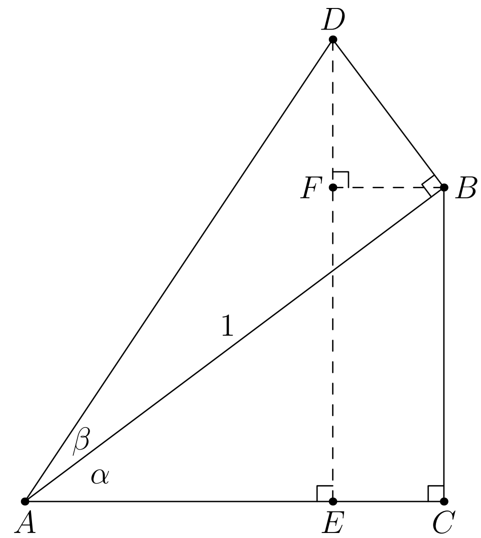

Let the intersection of DE and BA be point G. Then, we have that $\angle DFB = \angle ACB = 90$. Furthermore, it is trivial that $\angle CBA = 90 - \alpha$.

We notice that $\angle EGA = 180 - \angle AEG - \angle EAG = 180 - 90 - \alpha = 90 - \alpha$.

Then, we also notice that $\angle FGB = \angle EGA = 90 - \alpha$.

Thus, $\angle BDF = 180 - \angle DGB - \angle GBD = 180 - (90 - \alpha) - 90 = \alpha$. We can conclude that $\angle BDF = \angle BAC = \alpha$.

By AA Similarity, $\triangle DFB = \triangle BAC$ and by this reasoning the two triangles are similar.

$AC = \frac{AC}{1} = \frac{AC}{AB} = \boxed{\cos\alpha}$

$BC = \frac{BC}{1} = \frac{BC}{AB} = \boxed{\sin\alpha}$

$AD = \frac{AD}{1} = \frac{AD}{AB} = \boxed{\dfrac{1}{\cos\beta}}$

$BD = \frac{BD}{1} = \frac{BD}{AB} = \boxed{\tan\beta}$

BF and DF are trickier.

Note that $\sin\alpha = \frac{BC}{AB} = \frac{BF}{BD}$ because the two triangles are similar.

Then, we know that $BD = \tan\beta$ so $\sin\alpha = \frac{BF}{\tan\beta}$ so $BF = \boxed{\sin\alpha\tan\beta}$

Recall that $\angle BDF = \angle BAC = \alpha$. Thus, $\cos\angle BDF = \cos\angle BAC = \cos\alpha$.

We know that $\cos\angle BAC = \frac{AC}{AB} = \frac{DF}{DB}$ because of similar triangles, and we know that $DB = \tan\beta$.

Thus, we have that $\frac{DF}{\tan\beta} = \cos\alpha$ so $DF = \boxed{\cos\alpha\tan\beta}$

For $\sin\alpha + \beta$ we have:

$$
\begin{aligned}
\sin(\alpha+\beta) &= \frac{DE}{AD} \\
&= \frac{DF+FE}{AD} \\
&= \frac{DF+BC}{AD} \\
&= \frac{DF}{AD} + \frac{BC}{AD} \\
&= \frac{\cos\alpha\tan\beta}{\frac{1}{\cos\beta}} + \frac{\sin\alpha}{\frac{1}{\cos\beta}} \\
&= \frac{\cos\alpha\cdot\left(\frac{\sin\beta}{\cos\beta}\right)}{\frac{1}{\cos\beta}} + \frac{\sin\alpha}{\frac{1}{\cos\beta}} \\
&= \cos\beta\left(\cos\alpha\cdot\frac{\sin\beta}{\cos\beta} + \sin\alpha\right) \\
&= \cos\alpha\sin\beta + \sin\alpha\cos\beta \\
&= \boxed{\sin\alpha\cos\beta + \cos\alpha\sin\beta}
\end{aligned}
$$

For $\cos\alpha+\beta$ we have:

$$
\begin{aligned}
\cos(\alpha+\beta) &= \frac{AE}{AD} \\
&= \frac{AC-CE}{AD} \\
&= \frac{AC}{AD} - \frac{CE}{AD} \\
&= \frac{AC}{AD} - \frac{FB}{AD} \\
&= \frac{\cos\alpha}{\frac{1}{\cos\beta}} - \frac{\sin\alpha\tan\beta}{\frac{1}{\cos\beta}} \\
&= \cos\beta(\cos\alpha - \sin\alpha\tan\beta) \\
&= \cos\beta\left(\cos\alpha - \sin\alpha\frac{\sin\beta}{\cos\beta}\right) \\
&= \boxed{\cos\alpha\cos\beta - \sin\alpha\sin\beta}
\end{aligned}
$$