison approach known as the *analytic hierarchy process (AHP)*.

Table 16.1 summarizes the decision rules for both minimize and maximize objectives.

Figure 16.1

Innis Investments Risk-Reward Assessment

Table 16.1

Summary of Decision Strategies Under Uncertainty

Strategy/ Objective		Aggressive Strategy	Conservative Strategy	Opportunity-Loss Strategy
Minimize objective		Find the smallest payoff for each decision among all outcomes, and choose the decision with the smallest of these (*minimum*).	Find the largest payoff for each decision among all outcomes, and choose the decision with the smallest of these (*minimax*).	For each outcome, compute the opportunity loss for each decision as the absolute difference between its payoff and the *smallest* payoff for that outcome. Find the maximum opportunity loss for each decision, and choose the decision with the smallest opportunity loss (*minimax regret*).
Maximize objective	Choose the decision with the largest average payoff.	Find the largest payoff for each decision among all outcomes, and choose the decision with the largest of these (*maximax*).	Find the smallest payoff for each decision among all outcomes, and choose the decision with the largest of these (*maximin*).	For each outcome, compute the opportunity loss for each decision as the absolute difference between its payoff and the *largest* payoff for that outcome. Find the maximum opportunity loss for each decision, and choose the decision with the smallest opportunity loss (*minimax regret*).

Decision Strategies with Outcome Probabilities

The aggressive, conservative, and opportunity-loss strategies assume no knowledge of the probabilities associated with future outcomes. In many situations, we might have some assessment of these probabilities, either through some method of forecasting or reliance on expert opinions.

Average Payoff Strategy

If we can assess a probability for each outcome, we can choose the best decision based on the expected value using concepts that we introduced in Chapter 5. For any decision, the expected value is the summation of the payoffs multiplied by their probability, summed over all outcomes. The simplest case is to assume that each outcome is equally likely to occur; that is, the probability of each outcome is simply 1/N, where N is the number of possible outcomes. This is called the **average payoff strategy**. This approach was proposed by the French mathematician Laplace, who stated the *principle of insufficient reason*: if there is no reason for one outcome to be more likely than another, treat them as equally likely. Under this assumption, we evaluate each decision by simply averaging the payoffs. We then select the decision with the best average payoff.

Example 16.6 Mortgage Decision with the Average Payoff Strategy

For the mortgage-selection problem, computing the average payoffs results in the following:

	Outcome			
Decision	**Rates Rise**	**Rates Stable**	**Rates Fall**	**Average Payoff**
1-year ARM	$61,134	$46,443	$40,161	$49,246
3-year ARM	$56,901	$51,075	$46,721	$51,566
30-year fixed	$54,658	$54,658	$54,658	$54,658

Based on this criterion, we choose the decision having the smallest average payoff, or the 1-year ARM.

Expected Value Strategy

A more general case of the average payoff strategy is when the probabilities of the outcomes are not all the same. This is called the **expected value strategy**. We may use the expected value calculation that we introduced in formula (5.9) in Chapter 5.

Example 16.7 Mortgage Decision with the Expected Value Strategy

Suppose that we can estimate the probabilities of rates rising as 0.6, rates stable as 0.3, and rates falling as 0.1. The following table shows the expected payoffs associated with each decision. The smallest expected payoff, $54,135.20, occurs for the 3-year ARM, which represents the best expected value decision.

	Outcome			
	0.6	0.3	0.1	
Decision	Rates Rise	Rates Stable	Rates Fall	Expected Payoff
1-year ARM	$61,134	$46,443	$40,161	$54,629.40
3-year ARM	$56,901	$51,075	$46,721	$54,135.20
30-year fixed	$54,658	$54,658	$54,658	$54,658.00

Evaluating Risk

An implicit assumption in using the average payoff or expected value strategy is that the decision is repeated a large number of times. However, for any *one-time* decision (with the trivial exception of equal payoffs), the expected value outcome will *never occur*. In the previous example, for instance, even though the expected value of the 3-year ARM (the best decision) is $54,135.20, the actual result would be only one of three possible payoffs, depending on the outcome of the mortgage rate event: $56,901 if rates rise, $51,075 if rates remain stable, or $46,721 if rates fall. Thus, for a one-time decision, we must carefully weigh the risk associated with the decision in lieu of blindly choosing the expected value decision.

Example 16.8 Evaluating Risk in the Mortgage Decision

In the mortgage-selection example, although the average payoffs are fairly similar, note that the 1-year ARM has a larger variation in the possible outcomes. We may compute the standard deviation of the outcomes associated with each decision:

Decision	Standard Deviation
1-year ARM	$10,763.80
3-year ARM	$5,107.71
30-year fixed	$—

Based solely on the standard deviation, the 30-year fixed mortgage has no risk at all, whereas the 1-year ARM appears to be the riskiest. Although based only on three data points, the 3-year ARM is fairly symmetric about the mean, whereas the 1-year ARM is positively skewed—most of the variation around the average is driven by the upside potential (i.e., lower costs), not the downside risk of higher costs. Although none of the formal decision strategies chose the 3-year ARM, viewing risk from this perspective might lead to this decision. For instance, a conservative decision maker who is willing to tolerate a moderate amount of risk might choose the 3-year ARM over the 30-year fixed because the downside risk is relatively small (and is smaller than the 1-year ARM) and the upside potential is much larger. The larger upside potential associated with the 1-year ARM might even make this decision attractive.

Thus, it is important to understand that making decisions under uncertainty cannot be done using only simple rules, but by careful evaluation of risk versus rewards. This is why top executives make the big bucks. Evaluating risk in making a decision should also take into account the magnitude of potential gains and losses as well as their probabilities of occurrence, if this can be assessed. For example, a 70% chance of losing $10,000 against a 30% chance of gaining $500,000 might be viewed as an acceptable risk for a company, but a 10% chance of losing $250,000 against a 90% chance of gaining $500,000 might not.

Decision Trees

A useful approach to structuring a decision problem involving uncertainty is to use a graphical model called a **decision tree**. Decision trees consist of a set of **nodes** and **branches**. Nodes are points in time at which events take place. The event can be a selection of a decision from among several alternatives, represented by a **decision node**, or an outcome over which the decision maker has no control, an **event node**. Event nodes are conventionally depicted by circles, and decision nodes are expressed by squares. Branches are associated with decisions and events. Many decision makers find decision trees useful because *sequences* of decisions and outcomes over time can be modeled easily.

Decision trees may be created in Excel using *Analytic Solver Platform*. Click the *Decision Tree* button. To add a node, select *Add Node* from the *Node* drop-down list, as shown in Figure 16.2. Click on the radio button for the type of node you wish to create (decision or event). This displays one of the dialogs shown in Figure 16.3. For a decision node, enter the name of the node and names of the branches that emanate from the node (you may also add additional ones). The *Value* field can be used to input cash flows, costs, or revenues that result from choosing a particular branch. For an event node, enter the name of the node and branches. The *Chance* field allows you to enter the probabilities of the events.

Figure 16.2

Decision Tree Menu in *Analytic Solver Platform*

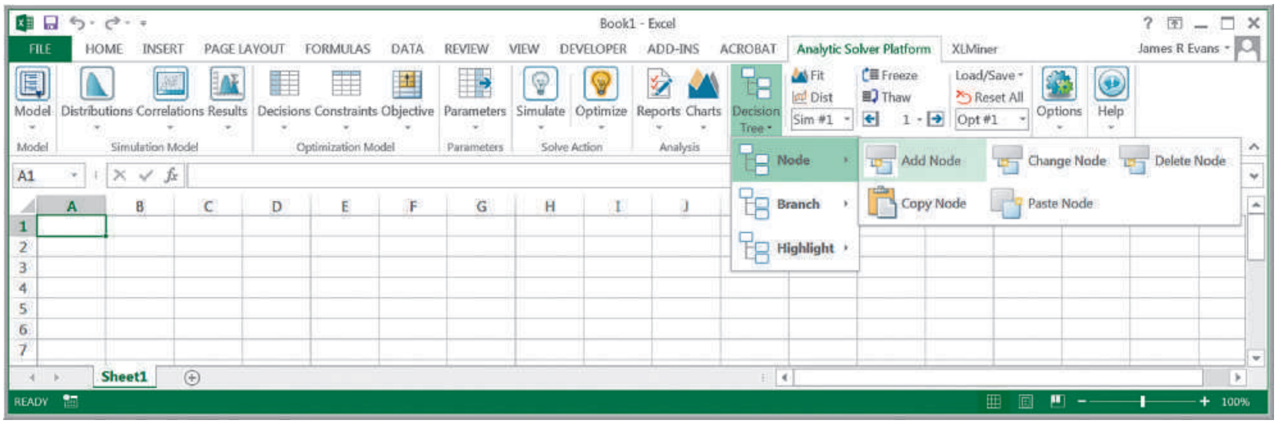
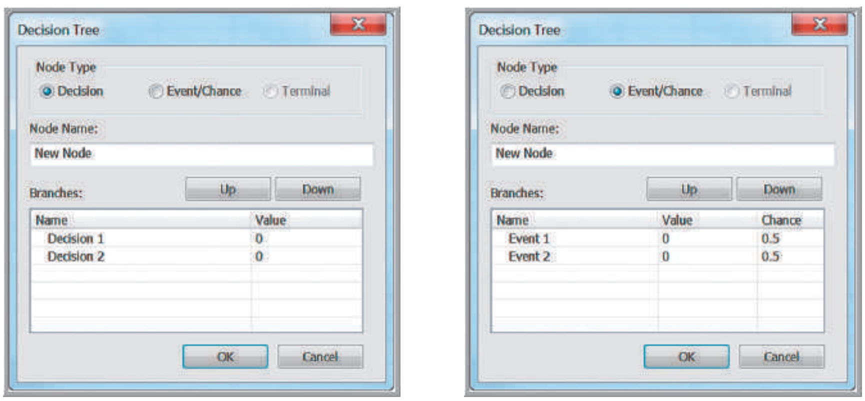


Figure 16.3

Decision Tree Dialogs for Decisions and Events



Example 16.9 Creating a Decision Tree

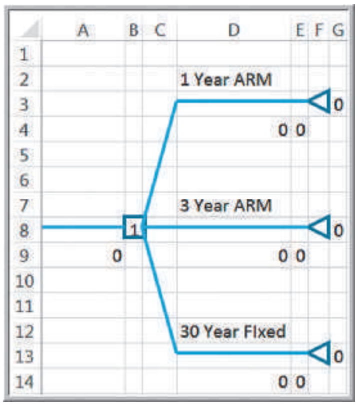
For the mortgage-selection problem, we will first create a decision node for the selection of one of the three mortgage instruments. In the dialog in Figure 16.3, we name the node “Mortgage Instrument” and name the branches from this node “1 Year ARM,” “3 Year ARM,” and “30 Year Fixed.” The result is shown in Figure 16.4. Next, select the node at the end of the 1-Year ARM branch (cell F3) and choose *Add Node.* In the dialog, click the radio button for *Event.* In this example, we name the node “Outcomes” with branches “Rates Rise,” “Rates Stable,” and “Rates Fall.” We assign the probabilities to these outcomes from Example 16.7. This creates the tree shown in Figure 16.5.

You may copy and paste a subtree rooted at the selected node at another position in the decision tree. Select cell F8, choose *Node* > *Copy Node*, and then select cell F18 (the end of the 3-Year ARM branch), and choose *Node* > *Paste Node*. Repeat this process to copy the outcomes subtree to cell F38.

Finally, enter the payoffs of the outcomes associated with each event in the cells immediately below the branches (column H in this example). Because the payoffs are costs, we enter them as negative values. (*Analytic Solver Platform* defaults to maximizing the expected value of the decision tree. We could have entered the costs as positive values and changed the objective in the Task Pane by clicking the *Model* button in the ribbon, choosing the *Platform* Tab, and changing the value of the field *Decision Node EV/CE* to Minimize.) The final decision tree is shown in Figure 16.6 (Excel file *Mortgage Selection Decision Tree*).

Figure 16.4

First Partial Decision Tree for Mortgage Selection



In Figure 16.6, the terminal values in column K are the sum of all the cash flows along the path leading to that terminal node; for example, the value in cell K3 is the sum of the values in cells D9 and H4. *Analytic Solver Platform* will automatically identify the best strategy that maximizes the expected value of the payoff. The tree is “rolled back” by computing expected values at event nodes and by selecting the optimal value of the alternative decisions at decision nodes. For example, if the 1-Year ARM is chosen, the expected value of the chance events is $0.6 \times (-\$61{,}134) + 0.3 \times (-\$46{,}443) + 0.1 \times (-\$40{,}161) = -\$54{,}629.40$ in cell E9. At the decision node (cell B23), the maximum expected value is chosen and shown in cell A24. The number inside the decision node represents the branch that corresponds to the best decision. In Figure 16.6, this is branch 2, or the 3-Year ARM, having an expected cost of \$54,135.20 (the same decision we found in Example 16.7). You can see this visually by choosing *Highlight* > *Highlight Best* from the *Decision Tree* menu.

Many decision problems have multiple sequences of decisions and events. Decision trees help managers better understand the structure of the decisions they face.

Figure 16.5

Second Partial Decision Tree for Mortgage Selection

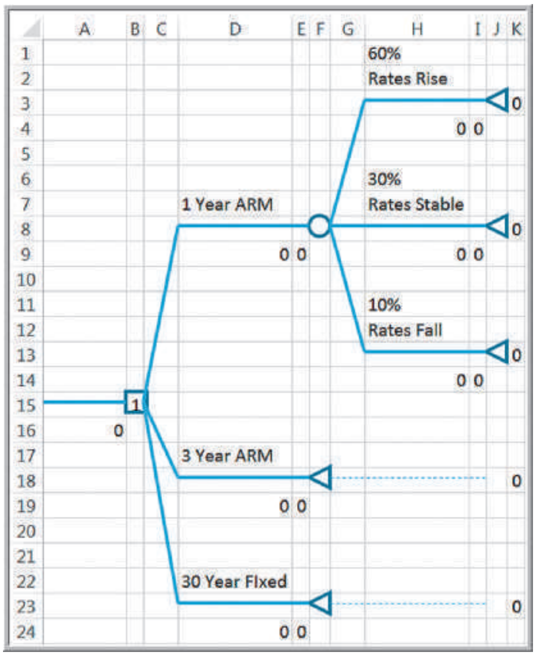
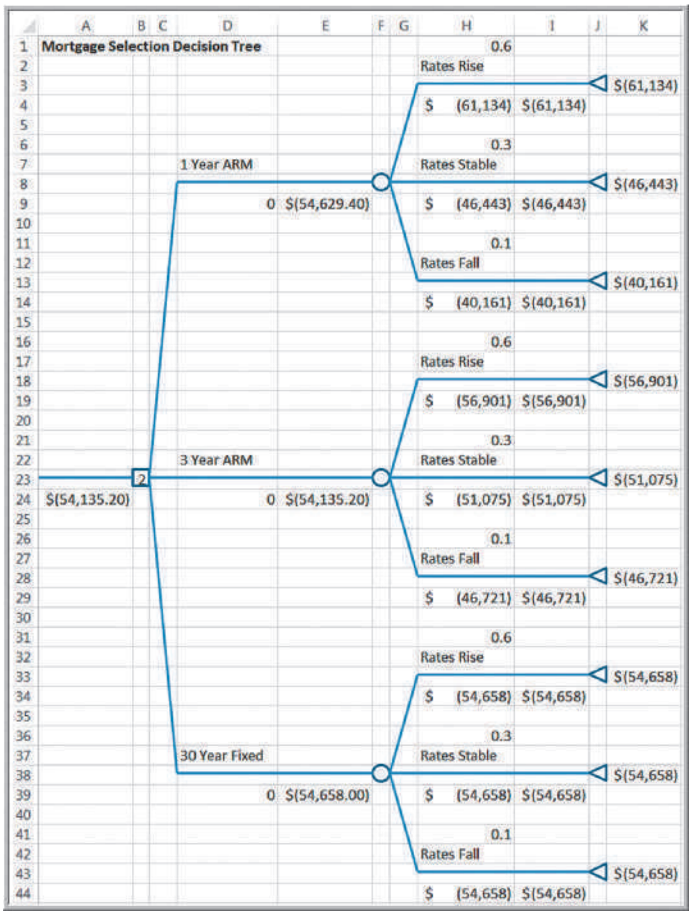


Figure 16.6

Mortgage-Selection Decision Tree



Example 16.10 A Pharmaceutical R&D Model

We will consider the R&D process for a new drug (you might recall the basic financial model we developed for the Moore Pharmaceuticals example in Chapter 11). Suppose that the company has spent $300 million to date in research expenses. The first decision is whether or not to proceed with clinical trials. We can either decide to conduct them, or stop development at this point, incurring the $300 million cost already spent on research. The cost of clinical trials is estimated to be $250 million, and the probability of a successful outcome is 0.3. Therefore, if we decide to conduct the trials, we face the chance events that the trials will either be successful or not successful. If they are not successful, then clearly the process stops at this point. If they are successful, the company may seek approval from the Food and Drug Administration or decide to stop the development process. The cost of seeking approval is $25 million, and there is a 60% chance of approval. If the company seeks approval, it faces the chance events that the FDA will approve the drug or not approve it. Finally, if the drug is approved and is released to the market, the market potential has been identified as either large, medium, or small, with the following characteristics:

Market Potential	Expected Revenues (millions of $)	Probability
Large	4,500	0.6
Medium	2,200	0.3
Small	1,500	0.1

A decision tree for this situation is shown in Figure 16.7 (Excel file *Drug Development Decision Tree*). When we have sequences of decisions and events, a **decision strategy** is a specification of an initial decision and subsequent decisions to make after knowing what events occur. We can identify the best strategy from the branch number in the decision nodes. For example, the best strategy is to conduct clinical trials and, if successful, seek FDA approval and, if approved, market the drug. The expected net revenue is calculated as $74.3 million.

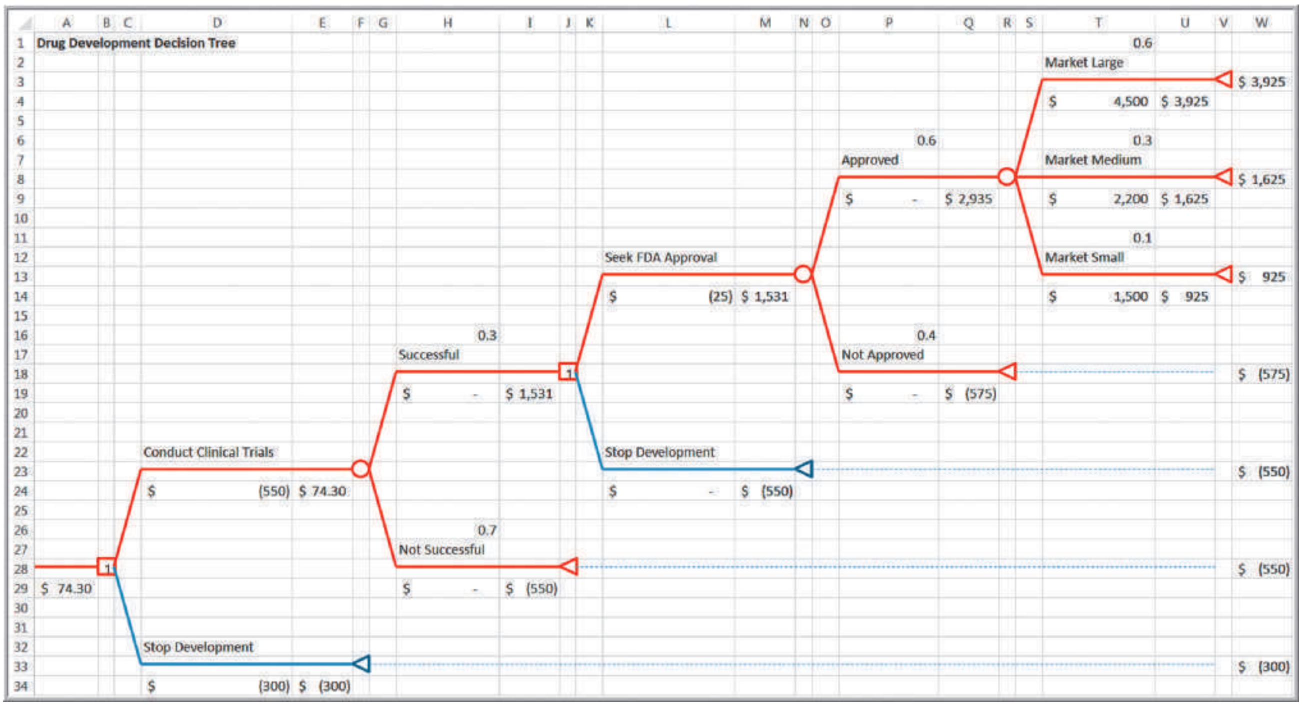


Figure 16.7

New-Drug-Development Decision Tree

Decision Trees and Monte Carlo Simulation

Because all computations use Excel formulas, you could easily perform what-if analyses or create data tables to analyze changes in the assumptions of the model. One of the interesting features of decision trees in *Analytic Solver Platform* is that you can also use the Excel model to develop a Monte Carlo simulation or an optimization model using the decision tree.

Example 16.11 Simulating the *Moore Pharmaceuticals* Decision Tree Model

Suppose that the payoffs for the market outcomes are uncertain. Let us assume that if the market is large, the payoff is lognormally distributed with a mean of $4,500 million and a standard deviation of $1,000 million; if the market is medium, the payoff is lognormally distributed with a mean of $2,200 million and a standard deviation of $500 million; and if the market is small, the payoff is normally distributed with a mean of $1,500 million and standard deviation of $200 million. Insert the formula =PsiLogNormal (4500,1000) into cell T4, =PsiLogNormal(2200,500) into cell T9, and =PsiNormal(1500, 200) into cell T14. Further, assume that the cost of clinical trials is uncertain and estimates are modeled using a triangular distribution with a minimum of −$700 million, most likely value of −550 million, and maximum value of −$500 million. Therefore, use the formula =PsiTriangular (−700, −550, −500) in cell D24.

Because of the way that *Analytic Solver Platform* performs decision tree calculations to ensure that rollback values are consistent, we cannot define cell A29 as an output cell to predict the expected value of the decision tree. However, all we need to do is to copy the expected value of the decision tree to another cell and set this as an output cell for the simulation. We will do this in cell A32; the formula is =A29 + PsiOutput( ). You may examine the Excel file *Drug Development Monte Carlo Simulation Model* to see how the model is implemented.

Figure 16.8 shows the results of a simulation of this scenario. We see that there is about a 40% chance that the development of the drug will result in a loss. This might be considered too risky and the company might decide to stop development rather than pursue the project.

Decision Trees and Risk

The decision tree approach is an example of expected value decision making. Thus, in the drug-development example, if the company's portfolio of drug-development projects has similar characteristics, then pursuing further development is justified on an expected value basis. However, this approach does not explicitly consider risk.

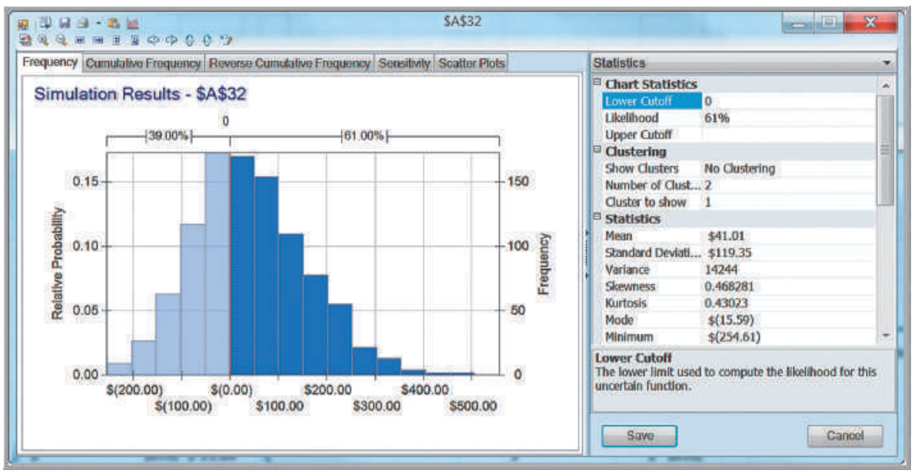


Figure 16.8 Simulation Results of the New-Drug-Development Decision Tree

From a classical decision analysis perspective, we may summarize the company's decision as the following payoff table:

	Unsuccessful Clinical Trials	Successful Clinical Trials; No FDA Approval	Successful Trials and Approval; Large Market	Successful Trials and Approval; Medium Market	Successful Trials and Approval; Small Market
Develop drug	($550)	($575)	$3,925	$1,625	$925
Stop development	($300)	($300)	($300)	($300)	($300)

If we apply the aggressive, conservative, and opportunity-loss decision strategies to these data (note that the payoffs are profits as opposed to costs, so it is important to use the correct rule, as discussed earlier in the chapter), we obtain the following.

Aggressive strategy (maximax):

	Maximum
Develop drug	$3,925
Stop development	($300)

The decision that maximizes the maximum payoff is to develop the drug.

Conservative strategy (maximin):

	Minimum
Develop drug	($575)
Stop development	($300)

The decision that maximizes the minimum payoff is to stop development.

Opportunity loss:

	Unsuccessful Clinical Trials	Successful Clinical Trials; No FDA Approval	Successful Trials and Approval; Large Market	Successful Trials and Approval; Medium Market	Successful Trials and Approval; Small Market	Maximum
Develop drug	$250	$275	$—	$—	$—	$275
Stop development	$—	$—	$4,225	$1,925	$1,225	$4,225

The decision that minimizes the maximum opportunity loss is to develop the drug. However, as we noted, we must evaluate risk by considering both the magnitude of the payoffs and their chances of occurrence. The aggressive, conservative, and opportunity-loss rules do not consider the probabilities of the outcomes.

Each decision strategy has an associated payoff distribution, called a **risk profile**. Risk profiles show the possible payoff values that can occur and their probabilities.

Example 16.12 Constructing a Risk Profile

In the drug-development example, consider the strategy of pursuing development. The possible outcomes that can occur and their probabilities are:

Terminal Outcome	Net Revenue	Probability
Market large	$3,925	0.108
Market medium	$1,625	0.054
Market small	$925	0.018
FDA not approved	($575)	0.120
Clinical trials not successful	($550)	0.700

The probabilities are computed by multiplying the probabilities on the event branches along the path to the terminal outcome. For example, the probability of getting to "Market large" is $0.3 \times 0.6 \times 0.6 = 0.108$. Thus, we see that the probability that the drug will not reach the market is $1 - (0.108 + 0.054 + 0.018) = 0.82$, and the company will incur a loss of more than $500 million. On the other hand, if they decide not to pursue clinical trials, the loss would be only $300 million, the cost of research to date. If this were a one-time decision, what decision would you make if you were a top executive of this company?

Sensitivity Analysis in Decision Trees

We may use Excel data tables to investigate the sensitivity of the optimal decision to changes in probabilities or payoff values. We illustrate this using the airline revenue management scenario we discussed in Example 5.22 in Chapter 5.

Example 16.13 Sensitivity Analysis for Airline Revenue Management Decision

Figure 16.9 shows the decision tree (Excel file *Airline Revenue Management Decision Tree*) for deciding whether or not to discount the fare with a data table for varying the probability of success with two output columns, one providing the expected value from cell A10 in the tree and the second providing the best decision. The formula in cell O3 is =IF(B9=1, "Full", "Discount"). However, we must first modify the worksheet prior to constructing the data table so that probabilities will always sum to 1. To do this, enter the formula =1−H1 in cell H6, corresponding to the probability of not selling the full-fare ticket. When constructing the data table, use cell H1 as the column input cell. From the results, we see that if the probability of selling the full-fare ticket is 0.7 or less, then the best decision is to discount the price. Two-way data tables may also be used in a similar fashion to study simultaneous changes in model parameters.

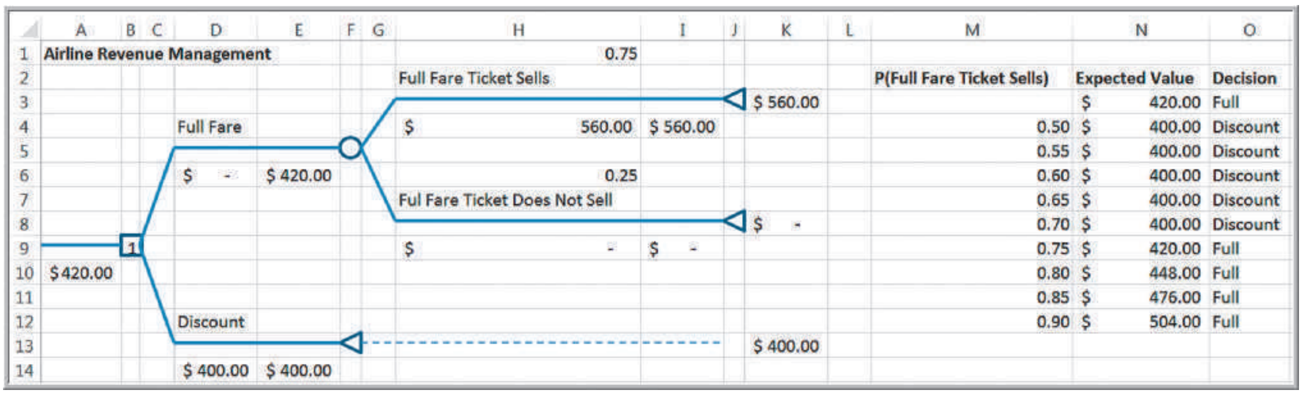


Figure 16.9

Airline Revenue Management Decision Tree and Data Table

The Value of Information

When we deal with uncertain outcomes, it is logical to try to obtain better information about their likelihood of occurrence before making a decision. The **value of information** represents the improvement in the expected return that can be achieved if the decision maker is able to acquire—before making a decision—additional information about the future event that will take place. In the ideal case, we would like to have **perfect information**, which tells us with certainty what outcome will occur. Although this will never occur, it is useful to know the value of perfect information because it provides an upper bound on the value of any information that we may acquire. The **expected value of perfect information (EVPI)** is the expected value with perfect information (assumed at no cost) minus the expected value without any information; again, it represents the most you should be willing to pay for perfect information.

The **expected opportunity loss** represents the average additional amount the decision maker would have achieved by making the right decision instead of a wrong one. To find the expected opportunity loss, we create an opportunity-loss table, as discussed earlier in this chapter, and then find the expected value for each decision. *It will always be true that the decision having the best expected value will also have the minimum expected opportunity loss.* The minimum expected opportunity loss is the EVPI.

Example 16.14 Finding EVPI for the Mortgage-Selection Decision

The following table shows the calculations of the expected opportunity losses for each decision (see Example 16.4 for calculation of the opportunity-loss matrix). The minimum expected opportunity loss occurs for the 3-year ARM (which was the best expected value decision) and is $3,391.40. This is the value of EVPI.

	Outcome			
	0.6	0.3	0.1	
Decision	Rates Rise	Rates Stable	Rates Fall	Expected Opportunity Loss
1-year ARM	$6,476	$—	$—	$3,885.60
3-year ARM	$2,243	$4,632	$6,560	$3,391.40
30-year fixed	$—	$8,215	$14,497	$3,914.20

Another way to understand this is to use the following logic. Suppose we know that rates will rise. Then, we should choose the 30-year fixed mortgage and incur a cost of $54,658. If we know that rates will be stable, then our best decision would be to choose the 1-year ARM, with a cost of $46,443. Finally, if we know that rates will fall, we should choose the 1-year ARM with a cost of $40,161. By weighting these values by the probabilities that their associated events will occur, under *perfect information*, our expected cost would be $0.6 \times \$54,658 + 0.3 \times \$46,443 + 0.1 \times \$40,161 = \$50,743.80$. If we did not have perfect information about the future, then we would choose the 3-year ARM no matter what happens and incur an expected cost of $54,135.20. By having perfect information, we would save $\$54,135.20 - \$50,743.80 = \$3,391.40$. This is the expected value of perfect information. We would never want to pay more than $3,391.40 for any information about the future event, no matter how good.

Decisions with Sample Information

Sample information is the result of conducting some type of experiment, such as a market research study or interviewing an expert. Sample information is always imperfect. Often, sample information comes at a cost. Thus, it is useful to know how much we should be willing to pay for it. The **expected value of sample information (EVSI)** is the expected value with sample information (assumed at no cost) minus the expected value without sample information; it represents the most you should be willing to pay for the sample information.

Example 16.15 Decisions with Sample Information

Suppose that a company is developing a new touch-screen cell phone. Historically, 70% of their new phones have resulted in high consumer demand, whereas 30% have resulted in low consumer demand. The company has the decision of choosing between two alternative models with different features that require different amounts of investment and also have different sales potential. Figure 16.10 shows a completed decision tree in which all cash flows are in thousands of dollars. For example, model 1 requires an initial investment for development of $200,000, and model 2 requires an investment of $175,000. If demand is high for model 1, the company will gain $500,000 in revenue, with a net profit of $300,000; it will receive only $160,000 if demand is low, resulting in a net profit of –$40,000. Based on the probabilities of demand, the expected profit is $198,000. For model 2, we see that the expected profit is only $188,000. Therefore, the best decision is to select model 1. Clearly there is risk in either decision, but on an expected value basis, model 1 is the best decision.

Now suppose that the firm conducts a market research study to obtain sample information and better understand the nature of consumer demand. Analysis of past market research studies, conducted prior to introducing similar products, has found that 90% of all products that resulted in high consumer demand had previously received a high survey response, whereas only 20% of all products with ultimately low consumer demand had previously received a high survey response. These probabilities show that the market research is not always accurate and can lead to a false indication of the true market potential. However, we should expect that a high survey response would increase the historical probability of high demand, whereas a low survey response would increase the historical probability of a low demand. Thus, we need to compute the conditional probabilities:

$P(\text{high demand} \mid \text{high survey response})$

$P(\text{high demand} \mid \text{low survey response})$

$P(\text{low demand} \mid \text{high survey response})$

$P(\text{low demand} \mid \text{low survey response})$

This can be accomplished using a formula called Bayes's rule.

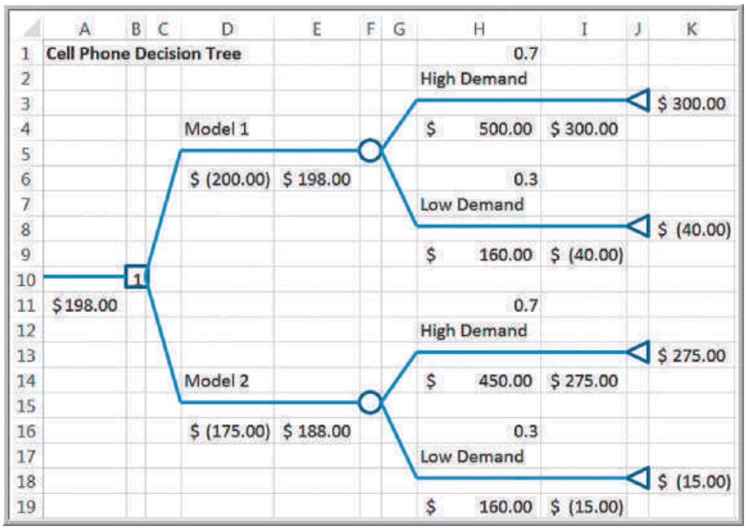
Bayes's Rule

Bayes's rule extends the concept of conditional probability to revise historical probabilities based on sample information. Suppose that $A_1, A_2, \ldots, A_k$ is a set of mutually exclusive and collectively exhaustive events, and we seek the probability that some event A_i occurs given that another event B has occurred. Bayes's rule is stated as follows:

$$P(A_i \mid B) = \frac{P(B \mid A_i)\,P(A_i)}{P(B \mid A_1)\,P(A_1) + P(B \mid A_2)\,P(A_2) + \cdots + P(B \mid A_k)\,P(A_k)} \tag{16.1}$$

Figure 16.10

Cell Phone Decision Tree



Example 16.16 Applying Bayes's Rule to Compute Conditional Probabilities

In the cell phone example, define the events:

$$A_1 = \text{high consumer demand}$$
$$A_2 = \text{low consumer demand}$$
$$B_1 = \text{high survey response}$$
$$B_2 = \text{low survey response}$$

We need to compute $P(A_i|B_j)$ for each i and j.

Using these definitions and the information presented in Example 16.15, we have

$$P(A_1) = 0.7$$
$$P(A_2) = 0.3$$
$$P(B_1|A_1) = 0.9$$
$$P(B_1|A_2) = 0.2$$

It is important to carefully distinguish between $P(A|B)$ and $P(B|A)$. As stated, *among all products that resulted in high consumer demand*, 90% received a high market survey response. Thus, the probability of a high survey response *given* high consumer demand is 0.90 and not the other way around. Because the probabilities $P(B_1|A_i) + P(B_2|A_i)$ must add to 1 for each A_i, we have

$$P(B_2|A_1) = 1 - P(B_1|A_1) = 0.1$$
$$P(B_2|A_2) = 1 - P(B_1|A_2) = 0.8$$

Now we may apply Bayes's rule to compute the conditional probabilities of demand given the survey response:

$$P(A_1|B_1) = \frac{P(B_1|A_1)\,P(A_1)}{P(B_1|A_1)\,P(A_1) + P(B_1|A_2)\,P(A_2)}$$

$$= \frac{(0.9)(0.7)}{(0.9)(0.7) + (0.2)(0.3)} = 0.913$$

Therefore, $P(A_2|B_1) = 1 - 0.913 = 0.087$.

$$P(A_1|B_2) = \frac{P(B_2|A_1)\,P(A_1)}{P(B_2|A_1)\,P(A_1) + P(B_2|A_2)\,P(A_2)}$$

$$= \frac{(0.1)(0.7)}{(0.1)(0.7) + (0.8)(0.3)} = 0.226$$

Therefore $P(A_2|B_2) = 1 - 0.226 = 0.774$.

Although 70% of all previous new models historically had high demand, knowing that the marketing report is favorable increases the likelihood to 91.3%, and if the marketing report is unfavorable, then the probability of low demand increases to 77%.

Finally, we need to compute the nonconditional (marginal) probabilities that the survey response will be either high or low—that is, $P(B_1)$ and $P(B_2)$. These are simply the denominators in Bayes's rule:

$$P(B_1) = P(B_1|A_1)\,P(A_1) + P(B_1|A_2)\,P(A_2)$$
$$= (0.9)(0.7) + (0.2)(0.3) = 0.69$$
$$P(B_2) = P(B_2|A_1)\,P(A_1) + P(B_2|A_2)\,P(A_2)$$
$$= (0.1)(0.7) + (0.8)(0.3) = 0.31$$

The marginal probabilities state that there is a 69% chance that the survey will return a high-demand response, and there is a 31% chance that the survey will result in a low-demand response.

Figure 16.11

Cell Phone Decision Tree with Sample Market Survey

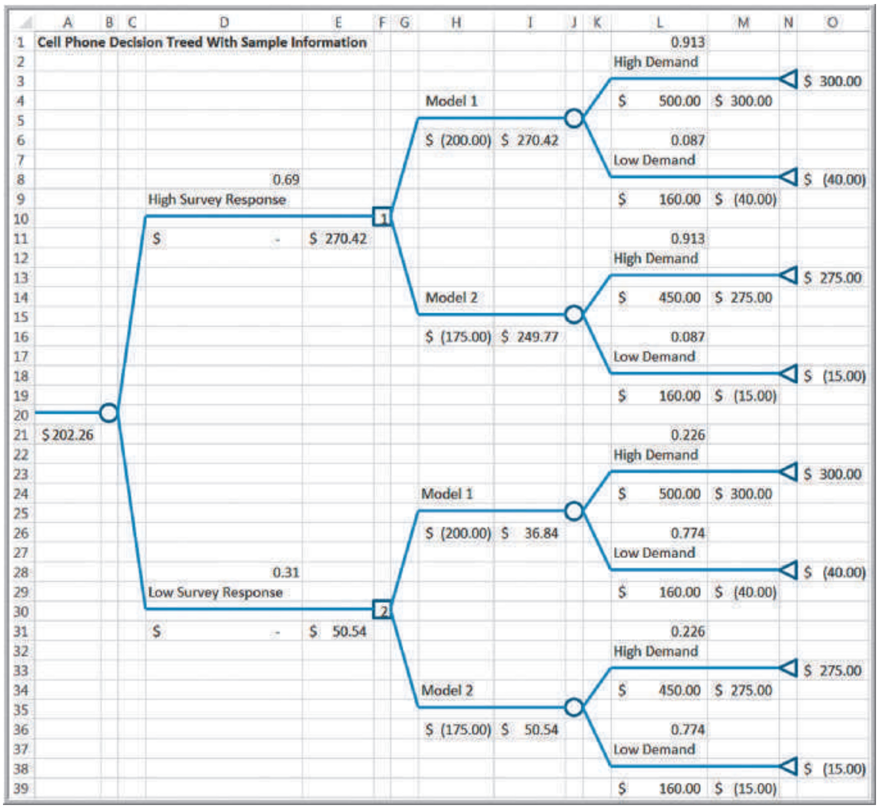


Figure 16.11 shows a decision tree that incorporates the market survey information and the probabilities we calculated in the previous example. The optimal decision strategy is to select model 1 if the survey response is high, and if the response is low, then select model 2. Note that the expected value (which includes the probabilities of obtaining the survey responses) is \$202,257. Comparing this to Figure 16.10, we see that the sample information increases the expected value by \$202,257 − \$198,000 = \$4,257. This is the value of EVSI. So we should not pay more than \$4,257 to conduct the market survey.

Utility and Decision Making

In Example 5.21 in Chapter 5, we discussed a charity raffle in which 1,000 \$50 tickets are sold to win a \$5,000 prize. The probability of winning is only 0.001, and the expected payoff is $(-\$0)(0.999) + (\$24{,}950)(0.001) = -\$25.00$. From a purely economic standpoint, this would be a poor gamble. Nevertheless, many people would take this chance because the financial risk is low (and it's for charity). On the other hand, if only 10 tickets were sold at \$5,000 with a chance to win \$100,000, even though the expected value would be $(-\$5000)(0.9) + (\$100{,}000)(0.1) = \$5{,}500$, most people would *not* take the chance because of the higher monetary risk involved.

An approach for assessing risk attitudes quantitatively is called **utility theory**. This approach quantifies a decision maker's relative preferences for particular outcomes. We can determine an individual's utility function by posing a series of decision scenarios. This is best illustrated with an example; we use a personal investment problem to do this.

Example 16.17 A Personal Investment Decision

Suppose that you have $10,000 to invest and are expecting to buy a new car in a year, so you can tie the money up for only 12 months. You are considering three options: a bank CD paying 4%, a bond mutual fund, and a stock fund. Both the bond and stock funds are sensitive to changing interest rates. If rates remain the same over the coming year, the share price of the bond fund is expected to remain the same, and you expect to earn $840. The stock fund would return about $600 in dividends and capital gains. However, if interest rates rise, you can anticipate losing about $500 from the bond fund after taking into account the drop in share price and, likewise, expect to lose $900 from the stock fund. If interest rates fall, however, the yield from the bond fund would be $1,000 and the stock fund would net $1,700. Table 16.2 summarizes the payoff table for this decision problem. The decision could result in a variety of payoffs, ranging from a profit of $1,700 to a loss of $900.

Table 16.2

Investment Return Payoff Table

Decision/Event	Rates Rise	Rates Stable	Rates Fall
Bank CD	$400	$400	$400
Bond fund	−$500	$840	$1,000
Stock fund	−$900	$600	$1,700

Constructing a Utility Function

The first step in determining a utility function is to rank-order the payoffs from highest to lowest. We conveniently assign a utility of 1.0 to the highest payoff and a utility of 0 to the lowest. Next, for each payoff between the highest and lowest, consider the following situation: Suppose you have the opportunity of achieving a *guaranteed return of x* or taking a chance of receiving the highest payoff with probability p or the lowest payoff with probability $1 - p$. (We use the term **certainty equivalent** to represent the amount that a decision maker feels is equivalent to an uncertain gamble.) What value of p would make you indifferent to these two choices? Then repeat this process for each payoff.

Example 16.18 Constructing a Utility Function for the Personal Investment Decision

First rank the payoffs from highest to lowest; assign a utility of 1.0 to the highest and a utility of 0 to the lowest:

Payoff, *X*	Utility, *U*(*X*)
$1,700	1.0
$1,000	
$840	
$600	
$400	
−$500	
−$900	0.0

Let us start with $x = \$1,000$. The decision is illustrated in the simple decision tree in Figure 16.12 (Excel file *Lottery Decision Tree*). Because this is a relatively high value, you decide that p would have to be at least 0.9 to take this risk. This represents the utility of a payoff of $1,000, denoted as *U*($1,000). For example, $1,000 is this decision maker's certainty equivalent for the uncertain situation of receiving $1,700 with probability 0.9 or −$900 with probability 0.1.

Repeating this process for each payoff, suppose we obtain the following utility function:

Payoff, *X*	Utility, *U*(*X*)
$1,700	1.0
$1,000	0.90
$840	0.85
$600	0.80
$400	0.75
−$500	0.35
−$900	0.0

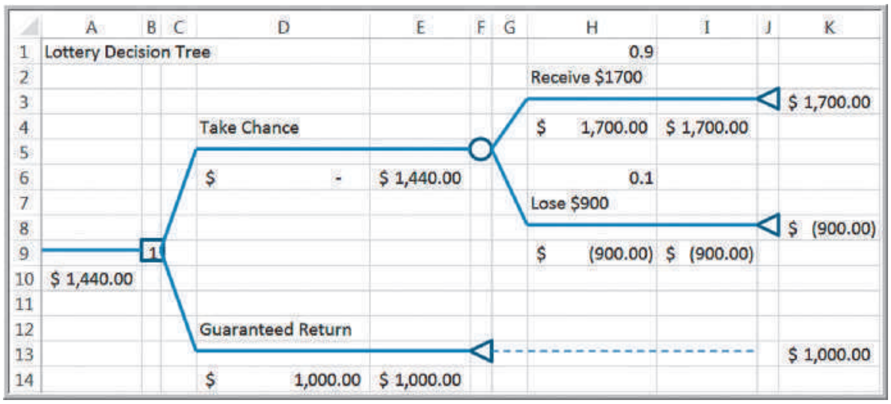


Figure 16.12

Decision Tree Lottery for Determining the Utility of $1,000

If we compute the expected value of each of the gambles for the chosen values of p, we see that they are higher than the corresponding payoffs. For example, for the payoff of $1,000 and the corresponding $p = 0.9$, the expected value of taking the gamble is

$$0.9(\$1{,}700) + 0.1(-\$900) = \$1{,}440$$

This is greater than accepting $1,000 outright. We can interpret this to mean that you require a risk premium of $\$1{,}440 - \$1{,}000 = \$440$ to feel comfortable enough to risk losing $900 if you take the gamble. In general, the **risk premium** is the amount an individual is willing to forgo to avoid risk. This indicates that you are a *risk-averse individual*, that is, relatively conservative.

Another way of viewing this is to find the *break-even probability* at which you would be indifferent to receiving the guaranteed return and taking the gamble. This probability is found by solving the equation

$$1{,}700\,p - 900(1 - p) = 1{,}000$$

resulting in $p = 19/26 = 0.73$. Because you require a higher probability of winning the gamble, it is clear that you are uncomfortable taking the risk.

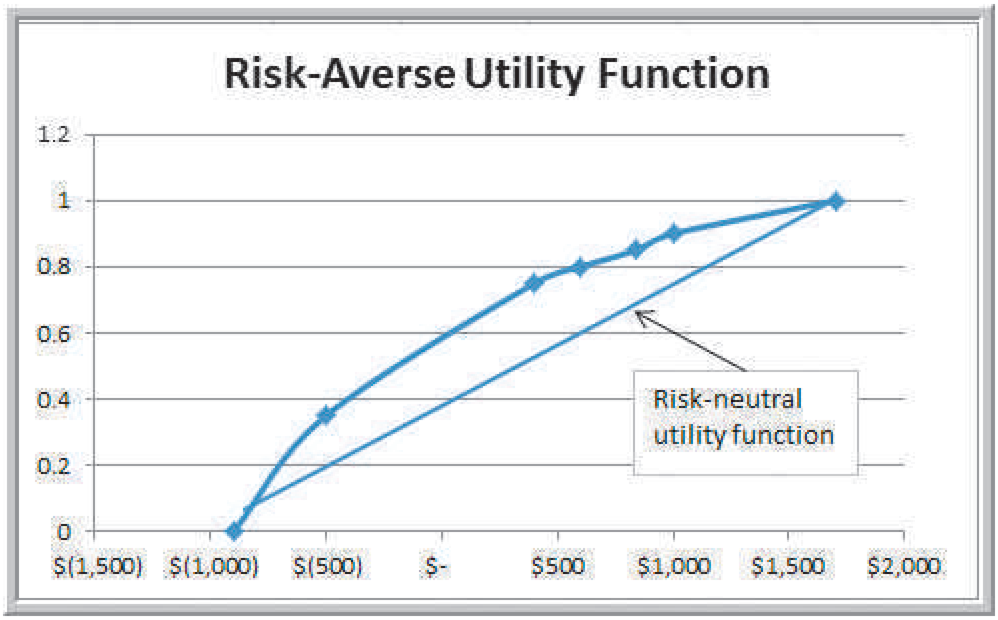
If we graph the utility versus the payoffs, we can sketch a utility function, as shown in Figure 16.13. This utility function is generally *concave downward.* This type of curve is characteristic of risk-averse individuals. Such decision makers avoid risk, choosing conservative strategies and those with high return-to-risk values. Thus, a gamble must have a higher expected value than a given payoff to be preferable or, equivalently, a higher probability of winning than the break-even value.

Other individuals might be risk takers. What would their utility functions look like? As you might suspect, they are *concave upward.* These individuals would take a gamble that offers higher rewards even if the expected value is less than a certain payoff. An example of a utility function for a risk-taking individual in this situation would be as follows:

Payoff, X	Utility, $U(X)$
$1,700	1.0
$1,000	0.6
$840	0.55
$600	0.45
$400	0.40
−$500	0.1
−$900	0.0

Figure 16.13

Example of a Risk-Averse Utility Function



For the payoff of \$1,000, this individual would be indifferent between receiving \$1,000 and taking a chance at \$1,700 with probability 0.6 and losing \$900 with probability 0.4. The expected value of this gamble is

$$0.6(\$1{,}700) + 0.4(-\$900) = \$660$$

Because this is considerably less than \$1,000, the individual is taking a larger risk to try to receive \$1,700. Note that the probability of winning is less than the break-even value. Risk takers generally prefer more aggressive strategies.

Finally, some individuals are risk neutral; they prefer neither taking risks nor avoiding them. Their utility function is linear and corresponds to the break-even probabilities for each gamble. For example, a payoff of \$600 would be equivalent to the gamble if

$$\$600 = p(\$1{,}700) + (1 - p)(-\$900)$$

Solving for p, we obtain $p = 15/26$, or 0.58, which represents the utility of this payoff. The decision of accepting \$600 outright or taking the gamble could be made by flipping a coin. These individuals tend to ignore risk measures and base their decisions on the average payoffs.

A utility function may be used instead of the actual monetary payoffs in a decision analysis by simply replacing the payoffs with their equivalent utilities and then computing expected values. The expected utilities and the corresponding optimal decision strategy then reflect the decision maker's preferences toward risk. For example, if we use the average payoff strategy (because no probabilities of events are given) for the data in Table 16.2, the best decision would be to choose the stock fund. However, if we replace the payoffs in Table 16.2 with the (risk-averse) utilities that we defined and again use the average payoff strategy, the best decision would be to choose the bank CD as opposed to the stock fund, as shown in the following table.

Decision/Event	Rates Rise	Rates Stable	Rates Fall	Average Utility
Bank CD	0.75	0.75	0.75	0.75
Bond fund	0.35	0.85	0.9	0.70
Stock fund	0	0.80	1.0	0.60

If assessments of event probabilities are available, these can be used to compute the expected utility and identify the best decision.

Exponential Utility Functions

It can be rather difficult to compute a utility function, especially for situations involving a large number of payoffs. Because most decision makers typically are risk averse, we may use an exponential utility function to approximate the true utility function. The exponential utility function is

$$U(x) = 1 - e^{-x/R} \tag{16.2}$$

where e is the base of the natural logarithm (2.71828 …) and R is a shape parameter that is a measure of risk tolerance. Figure 16.14 shows several examples of $U(x)$ for different values of R. Notice that all these functions are concave and that as R increases, the functions become flatter, indicating more tendency toward risk neutrality.

One approach to estimating a reasonable value of R is to find the maximum payoff $\$R$ for which the decision maker is willing to take an equal chance on winning $\$R$ or losing $\$R/2$. The smaller the value of R, the more risk averse is the individual. For instance, would you take a bet on winning \$10 versus losing \$5? How about winning \$10,000 versus losing \$5,000? Most people probably would not worry about taking the first gamble but might definitely think twice about the second. Finding one's maximum comfort level establishes the utility function.

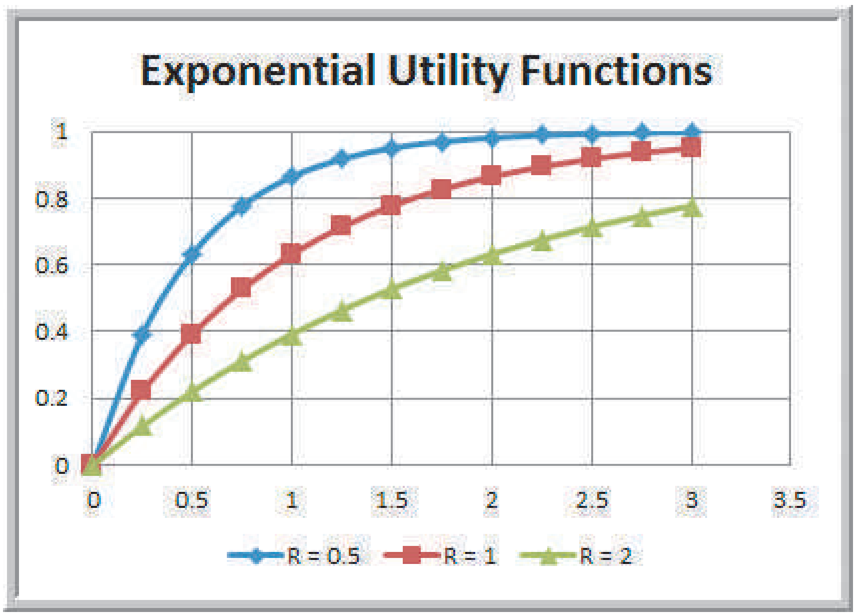


Figure 16.14

Examples of Exponential Utility Functions

Example 16.19 Using an Exponential Utility Function

For the personal investment decision example, suppose that $R = \$400$. The utility function is $U(x) = 1 - e^{-x/400}$, resulting in the following utility values:

Payoff, X	Utility, $U(X)$
\$1,700	0.9857
\$1,000	0.9179
\$840	0.8775
\$600	0.7769
\$400	0.6321
−\$500	−2.4903
−\$900	−8.4877

Using the utility values in the payoff table, we find that the bank CD remains the best decision, as shown in the following table, as it has the highest average utility.

Decision/Event	Rates Rise	Rates Stable	Rates Fall	Average Utility
Bank CD	0.6321	0.6321	0.6321	0.6321
Bond fund	−2.4903	0.8775	0.9179	−0.2316
Stock fund	−8.4877	0.7769	0.9857	−2.2417

Analytics in Practice: Using Decision Analysis in Drug Development

Drug development in the United States is time consuming, resource intensive, risky, and heavily regulated.[2] On average, it takes nearly 15 years to research and develop a drug in the United States, with an after-tax cost in 1990 dollars of approximately \$200 million.

In July 1999, the biological products leadership committee, composed of the senior managers within Bayer Biological Products (BP), a business unit of Bayer Pharmaceuticals (Pharma), made its newly formed strategic-planning department responsible for the commercial evaluation of a new blood-clot-busting drug. To ensure that it made the best drug-development decisions, Pharma used a structured process based on the principles of decision analysis to evaluate the technical feasibility and market potential of its new drugs. Previously, BP had analyzed a few business cases for review by Pharma. This commercial evaluation was BP's first decision analysis project.

Probability distributions of uncertain variables were assessed by estimating the 10th percentile and 90th percentile from experts, who were each asked to review the results to make sure they accurately reflect his or her judgment. Pharma used net present value (NPV) as its decision-making criterion. Given the complexity and inherent structure of decisions concerning new drugs, the new-drug-development decision making was defined as a sequence of six decision points, with identified key market-related and scientific deliverables so senior managers can assess the likelihood of success versus the company's exposure to risk, costs, and strategic fit. Decision point 1 was whether to begin preclinical development. After successful preclinical animal testing, Bayer can decide (decision point 2) to begin testing the drug in humans. Decision point 3 and decision point 4 (are both decisions to invest or not in continuing clinical devel-

[2]Based on Jeffrey S. Stonebraker, "How Bayer Makes Decisions to Develop New Drugs," *Interfaces*, 32, 6 (November–December 2002): 77–90.

opment. Following successful completion of development, Bayer can choose to file a biological license application with the FDA (decision point 5). If the FDA approves it, Bayer can decide (decision point 6) to launch the new drug in the marketplace.

The project team presented their input assumptions and recommendations for the commercial evaluation of the drug to the three levels of Pharma decision makers, who eventually approved preclinical development. External validation of the data inputs and assumptions demonstrated their rigor and defensibility. Senior managers could compare the evaluation results for the proposed drug with those for other development drugs with confidence. The international committees lauded the project team's effort as top-notch, and the decision-analysis approach set new standards for subsequent BP analyses.

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Key Terms

Average payoff strategy
Branches
Certainty equivalent
Decision alternatives
Decision making
Decision node
Decision strategy
Decision tree
Event node
Expected opportunity loss
Expected value of perfect information (EVPI)
Expected value of sample information (EVSI)
Expected value strategy
Laplace, or average payoff, strategy
Maximax strategy
Maximin strategy
Minimax regret strategy
Minimax strategy
Minimin strategy
Nodes
Outcomes
Payoffs
Payoff table
Perfect information
Risk premium
Risk profile
Sample information
States of nature
Uncertain events
Utility theory
Value of information

Problems and Exercises

Note: Data for selected problems can be found in the Excel file Chapter 16 Problem Data *to facilitate your problem-solving efforts. Worksheet tabs correspond to the problem numbers.*

1. Use the *Outsourcing Decision Model* Excel file to compute the cost of in-house manufacturing and outsourcing for the following levels of demand: 800, 1000, 1200, and 1400. Use this information to set up a payoff table for the decision problem, and apply the aggressive, conservative, and opportunity loss strategies.

2. The DoorCo Corporation is a leading manufacturer of garage doors. All doors are manufactured in their plant in Carmel, Indiana, and shipped to distribution centers or major customers. DoorCo recently acquired another manufacturer of garage doors, Wisconsin Door, and is considering moving its wood-door operations to the Wisconsin plant. Key

considerations in this decision are the transportation, labor, and production costs at the two plants. Complicating matters is the fact that marketing is predicting a decline in the demand for wood doors. The company developed three scenarios:

1. Demand falls slightly, with no noticeable effect on production.
2. Demand and production decline 20%.
3. Demand and production decline 40%.

The following table shows the total costs under each decision and scenario.

	Slight Decline	20% Decline	40% Decline
Stay in Carmel	$1,000,000	$900,000	$840,000
Move to Wisconsin	$1,200,000	$915,000	$800,000

What decision should DoorCo make using each strategy?

a. aggressive strategy

b. conservative strategy

c. opportunity-loss strategy

3. Suppose that a car-rental agency offers insurance for a week that costs $75. A minor fender bender will cost $2,000, whereas a major accident might cost $16,000 in repairs. Without the insurance, you would be personally liable for any damages. What should you do? Clearly, there are two decision alternatives: take the insurance, or do not take the insurance. The uncertain consequences, or events that might occur, are that you would not be involved in an accident, that you would be involved in a fender bender, or that you would be involved in a major accident. Develop a payoff table for this situation. What decision should you make using each strategy?

   a. aggressive strategy

   b. conservative strategy

   c. opportunity-loss strategy

4. Slaggert Systems is considering becoming certified to the ISO 9000 series of quality standards. Becoming certified is expensive, but the company could lose a substantial amount of business if its major customers suddenly demand ISO certification and the company does not have it. At a management retreat, the senior executives of the firm developed the following payoff table, indicating the net present value of profits over the next 5 years.

	Customer Response	
	Standards Required	Standards Not Required
Become certified	$575,000	$500,000
Stay uncertified	$450,000	$600,000

   What decision should the company make using each strategy?

   a. aggressive strategy

   b. conservative strategy

   c. opportunity-loss strategy

5. For the DoorCo Corporation decision in Problem 2, compute the standard deviation of the payoffs for each decision. What does this tell you about the risk in making the decision?
6. For the car-rental situation in Problem 3, compute the standard deviation of the payoffs for each decision. What does this tell you about the risk in making the decision?
7. For Slaggert Systems decision in Problem 4, compute the standard deviation of the payoffs for each decision. What does this tell you about the risk in making the decision?
8. What decisions should be made using the average payoff strategy in Problems 2, 3, and 4?
9. For the DoorCo Corporation decision in Problem 2, suppose that the probabilities of the three scenarios are estimated to be 0.15, 0.40, and 0.45, respectively. Find the best expected value decision.
10. For the car-rental situation described in Problem 3, assume that you researched insurance industry statistics and found out that the probability of a major accident is 0.05% and that the probability of a fender bender is 0.16%. What is the expected value decision? Would you choose this? Why or why not?
11. For the DoorCo Corporation decision in Problems 2 and 9, construct a decision tree and compute the rollback values to find the best expected value decision.
12. For the car-rental decision in Problems 3 and 10, construct a decision tree and compute the rollback values to find the best expected value decision.
13. For the car-rental decision in Problems 3 and 10, suppose that the cost of a minor fender bender is

normally distributed with a mean of $2000 and standard deviation of $100, and the cost of a major accident is triangular with a minimum of $10,000, maximum of $25,000, and most likely value of $16,000. Use *Analytic Solver Platform* to simulate the decision tree and find the distribution of the expected value of not taking the insurance.

14. An information system consultant is bidding on a project that involves some uncertainty. Based on past experience, if all went well (probability 0.1), the project would cost $1.2 million to complete. If moderate debugging were required (probability 0.7), the project would probably cost $1.4 million. If major problems were encountered (probability 0.2), the project could cost $1.8 million. Assume that the firm is bidding competitively and the expectation of successfully gaining the job at a bid of $2.2 million is 0, at $2.1 million is 0.1, at $2.0 million is 0.2, at $1.9 million is 0.3, at $1.8 million is 0.5, at $1.7 million is 0.8, and at $1.6 million is practically certain.

a. Calculate the expected value for the given bids.

b. What is the best bidding decision?

15. IM Retail deals in retail of all items of a popular cosmetic brand Beau. For a particular item, the price of stocking, selling, and cost price varies with the season. The cost price of the item in season is $12, while its selling price in season is $18. After the season, the bargain price is $9 and cost of stocking the item after season is $1. Gathering past data IM Retail has developed the following probability distribution for demand:

Demand (units)	Probability
7	.20
8	.20
9	.25
10	.15
11	.20

a. Construct a payoff table for IM Retail decision problem of how many units to be stocked. What is the best decision from an expected value basis?

b. Find the expected value of perfect information.

c. What is the expected demand? What is the expected profit if the retailer stocks the expected demand?

16. Bev's Bakery specializes in sourdough bread. Early each morning, Bev must decide how many loaves to bake for the day. Each loaf costs $1.25 to make and sells for $3.50. Bread left over at the end of the day can be sold the next day for $1.00. Past data indicate that demand is distributed as follows:

Number of Loaves	Probability
15	0.02
16	0.05
17	0.10
18	0.16
19	0.28
20	0.20
21	0.15
22	0.04

a. Construct a payoff table and determine the optimal quantity for Bev to bake each morning using expected values.

b. What is the optimal quantity for Bev to bake if the unsold loaves are sold the next day but are donated to a food bank?

17. Ravex Yacht has developed a new cabin cruiser which they have earmarked for the medium to large boat market. A market analysis suggests a 30% probability of annual sales being 5000 boats, 40% probability of 4000 annual sales, and 30% probability of 3000 annual sales. The firm can go into limited production where variable costs are 10000$ per boat and fixed costs are 800,000$ annually. Or the firm can go into full scale production where variable costs are $9000 per boat and fixed costs are 5,000,000$ annually.

a. Construct a decision tree for the situation.

b. Compute payoffs and probabilities.

c. If the boat is to be sold at $11,000, should the company go into limited or full scale production such that the profits are maximized?

18. Midwestern Hardware must decide how many snow shovels to order for the coming snow season. Each shovel costs $15.00 and is sold for $29.95. No inventory is carried from one snow season to the next. Shovels unsold after February are sold at a discount price of $10.00. Past data indicate that sales are highly dependent on the severity of the winter season. Past seasons have been classified as mild or harsh, and the following distribution of regular price demand has been tabulated:

Mild Winter		Harsh Winter	
No. of Shovels	**Probability**	**No. of Shovels**	**Probability**
250	0.5	1,500	0.2
300	0.4	2,500	0.3
350	0.1	3,000	0.5

Shovels must be ordered from the manufacturer in lots of 200; thus, possible order sizes are 200, 400, 1,400, 1,600, 2,400, 2,600, and 3,000 units. Construct a decision tree to illustrate the components of the decision model, and find the optimal quantity for Midwestern to order if the forecast calls for a 40% chance of a harsh winter.

19. Perform a sensitivity analysis of the Midwestern Hardware scenario (Problem 18). Find the optimal order quantity and optimal expected profit for probabilities of a harsh winter ranging from 0.2 to 0.8 in increments of 0.2. Plot optimal expected profit as a function of the probability of a harsh winter.

20. Dean Kuroff started a business of rehabbing old homes. He recently purchased a circa-1800 Victorian mansion and converted it into a three-family residence. Recently, one of his tenants complained that the refrigerator was not working properly. Dean's cash flow was not extensive, so he was not excited about purchasing a new refrigerator. He is considering two other options: purchase a used refrigerator or repair the current unit. He can purchase a new one for $400, and it will easily last 3 years. If he repairs the current one, he estimates a repair cost of $150, but he also believes that there is only a 30% chance that it will last a full 3 years and he will end up purchasing a new one anyway. If he buys a used refrigerator for $200, he estimates that there is a 0.6 probability that it will last at least 3 years. If it breaks down, he will still have the option of repairing it for $150 or buying a new one. Develop a decision tree for this situation and determine Dean's optimal strategy.

21. Many automobile dealers advertise lease options for new cars. Suppose that you are considering three alternatives:

1. Purchase the car outright with cash.
2. Purchase the car with 20% down and a 48-month loan.
3. Lease the car.

Select an automobile whose leasing contract is advertised in a local paper. Using current interest rates and advertised leasing arrangements, perform a decision analysis of these options. Make, but clearly define, any assumptions that may be required.

22. Drilling decisions by oil and gas operators involve intensive capital expenditures made in an environment characterized by limited information and high risk. A well site is dry, wet, or gushing. Historically, 50% of all wells have been dry, 30% wet, and 20% gushing. The value (net of drilling costs) for each type of well is as follows:

Dry	– $80.000
Wet	$100,000
Gushing	$200,000

Wildcat operators often investigate oil prospects in areas where deposits are thought to exist by making geological and geophysical examinations of the area before obtaining a lease and drilling permit. This often includes recording shock waves from detonations by a seismograph and using a magnetometer to measure the intensity of Earth's magnetic effect to detect rock formations below the surface. The cost of doing such studies is approximately $15,000. Of course, one may choose to drill in a location based on "gut feel" and avoid the cost of the study. The geological and geophysical examination classifies an area into one of three categories: no structure (NS), which is a bad sign; open structure (OS), which is an "OK" sign; and closed structure (CS), which is hopeful. Historically, 40% of the tests resulted in NS, 35% resulted in OS, and 25% resulted in CS readings. After the result of the test is known, the company may decide not to drill. The following table shows probabilities that the well will actually be dry, wet, or gushing based on the classification provided by the examination (in essence, the examination cannot accurately predict the actual event):

	Dry	Wet	Gushing
NS	0.73	0.22	0.05
OS	0.43	0.34	0.23
CS	0.23	0.372	0.398

a. Construct a decision tree of this problem that includes the decision of whether or not to perform the geological tests.

b. What is the optimal decision under expected value when no experimentation is conducted?

c. Find the overall optimal strategy by rolling back the tree.

23. Hahn Engineering is planning on bidding on a job and often competes against a major competitor, Sweigart and Associates (S&A), as well as other firms. Historically, S&A has bid for the same jobs 80% of the time; thus the probability that S&A will bid on this job is 0.80. If S&A bids on a job, the probability that Hahn Engineering will win it is 0.30. If S&A does not bid on a job, the probability that Hahn will win the bid is 0.60. Apply Bayes's rule to find the probability that Hahn Engineering will win the bid. If they do, what is the probability that S&A did bid on it?

24. MJ Logistics has decided to build a new warehouse to support its supply chain activities. They have the option of building either a large warehouse or a small one. Construction costs for the large facility are $8 million versus $3 million for the small facility. The profit (excluding construction cost) depends on the volume of work the company expects to contract for in the future. This is summarized in the following table (in millions of dollars):

	High Volume	Low Volume
Large warehouse	$35	$20
Small warehouse	$25	$15

The company believes that there is a 60% chance that the volume of demand will be high.

a. Construct a decision tree to identify the best choice.

b. Suppose that the company engages an economic expert to provide an opinion about the volume of work based on a forecast of economic conditions. Historically, the expert's upside predictions has been 75% accurate, whereas the downside predictions have been 90% accurate. In contrast to the company's assessment, the expert believes that the chance for high demand is 70%. Determine the best strategy if their predictions suggest that the economy will improve or will deteriorate. Given the information, what is the probability that the volume will be high?

25. Consider the car-rental insurance scenario in Problems 3 and 10. Use the approach described in this chapter to develop your personal utility function for the payoffs associated with this decision. Determine the decision that would result using the utilities instead of the payoffs. Is the decision consistent with your choice?

26. A college football team is trailing 14–0 late in the game. The team just made a touchdown. If they can, hold the opponent and score one more time, they can tie or win the game. The coach is wondering whether to go for an extra-point kick or a two-point conversion now and what to do if they can score again.

a. Develop a decision tree for the coach's decision.

b. Estimate probabilities for successful kicks or two-point conversions and a last minute score. (You might want to do this by doing some group brainstorming or by calling on experts, such as your school's coach or a sports journalist.) Using the probabilities from part (a), determine the optimal strategy.

c. Why would utility theory be a better approach than using the points for making a decision? Propose a utility function and compare your results.

Case: Performance Lawn Equipment

PLE has developed a prototype for a new snow blower for the consumer market. This can exploit the company's expertise in small-gasoline-engine technology and also balance seasonal demand cycles in the North American and European markets to provide additional revenues during the winter months. Initially, PLE faces two possible decisions: introduce the product globally at a cost of $850,000 or evaluate it in a North American test market at a cost of $200,000. If it introduces the product globally, PLE might find either a high or low response to the product. Probabilities of these events are estimated to be 0.6 and 0.4, respectively. With a high response, gross revenues of $2,000,000 are expected; with a low response, the figure is $450,000. If it starts with a North American test market, it might find a low response or a high response with probabilities 0.3 and 0.7, respectively. This may or may not reflect the global market potential. In any case, after conducting the marketing re-